

Problem Solutions

By
Dean Updike

EL SOLUCIONARIO

EL SOLUCIONARIO

<http://www.elsolucionario.net>



LIBROS UNIVERISTARIOS
Y SOLUCIONARIOS DE
MUCHOS DE ESTOS LIBROS

LOS SOLUCIONARIOS
CONTIENEN TODOS LOS
EJERCICIOS DEL LIBRO
RESUELTOS Y EXPLICADOS
DE FORMA CLARA

VISITANOS PARA
DESARGALOS GRATIS.

Chapter 1

Problem 1.1

1.1 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force *P* for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

$$A_{AB} = \frac{\pi}{4} (50)^2 = 1963.5 \text{ mm}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{1963.5} \\ = 509.3 \times 10^{-6} P$$

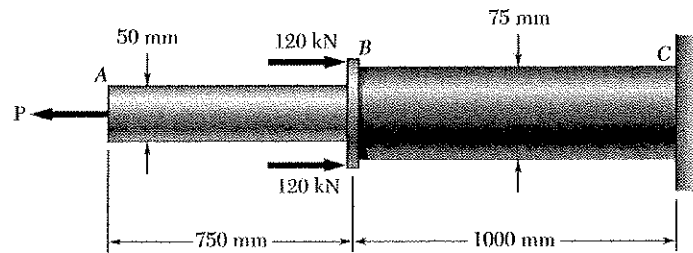
$$A_{BC} = \frac{\pi}{4} (75)^2 = 4417.9 \text{ mm}^2$$

$$\sigma_{BC} = \frac{(2)(120) - P}{A_{AB}} \\ = \frac{240 - P}{4417.9} = 0.0543 - 226.4 \times 10^{-6} P$$

Equating σ_{AB} to $2 \sigma_{BC}$

$$509.3 \times 10^{-6} P = 2(0.0543 - 226.4 \times 10^{-6} P)$$

$$P = 112.9 \text{ kN}$$



Problem 1.2

1.2 In Prob. 1.1, knowing that $P = 160 \text{ kN}$, determine the average normal stress at the midsection of (a) rod *AB*, (b) rod *BC*.

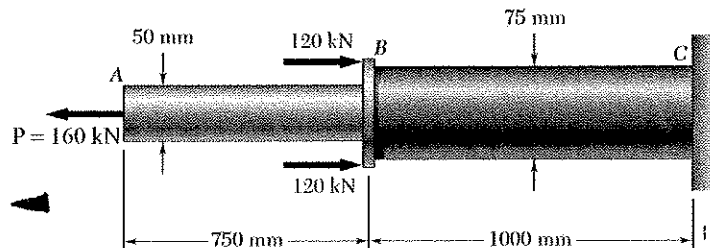
1.1 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force *P* for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

(a) Rod *AB*.

$$P = 160 \text{ kN (tension)}$$

$$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi (50)^2}{4} = 1963.5 \text{ mm}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{160 \times 10^3}{1963.5} = 81.5 \text{ MPa}$$



(b) Rod *BC*.

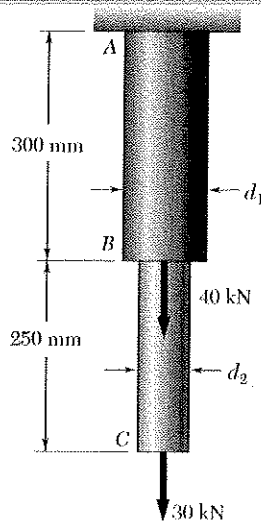
$$F = 160 - (2)(120) = -80 \text{ kN} \quad \text{i.e. } 80 \text{ kN compression.}$$

$$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi (75)^2}{4} = 4417.9 \text{ mm}^2$$

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-80 \times 10^3}{4417.9}$$

$$\sigma_{BC} = -18.1 \text{ MPa}$$

Problem 1.3



1.3 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod AB and 150 MPa in rod BC , determine the smallest allowable values of d_1 and d_2 .

Rod AB.

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4} d_1^2} = \frac{4P}{\pi d_1^2}$$

$$d_1 = \sqrt{\frac{4P}{\pi \sigma_{AB}}} = \sqrt{\frac{(4)(70 \times 10^3)}{\pi (175 \times 10^6)}} = 22.6 \times 10^{-3} \text{ m}$$

$$d_1 = 22.6 \text{ mm} \quad \blacktriangleleft$$

Rod BC.

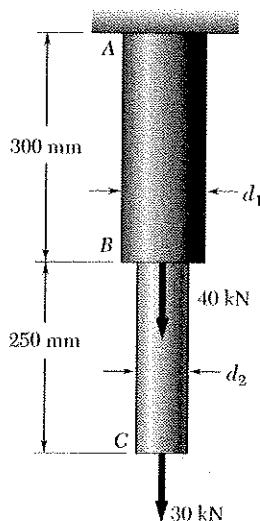
$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4} d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi \sigma_{BC}}} = \sqrt{\frac{(4)(30 \times 10^3)}{\pi (150 \times 10^6)}} = 15.98 \times 10^{-3} \text{ m}$$

$$d_2 = 15.98 \text{ mm} \quad \blacktriangleleft$$

Problem 1.4



1.4 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that $d_1 = 50 \text{ mm}$ and $d_2 = 30 \text{ mm}$, find average normal stress at the midsection of (a) rod AB , (b) rod BC .

(a) Rod AB.

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{70 \times 10^3}{1.9635 \times 10^{-3}} = 35.7 \times 10^6 \text{ Pa}$$

$$\sigma_{AB} = 35.7 \text{ MPa} \quad \blacktriangleleft$$

(b) Rod BC.

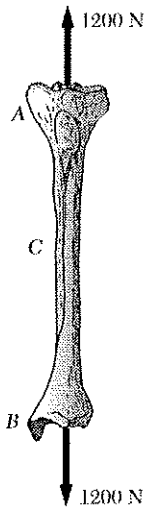
$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} (30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{30 \times 10^3}{706.86 \times 10^{-6}} = 42.4 \times 10^6 \text{ Pa}$$

$$\sigma_{BC} = 42.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 1.5



1.5 A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

$$\text{Geometry:} \quad A = \frac{\pi}{4} (d_1^2 - d_2^2)$$

$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

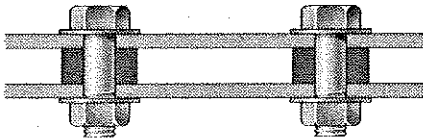
$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

$$= 222.9 \times 10^{-6} \text{ m}^2$$

$$d_2 = 14.93 \times 10^{-3} \text{ m}$$

$$d_2 = 14.93 \text{ mm} \quad \blacktriangleleft$$

Problem 1.6



1.6 Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time the spacer pushes that plate upward with a compressive force P_s . In order to maintain equilibrium

$$P_b = P_s$$

$$\text{For the bolt,} \quad \sigma_b = \frac{F_b}{A_b} = \frac{4P_b}{\pi d_b^2} \quad \text{or} \quad P_b = \frac{\pi}{4} \sigma_b d_b^2$$

$$\text{For the spacer,} \quad \sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)} \quad \text{or} \quad P_s = \frac{\pi}{4} \sigma_s (d_s^2 - d_b^2)$$

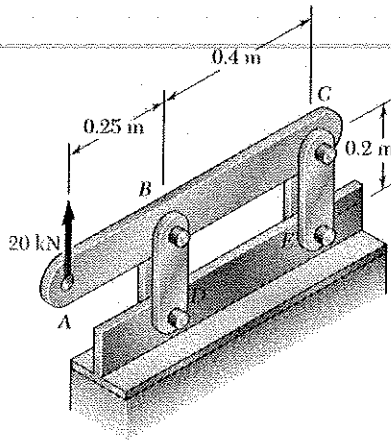
Equating P_b and P_s ,

$$\frac{\pi}{4} \sigma_b d_b^2 = \frac{\pi}{4} \sigma_s (d_s^2 - d_b^2)$$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)} d_b = \sqrt{1 + \frac{200}{130}} (16)$$

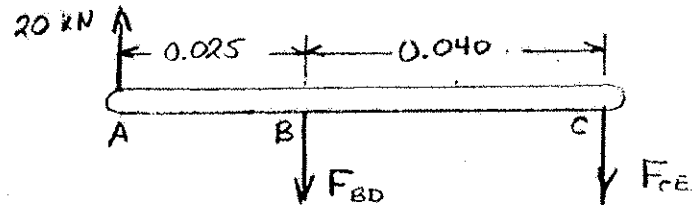
$$d_s = 25.2 \text{ mm} \quad \blacktriangleleft$$

Problem 1.7



1.7 Each of the four vertical links has an 8×36 -mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points B and D, (b) points C and E.

Use bar ABC as a free body.



$$\sum M_C = 0: (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N} \quad \text{Link BD is in tension.}$$

$$\sum M_B = 0: -(0.040)F_{CE} - (0.025)(20 \times 10^3) = 0$$

$$F_{CE} = -12.5 \times 10^3 \text{ N} \quad \text{Link CE is in compression.}$$

$$\text{Net area of one link for tension} = (0.008)(0.036 - 0.016)$$

$$= 160 \times 10^{-6} \text{ m}^2. \quad \text{For two parallel links, } A_{\text{net}} = 320 \times 10^{-6} \text{ m}^2$$

Tensile stress in link BD.

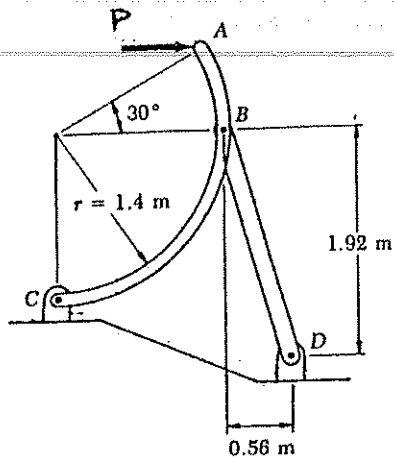
$$(a) \quad \sigma_{BD} = \frac{F_{BD}}{A_{\text{net}}} = \frac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.56 \times 10^6 \quad \sigma_{BD} = 101.6 \text{ MPa} \quad \blacktriangleleft$$

$$\text{Area for one link in compression} = (0.008)(0.036)$$

$$= 288 \times 10^{-6} \text{ m}^2. \quad \text{For two parallel links, } A = 576 \times 10^{-6} \text{ m}^2$$

$$(b) \quad \sigma_{CE} = \frac{F_{CE}}{A} = \frac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.70 \times 10^6 \quad \sigma_{CE} = -21.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 1.8



1.8 Knowing that the central portion of the link BD has a uniform cross-sectional area of 800 mm^2 , determine the magnitude of the load P for which the normal stress in that portion of BD is 50 MPa .

$$F_{BD} = \sigma A$$

$$= (50 \times 10^6)(800 \times 10^{-6})$$

$$= 40 \times 10^3 \text{ N}$$

$$BD = \sqrt{(0.56)^2 + (1.92)^2}$$

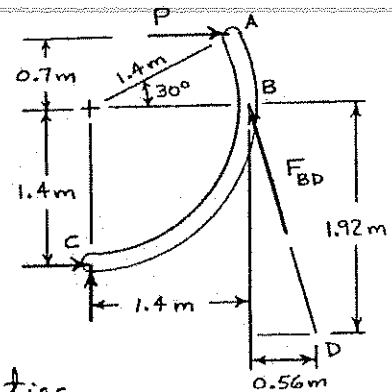
$$= 2.00 \text{ m}$$

Use Free Body AC for statics.

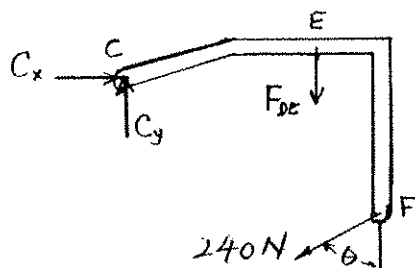
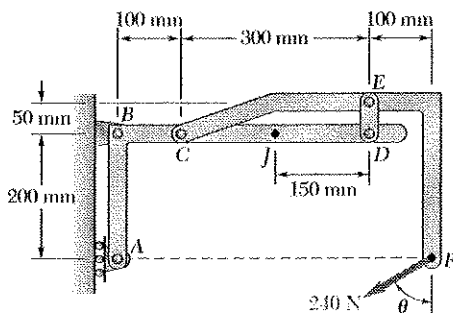
$$+\circlearrowleft \sum M_C = 0: \quad \frac{0.56}{2.00} (40 \times 10^3)(1.4) + \frac{1.92}{2.00} (40 \times 10^3)(1.4) - P(0.7 + 1.4) = 0$$

$$P = 33.1 \times 10^3 \text{ N}$$

$$P = 33.1 \text{ kN}$$



Problem 1.9



1.9 Knowing that link DE is 25 mm wide and 3 mm thick, determine the normal stress in the central portion of that link when (a) $\theta = 0^\circ$, (b) $\theta = 90^\circ$.

Use member CEF as a free body

$$+\circlearrowleft \sum M_C = 0$$

$$-0.3 F_{DE} - (0.2)(240 \sin \theta) - (0.4)(240 \cos \theta) = 0$$

$$F_{DE} = -160 \sin \theta - 320 \cos \theta \text{ N}$$

$$A_{DE} = (0.025)(0.003) = 75 \times 10^{-6} \text{ m}^2$$

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

$$(a) \theta = 0^\circ: F_{DE} = -320 \text{ N}$$

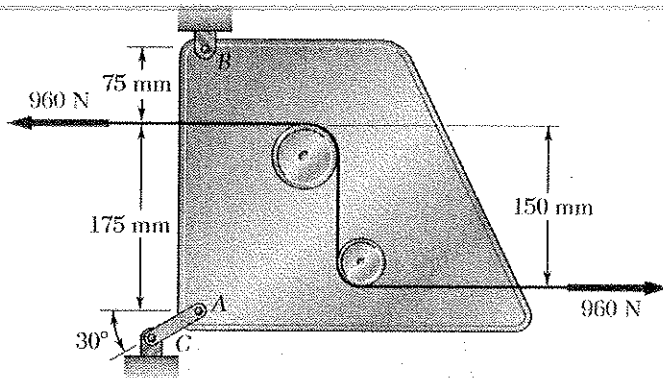
$$\sigma_{DE} = \frac{-320}{75 \times 10^{-6}} = -4.27 \text{ MPa}$$

$$(b) \theta = 90^\circ: F_{DE} = -160 \text{ N}$$

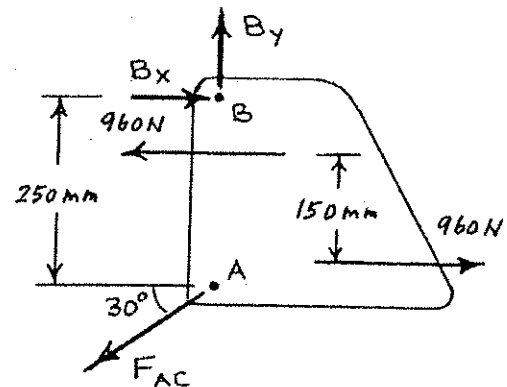
$$\sigma_{DE} = \frac{-160}{75 \times 10^{-6}} = -2.13 \text{ MPa}$$

Problem 1.10

1.10 Link AC has a uniform rectangular cross section 3 mm thick and 12 mm wide. Determine the normal stress in the central portion of the link.



Free Body Diagram of Plate



Note that the two 960-N forces form a couple of moment.

$$(960 \text{ N})(0.15 \text{ m}) = 144 \text{ N}\cdot\text{m}$$

$$\odot \sum M_B = 0 : 144 \text{ N}\cdot\text{m} - (F_{AC} \cos 30^\circ)(0.25 \text{ m}) = 0$$

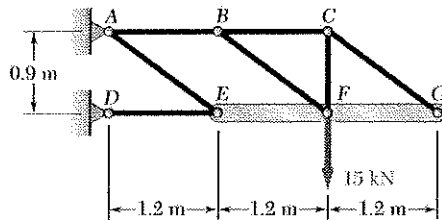
$$F_{AC} = 665.1 \text{ N}$$

$$\text{Area of link: } A_{AC} = (3 \text{ mm})(12 \text{ mm}) = 36 \text{ mm}^2$$

$$\text{Stress } \sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{665.1}{36} = 18.475 \text{ MPa} \quad \sigma_{AC} = 18.5 \text{ MPa}$$

Problem 1.11

1.11 The rigid bar EFG is supported by the truss system shown. Knowing that the member CG is a solid circular rod of 18 mm diameter, determine the normal stress in CG .



Using portion $EFGCB$ as a free body

$$\uparrow \sum F_y = 0: \frac{0.9}{1.5} F_{AB} - 15 = 0$$

$$F_{AE} = 25 \text{ kN}$$

Using beam EFG as a free body

$$\circlearrowleft M_F = 0: -(1.2) \frac{0.9}{1.2} F_{AE} + (1.2) \left(\frac{0.9}{1.2} F_{CG} \right) = 0$$

$$F_{CG} = F_{AE} = 25 \text{ kN}$$

Cross sectional area of member CG

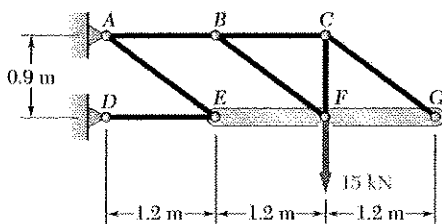
$$A_{CG} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.018)^2 = 254.4 \times 10^{-6} \text{ m}^2$$

Normal stress in CG .

$$\sigma_{CG} = \frac{F_{CG}}{A_{CG}} = \frac{25}{254.4 \times 10^{-6}} = 98.3 \text{ MPa}$$

Problem 1.12

1.12 The rigid bar EFG is supported by the truss system shown. Determine the cross-sectional area of member AE for which the normal stress in the member is 105 MPa.



Using portion $EFGCB$ as a free body

$$\uparrow \sum F_y = 0: \frac{0.9}{1.5} F_{AE} - 15 = 0$$

$$F_{AE} = 25 \text{ kN}$$

Stress in member AE

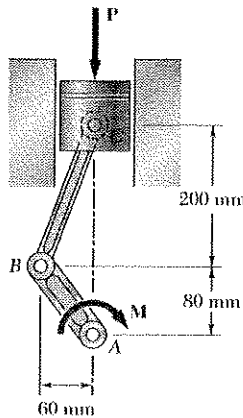
$$\sigma_{AE} = 105 \text{ MPa}$$

$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}}$$

$$A_{AE} = \frac{F_{AE}}{\sigma_{AE}} = \frac{25 \times 10^3}{105 \times 10^6} = 238.1 \times 10^{-6} \text{ m}^2$$

Problem 1.13

1.13 A couple M of magnitude $1500 \text{ N} \cdot \text{m}$ is applied to the crank of an engine. For the position shown, determine (a) the force P required to hold the engine system in equilibrium, (b) the average normal stress in the connecting rod BC , which has a 450-mm^2 uniform cross section.

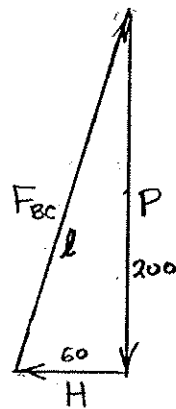
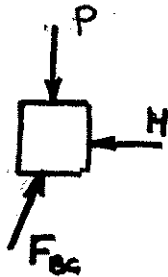
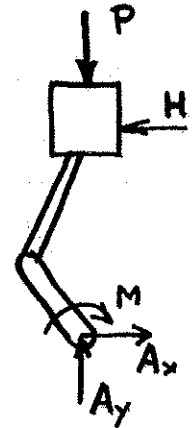


Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y .

$$\sum M_A = 0:$$

$$(0.280 \text{ m})H - 1500 \text{ N} \cdot \text{m} = 0$$

$$H = 5.3571 \times 10^3 \text{ N}$$



Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{bc} is known. Draw the force triangle and solve for P and F_{bc} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3 \text{ N}$$

$$P = 17.86 \text{ kN}$$

(a)

$$\frac{F_{bc}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{bc} = 18.643 \times 10^3 \text{ N}$$

Rod BC is a compression member. Its area is $450 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$

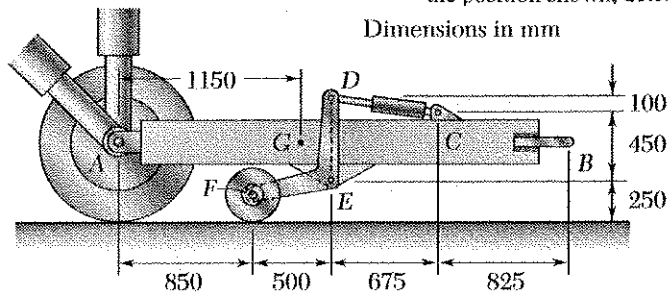
$$\text{Stress, } \sigma_{bc} = \frac{-F_{bc}}{A} = \frac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6 \text{ Pa}$$

(b)

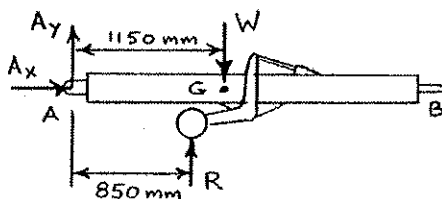
$$\sigma_{bc} = -41.4 \text{ MPa}$$

Problem 1.14

1.14 An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.



FREE BODY - ENTIRE TOW BAR:



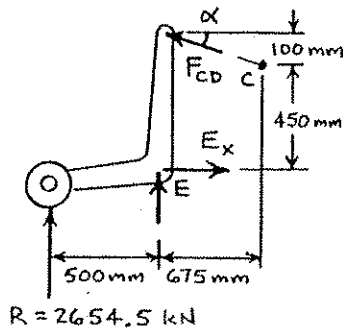
$$W = (200 \text{ kg})(9.81 \text{ m/s}^2) = 1962.00 \text{ N}$$

$$+\circlearrowleft \sum M_A = 0:$$

$$850R - 1150(1962.00 \text{ N}) = 0$$

$$R = 2654.5 \text{ N}$$

FREE BODY - BOTH ARM
WHEEL UNITS:



$$\tan \alpha = \frac{100}{675} \quad \alpha = 8.4270^\circ$$

$$+\circlearrowleft \sum M_E = 0:$$

$$(F_{cd} \cos \alpha)(550) - R(500) = 0$$

$$F_{cd} = \frac{500}{550 \cos 8.4270^\circ} (2654.5 \text{ N})$$

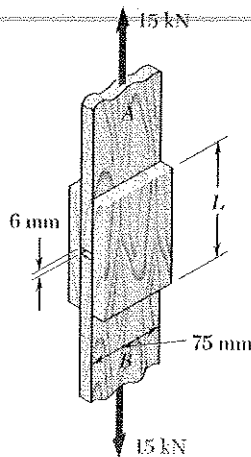
$$= 2439.5 \text{ N (COMP.)}$$

$$\sigma_{cd} = -\frac{F_{cd}}{A_{cd}} = -\frac{2439.5 \text{ N}}{\pi(0.0125 \text{ m})^2}$$

$$= -4.9697 \times 10^6 \text{ Pa}$$

$$\sigma_{cd} = -4.97 \text{ MPa} \quad \blacktriangleleft$$

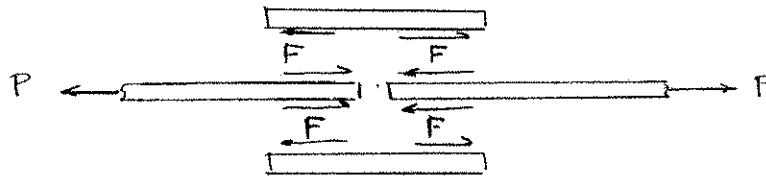
Problem 1.15



1.15 The wooden members *A* and *B* are to be joined by plywood splice plates that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be 6 mm, determine the smallest allowable length *L* if the average shearing stress in the glue is not to exceed 700 kPa.

There are four separate areas that are glued. Each of these areas transmits one half the 15 kN load. Thus

$$F = \frac{1}{2}P = \frac{1}{2}(15) = 7.5 \text{ kN} = 7500 \text{ N}$$



Let l = length of one glued area and $W = 75 \text{ mm} = 0.075 \text{ m}$ be its width.

For each glued area, $A = lW$

Average shearing stress: $\tau = \frac{F}{A} = \frac{F}{lW}$

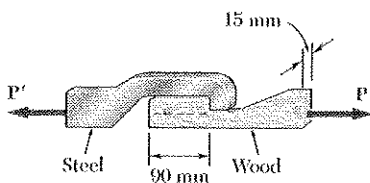
The allowable shearing stress is $\tau = 700 \times 10^3 \text{ Pa}$

Solving for l , $l = \frac{F}{\tau W} = \frac{7500}{(700 \times 10^3)(0.075)} = 0.14286 \text{ m} = 142.85 \text{ mm}$

Total length L : $L = l + (\text{gap}) + l = 142.85 + 6 + 142.85$

$$L = 292 \text{ mm} \quad \blacktriangleleft$$

Problem 1.16



1.16 When the force *P* reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

Area being sheared

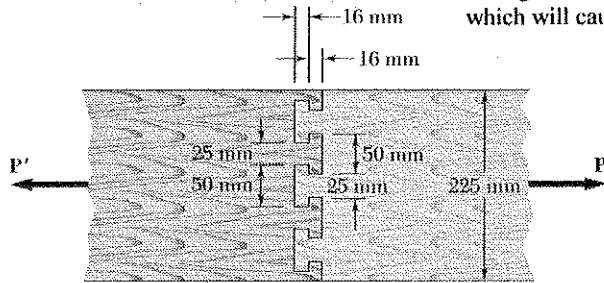
$$A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$$

Force $P = 8 \times 10^3 \text{ N}$

Shearing stress $\tau = \frac{P}{A} = \frac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^3 \text{ Pa} = 5.93 \text{ MPa} \quad \blacktriangleleft$

Problem 1.17

1.17 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.



Six areas must be sheared off when the joint fails. Each of these areas has dimensions 16 mm $\times$ 12 mm, its area being

$$A = (16)(12) = 192 \text{ mm}^2 = 192 \times 10^{-6} \text{ m}^2$$

At failure the force F carried by each of areas is

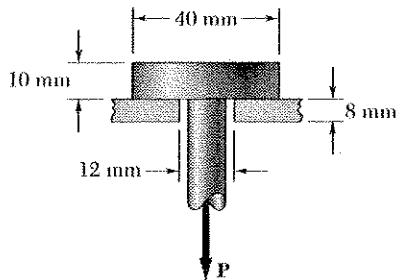
$$F = \tau A = (8 \times 10^6)(192 \times 10^{-6}) = 1536 \text{ N} = 1.536 \text{ kN}$$

Since there are six failure areas

$$P = 6F = (6)(1.536) = 9.22 \text{ kN} \quad \blacktriangleleft$$

Problem 1.18

1.18 A load P is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load P that can be applied to the rod.



For the steel rod,

$$A_1 = \pi d_1 t_1 = (\pi)(0.012)(0.010) = 376.99 \times 10^{-6} \text{ m}^2$$

$$\tau_1 = \frac{P}{A_1} \rightarrow P_1 = \tau_1 A_1$$

$$P_1 = (180 \times 10^6)(376.99 \times 10^{-6}) = 67.86 \times 10^3 \text{ N}$$

For the aluminum plate,

$$A_2 = \pi d_2 t_2 = (\pi)(0.040)(0.008) = 1.00531 \times 10^{-3} \text{ m}^2$$

$$\tau_2 = \frac{P}{A_2} \Rightarrow P_2 = \tau_2 A_2$$

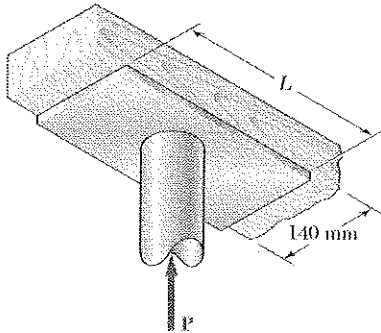
$$P_2 = (70 \times 10^6)(1.0053 \times 10^{-3}) = 70.372 \times 10^3 \text{ N}$$

The limiting value for the load P is the smaller of P_1 and P_2 .

$$P = 67.86 \times 10^3 \text{ N}$$

$$P = 67.9 \text{ kN} \quad \blacktriangleleft$$

Problem 1.19



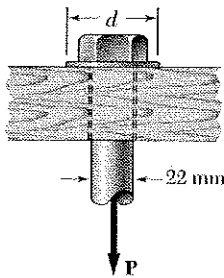
1.19 The axial force in the column supporting the timber beam shown is $P = 75 \text{ kN}$. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa .

SOLUTION

$$\sigma_b = \frac{P}{A} = \frac{P}{Lw}$$

$$\text{Solving for } L: \quad L = \frac{P}{\sigma_b w} = \frac{75 \times 10^3}{(3.0 \times 10^6)(0.140)} \\ 178.6 \times 10^{-3} \text{ m} \\ L = 178.6 \text{ mm}$$

Problem 1.20



1.20 The load P applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm , which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa .

$$\text{Steel rod: } A = \frac{\pi}{4} (0.022)^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$\sigma = 35 \times 10^6 \text{ Pa}$$

$$P = \sigma A = (35 \times 10^6)(380.13 \times 10^{-6}) \\ = 13.305 \times 10^3 \text{ N}$$

$$\text{Washer: } \sigma_b = 5 \times 10^6 \text{ Pa}$$

Required bearing area:

$$A_b = \frac{P}{\sigma_b} = \frac{13.305 \times 10^3}{5 \times 10^6} = 2.6609 \times 10^{-3} \text{ m}^2$$

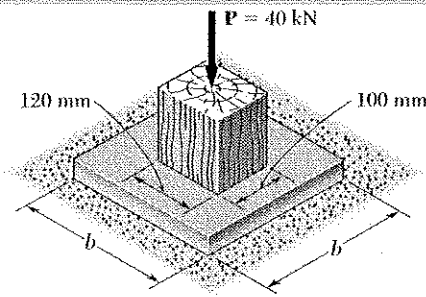
$$\text{But, } A_b = \frac{\pi}{4} (d^2 - d_i^2)$$

$$d^2 = d_i^2 + \frac{4A_b}{\pi} \\ = (0.025)^2 + \frac{(4)(2.6609 \times 10^{-3})}{\pi} \\ = 4.013 \times 10^{-3} \text{ m}^2$$

$$d = 63.3 \times 10^{-3} \text{ m}$$

$$d = 63.3 \text{ mm}$$

Problem 1.21



1.21 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (a) the maximum bearing stress on the concrete footing, (b) the size of the footing for which the average bearing stress in the soil is 145 kPa.

(a) Bearing stress on concrete footing.

$$P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$$

$$A = (100)(120) = 12 \times 10^3 \text{ mm}^2 = 12 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.33 \times 10^6 \text{ Pa}$$

$$3.33 \text{ MPa} \quad \blacktriangleleft$$

(b) Footing area.

$$P = 40 \times 10^3 \text{ N}$$

$$\sigma = 145 \text{ kPa} = 145 \times 10^3 \text{ Pa}$$

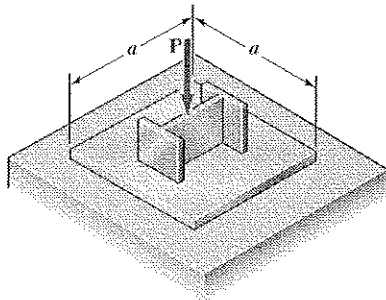
$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m}$$

$$b = 525 \text{ mm} \quad \blacktriangleleft$$

Problem 1.22



1.22 An axial load P is supported by a short W200 $\times$ 59 column of cross-sectional area $A = 7560 \text{ mm}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 200 MPa and that the bearing stress on the concrete foundation must not exceed 20 MPa, determine the side a of the plate that will provide the most economical and safe design.

$$\text{For the column} \quad \sigma = \frac{P}{A}$$

$$\text{or} \quad P = \sigma A = (200 \times 10^6)(7560 \times 10^{-6}) = 1512 \text{ kN}$$

$$\text{For the } a \times a \text{ plate,} \quad \sigma = 20 \text{ MPa}$$

$$A = \frac{P}{\sigma} = \frac{1512}{20} = 0.0756 \text{ m}^2$$

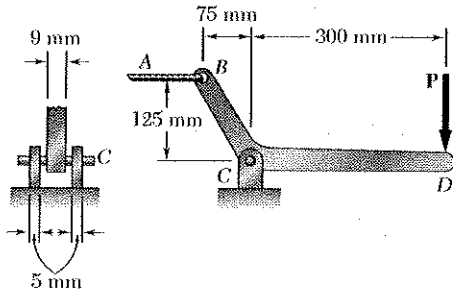
Since the plate is square $A = a^2$

$$a = \sqrt{A} = \sqrt{0.0756} = 0.275 \text{ m}$$

$$= 275 \text{ mm}$$

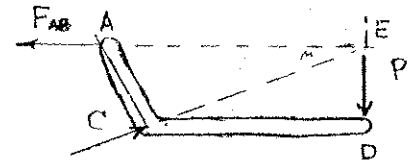
Problem 1.23

1.23 A 6-mm-diameter pin is used at connection C of the pedal shown. Knowing that $P = 500$ N, determine (a) the average shearing stress in the pin, (b) the nominal bearing stress in the pedal at C, (c) the nominal bearing stress in each support bracket at C.



Draw free body diagram of ACD.

Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm.

$$+\uparrow \Sigma F_y = 0: \frac{125}{325} C - P = 0 \quad C = 2.6 P = (2.6)(500) = 1300 \text{ N}$$

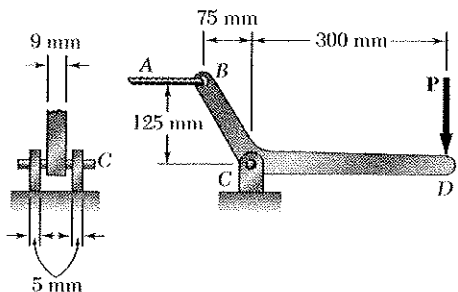
$$(a) \tau_{pin} = \frac{\frac{1}{2}C}{A_{pin}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \frac{2C}{\pi d^2} = \frac{(2)(1300)}{\pi(6 \times 10^{-3})^2} = 23.0 \times 10^6 \text{ Pa} \quad \tau_{pin} = 23.0 \text{ MPa} \blacktriangleleft$$

$$(b) \sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1300}{(6 \times 10^{-3})(9 \times 10^{-3})} = 24.1 \times 10^6 \text{ Pa} \quad \sigma_b = 24.1 \text{ MPa} \blacktriangleleft$$

$$(c) \sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1300}{(2)(6 \times 10^{-3})(5 \times 10^{-3})} = 21.7 \times 10^6 \text{ Pa} \quad \sigma_b = 21.7 \text{ MPa} \blacktriangleleft$$

Problem 1.24

1.24 Knowing that a force P of magnitude 750 N is applied to the pedal shown, determine (a) the diameter of the pin at C for which the average shearing stress in the pin is 40 MPa, (b) the corresponding bearing stress in the pedal at C, (c) the corresponding bearing stress in the each support bracket at C.



Draw free body diagram of ACD.

Since ACD is a 3-force member, the reaction at C is directed toward point E, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm

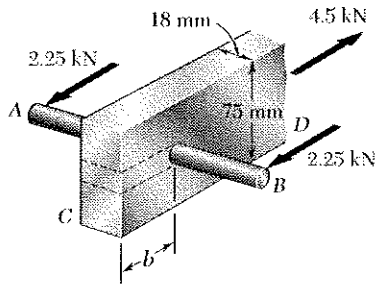
$$+\uparrow \Sigma F_y = 0: \frac{125}{325} C - P = 0 \quad C = 2.6 P = (2.6)(750) = 1950 \text{ N}$$

$$(a) \tau_{pin} = \frac{\frac{1}{2}C}{A_{pin}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} \quad d = \sqrt{\frac{2C}{\pi \tau_{pin}}} = \sqrt{\frac{(2)(1950)}{\pi(40 \times 10^6)}} = 5.57 \times 10^{-3} \text{ m} \quad d = 5.57 \text{ mm} \blacktriangleleft$$

$$(b) \sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1950}{(5.57 \times 10^{-3})(9 \times 10^{-3})} = 38.9 \times 10^6 \text{ Pa} \quad \sigma_b = 38.9 \text{ MPa} \blacktriangleleft$$

$$(c) \sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1950}{(2)(5.57 \times 10^{-3})(5 \times 10^{-3})} = 35.0 \times 10^6 \text{ Pa} \quad \sigma_b = 35.0 \text{ MPa} \blacktriangleleft$$

Problem 1.25



1.25 A 12-mm-diameter steel rod AB is fitted to a round hole near end C of the wooden member CD . For the loading shown, determine (a) the maximum average normal stress in the wood, (b) the distance b for which the average shearing stress is 620 kPa on the surfaces indicated by the dashed lines, (c) the average bearing stress on the wood.

(a) Maximum average normal stress in the wood.

$$A_{\text{net}} = (75 - 12)(18) = 1.134 \times 10^3 \text{ mm}^2 = 1.134 \times 10^{-3} \text{ m}^2$$

$$P = 4.50 \text{ kN} = 4.50 \times 10^3 \text{ N}$$

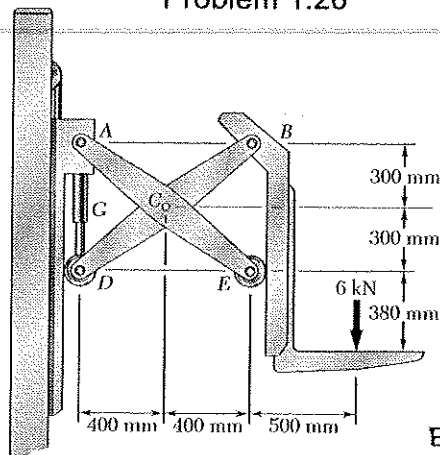
$$\sigma = \frac{P}{A_{\text{net}}} = \frac{4.50 \times 10^3}{1.134 \times 10^{-3}} = 3.97 \times 10^6 \text{ Pa} \quad 3.97 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \tau = \frac{P}{A} = \frac{P}{2bt} \quad b = \frac{P}{2t\tau} = \frac{4.50 \times 10^3}{(2)(18 \times 10^{-3})(620 \times 10^3)} = 202 \times 10^{-3} \text{ m}$$

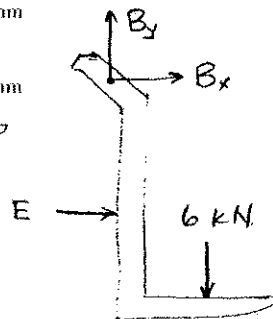
$$b = 202 \text{ mm} \quad \blacktriangleleft$$

$$(c) \quad \sigma_b = \frac{P}{dt} = \frac{4.50 \times 10^3}{(12 \times 10^{-3})(18 \times 10^{-3})} = 20.8 \times 10^6 \text{ Pa} \quad 20.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 1.26



1.26 Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 6 kN. Knowing that the thickness of member BD is 16 mm, determine (a) the average shearing stress in the 12-mm-diameter pin at B , (b) the bearing stress at B in member BD .



Use one fork as a free body.

$$+\circlearrowleft \sum M_B = 0:$$

$$0.6E - (0.5)(6) = 0$$

$$E = 5 \text{ kN} \rightarrow$$

$$+\rightarrow \sum F_x = 0$$

$$E + B_x = 0 \quad B_x = -E$$

$$B_x = 5 \text{ kN} \leftarrow$$

$$+\uparrow \sum F_y = 0: \quad B_y - 6 = 0 \quad B_y = 6 \text{ kN}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{5^2 + 6^2} = 7.81 \text{ kN}$$

(a) Shearing stress in pin at B .

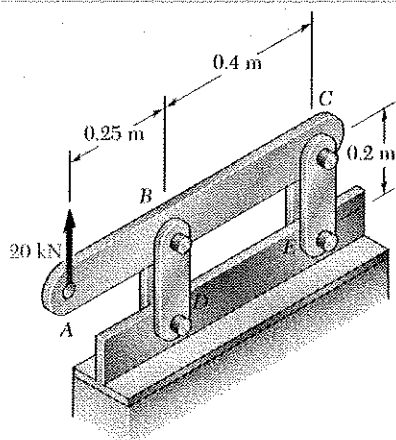
$$A_{pin} = \frac{\pi}{4} d_{pin}^2 = \frac{\pi}{4} (0.012)^2 = 113.1 \times 10^{-6} \text{ m}^2$$

$$\tau = \frac{B}{A_{pin}} = \frac{7.81 \times 10^3}{113.1 \times 10^{-6}} = 69 \text{ MPa}$$

(b) Bearing stress at B .

$$\sigma = \frac{B}{dt} = \frac{7.81 \times 10^3}{(0.012)(0.016)} = 40.6 \text{ MPa}$$

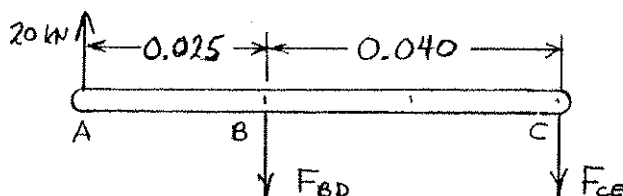
Problem 1.27



1.27 For the assembly and loading of Prob. 1.7, determine (a) the average shearing stress in the pin at B, (b) the average bearing stress at B in member BD, (c) the average bearing stress at B in member ABC, knowing that this member has a 10×50 -mm uniform rectangular cross section.

1.7 Each of the four vertical links has an 8×36 -mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points B and D, (b) points C and E.

Use bar ABC as a free body.



$$\sum M_C = 0: (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) Shear pin at B. $\tau = \frac{F_{BD}}{2A}$ for double shear,

where $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \quad \tau = 80.8 \text{ MPa} \blacktriangleleft$$

(b) Bearing: link BD. $A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \quad \sigma_b = 127.0 \text{ MPa} \blacktriangleleft$$

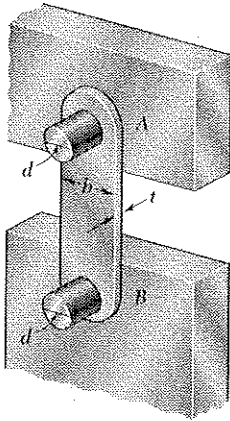
(c) Bearing in ABC at B.

$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \quad \sigma_b = 203 \text{ MPa} \blacktriangleleft$$

Problem 1.28

1.28 Link AB, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (a) the diameter d of the pins, (b) the average bearing stress in the link.



Rod AB is in compression.

$$A = bt \quad \text{where } b = 50 \text{ mm and } t = 6 \text{ mm}$$

$$A = (0.050)(0.006) = 300 \times 10^{-6} \text{ m}^2$$

$$P = -\sigma A = -(140 \times 10^6)(300 \times 10^{-6}) \\ = 42 \times 10^3 \text{ N}$$

$$\text{For the pin, } A_p = \frac{\pi}{4} d^2 \quad \text{and} \quad \tau = \frac{P}{A_p}$$

$$A_p = \frac{P}{\tau} = \frac{42 \times 10^3}{80 \times 10^6} = 525 \times 10^{-6} \text{ m}^2$$

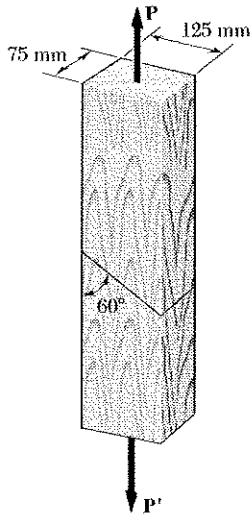
$$d = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{(4)(525 \times 10^{-6})}{\pi}} = 2.585 \times 10^{-3} \text{ m}$$

$$d = 2.59 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \sigma_b = \frac{P}{dt} = \frac{42 \times 10^3}{(2.585 \times 10^{-3})(0.006)} = 271 \times 10^6 \text{ Pa}$$

$$\sigma_b = 271 \text{ MPa} \quad \blacktriangleleft$$

Problem 1.29



1.29 The 5.6-kN load P is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$$P = 6227 \text{ N}$$

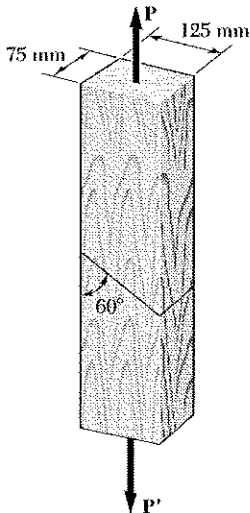
$$\theta = 90^\circ - 60^\circ = 30^\circ$$

$$A_o = (0.125)(0.075) = 9.375 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P \cos^2 \theta}{A_o} = \frac{(6.227 \times 10^3)(\cos 30^\circ)^2}{9.375 \times 10^{-3}} \quad \sigma = 0.498 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = \frac{P \sin 2\theta}{2A_o} = \frac{(6.227 \times 10^3) \sin 60^\circ}{(2)(9.375 \times 10^{-3})} \quad \tau = 0.288 \text{ MPa} \quad \blacktriangleleft$$

Problem 1.30



1.30 Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 525 kPa, determine (a) the largest load P that can be safely supported, (b) the corresponding tensile stress in the splice.

$$A_o = (0.125)(0.075) = 9.375 \times 10^{-3} \text{ m}^2$$

$$\theta = 90^\circ - 60^\circ = 30^\circ$$

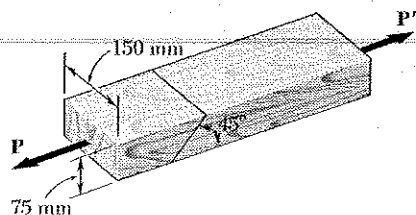
$$\sigma = \frac{P \cos^2 \theta}{A_o}$$

$$(a) \quad P = \frac{\sigma A_o}{\cos^2 \theta} = \frac{(525 \times 10^3)(9.375 \times 10^{-3})}{\cos^2 30^\circ} = 6562 \text{ N}$$

$$P = 6.562 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad \tau = \frac{P \sin 2\theta}{2A_o} = \frac{(6562) \sin 60^\circ}{(2)(9.375 \times 10^{-3})} \quad \tau = 0.303 \text{ MPa} \quad \blacktriangleleft$$

Problem 1.31



1.31 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11 \text{ kN}$, determine the normal and shearing stresses in the glued splice.

$$\theta = 90^\circ - 45^\circ = 45^\circ$$

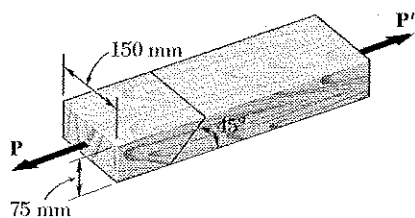
$$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P \cos^2 \theta}{A_o} = \frac{(11 \times 10^3) \cos^2 45^\circ}{11.25 \times 10^{-3}} = 489 \times 10^3 \text{ Pa} \quad \sigma = 489 \text{ kPa} \quad \blacktriangleleft$$

$$\tau = \frac{P \sin 2\theta}{2A_o} = \frac{(11 \times 10^3)(\sin 90^\circ)}{2(11.25 \times 10^{-3})} = 4.89 \times 10^3 \text{ Pa} \quad \tau = 489 \text{ kPa} \quad \blacktriangleleft$$

Problem 1.32



1.32 Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 560 kPa, determine (a) the largest load P that can be safely applied, (b) the corresponding shearing stress in the splice.

$$\theta = 90^\circ - 45^\circ = 45^\circ$$

$$A_o = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

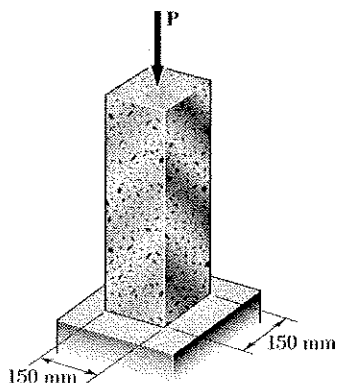
$$\sigma = 560 \text{ kPa} = 560 \times 10^3 \text{ Pa}$$

$$\sigma = \frac{P \cos^2 \theta}{A_o}$$

$$(a) \quad P = \frac{\sigma A_o}{\cos^2 \theta} = \frac{(560 \times 10^3)(11.25 \times 10^{-3})}{\cos^2 45^\circ} = 12.60 \times 10^3 \text{ N} \quad P = 12.60 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad \tau = \frac{P \sin \theta \cos \theta}{A_o} = \frac{(12.60 \times 10^3)(\sin 45^\circ)(\cos 45^\circ)}{11.25 \times 10^{-3}} = 560 \times 10^3 \text{ Pa} \quad \tau = 560 \text{ kPa} \quad \blacktriangleleft$$

Problem 1.33



1.33 A centric load P is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 17 MPa, determine (a) the magnitude of P , (b) the orientation of the surface on which the maximum shearing stress occurs, (c) the normal stress exerted on the surface, (d) the maximum value of the normal stress in the block.

$$A_o = (0.15)(0.15) = 0.0225 \text{ m}^2 \quad \tau_{\max} = 17 \text{ MPa}$$

$$\theta = 45^\circ \text{ for plane of } \tau_{\max}$$

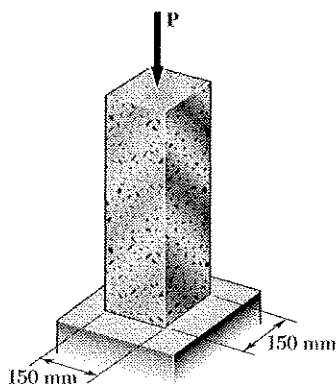
$$(a) \quad \tau_{\max} = \frac{|P|}{2A_o} \therefore |P| = 2A_o \tau_{\max} = (2)(0.0225)(17 \times 10^6) = 765 \text{ kN}$$

$$(b) \quad \sin 2\theta = 1 \quad 2\theta = 90^\circ \quad \theta = 45^\circ$$

$$(c) \quad \sigma_{45} = \frac{P}{A_o} \cos^2 45^\circ = \frac{P}{2A_o} = \frac{-765 \times 10^3}{2(0.0225)} = -17 \text{ MPa}$$

$$(d) \quad \sigma_{\max} = \frac{P}{A_o} = \frac{-765 \times 10^3}{0.0225} = -34 \text{ MPa}$$

Problem 1.34



1.34 A 960-kN load P is applied to the granite block shown. Determine the resulting maximum value of (a) the normal stress, (b) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

$$A_o = (0.15)(0.15) = 0.0225 \text{ m}^2$$

$$\sigma = \frac{P}{A_o} \cos^2 \theta = \frac{-960 \times 10^3}{0.0225} \cos^2 \theta = -42.67 \times 10^6 \cos^2 \theta$$

$$(a) \quad \text{max tensile stress} = 0 \text{ at } \theta = 90^\circ$$

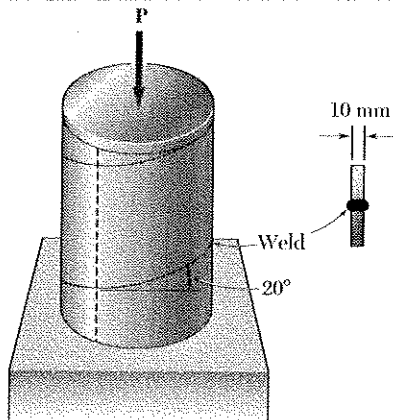
$$\text{max. compressive stress} = 42.7 \text{ MPa}$$

$$\text{at } \theta = 0^\circ$$

$$(b) \quad \tau_{\max} = \frac{P}{2A_o} = \frac{960 \times 10^3}{2(0.0225)} = 21.3 \text{ MPa}$$

$$\text{at } \theta = 45^\circ$$

Problem 1.35



1.35 A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 60$ MPa and $\tau = 36$ MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.

$$d_o = 0.400 \text{ m} \quad r_o = \frac{1}{2}d_o = 0.200 \text{ m}$$

$$r_i = r_o - t = 0.200 - 0.010 = 0.190 \text{ m}$$

$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2) = 12.25 \times 10^{-3} \text{ m}^2$$

$$\theta = 20^\circ$$

Based on $|\sigma| = 60$ MPa: $\sigma = \frac{P}{A_o} \cos^2 \theta$

$$P = \frac{A_o \sigma}{\cos^2 \theta} = \frac{(12.25 \times 10^{-3})(60 \times 10^6)}{\cos^2 20^\circ} = 833 \times 10^3 \text{ N}$$

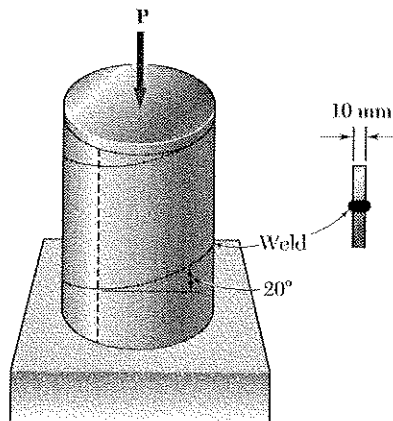
Based on $|\tau| = 30$ MPa: $\tau = \frac{P}{2A_o} \sin 2\theta$

$$P = \frac{2A_o \tau}{\sin 2\theta} = \frac{(2)(12.25 \times 10^{-3})(36 \times 10^6)}{\sin 40^\circ} = 1372 \times 10^3 \text{ N}$$

Smaller value is the allowable value of P .

$$P = 833 \text{ kN} \quad \leftarrow$$

Problem 1.36



1.36 A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force P is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

$$d_o = 0.400 \text{ m} \quad r_o = \frac{1}{2}d_o = 0.200 \text{ m}$$

$$r_i = r_o - t = 0.200 - 0.010 = 0.190 \text{ m}$$

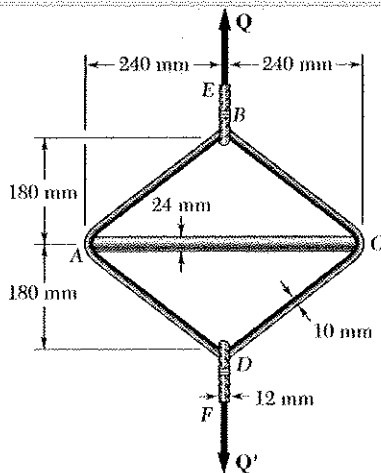
$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2) = 12.25 \times 10^{-3} \text{ m}^2$$

$$\theta = 20^\circ$$

$$\sigma = \frac{P}{A_o} \cos^2 \theta = \frac{-300 \times 10^3 \cos^2 20^\circ}{12.25 \times 10^{-3}} = -21.6 \times 10^6 \text{ Pa} \quad \sigma = -21.6 \text{ MPa} \quad \leftarrow$$

$$\tau = \frac{P}{2A_o} \sin 2\theta = \frac{-300 \times 10^3 \sin 40^\circ}{(2)(12.25 \times 10^{-3})} = -7.87 \times 10^6 \quad \tau = 7.87 \text{ MPa} \quad \leftarrow$$

Problem 1.37

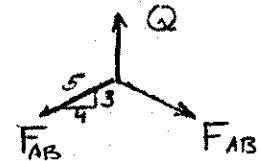


1.37 A steel loop $ABCD$ of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod AC . Cables BE and DF , each of 12-mm diameter, are used to apply the load Q . Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load Q that can be applied if an overall factor of safety of 3 is desired.

Using joint B as a free body and considering symmetry,

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

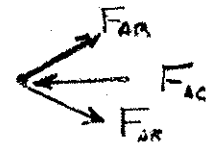
$$Q = \frac{6}{5} F_{AB}$$



Using joint A as a free body and considering symmetry,

$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore Q = \frac{3}{4} F_{AC}$$



Based on strength of cable BE ,

$$Q_u = \sigma_u A = \sigma_u \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop,

$$\begin{aligned} Q_u &= \frac{6}{5} F_{AB,u} = \frac{6}{5} \sigma_u A = \frac{6}{5} \sigma_u \frac{\pi}{4} d^2 \\ &= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N} \end{aligned}$$

Based on strength of rod AC ,

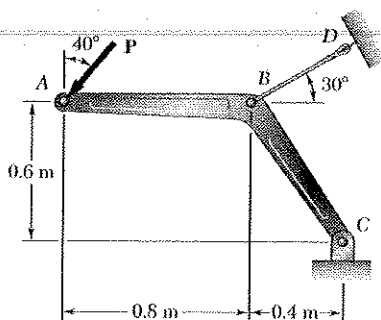
$$\begin{aligned} Q_u &= \frac{3}{4} F_{AC,u} = \frac{3}{4} \sigma_u A = \frac{3}{4} \sigma_u \frac{\pi}{4} d^2 \\ &= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N} \end{aligned}$$

Actual ultimate load Q_u is the smallest. $\therefore Q_u = 45.24 \times 10^3 \text{ N}$

$$\text{Allowable load } Q = \frac{Q_u}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3 \text{ N}$$

$$Q = 15.08 \text{ kN}$$

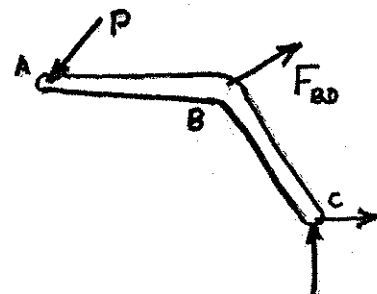
Problem 1.38



1.38 Member ABC , which is supported by a pin and bracket at C and a cable BD , was designed to support the 16-kN load P as shown. Knowing that the ultimate load for cable BD is 100 kN, determine the factor of safety with respect to cable failure.

Use member ABC as a free body, and note that member BD is a two-force member.

$$\sum M_C = 0:$$



$$(P \cos 40^\circ)(1.2) + (P \sin 40^\circ)(0.6) - (F_{BD} \cos 30^\circ)(0.6) - (F_{BD} \sin 30^\circ)(0.4) = 0$$

$$1.30493 P - 0.71962 F_{BD} = 0$$

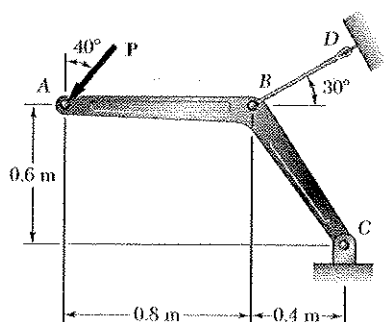
$$F_{BD} = 1.81335 P = (1.81335)(16 \times 10^3) = 2.9014 \times 10^3 \text{ N}$$

$$F_{ult} = 100 \times 10^3 \text{ N}$$

$$F.S. = \frac{F_{ult}}{F_{BD}} = \frac{100 \times 10^3}{2.9014 \times 10^3}$$

$$F.S. = 3.45 \quad \blacktriangleleft$$

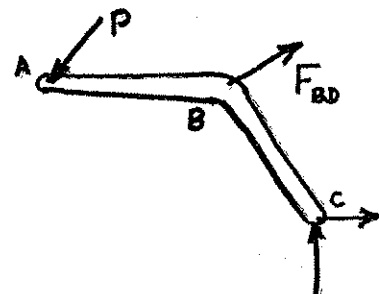
Problem 1.39



1.39 Knowing that the ultimate load for cable BD is 100 kN and that a factor of safety of 3.2 with respect to cable failure is required, determine the magnitude of the largest force P which can be safely applied as shown to member ABC .

Use member ABC as a free body, and note that member BD is a two-force member.

$$\sum M_C = 0:$$



$$(P \cos 40^\circ)(1.2) + (P \sin 40^\circ)(0.6) - (F_{BD} \cos 30^\circ)(0.6) - (F_{BD} \sin 30^\circ)(0.4) = 0$$

$$1.30493 P - 0.71962 F_{BD} = 0$$

$$P = 0.55404 F_{BD}$$

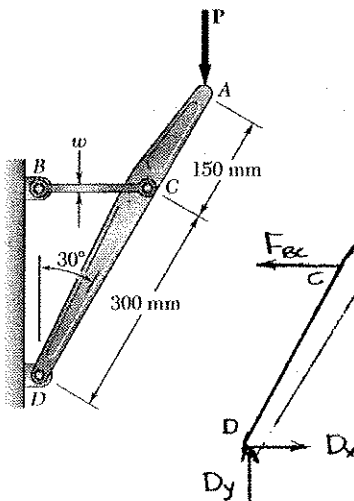
Allowable value of F_{BD} .

$$F_{BD} = \frac{F_{ult}}{F.S.} = \frac{100 \text{ kN}}{3.2} = 3.125 \text{ kN}$$

$$P_{all} = (0.55404)(3.125)$$

$$P_{all} = 1.732 \text{ kN} \quad \blacktriangleleft$$

Problem 1.40



1.40 The horizontal link BC is 6 mm thick, has a width $w = 30$ mm, and is made of a steel with a 450-MPa ultimate strength in tension. What is the factor of safety if the structure shown is designed to support a load $P = 40$ kN?

$$+\circlearrowleft \sum M_C = 0$$

$$(0.3 \cos 30^\circ) F_{AB} - (0.45 \sin 30^\circ)(40) = 0$$

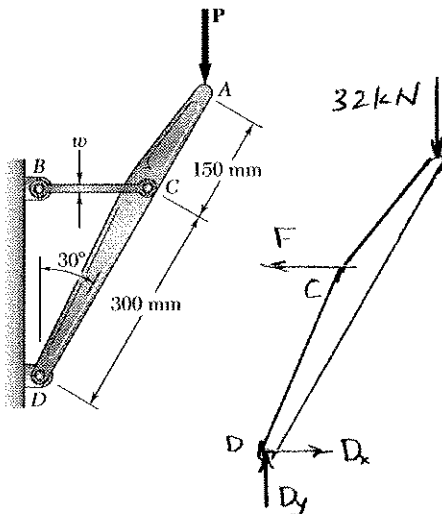
$$F_{BC} = 34.6 \text{ kN}$$

$$A_{BC} = (0.006)(0.03) = 1.8 \times 10^{-4} \text{ m}^2$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{\sigma_{ult}}{F.S.}$$

$$F.S. = \frac{A_{BC} \sigma_{ult}}{F_{BC}} = \frac{(1.8 \times 10^{-4})(450 \times 10^6)}{34.6 \times 10^3} = 2.34$$

Problem 1.41



1.41 The horizontal link BC is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be the width w of the link if the structure shown is to be designed to support a load $P = 32$ kN with a factor of safety equal to 3?

$$+\circlearrowleft \sum M_C = 0$$

$$(0.3 \cos 30^\circ) F_{AB} - (0.45 \sin 30^\circ)(32) = 0$$

$$F_{AB} = 27.7 \text{ kN}$$

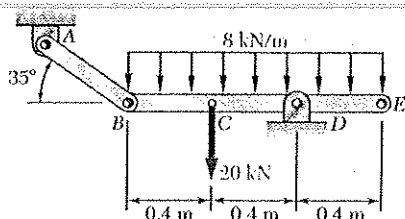
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{F_{AB}}{tw} = \frac{\sigma_{ult}}{F.S.}$$

$$w = \frac{(F.S.) F_{AB}}{t \sigma_{ult}} = \frac{(3)(27.7 \times 10^3)}{(0.006)(450 \times 10^6)}$$

$$w = 0.03078 \text{ m}$$

$$= 30.8 \text{ mm}$$

Problem 1.42



1.42 Link AB is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for AB for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at A and B .

$$P = (1.2)(8) = 9.6 \text{ kN}$$

$$+\circlearrowleft \Sigma M_D = 0:$$

$$-(0.8)(F_{AB} \sin 35^\circ) + (0.2)(9.6) + (0.4)(20) = 0$$

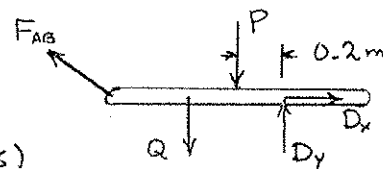
$$F_{AB} = 21.619 \text{ kN} = 21.619 \times 10^3 \text{ N}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{ult}}{F.S.}$$

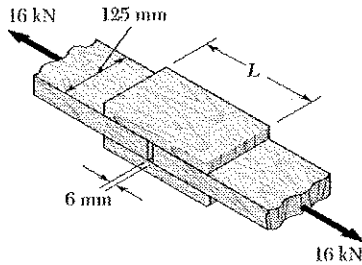
$$A_{AB} = \frac{(F.S.) F_{AB}}{\sigma_{ult}} = \frac{(3.50)(21.619 \times 10^3)}{450 \times 10^6}$$

$$= 168.1 \times 10^{-6} \text{ m}^2$$

$$A_{AB} = 168.1 \text{ mm}^2$$



Problem 1.43



1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.

There are 4 separate areas of glue.
Each glue area must transmit 8 kN of shear load.

$$P = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

Required ultimate load.

$$P_u = (\text{F.S.}) P = (2.75)(8 \times 10^3) = 22 \times 10^3 \text{ N}$$

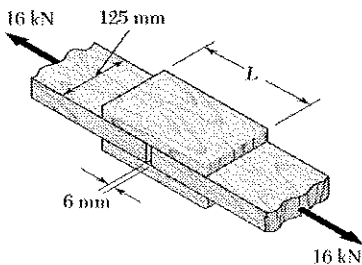
Required length of each glue area.

$$P_u = \tau_u A = \tau_u l w \quad l = \frac{P_u}{\tau_u w} = \frac{22 \times 10^3}{(2.5 \times 10^6)(0.125)} = 70.4 \times 10^{-3} \text{ m}$$

$$\text{Length of splice: } L = 2l + c = (2)(70.4 \times 10^{-3}) + 0.006 = 0.1468 \times 10^{-3} \text{ m}$$

$$L = 146.8 \text{ mm} \quad \blacktriangleleft$$

Problem 1.44



1.44 For the joint and loading of Prob. 1.43, determine the factor of safety, knowing that the length of each splice is $L = 180 \text{ mm}$.

1.43 The two wooden members shown, which support a 16-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.5 MPa and the clearance between the members is 6 mm. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.

There are 4 separate areas of glue.
Each glue area must transmit 8 kN of shear load.

$$P = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

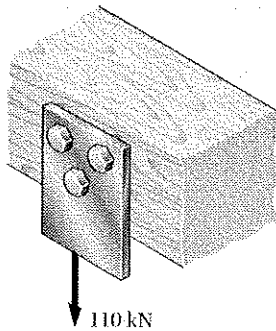
Length of splice. $L = 2l + c$ where l = length of glue and c = clearance.
 $l = \frac{1}{2}(L - c) = \frac{1}{2}(0.180 - 0.006) = 0.087 \text{ m}$

$$\text{Area of glue. } A = l w = (0.087)(0.125) = 10.875 \times 10^{-3} \text{ m}^2$$

$$\text{Ultimate load. } P_u = \tau_u A = (2.5 \times 10^6)(10.875 \times 10^{-3}) = 27.1875 \times 10^3 \text{ N}$$

$$\text{Factor of safety. } \text{F.S.} = \frac{P_u}{P} = \frac{27.1875 \times 10^3}{8 \times 10^3} \quad \text{F.S.} = 3.40 \quad \blacktriangleleft$$

Problem 1.45



1.45 Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.

$$\text{For each bolt, } A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (18)^2 = 254.47 \text{ mm}^2 \\ = 254.47 \times 10^{-6} \text{ m}^2$$

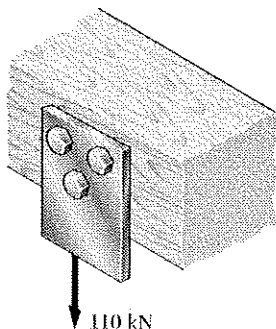
$$P_u = A \tau_u = (254.47 \times 10^{-6}) (360 \times 10^6) \\ = 91.609 \times 10^3 \text{ N}$$

$$\text{For the three bolts, } P_u = (3)(91.609 \times 10^3) \\ = 274.83 \times 10^3 \text{ N}$$

Factor of safety.

$$\text{F.S.} = \frac{P_u}{P} = \frac{274 \times 10^3}{110 \times 10^3} = \text{F.S.} = 2.50 \quad \blacktriangleleft$$

Problem 1.46



1.46 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110 kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.

$$\text{For each bolt, } P = \frac{110}{3} = 36.667 \text{ kN}$$

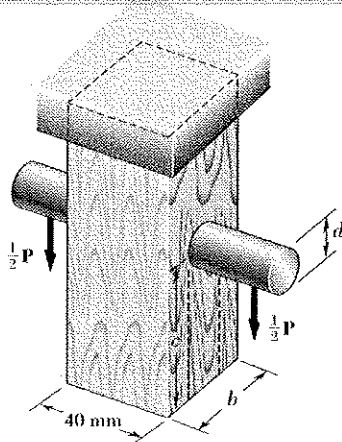
$$\text{Required } P_u = (\text{F.S.})P = (3.35)(36.667) = 122.83 \text{ kN}$$

$$\tau_u = \frac{P_u}{A} = \frac{P_u}{\frac{\pi}{4} d^2} = \frac{4P_u}{\pi d^2}$$

$$d = \sqrt{\frac{4P_u}{\pi \tau_u}} = \sqrt{\frac{(4)(122.83 \times 10^3)}{\pi (360 \times 10^6)}} = 20.8 \times 10^{-3} \text{ m}$$

$$d = 20.8 \text{ mm} \quad \blacktriangleleft$$

Problem 1.47



1.47 A load P is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load P if an overall factor of safety of 3.2 is desired.

1.48 For the support of Prob. 1.47, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20$ kN, determine (a) the factor of safety for the pin, (b) the required values of b and c if the factor of safety for the wooden members is the same as that found in part a for the pin.

$$P = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

$$(a) \text{ Pin: } A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$\text{Double shear } \tau = \frac{P}{2A} \quad \tau_u = \frac{P_u}{2A}$$

$$P_u = 2A\tau_u = (2)(201.06 \times 10^{-6})(145 \times 10^6)$$

$$= 58.336 \times 10^3 \text{ N}$$

$$F.S. = \frac{P_u}{P} = \frac{58.336 \times 10^3}{20 \times 10^3} = 2.92$$

$$(b) \text{ Tension in wood } P_u = 58.336 \times 10^3 \text{ N for same F.S.}$$

$$\sigma_u = \frac{P_u}{A} = \frac{P_u}{w(b-d)} \quad \text{where } w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_u}{w\sigma_u} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^{-3} \text{ m}$$

$$b = 40.3 \text{ mm}$$

$$\text{Shear in wood } P_u = 58.336 \times 10^3 \text{ N for same F.S.}$$

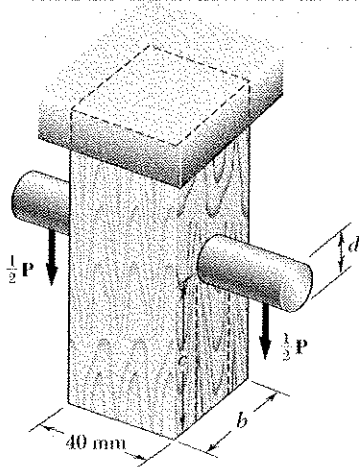
$$\text{Double shear; each area is } A = wc$$

$$\tau_u = \frac{P_u}{2A} = \frac{P_u}{2wc}$$

$$c = \frac{P_u}{2w\tau_u} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3} \text{ m}$$

$$c = 97.2 \text{ mm}$$

Problem 1.48



1.48 A load P is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load P if an overall factor of safety of 3.2 is desired.

Based on double shear in pin

$$P_u = 2A\tau_u = 2\frac{\pi}{4}d^2\tau_u$$

$$= \frac{\pi}{4}(2)(0.012)^2(145 \times 10^6) = 32.80 \times 10^3 \text{ N}$$

Based on tension in wood

$$P_u = A\sigma_u = w(b-d)\sigma_u$$

$$= (0.040)(0.040 - 0.012)(60 \times 10^6)$$

$$= 67.2 \times 10^3 \text{ N}$$

Based on double shear in the wood

$$P_u = 2A\tau_u = 2wc\tau_u = (2)(0.040)(0.055)(7.5 \times 10^6)$$

$$= 33.0 \times 10^3 \text{ N}$$

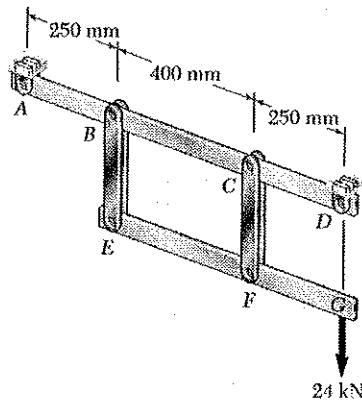
Use smallest

$$P_u = 32.8 \times 10^3 \text{ N}$$

$$\text{Allowable } P = \frac{P_u}{\text{F.S.}} = \frac{32.8 \times 10^3}{3.2} = 10.25 \times 10^3 \text{ N}$$

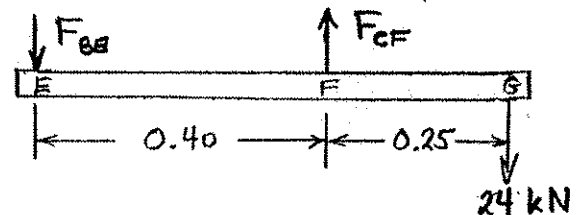
$$10.25 \text{ kN}$$

Problem 1.49



1.49 Each of the two vertical links CF connecting the two horizontal members AD and EG has a 10×40 -mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at C and F has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.

Use member EFG as free body.



$$\begin{aligned}\sum M_E &= 0 \\ 0.40 F_{CF} - (0.65)(24 \times 10^3) &= 0 \\ F_{CF} &= 39 \times 10^3 \text{ N}\end{aligned}$$

Based on tension in links CF

$$A = (b - d)t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6} \text{ m}^2 \text{ (one link)}$$

$$F_u = 2\sigma_u A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3 \text{ N}$$

Based on double shear in pins

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$F_u = 2\tau_u A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3 \text{ N}$$

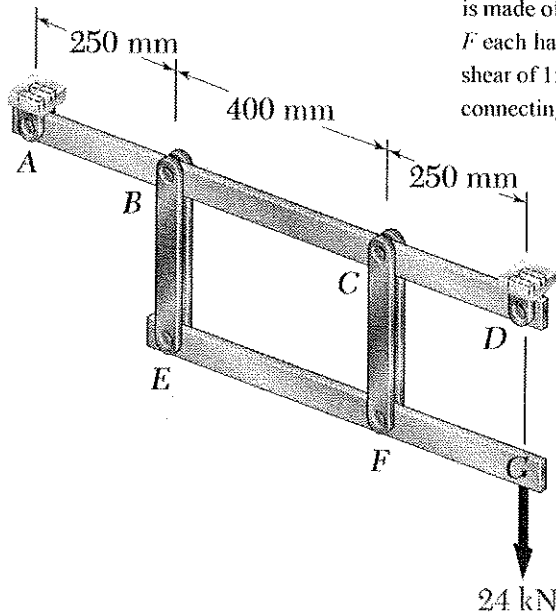
Actual F_u is smaller value, i.e. $F_u = 94.248 \times 10^3 \text{ N}$

$$\text{Factor of safety } F.S. = \frac{F_u}{F_{CF}} = \frac{94.248 \times 10^3}{39 \times 10^3} = 2.42$$

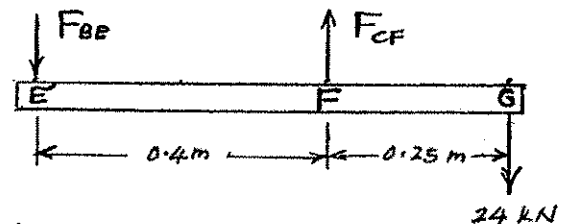
Problem 1.50

1.50 Solve Prob. 1.49, assuming that the pins at C and F have been replaced by pins with a 30 mm diameter.

1.49 Each of the two vertical links CF connecting the two horizontal members AD and EG has a uniform rectangular cross section 10 mm thick and 40 mm wide, and is made of a steel with an ultimate strength in tension of 400 MPa. The pins at C and F each have a 20 mm diameter and are made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.



Use member EFG as free body.



$$\sum M_E = 0:$$

$$0.4 F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \text{ kN}$$

Failure by tension in links CF. (2 parallel links)

$$\text{Net section area for 1 link: } A = (b - d)t = (0.04 - 0.02)(0.01) = 200 \times 10^{-6} \text{ m}^2$$

$$F_u = 2A\sigma_u = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160 \text{ kN}$$

Failure by double shear in pins.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.02)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

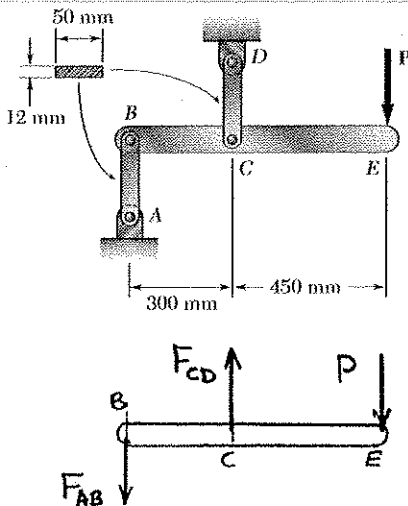
$$F_u = 2A\tau_u = (2)(314.16 \times 10^{-6})(150 \times 10^6) = 94.25 \text{ kN}$$

Actual ultimate load is the smaller value. $F_u = 94.25 \text{ kN}$

$$\text{Factor of safety: } F.S. = \frac{F_u}{F_{CF}} = \frac{94.25}{39}$$

$$F.S. = 2.42$$

Problem 1.51



1.51 Each of the steel links AB and CD is connected to a support and to member BCE by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load P if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)

Use member BCE as free body.

$$+\circlearrowleft \sum M_B = 0 : 0.3 F_{CD} - 0.75 P = 0$$

$$P = \frac{2}{5} F_{CD}$$

$$+\circlearrowleft \sum M_C = 0 : 0.3 F_{AB} - 0.45 P = 0$$

$$P = \frac{2}{3} F_{AB}$$

Both links have the same area, pin diameter and material, Therefore, they have the same ultimate load.

Failure by pin in single shear. $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.025)^2 = 490.9 \times 10^{-6} \text{ m}^2$

$$F_u = \tau_u A = (210 \times 10^6) (490.9 \times 10^{-6}) = 103.09 \text{ kN}$$

Failure by tension in link. $A = (b - d)t = (0.05 - 0.025) 0.012 = 3 \times 10^{-4} \text{ m}^2$

$$F_u = \sigma_u A = (490 \times 10^6) (3 \times 10^{-4}) = 147 \text{ kN}$$

Ultimate load for link and pin is the smaller.

$$F_u = 103.09 \text{ kN}$$

Allowable values of F_{CD} and F_{AB} .

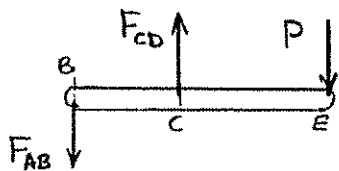
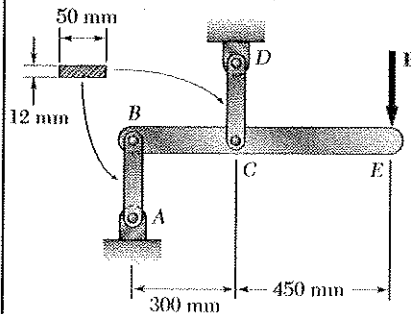
$$F_{all} = \frac{F_u}{F.S.} = \frac{103.09}{3.0} = 34.36 \text{ kN}$$

Allowable load for structure is the smaller of $\frac{2}{3} F_{all}$ and $\frac{2}{5} F_{all}$.

$$P = \frac{2}{5} (34.36)$$

$$P = 13.7 \text{ kN} \quad \blacktriangleleft$$

Problem 1.52



1.52 An alternative design is being considered to support member BCE of Prob. 1.51, in which link CD will be replaced by two links, each of 6×50-mm cross section, causing the pins at C and D to be in double shear. Assuming that all other specifications remain unchanged, determine the allowable load P if an overall factor of safety of 3.0 is desired.

1.51 Each of the steel links AB and CD is connected to a support and to member BCE by 25-mm-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 210 MPa for the steel used in the pins and that the ultimate normal stress is 490 MPa for the steel used in the links, determine the allowable load P if an overall factor of safety of 3.0 is desired. (Note that the links are not reinforced around the pin holes.)

Use member BCE as free body.

$$+\circlearrowleft \sum M_B = 0: 0.3 F_{CD} - 0.75 P = 0$$

$$P = \frac{2}{5} F_{CD}$$

$$+\circlearrowleft \sum M_C = 0: 0.3 F_{AB} - 0.45 P = 0$$

$$P = \frac{2}{3} F_{AB}$$

Area of all pins: $A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.025)^2 = 490.9 \times 10^{-6} \text{ m}^2$

Net section area of link AB: $A_{net} = (b - d) t_{AB} = (0.05 - 0.025)(0.012) = 3 \times 10^{-4} \text{ m}^2$

Net section area of the 2 links CD is the same.

Failure by pins A and B in single shear. $(F_{AB})_u = \tau_u A_{pin}$

$$(F_{AB})_u = (210 \times 10^6)(490.9 \times 10^{-6}) = 103.09 \text{ kN}$$

Failure by tension in link AB.

$$(F_{AB})_u = \sigma_u A_{net}$$

$$(F_{AB})_u = (490 \times 10^6)(3 \times 10^{-4}) = 147 \text{ kN}$$

Ultimate load for link and pins AB is the smaller: $(F_{AB})_u = 103.09 \text{ kN}$

Corresponding ultimate load, $P_u = \frac{2}{3} (F_{AB})_u = 68.73 \text{ kN}$

Failure by pins C and D in double shear.

$$(F_{CD})_u = 2 \tau_u A_{pin}$$

$$(F_{CD})_u = (2)(210 \times 10^6)(490.9 \times 10^{-6}) = 206.18 \text{ kN}$$

Failure by tension in links CD.

$$(F_{CD})_u = \sigma_u A_{net}$$

$$(F_{CD})_u = (490 \times 10^6)(3 \times 10^{-4}) = 147 \text{ kN}$$

Ultimate load for links and pins CD is the smaller: $(F_{CD})_u = 147 \text{ kN}$

Corresponding ultimate load: $P_u = \frac{2}{5} (F_{CD})_u = 58.8 \text{ kN}$

Actual ultimate load is the smaller. $P_u = 58.8 \text{ kN}$

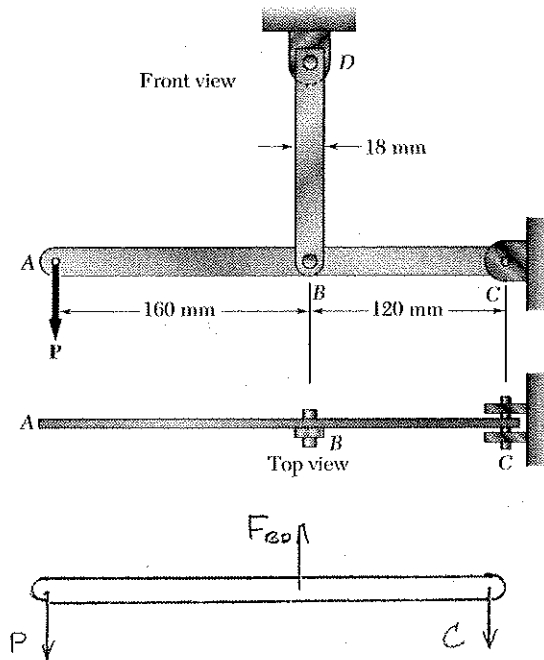
Allowable load P:

$$P = \frac{P_u}{F.S.} = \frac{58.8}{3.0}$$

$$P = 19.6 \text{ kN} \quad \blacktriangleleft$$

Problem 1.53

1.53 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load P that can be applied at A. Note that link BD is not reinforced around the pin holes.



Use free body ABC.

$$+\circlearrowleft \sum M_C = 0:$$

$$0.280 P - 0.120 F_{BD} = 0$$

$$P = \frac{3}{7} F_{BD} \quad (1)$$

$$+\circlearrowleft \sum M_B = 0:$$

$$0.160 P - 0.120 C = 0$$

$$P = \frac{3}{4} C \quad (2)$$

Tension on net section of link BD.

$$F_{BD} = \sigma A_{net} = \frac{\sigma_u}{F.S.} A_{net}$$

$$= \left(\frac{400 \times 10^6}{3} \right) (6 \times 10^{-3}) (18 - 10) (10^{-3})$$

$$= 6.40 \times 10^3 \text{ N}$$

Shear in pins at B and D.

$$F_{BD} = 2 A_{pin} = \frac{2 \tau_u}{F.S.} \left(\frac{\pi}{4} d^2 \right) = \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (10 \times 10^{-3})^2 = 3.9270 \times 10^3 \text{ N}$$

Smaller value of F_{BD} is $3.9270 \times 10^3 \text{ N}$.

$$\text{From (1)} \quad P = \left(\frac{3}{7} \right) (3.9270 \times 10^3) = 1.683 \times 10^3 \text{ N}$$

Shear in pin at C

$$C = 2 A_{pin} = 2 \left(\frac{2 \tau_u}{F.S.} \right) \left(\frac{\pi}{4} d^2 \right) = (2) \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (6 \times 10^{-3})^2 = 2.8274 \times 10^3 \text{ N}$$

$$\text{From (2)} \quad P = \left(\frac{3}{4} \right) (2.8274 \times 10^3) = 2.12 \times 10^3 \text{ N}$$

Smaller value of P is allowable value.

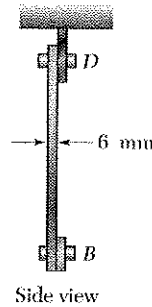
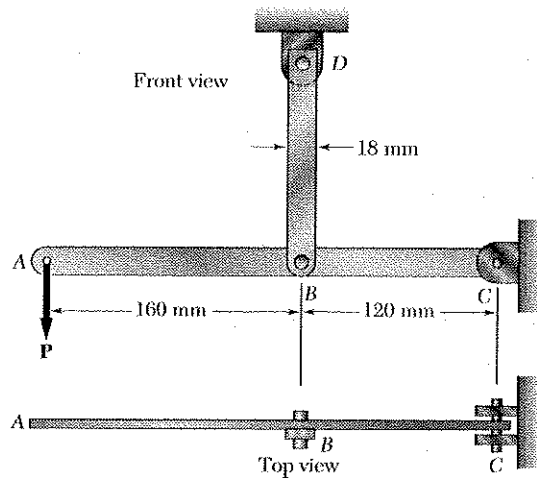
$$P = 1.683 \times 10^3 \text{ N}$$

$$P = 1.683 \text{ kN} \quad \blacktriangleleft$$

Problem 1.54

1.54 Solve Prob. 1.53, assuming that the structure has been redesigned to use 12-mm-diameter pins at B and D and no other change has been made.

1.53 In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D . The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD . Knowing that a factor of safety of 3.0 is desired, determine the largest load P that can be applied at A . Note that link BD is not reinforced around the pin holes.



Use free body ABC.

$$+\circlearrowleft \sum M_C = 0 =$$

$$0.280 P - 0.120 F_{BD} = 0$$

$$P = \frac{3}{7} F_{BD} \quad (1)$$

$$+\circlearrowleft \sum M_B = 0 =$$

$$0.160 P - 0.120 C = 0$$

$$P = \frac{3}{4} C \quad (2)$$

Tension on net section of link BD .

$$F_{BD} = \sigma A_{net} = \frac{\sigma_u}{F.S.} A_{net}$$

$$= \left(\frac{400 \times 10^6}{3} \right) (6 \times 10^{-3}) (18 - 12) (10^{-3})$$

$$= 4.80 \times 10^3 \text{ N}$$

Shear in pins at B and D .

$$F_{BD} = 2 A_{pin} = \frac{\tau_u}{F.S.} \frac{\pi}{4} d^2 = \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (12 \times 10^{-3})^2 = 5.6549 \times 10^3 \text{ N}$$

Smaller value of F_{BD} is $4.80 \times 10^3 \text{ N}$.

$$\text{From (1), } P = \left(\frac{3}{7} \right) (4.80 \times 10^3) = 2.06 \times 10^3 \text{ N}$$

Shear in pin at C .

$$C = 2 A_{pin} = 2 \frac{\tau_u}{F.S.} \frac{\pi}{4} d^2 = (2) \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (6 \times 10^{-3})^2 = 2.8274 \times 10^3 \text{ N}$$

$$\text{From (2), } P = \left(\frac{3}{4} \right) (2.8274 \times 10^3) = 2.12 \times 10^3 \text{ N}$$

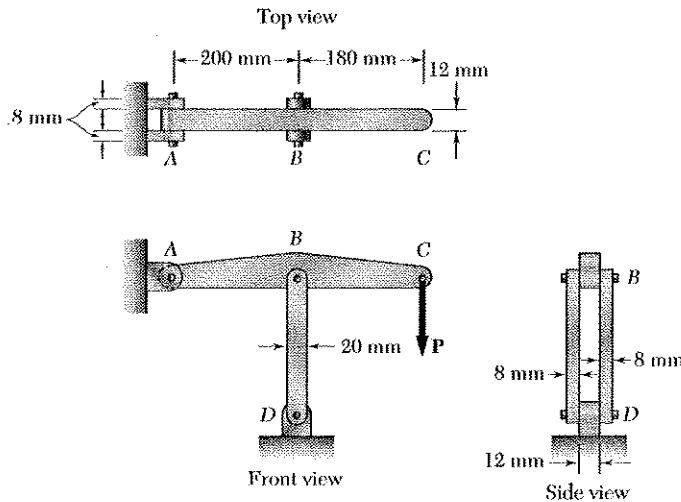
Smaller value of P is the allowable value.

$$P = 2.06 \times 10^3 \text{ N}$$

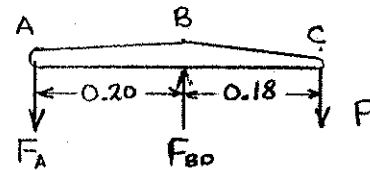
$$P = 2.06 \text{ kN}$$

Problem 1.55

1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load *P* if an overall factor of safety of 3.0 is desired.



Statics: Use ABC as free body.



$$\sum M_B = 0: 0.20 F_A - 0.18 P = 0$$

$$P = \frac{10}{9} F_A$$

$$\sum M_A = 0: 0.20 F_{BD} - 0.38 P = 0$$

$$P = \frac{10}{19} F_{BD}$$

Based on double shear in pin *A*.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.008)^2 = 50.266 \times 10^{-6} \text{ m}^2$$

$$F_A = \frac{2\tau_u A}{\text{F.S.}} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3 \text{ N}$$

$$P = \frac{10}{9} F_A = 3.72 \times 10^3 \text{ N}$$

Based on double shear in pins at *B* and *D*.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.012)^2 = 113.10 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2\tau_u A}{\text{F.S.}} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3 \text{ N}$$

Based on compression in links *BD*.

$$\text{For one link } A = (0.020)(0.008) = 160 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2\sigma_u A}{\text{F.S.}} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3 \text{ N}$$

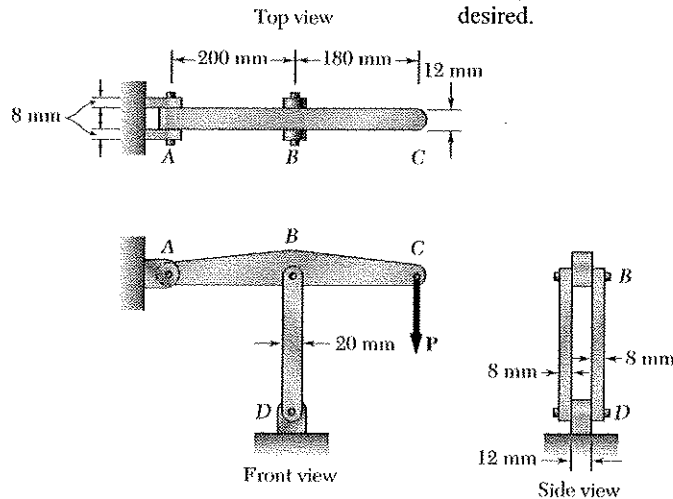
Allowable value of *P* is smallest. $\therefore P = 3.72 \times 10^3 \text{ N}$

$$P = 3.72 \text{ kN}$$

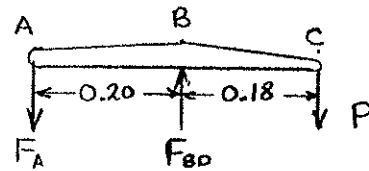
Problem 1.56

1.56 In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load P if an overall factor of safety of 3.0 is desired.

1.55 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load P if an overall factor of safety of 3.0 is desired.



Statics: Use ABC as free body.



$$+\sum M_B = 0: 0.20 F_A - 0.18 P = 0$$

$$P = \frac{10}{9} F_A$$

$$+\sum M_A = 0: 0.20 F_{BD} - 0.38 P = 0$$

$$P = \frac{10}{19} F_{BD}$$

Based on double shear in pin A.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.010)^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$F_A = \frac{2\tau_u A}{\text{F.S.}} = \frac{(2)(100 \times 10^6)(78.54 \times 10^{-6})}{3.0} = 5.236 \times 10^3 \text{ N}$$

$$P = \frac{10}{9} F_A = 5.82 \times 10^3 \text{ N}$$

Based on double shear in pins at B and D.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.012)^2 = 113.10 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2\tau_u A}{\text{F.S.}} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3 \text{ N}$$

Based on compression in links BD.

$$\text{For one link } A = (0.020)(0.008) = 160 \times 10^{-6} \text{ m}^2$$

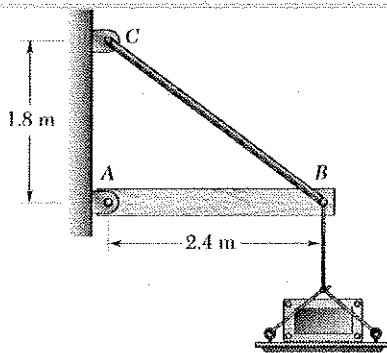
$$F_{BD} = \frac{2\sigma_u A}{\text{F.S.}} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3 \text{ N}$$

Allowable value of P is smallest. $\therefore P = 3.97 \times 10^3 \text{ N}$

$$P = 3.97 \text{ kN}$$

Problem 1.57



*1.57 A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (a) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (b) What is the corresponding conventional factor of safety for rod BC?

$$\sum M_A = 0: (2.4) \frac{3}{5} P - 2.4 W_1 - 1.2 W_2 \therefore P = \frac{5}{3} W_1 + \frac{5}{6} W_2$$

For dead loading, $W_1 = (40)(9.81) = 392.4 \text{ N}$

$$W_2 = (50)(9.81) = 490.5 \text{ N}$$

$$P_D = \left(\frac{5}{3}\right)(392.4) + \left(\frac{5}{6}\right)(490.5) = 1.0628 \times 10^3 \text{ N}$$

For live loading, $W_1 = mg$ $W_2 = 0$

$$P_L = \frac{5}{3} mg \quad \text{from which} \quad m = \frac{3}{5} \frac{P_L}{g}$$

Design criterion.

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^3)}{1.6}$$

$$= 5.920 \times 10^3 \text{ N}$$

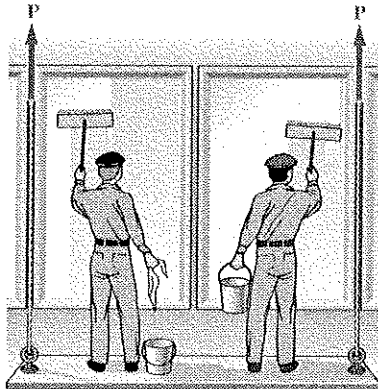
$$(a) \text{ Allowable load. } m = \frac{3}{5} \frac{5.92 \times 10^3}{9.81} \quad m = 362 \text{ kg} \quad \blacktriangleleft$$

Conventional factor safety.

$$P = P_D + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3 \text{ N}$$

$$(b) \text{ F.S.} = \frac{P_U}{P} = \frac{12 \times 10^3}{6.983 \times 10^3} \quad \text{F.S.} = 1.718 \quad \blacktriangleleft$$

Problem 1.58



*1.58 The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 72 kg and each of the window washers is assumed to weigh 88 kg with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (a) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (b) What is the conventional factor of safety for the selected cables?

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\phi}$$

$$= \frac{(1.2)(\frac{1}{2} \times 72) + (1.5)(\frac{3}{4} \times 2 \times 88)}{0.85}$$

$$= 283.76 \text{ kg} = 2.78 \text{ kN}$$

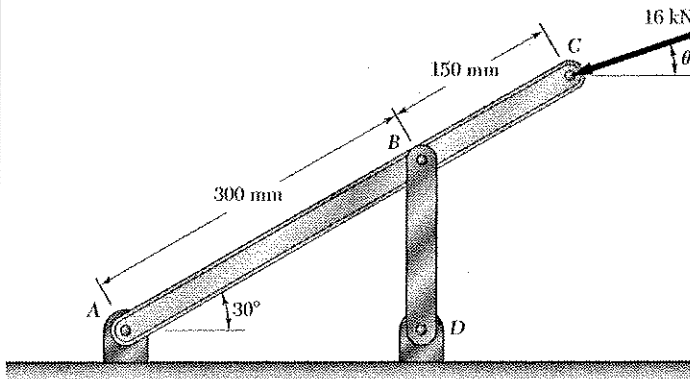
Conventional factor of safety

$$P = P_D + P_L = \frac{1}{2} \times 72 + 0.75 \times 2 \times 88 = 168 \text{ kg} = 1.648 \text{ kN}$$

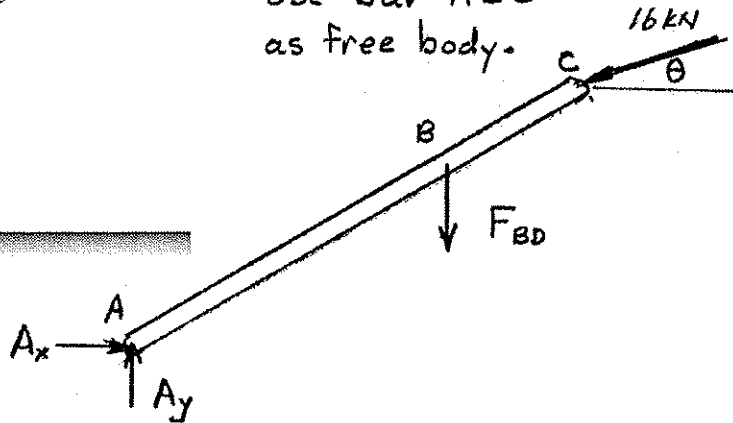
$$F.S. = \frac{P_U}{P} = \frac{2.784}{1.648} = 1.69$$

Problem 1.59

1.59 Link BD consists of a single bar 24 mm wide and 12 mm thick. Knowing that each pin has a 9 mm diameter, determine the maximum value of the average normal stress in link BD if (a) $\theta = 0^\circ$, (b) $\theta = 90^\circ$.



Use bar ABC
as free body.



(a) $\theta = 0^\circ$.

$$+\circlearrowleft \sum M_A = 0 = (0.45 \sin 30^\circ)(16) - (0.3 \cos 30^\circ) F_{BD} = 0$$

$$F_{BD} = 13.856 \text{ kN} \quad (\text{tension})$$

Area for tension loading: $A = (b - d)t = (24 - 9)(12) = 180 \text{ mm}^2$

$$\text{Stress: } \sigma = \frac{F_{BD}}{A} = \frac{13.856 \times 10^3}{180 \times 10^{-6}} \quad \sigma = 77 \text{ MPa}$$

(b) $\theta = 90^\circ$.

$$+\circlearrowleft \sum M_A = 0 = -(0.45 \cos 30^\circ)(16) + (0.3 \sin 30^\circ) F_{BD} = 0$$

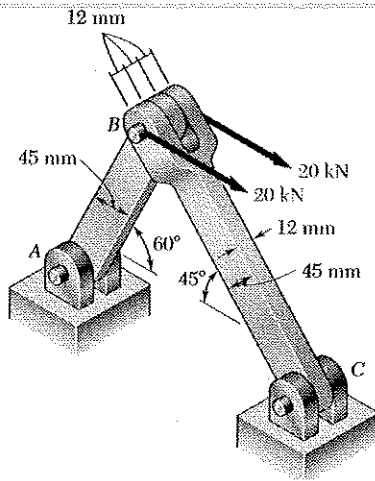
$$F_{BD} = -24 \text{ kN} \quad \text{i.e. compression.}$$

Area for compression loading: $A = bt = (24)(12) = 288 \text{ mm}^2$

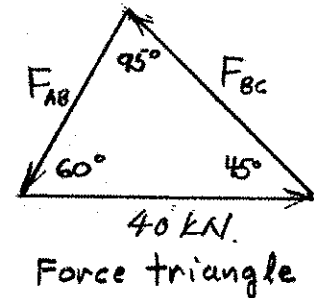
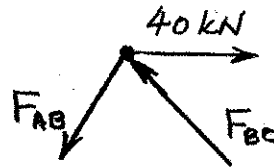
$$\text{Stress: } \sigma = \frac{F_{BD}}{A} = \frac{-24000}{288 \times 10^{-6}} \quad \sigma = -83.3 \text{ MPa}$$

Problem 1.60

1.60 Two horizontal 20 kN forces are applied to pin *B* of the assembly shown. Knowing that a pin of 20 mm diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.



Use joint *B* as free body.



Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{40}{\sin 95^\circ}$$

$$F_{AB} = 28.4 \text{ kN}$$

$$F_{BC} = 34.8 \text{ kN}$$

Link *AB* is a tension member

Minimum section at pin $A_{net} = (0.045 - 0.012)(0.012) = 300 \times 10^{-6} \text{ m}^2$

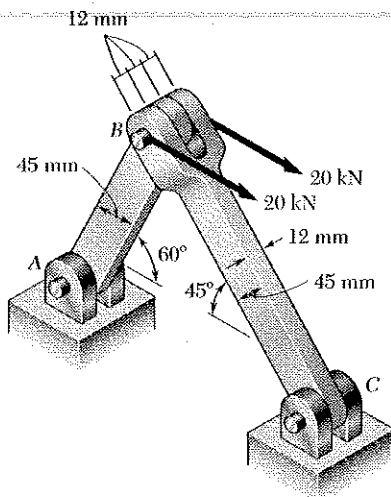
(a) Stress in *AB* $\sigma_{AB} = \frac{F_{AB}}{A_{net}} = \frac{28.4}{300 \times 10^{-6}} = 94.7 \text{ MPa}$ ◀

Link *BC* is a compression member

Cross sectional area is $A = (0.045)(0.012) = 540 \times 10^{-6} \text{ m}^2$

(b) Stress in *BC* $\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-34.8}{540 \times 10^{-6}} = -64.4 \text{ MPa}$ ◀

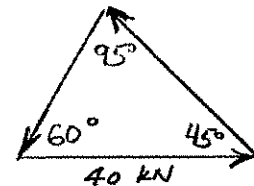
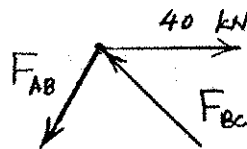
Problem 1.61



1.61 For the assembly and loading of Prob. 1.60, determine (a) the average shearing stress in the pin at C, (b) the average bearing stress at C in member BC, (c) the average bearing stress at B in member BC.

1.60 Two horizontal 20-kN forces are applied to pin B of the assembly shown. Knowing that a pin of 20-mm diameter is used at each connection, determine the maximum value of the average normal stress (a) in link AB, (b) in link BC.

Use joint B as free body.



Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{40}{\sin 95^\circ}$$

$$F_{BC} = 34.77 \text{ kN}$$

(a) Shearing stress in pin at C.

$$\tau = \frac{F_{BC}}{2A_p}$$

$$A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4} (20)^2 = 314.16 \text{ mm}^2$$

$$\tau = \frac{34770}{(2)(314.16 \times 10^{-6})} = 55.338 \times 10^6$$

$$\tau = 55.3 \text{ MPa}$$

(b) Bearing stress at C in member BC.

$$\sigma_b = \frac{F_{BC}}{A}$$

$$A = t d = (12)(20) = 240 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{240 \times 10^{-6}} = 144.875 \times 10^6$$

$$\sigma_b = 144.9 \text{ MPa}$$

(c) Bearing stress at B in member BC.

$$\sigma_b = \frac{F_{BC}}{A}$$

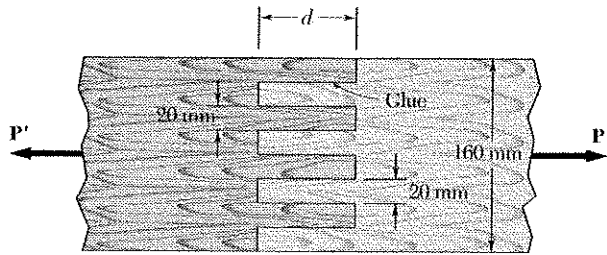
$$A = 2t d = 2(12)(20) = 480 \text{ mm}^2$$

$$\sigma_b = \frac{34770}{480 \times 10^{-6}} = 72.437 \times 10^6$$

$$\sigma_b = 72.4 \text{ MPa}$$

Problem 1.62

1.62 Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.



Seven surfaces carry the total load $P = 7.6 \text{ kN} = 7.6 \times 10^3$.

Let $t = 22 \text{ mm}$.

Each glue area is $A = dt$

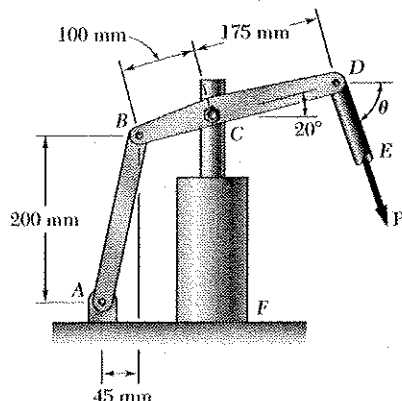
$$\tau = \frac{P}{7A} \quad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3} \text{ m}^2$$

$$= 1.32404 \times 10^3 \text{ mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2$$

$$d = 60.2 \text{ mm} \blacktriangleleft$$

Problem 1.63

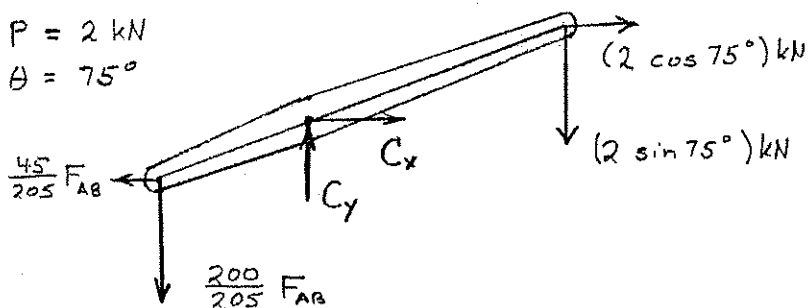


1.63 The hydraulic cylinder CF , which partially controls the position of rod DE , has been locked in the position shown. Member BD is 15 mm thick and is connected to the vertical rod by a 9-mm-diameter bolt. Knowing that $P = 2$ kN and $\theta = 75^\circ$, determine (a) the average shearing stress in the bolt, (b) the bearing stress at C in member BD .

Use member BCD as a free body, and note that AB is a two force member.

$$P = 2 \text{ kN}$$

$$\theta = 75^\circ$$



Length of member AB .

$$l_{AB} = \sqrt{200^2 + 45^2} = 205 \text{ mm}$$

$$\sum M_C = 0: \left(\frac{200}{205} F_{AB}\right)(100 \cos 20^\circ) - \left(\frac{45}{205} F_{AB}\right)(100 \sin 20^\circ) - (2 \cos 75^\circ)(175 \sin 20^\circ) - (2 \sin 75^\circ)(175 \cos 20^\circ) = 0$$

$$84.1696 F_{AB} - 348.668 = 0$$

$$F_{AB} = 4.1424 \text{ kN}$$

$$\sum F_x = 0: C_x - \frac{45}{205}(4.1424) + 2 \cos 75^\circ = 0$$

$$C_x = 0.3917 \text{ kN}$$

$$\sum F_y = 0: C_y - \frac{200}{205}(4.1424) - 2 \sin 75^\circ = 0$$

$$C_y = 5.9732 \text{ kN}$$

$$\text{Reaction at } C: C = \sqrt{C_x^2 + C_y^2}$$

$$C = 5.9860 \text{ kN}$$

(a) Shearing stress in bolt (single shear).

$$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.009)^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$\tau = \frac{C}{A_{bolt}} = \frac{5.9860 \times 10^3}{63.617 \times 10^{-6}} = 94.09 \times 10^6 \text{ Pa}$$

$$\tau = 94.1 \text{ MPa}$$

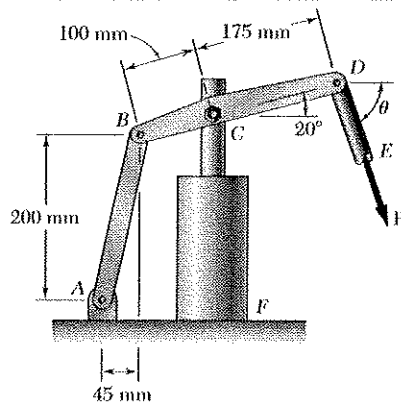
(b) Bearing stress at C in member BD .

$$A_b = dt = (0.009)(0.015) = 135 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{C}{A_b} = \frac{5.9860 \times 10^3}{135 \times 10^{-6}} = 44.34 \times 10^6 \text{ Pa}$$

$$\sigma_b = 44.3 \text{ MPa}$$

Problem 1.64



Length of member AB.

$$l_{AB} = \sqrt{200^2 + 45^2}$$

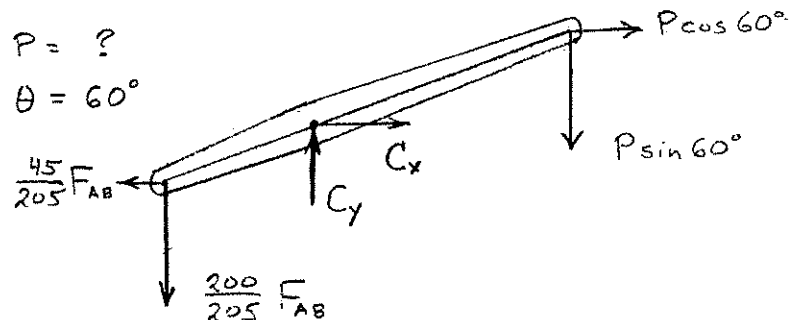
$$= 205 \text{ mm}$$

1.64 The hydraulic cylinder CF, which partially controls the position of rod DE, has been locked in the position shown. Link AB has a uniform rectangular cross section of $12 \times 25 \text{ mm}$ and is connected at B to member BD by an 8-mm-diameter pin. Knowing that the maximum allowable average shearing stress in the pin is 140 MPa, determine (a) the largest force P which may be applied at E when $\theta = 60^\circ$, (b) the corresponding bearing stress at B in link AB, (c) the corresponding maximum value of the normal stress in link AB.

Use member BCD as a free body, and note that AB is a two force member.

$$P = ?$$

$$\theta = 60^\circ$$



$$+\circlearrowleft \sum M_C = 0: \left(\frac{200}{205} F_{AB}\right)(100 \cos 20^\circ) - \left(\frac{45}{205} F_{AB}\right)(100 \sin 20^\circ) - (P \cos 60^\circ)(175 \sin 20^\circ) - (P \sin 60^\circ)(175 \cos 20^\circ) = 0$$

$$84.1696 F_{AB} - 172.3414 P = 0 \quad F_{AB} = 2.0475 P$$

(a) Allowable load P. Pin at A is in single shear.

$$A_{\text{pin}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.008)^2 = 50.2655 \times 10^{-6} \text{ m}^2$$

$$\tau = 140 \times 10^6 \text{ Pa}$$

$$\tau = \frac{F_{AB}}{A_{\text{pin}}}$$

$$140 \times 10^6 = \frac{2.0475 P}{50.2655 \times 10^{-6}}$$

$$P = 3.4370 \times 10^3 \text{ N}$$

$$P = 3.44 \text{ kN}$$

(b) Bearing stress at B in link AB. $d = 8 \text{ mm}$, $t = 12 \text{ mm}$

$$A_b = dt = (0.008)(0.012) = 96 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = (2.0475)(3.4375 \times 10^3) = 7.0383 \times 10^3 \text{ N}$$

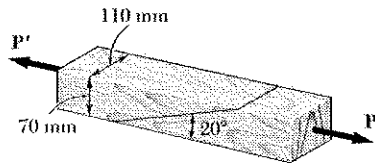
$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{7.0383 \times 10^3}{96 \times 10^{-6}} = 73.3 \times 10^6 \text{ Pa} \quad \sigma_b = 73.3 \text{ MPa}$$

(c) Maximum normal stress in link AB. $b = 25 \text{ mm}$, $t = 0.012 \text{ mm}$

$$A_{\text{net}} = (b - d)(t) = (0.025 - 0.008)(0.012) = 204 \times 10^{-6} \text{ m}^2$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{\text{net}}} = \frac{7.0383 \times 10^3}{204 \times 10^{-6}} = 34.5 \times 10^6 \text{ Pa} \quad \sigma_{AB} = 34.5 \text{ MPa}$$

Problem 1.65



1.65 Two wooden members of 70 × 110-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 500 kPa, determine the largest axial load P that can be safely applied.

$$A_o = (0.070 \text{ m})(0.110 \text{ m}) = 7.7 \times 10^{-3} \text{ m}^2$$

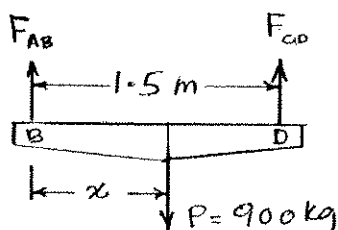
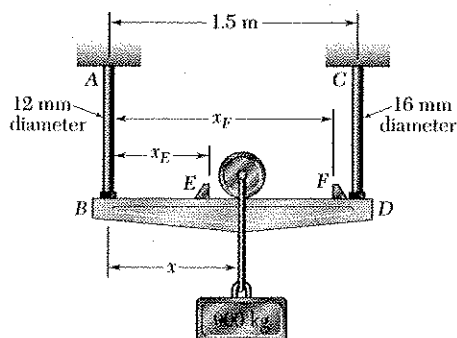
$$\theta = 90^\circ - 20^\circ = 70^\circ$$

$$\tau = \frac{P}{A_o} \sin \theta \cos \theta$$

$$P = \frac{A_o \tau}{\sin \theta \cos \theta} = \frac{(7.7 \times 10^{-3})(500 \times 10^3)}{\sin 70^\circ \cos 70^\circ} = 11.98 \times 10^3 \text{ N}$$

$$P = 11.98 \text{ kN}$$

Problem 1.66



1.66 The 900-kg load may be moved along the beam BD to any position between stops at E and F . Knowing that $\sigma_{all} = 42 \text{ MPa}$ for the steel used in rods AB and CD , determine where the stops should be placed if the permitted motion of the load is to be as large as possible.

Permitted member forces:

$$AB: (F_{AB})_{max} = \sigma_{all} A_{AB} = (42 \times 10^6) \left(\frac{\pi}{4} \right) (0.012)^2 = 4.75 \text{ kN}$$

$$CD: (F_{CD})_{max} = \sigma_{all} A_{CD} = (42 \times 10^6) \left(\frac{\pi}{4} \right) (0.016)^2 = 8.44 \text{ kN}$$

Use member $BEFD$ as a free body.

$$P = 900 \text{ kg} = 8.829 \text{ kN}$$

$$+\circlearrowleft \sum M_D = 0$$

$$-(1.5) F_{AB} + (1.5 - x_E) P = 0$$

$$1.5 - x_E = \frac{1.5 F_{AB}}{P} = \frac{(1.5)(4.75 \times 10^3)}{8.829 \times 10^3} = 0.807$$

$$x_E = 0.693 \text{ m} \quad \blacktriangleleft$$

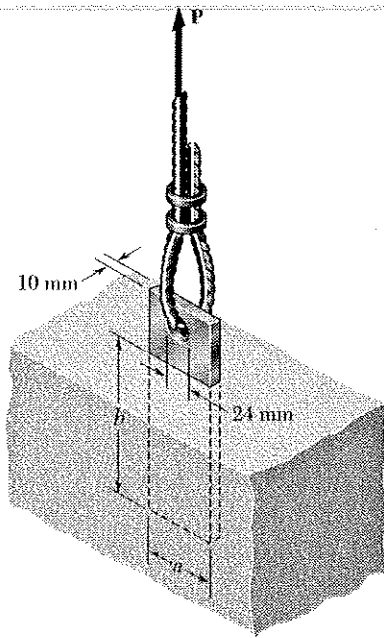
$$+\circlearrowleft \sum M_B = 0$$

$$1.5 F_{CD} - x P = 0$$

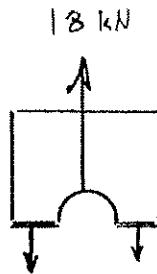
$$x_F = \frac{1.5 F_{CD}}{P} = \frac{(1.5)(8.44)}{8.829}$$

$$x_F = 1.43 \text{ m} \quad \blacktriangleleft$$

Problem 1.67



1.67 A steel plate 10 mm thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is 24 mm, the ultimate strength of the steel used is 250 MPa, and the ultimate bonding stress between plate and concrete is 2.1 MPa. Knowing that a factor of safety of 3.60 is desired when $P = 18$ kN, determine (a) the required width a of the plate, (b) the minimum depth b to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)



(a) Based on tension in the plate.

$$A = (a - d)t$$

$$P_u = \sigma_u A$$

$$F.S. = \frac{P_u}{P} = \frac{\sigma_u (a - d)t}{P}$$

Solving for a ,

$$a = d + \frac{(F.S.)P}{\sigma_u t} = 0.024 + \frac{(3.60)(18 \times 10^3)}{(250 \times 10^6)(0.010)}$$

$$a = 0.04992 \text{ m} \quad a = 49.9 \text{ mm} \quad \blacktriangleleft$$

(b) Based on shear between plate and concrete slab.

$$A = \text{perimeter} \times \text{depth} = (2a + 2t)b \quad \tau_u = 2.1 \times 10^6 \text{ Pa}$$

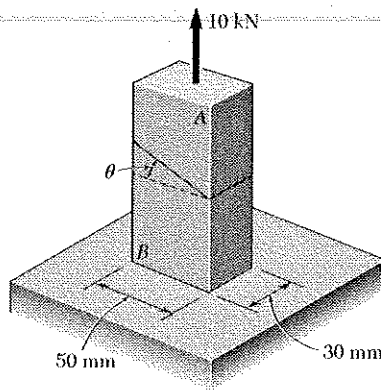
$$P_u = \tau_u A = 2\tau_u(a + t)b$$

$$F.S. = \frac{P_u}{P}$$

$$\text{Solving for } b, \quad b = \frac{(F.S.)P}{2(a + t)\tau_u} = \frac{(3.60)(18 \times 10^3)}{(2)(0.04992 + 0.010)(2.1 \times 10^6)}$$

$$b = 0.25748 \text{ m} \quad b = 257 \text{ mm} \quad \blacktriangleleft$$

Problem 1.68



1.68 The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

$$A_o = (0.05)(0.03) = 0.0015 \text{ m}^2$$

$$P = 10 \text{ kN}$$

$$P_o = (F.S.)P = 30 \text{ kN}$$

Based on tensile stress

$$\sigma_v = \frac{P_o}{A_o} \cos^2 \theta$$

$$\cos^2 \theta = \frac{\sigma_v A_o}{P_o} = \frac{(17 \times 10^6)(0.0015)}{30 \times 10^3} = 0.85$$

$$\theta = 22.8^\circ$$

$$\theta \geq 22.8^\circ$$

Based on shearing stress

$$\tau_v = \frac{P_o}{A_o} \sin \theta \cos \theta = \frac{P_o}{2A_o} \sin 2\theta$$

$$\sin 2\theta = \frac{2A_o \tau_v}{P_o} = \frac{(2)(0.0015)(9 \times 10^6)}{30 \times 10^3} = 0.9$$

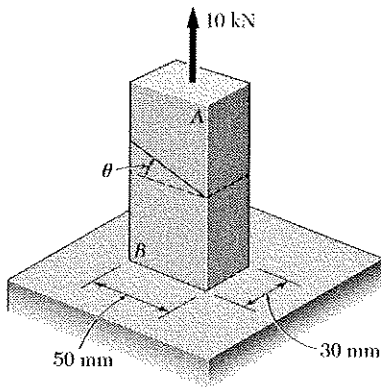
$$2\theta = 64.16^\circ$$

$$\theta = 32.1^\circ$$

$$\theta \leq 32.1^\circ$$

$$\text{Hence } 22.8^\circ \leq \theta \leq 32.1^\circ$$

Problem 1.69



1.69 The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine (a) the value of θ for which the factor of safety of the member is maximum, (b) the corresponding value of the factor of safety. (Hint: Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

$$A_o = (0.05)(0.03) = 0.0015 \text{ m}^2$$

At the optimum angle $(F.S.)_\sigma = (F.S.)_\tau$

$$\text{Normal stress: } \sigma = \frac{P}{A_o} \cos^2 \theta \therefore P_{o,\sigma} = \frac{\sigma_v A_o}{\cos^2 \theta}$$

$$(F.S.)_\sigma = \frac{P_{o,\sigma}}{P} = \frac{\sigma_v A_o}{P \cos^2 \theta}$$

$$\text{Shearing stress: } \tau = \frac{P}{A_o} \sin \theta \cos \theta \therefore P_{o,\tau} = \frac{\tau_v A_o}{\sin \theta \cos \theta}$$

$$(F.S.)_\tau = \frac{P_{o,\tau}}{P} = \frac{\tau_v A_o}{P \sin \theta \cos \theta}$$

$$\text{Equating: } \frac{\sigma_v A_o}{P \cos^2 \theta} = \frac{\tau_v A_o}{P \sin \theta \cos \theta}$$

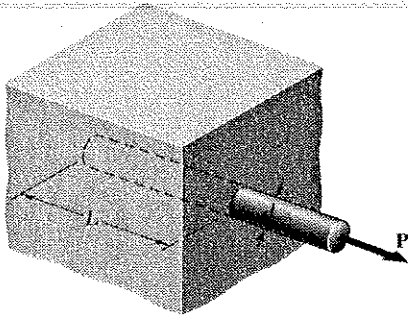
$$\text{Solving: } \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{\tau_v}{\sigma_v} = \frac{9}{17} = 0.529$$

$$(a) \theta_{opt} = 27.9^\circ$$

$$(b) P_o = \frac{\sigma_v A_o}{\cos^2 \theta} = \frac{(17 \times 10^6)(0.0015)}{\cos^2 27.9^\circ} = 32.65 \text{ kN}$$

$$F.S. = \frac{P_o}{P} = \frac{32.64}{10} = 3.26$$

Problem 1.70



1.70 A force P is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

For shear, $A = \pi d L$

$$P = \tau_{all} A = \tau_{all} \pi d L$$

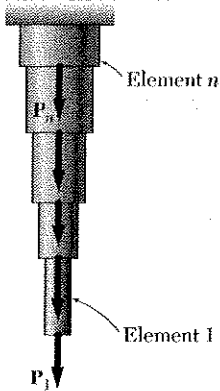
For tension, $A = \frac{\pi}{4} d^2$

$$P = \sigma_{all} A = \sigma_{all} \left(\frac{\pi}{4} d^2 \right)$$

Equating, $\tau_{all} \pi d L = \sigma_{all} \frac{\pi}{4} d^2$

Solving for L ,

$$L_{min} = \sigma_{all} d / 4 \tau_{all}$$

PROBLEM 1.C1

1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by P_i , with the magnitude P_i of this load being assumed positive if P_i is directed downward as shown and negative otherwise. (a) Write a computer program to determine the average stress in each element of the rod. (b) Use this program to solve Probs. 1.2 and 1.4.

SOLUTION**FORCE IN ELEMENT i :**

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^i P_k$$

AVERAGE STRESS IN ELEMENT i :

$$\text{Area} = A_i = \frac{1}{4} \pi d_i^2 \quad \text{Ave stress} = \frac{F_i}{A_i}$$

PROGRAM OUTPUTS

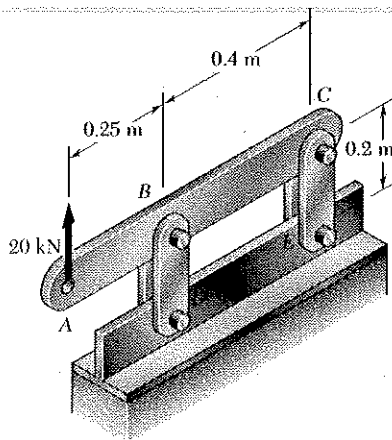
Problem 1.2
Element Stress (MPa)

1	81.487
2	-18.108

Problem 1.4
Element Stress (MPa)

1	35.651
2	42.441

PROBLEM 1.C2

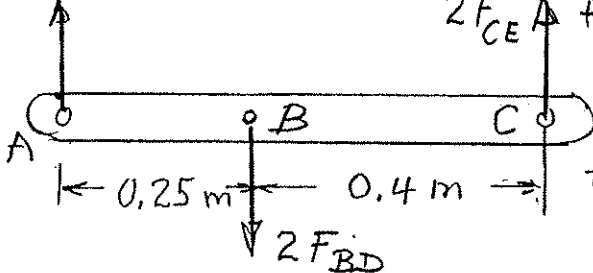


1.C2 A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (a) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (2) the average normal stress in the links connecting pins *C* and *E*, (3) the average shearing stress in pin *B*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *B* in member *ABC*, (6) the average bearing stress at *C* in member *ABC*. (b) Check your program by comparing the values obtained for *d* = 16 mm with the answers given for Probs. 1.7 and 1.27. (c) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively 150 MPa, 90 MPa, and 230 MPa. (d) Solve part c, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

FORCES IN LINKS

$$P = 20 \text{ kN}$$



F.B. DIAGRAM OF ABC:

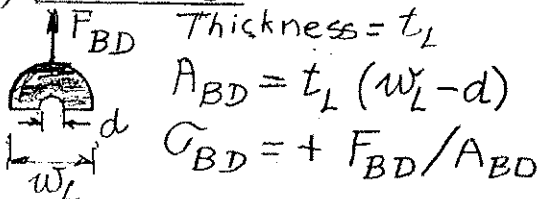
$$+ \circlearrowleft \sum M_C = 0: 2F_{BD}(BC) - P(AC) = 0$$

$$F_{BD} = P(AC)/2(BC) \text{ (TENSION)}$$

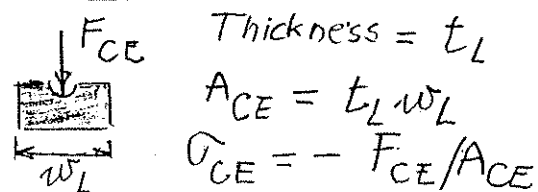
$$+ \circlearrowleft \sum M_B = 0: 2F_{CE}(BC) - P(AB) = 0$$

$$F_{CE} = P(AB)/2(BC) \text{ (COMP.)}$$

(1) LINK BD



(2) LINK CE



(3) PIN B

$$\tau_B = F_{BD}/(\pi d^2/4)$$

(4) PIN C

$$\tau_C = F_{CE}/(\pi d^2/4)$$

(5) BEARING STRESS AT B

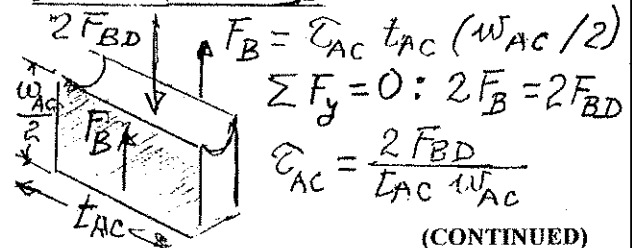
Thickness of member AC = t_{AC}

$$\text{Sig Bear B} = F_{BD}/(d t_{AC})$$

(6) BEARING STRESS AT C

$$\text{Sig Bear C} = F_{CE}/(d t_{AC})$$

SHEARING STRESS IN ABC UNDER PIN B



(CONTINUED)

PROBLEM 1.C2 CONTINUED
PROGRAM OUTPUTS

INPUT DATA FOR PARTS (a), (b), (c): $P = 20 \text{ kN}$, $AB = 0.25 \text{ m}$, $BC = 0.40 \text{ m}$,
 $AC = 0.65 \text{ m}$, $TL = 8 \text{ mm}$, $WL = 36 \text{ mm}$, $TAC = 10 \text{ mm}$, $WAC = 50 \text{ mm}$

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	325.00	125.00
11.00	81.25	-21.70	170.99	65.77	295.45	113.64
12.00	84.64	-21.70	143.68	55.26	270.83	104.17
13.00	88.32	-21.70	122.43	47.09	250.00	96.15
14.00	92.33	-21.70	105.56	40.60	232.14	89.29
15.00	96.73	-21.70	91.96	35.37	216.67	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	156.25	-21.70	39.11	15.04	141.30	54.35
24.00	169.27	-21.70	35.92	13.82	135.42	52.08
25.00	184.66	-21.70	33.10	12.73	130.00	50.00
26.00	203.13	-21.70	30.61	11.77	125.00	48.08
27.00	225.69	-21.70	28.38	10.92	120.37	46.30
28.00	253.91	-21.70	26.39	10.15	116.07	44.64
29.00	290.18	-21.70	24.60	9.46	112.07	43.10
30.00	338.54	-21.70	22.99	8.84	108.33	41.67

(c) ANSWER : $16 \text{ mm} \leq d \leq 22 \text{ mm}$

CHECK: For $d = 22 \text{ mm}$, $\text{Tau AC} = 65 \text{ MPa} < 90 \text{ MPa}$ O.K.

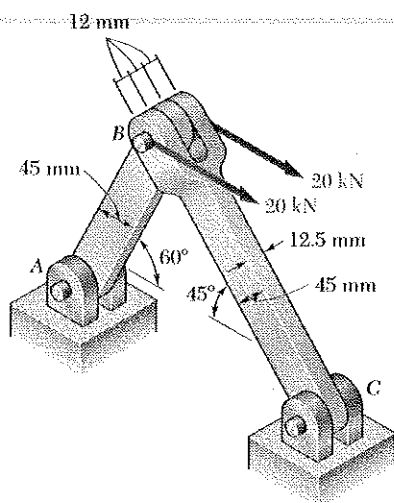
INPUT DATA FOR PART (d): $P = 20 \text{ kN}$, $AB = 0.25 \text{ m}$, $BC = 0.40 \text{ m}$,
 $AC = 0.65 \text{ m}$, $TL = 8 \text{ mm}$, $WL = 36 \text{ mm}$, $TAC = 8 \text{ mm}$, $WAC = 50 \text{ mm}$

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	406.25	156.25
11.00	81.25	-21.70	170.99	65.77	369.32	142.05
12.00	84.64	-21.70	143.68	55.26	338.54	130.21
13.00	88.32	-21.70	122.43	47.09	312.50	120.19
14.00	92.33	-21.70	105.56	40.60	290.18	111.61
15.00	96.73	-21.70	91.96	35.37	270.83	104.17
16.00	101.56	-21.70	80.82	31.08	253.91	97.66
17.00	106.91	-21.70	71.59	27.54	238.97	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	156.25	-21.70	39.11	15.04	176.63	67.93
24.00	169.27	-21.70	35.92	13.82	169.27	65.10
25.00	184.66	-21.70	33.10	12.73	162.50	62.50
26.00	203.13	-21.70	30.61	11.77	156.25	60.10
27.00	225.69	-21.70	28.38	10.92	150.46	57.87
28.00	253.91	-21.70	26.39	10.15	145.09	55.80
29.00	290.18	-21.70	24.60	9.46	140.09	53.88
30.00	338.54	-21.70	22.99	8.84	135.42	52.08

(d) ANSWER : $18 \text{ mm} \leq d \leq 22 \text{ mm}$

CHECK: For $d = 22 \text{ mm}$, $\text{Tau AC} = 81.25 \text{ MPa} < 90 \text{ MPa}$ O.K.

PROBLEM 1.C3

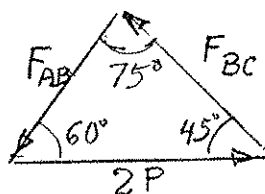
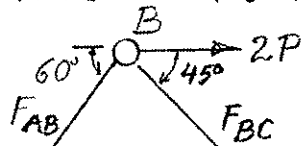


1.C3 Two horizontal 20 kN forces are applied to pin B of the assembly shown. Each of the three pins at A, B, and C has the same diameter d and is in double shear. (a) Write a computer program to calculate for values of d from 12.5 mm to 37.5 mm, using 1.25-mm increments, (1) the maximum value of the average normal stress in member AB, (2) the average normal stress in member BC, (3) the average shearing stress in pin A, (4) the average shearing stress in pin C, (5) the average bearing stress at A in member AB, (6) the average bearing stress at C in member BC, (7) the average bearing stress at B in member BC. (b) Check your program by comparing the values obtained for $d = 20$ mm with the answers given for Probs. 1.60 and 1.61. (c) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 250 MPa. (d) Solve part c, assuming that a new design is being investigated in which the thickness and width of the two members are changed, respectively, from 12.5 to 8 mm and from 45 mm to 60 mm.

SOLUTION

FORCES IN MEMBERS AB AND BC

FREE BODY: PIN B



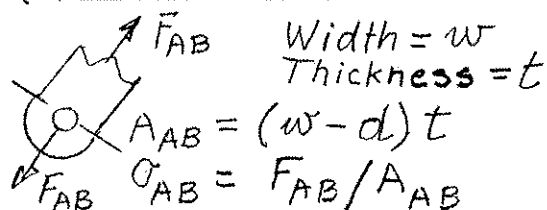
FROM FORCE TRIANGLE:

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{2P}{\sin 75^\circ}$$

$$F_{AB} = 2P(\sin 45^\circ / \sin 75^\circ)$$

$$F_{BC} = 2P(\sin 60^\circ / \sin 75^\circ)$$

(1) MAX. AVE. STRESS IN AB



(3) PIN A

$$\tau_A = (F_{AB}/2) / (\pi d^2/4)$$

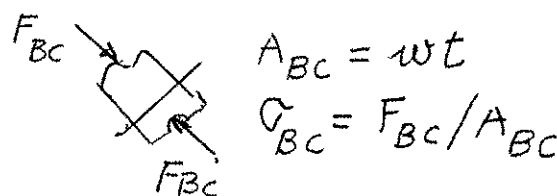
(5) BEARING STRESS AT A

$$\text{Sig Bear A} = F_{AB}/dt$$

(7) BEARING STRESS AT B IN MEMBER BC

$$\text{Sig Bear B} = F_{BC}/2dt$$

(2) AVE. STRESS IN BC



(4) PIN C

$$\tau_C = (F_{BC}/2) / (\pi d^2/4)$$

(6) BEARING STRESS AT C

$$\text{Sig Bear C} = F_{BC}/dt$$

(CONTINUED)

PROBLEM 1.C3 CONTINUED
PROGRAM OUTPUTS

 INPUT DATA FOR PARTS (a), (b), (c): $P = 20 \text{ kN}$ $w = 45 \text{ mm}$ $t = 12 \text{ mm}$

D mm	SIGAB MPa	SIGBC MPa	TAUA MPa	TAUC MPa	SIGBRGA MPa	SIGBRGC MPa	SIGBRGB MPa
12.5	77.651	-68.687	125.537	157.420	201.890	247.275	123.641
17.5	91.773	-68.687	65.578	80.320	144.216	176.622	88.311
20	100.950	-68.687	50.209	61.490	126.185	154.544	77.272 (b)
27.5	144.216	-68.687	26.560	32.524	91.773	112.395	56.201
37.5	336.497	-68.687	14.280	17.493	61.302	82.423	41.211

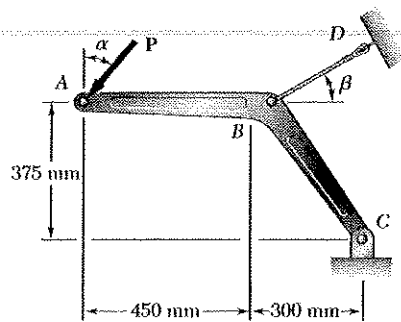
 (c) ANSWER: $17.5 \text{ mm} \leq d \leq 27.5 \text{ mm}$

 INPUT DATA FOR PART (d): $P = 20 \text{ kN}$ $w = 60 \text{ mm}$ $t = 8 \text{ mm}$

D mm	SIGAB MPa	SIGBC MPa	TAUA MPa	TAUC MPa	SIGBRA MPa	SIGBRC MPa	SIGBRB MPa
12.5	88.552	-85.857	128.537	157.420	336.497	412.128	206.064
20	105.156	-85.857	50.209	61.490	210.311	257.577	128.792
25	120.180	-85.857	31.131	39.257	168.252	206.064	103.032
31.25	146.305	-85.857	20.568	28.187	134.597	164.853	82.423
37.5	186.944	-85.857	14.280	17.493	112.168	137.376	68.688

 (d) ANSWER: $21.25 \text{ mm} \leq d \leq 31.25 \text{ mm}$

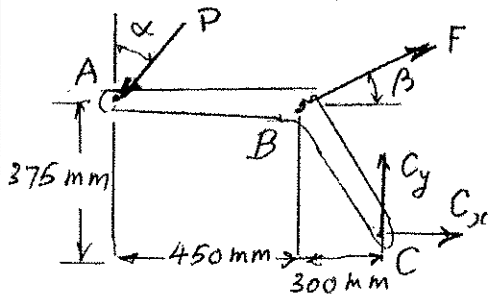
PROBLEM 1.C4



1.C4 A 16 kN force P forming an angle α with the vertical is applied as shown to member ABC , which is supported by a pin and bracket at C and by a cable BD forming an angle β with the horizontal. (a) Knowing that the ultimate load of the cable is 100 kN, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45° , using increments in α and β corresponding to 0.1 increments in $\tan \alpha$ and $\tan \beta$. (b) Check that for any given value of α the maximum value of the factor of safety is obtained for $\beta = 38.66^\circ$ and explain why. (c) Determine the smallest possible value of the factor of safety for $\beta = 38.66^\circ$, as well as the corresponding value of α , and explain the result obtained.

SOLUTION

(a) DRAW F.B. DIAGRAM OF ABC:



$$+\circlearrowleft \sum M_C = 0: (P \sin \alpha)(375 \text{ mm}) + (P \cos \alpha)(750 \text{ mm}) - (F \cos \beta)(375 \text{ mm}) - (F \sin \beta)(300 \text{ mm}) = 0$$

$$F = P \frac{15 \sin \alpha + 30 \cos \alpha}{15 \cos \beta + 12 \sin \beta}$$

$$F.S. = F_{ult} / F$$

OUTPUT FOR $P = 16 \text{ kN}$ AND $F_{ult} = 100 \text{ kN}$. ✓

VALUES OF FS
BETA

ALPHA	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

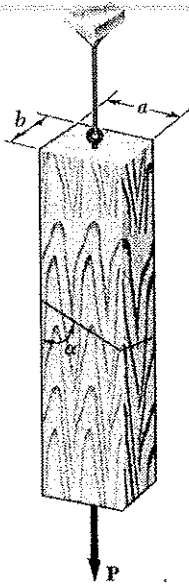
(b) When $\beta = 38.66^\circ$, $\tan \beta = 0.8$ and cable BD is perpendicular to the lever arm BC .

(c) $F.S. = 3.579$ for $\alpha = 26.6^\circ$, P is perpendicular to the lever arm AC

NOTE:

The value $F.S. = 3.579$ is the smallest of the values of $F.S.$ corresponding to $\beta = 38.66^\circ$ and the largest of those corresponding to $\alpha = 26.6^\circ$. The point $\alpha = 26.6^\circ$, $\beta = 38.66^\circ$ is a "saddle point", or "minimax" of the function $F.S.(\alpha, \beta)$.

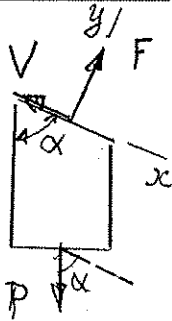
PROBLEM 1.C5



1.C5 A load P is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (a) Denoting by σ_U and τ_U , respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a , b , P , σ_U and τ_U , and for values of α from 5° to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (b) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that $\sigma_U = 1.26$ MPa and $\tau_U = 1.50$ MPa for the glue used in Prob. 1.29, and that $\sigma_U = 1.03$ MPa and $\tau_U = 1.47$ MPa for the glue used in Prob. 1.31. (c) Verify in each of these two cases that the shearing stress is maximum for $\alpha = 45^\circ$.

SOLUTION

(1) and (2)



Draw the F.B. diagram of lower member:

$$+\uparrow \sum F_x = 0: -V + P \cos \alpha = 0 \quad V = P \cos \alpha$$

$$+\rightarrow \sum F_y = 0: F - P \sin \alpha = 0 \quad F = P \sin \alpha$$

$$\text{Area} = ab / \sin \alpha$$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab) \sin^2 \alpha$$

$$\text{Shearing stress: } \tau = \frac{V}{\text{Area}} = (P/ab) \sin \alpha \cos \alpha$$

(3) F.S. for tension (normal stresses)

$$FSN = \sigma_U / \sigma$$

(4) F.S. for shear:

$$FSS = \tau_U / \tau$$

(c) OVERALL F.S.:

$$FS = \text{The smaller of } FSN \text{ and } FSS.$$

(CONTINUED)

PROBLEM 1.C5 CONTINUED

PROGRAM OUTPUTS

Problem 1.31

a = 150 mm
b = 75 mm
P = 11 kN
SIGU = 1.26 MPa
TAUU = 1.50 MPa

ALPHA	SIG (MPa)	TAU (MPa)	FSN	FSS	FS
5	.007	.085	169.644	17.669	17.669
10	.029	.167	42.736	8.971	8.971
15	.065	.244	19.237	6.136	6.136
20	.114	.314	11.016	4.773	4.773
25	.175	.375	7.215	4.005	4.005
30	.244	.423	5.155	3.543	3.543
35	.322	.459	3.917	3.265	3.265
40	.404	.481	3.119	3.116	3.116
45	.489	.489	2.577	3.068	2.577
50	.574	.481	2.196	3.116	2.196
55	.656	.459	1.920	3.265	1.920
60	.733	.423	1.718	3.543	1.718
65	.803	.375	1.569	4.005	1.569
70	.863	.314	1.459	4.773	1.459
75	.912	.244	1.381	6.136	1.381
80	.948	.167	1.329	8.971	1.329
85	.970	.085	1.298	17.669	1.298

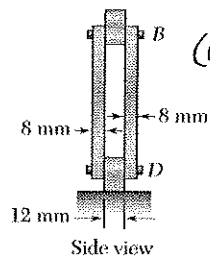
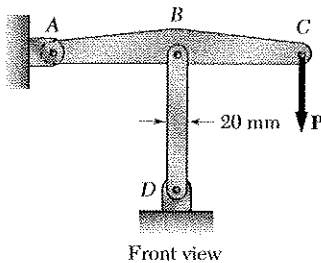
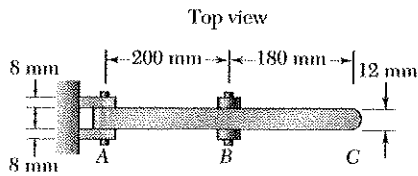
◀ (b), (c)

Problem 1.29

a = 125 mm
b = 75 mm
P = 5600 N
SIGU = 1.05 MPa
TAUU = 1.5 MPa

ALPHA	SIG (kPa)	Tau (kPa)	FSN	FSS	FS
5	4.693	56.728	211.574	26.408	26.408
25	116.69	250.243	8.998	5.986	5.986
45	326.669	321.769	3.214	4.586	3.214
60	490.000	282.905	2.143	5.295	2.143
85	648.368	55.877	1.619	26.408	1.619

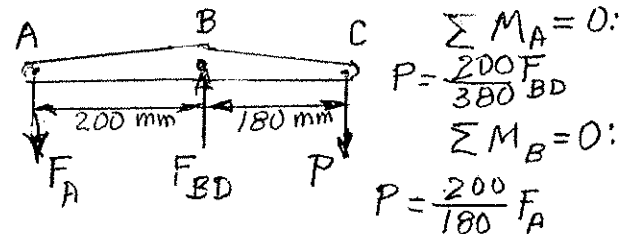
PROBLEM 1.C6



1.C6 Member ABC is supported by a pin and bracket at A and by two links, which are pin-connected to the member at B and to a fixed support at D . (a) Write a computer program to calculate the allowable load P_{all} for any given values of (1) the diameter d_1 of the pin at A , (2) the common diameter d_2 of the pins at B and D , (3) the ultimate normal stress σ_U in each of the two links, (4) the ultimate shearing stress τ_U in each of the three pins, (5) the desired overall factor of safety $F.S.$ Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at A , or the shearing stress in the pins at B and D . (b and c) Check your program by using the data of Probs. 1.55 and 1.56, respectively, and comparing the answers obtained for P_{all} with those given in the text. (d) Use your program to determine the allowable load P_{all} as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa for aluminum links, $\tau_U = 100$ MPa for steel pins, and $F.S. = 3.2$.

SOLUTION

(a) F.B. DIAGRAM OF ABC:



- (1) For given d_1 of pin A: $F_A = 2(\sigma_U / F.S.) (\pi d_1^2 / 4)$, $P_1 = \frac{200}{180} F_A$
 - (2) For given d_2 of pins B and D: $F_{BD} = 2(\sigma_U / F.S.) (\pi d_2^2 / 4)$, $P_2 = \frac{200}{380} F_{BD}$
 - (3) For ultimate stress in links BD: $F_{BD} = 2(\sigma_U / F.S.) (0.02)(0.008)$, $P_3 = \frac{200}{380} F_{BD}$
 - (4) For ult. shearing stress in pins: P_4 is the smaller of P_1 and P_2
 - (5) For desired overall F.S.: P_5 is the smaller of P_3 and P_4
- If $P_3 < P_4$, stress is critical in links
 If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in pin A
 If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in pins B and D

PROGRAM OUTPUTS

- (b) Prob. 1.53. DATA: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$
 $P_{all} = 3.72$ kN. Stress in pin A is critical $\blacktriangleleft$
- (c) Prob. 1.54 DATA: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$
 $P_{all} = 3.97$ kN. Stress in pins B and D is critical $\blacktriangleleft$
- (d) DATA: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.2$
 $P_{all} = 5.79$ kN. Stress in links is critical $\blacktriangleleft$

Chapter 2

Problem 2.1

2.1 Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod. Knowing that, with an axial load of 6000 N acting on the rod, the distance between the gage marks is 250.18 mm, determine the modulus of elasticity of the aluminum used in the rod.

$$\Delta L = L - L_0 = 250.18 - 250.00 = 0.18 \text{ mm}$$

$$\epsilon = \frac{\Delta L}{L_0} = \frac{0.18 \text{ mm}}{250 \text{ mm}} = 0.00072$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{6000}{113.097 \times 10^{-6}} = 53.052 \times 10^6 \text{ Pa}$$

$$E = \frac{\sigma}{\epsilon} = \frac{53.052 \times 10^6}{0.00072} = 73.683 \times 10^9 \text{ Pa} \quad E = 73.7 \text{ GPa} \quad \blacktriangleleft$$

Problem 2.2

2.2 A polystyrene rod of length 300 mm and diameter 12 mm is subjected to a 3-kN tensile load. Knowing that $E = 3.1 \text{ GPa}$, determine (a) the elongation of the rod, (b) the normal stress in the rod.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.012)^2 = 113.1 \times 10^{-6} \text{ m}^2$$

$$(a) \quad \delta = \frac{PL}{AE} = \frac{(3000)(0.3)}{(113.1 \times 10^{-6})(3.1 \times 10^9)} = 0.002567 \text{ m} \quad \delta = 2.6 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{P}{A} = \frac{3000}{113.1 \times 10^{-6}} = 26.525 \text{ Pa} \quad \sigma = 26.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.3

2.3 A 60-m-long steel wire is subjected to 6 kN tensile force. Knowing that $E = 200 \text{ GPa}$ and that the length of the rod increases by 48 mm, determine (a) the smallest diameter that may be selected for the wire, (b) the corresponding normal stress.

$$P = 6 \times 10^3 \text{ N}, \quad \delta = 48 \times 10^{-3} \text{ m}, \quad E = 200 \times 10^9 \text{ Pa} \quad \delta = \frac{PL}{AE}$$

$$A = \frac{PL}{E\delta} = \frac{(6 \times 10^3)(60)}{(200 \times 10^9)(48 \times 10^{-3})} = 37.5 \times 10^{-6} \text{ m}^2$$

$$(a) \quad A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(37.5 \times 10^{-6})}{\pi}} = 6.91 \times 10^{-3} \text{ m} \quad d = 6.91 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{P}{A} = \frac{6 \times 10^3}{37.5 \times 10^{-6}} = 160 \times 10^6 \text{ Pa} \quad \sigma = 160.0 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.4

2.4 A 9-m length of 6 mm-diameter steel wire is to be used in a hanger. It is noted that the wire stretches 11 mm when a tensile force P is applied. Knowing that $E = 200 \text{ GPa}$, determine (a) the magnitude of the force P , (b) the corresponding normal stress in the wire.

$$L = 9 \text{ m}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.006)^2 = 28.27 \times 10^{-6} \text{ m}^2$$

$$(a) \quad \delta = \frac{PL}{AE} \quad P = \frac{AES}{L} = \frac{(28.27 \times 10^{-6})(200 \times 10^9)(0.011)}{9} = 6.91 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad \sigma = E\epsilon = \frac{ES}{L} = \frac{(200 \times 10^9)(0.011)}{9} = 244.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.5

2.5 A cast-iron tube is used to support a compressive load. Knowing that $E = 69$ GPa and that the maximum allowable change in length is 0.025 %, determine (a) the maximum normal stress in the tube, (b) the minimum wall thickness for a load of 7.2 kN if the outside diameter of the tube is 50 mm.

$$E = 69 \text{ GPa} = 69 \times 10^9 \text{ Pa}$$

$$\epsilon = \frac{\delta}{L} = \frac{0.00025 L}{L} = 0.00025$$

$$(a) \sigma = E\epsilon = (69 \times 10^9)(0.00025) = 17.25 \times 10^6 \text{ Pa} \quad \sigma = 17.25 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{7.2 \times 10^3}{17.25 \times 10^6} = 417.39 \times 10^{-6} \text{ m}^2 = 417.39 \text{ mm}^2$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) \quad d_i^2 = d_o^2 - \frac{4A}{\pi} = 50^2 - \frac{(4)(417.39)}{\pi} = 1968.56 \text{ mm}^2$$

$$d_i = 44.368 \text{ mm} \quad t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(50 - 44.368)$$

$$t = 2.82 \text{ mm} \quad \blacktriangleleft$$

Problem 2.6

2.6 A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that $E = 105$ GPa and that the maximum allowable normal stress is 180 MPa, determine (a) the smallest diameter that can be selected for the rod, (b) the corresponding maximum length of the rod.

$$(a) \sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{4 \times 10^3}{180 \times 10^6} = 22.222 \times 10^{-6} \text{ m}^2$$

$$A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(22.222 \times 10^{-6})}{\pi}} = 5.32 \times 10^{-3} \text{ m}$$

$$d = 5.32 \text{ mm} \quad \blacktriangleleft$$

$$(b) \delta = \frac{PL}{AE} \quad L = \frac{AES}{P} = \frac{(22.222 \times 10^{-6})(105 \times 10^9)(3 \times 10^{-3})}{4 \times 10^3}$$

$$L = 1.750 \text{ m} \quad \blacktriangleleft$$

Problem 2.7

2.7 Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod with $E = 73$ GPa and an ultimate strength of 140 MPa. Knowing that the distance between the gage marks is 250.28 mm after a load is applied, determine (a) the stress in the rod, (b) the factor of safety.

$$E = 73 \times 10^9 \text{ Pa}$$

$$\delta = 250.28 - 250.00 = 0.28 \text{ mm} = 0.28 \times 10^{-3} \text{ m}$$

$$L = 250 \text{ mm} = 250 \times 10^{-3} \text{ m}$$

$$(a) \sigma = E\epsilon = \frac{ES}{L} = \frac{(73 \times 10^9)(0.28 \times 10^{-3})}{250 \times 10^{-3}} = 81.76 \times 10^6 \text{ Pa} \quad 81.8 \text{ MPa} \quad \blacktriangleleft$$

$$(b) F.S. = \frac{\sigma_u}{\sigma} = \frac{140 \times 10^6}{81.76 \times 10^6} = 1.712 \quad \blacktriangleleft$$

Problem 2.8

2.8 An 80-m-long wire of 5-mm diameter is made of a steel with $E = 200$ GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (a) the largest allowable tension in the wire, (b) the corresponding elongation of the wire.

$$(a) \quad \sigma_u = 400 \times 10^6 \text{ Pa} \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (5)^2 = 19.635 \text{ mm}^2 = 19.635 \times 10^{-6} \text{ m}^2$$

$$P_u = \sigma_u A = (400 \times 10^6)(19.635 \times 10^{-6}) = 7854 \text{ N}$$

$$P_{all} = \frac{P_u}{F.S.} = \frac{7854}{3.2} = 2454 \text{ N} \quad P_{all} = 2.45 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad \delta = \frac{PL}{AE} = \frac{(2454)(80)}{(19.635 \times 10^{-6})(200 \times 10^9)} = 50.0 \times 10^{-3} \text{ m}$$

$$\delta = 50.0 \text{ mm} \quad \blacktriangleleft$$

Problem 2.9

2.9 A block of 250-mm length and 50×40 mm cross section is to support a centric compressive load P . The material to be used is a bronze for which $E = 95$ GPa. Determine the largest load which can be applied, knowing that the normal stress must not exceed 80 MPa and that the decrease in length of the block should be at most 0.12% of its original length.

$$A = (50)(40) = 2000 \text{ mm}^2 \\ = 2 \times 10^{-3} \text{ m}^2$$

$$\sigma_u = 80 \text{ MPa} = 80 \times 10^6 \text{ Pa} \quad E = 95 \times 10^9 \text{ Pa}$$

Considering allowable stress:

$$\sigma = \frac{P}{A} \quad P = A\sigma = (2 \times 10^{-3})(80 \times 10^6) = 160 \times 10^3 \text{ N}$$

Considering allowable deformation:

$$\delta = \frac{PL}{AE} \quad P = AE\left(\frac{\delta}{L}\right) = (2 \times 10^{-3})(95 \times 10^9)(0.0012) = 228 \times 10^3 \text{ N}$$

$$\text{The smaller value governs.} \quad P = 160 \times 10^3 \text{ N} \quad P = 160.0 \text{ kN} \quad \blacktriangleleft$$

Problem 2.10

2.10 A 1.5-m-long aluminum rod must not stretch more than 1 mm and the normal stress must not exceed 40 MPa when the rod is subjected to a 3-kN axial load. Knowing that $E = 70$ GPa, determine the required diameter of the rod.

$$L = 1.5 \text{ m}$$

$$\delta = 1 \times 10^{-3} \text{ m}, \quad \sigma = 40 \times 10^6 \text{ Pa}, \quad E = 70 \times 10^9 \text{ Pa}, \quad P = 3 \times 10^3 \text{ N}$$

$$\text{Stress: } \sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{3 \times 10^3}{40 \times 10^6} = 75 \times 10^{-6} \text{ m}^2 = 75 \text{ mm}^2$$

$$\text{Deformation: } \delta = \frac{PL}{AE}$$

$$A = \frac{PL}{E\delta} = \frac{(3 \times 10^3)(1.5)}{(70 \times 10^9)(1 \times 10^{-3})} = 64.29 \times 10^{-6} \text{ m}^2 = 64.29 \text{ mm}^2$$

Larger value of A governs.

$$A = 75 \text{ mm}^2$$

$$A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(75)}{\pi}} \quad d = 9.77 \text{ mm} \quad \blacktriangleleft$$

Problem 2.11

2.11 An aluminum control rod must stretch 2 mm when a 2-kN tensile load is applied to it. Knowing that $\sigma_{all} = 154 \text{ MPa}$ and $E = 70 \text{ GPa}$, determine the smallest diameter and shortest length which may be selected for the rod.

$$P = 2 \text{ kN}, \quad \delta = 2 \text{ mm}$$

$$\sigma_{all} = 154 \text{ MPa}$$

$$\sigma = \frac{P}{A} \leq \sigma_{all}$$

$$A \geq \frac{P}{\sigma_{all}} = \frac{2000}{154} = 12.987 \text{ mm}^2$$

$$A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(12.987)}{\pi}}$$

$$d_{min} = 4.07 \text{ mm}$$

$$\sigma = E \epsilon = \frac{E \delta}{L} \leq \sigma_{all}$$

$$L \geq \frac{E \delta}{\sigma_{all}} = \frac{(70 \times 10^9)(0.002)}{154 \times 10^6} = 0.909 \text{ m}$$

$$L_{min} = 0.91 \text{ m}$$

Problem 2.12

2.12 A square aluminum bar should not stretch more than 1.4 mm when it is subjected to a tensile load. Knowing that $E = 70 \text{ GPa}$ and that the allowable tensile strength is 120 MPa, determine (a) the maximum allowable length of the pipe, (b) the required dimensions of the cross-section if the tensile load is 28 kN.

$$\sigma = 120 \times 10^6 \text{ Pa}$$

$$E = 70 \times 10^9 \text{ Pa}$$

$$\delta = 1.4 \times 10^{-3} \text{ m}$$

$$(a) \quad \delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$L = \frac{E \delta}{\sigma} = \frac{(70 \times 10^9)(1.4 \times 10^{-3})}{120 \times 10^6} = 0.817 \text{ m}$$

$$L = 817 \text{ mm}$$

$$(b) \quad \sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{28 \times 10^3}{120 \times 10^6} = 233.333 \times 10^{-6} \text{ m}^2 = 233.333 \text{ mm}^2$$

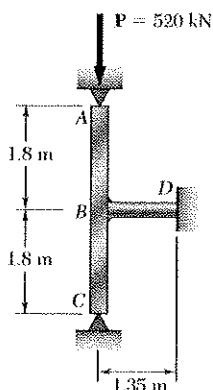
$$A = a^2$$

$$a = \sqrt{A} = \sqrt{233.333}$$

$$a = 15.28 \text{ mm}$$

Problem 2.13

2.13 Rod BD is made of steel ($E = 200 \text{ GPa}$) and is used to brace the axially compressed member ABC. The maximum force that can be developed in member BD is $0.02P$. If the stress must not exceed 126 MPa and the maximum change in length of BD must not exceed 0.001 times the length of ABC, determine the smallest-diameter rod that can be used for member BD.



$$F_{BD} = 0.02 P = (0.02)(520) = 10.4 \text{ kN}$$

$$\text{Considering stress: } \sigma = 126 \text{ MPa}$$

$$\sigma = \frac{F_{BD}}{A} \therefore A = \frac{F_{BD}}{\sigma} = \frac{10400}{126} = 82.54 \text{ mm}^2$$

$$\text{Considering deformation: } \delta = (0.001)(3600) = 3.6 \text{ mm}$$

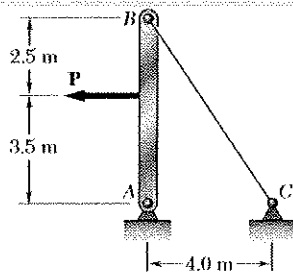
$$\delta = \frac{F_{BD} L_{BD}}{AE} \therefore A = \frac{F_{BD} L_{BD}}{E \delta} = \frac{(10.4 \times 10^3)(1350)}{(200 \times 10^9)(3.6)} = 19.5 \text{ mm}^2$$

$$\text{Larger area governs. } A = 19.5 \text{ mm}^2$$

$$A = \frac{\pi}{4} d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(19.5)}{\pi}}$$

$$d = 5 \text{ mm}$$

Problem 2.14



2.14 The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load P that can be applied as shown.

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar AB as a free body.

$$\sum M_A = 0 \quad 3.5P - (6)\left(\frac{4}{7.2111}\right)F_{BC} = 0$$

$$P = 0.9509 F_{BC}$$

Considering allowable stress: $\sigma = 190 \times 10^6 \text{ Pa}$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

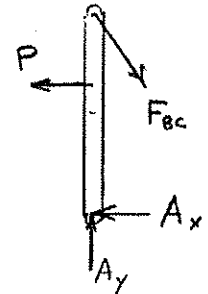
$$\sigma = \frac{F_{BC}}{A} \therefore F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

Considering allowable elongation: $\delta = 6 \times 10^{-3} \text{ m}$

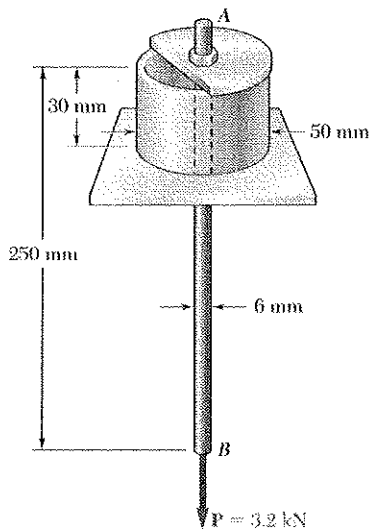
$$\delta = \frac{F_{BC} L_{BC}}{AE} \therefore F_{BC} = \frac{AES}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3 \text{ N}$

$$P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N} \quad P = 1.988 \text{ kN} \blacktriangleleft$$



Problem 2.15



2.15 A 3-mm-thick hollow polystyrene cylinder ($E = 3$ GPa) and a rigid circular plate (only part of which is shown) are used to support a 250-mm-long steel rod AB ($E = 200$ GPa) of 6 mm diameter. If an 3.2 kN load P is applied at B , determine (a) the elongation of rod AB , (b) the deflection of point B , (c) the average normal stress in rod AB .

$$\text{Rod } AB: P_{AB} = 3200 \text{ N} \quad L_{AB} = 0.25 \text{ m} \quad d = 6 \text{ mm}$$

$$A_{AB} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.006)^2 = 28.27 \times 10^{-6} \text{ m}^2$$

$$(a) \quad \delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(3200)(0.25)}{(200 \times 10^9)(28.27 \times 10^{-6})} = 0.1415 \times 10^{-3} \text{ m} \blacktriangleleft$$

$$\text{Hollow cylinder: } d_o = 0.05 \text{ m} \quad d_i = (0.05 - (0.003)(2)) = 0.044 \text{ m}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.05^2 - 0.044^2) = 0.000443 \text{ m}^2$$

$$L = 0.03 \text{ m}, \quad P = 3200 \text{ N}$$

$$\delta_{cyl} = \frac{PL}{EA} = \frac{(3200)(0.03)}{(3 \times 10^9)(0.000443)} = 0.0722 \times 10^{-3} \text{ m}$$

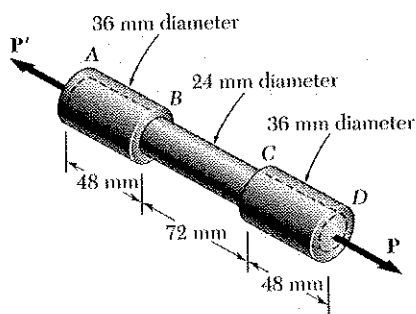
(b) Deflection of point B

$$\delta_B = \delta_{AB} + \delta_{cyl} = 0.214 \times 10^{-3} \text{ m} \blacktriangleleft$$

(c) Stress in rod AB

$$\sigma = \frac{P}{A_{AB}} = \frac{3200}{28.27 \times 10^{-6}} = 113.2 \text{ MPa} \blacktriangleleft$$

Problem 2.16



2.16 The specimen shown is made from a 24-mm-diameter cylindrical steel rod with two 36-mm-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 200$ GPa, determine (a) the load P so that the total deformation is 0.04 mm, (b) the corresponding deformation of the central portion BC.

$$(a) \quad \delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \sum \frac{L_i}{A_i}$$

$$P = E \delta \left(\sum \frac{L_i}{A_i} \right)^{-1} \quad A_i = \frac{\pi}{4} d_i^2$$

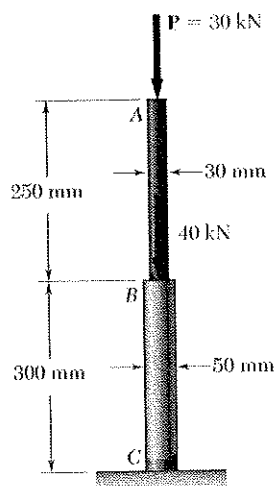
	L, mm	d, mm	A, mm^2	$L/A, \text{mm}^{-1}$
AB	48	36	1017.9	0.04716
BC	72	24	452.4	0.15915
CD	48	36	1017.9	0.04716
				0.25347 ← sum

$$P = (200 \times 10^3) (0.04) (0.25347)^{-1} = 31.562 \times 10^3 \text{ N}$$

$$P = 31.6 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad \delta_{BC} = \frac{P L_{BC}}{A_{BC} E} = \frac{P}{E} \frac{L_{BC}}{A_{BC}} = \frac{31.562 \times 10^3}{200 \times 10^3} (0.15915) \quad \delta = 0.025 \text{ mm} \quad \blacktriangleleft$$

Problem 2.17



2.17 Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel ($E = 200$ GPa) and rod BC of brass ($E = 105$ GPa). Determine (a) the total deformation of the composite rod ABC, (b) the deflection of point B.

$$\text{Rod AB:} \quad F_{AB} = -P = -30 \times 10^3 \text{ N}$$

$$L_{AB} = 0.250 \text{ m}$$

$$E_{AB} = 200 \times 10^9 \text{ Pa}$$

$$A_{AB} = \frac{\pi}{4} (30)^2 = 706.85 \text{ mm}^2 = 706.85 \times 10^{-6} \text{ m}^2$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E_{AB} A_{AB}} = - \frac{(30 \times 10^3) (0.250)}{(200 \times 10^9) (706.85 \times 10^{-6})} = -53.052 \times 10^{-6} \text{ m}$$

$$\text{Rod BC:} \quad F_{BC} = 30 + 40 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$L_{BC} = 0.300 \text{ m}$$

$$E_{BC} = 105 \times 10^9 \text{ Pa}$$

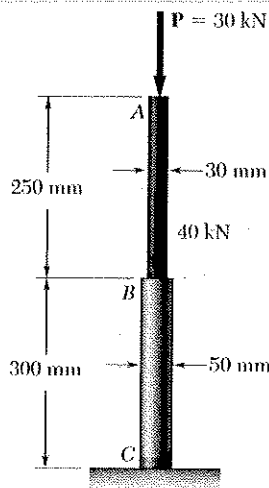
$$A_{BC} = \frac{\pi}{4} (50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E_{BC} A_{BC}} = - \frac{(70 \times 10^3) (0.300)}{(105 \times 10^9) (1.9635 \times 10^{-3})} = -101.859 \times 10^{-6} \text{ m}$$

$$(a) \quad \text{Total deformation:} \quad \delta_{\text{tot}} = \delta_{AB} + \delta_{BC} = -154.9 \times 10^{-6} \text{ m} = -0.1549 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \text{Deflection of point B:} \quad \delta_B = \delta_{BC} \quad \delta_B = 0.1019 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 2.18



2.18 For the composite rod of Prob. 2.17, determine (a) the load P for which the total deformation of the rod is -0.2 mm, (b) the corresponding deflection of point B.

2.17 Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel ($E = 200$ GPa) and rod BC of brass ($E = 105$ GPa). Determine (a) the total deformation of the composite rod ABC, (b) the deflection of point B.

Rod AB: $F_{AB} = -P$

$$L_{AB} = 0.250 \text{ m}$$

$$E_{AB} = 200 \times 10^9 \text{ Pa}$$

$$A_{AB} = \frac{\pi}{4} (30)^2 = 706.85 \text{ mm}^2 = 706.85 \times 10^{-6} \text{ m}^2$$

$$\begin{aligned} S_{AB} &= \frac{F_{AB} L_{AB}}{E_{AB} A_{AB}} = - \frac{P (0.250)}{(200 \times 10^9) (706.85 \times 10^{-6})} \\ &= -1.76841 P \end{aligned}$$

Rod BC: $F_{BC} = -(P + 40 \times 10^3)$

$$L_{BC} = 0.300 \text{ m}$$

$$E_{BC} = 105 \times 10^9 \text{ Pa}$$

$$A_{BC} = \frac{\pi}{4} (50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\begin{aligned} S_{BC} &= \frac{F_{BC} L_{BC}}{E_{BC} A_{BC}} = - \frac{(P + 40 \times 10^3) (0.300)}{(105 \times 10^9) (1.9635 \times 10^{-3})} \\ &= -1.45513 \times 10^{-9} P - 58.205 \times 10^{-6} \end{aligned} \quad (1)$$

(a) Total deformation: $S_{tot} = S_{AB} + S_{BC}$

$$-0.2 \times 10^{-3} = -1.76841 \times 10^{-9} P - 1.45513 \times 10^{-9} P - 58.205 \times 10^{-6}$$

$$P = 43.987 \times 10^3 \text{ N}$$

$$P = 44.0 \text{ kN} \quad \blacktriangleleft$$

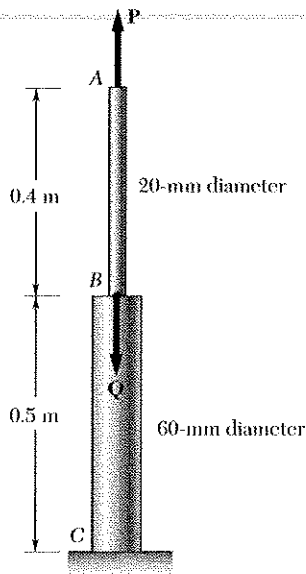
From (1), $S_{BC} = -(1.45513 \times 10^{-9}) (43.987 \times 10^3) - 58.205 \times 10^{-6}$

$$= -122.21 \times 10^{-6} \text{ m}$$

(b) Deflection of point B. $S_B = S_{BC}$

$$S_B = 0.1222 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 2.19



2.19 Both portions of the rod ABC are made of an aluminum for which $E = 70 \text{ GPa}$. Knowing that the magnitude of P is 4 kN , determine (a) the value of Q so that the deflection at A is zero, (b) the corresponding deflection of B .

$$(a) \quad A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (0.060)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

Force in member AB is P tension.

$$\text{Elongation. } \delta_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{(4 \times 10^3)(0.4)}{(70 \times 10^9)(314.16 \times 10^{-6})} = 72.756 \times 10^{-6} \text{ m}$$

Force in member BC is $Q - P$ compression.

$$\text{Shortening. } \delta_{BC} = \frac{(Q - P) L_{BC}}{E A_{BC}} = \frac{(Q - P)(0.5)}{(70 \times 10^9)(2.8274 \times 10^{-3})} = 2.5263 \times 10^{-9} (Q - P)$$

For zero deflection at A , $\delta_{BC} = \delta_{AB}$

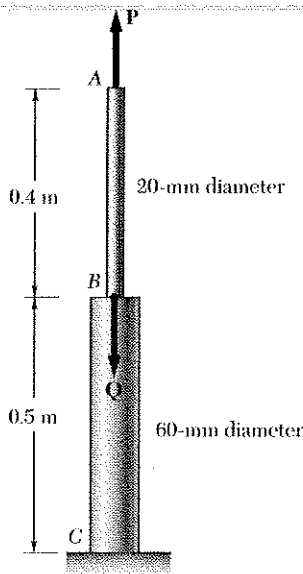
$$2.5263 \times 10^{-9} (Q - P) = 72.756 \times 10^{-6} \quad \therefore \quad Q - P = 28.8 \times 10^3 \text{ N}$$

$$Q = 28.8 \times 10^3 + 4 \times 10^3 = 32.8 \times 10^3 \text{ N} = 32.8 \text{ kN}$$

$$(b) \quad \delta_{AB} = \delta_{BC} - \delta_B = 72.756 \times 10^{-6} \text{ m} = 0.0728 \text{ mm} \downarrow$$

Problem 2.20

2.20 The rod ABC is made of an aluminum for which $E = 70$ GPa. Knowing that $P = 6$ kN and $Q = 42$ kN, determine the deflection of (a) point A , (b) point B :



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (0.060)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

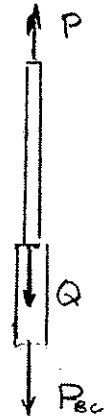
$$P_{AB} = P = 6 \times 10^3 \text{ N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3 \text{ N}$$

$$L_{AB} = 0.4 \text{ m} \quad L_{BC} = 0.5 \text{ m}$$

$$\begin{aligned} \delta_{AB} &= \frac{P_{AB} L_{AB}}{A_{AB} E} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)} \\ &= 109.135 \times 10^{-6} \text{ m} \end{aligned}$$

$$\begin{aligned} \delta_{BC} &= \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)} \\ &= -90.947 \times 10^{-6} \text{ m} \end{aligned}$$

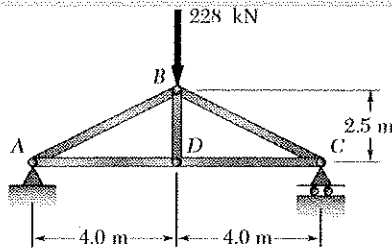


$$\begin{aligned} (a) \quad \delta_A &= \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6} \text{ m} = 18.19 \times 10^{-6} \text{ m} \\ &= 0.01819 \text{ mm} \uparrow \end{aligned}$$

$$(b) \quad \delta_B = \delta_{BC} = -90.9 \times 10^{-6} \text{ m} = -0.0909 \text{ mm} \text{ or } 0.0919 \text{ mm} \downarrow$$

Problem 2.21

2.21 For the steel truss ($E = 200 \text{ GPa}$) and loading shown, determine the deformations of the members AB and AD , knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2 , respectively.



Statics: Reactions are 114 kN upward at A & C.

Member BD is a zero force member.

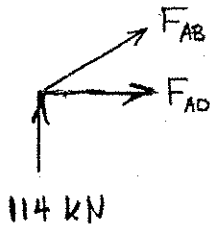
$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717 \text{ m}$$

Use joint A as a free body. $\uparrow \sum F_y = 0$: $114 + \frac{2.5}{4.717} F_{AB} = 0$

$$F_{AB} = -215.10 \text{ kN}$$

$$\pm \sum F_x = 0: F_{AD} + \frac{4}{4.717} F_{AB} = 0$$

$$F_{AD} = \frac{(4)(-215.10)}{4.717} = -182.4 \text{ kN}$$



Member AB:
$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-215.10 \times 10^3)(4.717)}{(200 \times 10^9)(2400 \times 10^{-6})}$$

$$= -2.11 \times 10^{-3} \text{ m}$$

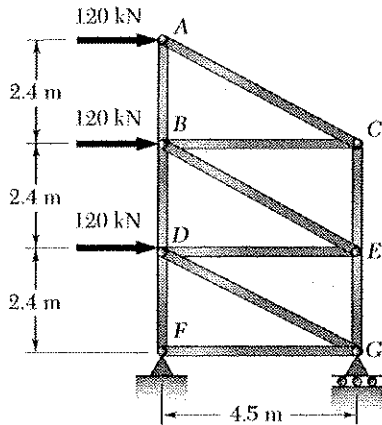
$$\delta_{AB} = 2.11 \text{ mm} \leftarrow$$

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(-182.4 \times 10^3)(4.0)}{(200 \times 10^9)(1800 \times 10^{-6})} = 2.03 \times 10^{-3} \text{ m}$$

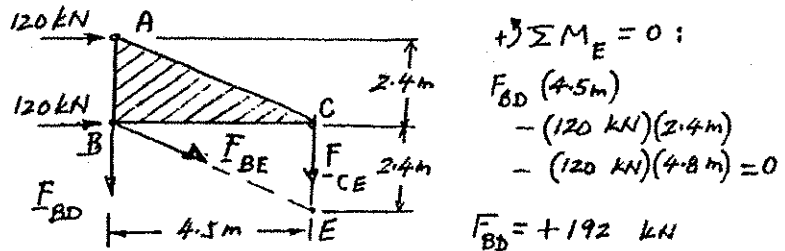
$$\delta_{AD} = 2.03 \text{ mm} \leftarrow$$

Problem 2.22

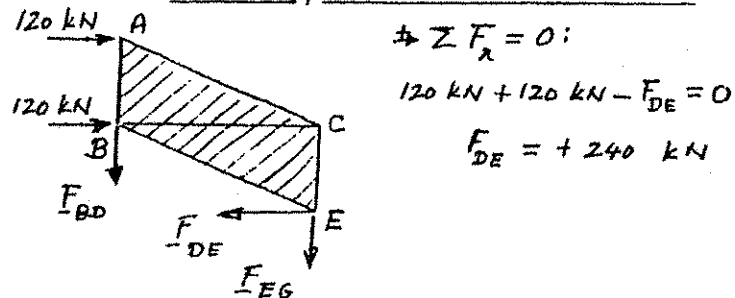
2.22 For the steel truss ($E = 200 \text{ GPa}$) and loading shown, determine the deformations of the members BD and DE , knowing that their cross-sectional areas are 1250 mm^2 and 1875 mm^2 , respectively.



Free Body: Portion ABC of truss



Free Body: Portion ABEC of truss



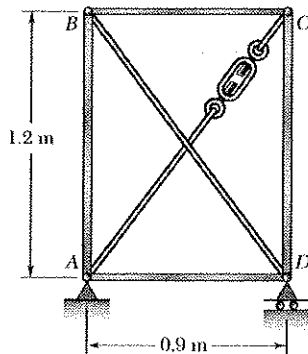
$$\delta_{BD} = \frac{PL}{AE} = \frac{(+192 \times 10^3 \text{ N})(2400 \text{ mm})}{(1250 \text{ mm}^2)(200000 \text{ MPa})}$$

$$\delta_{BD} = +1.84 \text{ mm} \quad \rightarrow$$

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+240 \times 10^3 \text{ N})(4500 \text{ mm})}{(1875 \text{ mm}^2)(200000 \text{ MPa})}$$

$$\delta_{DE} = +2.88 \text{ mm} \quad \rightarrow$$

Problem 2.23

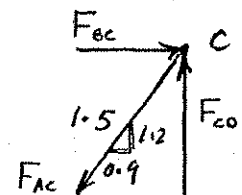


2.23 Members AB and CD are 30-mm-diameter steel rods, and members BC and AD are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 200$ GPa, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 1.0 mm.

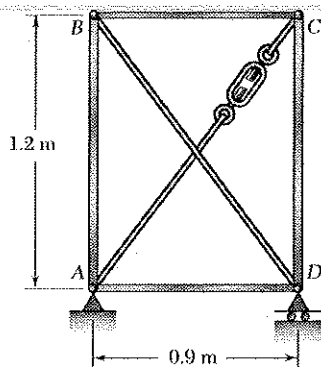
$$\begin{aligned} \delta_{AB} &= \delta_{CD} = 1 \text{ mm} & h &= 1.2 \text{ m} & & = L_{CD} \\ A_{CD} &= \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.022)^2 = 3.801 \times 10^{-4} \text{ m}^2 \\ \delta_{CD} &= \frac{F_{CD} L_{CD}}{E A_{CD}} \\ F_{CD} &= \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(200 \times 10^9)(3.801 \times 10^{-4})(0.001)}{1.2} \\ &= 117.8 \text{ kN} \end{aligned}$$

Use joint C as a free body

$$\begin{aligned} +\uparrow \sum F_y &= 0 : F_{CD} - \frac{1.2}{1.5} F_{AC} = 0 \therefore F_{AC} = \frac{1.5}{1.2} F_{CD} \\ F_{AC} &= \frac{1.5}{1.2} (117.8) = 147.3 \text{ kN} \end{aligned}$$



Problem 2.24



2.24 For the structure in Prob. 2.23, determine (a) the distance h so that the deformations in members AB , BC , CD , and AD are equal to 1 mm, (b) the corresponding tension in member AC .

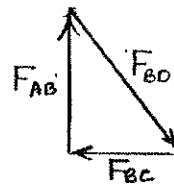
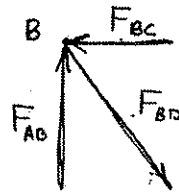
2.23 Members AB and CD are 30-mm-diameter steel rods, and members BC and AD are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 200$ GPa and $h = 1.2$ m, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 1 mm.

(a) Statics: Use joint B as a free body

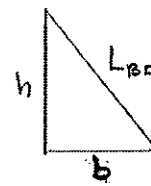
From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BD}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$



Force Triangle



Geometry

For equal deformations

$$\delta_{AB} = \delta_{BC} \therefore \frac{F_{AB} h}{E A_{AB}} = \frac{F_{BC} b}{E A_{BC}} \therefore F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

Equating expressions for F_{AB}

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC} \quad \frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{30}{22} = \frac{15}{11} \quad b = 0.9 \text{ m}$$

$$h = \frac{15}{11} b = \frac{15}{11} (0.9) = 1.227 \text{ m}$$

(b) Setting $\delta_{AB} = \delta_{BC} = 1 \text{ mm}$

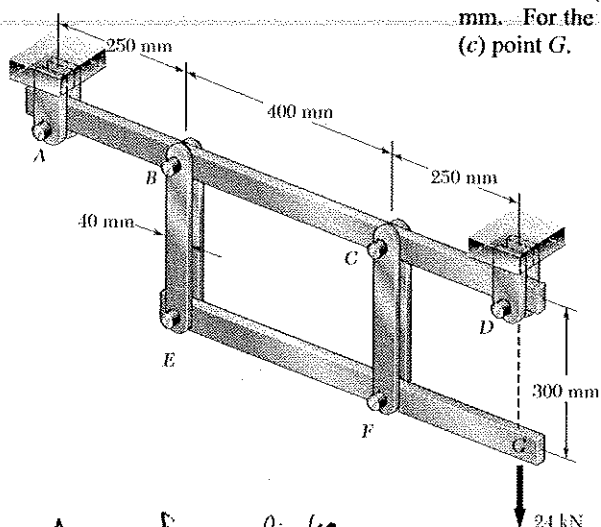
$$\delta_{BC} = \frac{F_{BC} b}{E A_{BC}} \therefore F_{BC} = \frac{E A_{BC} \delta_{BC}}{b} = \frac{(200 \times 10^9) \frac{\pi}{4} (0.022)^2 (0.001)}{0.9} = 84.5 \text{ kN}$$

$$F_{AB} = \frac{h}{b} F_{BC} = \frac{15}{11} (84.5) = 115.2 \text{ kN}$$

From the force triangle

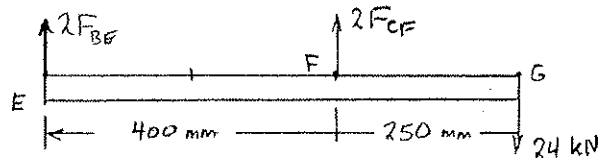
$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 142.9 \text{ kN}$$

Problem 2.25



2.25 Each of the four vertical links connecting the two horizontal members is made of aluminum ($E = 70 \text{ GPa}$) and has a uniform rectangular cross section of $10 \times 40 \text{ mm}$. For the loading shown, determine the deflection of (a) point E, (b) point F, (c) point G.

Statics. Free body EFG.



$$+\circlearrowleft \sum M_F = 0 :$$

$$-(400)(2F_{BE}) - (250)(24) = 0$$

$$F_{BE} = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

$$+\circlearrowleft \sum M_E = 0 :$$

$$(400)(2F_{CF}) - (650)(24) = 0$$

$$F_{CF} = 19.5 \text{ kN} = 19.5 \times 10^3 \text{ N}$$

Area of one link:

$$A = (10)(40) = 400 \text{ mm}^2 \\ = 400 \times 10^{-6} \text{ m}^2$$

$$\text{Length: } L = 300 \text{ mm} = 0.300 \text{ m}$$

Deformations, $\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = -80.357 \times 10^{-6} \text{ m}$

$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = 208.93 \times 10^{-6} \text{ m}$$

(a) Deflection of point E, $\delta_E = |\delta_{BE}|$

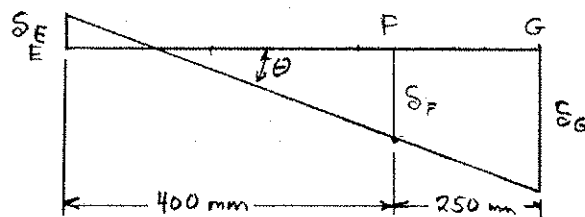
$$\delta_E = 80.4 \mu\text{m} \uparrow$$

(b) Deflection of point F, $\delta_F = \delta_{CF}$

$$\delta_F = 209 \mu\text{m} \downarrow$$

Geometry change.

Let θ be the small change in slope angle.



$$\theta = \frac{\delta_E + \delta_F}{L_{EG}} = \frac{80.357 \times 10^{-6} + 208.93 \times 10^{-6}}{0.400} = 723.22 \times 10^{-6} \text{ radians}$$

(c) Deflection of point G, $\delta_G = \delta_F + L_{FG} \theta$

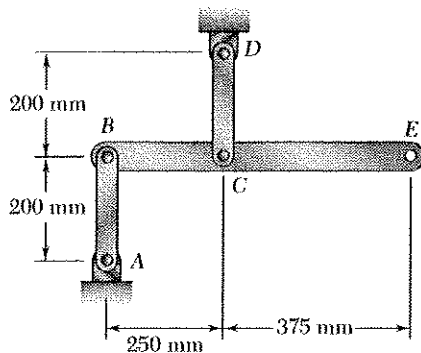
$$\delta_G = \delta_F + L_{FG} \theta = 208.93 \times 10^{-6} + (0.250)(723.22 \times 10^{-6})$$

$$= 389.73 \times 10^{-6} \text{ m}$$

$$\delta_G = 390 \mu\text{m} \downarrow$$

Problem 2.26

2.26 Each of the links AB and CD is made of steel ($E = 200 \text{ GPa}$) and has a uniform rectangular cross section of $6 \times 24 \text{ mm}$. Determine the largest load which can be suspended from point E if the deflection of E is not to exceed 0.25 mm .



Area of link:

$$A = (6)(24) = 144 \text{ mm}^2$$

Length: 200 mm

$L =$

Deformations.

$$\delta_{AB} = \frac{F_{AB} L}{EA} = \frac{(1.5P)(0.2)}{(200 \times 10^9)(144 \times 10^{-6})} = 1.0417 \times 10^{-8} P$$

$$\delta_{CD} = \frac{F_{CD} L}{EA} = \frac{(2.5P)(0.2)}{(200 \times 10^9)(144 \times 10^{-6})} = 1.7361 \times 10^{-8} P$$

Deflections.

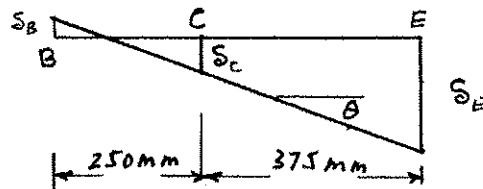
Point B. $\delta_B = \delta_{AB}$

$$\delta_B = 1.0417 \times 10^{-8} P \uparrow$$

Point C. $\delta_C = \delta_{CD}$

$$\delta_C = 1.7361 \times 10^{-8} P \downarrow$$

Geometry change.



$$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{1.0417 \times 10^{-8} P + 1.7361 \times 10^{-8} P}{0.25} = 0.11111 \times 10^{-6} P$$

$$\delta_E = \delta_C + L_{CE} \theta = 1.7361 \times 10^{-8} P + (0.375)(0.11111 \times 10^{-6} P) = 5.9027 \times 10^{-8} P \downarrow$$

Limiting value of δ_E : $\delta_E = 0.25 \times 10^{-3} \text{ m}$

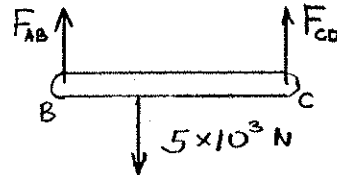
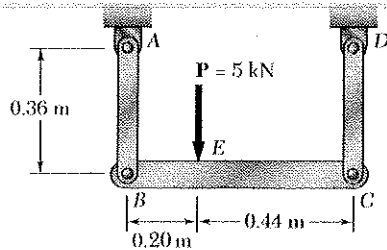
Limiting value of P: $5.9027 \times 10^{-8} P = 0.25 \times 10^{-3}$

$$P = 4235 \text{ N}$$

$$P = 4.24 \text{ kN} \quad \blacktriangleleft$$

Problem 2.27

2.27 Each of the links AB and CD is made of aluminum ($E = 75 \text{ GPa}$) and has a cross-sectional area of 125 mm^2 . Knowing that they support the rigid member BC , determine the deflection of point E .



Use member BC as a free body.

$$\sum M_C = 0: -(0.64) F_{AB} + (0.44)(5 \times 10^3) = 0$$

$$F_{AB} = 3.4375 \times 10^3 \text{ N}$$

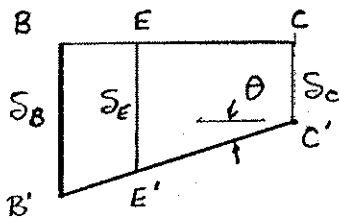
$$\sum M_B = 0: (0.64) F_{CD} - (0.20)(5 \times 10^3) = 0$$

$$F_{CD} = 1.5625 \times 10^3 \text{ N}$$

For links AB and CD , $A = 125 \text{ mm}^2 = 125 \times 10^{-6} \text{ m}^2$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{EA} = \frac{(3.4375 \times 10^3)(0.36)}{(75 \times 10^9)(125 \times 10^{-6})} = 132.00 \times 10^{-6} \text{ m} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{EA} = \frac{(1.5625 \times 10^3)(0.36)}{(75 \times 10^9)(125 \times 10^{-6})} = 60.00 \times 10^{-6} \text{ m} = \delta_C$$



Deformation diagram.

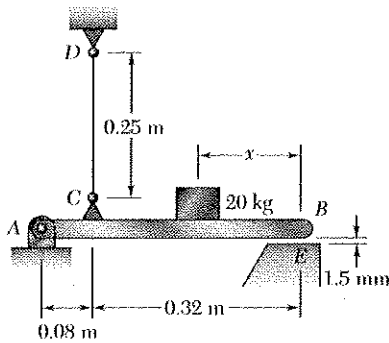
$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{BC}} = \frac{72.00 \times 10^{-6}}{0.64} = 112.5 \times 10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + l_{EC} \theta$$

$$= 60.00 \times 10^{-6} + (0.44)(112.5 \times 10^{-6})$$

$$= 109.5 \times 10^{-6} \text{ m} \quad \delta_E = 0.1095 \text{ mm} \downarrow$$

Problem 2.28



2.28 The length of the 2-mm-diameter steel wire CD has been adjusted so that with no load applied, a gap of 1.5 mm exists between the end B of the rigid beam ACB and a contact point E . Knowing that $E = 200$ GPa, determine where a 20-kg block should be placed on the beam in order to cause contact between B and E .

Rigid rod ACE rotates through angle θ to close gap.

$$\theta = \frac{1.5 \times 10^{-3}}{0.40} = 3.75 \times 10^{-3} \text{ rad}$$

Point C moves downward.

$$\delta_c = 0.08 \theta = (0.08)(3.75 \times 10^{-3}) = 300 \times 10^{-6} \text{ m}$$

$$\delta_{co} = \delta_c = 300 \times 10^{-6} \text{ m}$$

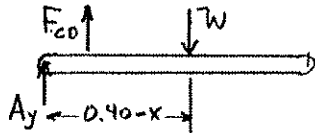
$$A_{co} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (2)^2 = 3.1416 \text{ mm}^2 = 3.1416 \times 10^{-6} \text{ m}^2$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$\delta_{cd} = \frac{F_{co} L_{co}}{E A_{co}}$$

$$F_{co} = \frac{E A_{co} \delta_{co}}{L_{co}} = \frac{(200 \times 10^9)(3.1416 \times 10^{-6})(300 \times 10^{-6})}{0.25} = 753.98 \text{ N}$$

Use beam ACB as a free body. $W = mg = (20)(9.81) = 196.2 \text{ N}$



$$+\circlearrowleft \sum M_A = 0: 0.08 F_{co} - (0.40 - x) W = 0$$

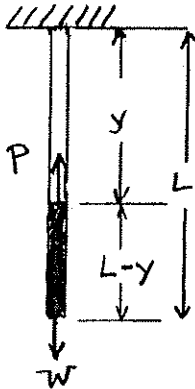
$$0.40 - x = \frac{(0.08)(753.98)}{196.2} = 0.30743 \text{ m}$$

$$x = 0.0926 \text{ m} = 92.6 \text{ mm}$$

For contact, $x < 92.6 \text{ mm}$

Problem 2.29

2.29 A homogeneous cable of length L and uniform cross section is suspended from one end. (a) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (b) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.



(a) For element at point identified by coordinate y

P = weight of portion below the point

$$= \rho g A (L - y)$$

$$dS = \frac{P dy}{EA} = \frac{\rho g A (L - y) dy}{EA} = \frac{\rho g (L - y)}{E} dy$$

$$S = \int_0^L \frac{\rho g (L - y)}{E} dy = \frac{\rho g}{E} \left(Ly - \frac{1}{2} y^2 \right) \Big|_0^L$$

$$= \frac{\rho g}{E} \left(L^2 - \frac{L^2}{2} \right)$$

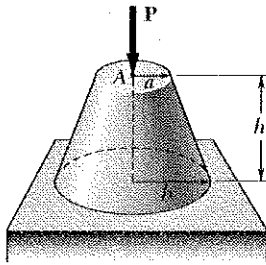
$$S = \frac{1}{2} \frac{\rho g L^2}{E} \quad \blacktriangle$$

(b) Total weight. $W = \rho g A L$

$$F = \frac{EAS}{L} = \frac{EA}{L} \cdot \frac{1}{2} \frac{\rho g L^2}{E} = \frac{1}{2} \rho g A L$$

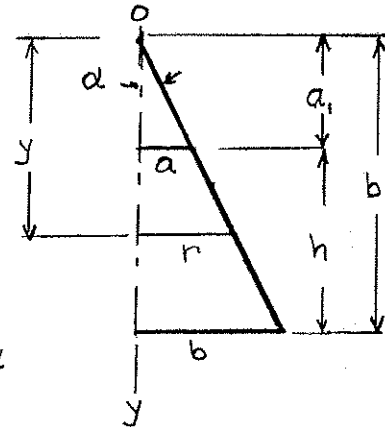
$$F = \frac{1}{2} W \quad \blacktriangle$$

Problem 2.30



2.30 A vertical load P is applied at the center A of the upper section of a homogeneous frustum of a circular cone of height h , minimum radius a , and maximum radius b . Denoting by E the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point A .

Extend the slant sides of the cone to meet at a point O and place the origin of the coordinate system there.



From geometry, $\tan \alpha = \frac{b-a}{h}$

$$a_1 = \frac{a}{\tan \alpha}, \quad b_1 = \frac{b}{\tan \alpha}, \quad r = y \tan \alpha$$

At coordinate point y , $A = \pi r^2$

Deformation of element of height dy : $dS = \frac{P dy}{AE}$

$$dS = \frac{P}{E\pi} \frac{dy}{r^2} = \frac{P}{\pi E \tan^2 \alpha} \frac{dy}{y^2}$$

Total deformation.

$$\begin{aligned} S_A &= \frac{P}{\pi E \tan^2 \alpha} \int_{a_1}^{b_1} \frac{dy}{y^2} = \frac{P}{\pi E \tan^2 \alpha} \left(-\frac{1}{y} \right) \Big|_{a_1}^{b_1} = \frac{P}{\pi E \tan^2 \alpha} \left(\frac{1}{a_1} - \frac{1}{b_1} \right) \\ &= \frac{P}{\pi E \tan^2 \alpha} \frac{b_1 - a_1}{a_1 b_1} = \frac{P(b_1 - a_1)}{\pi E a b} \end{aligned}$$

$$S_A = \frac{Ph}{\pi E a b} \downarrow \blacktriangleleft$$

Problem 2.31

2.31 Denoting by ϵ the "engineering strain" in a tensile specimen, show that the true strain is $\epsilon_t = \ln(1 + \epsilon)$.

$$\epsilon_t = \ln \frac{L}{L_0} = \ln \frac{L_0 + \delta}{L_0} = \ln \left(1 + \frac{\delta}{L_0} \right) = \ln(1 + \epsilon)$$

Thus

$$\epsilon_t = \ln(1 + \epsilon) \quad \blacktriangleleft$$

Problem 2.32

2.32 The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1 , show that when the diameter is d , the true strain is $\epsilon_t = 2 \ln(d_1/d)$.

If the volume is constant, $\frac{\pi}{4} d^2 L = \frac{\pi}{4} d_1^2 L_0$

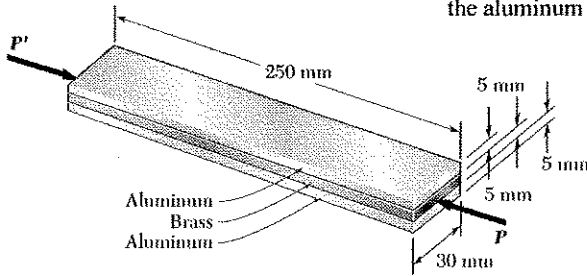
$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d} \right)^2$$

$$\epsilon_t = \ln \frac{L}{L_0} = \ln \left(\frac{d_1}{d} \right)^2$$

$$\epsilon_t = 2 \ln \frac{d_1}{d} \quad \blacktriangleleft$$

Problem 2.33

2.33 A 250-mm bar of 15×30 -mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (a) in the aluminum layers, (b) in the brass layer.



For each layer,

$$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{ m}^2$$

Let P_a = load on each aluminum layer.

P_b = load on brass layer.

Deformation. $\delta = \frac{P_a L}{E_a A} = \frac{P_b L}{E_b A}$ $P_b = \frac{E_b}{E_a} P_a = \frac{105}{70} P_a = 1.5 P_a$

Total force. $P = 2P_a + P_b = 3.5 P_a$

Solving for P_a and P_b , $P_a = \frac{2}{7} P$ $P_b = \frac{3}{7} P$

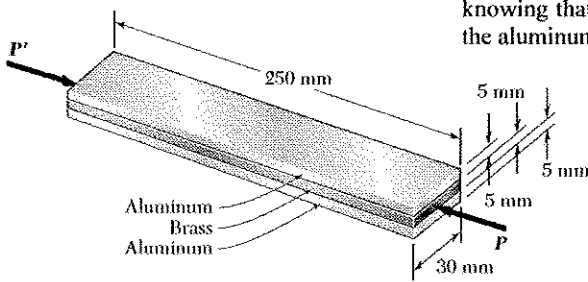
(a) $\sigma_a = -\frac{P_a}{A} = -\frac{2}{7} \frac{P}{A} = -\frac{2}{7} \frac{30 \times 10^3}{150 \times 10^{-6}} = -57.1 \times 10^6 \text{ Pa}$ $\sigma_a = -57.1 \text{ MPa}$ $\blacktriangleleft$

(b) $\sigma_b = -\frac{P_b}{A} = -\frac{3}{7} \frac{P}{A} = -\frac{3}{7} \frac{30 \times 10^3}{150 \times 10^{-6}} = -85.7 \times 10^6 \text{ Pa}$ $\sigma_b = -85.7 \text{ MPa}$ $\blacktriangleleft$

Problem 2.34

2.34 Determine the deformation of the composite bar of Prob. 2.33 if it is subjected to centric forces of magnitude $P = 45$ kN.

2.33 A 250-mm bar of 15×30 -mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (a) in the aluminum layers, (b) in the brass layer.



For each layer,

$$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{ m}^2$$

Let P_a = load on each aluminum layer.

P_b = load on brass layer.

Deformation. $\delta = -\frac{P_a L}{E_a A} = -\frac{P_b L}{E_b A}$ $P_b = \frac{E_b}{E_a} P_a = \frac{105}{70} P_a = 1.5 P_a$

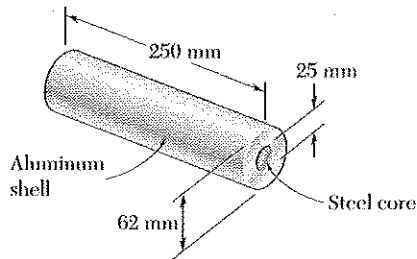
Total force. $P = 2P_a + P_b = 3.5 P_a$ $P_a = \frac{2}{7} P$

$$\delta = -\frac{P_a L}{E_a A} = -\frac{2}{7} \frac{PL}{E_a A} = -\frac{2}{7} \frac{(45 \times 10^3)(250 \times 10^{-3})}{(70 \times 10^9)(150 \times 10^{-6})} = -306 \times 10^{-6} \text{ m}$$

$$\delta = -0.306 \text{ mm} \quad \blacktriangleleft$$

Problem 2.35

2.35 Compressive centric forces of 160 kN are applied at both ends of the assembly shown by means of rigid plates. Knowing that $E_s = 200$ GPa and $E_a = 70$ GPa, determine (a) the normal stresses in the steel core and the aluminum shell, (b) the deformation of the assembly.



Let P_a = portion of axial force carried by shell.

P_s = portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}$$

$$P_a = \frac{E_a A_a}{L} \delta$$

$$\delta = \frac{P_s L}{E_s A_s}$$

$$P_s = \frac{E_s A_s}{L} \delta$$

Total force $P = P_a + P_s = (E_a A_a + E_s A_s) \frac{\delta}{L}$

$$\frac{\delta}{L} = \epsilon = \frac{P}{E_a A_a + E_s A_s}$$

Data: $P = 160$ kN

$$A_a = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (0.062^2 - 0.025^2) = 0.002528 \text{ m}^2$$

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.025)^2 = 0.000491 \text{ m}^2$$

$$\epsilon = \frac{-160000}{(70 \times 10^9)(0.002528) + (200 \times 10^9)(0.000491)} = -581.5 \times 10^{-6}$$

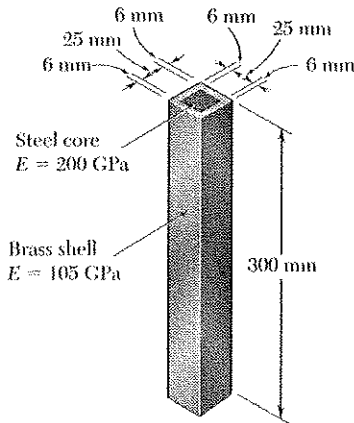
(a) $\sigma_s = E_s \epsilon = (200 \times 10^9)(-581.5 \times 10^{-6}) = -116.3 \text{ MPa}$ ▶

$\sigma_a = E_a \epsilon = (70 \times 10^9)(-581.5 \times 10^{-6}) = -40.7 \text{ MPa}$ ▶

(b) $\delta = L \epsilon = (0.25)(-581.5 \times 10^{-6}) = -145 \times 10^{-6} \text{ m} = -0.145 \text{ mm}$ ▶

Problem 2.36

2.36 The length of the assembly decreases by 0.15 mm when an axial force is applied by means of rigid end plates. Determine (a) the magnitude of the applied force, (b) the corresponding stress in the steel core.



Let P_b = portion of axial force carried by brass shell.

P_s = portion of axial force carried by steel core.

$$\delta = \frac{P_b L}{A_b E_b} \quad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \quad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s) \frac{\delta}{L}$$

$$A_s = (0.025)^2 = 625 \times 10^{-6} \text{ m}^2$$

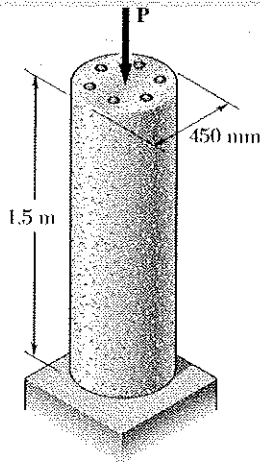
$$A_b = (0.037)^2 - (0.025)^2 = 744 \times 10^{-6} \text{ m}^2$$

$$(a) \quad P = [(105 \times 10^9)(744 \times 10^{-6}) + (200 \times 10^9)(625 \times 10^{-6})] \frac{0.00015}{0.3} = 101.6 \text{ kN}$$

$$(b) \quad \sigma_s = E_s \epsilon = E_s \frac{\delta}{L} = (200 \times 10^9) \frac{0.00015}{0.3} = 100 \text{ MPa}$$

Problem 2.37

2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force P is applied to the post.



Let P_c = portion of axial force carried by concrete.
 P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \quad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \quad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s) \frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_c A_c + E_s A_s}$$

$$A_s = 6 \frac{\pi}{4} d_s^2 = \frac{6\pi}{4} (28)^2 = 3.6945 \times 10^3 \text{ mm}^2 = 3.6945 \times 10^{-3} \text{ m}^2$$

$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4} (450)^2 - 3.6945 \times 10^3 = 155.349 \times 10^3 \text{ mm}^2 \\ = 153.349 \times 10^{-3} \text{ m}^2$$

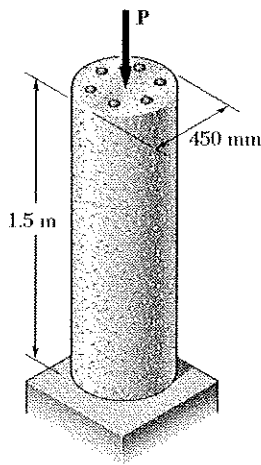
$$L = 1.5 \text{ m}$$

$$\varepsilon = \frac{1550 \times 10^3}{(25 \times 10^9)(153.349 \times 10^{-3}) + (200 \times 10^9)(3.6945 \times 10^{-3})} = 335.31 \times 10^{-6}$$

$$\sigma_s = E_s \varepsilon = (200 \times 10^9)(335.31 \times 10^{-6}) = 67.1 \times 10^6 \text{ Pa} = 67.1 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_c = E_c \varepsilon = (25 \times 10^9)(335.31 \times 10^{-6}) = 8.38 \times 10^6 \text{ Pa} = 8.38 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.38



2.38 For the post of Prob. 2.37, determine the maximum centric force which can be applied if the allowable normal stress is 160 MPa in the steel and 18 MPa in the concrete.

2.37 The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550 kN axial centric force P is applied to the post.

Determine allowable strain in each material.

$$\text{Steel: } \epsilon_s = \frac{\sigma_s}{E_s} = \frac{160 \times 10^6}{200 \times 10^9} = 800 \times 10^{-6}$$

$$\text{Concrete: } \epsilon_c = \frac{\sigma_c}{E_c} = \frac{18 \times 10^6}{25 \times 10^9} = 720 \times 10^{-6}$$

$$\text{Smaller value governs } \epsilon = \frac{\delta}{L} = 720 \times 10^{-6}$$

Let P_c = portion of load carried by concrete.

P_s = portion carried by six steel rods.

$$\delta = \frac{P_c L}{E_c A_c}, \quad P_c = E_c A_c \frac{\delta}{L} = E_c A_c \epsilon$$

$$\delta = \frac{P_s L}{E_s A_s}, \quad P_s = E_s A_s \frac{\delta}{L} = E_s A_s \epsilon$$

$$P = P_c + P_s = (E_c A_c + E_s A_s) \epsilon$$

$$A_s = 6 \frac{\pi}{4} d_s^2 = \frac{6\pi}{4} (28)^2 = 3.6945 \times 10^3 \text{ mm}^2 = 3.6945 \times 10^{-3} \text{ m}^2$$

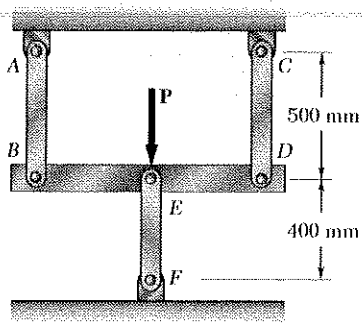
$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4} (450)^2 - 3.6945 \times 10^3 = 155.349 \times 10^3 \text{ mm}^2 \\ = 155.349 \times 10^{-3} \text{ m}^2$$

$$P = [(25 \times 10^9)(155.349 \times 10^{-3}) + (200 \times 10^9)(3.6945 \times 10^{-3})](720 \times 10^{-6})$$

$$= 3.33 \times 10^6 \text{ N}$$

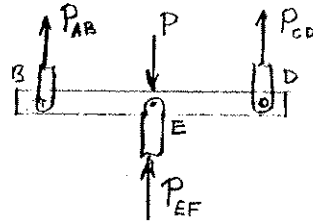
$$3330 \text{ kN} \blacktriangleleft$$

Problem 2.39



2.39 Three steel rods ($E = 200 \text{ GPa}$) support a 36-kN load P . Each of the rods AB and CD has a 200-mm^2 cross-sectional area and rod EF has a 625-mm^2 cross-sectional area. Neglecting the deformation of rod BED , determine (a) the change in length of rod EF , (b) the stress in each rod.

Use member BED as a free body.



By symmetry, or by $\sum M_E = 0$

$$P_{CD} = P_{AB}$$

$$\sum F_y = 0:$$

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E A_{AB}}, \quad \delta_{CD} = \frac{P_{CD} L_{CD}}{E A_{CD}}, \quad \delta_{EF} = \frac{P_{EF} L_{EF}}{E A_{EF}}$$

$$\text{Since } L_{AB} = L_{CD} \text{ and } A_{AB} = A_{CD}, \quad \delta_{AB} = \delta_{CD}$$

$$\text{Since points } A, C, \text{ and } E \text{ are fixed, } \delta_B = \delta_{AB}, \quad \delta_D = \delta_{CD}, \quad \delta_E = \delta_{EF}$$

$$\text{Since member } BED \text{ is rigid, } \delta_E = \delta_B = \delta_D$$

$$\frac{P_{AB} L_{AB}}{E A_{AB}} = \frac{P_{EF} L_{EF}}{E A_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF} = 0.256 P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2)(0.256)P_{EF} + P_{EF} = 1.512 P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3 \text{ N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3 \text{ N}$$

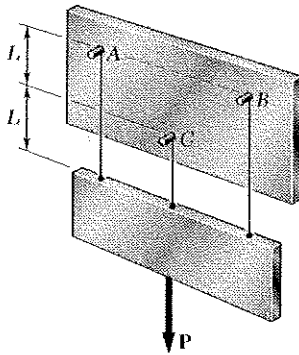
$$(a) \quad \delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m} = 0.0762 \text{ mm}$$

$$\text{or} \quad \delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

$$(b) \quad \sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6 \text{ Pa} = 30.5 \text{ MPa}$$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6 \text{ Pa} = 38.1 \text{ MPa}$$

Problem 2.40



2.40 Three wires are used to suspend the plate shown. Aluminum wires are used at A and B with a diameter of 3 mm and a steel wire is used at C with a diameter of 2 mm. Knowing that the allowable stress for aluminum ($E = 70$ GPa) is 98 MPa and that the allowable stress for steel ($E = 200$ GPa) is 126 MPa, determine the maximum load P that can be applied.

By symmetry $P_A = P_B$, and $\delta_A = \delta_B$

Also, $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire.

$$\epsilon_A = \epsilon_B = \frac{\delta}{2L}, \quad \epsilon_C = \frac{\delta}{L} = 2\epsilon_A$$

Determine allowable strain.

Wires A & B: $\epsilon_A = \frac{\sigma_A}{E_A} = \frac{98}{70 \times 10^3} = 1.4 \times 10^{-3}$

$$\epsilon_C = 2\epsilon_A = 2.8 \times 10^{-3}$$

Wire C: $\epsilon_C = \frac{\sigma_C}{E_C} = \frac{126}{200 \times 10^3} = 0.63 \times 10^{-3}$

$$\epsilon_A = \epsilon_B = \frac{1}{2}\epsilon_C = 0.315 \times 10^{-3}$$

Allowable strain for wire C governs $\therefore \sigma_C = 126$ MPa

$$\sigma_A = E_A \epsilon_A \quad P_A = A_A E_A \epsilon_A = \frac{\pi}{4} (0.003)^2 (70 \times 10^9) (0.315 \times 10^{-3})$$

$$= 155.87 \text{ N}$$

$$P_B = 155.87 \text{ N}$$

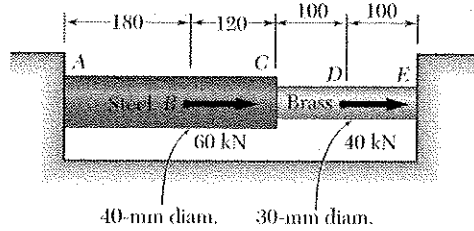
$$\sigma_C = E_C \epsilon_C \quad P_C = A_C \sigma_C = \frac{\pi}{4} (0.002)^2 (126 \times 10^6) = 395.84 \text{ N}$$

For equilibrium of the plate,

$$P = P_A + P_B + P_C = 707.6 \text{ N}$$

Problem 2.41

Dimensions in mm



2.41 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200 \text{ GPa}$ and $E_b = 105 \text{ GPa}$, determine (a) the reactions at A and E, (b) the deflection of point C.

A to C: $E = 200 \times 10^9 \text{ Pa}$

$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$EA = 251.327 \times 10^6 \text{ N}$$

C to E: $E = 105 \times 10^9 \text{ Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$EA = 74.220 \times 10^6 \text{ N}$$

A to B: $P = R_A$

$$L = 180 \text{ mm} = 0.180 \text{ m}$$

$$S_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6} = 716.20 \times 10^{-12} R_A$$

B to C: $P = R_A - 60 \times 10^3$

$$L = 120 \text{ mm} = 0.120 \text{ m}$$

$$S_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6} = 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

C to D: $P = R_A - 60 \times 10^3$

$$L = 100 \text{ mm} = 0.100 \text{ m}$$

$$S_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 80.841 \times 10^{-6}$$

D to E: $P = R_A - 100 \times 10^3$

$$L = 100 \text{ mm} = 0.100 \text{ m}$$

$$S_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 134.735 \times 10^{-6}$$

A to E: $S_{AE} = S_{AB} + S_{BC} + S_{CD} + S_{DE} = 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6}$

Since point E cannot move relative to A, $S_{AE} = 0$

(a) $3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0 \quad R_A = 62.831 \times 10^3 \text{ N} \quad 62.8 \text{ kN} \leftarrow$

$R_E = R_A - 100 \times 10^3 = 62.8 \times 10^3 - 100 \times 10^3 = -37.2 \times 10^3 \text{ N} \quad 37.2 \text{ kN} \leftarrow$

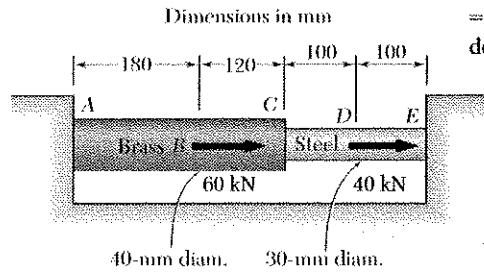
(b) $S_C = S_{AB} + S_{BC} = 1.16367 \times 10^{-9} R_A - 26.848 \times 10^{-6}$

$$= (1.16369 \times 10^{-9})(62.831 \times 10^3) - 26.848 \times 10^{-6}$$

$$= 46.3 \times 10^{-6} \text{ m}$$

$S_C = 46.3 \mu\text{m} \rightarrow$

Problem 2.42



2.42 Solve Prob. 2.41, assuming that rod AC is made of brass and rod CE is made of steel.

2.41 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (a) the reactions at A and E, (b) the deflection of point C.

A to C: $E = 105 \times 10^9 \text{ Pa}$

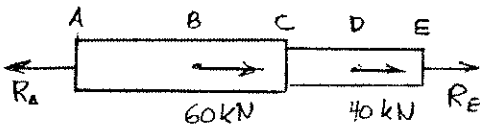
$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-5} \text{ m}^2$$

$$EA = 131.947 \times 10^6 \text{ N}$$

C to E: $E = 200 \times 10^9 \text{ Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$EA = 141.372 \times 10^6 \text{ N}$$



A to B: $P = R_A$

$$L = 180 \text{ mm} = 0.180 \text{ m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947 \times 10^6} = 1.36418 \times 10^{-9} R_A$$

B to C: $P = R_A - 60 \times 10^3$

$$L = 120 \text{ mm} = 0.120 \text{ m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{131.947 \times 10^6} = 909.456 \times 10^{-12} R_A - 54.567 \times 10^{-6}$$

C to D: $P = R_A - 60 \times 10^3$

$$L = 100 \text{ mm} = 0.100 \text{ m}$$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 42.441 \times 10^{-6}$$

D to E: $P = R_A - 100 \times 10^3$

$$L = 100 \text{ mm} = 0.100 \text{ m}$$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{141.372 \times 10^6} = 707.354 \times 10^{-12} R_A - 70.735 \times 10^{-6}$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6}$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.68834 \times 10^{-9} R_A - 167.743 \times 10^{-6} = 0 \quad R_A = 45.479 \times 10^3 \text{ N} \quad 45.5 \text{ kN} \leftarrow$

$$R_E = R_A - 100 \times 10^3 = 45.479 \times 10^3 - 100 \times 10^3 = -54.521 \times 10^3 \quad 54.5 \text{ kN} \leftarrow$$

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364 \times 10^{-9} R_A - 54.567 \times 10^{-6}$

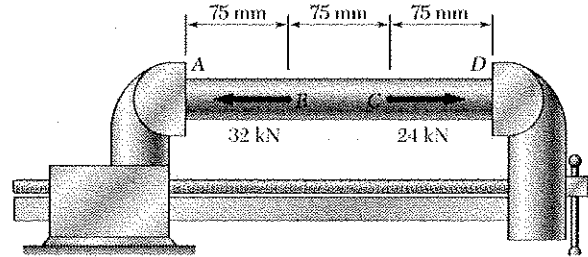
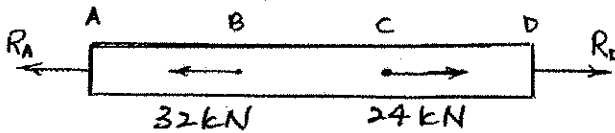
$$= (2.27364 \times 10^{-9})(45.479 \times 10^3) - 54.567 \times 10^{-6}$$

$$= 48.8 \times 10^{-6} \text{ m}$$

$$\delta_C = 48.8 \mu\text{m} \rightarrow$$

Problem 2.43

2.43 A steel tube ($E_s = 200 \text{ GPa}$) with a 30-mm outer diameter and a 3-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (a) the forces exerted by the vise on the tube at A and D, (b) the change in length of the portion BC of the tube.



For the tube $d_o = 30 \text{ mm}$

$$d_i = d_o - 2t = 0.03 - 2(0.003) = 0.024 \text{ m}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(0.03^2 - 0.024^2) = 254.5 \times 10^{-6} \text{ m}^2$$

A to B: $P = R_A \text{ kN}$ $L = 0.075 \text{ m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.075)}{(200 \times 10^9)(254.5 \times 10^{-6})} = 1.473 \times 10^{-9} R_A \text{ m}$$

B to C: $P = R_A + 32 \text{ kN}$, $L = 0.075 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 32000)(0.075)}{(200 \times 10^9)(254.5 \times 10^{-6})} = 1.473 \times 10^{-9} R_A + 47.15 \times 10^{-6} \text{ m}$$

C to D: $P = R_A + 8 \text{ kN}$, $L = 0.075 \text{ m}$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 8000)(0.075)}{(200 \times 10^9)(254.5 \times 10^{-6})} = 1.473 \times 10^{-9} R_A + 11.79 \times 10^{-6} \text{ m}$$

A to D: $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 4.419 \times 10^{-9} R_A + 58.94 \times 10^{-6} \text{ m}$

Given jaw movement $\delta_{AD} = -0.0002 \text{ m}$

(a) $4.419 \times 10^{-9} R_A + 58.94 \times 10^{-6} = -0.0002$

$$R_A = -58597 \text{ N}$$

$$R_A = 58.6 \text{ kN} \rightarrow$$

$$R_D = -R_A + 8000 = -50597 \text{ N}$$

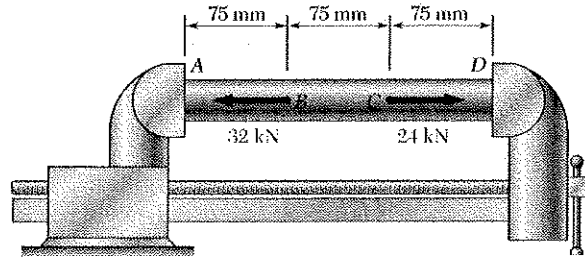
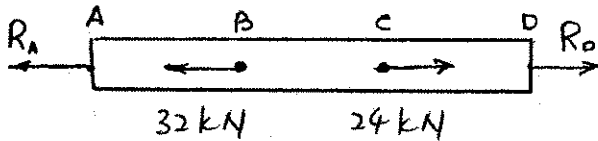
$$R_D = 50.6 \text{ kN} \leftarrow$$

(b) $\delta_{BC} = (1.473 \times 10^{-9})(-58597) + 47.15 \times 10^{-6} = 39.2 \times 10^{-6} \text{ m}$
 $= 39.2 \times 10^{-3} \text{ mm}$

Problem 2.44

2.44 Solve Prob. 2.43, assuming that after the forces have been applied, the vise is adjusted to increase the distance between its jaws by 0.1 mm.

2.43 A steel tube ($E_s = 200 \text{ GPa}$) with a 30-mm outer diameter and a 3-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (a) the forces exerted by the vise on the tube at A and D, (b) the change in length of the portion BC of the tube.



For the tube $d_o = 30 \text{ mm}$

$$d_i = d_o - 2t = 0.03 - 2(0.003) = 0.024 \text{ m}$$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (0.03^2 - 0.024^2) = 254.5 \times 10^{-6} \text{ m}^2$$

A to B: $P = R_A$ $L = 0.075 \text{ m}$

$$S_{AB} = \frac{PL}{EA} = \frac{R_A(0.075)}{(200 \times 10^9)(254.5 \times 10^{-6})} = 1.473 \times 10^{-9} R_A \text{ m}$$

B to C: $P = R_A + 32 \text{ kN}$, $L = 0.075 \text{ m}$

$$S_{BC} = \frac{PL}{EA} = \frac{(R_A + 32000)(0.075)}{(200 \times 10^9)(254.5 \times 10^{-6})} = 1.473 \times 10^{-9} R_A + 47.15 \times 10^{-6} \text{ m}$$

C to D: $P = R_A + 8000 \text{ N}$ $L = 0.075 \text{ m}$

$$S_{CD} = \frac{PL}{EA} = \frac{(R_A + 8000)(0.075)}{(200 \times 10^9)(254.5 \times 10^{-6})} = 1.473 \times 10^{-9} R_A + 11.79 \times 10^{-6} \text{ m}$$

A to D: $S_{AD} = S_{AB} + S_{BC} + S_{CD} = 4.419 \times 10^{-9} R_A + 58.94 \times 10^{-6}$

Given jaw movement $S_{AD} = -0.0001 \text{ m}$

(a) $4.419 \times 10^{-9} R_A + 58.94 \times 10^{-6} = -0.0001$

$$R_A = -35967 \text{ N}$$

$$R_A = 35.97 \text{ kN} \rightarrow \blacktriangleleft$$

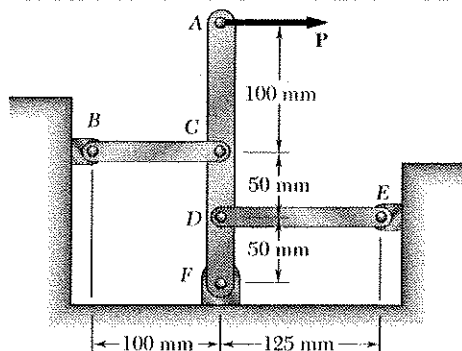
$$R_D = R_A + 8000 = -27967$$

$$R_D = 27.97 \text{ kN} \leftarrow \blacktriangleleft$$

(b) $S_{BC} = (1.473 \times 10^{-9})(-35967) + 47.15 \times 10^{-6} = -5.83 \times 10^{-6} \text{ m}$
 $= -5.83 \times 10^{-3} \text{ mm}$

Problem 2.45

2.45 Links BC and DE are both made of steel ($E = 200$ GPa) and are 12 mm wide and 6 mm thick. Determine (a) the force in each link when a 2.4-kN force P is applied to the rigid member AF shown, (b) the corresponding deflection of point A .



Let the rigid member $ACDF$ rotate through small angle θ clockwise about point F .

$$\text{Then } \delta_C = \delta_{BC} = 0.1\theta \text{ m} \rightarrow$$

$$\delta_D = -\delta_{DE} = 0.05\theta \text{ m} \rightarrow$$

$$\delta = \frac{FL}{EA} \quad \text{or} \quad F = \frac{EAS}{L}$$

$$\text{For links: } A = (0.012)(0.006) = 72 \times 10^{-6} \text{ m}^2, \quad L_{BC} = 0.1 \text{ m}, \quad L_{DE} = 0.125 \text{ m}$$

$$F_{BC} = \frac{EAS_{BC}}{L_{BC}} = \frac{(200 \times 10^9)(72 \times 10^{-6})(0.1\theta)}{0.1} = 14.4 \times 10^6 \theta$$

$$F_{DE} = \frac{EAS_{DE}}{L_{DE}} = \frac{(200 \times 10^9)(72 \times 10^{-6})(-0.05\theta)}{0.125} = -5.76 \times 10^6 \theta$$

Use member $ACDF$ as a free body.

$$(+\sum M_F = 0: 0.2P - 0.1F_{BC} + 0.05F_{DE} = 0$$

$$P = \frac{1}{2}F_{BC} - \frac{1}{4}F_{DE}$$

$$2400 = \frac{1}{2}(14.4 \times 10^6)\theta - \frac{1}{4}(-5.76 \times 10^6)\theta = 8.64 \times 10^6 \theta$$

$$\theta = 0.27778 \times 10^{-3} \text{ rad } \curvearrowright$$

$$(a) \quad F_{BC} = (14.4 \times 10^6)(0.27778 \times 10^{-3})$$

$$F_{BC} = 4 \text{ kN} \quad \leftarrow$$

$$F_{DE} = -(5.76 \times 10^6)(0.27778 \times 10^{-3})$$

$$F_{DE} = -1.6 \text{ kN} \quad \leftarrow$$

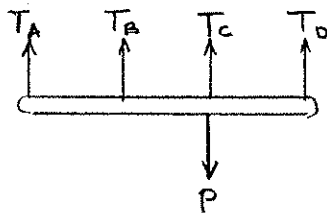
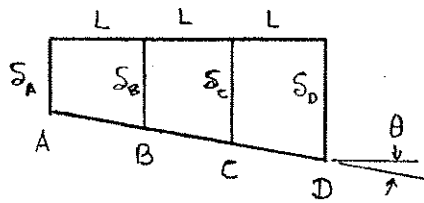
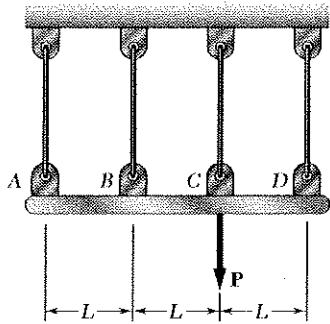
(b) Deflection at point A.

$$\delta_A = 0.2\theta = (0.2)(0.27778 \times 10^{-3})$$

$$\delta_A = 0.056 \text{ mm} \quad \rightarrow \leftarrow$$

Problem 2.46

2.46 The rigid bar $ABCD$ is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.



Deformations Let θ be the rotation of bar $ABCD$ and S_A, S_B, S_C and S_D be the deformations of wires A, B, C, and D.

From geometry, $\theta = \frac{S_D - S_A}{L}$

$$S_B = S_A + L\theta$$

$$S_C = S_A + 2L\theta = 2S_B - S_A \quad (1)$$

$$S_D = S_A + 3L\theta = 3S_B - 2S_A \quad (2)$$

Since all wires are identical, the forces in the wires are proportional to the deformations.

$$T_C = 2T_B - T_A \quad (1')$$

$$T_D = 3T_B - 2T_A \quad (2')$$

Use bar $ABCD$ as a free body.

$$+\circlearrowleft \sum M_C = 0: -2LT_A - LT_B + LT_D = 0 \quad (3)$$

$$+\uparrow \sum F_y = 0: T_A + T_B + T_C + T_D - P = 0 \quad (4)$$

Substituting (2') into (3) and dividing by L ,

$$-4T_A + 2T_B = 0 \quad T_B = 2T_A \quad (3')$$

Substituting (1'), (2'), and (3') into (4),

$$T_A + 2T_A + 3T_A + 4T_A - P = 0 \quad 10T_A = P$$

$$T_A = \frac{1}{10}P \quad \blacktriangleleft$$

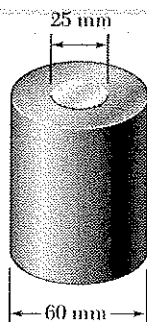
$$T_B = 2T_A = (2)\left(\frac{1}{10}P\right) \quad T_B = \frac{1}{5}P \quad \blacktriangleleft$$

$$T_C = (2)\left(\frac{1}{5}P\right) - \left(\frac{1}{10}P\right) \quad T_C = \frac{3}{10}P \quad \blacktriangleleft$$

$$T_D = (3)\left(\frac{1}{5}P\right) - (2)\left(\frac{1}{10}P\right) \quad T_D = \frac{2}{5}P \quad \blacktriangleleft$$

Problem 2.47

2.47 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15 °C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195 °C.



Brass core
 $E = 105 \text{ GPa}$
 $\alpha = 20.9 \times 10^{-6} / ^\circ\text{C}$

Aluminum shell
 $E = 70 \text{ GPa}$
 $\alpha = 23.6 \times 10^{-6} / ^\circ\text{C}$

Let L be the length of the assembly.

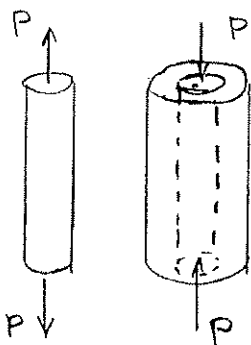
Free thermal expansion.

$$\Delta T = 195 - 15 = 180^\circ\text{C}$$

$$\text{Brass core: } (\sigma_T)_b = L \alpha_b (\Delta T)$$

$$\text{Aluminum shell: } (\sigma_T)_a = L \alpha_a (\Delta T)$$

Net expansion of shell with respect to the core. $\delta = L(\alpha_a - \alpha_b)(\Delta T)$



Let P be the tensile force in the core and the compressive force in the shell.

Brass core: $E_b = 105 \times 10^9 \text{ Pa}$

$$A_b = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$(\sigma_p)_b = \frac{PL}{E_b A_b}$$

Aluminum shell: $E_a = 70 \times 10^9 \text{ Pa}$

$$A_a = \frac{\pi}{4} (60^2 - 25^2) = 2.3366 \times 10^3 \text{ mm}^2 = 2.3366 \times 10^{-3} \text{ m}^2$$

$$\delta = (\sigma_p)_b + (\sigma_p)_a$$

$$L(\alpha_b - \alpha_a)(\Delta T) = \frac{PL}{E_b A_b} + \frac{PL}{E_a A_a} = K PL$$

$$\text{where } K = \frac{1}{E_b A_b} + \frac{1}{E_a A_a} = \frac{1}{(105 \times 10^9)(490.87 \times 10^{-6})} + \frac{1}{(70 \times 10^9)(2.3366 \times 10^{-3})}$$

$$= 25.516 \times 10^{-9} \text{ N}^{-1}$$

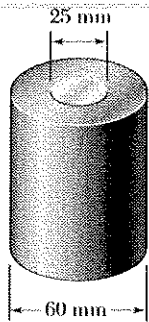
$$\text{Then } P = \frac{(\alpha_b - \alpha_a)(\Delta T)}{K} = \frac{(23.6 \times 10^{-6} - 20.9 \times 10^{-6})(180)}{25.516 \times 10^{-9}} = 19.047 \times 10^3 \text{ N}$$

Stress in aluminum.

$$\sigma_a = -\frac{P}{A_a} = -\frac{19.047 \times 10^3}{2.3366 \times 10^{-3}} = -8.15 \times 10^6 \text{ Pa}$$

$$\sigma_a = -8.15 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.48



Aluminum shell
 $E = 70 \text{ GPa}$
 $\alpha = 23.6 \times 10^{-6}/^\circ\text{C}$

2.48 Solve Prob. 2.47, assuming that the core is made of steel ($E = 200 \text{ GPa}$, $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$) instead of brass.

2.47 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C . Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C .

Let L be the length of the assembly.

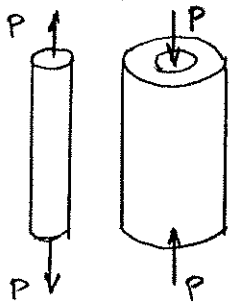
Free thermal expansion.

$$\Delta T = 195 - 15 = 180^\circ\text{C}$$

$$\text{Steel core: } (\delta_T)_s = L \alpha_s (\Delta T)$$

$$\text{Aluminum shell: } (\delta_T)_a = L \alpha_a (\Delta T)$$

Net expansion of shell with respect to the core. $\delta = L(\alpha_a - \alpha_s)(\Delta T)$



Let P be the tensile force in the core and the compressive force in the shell.

Steel core: $E_s = 200 \times 10^9 \text{ Pa}$

$$A_s = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$(\delta_p)_s = \frac{PL}{E_s A_s}$$

Aluminum shell: $E_a = 70 \times 10^9 \text{ Pa}$

$$(\delta_p)_a = \frac{PL}{E_a A_a}$$

$$A_a = \frac{\pi}{4}(60^2 - 25^2) = 2.3366 \times 10^3 \text{ mm}^2 = 2.3366 \times 10^{-3} \text{ m}^2$$

$$\delta = (\delta_p)_s + (\delta_p)_a$$

$$L(\alpha_a - \alpha_s)(\Delta T) = \frac{PL}{E_s A_s} + \frac{PL}{E_a A_a} = KPL$$

$$\text{where } K = \frac{1}{E_s A_s} + \frac{1}{E_a A_a} = \frac{1}{(200 \times 10^9)(490.87 \times 10^{-6})} + \frac{1}{(70 \times 10^9)(2.3366 \times 10^{-3})}$$

$$= 16.2999 \times 10^{-9} \text{ N}^{-1}$$

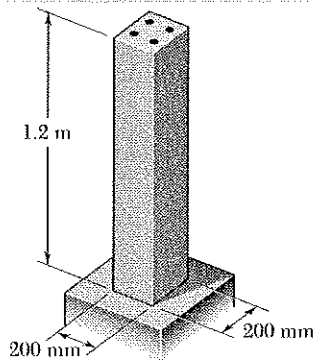
$$\text{Then } P = \frac{(\alpha_a - \alpha_s)(\Delta T)}{K} = \frac{(23.6 \times 10^{-6} - 11.7 \times 10^{-6})(180)}{16.2999 \times 10^{-9}} = 131.41 \times 10^3 \text{ N}$$

Stress in aluminum.

$$\sigma_a = -\frac{P}{A_a} = -\frac{131.19 \times 10^3}{2.3366 \times 10^{-3}} = -56.2 \times 10^6 \text{ Pa}$$

$$\sigma_a = -56.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.49



2.49 A 1.2-m concrete post is reinforced by four steel bars, each of 18 mm diameter. Knowing that $E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$ and $E_c = 25$ GPa and $\alpha_c = 9.9 \times 10^{-6}/^\circ\text{C}$, determine the normal stresses induced in the steel and in the concrete by a temperature rise of 27°C .

$$A_s = 4 \frac{\pi}{4} d^2 = (4 \frac{\pi}{4}) (0.018)^2 = 0.0010179 \text{ m}^2$$

$$A_c = A - A_s = 0.2^2 - 0.0010179 = 0.039 \text{ m}^2$$

Let P_c be the tensile in the concrete. For equilibrium with zero total force, the compressive force in the four steel rods is $-P_c$.

Strains: $\epsilon_s = \frac{P_s}{E_s A_s} + \alpha_s (\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$

$$\epsilon_c = \frac{P_c}{E_c A_c} + \alpha_c (\Delta T)$$

Matching: $\epsilon_c = \epsilon_s \quad \frac{P_c}{E_c A_c} + \alpha_c (\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s} \right) P_c = (\alpha_s - \alpha_c) (\Delta T)$$

$$\left(\frac{1}{(25 \times 10^9)(0.039)} + \frac{1}{(200 \times 10^9)(0.0010179)} \right) P_c = (11.7 - 9.9)(10^{-6})(27)$$

$$= (11.7 - 9.9)(10^{-6})(27)$$

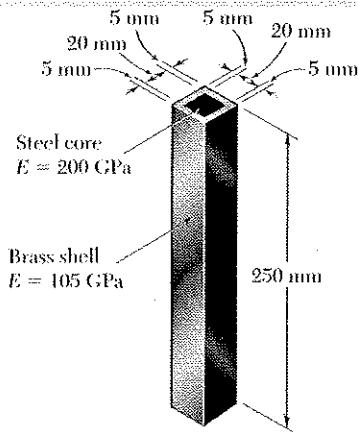
$$P_c = 8185 \text{ N}$$

$$P_s = -8185 \text{ N}$$

Stress in steel $\sigma_s = \frac{P_s}{A_s} = \frac{-8185 \times 10^3}{0.0010179} = -8.04 \text{ MPa}$

Stress in concrete $\sigma_c = \frac{P_c}{A_c} = \frac{8185 \times 10^3}{0.039} = 0.21 \text{ MPa}$

Problem 2.50



2.50 The brass shell ($\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$) is fully bonded to the steel core ($\alpha_s = 11.7 \times 10^{-6}/^\circ\text{F}$). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 55 MPa.

Let P_s = axial force developed in the steel core
For equilibrium with zero total force the compressive force in the brass shell is P_s .

$$\text{Strains } \epsilon_s = \frac{P_s}{E_s A_s} + \alpha_s (\Delta T)$$

$$\epsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b (\Delta T)$$

Matching, $\epsilon_s = \epsilon_b$

$$\frac{P_s}{E_s A_s} + \alpha_s (\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b (\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b} \right) P_s = (\alpha_b - \alpha_s) (\Delta T)$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$$

$$\alpha_b - \alpha_s = 9.2 \times 10^{-6} / ^\circ\text{C}$$

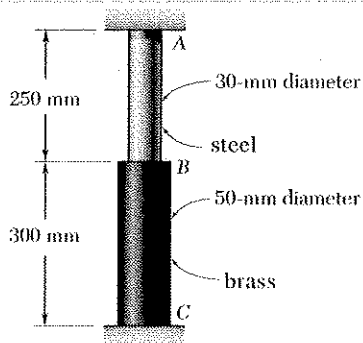
$$P_s = \sigma_s A_s = (55 \times 10^6)(400 \times 10^{-6}) = 22 \times 10^3 \text{ N}$$

$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(200 \times 10^9)(400 \times 10^{-6})} + \frac{1}{(105 \times 10^9)(500 \times 10^{-6})} = 31.55 \times 10^{-9} \text{ N}^{-1}$$

$$(31.55 \times 10^{-9})(22 \times 10^3) = (9.2 \times 10^{-6})(\Delta T)$$

$$\Delta T = 75.4 ^\circ\text{C} \quad \blacktriangleleft$$

Problem 2.51



2.51 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 200 \text{ GPa}$, $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$) and portion BC is made of brass ($E_b = 105 \text{ GPa}$, $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$). Knowing that the rod is initially unstressed, determine the compressive force induced in ABC when there is a temperature rise of 50°C .

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

Free thermal expansion.

$$\begin{aligned} S_T &= L_{AB} \alpha_s (\Delta T) + L_{BC} \alpha_b (\Delta T) \\ &= (0.250)(11.7 \times 10^{-6})(50) + (0.300)(20.9 \times 10^{-6})(50) \\ &= 459.75 \times 10^{-6} \text{ m} \end{aligned}$$

Shortening due to induced compressive force P .

$$\begin{aligned} S_P &= \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}} \\ &= \frac{0.250 P}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{0.300 P}{(105 \times 10^9)(1.9635 \times 10^{-3})} \\ &= 3.2235 \times 10^{-9} P \end{aligned}$$

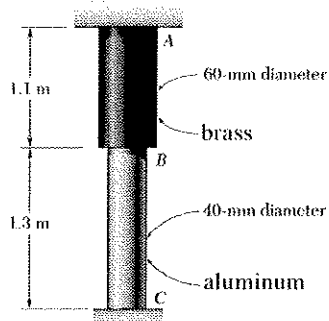
For zero net deflection, $S_P = S_T$

$$3.2235 \times 10^{-9} P = 459.75 \times 10^{-6}$$

$$P = 142.62 \times 10^3 \text{ N}$$

$$P = 142.6 \text{ kN} \quad \blacktriangleleft$$

Problem 2.52



2.52 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of brass ($E_b = 105 \text{ GPa}$, $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$) and portion BC is made of aluminum ($E_s = 72 \text{ GPa}$, $\alpha_s = 23.9 \times 10^{-6}/^\circ\text{C}$). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions AB and BC by a temperature rise of 42°C , (b) the corresponding deflection of point B .

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (60)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-5} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (40)^2 = 1.2566 \times 10^3 \text{ mm}^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

Free thermal expansion

$$\begin{aligned} \delta_T &= L_{AB} \alpha_b (\Delta T) + L_{BC} \alpha_a (\Delta T) \\ &= (1.1)(20.9 \times 10^{-6})(42) + (1.3)(23.9 \times 10^{-6})(42) \\ &= 2.2705 \times 10^{-3} \text{ m} \end{aligned}$$

Shortening due to induced compressive force

$$\begin{aligned} \delta_P &= \frac{P L_{AB}}{E_b A_{AB}} + \frac{P L_{BC}}{E_a A_{BC}} \\ &= \frac{1.1 P}{(105 \times 10^9)(2.8274 \times 10^{-3})} + \frac{1.3 P}{(72 \times 10^9)(1.2566 \times 10^{-3})} \\ &= 18.074 \times 10^{-9} P \end{aligned}$$

For zero net deflection $\delta_P = \delta_T$

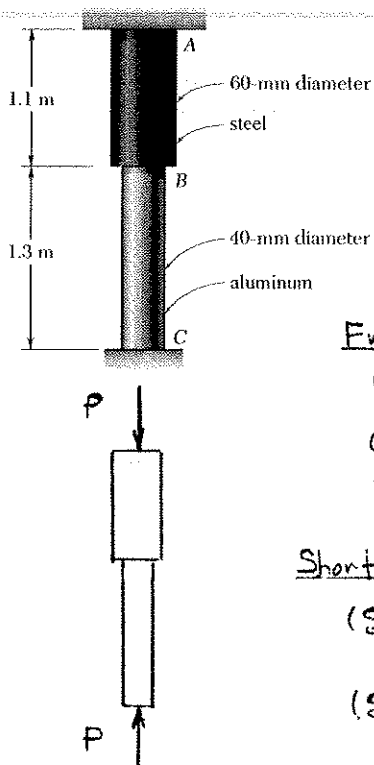
$$\begin{aligned} 18.074 \times 10^{-9} P &= 2.2705 \times 10^{-3} \\ P &= 125.62 \times 10^3 \text{ N} \end{aligned}$$

$$(a) \quad \sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{125.62 \times 10^3}{2.8274 \times 10^{-3}} = -44.4 \times 10^6 \text{ Pa} = -44.4 \text{ MPa}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{125.62 \times 10^3}{1.2566 \times 10^{-3}} = -100.0 \times 10^6 \text{ Pa} = -100.0 \text{ MPa}$$

$$\begin{aligned} (b) \quad \delta_B &= +\frac{P L_{AB}}{E_b A_{AB}} - L_{AB} \alpha_b (\Delta T) \\ &= \frac{(125.62 \times 10^3)(1.1)}{(105 \times 10^9)(2.8274 \times 10^{-3})} - (1.1)(20.9 \times 10^{-6})(42) \\ &= -500 \times 10^{-6} \text{ m} = -0.500 \text{ mm} \\ &\quad \text{i.e. } 0.500 \text{ mm } \downarrow \end{aligned}$$

Problem 2.53



2.53 Solve Prob. 2.52, assuming that portion *AB* of the composite rod is made of steel and portion *BC* is made of brass.

2.52 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$) and portion *BC* is made of aluminum ($E_a = 72$ GPa, $\alpha_a = 23.9 \times 10^{-6}/^\circ\text{C}$). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 42°C , (b) the corresponding deflection of point *B*.

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (0.06)^2 = 2.8274 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (0.04)^2 = 1.2566 \times 10^{-6} \text{ m}^2$$

Free thermal expansion. $\Delta T = 42^\circ$

$$(\delta_T)_{AB} = L_{AB} \alpha_b (\Delta T) = (1.1)(20.9 \times 10^{-6})(42) = 0.9541 \times 10^{-3} \text{ m}$$

$$(\delta_T)_{BC} = L_{BC} \alpha_a (\Delta T) = (1.3)(23.9 \times 10^{-6})(42) = 1.305 \times 10^{-3} \text{ m}$$

$$\text{Total } \delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 2.2591 \times 10^{-3} \text{ m}$$

Shortening due to induced compressive force *P*.

$$(\delta_P)_{AB} = \frac{P L_{AB}}{E_b A_{AB}} = \frac{1.1 P}{(105 \times 10^9)(2.8274 \times 10^{-6})} = 3.745 \times 10^{-9} P$$

$$(\delta_P)_{BC} = \frac{P L_{BC}}{E_a A_{BC}} = \frac{1.3 P}{(72 \times 10^9)(1.2566 \times 10^{-6})} = 1.4368 \times 10^{-9} P$$

$$\text{Total } \delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 5.1818 \times 10^{-9} P$$

For zero net deflection $\delta_P = \delta_T$

$$5.1818 \times 10^{-9} P = 2.2591 \times 10^{-3}$$

$$P = 436.16 \text{ N}$$

$$(a) \quad \sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{436.16}{2.8274 \times 10^{-6}} = -154.3 \text{ MPa}$$

$$\sigma_{AB} = -154.3 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{436.16}{1.2566 \times 10^{-6}} = -347.3 \text{ MPa}$$

$$\sigma_{BC} = -347.3 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad (\delta_P)_{AB} = (3.745 \times 10^{-9})(436.16) = 1.634 \times 10^{-3} \text{ m}$$

$$\delta_B = (\delta_T)_{AB} \downarrow + (\delta_P)_{AB} \uparrow = 0.9541 \times 10^{-3} \downarrow + 1.634 \times 10^{-3} \uparrow$$

$$\delta_B = 0.6799 \text{ mm} \quad \downarrow \quad \blacktriangleleft$$

$$\text{or } (\delta_P)_{BC} = (1.4368 \times 10^{-9})(436.16) = 0.6266 \times 10^{-3} \text{ m}$$

$$\delta_B = (\delta_T)_{BC} \uparrow + (\delta_P)_{BC} \downarrow = 1.305 \times 10^{-3} \uparrow + 0.6266 \times 10^{-3} \downarrow$$

$$\delta_B = 0.6784 \text{ mm} \downarrow \quad (\text{checks})$$

Problem 2.54

2.54 A steel railroad track ($E_s = 200 \text{ GPa}$, $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$) was laid out at a temperature of -1.0°C . Determine the normal stress in the rail when the temperature reaches 52°C , assuming that the rails (a) are welded to form a continuous track, (b) are 12 m long with 6-mm gaps between them.

$$L = 12 \text{ m}$$

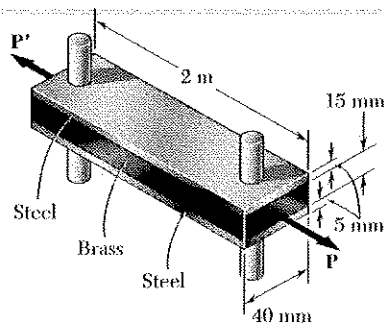
$$\delta_T = L \alpha (\Delta T) = (12)(11.7 \times 10^{-6})(52 - (-1)) = 0.00744 \text{ m.}$$

$$\delta_P = \frac{PL}{EA} = \frac{L}{E} \sigma = \frac{12}{200 \times 10^9} \sigma = 60 \times 10^{-12} \sigma$$

$$(a) \quad \delta_P + \delta_T = 0 \quad 60 \times 10^{-12} \sigma + 0.00744 = 0 \quad \sigma = -124 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \delta_P + \delta_T = 0.006 \quad 60 \times 10^{-12} \sigma + 0.00744 = 0.006 \quad \sigma = -24 \text{ MPa.} \quad \blacktriangleleft$$

Problem 2.55



2.55 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$) which is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

(a) Required temperature change for fabrication.

$$\delta_T = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

Temperature change required to expand steel bar by this amount

$$\delta_T = L \alpha_s \Delta T, \quad 0.5 \times 10^{-3} = (2.00)(11.7 \times 10^{-6})(\Delta T), \quad \Delta T =$$

$$0.5 \times 10^{-3} = (2)(11.7 \times 10^{-6})(\Delta T)$$

$$\Delta T = 21.368^\circ\text{C}$$

$$21.4^\circ\text{C} \quad \blacktriangleleft$$

(b) Once assembled, a tensile force P^* develops in the steel and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$(\delta_p)_s = \frac{F^* L}{A_s E_s} = \frac{P^*(2.00)}{(400 \times 10^{-6})(200 \times 10^9)} = 25 \times 10^{-9} P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$$(\delta_p)_b = \frac{P^* L}{A_b E_b} = \frac{P^*(2.00)}{(600 \times 10^{-6})(105 \times 10^9)} = 31.746 \times 10^{-9} P^*$$

But $(\delta_p)_s + (\delta_p)_b$ is equal to the initial amount of misfit

$$(\delta_p)_s + (\delta_p)_b = 0.5 \times 10^{-3}, \quad 56.746 \times 10^{-9} P^* = 0.5 \times 10^{-3}$$

$$P^* = 8.811 \times 10^3 \text{ N}$$

Stresses due to fabrication.

$$\text{Steel: } \sigma_s^* = \frac{P^*}{A_s} = \frac{8.811 \times 10^3}{400 \times 10^{-6}} = 22.03 \times 10^6 \text{ Pa} = 22.03 \text{ MPa}$$

$$\text{Brass: } \sigma_b^* = -\frac{P^*}{A_b} = -\frac{8.811 \times 10^3}{600 \times 10^{-6}} = -14.68 \times 10^6 \text{ Pa} = -14.68 \text{ MPa}$$

To these stresses must be added the stresses due to the 25 kN load.

continued

Problem 2.55 continued

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L} \delta' = \frac{(400 \times 10^{-6})(200 \times 10^9)}{2.00} \delta' = 40 \times 10^6 \delta'$$

$$P_b = \frac{A_b E_b}{L} \delta' = \frac{(600 \times 10^{-6})(105 \times 10^9)}{2.00} \delta' = 31.5 \times 10^6 \delta'$$

$$\text{Total } P = P_s + P_b = 25 \times 10^3 \text{ N}$$

$$40 \times 10^6 \delta' + 31.5 \times 10^6 \delta' = 25 \times 10^3 \quad \delta' = 349.65 \times 10^{-6} \text{ m}$$

$$P_s = (40 \times 10^6)(349.65 \times 10^{-6}) = 13.986 \times 10^3 \text{ N}$$

$$P_b = (31.5 \times 10^6)(349.65 \times 10^{-6}) = 11.140 \times 10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986 \times 10^3}{400 \times 10^{-6}} = 34.97 \times 10^6 \text{ Pa}$$

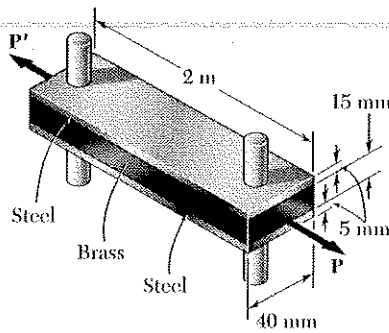
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140 \times 10^3}{600 \times 10^{-6}} = 18.36 \times 10^6 \text{ Pa}$$

Add stress due to fabrication. Total stresses.

$$\sigma_s = 34.97 \times 10^6 + 22.03 \times 10^6 = 57.0 \times 10^6 \text{ Pa} = 57.0 \text{ MPa}$$

$$\sigma_b = 18.36 \times 10^6 - 14.68 \times 10^6 = 3.68 \times 10^6 \text{ Pa} = 3.68 \text{ MPa} \blacktriangleleft$$

Problem 2.56



2.56 Determine the maximum load P that may be applied to the brass bar of Prob. 2.55 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

2.55 Two steel bars ($E_s = 200 \text{ GPa}$ and $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$) are used to reinforce a brass bar ($E_b = 105 \text{ GPa}$, $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$) which is subjected to a load $P = 25 \text{ kN}$. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

See solution to Problem 2.55 to obtain the fabrication stresses.

$$\sigma_s^* = 22.03 \text{ MPa}$$

$$\sigma_b^* = -14.68 \text{ MPa}$$

Allowable stresses: $\sigma_{s,all} = 30 \text{ MPa}$, $\sigma_{b,all} = 25 \text{ MPa}$

Available stress increase from load.

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$

$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains.

$$\epsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97 \times 10^6}{200 \times 10^9} = 39.85 \times 10^{-6}$$

$$\epsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68 \times 10^6}{105 \times 10^9} = 377.9 \times 10^{-6}$$

Smaller value governs $\therefore \epsilon = 39.85 \times 10^{-6}$

$$\text{Areas: } A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$$

$$P_s = E_s A_s \epsilon = (200 \times 10^9)(400 \times 10^{-6})(39.85 \times 10^{-6}) = 3.188 \times 10^3 \text{ N}$$

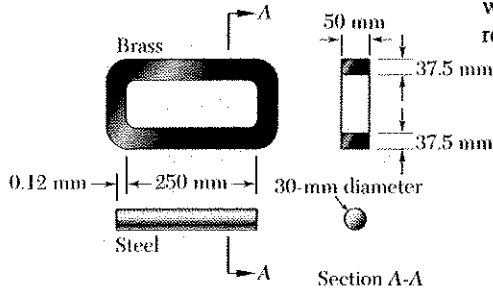
$$P_b = E_b A_b \epsilon = (105 \times 10^9)(600 \times 10^{-6})(39.85 \times 10^{-6}) = 2.511 \times 10^3 \text{ N}$$

Total allowable additional force.

$$P = P_s + P_b = 3.188 \times 10^3 + 2.511 \times 10^3 = 5.70 \times 10^3 \text{ N} = 5.70 \text{ kN}$$

Problem 2.57

2.57 An brass link ($E_b = 105 \text{ GPa}$, $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$) and a steel rod ($E_s = 200 \text{ GPa}$, $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$) have the dimensions shown at a temperature of 20°C . The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 45°C . Determine (a) the final stress in the steel rod, (b) the final length of the steel rod.



Initial dimensions at $T = 20^\circ\text{C}$.

Final dimensions at $T = 45^\circ\text{C}$.

$$\Delta T = 45 - 20 = 25^{\circ}\text{C}$$

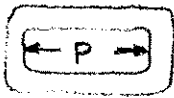
Free thermal expansion of each part.

Brass link: $(S_T)_b = \alpha_b (\Delta T) L' = (20.9 \times 10^{-6})(25)(0.250) = 130.625 \times 10^{-6} \text{ m}$

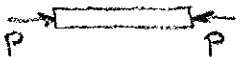
Steel rod: $(\delta_T)_s = \alpha_s (\Delta T) L = (11.7 \times 10^{-6})(25)(0.250) = 73.125 \times 10^{-6} \text{ m}$

At the final temperature the difference between the free length of the steel rod and the brass link is

$$S = 120 \times 10^{-6} + 73.125 \times 10^{-6} - 130.625 \times 10^{-6} = 62.5 \times 10^{-6} \text{ m}$$



Add equal but opposite forces P to elongate the brass link and contract the steel rod.



Brass link: $E = 105 \times 10^9 \text{ Pa}$

$$A_b = (2)(50)(37.5) = 3750 \text{ mm}^2 = 3.750 \times 10^{-3} \text{ m}^2$$

$$(S_p) = \frac{PL}{EA} = \frac{P(0.250)}{(105 \times 10^9)(3.750 \times 10^{-3})} = 634.92 \times 10^{-12} P$$

Steel rod: $E = 200 \times 10^9 \text{ Pa}$ $A_s = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$

$$(\delta_p)_s = \frac{PL}{E_s A_s} = \frac{P(0.250)}{(200 \times 10^9)(706.86 \times 10^{-6})} = 1.76838 \times 10^{-9} P$$

$$(S_p)_b + (S_p)_s = S \quad 2.4033 \times 10^{-9} \text{ P} = 62.5 \times 10^{-6}$$

$$P = 26.006 \times 10^3 \text{ N}$$

(a) Stress in steel rod. $\sigma_s = -\frac{P}{A_s} = -\frac{(26.006 \times 10^3)}{706.86 \times 10^{-6}}$
 $= -36.8 \times 10^6 \text{ Pa} \quad \sigma_s = -36.8 \text{ MPa} \blacktriangleleft$

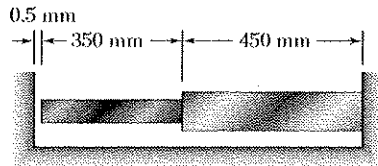
(b) Final length of steel rod. $L_f = L_o + (\delta_T)_s - (\delta_P)_s$

$$L_p = 0.250 + 1120 \times 10^{-6} + 73.125 \times 10^{-6} - (1.76838 \times 10^{-9})(26.003 \times 10^3)$$

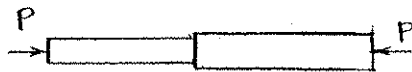
$$= 0.250147 \text{ m}$$

$$L_f = 250.147 \text{ mm}$$

Problem 2.58



Bronze	Aluminum
$A = 1500 \text{ mm}^2$	$A = 1800 \text{ mm}^2$
$E = 105 \text{ GPa}$	$E = 73 \text{ GPa}$
$\alpha = 21.6 \times 10^{-6} / ^\circ\text{C}$	$\alpha = 23.2 \times 10^{-6} / ^\circ\text{C}$



2.58 Knowing that a 0.5-mm gap exists when the temperature is 24°C , determine (a) the temperature at which the normal stress in the aluminum bar will be equal to -75 MPa , (b) the corresponding exact length of the aluminum bar.

$$\sigma_a = -75 \text{ MPa}$$

$$P = -\sigma_a A_a = -(75 \times 10^6)(1800 \times 10^{-6}) = -135 \text{ kN}$$

Shortening due to P

$$\begin{aligned} \delta_P &= \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a} \\ &= \frac{(135000)(0.35)}{(105 \times 10^9)(1500 \times 10^{-6})} + \frac{(135000)(0.45)}{(73 \times 10^9)(1800 \times 10^{-6})} \\ &= 762.3 \times 10^{-6} \text{ m} \end{aligned}$$

Available elongation for thermal expansion

$$\delta_T = (0.0005 + 762.3 \times 10^{-6}) = 1.2623 \times 10^{-3} \text{ m}$$

$$\text{But } \delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$$

$$= (0.35)(21.6 \times 10^{-6})(\Delta T) + (0.45)(23.2 \times 10^{-6})(\Delta T) = (18 \times 10^{-6}) \Delta T$$

$$\text{Equating } (18 \times 10^{-6}) \Delta T = 1.2623 \times 10^{-3} \quad \Delta T = 70.1^\circ\text{C}$$

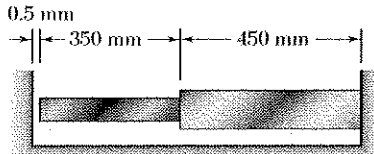
$$(a) \quad T_{\text{hot}} = T_{\text{cold}} + \Delta T = 24 + 70.1 = 94.1^\circ\text{C}$$

$$\begin{aligned} (b) \quad \delta_a &= L_a \alpha_a (\Delta T) - \frac{PL_a}{E_a A_a} \\ &= (0.45)(23.2 \times 10^{-6})(70.1) - \frac{(135000)(0.45)}{(73 \times 10^9)(1800 \times 10^{-6})} = 0.27 \times 10^{-3} \text{ m} \end{aligned}$$

$$L_{\text{exact}} = 0.45 + 0.27 \times 10^{-3} = 0.45027 \text{ m}$$

Problem 2.59

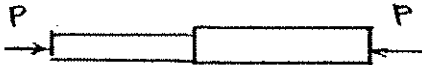
2.59 Determine (a) the compressive force in the bars shown after a temperature rise of 82°C ; (b) the corresponding change in length of the bronze bar.



Bronze	Aluminum
$A = 1500 \text{ mm}^2$	$A = 1800 \text{ mm}^2$
$E = 105 \text{ GPa}$	$E = 73 \text{ GPa}$
$\alpha = 21.6 \times 10^{-6}/^\circ\text{C}$	$\alpha = 23.2 \times 10^{-6}/^\circ\text{C}$

Thermal expansion if free of constraint

$$\begin{aligned} S_T &= L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T) \\ &= (0.35)(21.6 \times 10^{-6})(82) + (0.45)(23.2 \times 10^{-6})(82) \\ &= 1.476 \times 10^{-3} \text{ m} \end{aligned}$$



Constrained expansion. $S = 0.5 \text{ mm}$

Shortening due to induced compressive force P

$$S_p = 1.476 \times 10^{-3} - 0.0005 = 0.976 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \text{But } S_p &= \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a} = \left(\frac{L_b}{E_b A_b} + \frac{L_a}{E_a A_a} \right) P \\ &= \left(\frac{0.35}{(105 \times 10^9)(1500 \times 10^{-6})} + \frac{0.45}{(73 \times 10^9)(1800 \times 10^{-6})} \right) P = 5.647 \times 10^{-9} P \end{aligned}$$

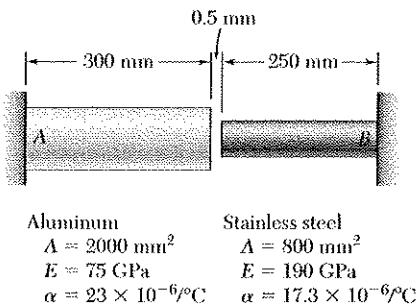
$$(a) \text{ Equating } 5.647 \times 10^{-9} P = 0.976 \times 10^{-3} \quad P = 172835 \text{ N}$$

$$172.8 \text{ kN} \blacktriangleleft$$

$$\begin{aligned} (b) S_b &= L_b \alpha_b (\Delta T) - \frac{PL_b}{E_b A_b} \\ &= (0.35)(21.6 \times 10^{-6})(82) - \frac{(172835)(0.35)}{(105 \times 10^9)(1500 \times 10^{-6})} = 0.2358 \times 10^{-3} \text{ m} = 0.236 \text{ mm} \blacktriangleleft \end{aligned}$$

Problem 2.60

2.60 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (a) the normal stress in the aluminum rod, (b) the change in length of the aluminum rod.



$$\Delta T = 140 - 20 = 120^\circ\text{C}$$

Free thermal expansion.

$$\begin{aligned}\delta_T &= L_a \alpha_a (\Delta T) + L_s \alpha_s (\Delta T) \\ &= (0.300)(23 \times 10^{-6})(120) + (0.250)(17.3 \times 10^{-6})(120) \\ &= 1.347 \times 10^{-3} \text{ m}\end{aligned}$$

Shortening due to P to meet constraint.

$$\delta_P = 1.347 \times 10^{-3} - 0.5 \times 10^{-3} = 0.847 \times 10^{-3} \text{ m}$$

$$\begin{aligned}\delta_P &= \frac{P L_a}{E_a A_a} + \frac{P L_s}{E_s A_s} = \left(\frac{L_a}{E_a A_a} + \frac{L_s}{E_s A_s} \right) P \\ &= \left(\frac{0.300}{(75 \times 10^9)(2000 \times 10^{-6})} + \frac{0.250}{(190 \times 10^9)(800 \times 10^{-6})} \right) P = 3.6447 \times 10^{-9} P\end{aligned}$$

Equating; $3.6447 \times 10^{-9} P = 0.847 \times 10^{-3}$ $P = 232.39 \times 10^3 \text{ N}$

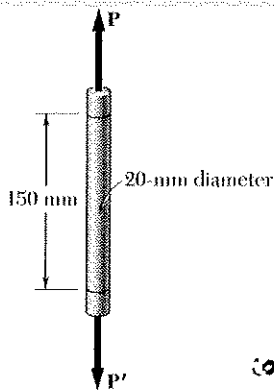
(a) $\sigma_a = -\frac{P}{A_a} = -\frac{232.39 \times 10^3}{2000 \times 10^{-6}} = -116.2 \times 10^6 \text{ Pa} \quad -116.2 \text{ MPa} \quad \blacktriangleleft$

(b) $\delta_a = L_a \alpha_a (\Delta T) - \frac{P L_a}{E_a A_a}$

$$= (0.300)(23 \times 10^{-6})(120) - \frac{(232.39 \times 10^3)(0.300)}{(75 \times 10^9)(2000 \times 10^{-6})} = 363 \times 10^{-6} \text{ m}$$

0.363 mm $\blacktriangleleft$

Problem 2.61



2.61 In a standard tensile test, an aluminum rod of 20-mm diameter is subjected to a tension force of $P = 30$ kN. Knowing that $\nu = 0.35$ and $E = 70$ GPa, determine (a) the elongation of the rod in an 150-mm gage length, (b) the change in diameter of the rod.

$$P = 30 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (20)^2 = 314.159 \text{ mm}^2 = 314.159 \times 10^{-6} \text{ m}^2$$

$$\sigma_y = \frac{P}{A} = \frac{30 \times 10^3}{314.159 \times 10^{-6}} = 95.493 \times 10^6 \text{ Pa}$$

$$\epsilon_y = \frac{\sigma_y}{E} = \frac{95.493 \times 10^6}{70 \times 10^9} = 1.36418 \times 10^{-3}$$

$$(a) \quad \delta_y = L \epsilon_y = (150 \times 10^{-3})(1.36418 \times 10^{-3}) = 205 \times 10^{-6} \text{ m}$$

$$0.205 \text{ mm} \quad \blacktriangleleft$$

$$\epsilon_x = -\nu \epsilon_y = -(0.35)(1.36418 \times 10^{-3}) = -477.46 \times 10^{-6}$$

$$(b) \quad \delta_x = d \epsilon_x = (20 \times 10^{-3})(-477.46 \times 10^{-6}) = -9.55 \times 10^{-6} \text{ m}$$

$$-0.00955 \text{ mm} \quad \blacktriangleleft$$

Problem 2.62

2.62 A 20-mm-diameter rod made of an experimental plastic is subjected to a tensile force of magnitude $P = 6$ kN. Knowing that an elongation of 14 mm and a decrease in diameter of 0.85 mm are observed in a 150-mm length, determine the modulus of elasticity, the modulus of rigidity and Poisson's ratio for the material.

Let the y-axis be along the length of the rod and the x-axis be transverse.

$$A = \frac{\pi}{4} (20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2 \quad P = 6 \times 10^3 \text{ N}$$

$$\sigma_y = \frac{P}{A} = \frac{6 \times 10^3}{314.16 \times 10^{-6}} = 19.0985 \times 10^6 \text{ Pa}$$

$$\epsilon_y = \frac{\delta_y}{L} = \frac{14 \text{ mm}}{150 \text{ mm}} = 0.093333$$

$$\text{Modulus of elasticity: } E = \frac{\sigma_y}{\epsilon_y} = \frac{19.0985 \times 10^6}{0.093333} = 204.63 \times 10^6 \text{ Pa}$$

$$E = 205 \text{ MPa} \quad \blacktriangleleft$$

$$\epsilon_x = \frac{\delta_x}{d} = -\frac{0.85}{20} = -0.0425$$

$$\text{Poisson's ratio: } \nu = -\frac{\epsilon_x}{\epsilon_y} = -\frac{-0.0425}{0.093333}$$

$$\nu = 0.455 \quad \blacktriangleleft$$

$$\text{Modulus of rigidity: } G = \frac{E}{2(1+\nu)} = \frac{204.63 \times 10^6}{2(1+0.455)} = 70.31 \times 10^6 \text{ Pa}$$

$$G = 70.3 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.63

2.63 A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200 \text{ GPa}$, $\nu = 0.30$). Determine the resulting change (a) in the 50-mm gage length, (b) in the width of portion AB of the test coupon, (c) in the thickness of portion AB, (d) in the cross-sectional area of portion AB.

$$A = (1.6)(12) = 19.2 \text{ mm}^2 \\ = 19.2 \times 10^{-6} \text{ m}^2$$

$$P = 2.75 \times 10^3 \text{ N}$$

$$\sigma_x = \frac{P}{A} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} \\ = 143.229 \times 10^6 \text{ Pa}$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\epsilon_y = \epsilon_z = -\nu \epsilon_x = -(0.30)(716.15 \times 10^{-6}) = -214.84 \times 10^{-6}$$

$$(a) \quad L = 0.050 \text{ m} \quad \delta_x = L \epsilon_x = (0.050)(716.15 \times 10^{-6}) = 35.81 \times 10^{-6} \text{ m}$$

$$0.0358 \text{ mm}$$

$$(b) \quad W = 0.012 \text{ m} \quad \delta_y = W \epsilon_y = (0.012)(-214.84 \times 10^{-6}) = -2.578 \times 10^{-6} \text{ m}$$

$$-0.00258 \text{ mm}$$

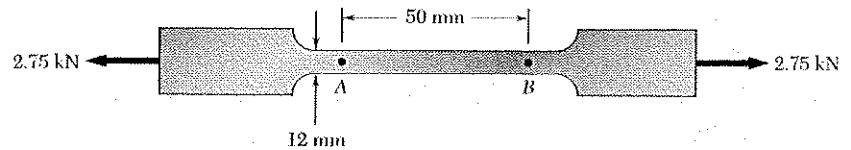
$$(c) \quad t = 0.0016 \text{ m} \quad \delta_z = t \epsilon_z = (0.0016)(-214.84 \times 10^{-6}) = -3437 \times 10^{-9} \text{ m}$$

$$-0.0003437 \text{ mm}$$

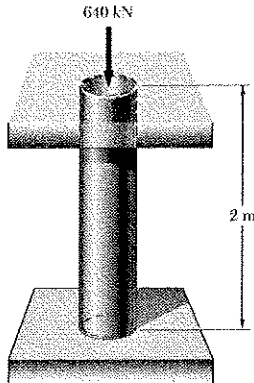
$$(d) \quad A = w_0(1 + \epsilon_y)t_0(1 + \epsilon_z) = w_0 t_0(1 + \epsilon_y + \epsilon_z + \epsilon_y \epsilon_z) \quad A_0 = w_0 t_0$$

$$\Delta A = A - A_0 = w_0 t_0(\epsilon_y + \epsilon_z + \text{negligible term}) \approx 2w_0 t_0 \epsilon_y$$

$$= (2)(0.012)(0.0016)(-214.84 \times 10^{-6}) = -8.25 \times 10^{-9} \text{ m}^2 = -0.00825 \text{ mm}^2$$



Problem 2.64



2.64 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (a) the change in length of the pipe, (b) the change in its outer diameter, (c) the change in its wall thickness.

$$d_o = 240 \text{ mm} \quad t = 10 \text{ mm} \quad d_i = d_o - 2t = 220 \text{ mm}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3 \text{ mm}^2 \\ = 7.2257 \times 10^{-3} \text{ m}^2$$

$$P = 640 \times 10^3 \text{ N}$$

$$(a) \delta = -\frac{PL}{AE} = -\frac{(640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3} \text{ m} \\ = -2.43 \text{ mm} \quad \blacktriangleleft$$

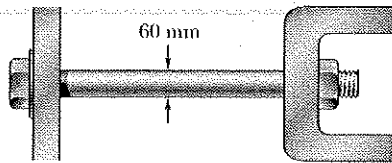
$$\epsilon = \frac{\delta}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$$

$$\epsilon_{\text{lat}} = -\nu \epsilon = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$$

$$(b) \Delta d_o = d_o \epsilon_{\text{lat}} = (240)(400.4 \times 10^{-6}) = 0.0961 \text{ mm} \quad \blacktriangleleft$$

$$(c) \Delta t = t \epsilon_{\text{lat}} = (10)(400.4 \times 10^{-6}) = 0.00400 \text{ mm} \quad \blacktriangleleft$$

Problem 2.65



2.65 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 200 \text{ GPa}$ and $\nu = 0.29$, determine the internal force in the bolt, if the diameter is observed to decrease by $13 \mu\text{m}$.

$$\delta_y = -13 \times 10^{-6} \text{ m} \quad d = 60 \times 10^{-3} \text{ m}$$

$$\epsilon_y = \frac{\delta_y}{d} = -\frac{13 \times 10^{-6}}{60 \times 10^{-3}} = -216.67 \times 10^{-6}$$

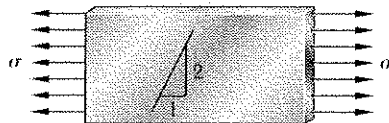
$$\nu = -\frac{\epsilon_y}{\epsilon_x} \quad \therefore \quad \epsilon_x = -\frac{\epsilon_y}{\nu} = \frac{216.67 \times 10^{-6}}{0.29} = 747.13 \times 10^{-6}$$

$$\sigma_x = E \epsilon_x = (200 \times 10^9)(747.13 \times 10^{-6}) = 149.43 \times 10^6 \text{ Pa}$$

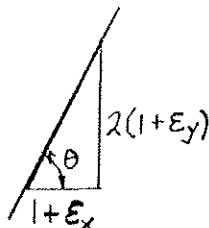
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (60)^2 = 2.827 \times 10^3 \text{ mm}^2 = 2.827 \times 10^{-3} \text{ m}^2$$

$$F = \sigma_x A = (149.43 \times 10^6)(2.827 \times 10^{-3}) = 422 \times 10^3 \text{ N} \\ = 422 \text{ kN}$$

Problem 2.66



2.66 An aluminum plate ($E = 74 \text{ GPa}$ and $\nu = 0.33$) is subjected to a centric axial load that causes a normal stress σ . Knowing that, before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when $\sigma = 125 \text{ MPa}$.



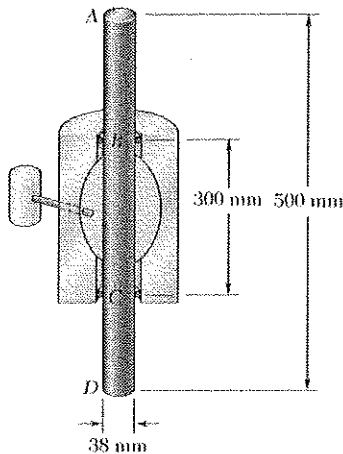
$$\text{The slope after deformation is } \tan \theta = \frac{2(1 + \epsilon_y)}{1 + \epsilon_x}$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{125 \times 10^6}{74 \times 10^9} = 1.6892 \times 10^{-3}$$

$$\epsilon_y = -\nu \epsilon_x = -(0.33)(1.6892 \times 10^{-3}) = -0.5574 \times 10^{-3}$$

$$\tan \theta = \frac{2(1 - 0.0005574)}{1 + 0.0016892} = 1.99551$$

Problem 2.67



2.67 The aluminum rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 42 MPa to the 300-mm portion BC of the rod. Knowing that $E = 70$ GPa and $\nu = 0.36$, determine (a) the change in the total length AD, (b) the change in diameter at the middle of the rod.

$$\sigma_x = \sigma_z = -p = -42 \text{ MPa} \quad \sigma_y = 0$$

$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) \\ &= \frac{1}{70 \times 10^9} [-42 \times 10^6 - (0.36)(0) - (0.36)(-42 \times 10^6)] \\ &= -384 \times 10^{-6} \end{aligned}$$

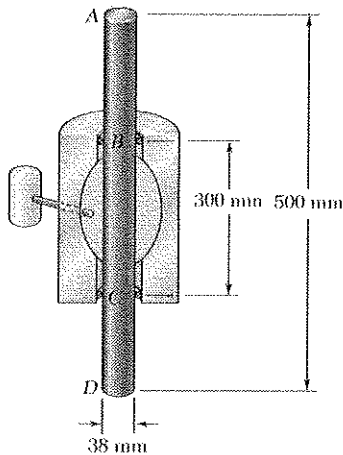
$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) \\ &= \frac{1}{70 \times 10^9} [-(0.36)(-42 \times 10^6) + 0 - (0.36)(-42 \times 10^6)] \\ &= 432 \times 10^{-6} \end{aligned}$$

Length subject to strain ϵ_x : $L = 0.3 \text{ m}$

$$(a) \quad \delta_y = L \epsilon_y = (0.3)(432 \times 10^{-6}) = 0.1296 \times 10^{-3} \text{ m} = 0.13 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \delta_x = d \epsilon_x = (0.038)(-384 \times 10^{-6}) = -0.01459 \times 10^{-3} \text{ m} = -0.0146 \text{ mm} \quad \blacktriangleleft$$

Problem 2.68



2.68 For the rod of Prob. 2.67, determine the forces that should be applied to the ends A and D of the rod (a) if the axial strain in portion BC of the rod is to remain zero as the hydrostatic pressure is applied, (b) if the total length AD of the rod is to remain unchanged.

2.67 The aluminum rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 42 MPa to the 300-mm portion BC of the rod. Knowing that $E = 70$ GPa and $\nu = 0.36$, determine (a) the change in the total length AD, (b) the change in diameter at the middle of the rod.

Over the pressurized portion BC

$$\sigma_x = \sigma_z = -p \quad \sigma_y = \sigma_y$$

$$(\epsilon_y)_{BC} = \frac{1}{E} (-2\sigma_x + \sigma_y - 2\sigma_z)$$

$$= \frac{1}{E} (2\nu p + \sigma_y)$$

$$(a) (\epsilon_y)_{BC} = 0 \quad 2\nu p + \sigma_y = 0$$

$$\sigma_y = -2\nu p = -(2)(0.36)(42 \times 10^6)$$

$$= -30.24 \text{ MPa}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.038)^2 = 1.134 \times 10^{-3} \text{ m}^2$$

$$F = A \sigma_y = (1.134 \times 10^{-3})(-30.24 \times 10^6) = -34292 \text{ N}$$

i.e. 34.3 kN compression

(b) Over unpressurized portions AB and CD $\sigma_x = \sigma_z = 0$

$$(\epsilon_y)_{AB} = (\epsilon_y)_{CD} = \frac{\sigma_y}{E}$$

For no change in length

$$\delta = L_{AB}(\epsilon_y)_{AB} + L_{BC}(\epsilon_y)_{BC} + L_{CD}(\epsilon_y)_{CD} = 0$$

$$(L_{AB} + L_{CD})(\epsilon_y)_{AB} + L_{BC}(\epsilon_y)_{BC} = 0$$

$$(0.5 - 0.3) \frac{\sigma_y}{E} + \frac{0.3}{E} (2\nu p + \sigma_y) = 0$$

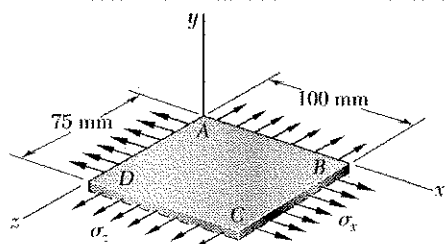
$$\sigma_y = -\frac{0.6\nu p}{0.1} = -\frac{(0.6)(0.36)(42 \times 10^6)}{0.1} = -90.72 \text{ MPa}$$

$$P = A \sigma_y = (1.134 \times 10^{-3})(-90.72 \times 10^6) = -102876 \text{ N}$$

i.e. 102.9 kN compression

Problem 2.69

2.69 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (a) side AB , (b) side BC , (c) diagonal AC .



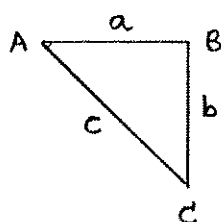
$$\sigma_x = 120 \times 10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160 \times 10^6 \text{ Pa}$$

$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) \\ &= \frac{1}{87 \times 10^9} [120 \times 10^6 - (0.34)(160 \times 10^6)] \\ &= 754.02 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} \epsilon_z &= \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_z) = \frac{1}{87 \times 10^9} [-(0.34)(120 \times 10^6) + 160 \times 10^6] \\ &= 1.3701 \times 10^{-3} \end{aligned}$$

$$(a) \quad \delta_{AB} = (\overline{AB}) \epsilon_x = (100 \text{ mm})(754.02 \times 10^{-6}) = 0.0754 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \delta_{BC} = (\overline{BC}) \epsilon_z = (75 \text{ mm})(1.3701 \times 10^{-3}) = 0.1028 \text{ mm} \quad \blacktriangleleft$$



Label sides of right triangle ABC as a , b , and c .

$$c^2 = a^2 + b^2$$

Obtain differentials by calculus.

$$2c \, dc = 2a \, da + 2b \, db$$

$$dc = \frac{a}{c} da + \frac{b}{c} db$$

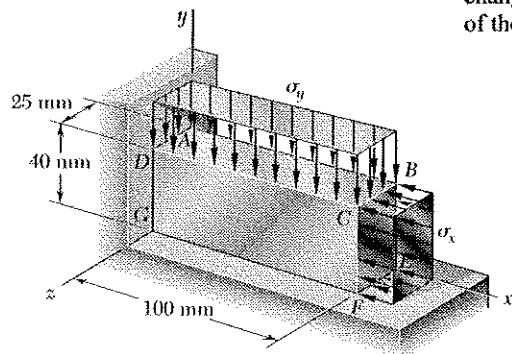
$$\text{But, } a = 100 \text{ mm, } b = 75 \text{ mm, } c = \sqrt{100^2 + 75^2} = 125 \text{ mm}$$

$$da = \delta_{AB} = 0.0754 \text{ mm} \quad db = \delta_{BC} = 0.1028 \text{ mm}$$

$$(c) \quad \delta_{AC} = dc = \frac{100}{125} (0.0754) + \frac{75}{125} (0.1028) = 0.1220 \text{ mm} \quad \blacktriangleleft$$

Problem 2.70

2.70 The block shown is made of a magnesium alloy for which $E = 45 \text{ GPa}$ and $\nu = 0.35$. Knowing that $\sigma_x = -180 \text{ MPa}$, determine (a) the magnitude of σ_y for which the change in the height of the block will be zero, (b) the corresponding change in the area of the face $ABCD$, (c) the corresponding change in the volume of the block.



$$\epsilon_y = 0 \quad \epsilon_y = 0 \quad \sigma_z = 0$$

$$\epsilon_y = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z)$$

$$\sigma_y = \nu \sigma_x = (0.35)(-180 \times 10^6)$$

$$= -63 \times 10^6 \text{ Pa} \quad \sigma_y = -63 \text{ MPa} \quad \blacktriangleleft$$

$$\epsilon_z = \frac{1}{E} (\sigma_z - \nu \sigma_x - \nu \sigma_y) = -\frac{\nu}{E} (\sigma_x + \sigma_y) = \frac{(0.35)(-243 \times 10^6)}{45 \times 10^9} = -1.89 \times 10^{-3}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{\sigma_x - \nu \sigma_y}{E} = \frac{157.95 \times 10^6}{45 \times 10^9} = -3.51 \times 10^{-3}$$

(b) $A_0 = L_x L_z$

$$A = L_x (1 + \epsilon_x) L_z (1 + \epsilon_z) = L_x L_z (1 + \epsilon_x + \epsilon_z + \epsilon_x \epsilon_z)$$

$$\Delta A = A - A_0 = L_x L_z (\epsilon_x + \epsilon_z + \epsilon_x \epsilon_z) \approx L_x L_z (\epsilon_x + \epsilon_z)$$

$$\Delta A = (100 \text{ mm})(25 \text{ mm})(-3.51 \times 10^{-3} - 1.89 \times 10^{-3}) \quad \Delta A = -13.50 \text{ mm}^2 \quad \blacktriangleleft$$

(c) $V_0 = L_x L_y L_z$

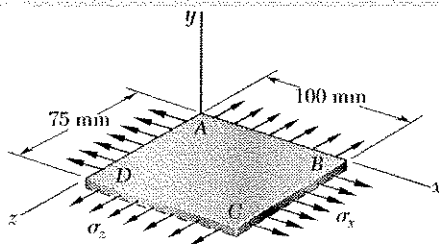
$$V = L_x (1 + \epsilon_x) L_y (1 + \epsilon_y) L_z (1 + \epsilon_z)$$

$$= L_x L_y L_z (1 + \epsilon_x + \epsilon_y + \epsilon_z + \epsilon_x \epsilon_y + \epsilon_y \epsilon_z + \epsilon_z \epsilon_x + \epsilon_x \epsilon_y \epsilon_z)$$

$$\Delta V = V - V_0 = L_x L_y L_z (\epsilon_x + \epsilon_y + \epsilon_z + \text{small terms})$$

$$\Delta V = (100)(40)(25)(-3.51 \times 10^{-3} + 0 - 1.89 \times 10^{-3}) \quad \Delta V = -540 \text{ mm}^3 \quad \blacktriangleleft$$

Problem 2.71



2.71 The homogeneous plate $ABCD$ is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\epsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ratio, determine (a) the required magnitude of σ_x , (b) the ratio σ_0 / ϵ_z .

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \epsilon_x = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{1}{E} (\sigma_x - \nu \sigma_0)$$

$$(a) \quad \sigma_x = \nu \sigma_0$$

$$(b) \quad \epsilon_z = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_z) = \frac{1}{E} (-\nu^2 \sigma_0 - 0 + \sigma_0) = \frac{1-\nu^2}{E} \sigma_0$$

$$\frac{\sigma_0}{\epsilon_z} = \frac{E}{1-\nu^2}$$

Problem 2.72

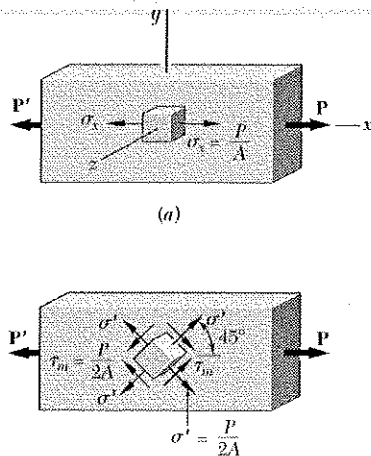


Fig. 1.40 (b)

2.72 For a member under axial loading, express the normal strain ϵ' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ϵ_x by (a) comparing the hypotenuses of the triangles shown in Fig. 2.54, which represent respectively an element before and after deformation, (b) using the values of the corresponding stresses σ' and σ_x shown in Fig. 1.40, and the generalized Hooke's law.

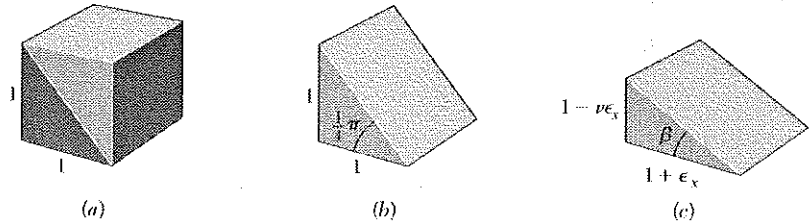
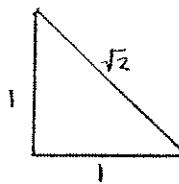
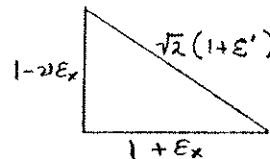


Fig. 2.54

(a)



Before deformation



After deformation

$$[\sqrt{2}(1 + \epsilon')]^2 = (1 + \epsilon_x)^2 + (1 - \nu\epsilon_x)^2$$

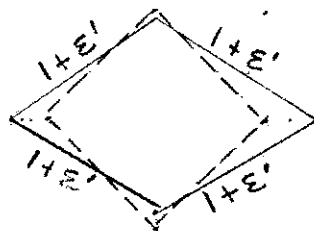
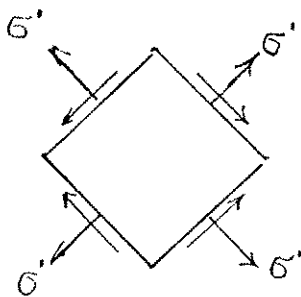
$$2(1 + 2\epsilon' + \epsilon'^2) = 1 + 2\epsilon_x + \epsilon_x^2 + 1 - 2\nu\epsilon_x + \nu^2\epsilon_x^2$$

$$4\epsilon' + 2\epsilon'^2 = 2\epsilon_x + \epsilon_x^2 - 2\nu\epsilon_x + \nu^2\epsilon_x^2$$

Neglect squares as small $4\epsilon' = 2\epsilon_x - 2\nu\epsilon_x$

$$\epsilon' = \frac{1 - \nu}{2} \epsilon_x$$

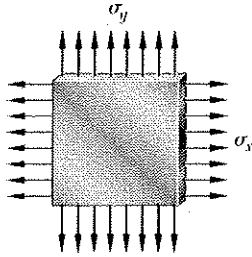
(b)



$$\begin{aligned} \epsilon' &= \frac{\sigma'}{E} - \frac{\nu\sigma'}{E} \\ &= \frac{1 - \nu}{E} \cdot \frac{P}{2A} \\ &= \frac{1 - \nu}{2E} \sigma_x \\ &= \frac{1 - \nu}{2} \epsilon_x \end{aligned}$$

Problem 2.73

2.73 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ϵ_x and ϵ_y have been determined experimentally, we can express σ_x , σ_y and ϵ_z as follows:



$$\sigma_x = E \frac{\epsilon_x + \nu \epsilon_y}{1 - \nu^2}$$

$$\sigma_y = E \frac{\epsilon_y + \nu \epsilon_x}{1 - \nu^2}$$

$$\epsilon_z = -\frac{\nu}{1 - \nu} (\epsilon_x + \epsilon_y)$$

$$\sigma_z = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) \quad (1)$$

$$\epsilon_y = \frac{1}{E} (-\nu \sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by ν and adding to (1)

$$\epsilon_x + \nu \epsilon_y = \frac{1 - \nu^2}{E} \sigma_x \quad \text{or} \quad \sigma_x = \frac{E}{1 - \nu^2} (\epsilon_x + \nu \epsilon_y)$$

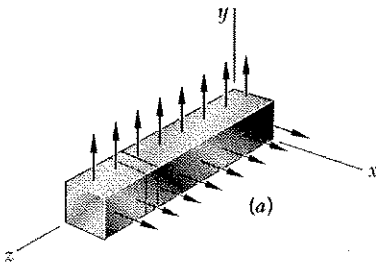
Multiplying (1) by ν and adding to (2)

$$\epsilon_y + \nu \epsilon_x = \frac{1 - \nu^2}{E} \sigma_y \quad \text{or} \quad \sigma_y = \frac{E}{1 - \nu^2} (\epsilon_y + \nu \epsilon_x)$$

$$\begin{aligned} \epsilon_z &= \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y) = -\frac{\nu}{E} \cdot \frac{E}{1 - \nu^2} (\epsilon_x + \nu \epsilon_y + \epsilon_y + \nu \epsilon_x) \\ &= -\frac{\nu(1 + \nu)}{1 - \nu^2} (\epsilon_x + \epsilon_y) = -\frac{\nu}{1 - \nu} (\epsilon_x + \epsilon_y) \end{aligned}$$

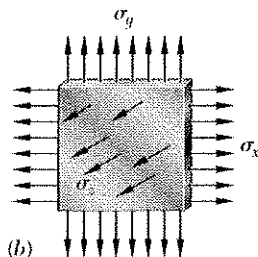
Problem 2.74

2.74 In many situations physical constraints prevent strain from occurring in a given direction, for example $\epsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express σ_z , ϵ_x and ϵ_y as follows:



$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\epsilon_x = \frac{1}{E} [(1 - \nu^2)\sigma_x - \nu(1 + \nu)\sigma_y] \quad \epsilon_y = \frac{1}{E} [(1 - \nu^2)\sigma_y - \nu(1 + \nu)\sigma_x]$$

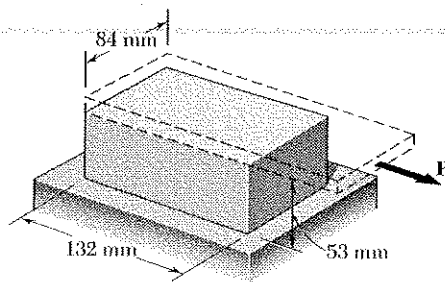


$$\epsilon_z = 0 = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = \nu(\sigma_x + \sigma_y)$$

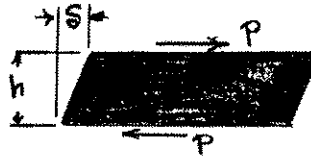
$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{1}{E} [\sigma_x - \nu \sigma_y - \nu^2 (\sigma_x + \sigma_y)] \\ &= \frac{1}{E} [(1 - \nu^2)\sigma_x - \nu(1 + \nu)\sigma_y] \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) = \frac{1}{E} [-\nu \sigma_x + \sigma_y - \nu^2 (\sigma_x + \sigma_y)] \\ &= \frac{1}{E} [(1 - \nu^2)\sigma_y - \nu(1 + \nu)\sigma_x] \end{aligned}$$

Problem 2.75



2.75 The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force P is applied. Knowing that for the plastic used $G = 385 \text{ MPa}$, determine the deflection of the plate when $P = 36 \text{ kN}$.



Consider the plastic block. The shearing force carried is $P = 36 \text{ kN}$

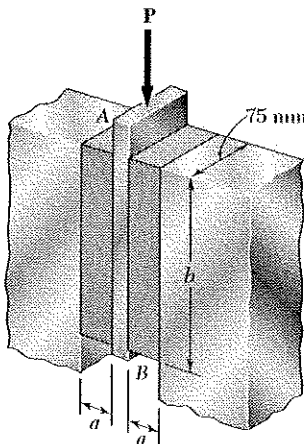
The area is $A = (0.084)(0.132) = 11.088 \times 10^{-3} \text{ m}^2$

Shearing stress, $\tau = \frac{P}{A} = \frac{36 \times 10^3}{11.088 \times 10^{-3}} = 3.24675 \text{ MPa}$.

Shearing strain, $\gamma = \frac{\tau}{G} = \frac{3.24675}{385} = 0.008433$

But $\gamma = \frac{S}{h} \therefore S = h \gamma = (53)(0.008433) = 0.45 \text{ mm}$

Problem 2.76



2.76 A vibration isolation unit consists of two blocks of hard rubber bonded to plate AB and to rigid supports as shown. For the type and grade of rubber used $\tau_{all} = 1.54 \text{ MPa}$ and $G = 12.6 \text{ MPa}$. Knowing that a centric vertical force of magnitude $P = 13 \text{ kN}$ must cause a 2.5-mm vertical deflection of the plate AB , determine the smallest allowable dimensions a and b of the block.

Consider the rubber block on the right. It carries a shearing force equal to $\frac{1}{2}P$

The shearing stress is $\tau = \frac{\frac{1}{2}P}{A}$

or required $A = \frac{P}{2\tau} = \frac{13 \times 10^3}{(2)(1.54 \times 10^6)} = 4.221 \times 10^{-3} \text{ m}^2$

But $A = (3.0) b$

Hence $b = \frac{A}{0.075} = 0.05628 \text{ m}$ $b_{min} = 56.3 \text{ mm}$

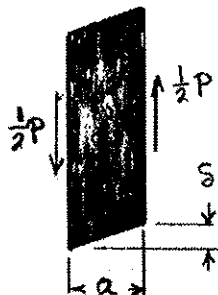
Use $b = 56.3 \text{ mm}$ and $\tau = 1.54 \text{ MPa}$

Shearing strain, $\gamma = \frac{\tau}{G} = \frac{1.54}{12.6} = 0.12222$

But, $\gamma = \frac{S}{a}$

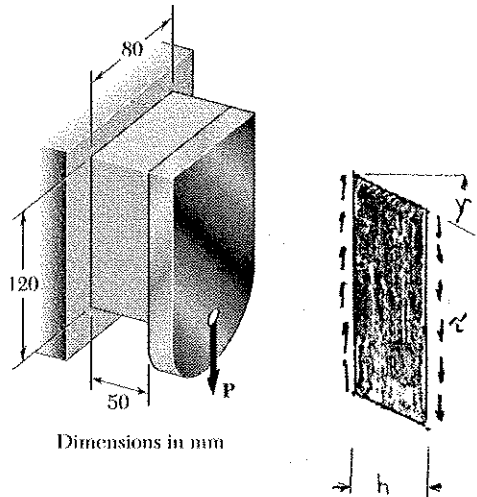
Hence, $a = \frac{S}{\gamma} = \frac{2.5}{0.12222} = 20.45 \text{ mm}$

$a_{min} = 20.5 \text{ mm}$



Problem 2.77

2.77 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load P is applied. Knowing that for the plastic used $G = 1050$ MPa, determine the deflection of the plate.



$$A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$P = 240 \times 10^3 \text{ N}$$

$$\tau = \frac{P}{A} = \frac{240 \times 10^3}{9.6 \times 10^{-3}} = 25 \times 10^6 \text{ Pa}$$

$$G = 1050 \times 10^6 \text{ Pa}$$

$$\gamma = \frac{\tau}{G} = \frac{25 \times 10^6}{1050 \times 10^6} = 23.810 \times 10^{-3}$$

$$h = 50 \text{ mm} = 0.050 \text{ m}$$

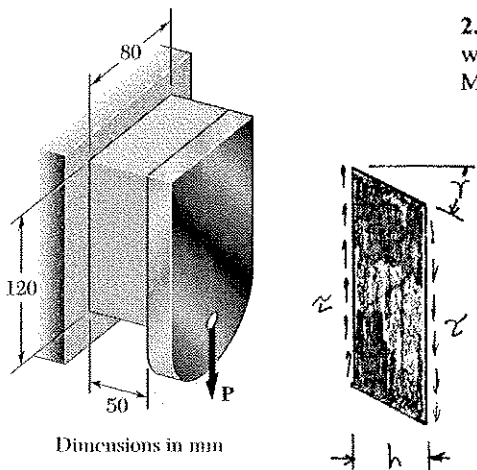
$$\delta = h\gamma = (0.050)(23.810 \times 10^{-3})$$

$$= 1.190 \times 10^{-3} \text{ m} \quad 1.091 \text{ mm} \downarrow$$

Problem 2.78

2.78 What load P should be applied to the plate of Prob. 2.77 to produce a 1.5-mm deflection?

2.77 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 240-kN load P is applied. Knowing that for the plastic used $G = 1050$ MPa, determine the deflection of the plate.



$$\delta = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$h = 50 \text{ mm} = 50 \times 10^{-3} \text{ m}$$

$$\gamma = \frac{\delta}{h} = \frac{1.5 \times 10^{-3}}{50 \times 10^{-3}} = 30 \times 10^{-3}$$

$$G = 1050 \times 10^6 \text{ Pa}$$

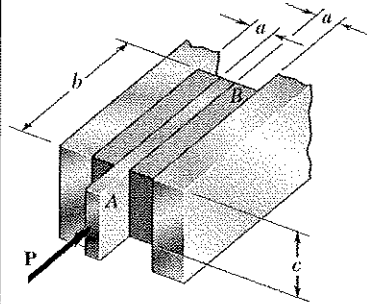
$$\tau = G\gamma = (1050 \times 10^6)(30 \times 10^{-3}) = 31.5 \times 10^6 \text{ Pa}$$

$$A = (120)(80) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

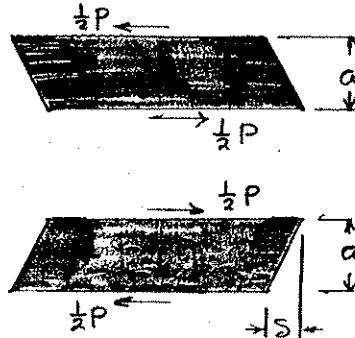
$$P = \tau A = (31.5 \times 10^6)(9.6 \times 10^{-3}) = 302 \times 10^3 \text{ N} \quad 302 \text{ kN} \blacktriangleleft$$

NOTE: In problems 2.75 through 2.82, 2.87, 2.88, and 2.132 it is assumed that γ is the tangent of the angle change. This makes the relative displacement proportional to the force.

Problem 2.79



2.79 Two blocks of rubber with a modulus of rigidity $G = 12 \text{ MPa}$ are bonded to rigid supports and to a plate AB . Knowing that $c = 100 \text{ mm}$ and $P = 40 \text{ kN}$, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm .



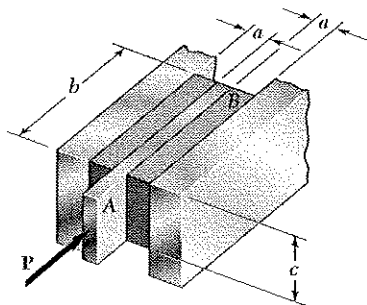
Shearing strain $\gamma = \frac{\delta}{a} = \frac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(12 \times 10^6)(0.005)}{1.4 \times 10^6} = 0.04286 \text{ m} = 43 \text{ mm}$$

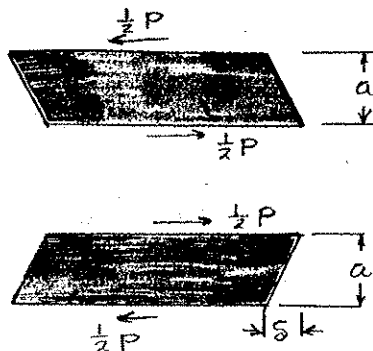
Shearing stress $\tau = \frac{1/2 P}{A} = \frac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{40 \times 10^3}{(2)(0.1)(1.4 \times 10^6)} = 0.143 \text{ m} = 143 \text{ mm}$$

Problem 2.80



2.80 Two blocks of rubber with a modulus of rigidity $G = 10 \text{ MPa}$ are bonded to rigid supports and to a plate AB . Knowing that $b = 200 \text{ mm}$ and $c = 125 \text{ mm}$, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm .



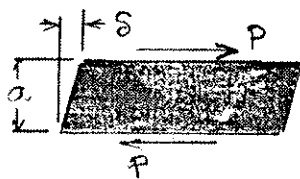
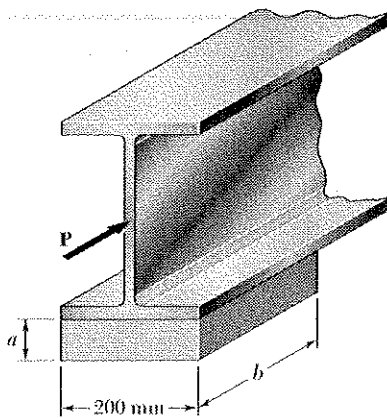
Shearing stress $\tau = \frac{1/2 P}{A} = \frac{P}{2bc}$

$$P = 2bc\tau = (2)(0.2)(0.125)(1.5 \times 10^6) = 75 \text{ kN}$$

Shearing strain $\gamma = \frac{\delta}{a} = \frac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(10 \times 10^6)(0.006)}{1.5 \times 10^6} = 0.04 \text{ m} = 40 \text{ mm}$$

Problem 2.81



2.81 An elastomeric bearing ($G = 0.9 \text{ MPa}$) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22-kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine (a) the smallest allowable dimension b , (b) the smallest required thickness a .

Shearing force $P = 22 \times 10^3 \text{ N}$

Shearing stress $\tau = 420 \times 10^3 \text{ Pa}$

$$\tau = \frac{P}{A} \therefore A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3} \text{ m}^2$$

$$= 52.381 \times 10^3 \text{ mm}^2$$

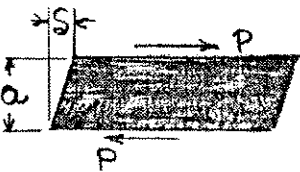
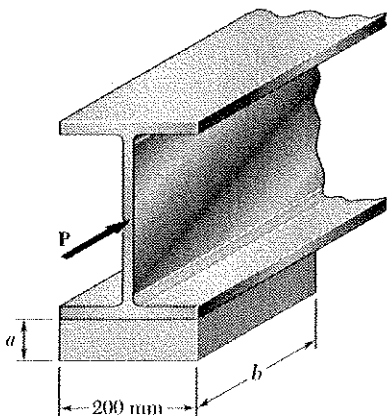
$$A = (200 \text{ mm})(b)$$

(a) $b = \frac{A}{200} = \frac{52.381 \times 10^3}{200} = 262 \text{ mm}$

$$\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$$

(b) But $\gamma = \frac{\delta}{a} \therefore a = \frac{\delta}{\gamma} = \frac{10 \text{ mm}}{466.67 \times 10^{-3}} = 21.4 \text{ mm}$

Problem 2.82



2.82 For the elastomeric bearing in Prob. 2.81 with $b = 220 \text{ mm}$ and $a = 30 \text{ mm}$, determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 19 \text{ kN}$ and a maximum displacement $\delta = 12 \text{ mm}$.

Shearing force $P = 19 \times 10^3 \text{ N}$

Area $A = (200 \text{ mm})(220 \text{ mm}) = 44 \times 10^3 \text{ mm}^2$

$$= 44 \times 10^{-3} \text{ m}^2$$

$$\tau = \frac{P}{A} = \frac{19 \times 10^3}{44 \times 10^{-3}} = 431.81 \times 10^3 \text{ Pa}$$

$$= 431 \text{ kPa}$$

Shearing strain $\gamma = \frac{\delta}{a} = \frac{12 \text{ mm}}{30 \text{ mm}} = 0.400$

Shearing modulus

$$G = \frac{\tau}{\gamma} = \frac{431.81 \times 10^3}{0.4} = 1.080 \times 10^6 \text{ Pa}$$

$$= 1.080 \text{ MPa}$$

NOTE: In problems 2.75 through 2.82, 2.87, 2.88, and 2.132 it is assumed that γ is the tangent of the angle change. This makes the relative displacement proportional to the force.

Problem 2.83

*2.83 A 150-mm diameter solid steel sphere is lowered into the ocean to a point where the pressure is 50 MPa (about 5 km below the surface). Knowing that $E = 200$ GPa and $\nu = 0.30$, determine (a) the decrease in diameter of the sphere, (b) the decrease in volume of the sphere, (c) the percent increase in the density of the sphere.

For a solid sphere $V_0 = \frac{\pi}{6} d_0^3 = \frac{\pi}{6} (0.15)^3 = 1.767 \times 10^{-3} \text{ m}^3$.

$$\sigma_x = \sigma_y = \sigma_z = -p = 50 \text{ MPa}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = -\frac{(1-2\nu)p}{E} = -\frac{(0.4)(50 \times 10^6)}{200 \times 10^9} = -100 \times 10^{-6}$$

Likewise $\epsilon_y = \epsilon_z = -100 \times 10^{-6}$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = -300 \times 10^{-6}$$

(a) $-\Delta d = -d_0 \epsilon_x = -(0.15)(100 \times 10^{-6}) = 15 \times 10^{-6} \text{ m}$ ▶

(b) $-\Delta V = -V_0 e = -(1.767 \times 10^{-3})(-300 \times 10^{-6}) = 530 \times 10^{-9} \text{ m}^3$ ▶

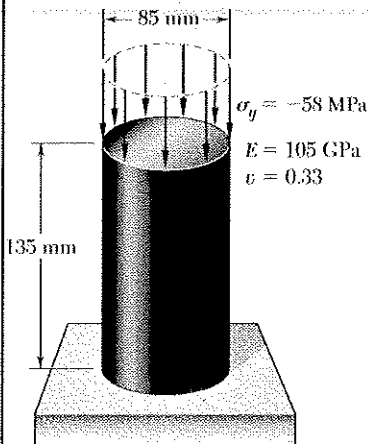
(c) Let $m = \text{mass of sphere}$, $m = \text{constant}$.

$$m = \rho_0 V_0 = \rho V = \rho V_0 (1+e)$$

$$\begin{aligned} \frac{\rho - \rho_0}{\rho_0} &= \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)} \cdot \frac{V_0}{m} - 1 = \frac{1}{1+e} - 1 \\ &= (1 - e + e^2 - e^3 + \dots) - 1 = -e + e^2 - e^3 + \dots \\ &\approx -e = 300 \times 10^{-6} \end{aligned}$$

$$\frac{\rho - \rho_0}{\rho_0} \times 100\% = (300 \times 10^{-6})(100\%) = 0.03\%$$
 ▶

Problem 2.84



*2.84 (a) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (b) Solve part a, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

$$h_o = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4} (85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_o = A_o h_o = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

$$(a) \quad \sigma_x = 0, \quad \sigma_y = -58 \times 10^6 \text{ Pa}, \quad \sigma_z = 0$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) = \frac{\sigma_y}{E} \\ &= -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6} \end{aligned}$$

$$\Delta h = h_o \epsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) = -0.0746 \text{ mm} \quad \blacktriangleleft$$

$$\begin{aligned} e &= \frac{1-2\nu}{E} (\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9} \\ &= -187.81 \times 10^{-6} \end{aligned}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) = -143.9 \text{ mm}^3 \quad \blacktriangleleft$$

$$(b) \quad \sigma_x = \sigma_y = \sigma_z = -70 \times 10^6 \text{ Pa} \quad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6 \text{ Pa}$$

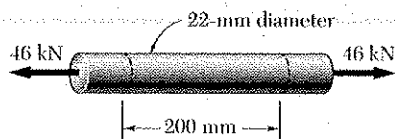
$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) = \frac{1-2\nu}{E} \sigma_y \\ &= \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6} \end{aligned}$$

$$\Delta h = h_o \epsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) = -0.0306 \text{ mm} \quad \blacktriangleleft$$

$$e = \frac{1-2\nu}{E} (\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) = -521 \text{ mm}^3 \quad \blacktriangleleft$$

Problem 2.85



*2.85 Determine the dilatation e and the change in volume of the 200-mm length of the rod shown if (a) the rod is made of steel with $E = 200$ GPa and $\nu = 0.30$, (b) the rod is made of aluminum with $E = 70$ GPa and $\nu = 0.35$.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$P = 46 \times 10^3 \text{ N} \quad \sigma_x = \frac{P}{A} = 121.01 \times 10^6 \text{ Pa} \quad \sigma_y = \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{\sigma_x}{E} \quad \epsilon_y = \epsilon_z = -\nu \epsilon_x = -\nu \frac{\sigma_x}{E}$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{(1 - 2\nu) \sigma_x}{E}$$

$$\text{Volume } V = AL = (380.13 \text{ mm}^2)(200 \text{ mm}) = 76.026 \times 10^3 \text{ mm}^3$$

$$\Delta V = V e$$

$$(a) \text{ steel: } e = \frac{(0.4)(121.01 \times 10^6)}{200 \times 10^9} = 242 \times 10^{-6}$$

$$\Delta V = (76.026 \times 10^3)(242 \times 10^{-6}) = 18.40 \text{ mm}^3$$

$$(b) \text{ aluminum: } e = \frac{(0.3)(121.01 \times 10^6)}{70 \times 10^9} = 519 \times 10^{-6}$$

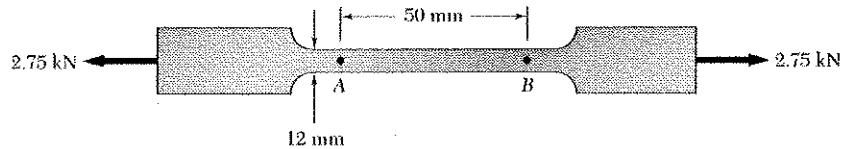
$$\Delta V = (76.026 \times 10^3)(519 \times 10^{-6}) = 39.4 \text{ mm}^3$$

Problem 2.86

*2.86 Determine the change in volume of the 50-mm gage length segment AB in Prob. 2.63 (a) by computing the dilatation of the material, (b) by subtracting the original volume of portion AB from its final volume.

2.63 A 2.75 kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200$ GPa, $\nu = 0.30$). Determine the resulting change (a) in the 50-mm gage length, (b) in the width of portion AB of the test coupon, (c) in the thickness of portion AB , (d) in the cross-sectional area of portion AB .

All calculations here
are in mm



$$(a) \quad A_0 = (12)(1.6) = 19.2 \text{ mm}^2 = 19.2 \times 10^{-6} \text{ m}^2$$

$$\text{Volume} \quad V_0 = L_0 A_0 = (50)(19.2) = 960 \text{ mm}^3$$

$$\sigma_x = \frac{P}{A_0} = \frac{2.75 \times 10^3}{19.2 \times 10^{-6}} = 143.229 \times 10^6 \text{ Pa} \quad \sigma_y = \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{\sigma_x}{E} = \frac{143.229 \times 10^6}{200 \times 10^9} = 716.15 \times 10^{-6}$$

$$\epsilon_y = \epsilon_z = -\nu \epsilon_x = -(0.30)(716.15 \times 10^{-6}) = -214.84 \times 10^{-6}$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = 286.46 \times 10^{-6}$$

$$\Delta V = V_0 e = (960)(286.46 \times 10^{-6}) = 0.275 \text{ mm}^3$$

(b) From the solution to Problem 2.62

$$\delta_x = 0.03581 \text{ mm} \quad \delta_y = -0.002578 \text{ mm} \quad \delta_z = -0.0003437 \text{ mm}$$

The dimensions when under the 2.75 kN load are:

$$\text{length} \quad L = L_0 + \delta_x = 50 + 0.03581 = 50.03581 \text{ mm}$$

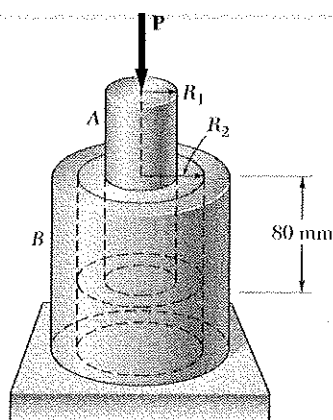
$$\text{width} \quad w = w_0 + \delta_y = 12 - 0.002578 = 11.997422 \text{ mm}$$

$$\text{thickness} \quad t = t_0 + \delta_z = 1.6 - 0.0003437 = 1.5996563 \text{ mm}$$

$$\text{volume} \quad V = Lwt = (50.03581)(11.997422)(1.5996563) = 960.275 \text{ mm}^3$$

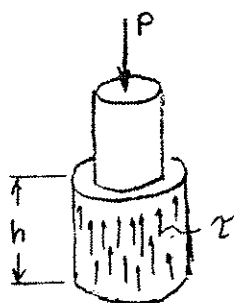
$$\Delta V = V - V_0 = 960.275 - 960 = 0.275 \text{ mm}^3$$

Problem 2.87



*2.87 A vibration isolation support consists of a rod A of radius $R_1 = 10$ mm and a tube B of inner radius $R_2 = 25$ mm bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 12$ MPa. Determine the largest allowable force P which can be applied to rod A if its deflection is not to exceed 2.50 mm.

Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \leq r \leq R_2$



Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$

Shearing deformation over radial length dr

$$\frac{dS}{dr} = \gamma$$

$$dS = \gamma dr = \frac{P}{2\pi G h} \frac{dr}{r}$$

Total deformation

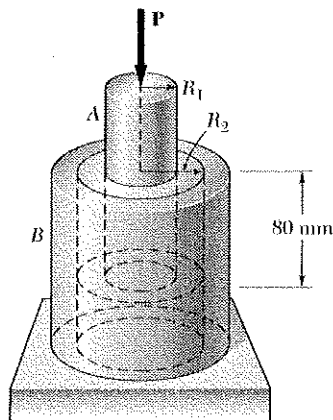
$$\begin{aligned} S &= \int_{R_1}^{R_2} dS = \frac{P}{2\pi G h} \int_{R_1}^{R_2} \frac{dr}{r} \\ &= \frac{P}{2\pi G h} \ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi G h} (\ln R_2 - \ln R_1) \\ &= \frac{P}{2\pi G h} \ln \frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi G h S}{\ln(R_2/R_1)} \end{aligned}$$

Data: $R_1 = 10 \text{ mm} = 0.010 \text{ m}$, $R_2 = 25 \text{ mm} = 0.025 \text{ m}$, $h = 80 \text{ mm} = 0.080 \text{ m}$

$$G = 12 \times 10^6 \text{ Pa} \quad S = 2.50 \times 10^{-3} \text{ m}$$

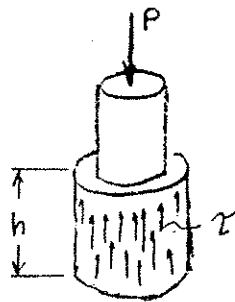
$$P = \frac{(2\pi)(12 \times 10^6)(0.080)(2.50 \times 10^{-3})}{\ln(0.025/0.010)} = 16.46 \times 10^3 \text{ N} \quad 16.46 \text{ kN} \quad \blacktriangleleft$$

Problem 2.88



*2.88 A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93$ MPa. Determine the required value of the ratio R_2/R_1 if a 10-kN force P is to cause a 2-mm deflection of rod A .

Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \leq r \leq R_2$



Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$

Shearing deformation over radial length dr

$$\frac{dS}{dr} = \gamma$$

$$dS = \gamma dr = \frac{P}{2\pi G h} \frac{dr}{r}$$

Total deformation

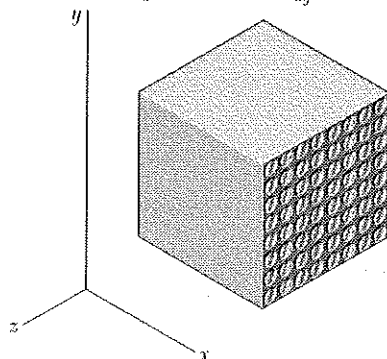
$$\begin{aligned} S &= \int_{R_1}^{R_2} dS = \frac{P}{2\pi G h} \int_{R_1}^{R_2} \frac{dr}{r} \\ &= \frac{P}{2\pi G h} \ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi G h} (\ln R_2 - \ln R_1) \\ &= \frac{P}{2\pi G h} \ln \frac{R_2}{R_1} \end{aligned}$$

$$\ln \frac{R_2}{R_1} = \frac{2\pi G h S}{P} = \frac{(2\pi)(10.93 \times 10^6)(0.080)(0.002)}{10 \times 10^3} = 1.0988$$

$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

Problem 2.89

$$\begin{aligned} E_x &= 50 \text{ GPa} & \nu_{xz} &= 0.254 \\ E_y &= 15.2 \text{ GPa} & \nu_{xy} &= 0.254 \\ E_z &= 15.2 \text{ GPa} & \nu_{zy} &= 0.428 \end{aligned}$$



*2.89 A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (a) the change in the length of the cube in the x direction, (b) the stresses σ_x , σ_y , and σ_z .

Stress-to-strain equations are

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1) \qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2) \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3) \qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

The constraint conditions are $\epsilon_y = 0$ and $\epsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y} \sigma_y - \frac{\nu_{zy}}{E_z} \sigma_z = \frac{\nu_{xy}}{E_x} \sigma_x \quad (7)$$

$$-\frac{\nu_{yz}}{E_y} \sigma_y + \frac{1}{E_z} \sigma_z = \frac{\nu_{xz}}{E_x} \sigma_x \quad (8)$$

$$\begin{aligned} \frac{1}{15.2} \sigma_y - \frac{0.428}{15.2} \sigma_z &= \frac{0.254}{50} \sigma_x \quad \text{or} \quad \sigma_y - 0.428 \sigma_z = 0.077216 \sigma_x \\ -\frac{0.428}{15.2} \sigma_y + \frac{1}{15.2} \sigma_z &= \frac{0.254}{50} \sigma_x \quad \text{or} \quad -0.428 \sigma_y + \sigma_z = 0.077216 \sigma_x \end{aligned}$$

Solving simultaneously, $\sigma_y = \sigma_z = 0.134993 \sigma_x$

Using (4) and (5) in (1), $\epsilon_x = \frac{1}{E_x} \sigma_x - \frac{\nu_{xy}}{E_x} \sigma_y - \frac{\nu_{xz}}{E_x} \sigma_z$

$$\begin{aligned} \epsilon_x &= \frac{1}{E_x} \left[1 - (0.254)(0.134993) - (0.254)(0.134993) \right] \sigma_x \\ &= \frac{0.93142}{E_x} \sigma_x \end{aligned}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600 \times 10^{-6} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65 \times 10^3}{1600 \times 10^{-6}} = 40.625 \times 10^6 \text{ Pa}$$

continued

Problem 2.89 continued

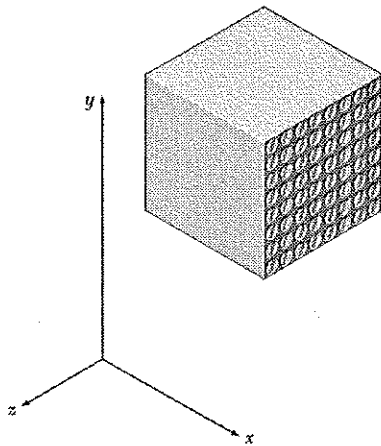
$$\epsilon_x = \frac{(0.93142)(40.625 \times 10^3)}{50 \times 10^9} = 756.78 \times 10^{-6}$$

$$(a) \delta_x = L_x \epsilon_x = (40 \text{ mm})(756.78 \times 10^{-6}) = 0.0303 \text{ mm}$$

$$(b) \sigma_x = 40.625 \times 10^6 \text{ Pa} = 40.6 \text{ MPa}$$

$$\sigma_y = \sigma_z = (0.134993)(40.625 \times 10^6) = 5.48 \times 10^6 \text{ Pa} = 5.48 \text{ MPa}$$

Problem 2.90



*2.90 The composite cube of Prob. 2.89 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (a) the stresses σ_x , σ_y , and σ_z , (b) the change in the dimension in the y direction.

$$\begin{aligned} E_x &= 50 \text{ GPa} & \nu_{xz} &= 0.254 \\ E_y &= 15.2 \text{ GPa} & \nu_{xy} &= 0.254 \\ E_z &= 15.2 \text{ GPa} & \nu_{zy} &= 0.428 \end{aligned}$$

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1)$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2)$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3)$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

Constraint condition. $\epsilon_z = 0$

Load condition. $\sigma_y = 0$

From equation (3), $0 = -\frac{\nu_{xz}}{E_x} \sigma_x + \frac{1}{E_z} \sigma_z$

$$\sigma_z = \frac{\nu_{xz} E_z}{E_x} \sigma_x = \frac{(0.254)(15.2)}{50} \sigma_x = 0.077216 \sigma_x$$

continued

Problem 2.90 continued

From equation (1) with $\sigma_y = 0$,

$$\begin{aligned}\epsilon_x &= \frac{1}{E_x} \sigma_x - \frac{\nu_{zx}}{E_z} \sigma_z = \frac{1}{E_x} \sigma_x - \frac{\nu_{xz}}{E_x} \sigma_z \\ &= \frac{1}{E_x} [\sigma_x - 0.254 \sigma_z] = \frac{1}{E_x} [1 - (0.254)(0.077216)] \sigma_x \\ &= \frac{0.98039}{E_x} \sigma_x\end{aligned}$$

$$\sigma_x = \frac{E_x \epsilon_x}{0.98039}$$

But, $\epsilon_x = \frac{\delta_x}{L_x} = \frac{0.035 \text{ mm}}{40 \text{ mm}} = 875 \times 10^{-6}$

(a) $\sigma_x = \frac{(50 \times 10^9)(875 \times 10^{-6})}{0.98039} = 44.625 \times 10^3 \text{ Pa} = 44.6 \text{ MPa} \quad \blacktriangleleft$

$\sigma_y = 0 \quad \blacktriangleleft$

$\sigma_z = (0.077216)(44.625 \times 10^3) = 3.446 \times 10^3 \text{ Pa} = 3.45 \text{ MPa} \quad \blacktriangleleft$

From (2),
$$\begin{aligned}\epsilon_y &= -\frac{\nu_{xy}}{E_x} \sigma_x + \frac{1}{E_y} \sigma_y - \frac{\nu_{yz}}{E_z} \sigma_z \\ &= -\frac{(0.254)(44.625 \times 10^3)}{50 \times 10^9} + 0 - \frac{(0.428)(3.446 \times 10^3)}{15.2 \times 10^9} \\ &= -323.73 \times 10^{-6}\end{aligned}$$

(b) $\delta_y = L_y \epsilon_y = (40 \text{ mm})(-323.73 \times 10^{-6}) = -0.0129 \text{ mm} \quad \blacktriangleleft$

Problem 2.91

*2.91 The material constants E , G , k , and ν are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (a) $k = GE/(9G - 3E)$ and (b) $\nu = (3k - 2G)/(6k + 2G)$.

$$k = \frac{E}{3(1-2\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}$$

$$(a) \quad 1 + \nu = \frac{E}{2G} \quad \text{or} \quad \nu = \frac{E}{2G} - 1$$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$= \frac{EG}{9G - 3E}$$

$$(b) \quad \frac{k}{G} = \frac{2(1+\nu)}{3(1-2\nu)}$$

$$3k - 6k\nu = 2G + 2G\nu$$

$$3k - 2G = 2G + 6k\nu$$

$$\nu = \frac{3k - 2G}{6k + 2G}$$

Problem 2.92

*2.92 Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [Hint: Refer to Eq. (2.43) and to Sec. 2.13.]

$$G = \frac{E}{2(1+\nu)} \quad \text{or} \quad \frac{G}{E} = \frac{1}{2(1+\nu)}$$

Assume $\nu \geq 0$ for almost all materials and $\nu < \frac{1}{2}$ for a positive bulk modulus.

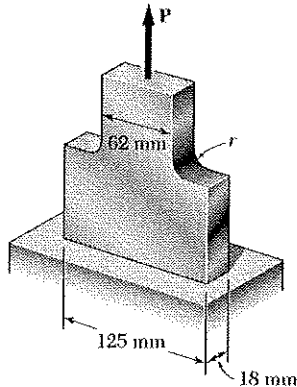
$$\text{Applying the bounds,} \quad 2 \leq \frac{E}{G} < 2\left(1 + \frac{1}{2}\right) = 3$$

$$\text{Taking the reciprocals,} \quad \frac{1}{2} \geq \frac{G}{E} > \frac{1}{3}$$

$$\text{or} \quad \frac{1}{3} \leq \frac{G}{E} < \frac{1}{2}$$

Problem 2.93

2.93 Knowing that $P = 40$ kN, determine the maximum stress when (a) $r = 12$ mm, (b) $r = 15$ mm.



$$P = 40 \text{ kN} \quad D = 125 \text{ mm} \quad d = 62 \text{ mm}$$

$$\frac{D}{d} = \frac{125}{62} = 2.016$$

$$A_{\min} = dt = (0.062)(0.018) = 1.116 \times 10^{-3} \text{ m}^2$$

$$(a) \quad r = 12 \text{ mm} \quad \frac{r}{d} = \frac{12}{62} = 0.1935$$

$$\text{From Fig. 2.64 b} \quad K = 1.94$$

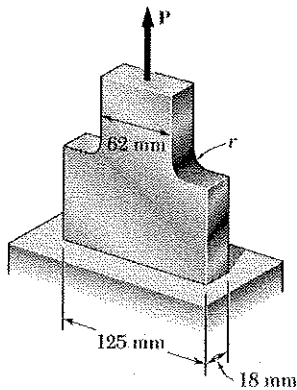
$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{(1.94)(40 \times 10^3)}{1.116 \times 10^{-3}} = 69.5 \text{ MPa}$$

$$(b) \quad r = 15 \text{ mm} \quad \frac{r}{d} = \frac{15}{62} = 0.24 \quad K = 1.86$$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{(1.86)(40 \times 10^3)}{1.116 \times 10^{-3}} = 66.7 \text{ MPa}$$

Problem 2.94

2.94 Knowing that, for the plate shown, the allowable stress is 110 MPa, determine the maximum allowable value of P when (a) $r = 10$ mm, (b) $r = 18$ mm.



$$\sigma_{\max} = 110 \text{ MPa}$$

$$D = 125 \text{ mm} \quad d = 62 \text{ mm} \quad \frac{D}{d} = 2.016$$

$$A_{\min} = dt = (0.062)(0.018) = 1.116 \times 10^{-3} \text{ m}^2$$

$$(a) \quad r = 10 \text{ mm} \quad \frac{r}{d} = \frac{10}{62} = 0.161$$

$$\text{From Fig. 2.64 b} \quad K = 2.09$$

$$\sigma_{\max} = \frac{KP}{A_{\min}}$$

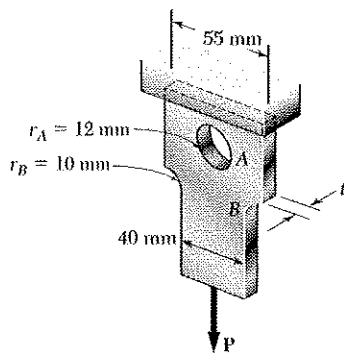
$$P = \frac{A_{\min} \sigma_{\max}}{K} = \frac{(1.116 \times 10^{-3})(110 \times 10^6)}{2.09} = 58.7 \text{ kN}$$

$$(b) \quad r = 18 \text{ mm} \quad \frac{r}{d} = \frac{18}{62} = 0.29 \quad K = 1.76$$

$$P = \frac{A_{\min} \sigma_{\max}}{K} = \frac{(1.116 \times 10^{-3})(110 \times 10^6)}{1.76} = 69.75 \text{ kN}$$

Problem 2.95

2.95 For $P = 35 \text{ kN}$, determine the minimum plate thickness t required if the allowable stress is 125 MPa .



At the hole: $r_A = 12 \text{ mm}$ $d_A = 55 - 24 = 31$

$$\frac{r_A}{d_A} = \frac{12}{31} = 0.39$$

From Fig 2.64 a $K = 2.26$

$$\sigma_{\max} = \frac{KP}{A_{\text{net}}} = \frac{KP}{d_A t} \therefore t = \frac{KP}{d_A \sigma_{\max}}$$

$$t = \frac{(2.26)(35000)}{(0.031)(125 \times 10^6)} = 0.0204 \text{ m} = 20.4 \text{ mm}$$

At the fillet $D = 55 \text{ mm}$ $d_B = 40 \text{ mm}$ $\frac{D}{d_B} = \frac{55}{40} = 1.375$

$$r_B = \frac{3}{8} = 10 \text{ mm}$$

$$\frac{r_B}{d_B} = \frac{10}{40} = 0.25$$

From Fig 2.64 b $K = 1.70$ $\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{KP}{d_B t}$

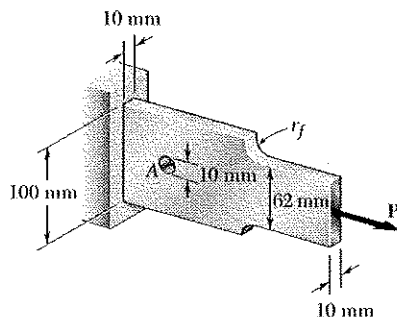
$$t = \frac{KP}{d_B \sigma_{\max}} = \frac{(1.70)(35000)}{(0.04)(125 \times 10^6)} = 0.0119 \text{ m} = 11.9 \text{ mm}$$

The larger value is the required minimum plate thickness

$$t = 20.4 \text{ mm} \blacktriangleleft$$

Problem 2.96

2.96 Knowing that the hole has a diameter of 10 mm , determine (a) the radius r_f of the fillets for which the same maximum stress occurs at the hole A and at the fillets, (b) the corresponding maximum allowable load P if the allowable stress is 105 MPa .



For the circular hole $r = \frac{10}{2} = 5 \text{ mm}$

$$d = 100 - 10 = 90 \text{ mm} \quad \frac{r}{d} = \frac{5}{90} = 0.056$$

$$A_{\text{net}} = dt = (0.09)(0.01) = 0.0009 \text{ m}^2$$

From Fig 2.64 a $K_{\text{hole}} = 2.8$

$$\sigma_{\max} = \frac{K_{\text{hole}} P}{A_{\text{net}}}$$

$$(b) \quad P = \frac{A_{\text{net}} \sigma_{\max}}{K_{\text{hole}}} = \frac{(0.0009)(105 \times 10^6)}{2.8} = 33.75 \text{ kN} \blacktriangleleft$$

(a) For fillet $D = 100 \text{ mm}$ $d = 62 \text{ mm}$ $\frac{D}{d} = \frac{100}{62} = 1.61$

$$A_{\min} = dt = (0.062)(0.01) = 0.00062 \text{ m}^2$$

$$\sigma_{\max} = \frac{K_{\text{fillet}} P}{A_{\min}} \therefore K_{\text{fillet}} = \frac{A_{\min} \sigma_{\max}}{P} = \frac{(0.00062)(105 \times 10^6)}{33750} = 1.93$$

From Fig 2.64 b $\frac{r_f}{d} \approx 0.18 \therefore r_f \approx 0.18d = (0.18)(62) = 11.16 \text{ mm} \blacktriangleleft$

Problem 2.97

2.97 A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from 12 to 24 mm in 3-mm increments. (a) Determine the diameter d of the largest bit that can be used if the allowable load at the hole is to exceed that at the fillets. (b) If the allowable stress in the plate is 145 MPa, what is the corresponding allowable load P ?

At the fillets, $r = 9 \text{ mm}$ $d = 75 \text{ mm}$

$$D = 112.5 \text{ mm} \quad \frac{D}{d} = \frac{112.5}{75} = 1.5$$

$$\frac{r}{d} = \frac{9}{75} = 0.12 \quad \text{From Fig 2.64 b} \quad K = 2.10$$

$$A_{\min} = (75)(12) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\max} = K \frac{P_{\text{all}}}{A_{\min}} = \sigma_{\text{all}} \therefore P_{\text{all}} = \frac{A_{\min} \sigma_{\text{all}}}{K} = \frac{(900 \times 10^{-6})(145 \times 10^6)}{2.10} = 62.1 \times 10^3 \text{ N} = 62.1 \text{ kN}$$

At the hole, $A_{\text{net}} = (D - 2r)t$, $\frac{r}{d} = \frac{r}{D - 2r}$

where $D = 112.5 \text{ mm}$ $r = \text{radius of circle}$ $t = 12 \text{ mm}$

K is taken from Fig 2.64 a.

$$\sigma_{\max} = K \frac{P}{A_{\text{net}}} = \sigma_{\text{all}} \therefore P_{\text{all}} = \frac{A_{\text{net}} \sigma_{\text{all}}}{K}$$

Hole diam	r	$d = D - 2r$	r/d	K	$A_{\text{net}} (\text{m}^2)$	$P_{\text{all}} (\text{N})$
12 mm	6.0 mm	100.5	0.060	2.80	1206×10^{-6}	62.5×10^3
15 mm	7.5 mm	97.5 mm	0.077	2.75	1170×10^{-6}	61.7×10^3
18 mm	9.0 mm	94.5 mm	0.095	2.71	1134×10^{-6}	60.7×10^3
21 mm	10.5 mm	91.5 mm	0.115	2.67	1098×10^{-6}	59.6×10^3
24 mm	12.0 mm	88.5 mm	0.136	2.62	1062×10^{-6}	58.8×10^3

(a) Bit diameter,

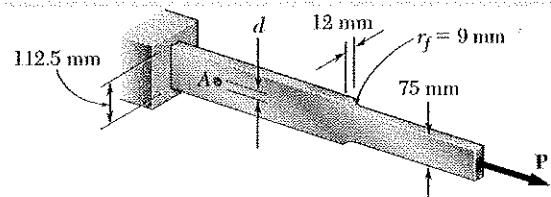
$t = 12 \text{ mm}$

(b) Allowable load.

62.1 kN

Problem 2.98

2.98 (a) For $P = 58 \text{ kN}$ and $d = 12 \text{ mm}$, determine the maximum stress in the plate shown. (b) Solve part a, assuming that the hole at A is not drilled.



Maximum stress at hole.

Use Fig. 2.64 a for values of K

$$\frac{r}{d} = \frac{9}{12} = 0.75, \quad K = 2.80$$

$$A_{\text{net}} = (12)(112.5 - 12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\text{max}} = K \frac{P}{A_{\text{net}}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6 \text{ Pa}$$

Maximum stress at fillets

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{12} = 0.75, \quad \frac{D}{d} = \frac{112.5}{12} = 9.375, \quad K = 2.10$$

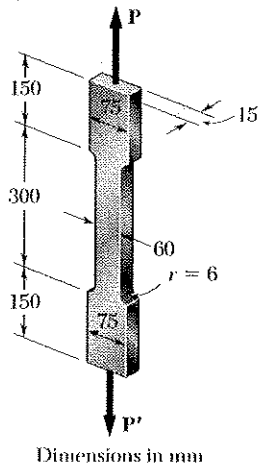
$$A_{\text{min}} = (12)(75) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\text{max}} = K \frac{P}{A_{\text{min}}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6 \text{ Pa}$$

(a) With hole and fillets. $\sigma_{\text{max}} = 134.7 \text{ MPa}$

(b) Without hole. $\sigma_{\text{max}} = 135.3 \text{ MPa}$

Problem 2.99



2.99 The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P . (a) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (b) Solve part a, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60×15 -mm rectangular cross section.

$$\sigma_{all} = 200 \times 10^6 \text{ Pa} \quad E = 70 \times 10^9 \text{ Pa}$$

$$A_{min} = (60 \text{ mm})(15 \text{ mm}) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

(a) Test specimen. $D = 75 \text{ mm}, d = 60 \text{ mm}, r = 6 \text{ mm}$

$$\frac{D}{d} = \frac{75}{60} = 1.25 \quad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.64(b) $K = 1.95$

$$\sigma_{max} = K \frac{P}{A}$$

$$P = \frac{A \sigma_{max}}{K} = \frac{(900 \times 10^{-6})(200 \times 10^6)}{1.95} = 92.308 \times 10^3 \text{ N}$$

$$P = 92.3 \text{ kN} \quad \blacktriangleleft$$

Wide area. $A^* = (75 \text{ mm})(15 \text{ mm}) = 1125 \text{ mm}^2 = 1.125 \times 10^{-3} \text{ m}^2$

$$\delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \sum \frac{L_i}{A_i} = \frac{92.308 \times 10^3}{70 \times 10^9} \left[\frac{0.150}{1.125 \times 10^{-3}} + \frac{0.300}{900 \times 10^{-6}} + \frac{0.150}{1.125 \times 10^{-3}} \right]$$

$$= 791 \times 10^{-6} \text{ m} \quad \delta = 0.791 \text{ mm} \quad \blacktriangleleft$$

(b) Uniform bar.

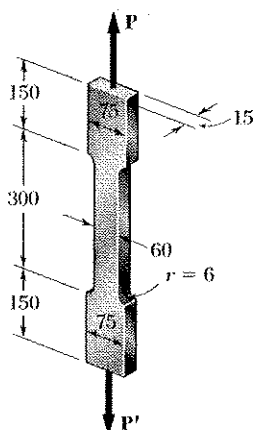
$$P = A \sigma_{all} = (900 \times 10^{-6})(200 \times 10^6) = 180 \times 10^3 \text{ N}$$

$$P = 180.0 \text{ kN} \quad \blacktriangleleft$$

$$\delta = \frac{PL}{AE} = \frac{(180 \times 10^3)(0.600)}{(900 \times 10^{-6})(70 \times 10^9)} = 1.714 \times 10^{-3} \text{ m}$$

$$\delta = 1.714 \text{ mm} \quad \blacktriangleleft$$

Problem 2.100



Dimensions in mm

2.100 For the test specimen of Prob. 2.99, determine the maximum value of the normal stress corresponding to a total elongation of 0.75 mm.

2.99 The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P . (a) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (b) Solve part a, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60×15 -mm rectangular cross section.

$$\delta = \sum \frac{P_i L_i}{E_i A_i} = \frac{P}{E} \sum \frac{L_i}{A_i} \quad \delta = 0.75 \times 10^{-3} \text{ m}$$

$$L_1 = L_3 = 150 \text{ mm} = 0.150 \text{ m}, \quad L_2 = 300 \text{ mm} = 0.300 \text{ m}$$

$$A_1 = A_3 = (75 \text{ mm})(15 \text{ mm}) = 1125 \text{ mm}^2 = 1.125 \times 10^{-3} \text{ m}^2$$

$$A_2 = (60 \text{ mm})(15 \text{ mm}) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sum \frac{L_i}{A_i} = \frac{0.150}{1.125 \times 10^{-3}} + \frac{0.300}{900 \times 10^{-6}} + \frac{0.150}{1.125 \times 10^{-3}} = 600 \text{ m}^{-1}$$

$$P = \frac{E \delta}{\sum \frac{L_i}{A_i}} = \frac{(70 \times 10^9)(0.75 \times 10^{-3})}{600} = 87.5 \times 10^3 \text{ N}$$

Stress concentration.

$$D = 75 \text{ mm}, \quad d = 60 \text{ mm}, \quad r = 6 \text{ mm}$$

$$\frac{D}{d} = \frac{75}{60} = 1.25$$

$$\frac{r}{d} = \frac{6}{60} = 0.10$$

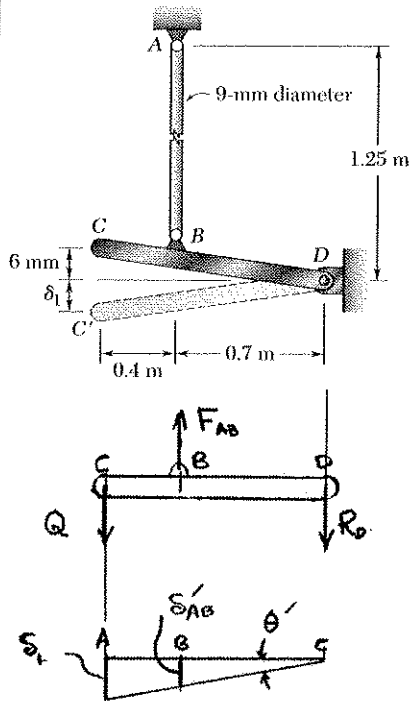
From fig. 2.59

$$K = 1.95$$

$$\sigma_{max} = K \frac{P}{A_{min}} = \frac{(1.95)(87.5 \times 10^3)}{900 \times 10^{-6}} = 189.6 \times 10^6 \text{ Pa} \quad \sigma_{max} = 189.6 \text{ MPa}$$

Note that $\sigma_{max} \leq \sigma_{all}$.

Problem 2.101



2.101 Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 345 \text{ MPa}$. After the rod has been attached to the rigid lever CD , it is found that end C is 6 mm too high. A vertical force Q is then applied at C until this point has moved to position C' . Determine the required magnitude of Q and the deflection δ_1 if the lever is to *snap* back to a horizontal position after Q is removed.

$$A_{AB} = \frac{\pi}{4} (9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

Since rod AB is to be stretched permanently,

$$(F_{AB})_{\max} = A_{AB} \sigma_Y = (63.617 \times 10^{-6}) (345 \times 10^6) = 21.948 \times 10^3 \text{ N}$$

$$+\circlearrowleft \sum M_D = 0: 1.1 Q - 0.7 F_{AB} = 0$$

$$Q_{\max} = \frac{0.7}{1.1} (21.948 \times 10^3) = 13.967 \times 10^3 \text{ N} = 13.97 \text{ kN}$$

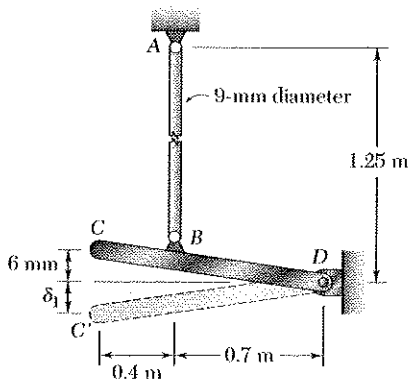
$$\delta_{AB}' = \frac{(F_{AB})_{\max} L_{AB}}{E A_{AB}} = \frac{(21.948 \times 10^3) (1.25)}{(200 \times 10^9) (63.617 \times 10^{-6})} = 2.15625 \times 10^{-3} \text{ m}$$

$$\theta' = \frac{\delta_{AB}'}{0.7} = 3.0804 \times 10^{-3} \text{ rad}$$

$$\delta_1 = 1.1 \theta' = 3.39 \times 10^{-3} \text{ m}$$

$$3.39 \text{ mm}$$

Problem 2.102



2.102 Solve Prob. 2.101, assuming that the yield point of the mild steel is 250 MPa.

2.101 Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 345 \text{ MPa}$. After the rod has been attached to the rigid lever CD , it is found that end C is 6 mm too high. A vertical force Q is then applied at C until this point has moved to position C' . Determine the required magnitude of Q and the deflection δ_1 if the lever is to *snap* back to a horizontal position after Q is removed.

$$A_{AB} = \frac{\pi}{4} (9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

Since rod AB is to be stretched permanently,

$$(F_{AB})_{\max} = A_{AB} \sigma_Y = (63.617 \times 10^{-6}) (250 \times 10^6) = 15.9043 \times 10^3 \text{ N}$$

$$+\circlearrowleft \sum M_D = 0: 1.1 Q - 0.7 F_{AB} = 0$$

$$Q_{\max} = \frac{0.7}{1.1} (15.9043 \times 10^3) = 10.12 \times 10^3 \text{ N} = 10.12 \text{ kN}$$

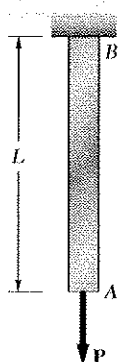
$$\delta_{AB}' = \frac{(F_{AB})_{\max} L_{AB}}{E A_{AB}} = \frac{(15.9043 \times 10^3) (1.25)}{(200 \times 10^9) (63.617 \times 10^{-6})} = 1.5625 \times 10^{-3} \text{ m}$$

$$\theta' = \frac{\delta_{AB}'}{0.7} = 2.2321 \times 10^{-3} \text{ rad}$$

$$\delta_1 = 1.1 \theta' = 2.46 \times 10^{-3} \text{ m}$$

$$2.46 \text{ mm}$$

Problem 2.103



2.103 The 30-mm square bar AB has a length $L = 2.2$ m; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force P is applied to the bar until end A has moved down by an amount δ_m . Determine the maximum value of the force P and the permanent set of the bar after the force has been removed, knowing that (a) $\delta_m = 4.5$ mm, (b) $\delta_m = 8$ mm.

$$A = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\delta_Y = L \epsilon_Y = \frac{L \sigma_Y}{E} = \frac{(2.2)(345 \times 10^6)}{200 \times 10^9} = 3.795 \times 10^{-3} = 3.795 \text{ mm}$$

$$\text{If } \delta_m \geq \delta_Y \quad P_m = A \sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3 \text{ N}$$

$$\text{Unloading: } \delta' = \frac{P_m L}{AE} = \frac{\sigma_Y L}{E} = \delta_Y = 3.795 \text{ mm}$$

$$\delta_f = \delta_m - \delta'$$

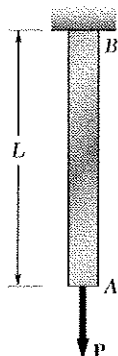
$$(a) \quad \delta_m = 4.5 \text{ mm} > \delta_Y \quad P_m = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$$

$$\delta_{\text{perm}} = 4.5 \text{ mm} - 3.795 \text{ mm} = 0.705 \text{ mm}$$

$$(b) \quad \delta_m = 8 \text{ mm} > \delta_Y \quad P_m = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$$

$$\delta_p = 8.0 \text{ mm} - 3.795 \text{ mm} = 4.205 \text{ mm}$$

Problem 2.104



2.104 The 30-mm square bar AB has a length $L = 2.5$ m; it is made of mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force P is applied to the bar and then removed to give it a permanent set δ_p . Determine the maximum value of the force P and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (a) 3.5 mm, (b) 6.5 mm.

$$A = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\delta_Y = L \epsilon_Y = \frac{L \sigma_Y}{E} = \frac{(2.5)(345 \times 10^6)}{200 \times 10^9} = 4.3125 \times 10^{-3} \text{ m} = 4.3125 \text{ mm}$$

When δ_m exceeds δ_Y , thus producing a permanent stretch of δ_p , the maximum force is

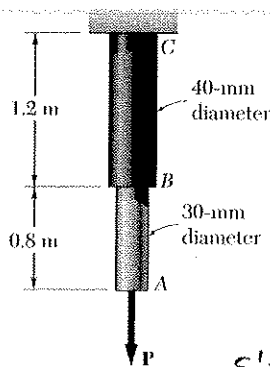
$$P_m = A \sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$$

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$$

$$(a) \quad \delta_p = 3.5 \text{ mm} \quad \delta_m = 3.5 \text{ mm} + 4.3125 \text{ mm} = 7.81 \text{ mm}$$

$$(b) \quad \delta_p = 6.5 \text{ mm} \quad \delta_m = 6.5 \text{ mm} + 4.3125 \text{ mm} = 10.81 \text{ mm}$$

Problem 2.105



2.105 Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$. A force P is applied to the rod and then removed to give it a permanent set $\delta_p = 2 \text{ mm}$. Determine the maximum value of the force P and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

$$A_{AB} = \frac{\pi}{4} (30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$P_{max} = A_{min} \sigma_Y = (706.86 \times 10^{-6}) (250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

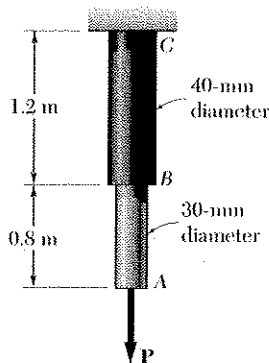
$$P_{max} = 176.7 \text{ kN} \quad \blacktriangleleft$$

$$\begin{aligned} \delta' &= \frac{P' L_{AB}}{E A_{AB}} + \frac{P' L_{BC}}{E A_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})} \\ &= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm} \end{aligned}$$

$$\delta_p = \delta_m - \delta' \quad \text{or} \quad \delta_m = \delta_p + \delta' = 2 + 1.84375$$

$$\delta_m = 3.84 \text{ mm} \quad \blacktriangleleft$$

Problem 2.106



2.106 Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$. A force P is applied to the rod until its end *A* has moved down by an amount $\delta_m = 5 \text{ mm}$. Determine the maximum value of the force P and the permanent set of the rod after the force has been removed.

$$A_{AB} = \frac{\pi}{4} (30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$$

$$P_{max} = A_{min} \sigma_Y = (706.86 \times 10^{-6}) (250 \times 10^6) = 176.715 \times 10^3 \text{ N}$$

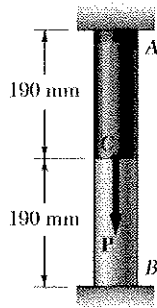
$$P_{max} = 176.7 \text{ kN} \quad \blacktriangleleft$$

$$\begin{aligned} \delta' &= \frac{P' L_{AB}}{E A_{AB}} + \frac{P' L_{BC}}{E A_{BC}} = \frac{(176.715 \times 10^3)(0.8)}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{(176.715 \times 10^3)(1.2)}{(200 \times 10^9)(1.25664 \times 10^{-3})} \\ &= 1.84375 \times 10^{-3} \text{ m} = 1.84375 \text{ mm} \end{aligned}$$

$$\delta_p = \delta_m - \delta' = 5 - 1.84375 = 3.16 \text{ mm}$$

$$\delta_p = 3.16 \text{ mm} \quad \blacktriangleleft$$

Problem 2.107



2.107 Rod AB consists of two cylindrical portions AC and BC , each with a cross-sectional area of 1750 mm^2 . Portion AC is made of a mild steel with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$, and portion CB is made of a high-strength steel with $E = 200 \text{ GPa}$ and $\sigma_Y = 345 \text{ MPa}$. A load P is applied at C as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C .

Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC} \epsilon_{Y,AC} = \frac{L_{AC} \sigma_{Y,AC}}{E} = \frac{(0.190)(250 \times 10^6)}{200 \times 10^9} = 0.2375 \times 10^{-3} \text{ m}$$

Corresponding force

$$F_{AC} = A \sigma_{Y,AC} = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^3 \text{ N}$$

$$F_{CB} = -\frac{E A \delta_C}{L_{CB}} = -\frac{(200 \times 10^9)(1750 \times 10^{-6})(0.2375 \times 10^{-3})}{0.190} = -437.5 \times 10^3 \text{ N}$$



For equilibrium of element at C

$$F_{AC} - (F_{CB} + P_Y) = 0 \quad P_Y = F_{AC} - F_{CB} = 875 \times 10^3 \text{ N}$$

Since applied load $P = 975 \times 10^3 \text{ N} > 875 \times 10^3 \text{ N}$, portion AC yields.

$$F_{CB} = F_{AC} - P = 437.5 \times 10^3 - 975 \times 10^3 \text{ N} = -537.5 \times 10^3 \text{ N}$$

$$(a) \quad \delta_C = -\frac{F_{CB} L_{CB}}{E A} = \frac{(537.5 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.29179 \times 10^{-3} \text{ m} = 0.292 \text{ mm}$$

$$(b) \quad \text{Maximum stresses:} \quad \sigma_{AB} = \sigma_{Y,AB} = 250 \text{ MPa}$$

$$\sigma_{BC} = \frac{F_{CB}}{A} = -\frac{537.5 \times 10^3}{1750 \times 10^{-6}} = -307.14 \times 10^6 \text{ Pa} = -307 \text{ MPa}$$

(c) Deflection and forces for unloading.

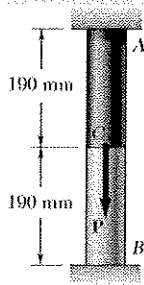
$$\delta' = \frac{F_{AC}' L_{AC}}{E A} = -\frac{F_{CB}' L_{CB}}{E A} \quad \therefore F_{CB}' = -F_{AC}' \frac{L_{AC}}{L_{CB}} = -F_{AC}'$$

$$P' = 975 \times 10^3 = P_{AC}' - P_{CB}' = 2 P_{AC}' \quad P_{AC}' = 487.5 \times 10^3 \text{ N}$$

$$\delta' = \frac{(487.5 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26464 \times 10^{-3} \text{ m}$$

$$\delta_p = \delta_m - \delta' = 0.29179 \times 10^{-3} - 0.26464 \times 10^{-3} = 0.02715 \times 10^{-3} \text{ m} = 0.027 \text{ mm}$$

Problem 2.108



2.108 For the composite rod of Prob. 2.107, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.03$ mm and then decreased back to zero, determine, (a) the maximum value of P , (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C after the load is removed.

2.107 Rod AB consists of two cylindrical portions AC and BC , each with a cross-sectional area of 1750 mm^2 . Portion AC is made of a mild steel with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$, and portion CB is made of a high-strength steel with $E = 200 \text{ GPa}$ and $\sigma_Y = 345 \text{ MPa}$. A load P is applied at C as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C .

Displacement at C is $\delta_m = 0.30 \text{ mm}$. The corresponding strains are

$$\epsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\epsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding

$$\epsilon_{Y,AC} = \frac{\sigma_{Y,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \quad (\text{yielding})$$

$$\epsilon_{Y,CB} = -\frac{\sigma_{Y,CB}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \quad (\text{elastic})$$

(a) Forces: $F_{AC} = A\sigma_Y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^3 \text{ N}$

$$F_{CB} = EA\epsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^3 \text{ N}$$

For equilibrium of element at C , $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CB} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{ N} = 990 \text{ kN} \quad \blacktriangleleft$$

(b) Stresses: AC $\sigma_{AC} = \sigma_{Y,AC} = 250 \text{ MPa}$ $\blacktriangleleft$

CB $\epsilon_{CB} = \frac{F_{CB}}{A} = -\frac{552.6 \times 10^3}{1750 \times 10^{-6}} = -316 \times 10^6 \text{ Pa} = -316 \text{ MPa}$ $\blacktriangleleft$

(c) Deflection and forces for unloading.

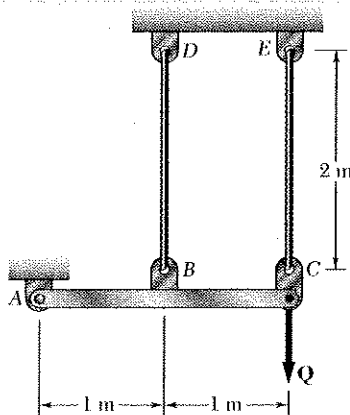
$$\delta' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} \quad \therefore P_{CB}' = -P_{AC}' \frac{L_{AC}}{L_{CB}} = -P_{AC}'$$

$$P' = P_{AC}' - P_{CB}' = 2P_{AC}' = 990.1 \times 10^3 \text{ N} \quad \therefore P_{AC}' = 495.05 \times 10^3 \text{ N}$$

$$\delta' = \frac{(495.05 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26874 \times 10^{-3} \text{ m} = 0.26874 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30 \text{ mm} - 0.26874 \text{ mm} = 0.031 \text{ mm} \quad \blacktriangleleft$$

Problem 2.109



2.109 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$. A force Q is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (a) the maximum stress that occurs in cable BD , (b) the maximum deflection of point C , (c) the final displacement of point C . (Hint: In part c, cable CE is not taut.)

Elongation constraints for taut cables.

Let θ = rotation angle of rigid bar ABC

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}} \delta_{CE} = \frac{1}{2} \delta_{CE} \quad (1)$$

Equilibrium of bar ABC .

$$+\circlearrowleft M_A = 0: L_{AB} F_{BD} + L_{AC} F_{CE} - L_{AC} Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}} F_{BD} = F_{CE} + \frac{1}{2} F_{BD} \quad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_Y = (100 \times 10^{-6})(345 \times 10^6) = 34.5 \times 10^3 \text{ N}$

From (2), $F_{BD} = 2(Q - F_{CE}) = 2(50 \times 10^3 - 34.5 \times 10^3) = 31.0 \times 10^3 \text{ N}$

Since $F_{BD} \leq A\sigma_Y = 34.5 \times 10^3 \text{ N}$, cable BD is elastic when $Q = 50 \text{ kN}$.

(a) Maximum stresses.

$$\sigma_{CE} = \sigma_Y = 345 \text{ MPa}$$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0 \times 10^3}{100 \times 10^{-6}} = 310 \times 10^6 \text{ Pa}$$

$$\sigma_{BD} = 310 \text{ MPa} \quad \blacktriangleleft$$

(b) Maximum of deflection of point C .

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{EA} = \frac{(31.0 \times 10^3)(2)}{(200 \times 10^9)(100 \times 10^{-6})} = 3.1 \times 10^{-3} \text{ m}$$

$$\text{From (1)} \quad \delta_C = \delta_{CE} = 2 \delta_{BD} = 6.2 \times 10^{-3} \text{ m} \quad 6.20 \text{ mm} \downarrow \quad \blacktriangleleft$$

Permanent elongation of cable CE : $(\delta_{CE})_p = (\delta_{CE}) - \frac{\sigma_Y L_{CE}}{E}$

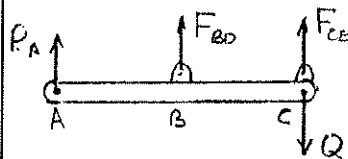
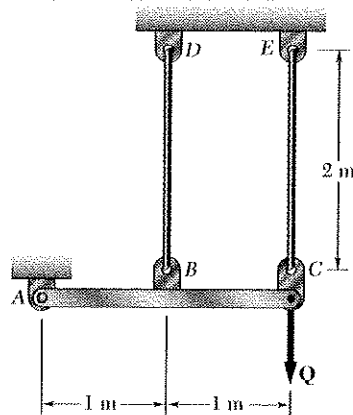
$$\begin{aligned} (\delta_{CE})_p &= (\delta_{CE})_{\max} - \frac{F_{CE} L_{CE}}{EA} = (\delta_{CE})_{\max} - \frac{\sigma_Y L_{CE}}{E} = \\ &= 6.20 \times 10^{-3} - \frac{(345 \times 10^6)(2)}{200 \times 10^9} = 2.75 \times 10^{-3} \text{ m} \end{aligned}$$

(c) Unloading. Cable CE is slack ($F_{CE} = 0$) at $Q = 0$.

$$\text{From (2), } F_{BD} = 2(Q - F_{CE}) = 2(0 - 0) = 0$$

$$\text{Since cable } BD \text{ remained elastic, } \sigma_{BD} = \frac{F_{BD} L_{BD}}{EA} = 0 \quad \blacktriangleleft$$

Problem 2.110



2.110 Solve Prob. 2.109, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

2.109 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$. A force Q is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (a) the maximum stress that occurs in cable BD , (b) the maximum deflection of point C , (c) the final displacement of point C . (Hint: In part c, cable CE is not taut.)

Elongation constraints.

Let θ = rotation angle of rigid bar ABC .

$$\theta = \frac{\delta_{BC}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}} \delta_{CE} = \frac{1}{2} \delta_{CE} \quad (1)$$

Equilibrium of bar ABC .

$$+\circlearrowleft \sum M_A = 0: L_{AB} F_{BD} + L_{AC} F_{CE} - L_{AC} Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}} F_{BD} = F_{CE} + \frac{1}{2} F_{BD} \quad (2)$$

Assume cable CE is yielded. $F_{CE} = A\sigma_Y = (100 \times 10^{-6})(345 \times 10^6) = 34.5 \times 10^3 \text{ N}$

From (2) $F_{BD} = 2(Q - F_{CE}) = (2)(50 \times 10^3 - 34.5 \times 10^3) = 31.0 \times 10^3 \text{ N}$

Since $F_{BD} < A\sigma_Y = 34.5 \times 10^3 \text{ N}$, cable BD is elastic when $Q = 50 \text{ kN}$.

(a) Maximum stresses.

$$\sigma_{CE} = \sigma_Y = 345 \text{ MPa}$$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0 \times 10^3}{100 \times 10^{-6}} = 310 \times 10^6 \text{ Pa}$$

$$\sigma_{BD} = 310 \text{ MPa}$$

(b) Maximum deflection of point C .

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{EA} = \frac{(31.0 \times 10^3)(2)}{(200 \times 10^9)(100 \times 10^{-6})} = 3.1 \times 10^{-3} \text{ m}$$

$$\text{From (1)} \quad (\delta_C)_m = \delta_{CE} = 2\delta_{BD} = 6.2 \times 10^{-3} \text{ m}$$

$$6.20 \text{ mm} \downarrow$$

Unloading. $Q' = 50 \times 10^3 \text{ N}$, $\delta_{CE}' = \delta_C'$ From (1) $\delta_{BD}' = \frac{1}{2} \delta_C'$

$$\text{Elastic} \quad F_{BD}' = \frac{EA\delta_{BD}'}{L_{BD}} = \frac{(200 \times 10^9)(100 \times 10^{-6})(\frac{1}{2}\delta_C')}{2} = 5 \times 10^6 \delta_C'$$

$$F_{CE}' = \frac{EA\delta_{CE}'}{L_{CE}} = \frac{(200 \times 10^9)(100 \times 10^{-6})(\delta_C')}{2} = 10 \times 10^6 \delta_C'$$

$$\text{From (2)} \quad Q' = F_{CE}' + \frac{1}{2} F_{BD}' = 12.5 \times 10^6 \delta_C'$$

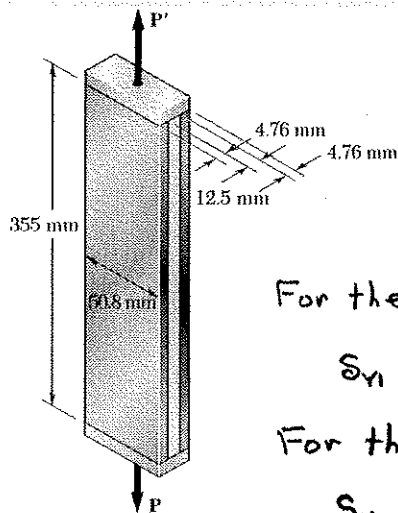
$$\text{Equating expressions for } Q' \quad 12.5 \times 10^6 \delta_C' = 50 \times 10^3$$

$$\delta_C' = 4 \times 10^{-3} \text{ m}$$

(c) Final displacement. $\delta_C = (\delta_C)_m - \delta_C' = 6.2 \times 10^{-3} - 4 \times 10^{-3} = 2.2 \times 10^{-3} \text{ m}$

$$2.20 \text{ mm} \downarrow$$

Problem 2.111



2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P . Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (a) the maximum value of P , (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

For the mild steel $A_1 = (0.0125)(0.0508) = 635 \times 10^{-6} \text{ m}^2$

$$S_{r1} = \frac{L \sigma_{r1}}{E} = \frac{(0.355)(345 \times 10^6)}{200 \times 10^9} = 0.612 \times 10^{-3} \text{ m}$$

For the tempered steel $A_2 = 2(0.00476)(0.0508) = 483.6 \times 10^{-6} \text{ m}^2$

$$S_{r2} = \frac{L \sigma_{r2}}{E} = \frac{(0.355)(690 \times 10^6)}{200 \times 10^9} = 1.225 \times 10^{-3} \text{ m}$$

Total area: $A = A_1 + A_2 = 1118.6 \times 10^{-6} \text{ m}^2$

$S_{r1} < S_m < S_{r2}$ The mild steel yields. Tempered steel is elastic.

(a) Forces $P_1 = A_1 \sigma_{r1} = (635 \times 10^{-6})(345 \times 10^6) = 219.075 \text{ kN}$

$$P_2 = \frac{EA_2 S_m}{L} = \frac{(200 \times 10^9)(483.6 \times 10^{-6})(0.001016)}{0.355} = 276.81 \text{ kN}$$

$$P = P_1 + P_2 = 495.9 \text{ kN}$$

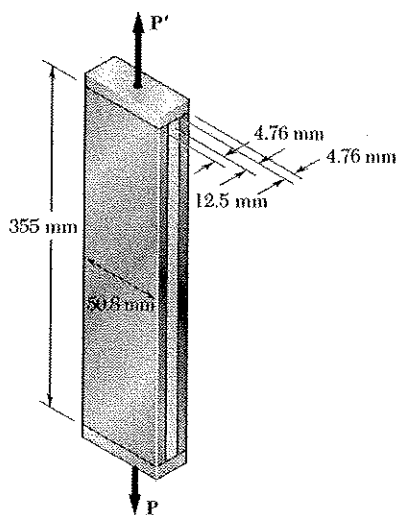
(b) Stresses $\sigma_1 = \frac{P_1}{A_1} = \sigma_{r1} = 345 \text{ MPa}$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{276.810}{483.6 \times 10^{-6}} = 572.4 \text{ MPa}$$

Unloading $S' = \frac{PL}{EA} = \frac{(495.900)(0.355)}{(200 \times 10^9)(1118.6 \times 10^{-6})} = 0.787 \times 10^{-3} \text{ m}$

(c) Permanent set $S_p = S_m - S' = 1.016 - 0.787 = 0.229 \text{ mm}$
 $\approx 0.23 \text{ mm}$

Problem 2.112



2.112 For the composite bar of Prob. 2.111, if P is gradually increased from zero to 436 kN and then decreased back to zero, determine (a) the maximum deformation of the bar, (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P . Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (a) the maximum value of P , (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

Mild steel $A_1 = (0.0125)(0.0508) = 635 \times 10^{-6} \text{ m}^2$

Tempered steel $A_2 = 2(0.00476)(0.0508) = 483.6 \times 10^{-6} \text{ m}^2$

Total: $A = A_1 + A_2 = 1118.6 \times 10^{-6} \text{ m}^2$

force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \therefore P_Y = A \sigma_{Y1} = (1118.6 \times 10^{-6})(345 \times 10^6) = 385.9 \text{ kPa}$$

$P > P_Y$, therefore mild steel yields.

Let P_1 = force carried by mild steel

P_2 = force carried by tempered steel

$$P_1 = A_1 \sigma_1 = (635 \times 10^{-6})(345 \times 10^6) = 219075 \text{ N}$$

$$P_1 + P_2 = P \quad P_2 = P - P_1 = 436000 - 219075 = 216925 \text{ N}$$

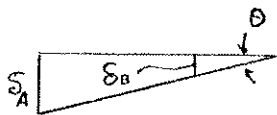
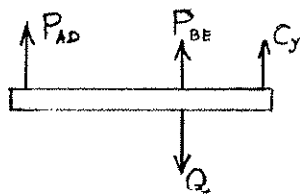
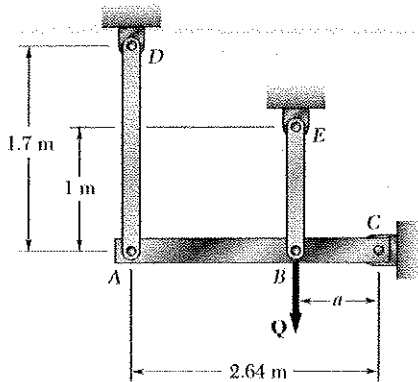
$$(a) \quad \delta_m = \frac{P_2 L}{E A_2} = \frac{(216925)(0.355)}{(200 \times 10^9)(483.6 \times 10^{-6})} = 0.796 \times 10^{-3} \text{ m} = 0.8 \text{ mm}$$

$$(b) \quad \sigma_2 = \frac{P_2}{A_2} = \frac{(216925)}{483.6 \times 10^{-6}} = 448.562 \text{ MPa}$$

$$\text{Unloading} \quad \delta' = \frac{PL}{EA} = \frac{(436000)(0.355)}{(200 \times 10^9)(1118.6 \times 10^{-6})} = 0.692 \times 10^{-3} \text{ m} = 0.692 \text{ mm} \\ = 0.7 \text{ mm}$$

$$(c) \quad \delta_p = \delta_m - \delta' = 0.796 - 0.692 = 0.104 \text{ mm}$$

Problem 2.113



2.113 The rigid bar ABC is supported by two links, AD and BE , of uniform 37.5 × 6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force Q applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B .

Statics: $\sum M_C = 0: 0.640(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $S_A = 2.64 \theta$, $S_B = a\theta = 0.640 \theta$

Elastic Analysis:

$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}} S_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7} S_A = 26.47 \times 10^6 S_A$$

$$= (26.47 \times 10^6)(2.64 \theta) = 69.88 \times 10^6 \theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$$

$$P_{BE} = \frac{EA}{L_{BE}} S_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0} S_B = 45 \times 10^6 S_B$$

$$= (45 \times 10^6)(0.640 \theta) = 28.80 \times 10^6 \theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128 \times 10^9 \theta$$

From Statics, $Q = P_{BE} + \frac{2.64}{0.640} P_{AD} = P_{BE} + 4.125 P_{AD}$

$$= [28.80 \times 10^6 + (4.125)(69.88 \times 10^6)] \theta = 317.06 \times 10^6 \theta$$

θ_Y at yielding of link AD $\sigma_{AD} = \sigma_Y = 250 \times 10^6 = 310.6 \times 10^9 \theta$

$$\theta_Y = 804.89 \times 10^{-6}$$

$$Q_Y = (317.06 \times 10^6)(804.89 \times 10^{-6}) = 255.2 \times 10^3 \text{ N}$$

(a) Since $Q = 260 \times 10^3 > Q_Y$, link AD yields. $\sigma_{AD} = 250 \text{ MPa}$ $\blackleftarrow$

$$P_{AD} = A \sigma_Y = (225 \times 10^{-6})(250 \times 10^6) = 56.25 \times 10^3 \text{ N}$$

From Statics, $P_{BE} = Q - 4.125 P_{AD} = 260 \times 10^3 - (4.125)(56.25 \times 10^3)$

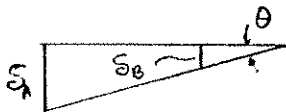
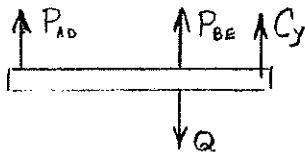
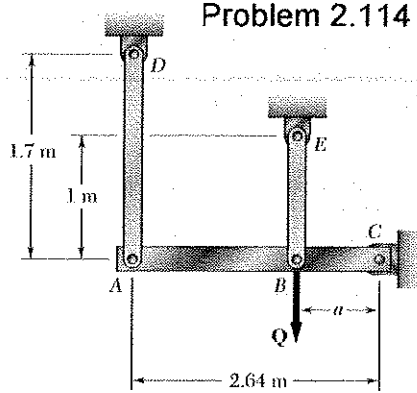
$$P_{BE} = 27.97 \times 10^3 \text{ N}$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97 \times 10^3}{225 \times 10^{-6}} = 124.3 \times 10^6 \text{ Pa} = 124.3 \text{ MPa} \quad \blackleftarrow$$

(b) $S_B = \frac{P_{BE} L_{BE}}{EA} = \frac{(27.97 \times 10^3)(1.0)}{(200 \times 10^9)(225 \times 10^{-6})} = 621.53 \times 10^{-6} \text{ m}$

$$S_B = 0.622 \text{ mm} \downarrow \quad \blackleftarrow$$

Problem 2.114



2.114 Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force Q applied at B is gradually increased from zero to 135 kN.

2.113 The rigid bar ABC is supported by two links, AD and BE , of uniform 37.5×6 -mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force Q applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B .

Statics: $\sum M_C = 0: 1.76(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64 \theta, \delta_B = 1.76 \theta$

Elastic Analysis:

$$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}} \delta_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7} \delta_A = 26.47 \times 10^6 \delta_A$$

$$= (26.47 \times 10^6)(2.64 \theta) = 69.88 \times 10^6 \theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$$

$$P_{BE} = \frac{EA}{L_{BE}} \delta_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0} \delta_B = 45 \times 10^6 \delta_B = (45 \times 10^6)(1.76 \theta)$$

$$= 79.2 \times 10^6 \theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 352 \times 10^9 \theta$$

From Statics, $Q = P_{BE} + \frac{2.64}{1.76} P_{AD} = P_{BE} + 1.500 P_{AD}$

$$= [79.2 \times 10^6 + (1.500)(69.88 \times 10^6)] \theta = 178.62 \times 10^6 \theta$$

θ_Y at yielding of link BE $\sigma_{BE} = \sigma_Y = 250 \times 10^6 = 352 \times 10^9 \theta_Y$

$$\theta_Y = 710.23 \times 10^{-6}$$

$$Q_Y = (178.62 \times 10^6)(710.23 \times 10^{-6}) = 126.86 \times 10^3 \text{ N}$$

Since $Q = 135 \times 10^3 \text{ N} > Q_Y$, link BE yields $\sigma_{BE} = \sigma_Y = 250 \text{ MPa}$

$$P_{BE} = A \sigma_Y = (225 \times 10^{-6})(250 \times 10^6) = 56.25 \times 10^3 \text{ N}$$

From Statics, $P_{AD} = \frac{1}{1.500} (Q - P_{BE}) = 52.5 \times 10^3 \text{ N}$

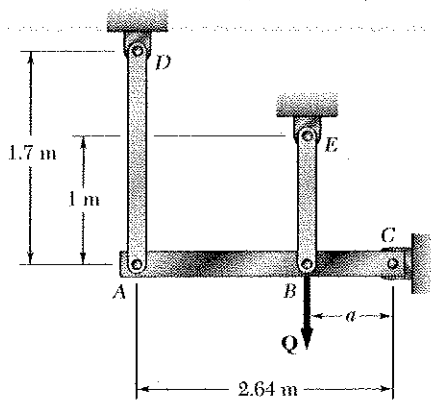
$$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5 \times 10^3}{225 \times 10^{-6}} = 233.3 \times 10^6 = 233 \text{ MPa}$$

From elastic analysis of AD , $\theta = \frac{P_{AD}}{69.88 \times 10^6} = 751.29 \times 10^{-6} \text{ rad}$

$$\delta_B = 1.76 \theta = 1.322 \times 10^{-3} \text{ m}$$

$$\delta_B = 1.322 \text{ mm} \downarrow$$

Problem 2.115



*2.115 Solve Prob. 2.113, assuming that the magnitude of the force Q applied at B is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (a) the residual stress in each link, (b) the final deflection of point B . Assume that the links are braced so that they can carry compressive forces without buckling.

2.113 The rigid bar ABC is supported by two links, AD and BE , of uniform 37.5×6 -mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force Q applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B .

See solution to Problem 2.113 for the normal stresses in each link and the deflection of point B after loading

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa} \quad \sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$

$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to PROBLEM 2.113 applies to the unloading

$$Q' = 317.06 \times 10^3 \theta'$$

$$\theta' = \frac{Q}{317.06 \times 10^3} = \frac{260 \times 10^3}{317.06 \times 10^3} = 820.03 \times 10^{-6}$$

$$\sigma'_{AD} = 310.6 \times 10^9 \theta' = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$

$$\sigma'_{BE} = 128 \times 10^9 \theta' = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$

$$\delta'_B = 0.640 \theta' = 524.82 \times 10^{-6} \text{ m}$$

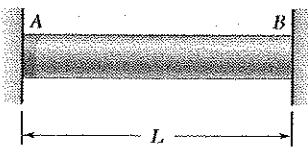
(a) Residual stresses

$$\begin{aligned} \sigma_{AD, \text{res}} &= \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^6 = -4.70 \times 10^6 \text{ Pa} \\ &= -4.70 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \sigma_{BE, \text{res}} &= \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} \\ &= 19.34 \text{ MPa} \end{aligned}$$

$$\begin{aligned} (b) \quad \delta_{B, p} &= \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} \\ &= 96.71 \times 10^{-6} \text{ m} = 0.0967 \text{ mm} \downarrow \end{aligned}$$

Problem 2.116



2.116 A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 7°C . The steel is assumed to be elastoplastic with $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$. Knowing that $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$, determine the stress in the bar (a) when the temperature is raised to 160°C , (b) after the temperature has returned to 7°C .

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_y L}{E} + L\alpha(\Delta T)_y = 0$$

$$(\Delta T)_y = \frac{\sigma_y}{E\alpha} = \frac{250 \times 10^6}{(200 \times 10^9)(11.7 \times 10^{-6})} = -106.8^\circ\text{C}$$

$$\text{But } \Delta T = 160 - 7 = 153^\circ\text{C} > (\Delta T)_y$$

(a) Yielding occurs

$$\sigma = -\sigma_y = 250 \text{ MPa} \quad \blacktriangleleft$$

Cooling: $(\Delta T)' = 153^\circ\text{C}$.

$$\delta' = \delta_p' + \delta_r' = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

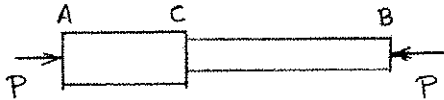
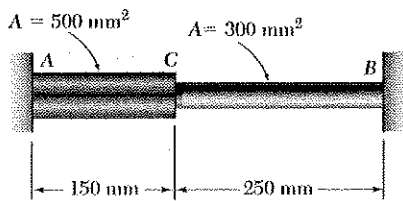
$$= -(200 \times 10^9)(11.7 \times 10^{-6})(153) = -358 \text{ MPa}$$

(b) Residual stress

$$\sigma_{\text{res}} = -\sigma_y - \sigma' = -250 \times 10^6 + 358 \times 10^6 = 108 \text{ MPa}$$

$$108 \text{ MPa} \quad \blacktriangleleft$$

Problem 2.117



2.117 The steel rod ABC is attached to rigid supports and is unstressed at a temperature of 25°C . The steel is assumed elastoplastic, with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$. The temperature of both portions of the rod is then raised to 150°C . Knowing that $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$, determine (a) the stress in both portions of the rod, (b) the deflection of point C .

$$A_{AC} = 500 \times 10^{-6} \text{ m}^2 \quad L_{AC} = 0.150 \text{ m}$$

$$A_{CB} = 300 \times 10^{-6} \text{ m}^2 \quad L_{CB} = 0.250 \text{ m}$$

Constraint: $S_p + S_T = 0$

Determine ΔT to cause yielding in portion CB .

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha} \left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300 \times 10^{-6})(250 \times 10^6) = 75 \times 10^3 \text{ N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha} \left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right)$$

$$= \frac{75 \times 10^3}{(0.400)(200 \times 10^9)(11.7 \times 10^{-6})} \left(\frac{0.150}{500 \times 10^{-6}} + \frac{0.250}{300 \times 10^{-6}} \right) = 90.812^\circ\text{C}$$

Actual $\Delta T: 150^\circ\text{C} - 25^\circ\text{C} = 125^\circ\text{C} > (\Delta T)_Y$ Yielding occurs.

For $\Delta T > (\Delta T)_Y$ $P = P_Y = 75 \times 10^3 \text{ N}$

(a) $\sigma_{AC} = -\frac{P_Y}{A_{AC}} = -\frac{75 \times 10^3}{500 \times 10^{-6}} = -150 \times 10^6 \text{ Pa} \quad \sigma_{AC} = -150 \text{ MPa} \quad \blacktriangleleft$

$\sigma_{CB} = -\frac{P_Y}{A_{CB}} = -\sigma_Y \quad \sigma_{CB} = -250 \text{ MPa} \quad \blacktriangleleft$

(b) For $\Delta T > (\Delta T)_Y$, portion AC remains elastic.

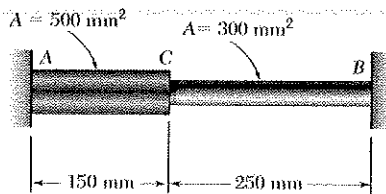
$$S_{C/A} = -\frac{P_Y L_{AC}}{EA_{AC}} + L_{AC}\alpha(\Delta T)$$

$$= -\frac{(75 \times 10^3)(0.150)}{(200 \times 10^9)(500 \times 10^{-6})} + (0.150)(11.7 \times 10^{-6})(125) = 106.9 \times 10^{-6} \text{ m}$$

Since point A is stationary, $S_C = S_{C/A} = 106.9 \times 10^{-6} \text{ m}$

$S_C = 0.1069 \text{ mm} \rightarrow \blacktriangleleft$

Problem 2.118



2.118 Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150 °C and then returned to 25 °C.

2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25 °C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150 °C. Knowing that $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$, determine (a) the stress in both portions of the rod, (b) the deflection of point *C*.

$$A_{AC} = 500 \times 10^{-6} \text{ m}^2 \quad L_{AC} = 0.150 \text{ m}$$

$$A_{CB} = 300 \times 10^{-6} \text{ m}^2 \quad L_{CB} = 0.250 \text{ m}$$

Constraint: $S_P + S_T = 0$

Determine ΔT to cause yielding in portion *CB*.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB} \alpha (\Delta T)$$

$$\Delta T = \frac{P}{L_{AB} E \alpha} \left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right)$$

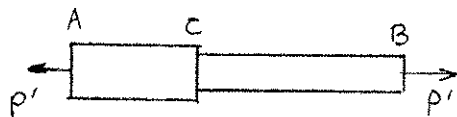
At yielding, $P = P_Y = A_{CB} \sigma_Y = (300 \times 10^{-6})(250 \times 10^6) = 75 \times 10^3 \text{ N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB} E \alpha} \left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right)$$

$$= \frac{75 \times 10^3}{(0.400)(200 \times 10^9)(11.7 \times 10^{-6})} \left(\frac{0.150}{500 \times 10^{-6}} + \frac{0.250}{300 \times 10^{-6}} \right) = 90.812^\circ\text{C}$$

Actual $\Delta T: 150^\circ\text{C} - 25^\circ\text{C} = 125^\circ\text{C} > (\Delta T)_Y$ Yielding occurs.

For $\Delta T > (\Delta T)_Y$ $P = P_Y = 75 \times 10^3 \text{ N}$



Cooling: $(\Delta T)' = 125^\circ\text{C}$

$$P' = \frac{EL_{AB} \alpha (\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right)}$$

$$P' = \frac{(200 \times 10^9)(0.400)(11.7 \times 10^{-6})(125)}{\frac{0.150}{500 \times 10^{-6}} + \frac{0.250}{300 \times 10^{-6}}} = 103.235 \times 10^3 \text{ N}$$

Residual force: $P_{res} = P' - P_Y = 103.235 \times 10^3 - 75 \times 10^3 = 28.235 \times 10^3 \text{ N (tension)}$

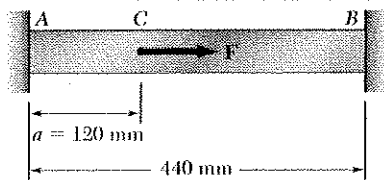
(a) Residual stresses. $\sigma_{AC} = \frac{P_{res}}{A_{AC}} = \frac{28.235 \times 10^3}{500 \times 10^{-6}} \quad \sigma_{AC} = 56.5 \text{ MPa} \leftarrow$

$\sigma_{CB} = \frac{P_{res}}{A_{CB}} = \frac{28.235 \times 10^3}{300 \times 10^{-6}} \quad \sigma_{CB} = 94.1 \text{ MPa} \leftarrow$

(b) Permanent deflection of point C. $s_C = \frac{P_{res} L_{AC}}{E A_{AC}} \quad s_C = 0.0424 \text{ mm} \rightarrow \leftarrow$

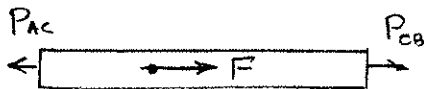
Problem 2.119

*2.119 Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$. Knowing that the force F increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point C , (b) the residual stress in the bar.



$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion AC : $P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$
 $= 300 \times 10^3 \text{ N}$



For equilibrium, $F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$

$$= -220 \times 10^3 \text{ N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.293333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading.

$$\delta'_C = \frac{P'_{AC} L_{AC}}{EA} = -\frac{P'_{CB} L_{CB}}{EA} = \frac{(F - P'_{AC}) L_{CB}}{EA}$$

$$P'_{AC} \left(\frac{L_{AC}}{EA} + \frac{L_{CB}}{EA} \right) = \frac{F L_{CB}}{EA}$$

$$P'_{AC} = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.120)}{0.440} = 378.182 \times 10^3 \text{ N}$$

$$P'_{CB} = P'_{AC} - F = 378.182 \times 10^3 - 520 \times 10^3 = -141.818 \times 10^3 \text{ N}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{378.182 \times 10^3}{1200 \times 10^{-6}} = 315.152 \times 10^6 \text{ Pa}$$

$$\sigma'_{BC} = \frac{P'_{BC}}{A} = -\frac{141.818 \times 10^3}{1200 \times 10^{-6}} = -118.182 \times 10^6 \text{ Pa}$$

$$\delta'_C = \frac{(378.182)(0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.189091 \times 10^{-3} \text{ m}$$

(a) $\delta_{Cp} = \delta_C - \delta'_C = 0.293333 \times 10^{-3} - 0.189091 \times 10^{-3} = 0.1042 \times 10^{-3} \text{ m}$
 $= 0.1042 \text{ mm}$

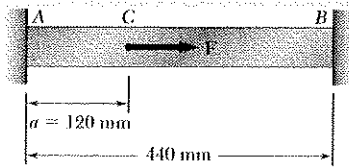
(b) $\sigma_{AC, \text{res}} = \sigma_Y - \sigma'_{AC} = 250 \times 10^6 - 315.152 \times 10^6 = -65.2 \times 10^6 \text{ Pa}$
 $= -65.2 \text{ MPa}$

$$\sigma_{CB, \text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333 \times 10^6 + 118.182 \times 10^6 = -65.2 \times 10^6 \text{ Pa}$$

$$= -65.2 \text{ MPa}$$

Problem 2.120

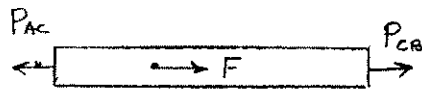
*2.120 Solve Prob. 2.119, assuming that $a = 180$ mm.



*2.119 Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$. Knowing that the force F increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point C , (b) the residual stress in the bar.

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

$$\text{Force to yield portion AC: } P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6) = 300 \times 10^3 \text{ N}$$



$$\text{For equilibrium } F + P_{CB} - P_{AC} = 0$$

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3 = -220 \times 10^3 \text{ N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.238333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading

$$\delta_C' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} = \frac{(F - P_{AC}') L_{CB}}{EA} \therefore P_{AC}' \left(\frac{L_{AC}}{EA} + \frac{L_{CB}}{EA} \right) = \frac{F L_{CB}}{EA}$$

$$P_{AC}' = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.180)}{0.440} = 307.273 \times 10^3 \text{ N}$$

$$P_{CB}' = P_{AC}' - F = 307.273 \times 10^3 - 520 \times 10^3 = -212.727 \times 10^3 \text{ N}$$

$$\delta_C' = \frac{(307.273 \times 10^3)(0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.230455 \times 10^{-3} \text{ m}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{307.273 \times 10^3}{1200 \times 10^{-6}} = 256.061 \times 10^6 \text{ Pa}$$

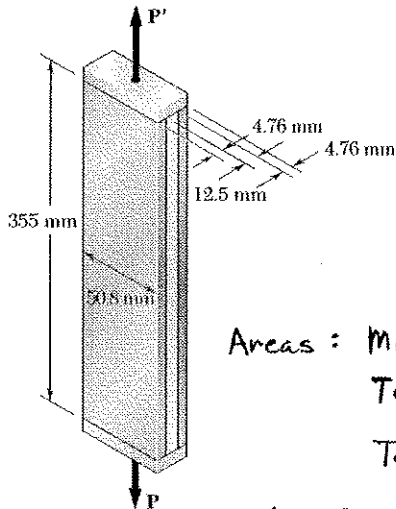
$$\sigma_{CB}' = \frac{P_{CB}'}{A} = \frac{-212.727 \times 10^3}{1200 \times 10^{-6}} = -177.273 \times 10^6 \text{ Pa}$$

$$(a) \delta_{cp} = \delta_C - \delta_C' = 0.238333 \times 10^{-3} - 0.230455 \times 10^{-3} = 0.00788 \times 10^{-3} \text{ m} = 0.00788 \text{ mm}$$

$$(b) \sigma_{AC, \text{res}} = \sigma_{AC} - \sigma_{AC}' = 250 \times 10^6 - 256.061 \times 10^6 = -6.06 \times 10^6 \text{ Pa} = -6.06 \text{ MPa}$$

$$\sigma_{CB, \text{res}} = \sigma_{CB} - \sigma_{CB}' = -183.333 \times 10^6 + 177.273 \times 10^6 = -6.06 \times 10^6 \text{ Pa} = -6.06 \text{ MPa}$$

Problem 2.121



***2.121** For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 436 kN and then decreased back to zero.

2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P . Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (a) the maximum value of P , (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

$$\begin{aligned} \text{Areas: Mild steel } A_1 &= (0.0125)(0.0508) = 635 \times 10^{-6} \text{ m}^2 \\ \text{Tempered steel } A_2 &= (2)(0.00476)(0.0508) = 483.6 \times 10^{-6} \text{ m}^2 \\ \text{Total: } A &= A_1 + A_2 = 1118.6 \times 10^{-6} \text{ m}^2 \end{aligned}$$

Total force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \therefore P_Y = A \sigma_{Y1} = (1118.6 \times 10^{-6})(345 \times 10^6) = 385.917 \text{ kN}$$

$P > P_Y$; therefore mild steel yields

Let P_1 = force carried by mild steel
 P_2 = force carried by tempered steel

$$P_1 = A_1 \sigma_{Y1} = (635 \times 10^{-6})(345 \times 10^6) = 219.075 \text{ kN}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 436 \times 10^3 - 219.075 \times 10^3 = 217 \text{ kN}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{217 \times 10^3}{483.6 \times 10^{-6}} = 448.7 \text{ MPa}$$

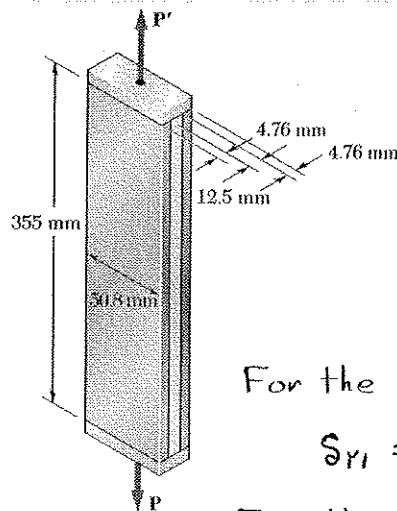
$$\text{Unloading } \sigma' = \frac{P}{A} = \frac{436 \times 10^3}{1118.6 \times 10^{-6}} = 389.8 \text{ MPa}$$

Residual stresses

$$\text{mild steel } \sigma_{1, \text{res}} = \sigma_1 - \sigma' = 345 \text{ MPa} - 389.8 \text{ MPa} = -44.8 \text{ MPa}$$

$$\begin{aligned} \text{tempered steel } \sigma_{2, \text{res}} &= \sigma_2 - \sigma' = 448.7 - 389.8 \\ &= 58.9 \text{ MPa} \end{aligned}$$

Problem 2.122



***2.122** For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 10$ mm and is then decreased back to zero.

2.111 Two tempered-steel bars, each 4.76 mm thick, are bonded to a 12.5-mm mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P . Both steels are elastoplastic with $E = 200$ GPa and with yield strengths equal to 690 MPa and 345 MPa, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 1.016$ mm and then decreased back to zero. Determine (a) the maximum value of P , (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

For the mild steel $A_1 = (0.0125)(0.0508) = 635 \times 10^{-6} \text{ m}^2$

$$S_{r1} = \frac{L \sigma_{r1}}{E} = \frac{(0.355)(345 \times 10^6)}{200 \times 10^9} = 0.612 \times 10^{-3} \text{ m}$$

For the tempered steel $A_2 = 2(0.00476)(0.0508) = 483.6 \times 10^{-6} \text{ m}^2$

$$S_{r2} = \frac{L \sigma_{r2}}{E} = \frac{(0.355)(690 \times 10^6)}{200 \times 10^9} = 1.225 \times 10^{-3} \text{ m}$$

Total area: $A = A_1 + A_2 = 1118.6 \times 10^{-6} \text{ m}^2$

$S_{r1} < S_m < S_{r2}$ The mild steel yields. Tempered steel is elastic.

Forces $P_1 = A_1 \sigma_{r1} = (635 \times 10^{-6})(345 \times 10^6) = 219.075 \text{ kN}$

$$P_2 = \frac{E A_2 S_m}{L} = \frac{(200 \times 10^9)(483.6 \times 10^{-6})(0.001016)}{0.355} = 276.81 \text{ kN}$$

Stresses $\sigma_1 = \frac{P_1}{A_1} = \sigma_{r1} = 345 \text{ MPa}$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{276.81 \times 10^3}{483.6 \times 10^{-6}} = 572.4 \text{ MPa}$$

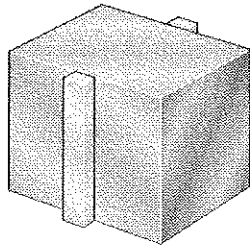
Unloading $\sigma' = \frac{P}{A} = \frac{495900}{1118.6 \times 10^{-6}} = 443.3 \text{ MPa}$

Residual stresses

$$\sigma_{1, \text{res}} = \sigma_1 - \sigma' = 345 - 443.3 = -98.3 \text{ MPa}$$

$$\sigma_{2, \text{res}} = \sigma_2 - \sigma' = 572.4 - 443.3 = 129.1 \text{ MPa}$$

Problem 2.123



***2.123** A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at $T_1 = 21^\circ\text{C}$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1 , determine (a) the highest temperature T_2 that does *not* result in residual stresses, (b) the temperature T_2 that will result in a residual stress in the aluminum equal to 400 MPa. Assume $\alpha_a = 23 \times 10^{-6}/^\circ\text{C}$ for the aluminum and $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 75 \text{ GPa}$ and $\sigma_Y = 400 \text{ MPa}$. (Hint: Neglect the small stresses in the plate.)

Determine temperature change to cause yielding

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{400 \times 10^6}{(75 \times 10^9)(23 - 11.7)(10^{-6})} = 472^\circ\text{C}$$

$$(a) \quad T_{2Y} = T_1 + (\Delta T)_Y = 21 + 472 = 493^\circ\text{C}$$

After yielding

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{\text{res}} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\text{Set } \sigma_{\text{res}} = -\sigma_Y$$

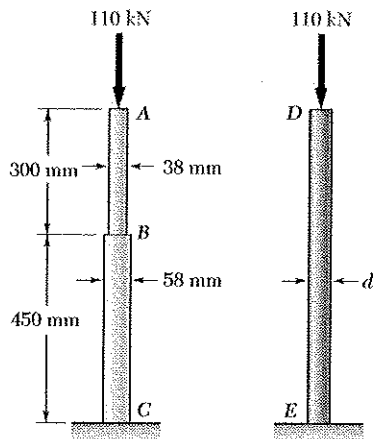
$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{400 \times 10^6}{(75 \times 10^9)(23 - 11.7)(10^{-6})} = 944^\circ\text{C}$$

$$(b) \quad T_2 = T_1 + \Delta T = 21 + 944 = 965^\circ\text{C}$$

If $T_2 > 965^\circ\text{C}$ the aluminum bar will most likely yield in compression.

Problem 2.124



2.124 The aluminum rod ABC ($E = 70 \text{ GPa}$), which consists of two cylindrical portions AB and BC , is to be replaced with a cylindrical steel rod DE ($E = 200 \text{ GPa}$) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 165 MPa .

Deformation of aluminum rod

$$\begin{aligned} \delta_A &= \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E} \left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right) \\ &= \frac{110 \times 10^3}{70 \times 10^9} \left(\frac{0.3}{\frac{\pi}{4}(0.038)^2} + \frac{0.45}{\frac{\pi}{4}(0.058)^2} \right) = 0.683 \times 10^{-3} \text{ m} \end{aligned}$$

Steel rod $\delta = 0.683 \times 10^{-3} \text{ m}$

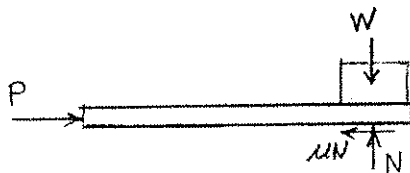
$$\delta = \frac{PL}{EA} \quad \therefore A = \frac{PL}{E\delta} = \frac{(110 \times 10^3)(0.75)}{(200 \times 10^9)(0.683 \times 10^{-3})} = 604 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} \quad \therefore A = \frac{P}{\sigma} = \frac{110 \times 10^3}{165 \times 10^6} = 0.667 \times 10^{-3} \text{ m}^2$$

Required area is the larger value $A = 0.667 \times 10^{-3} \text{ m}^2$

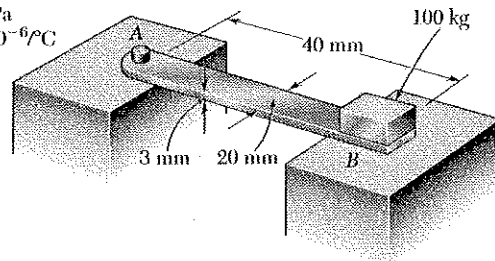
$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.667 \times 10^{-3})}{\pi}} = 0.029 \text{ m} = 29 \text{ mm}$$

Problem 2.125



2.125 The brass strip AB has been attached to a fixed support at A and rests on a rough support at B . Knowing that the coefficient of friction is 0.60 between the strip and the support at B , determine the decrease in temperature for which slipping will impend.

Brass strip:
 $E = 105 \text{ GPa}$
 $\alpha = 20 \times 10^{-6} / ^\circ\text{C}$



$$+\uparrow \Sigma F_y = 0: N - W = 0 \quad N = W$$

$$+\rightarrow \Sigma F_x = 0: P - \mu N = 0 \quad P = \mu W = \mu mg$$

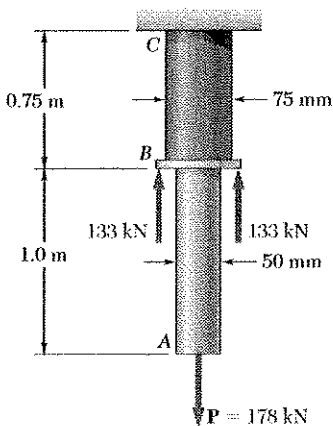
$$\delta = -\frac{PL}{EA} + L\alpha(\Delta T) = 0$$

$$\Delta T = \frac{P}{E\alpha d} = \frac{\mu mg}{E\alpha d}$$

Data: $\mu = 0.60$ $A = (20)(3) = 60 \text{ mm}^2 = 60 \times 10^{-6} \text{ m}^2$
 $m = 100 \text{ kg}$ $g = 9.81 \text{ m/s}^2$ $E = 105 \times 10^9 \text{ Pa}$

$$\Delta T = \frac{(0.60)(100)(9.81)}{(105 \times 10^9)(60 \times 10^{-6})(20 \times 10^{-6})} = 4.67^\circ\text{C}$$

Problem 2.126



2.126 Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel ($E = 200 \text{ GPa}$), and rod BC of brass ($E = 105 \text{ GPa}$). Determine (a) the total deformation of the composite rod ABC , (b) the deflection of point B .

Portion AB : $P_{AB} = 178 \text{ kN}$ $L_{AB} = 1 \text{ m}$ $d = 50 \text{ mm}$

$$A_{AB} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.05)^2 = 0.001963 \text{ m}^2 \quad E_{AB} = 200 \text{ GPa}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{(178 \times 10^3)(1)}{(200 \times 10^9)(0.001963)} = 0.45 \times 10^{-3} \text{ m}$$

Portion BC : $P_{BC} = -88 \text{ kN}$ $L_{BC} = 0.75 \text{ m}$, $d = 75 \text{ mm}$

$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.075)^2 = 0.00442 \text{ m}^2 \quad E_{BC} = 105 \text{ GPa}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{E_{BC} A_{BC}} = \frac{(-88 \times 10^3)(0.75)}{(105 \times 10^9)(0.00442)} = -0.142 \times 10^{-3} \text{ m}$$

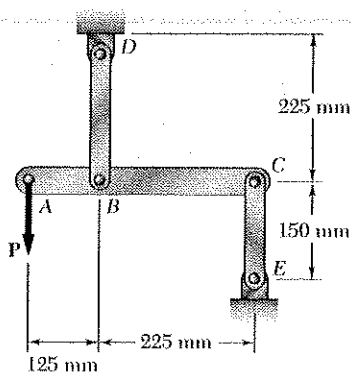
(a) $\delta = \delta_{AB} + \delta_{BC} = 0.45 - 0.142 = 0.308 \text{ mm}$

$\delta = 0.308 \text{ mm}$

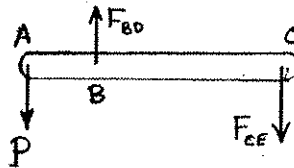
(b) $\delta_B = -\delta_{BC}$

$\delta_B = 0.142 \text{ mm}$

Problem 2.127



2.127 Link BD is made of brass ($E = 105 \text{ GPa}$) and has a cross-sectional area of 250 mm^2 . Link CE is made of aluminum ($E = 72 \text{ GPa}$) and has a cross-sectional area of 450 mm^2 . Determine the maximum force P that can be applied vertically at point A if the deflection of A is not to exceed 0.35 mm .



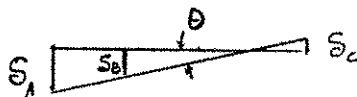
Use member ABC as a free body

$$\sum M_C = 0, \quad 350P - 225F_{BD} = 0, \quad F_{BD} = 1.5556P$$

$$\sum M_B = 0, \quad 125P - 225F_{CE} = 0, \quad F_{CE} = 0.5556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}} = \frac{(1.5556P)(0.225)}{(105 \times 10^9)(250 \times 10^{-6})} = 13.3334 \times 10^{-9} P \downarrow$$

$$\delta_C = \delta_{CE} = \frac{F_{CE} L_{CE}}{E_{CE} A_{CE}} = \frac{(0.5556P)(0.15)}{(72 \times 10^9)(450 \times 10^{-6})} = 2.5722 \times 10^{-9} P \uparrow$$



Deformation Diagram

From the deformation diagram

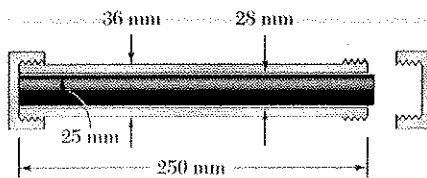
$$\text{slope } \theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{15.9056 \times 10^{-9} P}{0.225} = 70.6916 \times 10^{-9} P$$

$$\begin{aligned} \delta_A &= \delta_B + L_{AB} \theta \\ &= 13.3334 \times 10^{-9} P + (0.125)(70.69 \times 10^{-9}) P \\ &= 22.17 \times 10^{-9} P \end{aligned}$$

Apply displacement limit $\delta_A = 0.00035 = 22.17 \times 10^{-9} P$.

$$P = \frac{0.00035}{22.17 \times 10^{-9}} = 15.787 \text{ kN} \approx 15.79 \text{ kN}$$

Problem 2.128



2.128 A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

$$A_{\text{tube}} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\delta_{\text{tube}} = \frac{PL}{E_{\text{tube}}A_{\text{tube}}} = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} = 8.8815 \times 10^{-9} P$$

$$\delta_{\text{rod}} = -\frac{PL}{E_{\text{rod}}A_{\text{rod}}} = -\frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} = -4.8505 \times 10^{-9} P$$

$$\delta^* = \left(\frac{1}{4} \text{ turn}\right) \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{\text{tube}} = \delta^* + \delta_{\text{rod}} \quad \text{or} \quad \delta_{\text{tube}} - \delta_{\text{rod}} = \delta^*$$

$$8.8815 \times 10^{-9} P + 4.8505 \times 10^{-9} P = 375 \times 10^{-6}$$

$$P = \frac{0.375 \times 10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308 \times 10^3 \text{ N}$$

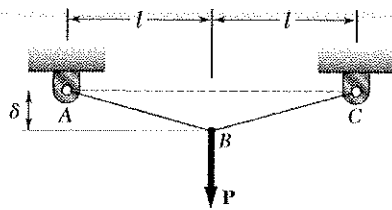
$$(a) \quad \sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{27.308 \times 10^3}{402.12 \times 10^{-6}} = 67.9 \times 10^6 \text{ Pa} = 67.9 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{27.308 \times 10^3}{490.87 \times 10^{-6}} = -55.6 \times 10^6 \text{ Pa} = -55.6 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \delta_{\text{tube}} = (8.8815 \times 10^{-9})(27.308 \times 10^3) = 242.5 \times 10^{-6} \text{ m} = 0.2425 \text{ mm} \quad \blacktriangleleft$$

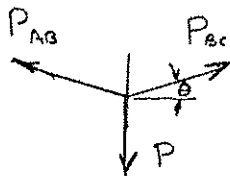
$$\delta_{\text{rod}} = -(4.8505 \times 10^{-9})(27.308 \times 10^3) = -132.5 \times 10^{-6} \text{ m} = -0.1325 \text{ mm} \quad \blacktriangleleft$$

Problem 2.129



2.129 The uniform wire ABC , of unstretched length $2l$, is attached to the supports shown and a vertical load P is applied at the midpoint B . Denoting by A the cross-sectional area of the wire and by E the modulus of elasticity, show that, for $\delta \ll l$, the deflection at the midpoint B is

$$\delta = l \sqrt[3]{\frac{P}{AE}}$$



Use approximation,

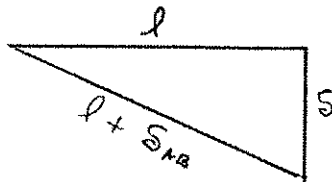
$$\sin \theta \approx \tan \theta \approx \frac{\delta}{l}$$

Statics $\uparrow \sum F_y = 0$: $2P_{AB} \sin \theta - P = 0$

$$P_{AB} = \frac{P}{2 \sin \theta} \approx \frac{Pl}{2\delta}$$

Elongation. $S_{AB} = \frac{P_{AB}l}{AE} = \frac{Pl^2}{2AES}$

Deflection



From the right triangle

$$(l + S_{AB})^2 = l^2 + S^2$$

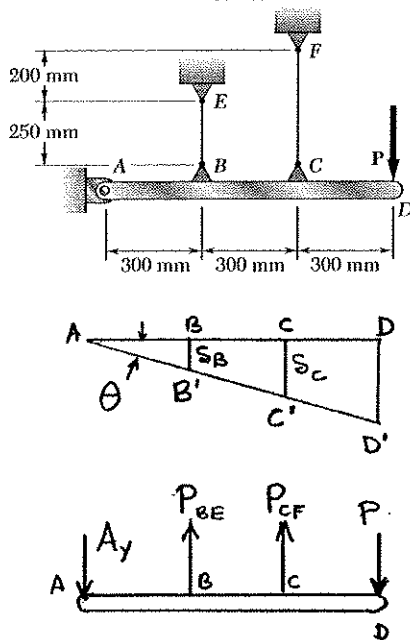
$$S^2 = l^2 + 2l S_{AB} + S_{AB}^2 - l^2$$

$$= 2l S_{AB} \left(1 + \frac{1}{2} \frac{S_{AB}}{l}\right) \approx 2l S_{AB}$$

$$\approx \frac{Pl^3}{AES}$$

$$S^3 \approx \frac{Pl^3}{AE} \therefore S \approx l \sqrt[3]{\frac{P}{AE}}$$

Problem 2.130



2.130 The rigid bar AD is supported by two steel wires of 1.5 mm diameter ($E = 200 \text{ GPa}$) and a pin and bracket at D. Knowing that the wires were initially taut, determine (a) the additional tension in each wire when a 900 N load P is applied at D, (b) the corresponding deflection of point D.

Let θ be the rotation of bar ABCD

Then $S_B = 0.3 \theta$

$S_C = 0.6 \theta$

$$S_B = \frac{P_{BE} L_{BE}}{AE}$$

$$P_{BE} = \frac{EA S_B}{L_{BE}} = \frac{(200 \times 10^9) \frac{\pi}{4} (0.0015)^2 (0.3 \theta)}{0.25}$$

$$= 424115 \theta$$

$$S_C = \frac{P_{CF} L_{CF}}{EA}$$

$$P_{CF} = \frac{EA S_C}{L_{CF}} = \frac{(200 \times 10^9) \frac{\pi}{4} (0.0015)^2 (0.6)}{0.45}$$

$$= 471239 \theta$$

Using free body ABCD

$$\sum M_A = 0 \quad 0.3 P_{BE} + 0.6 P_{CF} - 0.9 P = 0$$

$$(0.3)(424115 \theta) + (0.6)(471239 \theta) - (0.9)(900) = 0$$

$$409978 \theta = 810$$

$$\theta = 1.976 \times 10^{-3} \text{ rad}$$

(a) $P_{BE} = (424115)(1.976 \times 10^{-3}) = 838 \text{ N}$

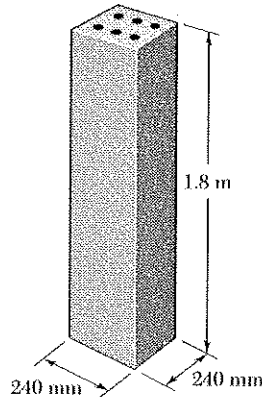
$P_{CF} = (471239)(1.976 \times 10^{-3}) = 931 \text{ N}$

(b) $S_D = 0.9 \theta = (0.9)(1.976 \times 10^{-3}) = 1.78 \times 10^{-3} \text{ m}$

$$= 1.78 \text{ mm}$$

Problem 2.131

2.131 The concrete post ($E_c = 25 \text{ GPa}$ and $\alpha_c = 9.9 \times 10^{-6}/^\circ\text{C}$) is reinforced with six steel bars, each of 22-mm diameter ($E_s = 200 \text{ GPa}$ and $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 35°C .



$$A_s = 6 \cdot \frac{\pi}{4} d^2 = 6 \cdot \frac{\pi}{4} (22)^2 = 2.2808 \times 10^3 \text{ mm}^2 = 2.2808 \times 10^{-3} \text{ m}^2$$

$$A_c = 240^2 - A_s = 240^2 - 2.2808 \times 10^3 = 55.32 \times 10^3 \text{ mm}^2 = 55.32 \times 10^{-3} \text{ m}^2$$

Let P_c = tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c .

Strains: $\epsilon_s = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$, $\epsilon_c = \frac{P_c}{E_c A_c} + \alpha_c (\Delta T)$

Matching: $\epsilon_c = \epsilon_s$ $\frac{P_c}{E_c A_c} + \alpha_c (\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s} \right) P_c = (\alpha_s - \alpha_c) (\Delta T)$$

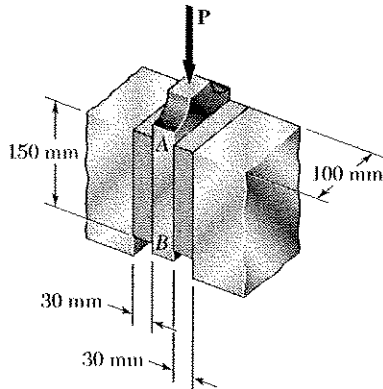
$$\left[\frac{1}{(25 \times 10^9)(55.32 \times 10^{-3})} + \frac{1}{(200 \times 10^9)(2.2808 \times 10^{-3})} \right] P_c = (1.8 \times 10^{-6})(35)$$

$$P_c = 21.61 \times 10^3 \text{ N}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{21.61 \times 10^3}{55.32 \times 10^{-3}} = 0.391 \times 10^6 \text{ Pa} = 0.391 \text{ MPa}$$

$$\sigma_s = -\frac{P_c}{A_s} = \frac{21.61 \times 10^3}{2.2808 \times 10^{-3}} = -9.47 \times 10^6 \text{ Pa} = -9.47 \text{ MPa}$$

Problem 2.132



2.132 A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19 \text{ MPa}$ bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.

Shearing strain $\gamma = \frac{\delta}{h}$

Shearing stress $\tau = G\gamma = \frac{G\delta}{h}$

Force $\frac{1}{2}P = A\tau = \frac{GA\delta}{h}$

$P = \frac{2GA\delta}{h}$

Effective spring constant

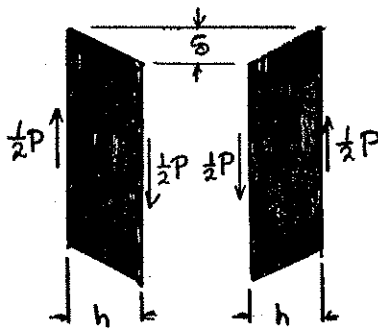
$k = \frac{P}{\delta} = \frac{2GA}{h}$

Data: $G = 19 \text{ MPa}$

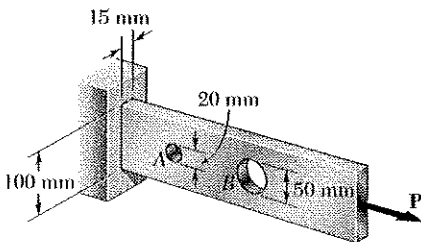
$A = (0.15)(0.1) = 0.015 \text{ m}^2$

$h = 30 \text{ mm}$

$k = \frac{2(19 \times 10^6)(0.015)}{0.03} = 19 \text{ N/m}$



Problem 2.133



2.133 Knowing that $\sigma_{\text{all}} = 120 \text{ MPa}$, determine the maximum allowable value of the centric axial load P .

At hole A: $r = \frac{1}{2}(20) = 10 \text{ mm}$

$d = D - 2r = 100 - 20 = 80 \text{ mm}$

$A_{\text{net}} = dt = (80)(15) = 1200 \text{ mm}^2 = 1.20 \times 10^{-3} \text{ m}^2$

$\frac{r}{d} = \frac{10}{80} = 0.125$

From Fig. 2.64a, $K = 2.65$

$\sigma_{\text{max}} = \frac{KP}{A_{\text{net}}} \quad P = \frac{A_{\text{net}}\sigma_{\text{max}}}{K} = \frac{(1.20 \times 10^{-3})(120 \times 10^6)}{2.65} = 54.3 \times 10^3 \text{ N}$

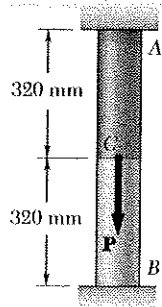
At hole B: $r = \frac{1}{2}(50) = 25 \text{ mm} \quad d = 100 - 50 = 50 \text{ mm}$

$A_{\text{net}} = (50)(15) = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2, \quad \frac{r}{d} = \frac{25}{50} = 0.50, \quad K = 2.16$

$P = \frac{(750 \times 10^{-6})(120 \times 10^6)}{2.16} = 41.7 \times 10^3 \text{ N}$

Allowable value of P is the smaller. $P = 41.7 \times 10^3 \text{ N} = 41.7 \text{ kN}$

Problem 2.134



2.134 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 2950 mm^2 . Portion AC is made of a mild steel with $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$, and portion CB is made of a high-strength steel with $E = 200 \text{ GPa}$ and $\sigma_Y = 345 \text{ MPa}$. A load P is applied at C as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of C if P is gradually increased from zero to 1625 kN and then reduced back to zero, (b) the permanent deflection of C.

Displacement at C to cause yielding of AC

$$S_{C,Y} = L_{AC} \epsilon_{Y,AC} = \frac{L_{AC} \sigma_{Y,AC}}{E} = \frac{(0.320)(250 \times 10^6)}{200 \times 10^9} = 0.400 \times 10^{-3} \text{ m}$$

Corresponding force $F_{AC} = A \sigma_{Y,AC} = (2950 \times 10^{-6})(250 \times 10^6) = 737.5 \times 10^3 \text{ N}$

$$F_{CB} = -\frac{E A S_{C,Y}}{L_{CB}} = -\frac{(200 \times 10^9)(2950 \times 10^{-6})(0.400 \times 10^{-3})}{0.320} = -737.5 \times 10^3 \text{ N}$$



For equilibrium of element at C,

$$F_{AC} - (F_{CB} + P) = 0 \quad P_Y = F_{AC} - F_{CB} = 1475 \times 10^3 \text{ N}$$

Since applied load $P = 1625 \times 10^3 \text{ N} > 1475 \times 10^3 \text{ N}$, portion AC yields.

$$F_{CB} = F_{AC} - P = 737.5 \times 10^3 - 1625 \times 10^3 \text{ N} = -887.5 \times 10^3 \text{ N}$$

$$(a) \quad S_C = -\frac{F_{CB} L_{CB}}{E A} = \frac{(887.5 \times 10^3)(0.320)}{(200 \times 10^9)(2950 \times 10^{-6})} = 0.48136 \times 10^{-3} \text{ m} = 0.481 \text{ mm} \downarrow$$

Maximum stresses. $\sigma_{AC} = \sigma_{Y,AC} = 250 \text{ MPa}$

$$\sigma_{BC} = \frac{F_{CB}}{A} = -\frac{887.5 \times 10^3}{2950 \times 10^{-6}} = -300.81 \times 10^6 \text{ Pa} = -301 \text{ MPa}$$

(b) Deflection and forces for unloading.

$$S' = \frac{P_{AC}' L_{AC}}{E A} = -\frac{P_{CB}' L_{CB}}{E A} \quad \therefore P_{CB}' = -P_{AC}' \frac{L_{AC}}{L_{CB}} = -P_{AC}'$$

$$P' = 1625 \times 10^3 = P_{AC}' - P_{CB}' = 2 P_{AC}' \quad P_{AC}' = 812.5 \times 10^3 \text{ N}$$

$$S' = \frac{(812.5 \times 10^3)(0.320)}{(200 \times 10^9)(2950 \times 10^{-6})} = 0.44068 \times 10^{-3} \text{ m}$$

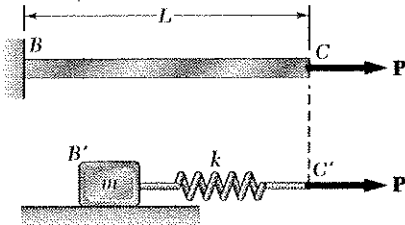
$$S_p = S_m - S' = 0.48136 \times 10^{-3} - 0.44068 \times 10^{-3} = 0.04068 \times 10^{-3} \text{ m}$$

Permanent deflection of C

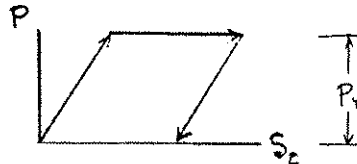
$$S_p = 0.0407 \text{ mm} \downarrow$$

Problem 2.135

2.135 The uniform rod BC has a cross-sectional area A and is made of a mild steel which can be assumed to be elastoplastic with a modulus of elasticity E and a yield strength σ_Y . Using the block-and-spring system shown, it is desired to simulate the deflection of end C of the rod as the axial force P is gradually applied and removed, that is, the deflection of points C and C' should be the same for all values of P . Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (a) the required mass m of the block, (b) the required constant k of the spring.



Force-deflection diagram for point C of rod BC .

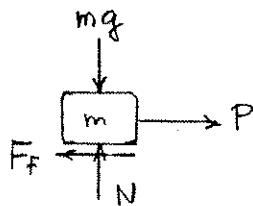


For $P < P_Y = A\sigma_Y$

$$s_c = \frac{PL}{EA}$$

$$P = \frac{EA}{L} s_c$$

$$P_{max} = P_Y = A\sigma_Y$$



Force-deflection diagram for point C' of block and spring system.

$$+\uparrow \sum F_y = 0: N - mg = 0 \quad N = mg$$

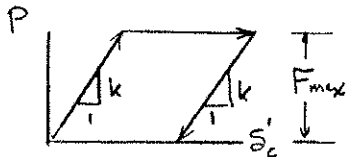
$$\pm \sum F_x = 0: P - F_f = 0 \quad P = F_f$$

If block does not move, i.e. $F_f < \mu N = \mu mg$ or $P < \mu mg$,

$$\text{then } s_c' = \frac{P}{k} \quad \text{or} \quad P = k s_c'$$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

If the force P is then removed, the spring returns to its initial length.



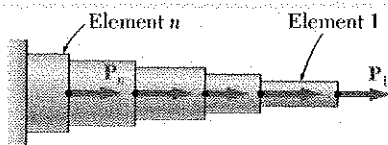
(a) Equating P_Y and F_{max} ,

$$A\sigma_Y = \mu mg \quad m = \frac{A\sigma_Y}{\mu g}$$

(b) Equating slopes,

$$k = \frac{EA}{L}$$

PROBLEM 2.C1



2.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i , its cross-sectional area by A_i , modulus of elasticity by E_i , and the load applied to its right end by P_i , the magnitude P_i of this load being assumed to be positive if P_i is directed to the right and negative otherwise. (a) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (b) Use this program to solve Probs. 2.20 and 2.126.

SOLUTION

FOR EACH ELEMENT, ENTER
 L_i, A_i, E_i

COMPUTE DEFORMATION

UPDATE AXIAL LOAD $P = P + P_i$

COMPUTE FOR EACH ELEMENT

$$\sigma_i = P/A_i$$

$$\delta_i = PL_i/A_i E_i$$

TOTAL DEFORMATION:

UPDATE THROUGH n ELEMENTS

$$\delta = \delta + \delta_i$$

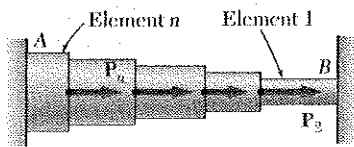
PROGRAM OUTPUT

Problem 2.20

Element	Stress (MPa)	Deformation (mm)
1	19.0986	.1091
2	-12.7324	-.0909
Total Deformation =		.0182 mm

Problem 2.126

Element	Stress (MPa)	Deformation (mm)
1	90.6775	0.4534
2	-19.9095	0.1422
Total Deformation =		0.3112

PROBLEM 2.C2

2.C2 Rod AB is horizontal with both ends fixed; it consists of n elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element i is denoted by L_i , its cross-sectional area by A_i , its modulus of elasticity by E_i , and the load applied to its right end by P_i , the magnitude P_i of this load being assumed to be positive if P_i is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (a) Write a computer program which can be used to determine the reactions at A and B , the average normal stress in each element, and the deformation of each element. (b) Use this program to solve Probs. 2.41 and 2.42.

SOLUTION

WE CONSIDER THE REACTION AT B REDUNDANT AND RELEASE THE ROD AT B

COMPUTE δ_B WITH $R_B = 0$

FOR EACH ELEMENT, ENTER

$$L_i, A_i, E_i$$

UPDATE AXIAL LOAD

$$P = P + P_i$$

COMPUTE FOR EACH ELEMENT

$$\sigma_i = P/A_i$$

$$\delta_i = PL_i/A_i E_i$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE δ_B DUE TO UNIT LOAD AT B

$$\text{UNIT } \sigma_i = 1/A_i$$

$$\text{UNIT } \delta_i = L_i/A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

SUPERPOSITION

FOR TOTAL DISPLACEMENT AT $B = \text{ZERO}$

$$\delta_B + R_B \text{ UNIT } \delta_B = 0$$

SOLVING:

$$R_B = -\delta_B / \text{UNIT } \delta_B$$

THEN:

$$R_A = \sum P_i + R_B$$

CONTINUED

PROBLEM 2.C2 CONTINUED

FOR EACH ELEMENT

$$\sigma = \sigma_i + R_B \text{ UNIT } \sigma_i$$

$$\delta = \delta_i + R_B \text{ UNIT } \delta_i$$

PROGRAM OUTPUT

Problem 2.41

RA = -62.809 kN

RB = -37.191 kN

Element	Stress (MPa)	Deformation (mm)
---------	--------------	------------------

1	-52.615	-.05011
2	3.974	.00378
3	2.235	.00134
4	49.982	.04498

Problem 2.42

RA = -45.479 kN

RB = -54.521 kN

Element	Stress (MPa)	Deformation (mm)
---------	--------------	------------------

1	-77.131	-.03857
2	-20.542	-.01027
3	-11.555	-.01321
4	36.191	.06204

PROBLEM 2.C3

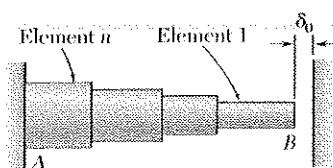


Fig. P2.C3

2.C3 Rod AB consists of n elements, each of which is homogeneous and of uniform cross section. End A is fixed, while initially there is a gap δ_0 between end B and the fixed vertical surface on the right. The length of element i is denoted by L_i , its cross-sectional area by A_i , its modulus of elasticity by E_i , and its coefficient of thermal expansion by α_i . After the temperature of the rod has been increased by ΔT , the gap at B is closed and the vertical surfaces exert equal and opposite forces on the rod. (a) Write a computer program which can be used to determine the magnitude of the reactions at A and B , the normal stress in each element, and the deformation of each element. (b) Use this program to solve Probs. 2.51, 2.59, and 2.60.

SOLUTION

WE COMPUTE THE DISPLACEMENTS AT B
 ASSUMING THERE IS NO SUPPORT AT B :

ENTER L_i, A_i, E_i, α_i

ENTER TEMPERATURE CHANGE T

COMPUTE FOR EACH ELEMENT

$$\delta_i = \alpha_i L_i T$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE δ_B DUE TO UNIT LOAD AT B

$$\text{UNIT } \delta_i = L_i / A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

COMPUTE REACTIONS

FROM SUPERPOSITION

$$R_B = (\delta_B - \delta_0) / \text{UNIT } \delta_B$$

THEN

$$R_A = -R_B$$

FOR EACH ELEMENT

$$\sigma_i = -R_B / A_i$$

$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

CONTINUED

PROBLEM 2.C3 CONTINUED

PROGRAM OUTPUT

Problem 2.51

R = 125.628 kN

Element	Stress (MPa)	Deform. (microm)
1	-44.432	.500
2	-99.972	-.500

Problem 2.59

R = 172.835 kN

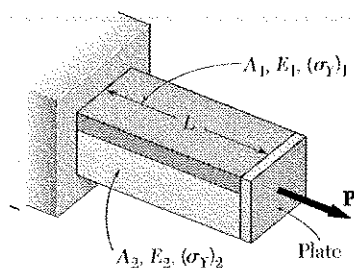
Element	Stress (MPa)	Deform. (mm)
1	-115.223	0.236
2	-96.019	0.24

Problem 2.60

R = 232.390 kN

Element	Stress (MPa)	Deform. (microm)
1	-116.195	363.220
2	-290.487	136.780

PROBLEM 2.C4



2.C4 Bar AB has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load P which is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (a) Write a computer program which, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (b) Use this program to solve Probs. 2.111, and 2.112.

SOLUTION

NOTE: THE FOLLOWING ASSUMES $(\sigma_Y)_1 < (\sigma_Y)_2$
DISPLACEMENT INCREMENT

$$\delta_m = 0.05 (\sigma_Y)_2 L / E_2$$

DISPLACEMENTS AT YIELDING

$$\delta_A = (\sigma_Y)_1 L / E_1 \quad \delta_B = (\sigma_Y)_2 L / E_2$$

FOR EACH DISPLACEMENT

IF $\delta_m < \delta_A$:

$$\sigma_1 = \delta_m E_1 / L$$

$$\sigma_2 = \delta_m E_2 / L$$

$$P_m = (\delta_m / L) (A_1 E_1 + A_2 E_2)$$

IF $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$

$$\sigma_2 = \delta_m E_2 / L$$

$$P_m = A_1 \sigma_1 + (\delta_m / L) A_2 E_2$$

IF $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1 \quad \sigma_2 = (\sigma_Y)_2$$

$$P_m = A_1 \sigma_1 + A_2 \sigma_2$$

PERMANENT DEFORMATIONS, RESIDUAL STRESSES

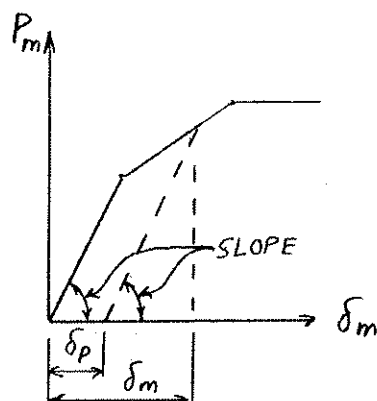
SLOPE OF FIRST (ELASTIC) SEGMENT

$$\text{SLOPE} = (A_1 E_1 + A_2 E_2) / L$$

$$\delta_p = \delta_m - (P_m / \text{SLOPE})$$

$$(\sigma_1)_{\text{res}} = \sigma_1 - (E_1 P_m / (L \text{ SLOPE}))$$

$$(\sigma_2)_{\text{res}} = \sigma_2 - (E_2 P_m / (L \text{ SLOPE}))$$



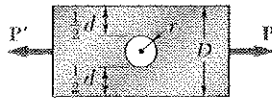
CONTINUED

PROBLEM 2.C4 CONTINUED

PROGRAM OUTPUT

Problems 2.109 and 2.110

DM mm	PM kN	SIGM(1) MPa	SIGM(2) MPa	DP mm	SIGR(1) MPa	SIGR(2) MPa
0	0	0	0	0	0	0
0.368	233.571	207.000	207.000	0	0	0
0.796	436.000	345.000	448.562	0.104	-44.892	58.580 P2.110
1.016	495.885	345.000	572.395	0.229	-98.560	128.835 P2.109
1.471	556.000	345.000	700.000	0.593	-150.248	198.305

PROBLEM 2.C5

2.C5 The plate has a hole centered across the width. The stress concentration factor for a flat bar under axial loading with a centric hole is:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. Write a computer program to determine the allowable load P for the given values of r , D , the thickness t of the bar, and the allowable stress σ_{all} of the material. Knowing that $r = 6$ mm, $D = 75$ mm and $\sigma_{all} = 110$ MPa, determine the allowable load P for values of t from 3 mm to 18 mm, using 3 mm increments.

SOLUTION

ENTER

$$r, D, t, \sigma_{all}$$

COMPUTE K

$$RD = 2.0 \, r/D$$

$$K = 3.00 - 3.13 \, RD + 3.66 \, RD^2 - 1.53 \, RD^3$$

COMPUTE AVERAGE STRESS

$$\sigma_{ave} = \sigma_{all}/K$$

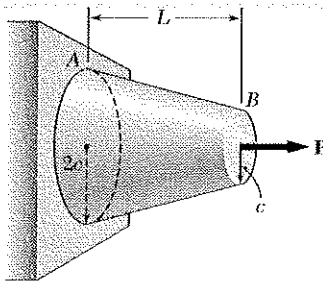
ALLOWABLE LOAD

$$P_{all} = \sigma_{ave} (D - 2.0 \, r) t$$

PROGRAM OUTPUT

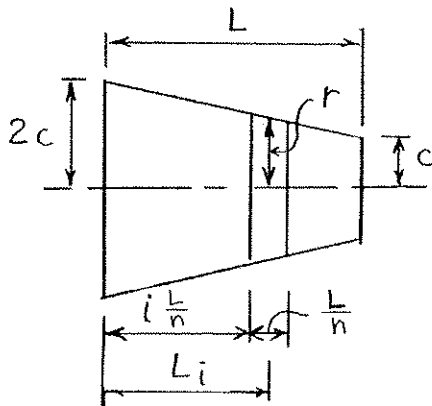
Radius (mm)	Allowable Load (kN)
3	16.405
6	16.023
9	15.368
12	14.391
15	13.165
18	11.721

PROBLEM 2.C6



2.C6 A solid truncated cone is subjected to an axial force P as shown. The exact elongation is $(PL)/(2\pi c^2 E)$. By replacing the cone by n circular cylinders of equal thickness, write a computer program that can be used to calculate the elongation of the truncated cone. What is the percentage error in the answer obtained from the program using (a) $n = 6$, (b) $n = 12$, (c) $n = 60$.

SOLUTION



FOR $i = 1$ TO n :

$$L_i = (i + 0.5)(L/n)$$

$$r_i = 2c - c(L_i/L)$$

AREA:

$$A = \pi r_i^2$$

DISPLACEMENT:

$$\delta = \delta + P(L/n)/(AE)$$

EXACT DISPLACEMENT:

$$\delta_{\text{EXACT}} = PL/(2.0\pi c^2 E)$$

PERCENTAGE ERROR:

$$\text{PERCENT} = 100(\delta - \delta_{\text{EXACT}})/\delta_{\text{EXACT}}$$

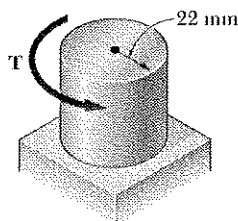
PROGRAM OUTPUT

n	Approximate	Exact	Percent
6	0.15852	0.15915	-.40083
12	0.15899	0.15915	-.10100
60	0.15915	0.15915	-.00405

Chapter 3

Problem 3.1

3.1 For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T = 1.5 \text{ kN} \cdot \text{m}$.



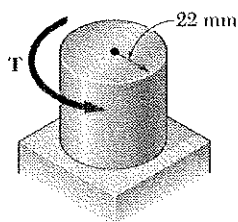
$$\tau_{\max} = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4$$

$$\tau_{\max} = \frac{2T}{\pi c^3} = \frac{(2)(1500)}{\pi (0.022)^3} = 89.682 \times 10^6$$

$$\tau_{\max} = 89.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.2

3.2 Determine the torque T that causes a maximum shearing stress of 80 MPa in the steel cylindrical shaft shown.



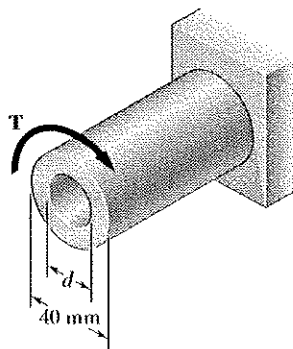
$$\tau_{\max} = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4$$

$$T = \frac{\pi}{2} c^3 \tau_{\max} = \frac{\pi}{2} (0.022)^3 (80 \times 10^6)$$

$$= 1338.1 \text{ N} \cdot \text{m} \quad T = 133.8 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

Problem 3.3

3.3 Knowing that the internal diameter of the hollow shaft shown is $d = 22 \text{ mm}$, determine the maximum shearing stress caused by a torque of magnitude $T = 900 \text{ N} \cdot \text{m}$.



$$c_2 = \frac{1}{2} d_2 = \left(\frac{1}{2}\right)(40) = 20 \text{ mm} \quad C = 0.02 \text{ m}$$

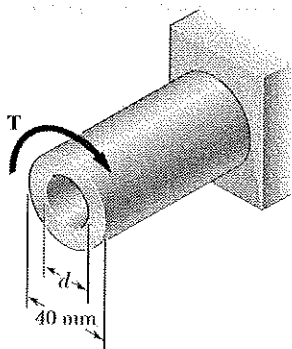
$$c_1 = \frac{1}{2} d_1 = \left(\frac{1}{2}\right)(22) = 11 \text{ mm}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (20^4 - 11^4) = 228329 \text{ mm}^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{(900)(0.02)}{228329 \times 10^{-12}} = 78.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.4

3.4 Knowing that $d = 30$ mm, determine the torque T that causes a maximum shearing stress of 52 MPa in the hollow shaft shown.



$$C_2 = \frac{1}{2} d_2 = \left(\frac{1}{2}\right)(40) = 20 \text{ mm}$$

$$C = 0.02 \text{ m}$$

$$C_1 = \frac{1}{2} d_1 = \left(\frac{1}{2}\right)(30) = 15 \text{ mm}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (20^4 - 15^4) = 171806 \text{ mm}^4$$

$$\tau_{\max} = \frac{T C}{J}$$

$$T = \frac{J \tau_{\max}}{C} = \frac{(171806 \times 10^{-12})(52 \times 10^6)}{0.02} = 446.7 \text{ Nm}$$

Problem 3.5

3.5 (a) Determine the torque that can be applied to a solid shaft of 20-mm diameter without exceeding an allowable shearing stress of 80 MPa. (b) Solve part a, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half of its own outer diameter.

(a) Solid shaft: $C = \frac{1}{2} d = \frac{1}{2}(0.020) = 0.010 \text{ m}$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.010)^4 = 15.7080 \times 10^{-9} \text{ m}^4$$

$$T = \frac{J \tau_{\max}}{C} = \frac{(15.7080 \times 10^{-9})(80 \times 10^6)}{0.010} = 125.664$$

$$T = 125.7 \text{ N}\cdot\text{m}$$

(b) Hollow shaft: Same area as solid shaft.

$$A = \pi (C_2^2 - C_1^2) = \pi [C_2^2 - (\frac{1}{2} C_2)^2] = \frac{3}{4} \pi C_2^2 = \pi C^2$$

$$C_2 = \frac{2}{\sqrt{3}} C = \frac{2}{\sqrt{3}} (0.010) = 0.0115470 \text{ m}$$

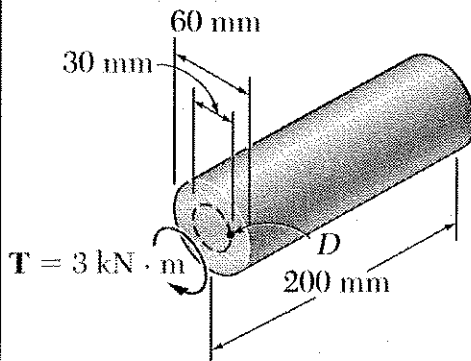
$$C_1 = \frac{1}{2} C_2 = 0.0057735 \text{ m}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.0115470^4 - 0.0057735^4) = 26.180 \times 10^{-9} \text{ m}^4$$

$$T = \frac{\tau_{\max} J}{C_2} = \frac{(80 \times 10^6)(26.180 \times 10^{-9})}{0.0115470} = 181.38$$

$$T = 181.4 \text{ N}\cdot\text{m}$$

Problem 3.6



3.6 A torque $T = 3 \text{ kN} \cdot \text{m}$ is applied to the solid bronze cylinder shown. Determine (a) the maximum shearing stress, (b) the shearing stress at point D which lies on a 15-mm-radius circle drawn on the end of the cylinder, (c) the percent of the torque carried by the portion of the cylinder within the 15-mm radius.

$$(a) \quad c = \frac{1}{2}d = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30 \times 10^{-3})^4 = 1.27235 \times 10^{-6} \text{ m}^4$$

$$T = 3 \text{ kN} = 3 \times 10^3 \text{ N}$$

$$\tau_m = \frac{Tc}{J} = \frac{(3 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}} = 70.736 \times 10^6 \text{ Pa}$$

$$\tau_m = 70.7 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \rho_D = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$\tau_D = \frac{\rho_D}{c} \tau_m = \frac{(15 \times 10^{-3})(70.736 \times 10^6)}{(30 \times 10^{-3})}$$

$$\tau_D = 35.4 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \tau_D = \frac{T_D \rho_D}{J_D}$$

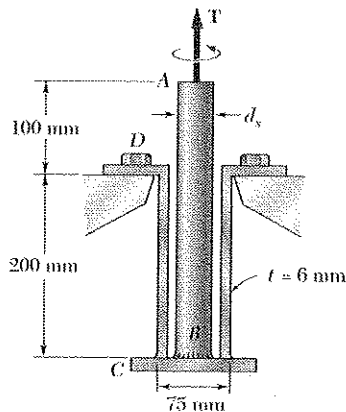
$$T_D = \frac{J_D \tau_D}{\rho_D} = \frac{\pi}{2} \rho_D^3 \tau_D$$

$$T_D = \frac{\pi}{2} (15 \times 10^{-3})^3 (35.368 \times 10^6) = 187.5 \text{ N} \cdot \text{m}$$

$$\frac{T_D}{T} \times 100\% = \frac{187.5}{3 \times 10^3} (100\%) =$$

$$6.25\% \quad \blacktriangleleft$$

Problem 3.7



3.7 The solid spindle AB is made of a steel with an allowable shearing stress of 84 MPa, and sleeve CD is made of a brass with an allowable shearing stress of 50 MPa. Determine (a) the largest torque T that can be applied at A if the allowable shearing stress is not to be exceeded in sleeve CD , (b) the corresponding required value of the diameter d_s of spindle AB .

(a) Sleeve CD : $c_2 = \frac{1}{2} d_2 = \frac{1}{2} (75) = 37.5 \text{ mm}$

$$c_1 = c_2 - t = 37.5 - 6 = 31.5 \text{ mm}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (0.0375^4 - 0.0315^4) = 1.56 \times 10^{-6} \text{ m}^4$$

$$\tau_{\max} = \frac{T c_2}{J}$$

$$T_{CD} = \frac{J \tau_{CD}}{c_2} = \frac{(1.56 \times 10^{-6})(50 \times 10^6)}{0.0375} = 2080 \text{ N}\cdot\text{m}$$

For equilibrium $T = 2.08 \text{ kN}$

(b) Solid spindle AB :

$$T = 2080 \text{ N}\cdot\text{m}$$

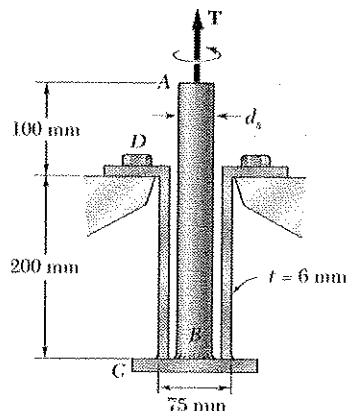
$$\tau = \frac{T c}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(2080)}{\pi (84 \times 10^6)}} = 0.025 \text{ m} = 25 \text{ mm}$$

$$d_s = 2c = (2)(25)$$

$$d_s = 50 \text{ mm}$$

Problem 3.8



3.8 The solid spindle AB has a diameter $d_s = 38 \text{ mm}$ and is made of a steel with an allowable shearing stress of 84 MPa, while sleeve CD is made of a brass with an allowable shearing stress of 50 MPa. Determine the largest torque T that can be applied at A .

Solid spindle AB : $c = \frac{1}{2} d_s = \frac{1}{2} (38) = 19 \text{ mm}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.019)^4 = 204.7 \times 10^{-9}$$

$$\tau_{\max} = \frac{T c}{J}$$

$$T_{AB} = \frac{J \tau_{AB}}{c} = \frac{(204.7 \times 10^{-9})(84 \times 10^6)}{0.019} = 905 \text{ N}\cdot\text{m}$$

Sleeve CD : $c_2 = \frac{1}{2} d_2 = \frac{1}{2} (75) = 37.5 \text{ mm}$

$$c_1 = c_2 - t = 37.5 - 6 = 31.5 \text{ mm}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (0.0375^4 - 0.0315^4) = 1.56 \times 10^{-6} \text{ m}^4$$

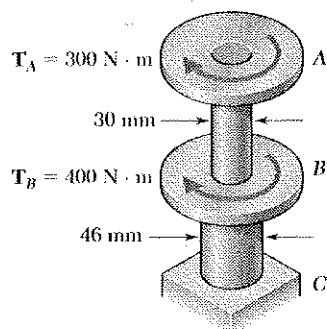
$$T_{CD} = \frac{J \tau_{CD}}{c_2} = \frac{(1.56 \times 10^{-6})(50 \times 10^6)}{0.0375} = 2080 \text{ N}\cdot\text{m}$$

Allowable value of torque T is the smaller.

$$T = 2.08 \text{ kN}\cdot\text{m}$$

Problem 3.9

3.9 The torques shown are exerted on pulleys *A* and *B*. Knowing that each shaft is solid, determine the maximum shearing stress (a) in shaft *AB*, (b) in shaft *BC*.



(a) Shaft AB: $T_{AB} = 300 \text{ N}\cdot\text{m}$, $d = 0.030 \text{ m}$, $c = 0.015 \text{ m}$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi (0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa} \quad \blacktriangleleft$$

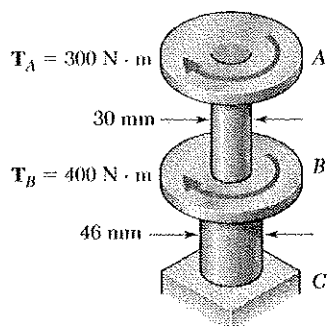
(b) Shaft BC: $T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}$

$d = 0.046 \text{ m}$, $c = 0.023 \text{ m}$ $\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi (0.023)^3}$

$$= 36.626 \times 10^6 \text{ Pa} \quad 36.6 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.10

3.10 The torques shown are exerted on pulleys *A* and *B* which are attached to solid circular shafts *AB* and *BC*. In order to reduce the total mass of the assembly, determine the smallest diameter of shaft *BC* for which the largest shearing stress in the assembly is not increased.



Shaft AB: $T_{AB} = 300 \text{ N}\cdot\text{m}$, $d = 0.030 \text{ m}$, $c = 0.015 \text{ m}$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi (0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

Shaft BC: $T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}$

$d = 0.046 \text{ m}$, $c = 0.023 \text{ m}$ $\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi (0.023)^3}$

$$= 36.626 \times 10^6 \text{ Pa} = 36.6 \text{ MPa}$$

The largest stress ($56.588 \times 10^6 \text{ Pa}$) occurs in portion AB

Reduce the diameter of *BC* to provide the same stress.

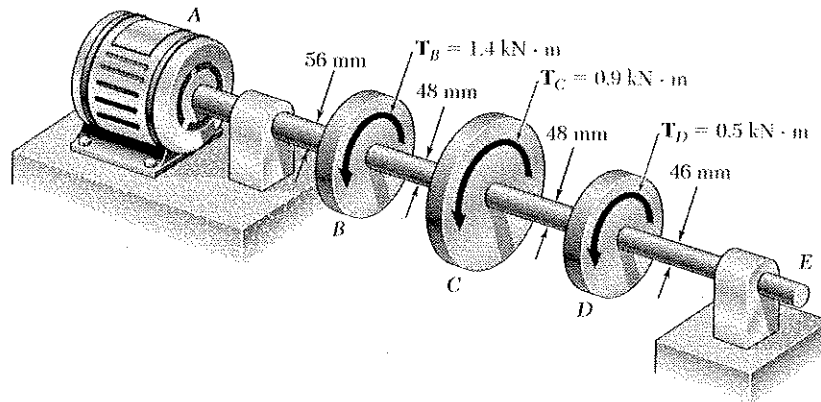
$$T_{BC} = 700 \text{ N}\cdot\text{m} \quad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau_{\max}} = \frac{(2)(700)}{\pi (56.588 \times 10^6)} = 7.875 \times 10^{-6} \text{ m}^3$$

$$c = 19.895 \times 10^{-3} \text{ m} \quad d = 2c = 39.79 \times 10^{-3} \text{ m} \quad 39.8 \text{ mm} \quad \blacktriangleleft$$

Problem 3.11

3.11 Under normal operating conditions, the electric motor exerts a torque of $2.8 \text{ kN} \cdot \text{m}$ on shaft AB . Knowing that each shaft is solid, determine the maximum shearing stress in (a) shaft AB , (b) shaft BC , (c) shaft CD .



- (a) Shaft AB: $T_{AB} = 2.8 \text{ kN} \cdot \text{m} = 2.8 \times 10^3 \text{ N} \cdot \text{m}$, $c = \frac{1}{2}d = 28 \text{ mm} = 0.028 \text{ m}$
 $\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.8 \times 10^3)}{\pi (0.028)^3} = 81.20 \times 10^6 \text{ Pa}$ $\tau_{AB} = 81.2 \text{ MPa}$ $\blacktriangleleft$
- (b) Shaft BC: $T_{BC} = 1.4 \text{ kN} \cdot \text{m} = 1.4 \times 10^3 \text{ N} \cdot \text{m}$, $c = \frac{1}{2}d = 24 \text{ mm} = 0.024 \text{ m}$
 $\tau_{BC} = \frac{2T}{\pi c^3} = \frac{(2)(1.4 \times 10^3)}{\pi (0.024)^3} = 64.47 \times 10^6 \text{ Pa}$ $\tau_{BC} = 64.5 \text{ MPa}$ $\blacktriangleleft$
- (c) Shaft CD: $T_{CD} = 0.5 \text{ kN} \cdot \text{m} = 0.5 \times 10^3 \text{ N} \cdot \text{m}$, $c = \frac{1}{2}d = 23 \text{ mm} = 0.023 \text{ m}$
 $\tau_{CD} = \frac{2T}{\pi c^3} = \frac{(2)(0.5 \times 10^3)}{\pi (0.023)^3} = 23.03 \times 10^6 \text{ Pa}$ $\tau_{CD} = 23.0 \text{ MPa}$ $\blacktriangleleft$

Problem 3.12

3.12 In order to reduce the total mass of the assembly of Prob 3.11, a new design is being considered in which the diameter of shaft BC will be smaller. Determine the smallest diameter of shaft BC for which the maximum value of the shearing stress in the assembly will not be increased.

See the solution to Problem 3.11 for figure and for maximum shearing stresses in shafts AB , BC , and CD . The largest shearing occurs in shaft AB . Its magnitude is 81.2 MPa . Adjust the diameter of shaft BC so that its maximum shearing stress is 81.2 MPa .

$$\tau_{BC} = 81.2 \times 10^6 \text{ Pa} \quad T_{BC} = 1.4 \times 10^3 \text{ N} \cdot \text{m}$$

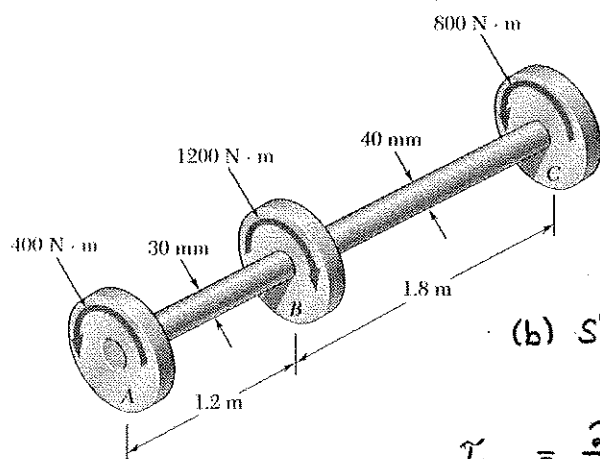
$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T_{BC}}{\pi \tau_{BC}}} = \sqrt[3]{\frac{(2)(1.4 \times 10^3)}{\pi (81.2 \times 10^6)}} = 22.22 \times 10^{-3} \text{ m} = 22.22 \text{ mm}$$

$$d = 2c = 44.4 \text{ mm} \quad \blacktriangleleft$$

Problem 3.13

3.13 The torques shown are exerted on pulleys A, B, and C. Knowing that both shafts are solid, determine the maximum shearing stress in (a) shaft AB, (b) shaft BC.



(a) Shaft AB: $T = 400 \text{ N}\cdot\text{m}$

$$c = \frac{1}{2}d = \frac{1}{2}(0.030) = 0.015 \text{ m}$$

$$J = \frac{\pi}{2}c^4 \quad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)(400)}{\pi(0.015)^3} = 75.5 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 75.5 \text{ MPa} \quad \blacktriangleleft$$

(b) Shaft BC: $T = 800 \text{ N}\cdot\text{m}$

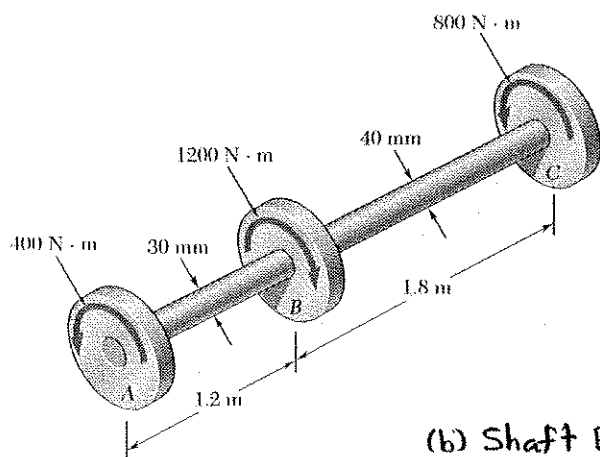
$$c = \frac{1}{2}d = 0.020 \text{ m}$$

$$\tau_{\max} = \frac{2T}{\pi c^3} = \frac{(2)(800)}{\pi(0.020)^3} = 63.7 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 63.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.14

3.14 The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 60 MPa, determine the smallest allowable diameter of (a) shaft AB, (b) shaft BC.



(a) Shaft AB: $T = 400 \text{ N}\cdot\text{m}$

$$\tau_{\max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

$$J = \frac{\pi}{2}c^4 \quad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(400)}{\pi(60 \times 10^6)}}$$

$$= 16.19 \times 10^{-3} \text{ m} = 16.19 \text{ mm}$$

$$d_{AB} = 2c = 32.4 \text{ mm} \quad \blacktriangleleft$$

(b) Shaft BC: $T = 800 \text{ N}\cdot\text{m}$

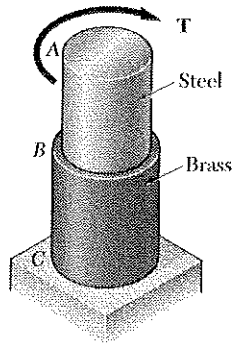
$$\tau_{\max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(800)}{\pi(60 \times 10^6)}}$$

$$= 20.40 \times 10^{-3} \text{ m} = 20.40 \text{ mm}$$

$$d_{BC} = 2c = 40.8 \text{ mm} \quad \blacktriangleleft$$

Problem 3.15



3.15 The allowable shearing stress is 100 MPa in the steel rod *AB* and 60 MPa in the brass rod *BC*. Knowing that a torque of magnitude $T = 900 \text{ N} \cdot \text{m}$ is applied at *A* and neglecting the effect of stress concentrations, determine the required diameter of (a) rod *AB*, (b) rod *BC*.

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2} c^4, \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

Shaft *AB*: $\tau_{max} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(100 \times 10^6)}} = 17.894 \times 10^{-3} \text{ m} = 17.894 \text{ mm}$$

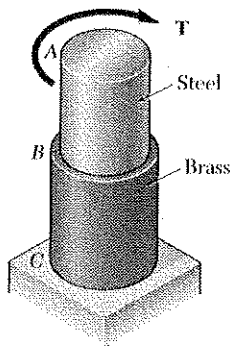
$$d_{AB} = 2c = 35.8 \text{ mm} \quad \blacktriangleleft$$

Shaft *BC*: $\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$

$$c = \sqrt[3]{\frac{(2)(900)}{\pi(60 \times 10^6)}} = 21.216 \times 10^{-3} \text{ m} = 21.216 \text{ mm}$$

$$d_{BC} = 2c = 42.4 \text{ mm} \quad \blacktriangleleft$$

Problem 3.16



3.16 The allowable shearing stress is 100 MPa in the 36-mm-diameter steel rod *AB* and 60 MPa in the 40-mm-diameter rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2} c^4, \quad T = \frac{\pi}{2} \tau_{max} c^3$$

Shaft *AB*: $\tau_{max} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$

$$c = \frac{1}{2} d_{AB} = \frac{1}{2}(36) = 18 \text{ mm} = 0.018 \text{ m}$$

$$T_{AB} = \frac{\pi}{2} (100 \times 10^6) (0.018)^3 = 916 \text{ N} \cdot \text{m}$$

Shaft *BC*: $\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$

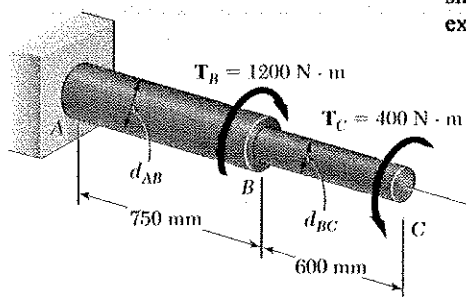
$$c = \frac{1}{2} d_{BC} = \frac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

$$T_{BC} = \frac{\pi}{2} (60 \times 10^6) (0.020)^3 = 1754 \text{ N} \cdot \text{m}$$

The allowable torque is the smaller of T_{AB} and T_{BC} .

$$T = 916 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

Problem 3.17



3.17 The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

$$\tau_{max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{T_C}{J} = \frac{2T}{\pi C^3} \quad C = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$\text{Shaft AB: } T_{AB} = 1200 - 400 = 800 \text{ N}\cdot\text{m}$$

$$C = \sqrt[3]{\frac{(2)(800)}{\pi(55 \times 10^6)}} = 21.00 \times 10^{-3} \text{ m} = 21.0 \text{ mm}$$

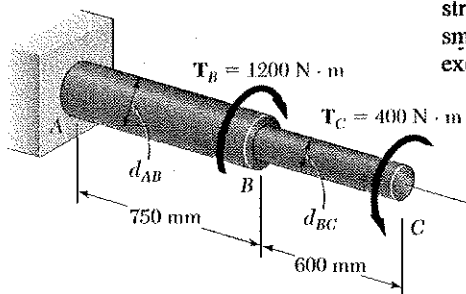
$$\text{minimum } d_{AB} = 2C = 42.0 \text{ mm} \quad \blacktriangleleft$$

$$\text{Shaft BC: } T_{BC} = 400 \text{ N}\cdot\text{m}$$

$$C = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

$$\text{minimum } d_{BC} = 2C = 33.3 \text{ mm} \quad \blacktriangleleft$$

Problem 3.18



3.18 Solve Prob. 3.17 assuming that the direction of T_C is reversed.

3.17 The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

Note that the direction of T_C has been reversed in the figure to the left.

$$\tau_{max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{T_C}{J} = \frac{2T}{\pi C^3} \quad C = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$\text{Shaft AB: } T_{AB} = 1200 + 400 = 1600 \text{ N}\cdot\text{m}$$

$$C = \sqrt[3]{\frac{(2)(1600)}{\pi(55 \times 10^6)}} = 26.46 \times 10^{-3} \text{ m} = 26.46 \text{ mm}$$

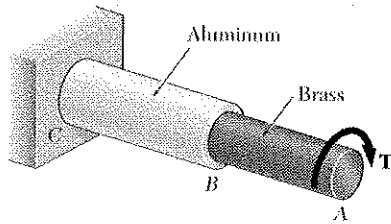
$$\text{minimum } d_{AB} = 2C = 52.9 \text{ mm} \quad \blacktriangleleft$$

$$\text{Shaft BC: } T_{BC} = 400 \text{ N}\cdot\text{m}$$

$$C = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

$$\text{minimum } d_{BC} = 2C = 33.3 \text{ mm} \quad \blacktriangleleft$$

Problem 3.19



3.19 The allowable stress is 50 MPa in the brass rod AB and 25 MPa in the aluminum rod BC. Knowing that a torque of magnitude $T = 125 \text{ N} \cdot \text{m}$ is applied at A, determine the required diameter of (a) rod AB, (b) rod BC.

$$\tau_{\max} = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4 \quad c^3 = \frac{2T}{\pi \tau_{\max}}$$

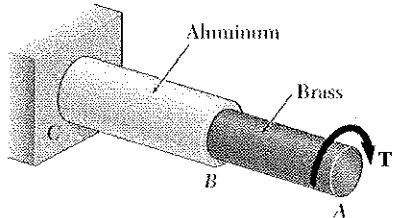
(a) Rod AB: $c^3 = \frac{(2)(1250)}{\pi (50 \times 10^6)} = 15.915 \times 10^{-6} \text{ m}^3$

$$c = 25.15 \times 10^{-3} \text{ m} = 25.15 \text{ mm} \quad d_{AB} = 2c = 50.3 \text{ mm}$$

(b) Rod BC: $c^3 = \frac{(2)(1250)}{\pi (25 \times 10^6)} = 31.831 \times 10^{-6} \text{ m}^3$

$$c = 31.69 \times 10^{-3} \text{ m} = 31.69 \text{ mm} \quad d_{BC} = 2c = 63.4 \text{ mm}$$

Problem 3.20



3.20 The solid rod BC has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod AB is hollow and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (a) the largest inner diameter of rod AB for which the factor of safety is the same for each rod, (b) the largest torque that can be applied at A.

Solid rod BC: $\tau = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4$

$$\tau_{\text{all}} = 25 \times 10^6 \text{ Pa} \quad c = \frac{1}{2} d = 0.015 \text{ m}$$

$$T_{\text{all}} = \frac{\pi}{2} c^3 \tau_{\text{all}} = \frac{\pi}{2} (0.015)^3 (25 \times 10^6) = 132.536 \text{ N} \cdot \text{m}$$

Hollow rod AB: $\tau_{\text{all}} = 50 \times 10^6 \text{ Pa} \quad T_{\text{all}} = 132.536 \text{ N} \cdot \text{m}$

$$c_2 = \frac{1}{2} d_2 = \frac{1}{2} (0.025) = 0.0125 \text{ m} \quad c_1 = ?$$

$$T_{\text{all}} = \frac{J \tau_{\text{all}}}{c_2} = \frac{\pi}{2} (c_2^4 - c_1^4) \frac{\tau_{\text{all}}}{c_2}$$

$$c_1^4 = c_2^4 - \frac{2 T_{\text{all}} c_2}{\pi \tau_{\text{all}}} = 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi (50 \times 10^6)} = 3.3203 \times 10^{-9} \text{ m}^4$$

(a) $c_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm} \quad d_1 = 2c_1 = 15.18 \text{ mm}$

(b) Allowable torque.

$$T_{\text{all}} = 132.5 \text{ N} \cdot \text{m}$$

Problem 3.21

3.21 A torque of magnitude $T = 900 \text{ N} \cdot \text{m}$ is applied at D as shown. Knowing that the allowable shearing stress is 50 MPa in each shaft, determine the required diameter of (a) shaft AB , (b) shaft CD .

$$T_{CD} = 900 \text{ N} \cdot \text{m}.$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100(900)}{40} = 2250 \text{ N} \cdot \text{m}.$$

$$\tau_{\max} = 50 \text{ MPa}.$$

$$\tau_{\max} = \frac{T_C}{J} = \frac{2T}{\pi C^3}$$

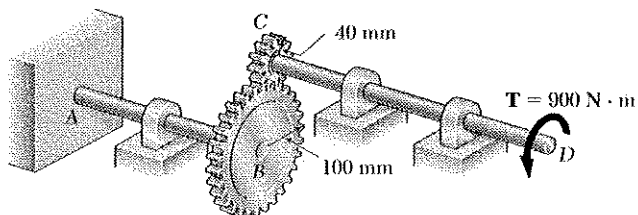
$$C = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}}$$

$$(a) \text{ Shaft } AB: \quad C = \sqrt[3]{\frac{(2)(2250)}{\pi(50 \times 10^6)}} = 0.0306 \text{ m} = 30.6 \text{ mm}$$

$$d_{AB} = 2C = 61.2 \text{ mm} \quad \blacktriangleleft$$

$$(b) \text{ Shaft } CD: \quad C = \sqrt[3]{\frac{(2)(900)}{\pi(50 \times 10^6)}} = 0.0225 \text{ m} = 22.5 \text{ mm}$$

$$d_{CD} = 2C = 45 \text{ mm} \quad \blacktriangleleft$$



Problem 3.22

3.22 A torque of magnitude $T = 900 \text{ N} \cdot \text{m}$ is applied at D as shown. Knowing that the diameter of shaft AB is 60 mm and that the diameter of shaft CD is 45 mm , determine the maximum shearing stress in (a) shaft AB , (b) shaft CD .

$$T_{CD} = 900 \text{ N} \cdot \text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(900) = 2250 \text{ N} \cdot \text{m}.$$

$$(a) \text{ Shaft } AB: \quad C = \frac{1}{2} d_{AB} = 30 \text{ mm}$$

$$\tau_{\max} = \frac{T_C}{J} = \frac{2T}{\pi C^3}$$

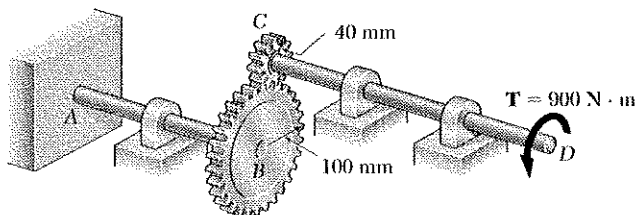
$$\tau_{\max} = \frac{(2)(2250)}{\pi (0.03)^3} = 53.05 \text{ MPa}.$$

$$(a) \tau_{\max} = 53.05 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \text{ Shaft } CD: \quad C = \frac{1}{2} d_{CD} = 22.5 \text{ mm}$$

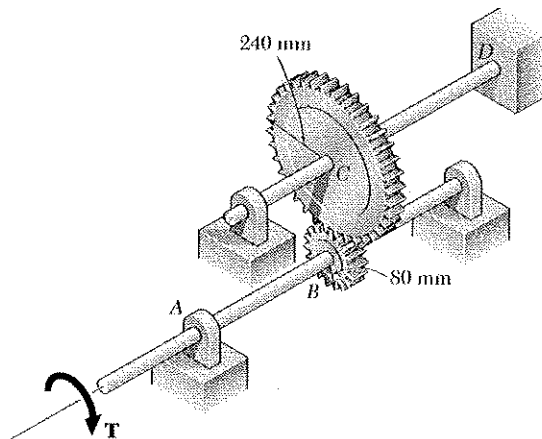
$$\tau_{\max} = \frac{2T}{\pi C^3} = \frac{(2)(900)}{\pi (0.0225)^3} = 50.3 \text{ MPa}$$

$$(b) \tau_{\max} = 50.3 \text{ MPa} \quad \blacktriangleleft$$



Problem 3.23

3.23 Two solid steel shafts are connected by the gears shown. A torque of magnitude $T = 900 \text{ N} \cdot \text{m}$ is applied to shaft AB . Knowing that the allowable shearing stress is 50 MPa and considering only stresses due to twisting, determine the required diameter of (a) shaft AB , (b) shaft CD .



$$T_{AB} = T = 900 \text{ N} \cdot \text{m}$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{240}{80} (900) = 2700 \text{ N} \cdot \text{m}$$

(a) Shaft AB: $\tau_{max} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$

$$\tau_{max} = \frac{T_C}{J} = \frac{2T}{\pi C^3}, \quad C = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$C = \sqrt[3]{\frac{(2)(900)}{\pi(50 \times 10^6)}} = 22.55 \times 10^{-3} \text{ m} = 22.55 \text{ mm}$$

$$d_{AB} = 2C = 45.1 \text{ mm} \quad \blacktriangleleft$$

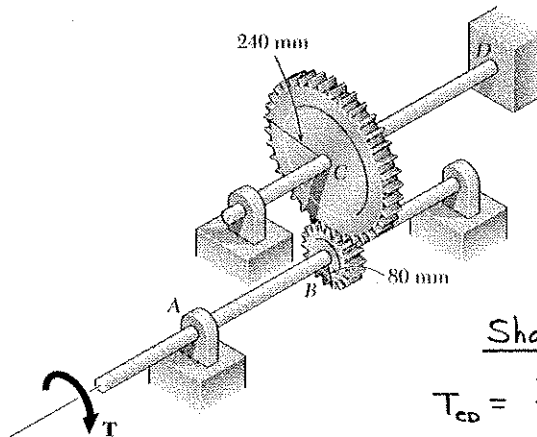
(b) Shaft CD: $\tau_{max} = 50 \text{ MPa}$

$$C = \sqrt[3]{\frac{(2)(2700)}{\pi(50 \times 10^6)}} = 32.52 \times 10^{-3} \text{ m} = 32.52 \text{ mm}$$

$$d_{CD} = 2C = 65.0 \text{ mm} \quad \blacktriangleleft$$

Problem 3.24

3.24 Shaft CD is made from a 66-mm-diameter rod and is connected to the 48-mm-diameter shaft AB as shown. Considering only stresses due to twisting and knowing that the allowable shearing stress is 60 MPa for each shaft, determine the largest torque T that can be applied.



$$\tau_{max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

Shaft AB: $C = \frac{1}{2} d_{AB} = 24 \text{ mm} = 0.024 \text{ m}$

$$\tau_{max} = \frac{T_C}{J} = \frac{2T}{\pi C^3} \quad T = \frac{\pi}{2} \tau_{max} C^3$$

$$T = \frac{\pi}{2} (60 \times 10^6) (0.024)^3 = 1303 \text{ N} \cdot \text{m}$$

Shaft CD: $C = \frac{1}{2} d_{CD} = 33 \text{ mm} = 0.033 \text{ m}$

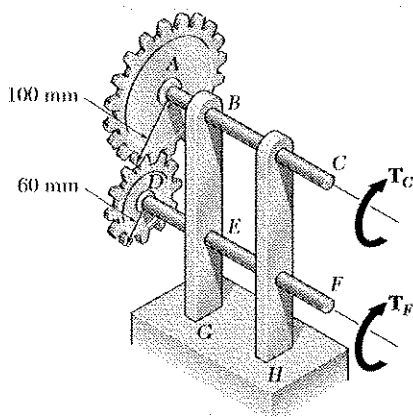
$$T_{CD} = \frac{\pi}{2} (60 \times 10^6) (0.033)^3 = 3387 \text{ N} \cdot \text{m}$$

$$T = \frac{r_B}{r_C} T_{CD} = \frac{80}{240} (3387) = 1129 \text{ N} \cdot \text{m}$$

The allowable torque is the smaller of the two calculated values.

$$T = 1129 \text{ N} \cdot \text{m} = 1.129 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

Problem 3.25



3.25 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 50 MPa. Knowing the diameters of the two shafts are, respectively, $d_{BC} = 40 \text{ mm}$ and $d_{EF} = 30 \text{ mm}$ determine the largest torque T_C that can be applied at C.

$$\tau_{max} = 50 \text{ MPa}$$

Shaft BC: $d_{BC} = 40 \text{ mm}$ $c = \frac{1}{2}d = 20 \text{ mm}$.

$$T_C = \frac{J \tau_{max}}{c} = \frac{\pi}{2} \tau_{max} c^3 = \frac{\pi}{2} (50 \times 10^6) (0.02)^3 = 628 \text{ N}\cdot\text{m}$$

Shaft EF: $d_{EF} = 30 \text{ mm}$ $c = \frac{1}{2}d = 15 \text{ mm}$.

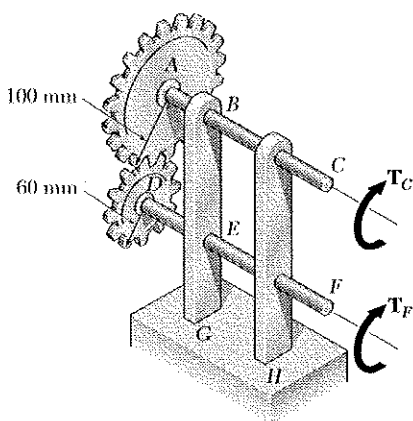
$$T_F = \frac{J \tau_{max}}{c} = \frac{\pi}{2} \tau_{max} c^3 = \frac{\pi}{2} (50 \times 10^6) (0.015)^3 = 265 \text{ N}\cdot\text{m}$$

By statics $T_C = \frac{r_A}{r_D} T_F = \frac{10}{6} (265) = 441.7 \text{ N}\cdot\text{m}$.

Allowable value of T_C is the smaller.

$$T_C = 441.7 \text{ N}\cdot\text{m} \blacktriangleleft$$

Problem 3.26



3.26 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 60 MPa. Knowing that a torque of magnitude $T_C = 600 \text{ N}\cdot\text{m}$ is applied at C and that the assembly is in equilibrium, determine the required diameter of (a) shaft BC, (b) shaft EF.

$$\tau_{max} = 60 \text{ MPa}$$

(a) Shaft BC: $T_C = 600 \text{ N}\cdot\text{m}$

$$\tau_{max} = \frac{T_C}{J} = \frac{2T}{\pi c^3} \quad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$c = \sqrt[3]{\frac{2(600)}{\pi (60 \times 10^6)}} = 0.0185 \text{ m}$$

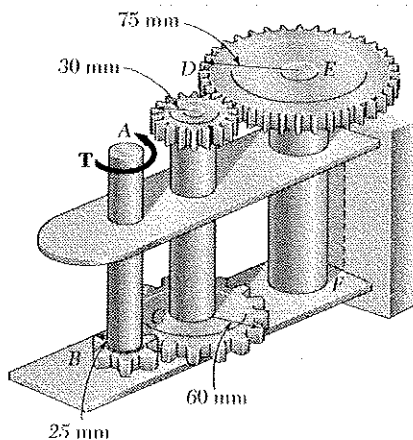
$$d_{BC} = 2c = 37 \text{ mm} \blacktriangleleft$$

Shaft EF: $T_F = \frac{r_D}{r_A} T_C = \frac{60}{100} (600) = 360 \text{ N}\cdot\text{m}$.

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{2(360)}{\pi (60 \times 10^6)}} = 0.0156 \text{ m}$$

$$d_{EF} = 2c = 31.2 \text{ mm} \blacktriangleleft$$

Problem 3.27



3.27 A torque of magnitude $T = 120 \text{ N} \cdot \text{m}$ is applied to shaft AB of the gear train shown. Knowing that the allowable shearing stress is 75 MPa in each of the three solid shafts, determine the required diameter of (a) shaft AB , (b) shaft CD , (c) shaft EF .

STATICS

Shaft AB. $T_{AB} = T_A = T_B = T$

Gears B and C. $r_B = 25 \text{ mm}$, $r_C = 60 \text{ mm}$

Force on gear circles. $F_{BC} = \frac{T_B}{r_B} = \frac{T_C}{r_C}$

$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4 T$

Shaft CD. $T_{CD} = T_C = T_D = 2.4 T$

Gears D and E. $r_D = 30 \text{ mm}$, $r_E = 75 \text{ mm}$

Force on gear circles. $F_{DE} = \frac{T_D}{r_D} = \frac{T_E}{r_E}$

$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30} (2.4 T) = 6 T$

Shaft EF. $T_{EF} = T_E = T_F = 6 T$

REQUIRED DIAMETERS

$$\tau_{\max} = \frac{T_C}{J} = \frac{2T}{\pi C^3} \quad C = \sqrt[3]{\frac{2T}{\pi \tau}} \quad d = 2C = 2 \sqrt[3]{\frac{2T}{\pi \tau_{\max}}}$$

$\tau_{\max} = 75 \times 10^6 \text{ Pa}$

(a) Shaft AB. $T_{AB} = T = 120 \text{ N} \cdot \text{m}$

$d_{AB} = 2 \sqrt[3]{\frac{2(120)}{\pi(75 \times 10^6)}} = 20.1 \times 10^{-3} \text{ m}$

$d_{AB} = 20.1 \text{ mm}$

(b) Shaft CD. $T_{CD} = (2.4)(120) = 288 \text{ N} \cdot \text{m}$

$d_{CD} = 2 \sqrt[3]{\frac{(2)(288)}{\pi(75 \times 10^6)}} = 26.9 \times 10^{-3} \text{ m}$

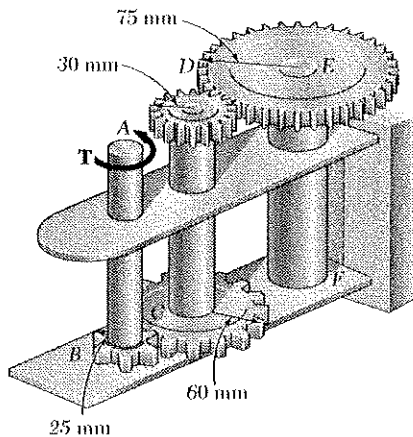
$d_{CD} = 26.9 \text{ mm}$

(c) Shaft EF. $T_{EF} = (6)(120) = 720 \text{ N} \cdot \text{m}$

$d_{EF} = 2 \sqrt[3]{\frac{(2)(720)}{\pi(75 \times 10^6)}} = 36.6 \times 10^{-3} \text{ m}$

$d_{EF} = 36.6 \text{ mm}$

Problem 3.28



3.28 A torque of magnitude $T = 100 \text{ N} \cdot \text{m}$ is applied to shaft AB of the gear train shown. Knowing that the diameters of the three solid shafts are, respectively, $d_{AB} = 21 \text{ mm}$, $d_{CD} = 30 \text{ mm}$, and $d_{EF} = 40 \text{ mm}$, determine the maximum shearing stress in (a) shaft AB , (b) shaft CD , (c) shaft EF .

STATICS

Shaft AB. $T_{AB} = T_A = T_B = T$

Gears B and C. $r_B = 25 \text{ mm}$, $r_C = 60 \text{ mm}$

Force on gear circles: $F_{BC} = \frac{T_B}{r_B} = \frac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4 T$$

Shaft CD. $T_{CD} = T_C = T_D = 2.4 T$

Gears D and E. $r_D = 30 \text{ mm}$, $r_E = 75 \text{ mm}$

Force on gear circles: $F_{DE} = \frac{T_D}{r_D} = \frac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30} (2.4 T) = 6 T$$

Shaft EF. $T_{EF} = T_E = T_F = 6 T$

MAXIMUM SHEARING STRESSES

$$\tau_{max} = \frac{T_C}{J} = \frac{2T}{\pi C^3}$$

(a) Shaft AB. $T = 100 \text{ N} \cdot \text{m}$ $C = \frac{1}{2} d = 10.5 \text{ mm} = 10.5 \times 10^{-3} \text{ m}$

$$\tau_{max} = \frac{(2)(100)}{\pi (10.5 \times 10^{-3})^3} = 55.0 \times 10^6 \text{ Pa} \quad \tau_{max} = 55.0 \text{ MPa} \quad \blacktriangleleft$$

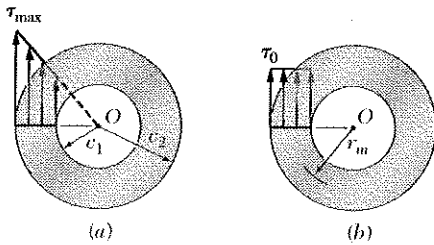
(b) Shaft CD. $T = (2.4)(100) = 240 \text{ N} \cdot \text{m}$
 $C = \frac{1}{2} d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$

$$\tau_{max} = \frac{(2)(240)}{\pi (15 \times 10^{-3})^3} = 45.3 \times 10^6 \text{ Pa} \quad \tau_{max} = 45.3 \text{ MPa} \quad \blacktriangleleft$$

(c) Shaft EF. $T = (6)(100) = 600 \text{ N} \cdot \text{m}$
 $C = \frac{1}{2} d = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$

$$\tau_{max} = \frac{(2)(600)}{\pi (20 \times 10^{-3})^3} = 47.7 \times 10^6 \text{ Pa} \quad \tau_{max} = 47.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.29



3.29 While the exact distribution of the shearing stresses in a hollow-cylindrical shaft is as shown in Fig. P3.29a, an approximate value can be obtained for $\tau_{\max}$ by assuming that the stresses are uniformly distributed over the area A of the cross section, as shown in Fig. P 3.29b, and then further assuming that all of the elementary shearing forces act at a distance from O equal to the mean radius $\frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio $\tau_{\max}/\tau_0$ of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2 respectively equal to 1.00, 0.95, 0.75, 0.50 and 0.

$$\text{For a hollow shaft: } \tau_{\max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)} = \frac{2Tc_2}{\pi(c_2^2 - c_1^2)(c_2^2 + c_1^2)}$$

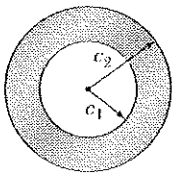
$$= \frac{2Tc_2}{A(c_2^2 + c_1^2)}$$

$$\text{By definition, } \tau_0 = \frac{T}{Ar_m} = \frac{2T}{A(c_2 + c_1)}$$

$$\text{Dividing, } \frac{\tau_{\max}}{\tau_0} = \frac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2}$$

c_1/c_2	1.0	0.95	0.75	0.5	0.0
$\tau_{\max}/\tau_0$	1.0	1.025	1.120	1.200	1.0

Problem 3.30



3.30 (a) For a given allowable shearing stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (b) Denoting by $(T/w)_0$ the value of this ratio for a solid shaft of the same radius c_2 , express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2 .

w = weight per unit length, ρg = specific weight

W = total weight, L = length

$$w = \frac{W}{L} = \frac{\rho g L A}{L} = \rho g A = \rho g \pi (c_2^2 - c_1^2)$$

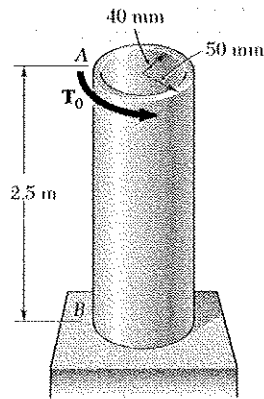
$$T = \frac{J\tau_{\text{all}}}{c_2} = \frac{\pi}{2} \frac{c_2^4 - c_1^4}{c_2} \tau_{\text{all}} = \frac{\pi}{2} \frac{(c_2^2 + c_1^2)(c_2^2 - c_1^2)}{c_2} \tau_{\text{all}}$$

$$(a) \quad \frac{T}{w} = (c_1^2 + c_2^2) \tau_{\text{all}} \quad \frac{T}{W} = \frac{(c_1^2 + c_2^2) \tau_{\text{all}}}{2\rho g c_2} \quad (\text{hollow shaft})$$

$$c_1 = 0 \text{ for solid shaft: } \left(\frac{T}{w}\right)_0 = \frac{c_2 \tau_{\text{all}}}{2\rho g} \quad (\text{solid shaft})$$

$$(b) \quad \frac{(T/w)_h}{(T/w)_0} = 1 + \frac{c_1^2}{c_2^2} \quad \left(\frac{T}{w}\right) = \left(\frac{T}{w}\right)_0 \left(1 + \frac{c_1^2}{c_2^2}\right)$$

Problem 3.31



3.31 (a) For the aluminum pipe shown ($G = 27 \text{ GPa}$), determine the torque T_0 causing an angle of twist of 2° . (b) Determine the angle of twist if the same torque T_0 is applied to a solid cylindrical shaft of the same length and cross-sectional area.

$$(a) c_o = 50 \text{ mm} = 0.050 \text{ m}, \quad c_i = 40 \text{ mm} = 0.040 \text{ m}$$

$$J = \frac{\pi}{2} (c_o^4 - c_i^4) = \frac{\pi}{2} (0.050^4 - 0.040^4) = 5.7962 \times 10^{-6} \text{ m}^4$$

$$\phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad} \quad L = 2.5 \text{ m}$$

$$G = 27 \times 10^9 \text{ Pa}$$

$$\phi = \frac{TL}{GJ} \quad T_0 = \frac{GJ\phi}{L} = \frac{(27 \times 10^9)(5.7962 \times 10^{-6})(34.907 \times 10^{-3})}{2.5}$$

$$= 2.1851 \times 10^3 \text{ N}\cdot\text{m}$$

$$T_0 = 2.19 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

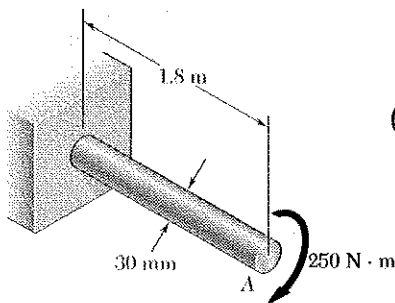
$$\text{Area of pipe: } A = \pi (c_o^2 - c_i^2) = \pi (0.050^2 - 0.040^2) = 2.8274 \text{ m}^2$$

$$(b) \text{ Radius of solid of same area: } c_s = \sqrt{\frac{A}{\pi}} = 0.030 \text{ m}$$

$$J_s = \frac{\pi}{2} c_s^4 = \frac{\pi}{2} (0.030)^4 = 1.27235 \times 10^{-6} \text{ m}^4$$

$$\phi_s = \frac{T_0 L}{GJ} = \frac{(2.1851 \times 10^3)(2.5)}{(27 \times 10^9)(1.27235 \times 10^{-6})} = 0.15902 \text{ rad} \quad \phi_s = 9.11^\circ \quad \blacktriangleleft$$

Problem 3.32



3.32 (a) For the solid steel shaft shown ($G = 77 \text{ GPa}$), determine the angle of twist at A. (b) Solve part a, assuming that the steel shaft is hollow with a 30-mm-outer diameter and a 20-mm-inner diameter.

$$(a) c = \frac{1}{2} d = 0.015 \text{ m}, \quad J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.015)^4$$

$$J = 79.522 \times 10^{-9} \text{ m}^4, \quad L = 1.8 \text{ m}, \quad G = 77 \times 10^9 \text{ Pa}$$

$$T = 250 \text{ N}\cdot\text{m}$$

$$\phi = \frac{TL}{GJ}$$

$$\phi = \frac{(250)(1.8)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 73.49 \times 10^{-3} \text{ rad}$$

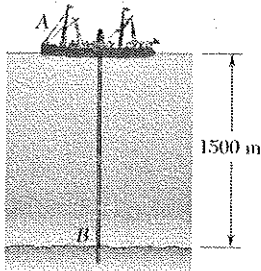
$$\phi = \frac{(73.49 \times 10^{-3}) 180}{\pi} = 4.21^\circ \quad \blacktriangleleft$$

$$(b) c_2 = 0.015 \text{ m}, \quad c_1 = \frac{1}{2} d_1 = 0.010 \text{ m}, \quad J = \frac{\pi}{2} (c_2^4 - c_1^4)$$

$$J = \frac{\pi}{2} (0.015^4 - 0.010^4) = 63.814 \times 10^{-9} \text{ m}^4 \quad \phi = \frac{TL}{GJ}$$

$$\phi = \frac{(250)(1.8)}{(77 \times 10^9)(63.814 \times 10^{-9})} = 91.58 \times 10^{-3} \text{ rad} = \frac{180}{\pi} (91.58 \times 10^{-3}) = 5.25^\circ \quad \blacktriangleleft$$

Problem 3.33



3.33 The ship at A has just started to drill for oil on the ocean floor at a depth of 1500 m. Knowing that the top of the 200-mm-diameter steel drill pipe ($G = 77.2$ GPa) rotates through two complete revolutions before the drill bit at B starts to operate, determine the maximum shearing stress caused in the pipe by torsion.

$$\phi = \frac{TL}{GJ}$$

$$T = \frac{GJ\phi}{L}$$

$$\tau = \frac{Tc}{J} = \frac{GJ\phi c}{JL} = \frac{G\phi c}{L}$$

$$\phi = 2 \text{ rev} = 2(2\pi) \text{ rad} = 12.566 \text{ rad}$$

$$c = \frac{1}{2}d = 100 \text{ mm} = 0.100 \text{ m}$$

$$G = 77.2 \text{ GPa} = 77.2 \times 10^9 \text{ Pa}$$

$$\tau = \frac{(77.2 \times 10^9)(12.566)(0.100)}{1500} = 64.7 \times 10^6 \text{ Pa}$$

$$\tau = 64.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.34

3.34 Determine the largest allowable diameter of a 3 m-long steel rod ($G = 77$ GPa) if the rod is to be twisted through 30° without exceeding a shearing stress of 84 MPa.

$$L = 3 \text{ m}$$

$$\tau = 84 \text{ MPa}$$

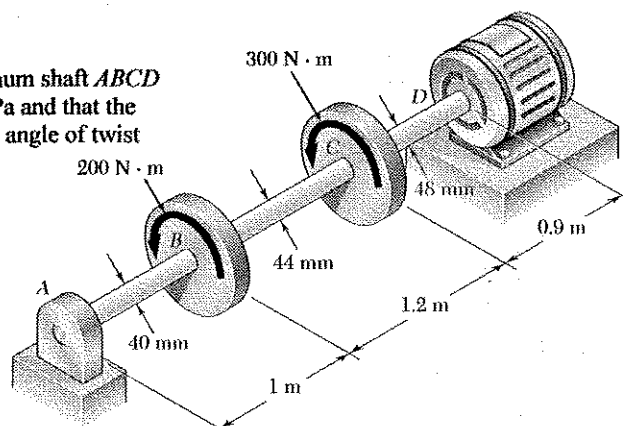
$$\phi = 30^\circ = \frac{30\pi}{180} = 0.52360 \text{ rad}$$

$$\phi = \frac{TL}{GJ}, \quad T = \frac{GJ\phi}{L}, \quad \tau = \frac{Tc}{J} = \frac{GJ\phi c}{JL} = \frac{G\phi c}{L}, \quad c = \frac{\tau L}{G\phi}$$

$$c = \frac{(84 \times 10^6)(3)}{(77 \times 10^9)(0.5236)} = 0.00625 \text{ m} = 6.25 \text{ mm} \quad d = 2c = 12.5 \text{ mm} \quad \blacktriangleleft$$

Problem 3.35

3.35 The electric motor exerts a 500 N·m torque on the aluminum shaft ABCD when it is rotating at a constant speed. Knowing that $G = 27$ GPa and that the torques exerted on pulleys B and C are as shown, determine the angle of twist between (a) B and C, (b) B and D.



(a) Angle of twist between B and C.

$$T_{BC} = 200 \text{ N}\cdot\text{m}, L_{BC} = 1.2 \text{ m}$$

$$c = \frac{1}{2}d = 0.022 \text{ m}, G = 27 \times 10^9 \text{ Pa}$$

$$J_{BC} = \frac{\pi}{2} c^4 = 367.97 \times 10^{-9} \text{ m}^4$$

$$\phi_{B/C} = \frac{TL}{GJ} = \frac{(200)(1.2)}{(27 \times 10^9)(367.97 \times 10^{-9})} = 24.157 \times 10^{-3} \text{ rad}$$

$$\phi_{B/C} = 1.384^\circ$$

(b) Angle of twist between B and D.

$$T_{CD} = 500 \text{ N}\cdot\text{m}, L_{CD} = 0.9 \text{ m}, c = \frac{1}{2}d = 0.024 \text{ m}, G = 27 \times 10^9 \text{ Pa}$$

$$J_{CD} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.024)^4 = 521.153 \times 10^{-9} \text{ m}^4$$

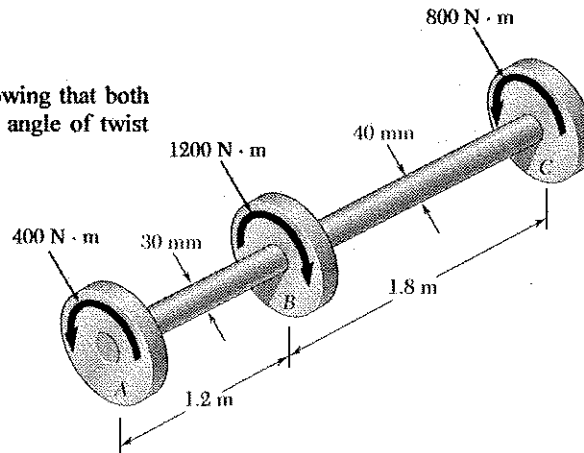
$$\phi_{C/D} = \frac{(500)(0.9)}{(27 \times 10^9)(521.153 \times 10^{-9})} = 31.980 \times 10^{-3} \text{ rad}$$

$$\phi_{B/D} = \phi_{B/C} + \phi_{C/D} = 24.157 \times 10^{-3} + 31.980 \times 10^{-3} = 56.137 \times 10^{-3} \text{ rad}$$

$$\phi_{B/D} = 3.22^\circ$$

Problem 3.36

3.36 The torques shown are exerted on pulleys A, B, and C. Knowing that both shafts are solid and made of brass ($G = 39$ GPa), determine the angle of twist between (a) A and B, (b) A and C.



(a) Angle of twist between A and B.

$$T_{AB} = 400 \text{ N}\cdot\text{m}, L_{AB} = 1.2 \text{ m}$$

$$c = \frac{1}{2}d = 0.015 \text{ m}, G = 39 \times 10^9 \text{ Pa}$$

$$J_{AB} = \frac{\pi}{2} c^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(400)(1.2)}{(39 \times 10^9)(79.52 \times 10^{-9})}$$

$$= 0.154772 \text{ rad } \curvearrowright$$

$$\phi_{A/B} = 8.87^\circ \curvearrowright$$

(b) Angle of twist between A and C.

$$T_{BC} = 800 \text{ N}\cdot\text{m}, L_{BC} = 1.8 \text{ m}, c = \frac{1}{2}d = 0.020 \text{ m}, G = 39 \times 10^9 \text{ Pa}$$

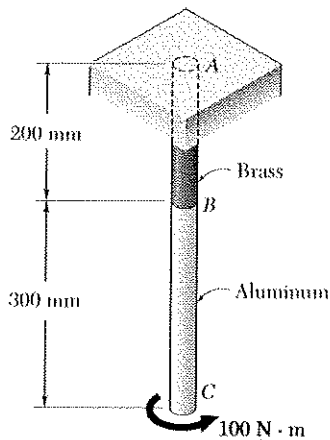
$$J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.020)^4 = 251.327 \times 10^{-9} \text{ m}^4$$

$$\phi_{B/C} = \frac{(800)(1.8)}{(39 \times 10^9)(251.327 \times 10^{-9})} = 0.146912 \text{ rad } \curvearrowright$$

$$\phi_{A/C} = \phi_{A/B} + \phi_{B/C} = 0.154772 + 0.146912 = 0.301684 \text{ rad } \curvearrowright$$

$$\phi_{A/C} = 17.28^\circ \curvearrowright$$

Problem 3.37



3.37 The aluminum rod BC ($G = 26$ GPa) is bonded to the brass rod AB ($G = 39$ GPa). Knowing that each rod is solid and has a diameter of 12 mm, determine the angle of twist (a) at B , (b) at C .

Both portions $C = \frac{1}{2}d = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (6 \times 10^{-3})^4 = 2.03575 \times 10^{-9} \text{ m}^4$$

$$T = 100 \text{ N} \cdot \text{m}$$

Rod AB : $G_{AB} = 39 \times 10^9 \text{ Pa}$, $L_{AB} = 0.200 \text{ m}$

$$(a) \quad \phi_B = \phi_{AB} = \frac{T L_{AB}}{G_{AB} J} = \frac{(100)(0.200)}{(39 \times 10^9)(2.03575 \times 10^{-9})} = 0.25191 \text{ rad} \quad \phi_B = 14.43^\circ$$

Rod BC : $G_{BC} = 26 \times 10^9 \text{ Pa}$, $L_{BC} = 0.300 \text{ m}$

$$\phi_{BC} = \frac{T L_{BC}}{G_{BC} J} = \frac{(100)(0.300)}{(26 \times 10^9)(2.03575 \times 10^{-9})} = 0.56679 \text{ rad}$$

$$(b) \quad \phi_C = \phi_B + \phi_{BC} = 0.25191 + 0.56679 = 0.81870 \text{ rad} \quad \phi_C = 46.9^\circ$$

Problem 3.38

3.38 The aluminum rod AB ($G = 27$ GPa) is bonded to the brass rod BD ($G = 39$ GPa). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at A .

Rod AB : $G = 27 \times 10^9 \text{ Pa}$, $L = 0.400 \text{ m}$

$$T = 800 \text{ N} \cdot \text{m} \quad C = \frac{1}{2}d = 0.018 \text{ m}$$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.018)^4 = 164.896 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{T L}{G J} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$

Part BC : $G = 39 \times 10^9 \text{ Pa}$, $L = 0.375 \text{ m}$, $C = \frac{1}{2}d = 0.030 \text{ m}$

$$T = 800 + 1600 = 2400 \text{ N} \cdot \text{m}, \quad J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\phi_{B/C} = \frac{T L}{G J} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3} \text{ rad}$$

Part CD : $C_1 = \frac{1}{2}d_1 = 0.020 \text{ m}$, $C_2 = \frac{1}{2}d_2 = 0.030 \text{ m}$, $L = 0.250 \text{ m}$

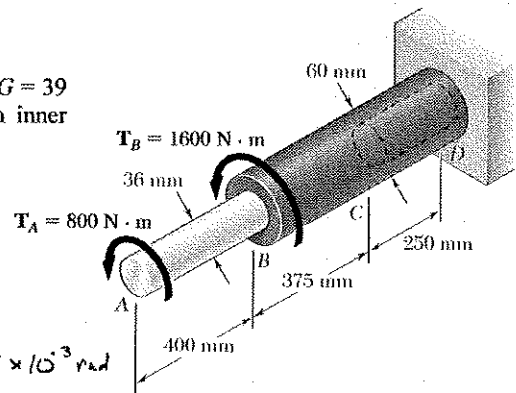
$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.030^4 - 0.020^4) = 1.02102 \times 10^{-6} \text{ m}^4$$

$$\phi_{C/D} = \frac{T L}{G J} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3} \text{ rad}$$

Angle of twist at A , $\phi_A = \phi_{A/B} + \phi_{B/C} + \phi_{C/D}$

$$= 105.080 \times 10^{-3} \text{ rad}$$

$$\phi_A = 6.02^\circ$$



Problem 3.39

3.39 Three solid shafts, each of 18-mm diameter, are connected by the gears shown. Knowing that $G = 77$ GPa, determine (a) the angle through which end A of shaft AB rotates, (b) the angle through which end E of shaft EF rotates.

Geometry:

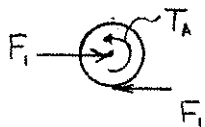
$$r_B = 0.036 \text{ m}, \quad r_C = 0.144 \text{ m}, \quad r_F = 0.048 \text{ m}$$

$$L_{AB} = 1.2 \text{ m}, \quad L_{CD} = 0.9 \text{ m}, \quad L_{EF} = 1.2 \text{ m}$$

Statics: $T_A = 10 \text{ N}\cdot\text{m}$ ↺

$$T_E = 20 \text{ N}\cdot\text{m}$$

Gear B. $+\circlearrowleft \sum M_B = 0:$

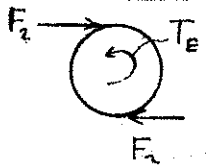


$$-r_B F_1 + T_A = 0$$

$$-0.036 F_1 + 10 = 0$$

$$F_1 = 277.8 \text{ N}$$

Gear F. $+\circlearrowleft \sum M_F = 0:$



$$-r_F F_2 + T_E = 0$$

$$-0.048 F_2 + 20 = 0$$

$$F_2 = 416.7 \text{ N}$$

Deformations:

For all shafts $c = \frac{1}{2}d = 0.009 \text{ m}$

$$J = \frac{\pi}{2} c^4 = 10.306 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{T_{AB} L_{AB}}{GJ} = \frac{(10)(1.2)}{(77 \times 10^9)(10.306 \times 10^{-9})} = 0.01512 \text{ rad} \quad \curvearrowright$$

$$\phi_{E/F} = \frac{T_{EF} L_{EF}}{GJ} = \frac{(20)(1.2)}{(77 \times 10^9)(10.306 \times 10^{-9})} = 0.03024 \text{ rad} \quad \curvearrowright$$

$$\phi_{C/D} = \frac{T_{CD} L_{CD}}{GJ} = \frac{(41.4)(0.9)}{(77 \times 10^9)(10.306 \times 10^{-9})} = 0.04695 \text{ rad} \quad \curvearrowright$$

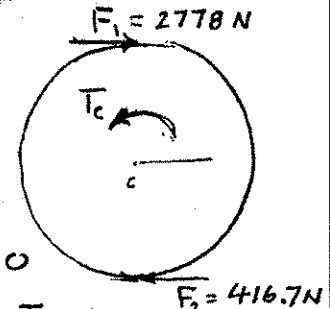
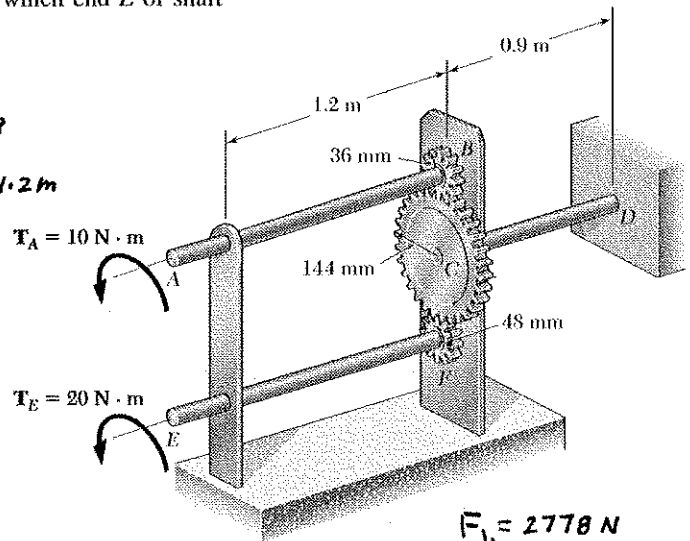
Kinematics: $\phi_C = \phi_{C/D} = 0.04695 \text{ rad} \quad \curvearrowright$

$$r_B \phi_B = r_C \phi_C \quad \phi_B = \frac{r_C}{r_B} \phi_C = \frac{0.144}{0.036} (0.04695) = 0.1878 \text{ rad} \quad \curvearrowright$$

(a) $\phi_{A/E} = \phi_B + \phi_{A/B} = 0.1878 + 0.01512 = 0.20292 \text{ rad} \quad \curvearrowright \quad \phi_B = 11.6^\circ \quad \curvearrowright$

$$r_F \phi_F = r_C \phi_C \quad \phi_F = \frac{r_C}{r_F} \phi_C = \frac{0.144}{0.048} (0.04695) = 0.14085 \text{ rad} \quad \curvearrowright$$

(b) $\phi_E = \phi_F + \phi_{E/F} = 0.14085 + 0.03024 = 0.17109 \text{ rad} \quad \phi_F = 9.8^\circ \quad \curvearrowright$



Gear C.

$$+\circlearrowleft \sum M_C = 0$$

$$-r_C F_1 + r_C F_2 + T_C = 0$$

$$-(0.144)(277.8) - (0.144)(10) + T_C = 0$$

$$T_C = 41.4 \text{ N}\cdot\text{m} \quad \curvearrowright$$

Problem 3.40

3.40 Two shafts, each of 22-mm diameter are connected by the gears shown. Knowing that $G = 77 \text{ GPa}$ and that the shaft at F is fixed, determine the angle through which end A rotates when a $130 \text{ N} \cdot \text{m}$ torque is applied at A .

Calculation of torques.

Circumferential contact force between gears B and E .

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E} \quad T_{EF} = \frac{r_E}{r_B} T_{AB}$$

$$T_{AB} = 130 \text{ N} \cdot \text{m}.$$

$$T_{EF} = \frac{150}{110} (130) = 177.3 \text{ N} \cdot \text{m}$$

Twist in shaft FE .

$$L = 300 \text{ mm}, \quad c = \frac{1}{2}d = 11 \text{ mm}, \quad G = 77 \text{ GPa}.$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.011)^4 = 23 \times 10^{-9} \text{ m}^4.$$

$$\phi_{E/F} = \frac{TL}{GJ} = \frac{(177.3)(0.3)}{(77 \times 10^9)(23 \times 10^{-9})} = 0.03 \text{ rad}$$

Rotation at E . $\phi_E = \phi_{E/F} = 0.03 \text{ rad}$

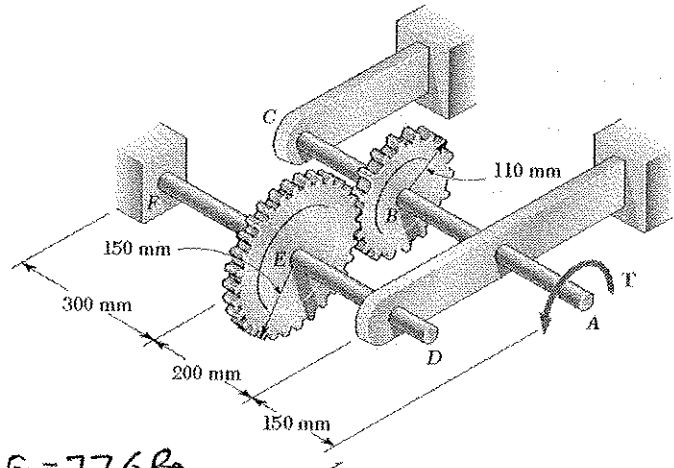
Tangential displacement at gear circle $s = r_E \phi_E = r_B \phi_B$

Rotation at B $\phi_B = \frac{r_E}{r_B} \phi_E = \frac{15}{11} (0.03) = 0.0409 \text{ rad}$

Twist in shaft BA . $L = 0.35 \text{ m}$ $J = 23 \times 10^{-9} \text{ m}^4$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(130)(0.35)}{(77 \times 10^9)(23 \times 10^{-9})} = 0.0257 \text{ rad}$$

Rotation at A . $\phi_A = \phi_B + \phi_{A/B} = 0.0666 \text{ rad} \quad \phi_A = 3.82^\circ$



Problem 3.41

3.41 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2 \text{ GPa}$ for each shaft, determine the angle through which end A rotates when $T_A = 1200 \text{ N}\cdot\text{m}$.

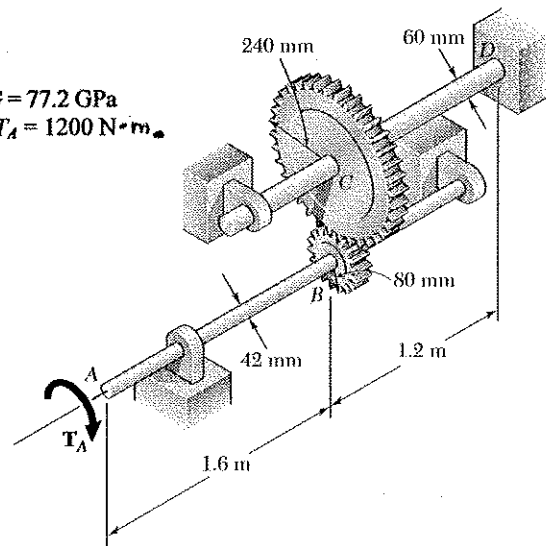
Calculation of torques.

Circumferential contact force between gears B and C.

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CB}}{r_C} \quad T_{CB} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = 1200 \text{ N}\cdot\text{m}$$

$$T_{CB} = \frac{240}{80} (1200) = 3600 \text{ N}\cdot\text{m}$$



Twist in shaft CD: $c = \frac{1}{2}d = 0.030 \text{ m}$, $L = 1.2 \text{ m}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\phi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(1.27234 \times 10^{-6})} = 43.981 \times 10^{-3} \text{ rad}$$

Rotation angle at C, $\phi_C = \phi_{C/D} = 43.981 \times 10^{-3} \text{ rad}$

Circumferential displacement at contact points of gears B and C

$$s = r_C \phi_C = r_B \phi_B$$

Rotation angle at B, $\phi_B = \frac{r_C}{r_B} \phi_C = \frac{240}{80} (43.981 \times 10^{-3}) = 131.942 \times 10^{-3} \text{ rad}$

Twist in shaft AB: $c = \frac{1}{2}d = 0.021 \text{ m}$, $L = 1.6 \text{ m}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.021)^4 = 305.49 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(305.49 \times 10^{-9})} = 81.412 \times 10^{-3} \text{ rad}$$

Rotation angle at A, $\phi_A = \phi_B + \phi_{A/B} = 213.354 \times 10^{-3} \text{ rad}$

$$\phi_A = 12.22^\circ$$

Problem 3.42

3.42 Solve Prob. 3.41, assuming that the diameter of each shaft is 54 mm.

3.41 Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200$ N·m.

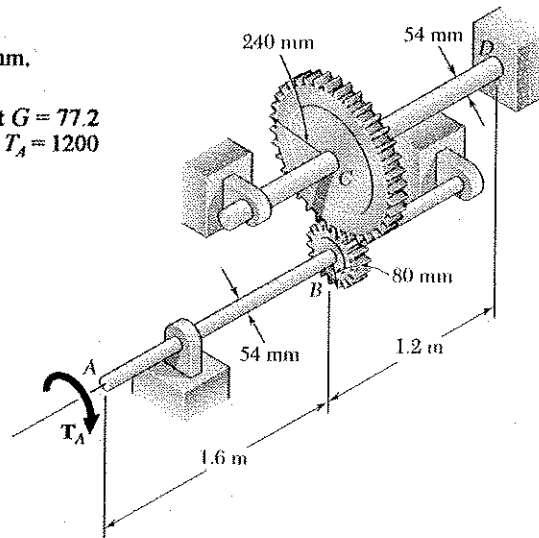
Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = 1200 \text{ N}\cdot\text{m}$$

$$T_{CD} = \frac{240}{80} (1200) = 3600 \text{ N}\cdot\text{m}$$



Twist in shaft CD: $c = \frac{1}{2}d = 0.027 \text{ m}$, $L = 1.2 \text{ m}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.027)^4 = 834.79 \times 10^{-9} \text{ m}^4$$

$$\phi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(834.79 \times 10^{-9})} = 67.033 \times 10^{-3} \text{ rad}$$

Rotation angle at C. $\phi_C = \phi_{C/D} = 67.033 \times 10^{-3} \text{ rad}$

Circumferential displacement at contact points of gears B and C

$$s = r_C \phi_C = r_B \phi_B$$

Rotation angle at B. $\phi_B = \frac{r_C}{r_B} \phi_C = \frac{240}{80} (67.033 \times 10^{-3}) = 201.10 \times 10^{-3} \text{ rad}$

Twist in shaft AB: $c = \frac{1}{2}d = 0.027 \text{ m}$, $L = 1.6 \text{ m}$, $G = 77.2 \times 10^9 \text{ Pa}$

$$J = 834.79 \times 10^{-9} \text{ m}^4$$

$$\phi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(834.79 \times 10^{-9})} = 29.792 \times 10^{-3} \text{ rad}$$

Rotation angle at A. $\phi_A = \phi_B + \phi_{A/B} = 230.89 \times 10^{-3} \text{ rad}$

$$\phi_A = 13.23^\circ$$

Problem 3.43

3.43 A coder F , used to record in digital form the rotation of shaft A , is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d . Two of the gears have a radius r and the other two a radius nr . If the rotation of the coder F is prevented, determine in terms of T , l , G , J , and n the angle through which end A rotates.

$$T_{AB} = T_A$$

$$T_{CD} = \frac{r_C}{r_D} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$$

$$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{n} = \frac{T_A}{n^2}$$

$$\phi_E = \phi_{EF} = \frac{T_{EF} l_{EF}}{GJ} = \frac{T_A l}{n^2 GJ}$$

$$\phi_D = \frac{r_E}{r_D} \phi_E = \frac{\phi_E}{n} = \frac{T_A l}{n^3 GJ}$$

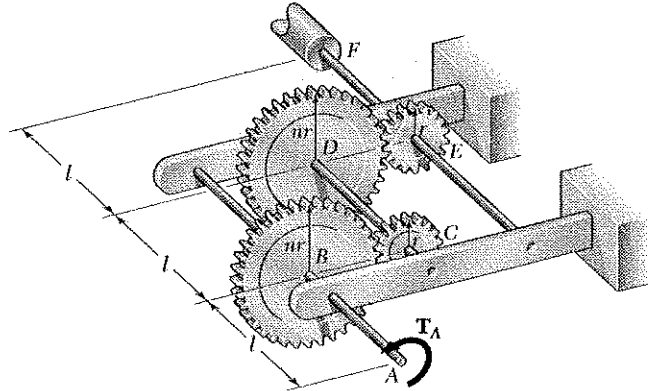
$$\phi_{CD} = \frac{T_{CD} l_{CD}}{GJ} = \frac{T_A l}{n GJ}$$

$$\phi_C = \phi_D + \phi_{CD} = \frac{T_A l}{n^3 GJ} + \frac{T_A l}{n GJ} = \frac{T_A l}{GJ} \left(\frac{1}{n^3} + \frac{1}{n} \right)$$

$$\phi_B = \frac{r_C}{r_B} \phi_C = \frac{\phi_C}{n} = \frac{T_A l}{GJ} \left(\frac{1}{n^4} + \frac{1}{n^2} \right)$$

$$\phi_{AB} = \frac{T_{AB} l_{AB}}{GJ} = \frac{T_A l}{GJ}$$

$$\phi_A = \phi_B + \phi_{AB} = \frac{T_A l}{GJ} \left(\frac{1}{n^4} + \frac{1}{n^2} + 1 \right)$$



Problem 3.44

3.44 For the gear train described in Prob. 3.43, determine the angle through which end A rotates when $T = 0.6 \text{ N} \cdot \text{m}$, $l = 60 \text{ mm}$, $d = 2 \text{ mm}$, $G = 77 \text{ GPa}$, and $n = 2$.

See solution to PROBLEM 3.43 for development of equation for ϕ_A

$$\phi_A = \frac{Tl}{GJ} \left(1 + \frac{1}{n^2} + \frac{1}{n^4} \right)$$

Data: $T = 0.6 \text{ Nm}$ $l = 60 \text{ mm}$, $c = \frac{1}{2}d = 1 \text{ mm}$ $G = 77 \times 10^9 \text{ Pa}$

$n = 2$, $J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.001)^4 = 1.57 \times 10^{-12} \text{ m}^4$

$$\phi_A = \frac{(0.6)(0.06)}{(77 \times 10^9)(1.57 \times 10^{-12})} \left(1 + \frac{1}{4} + \frac{1}{16} \right) = 0.391 \text{ rad}$$

$$\phi_A = 22.4^\circ$$

Problem 3.45

3.45 The design specifications of a 1.2-m-long solid transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of $750 \text{ N} \cdot \text{m}$ is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77.2 GPa .

$$T = 750 \text{ N} \cdot \text{m}, \quad \phi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}, \quad L = 1.2 \text{ m}, \quad J = \frac{\pi}{2} C^4$$

$$\tau = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa} \quad G = 77.2 \text{ GPa} = 77.2 \times 10^9 \text{ Pa}$$

Based on angle of twist. $\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G C^4}$

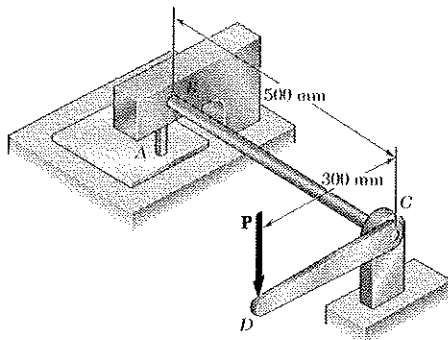
$$C = \sqrt[4]{\frac{2TL}{\pi G \phi}} = \sqrt[4]{\frac{(2)(750)(1.2)}{\pi (77.2 \times 10^9)(69.813 \times 10^{-3})}} = 18.06 \times 10^{-3} \text{ m}$$

Based on shearing stress. $\tau = \frac{TC}{J} = \frac{2T}{\pi C^3}$

$$C = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(750)}{\pi (90 \times 10^6)}} = 17.44 \times 10^{-3} \text{ m}$$

Use larger value. $C = 18.06 \times 10^{-3} \text{ m} = 18.06 \text{ mm} \quad d = 2C = 36.1 \text{ mm} \blacktriangleleft$

Problem 3.46



3.46 A hole is punched at A in a plastic sheet by applying a 600-N force P to end D of lever CD , which is rigidly attached to the solid cylindrical shaft BC . Design specifications require that the displacement of D should not exceed 15 mm from the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft BC if the shaft is made of a steel with $G = 77 \text{ GPa}$ and $\tau_{\text{all}} = 80 \text{ MPa}$.

$$T = VP = (0.300 \text{ m})(600 \text{ N}) = 180 \text{ N} \cdot \text{m}$$

Shaft diameter based on displacement limit.

$$\phi = \frac{\delta}{L} = \frac{15 \text{ mm}}{300 \text{ mm}} = 0.05 \text{ rad}$$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G C^4}$$

$$C^4 = \frac{2TL}{\pi G \phi} = \frac{(2)(180)(0.500)}{\pi (77 \times 10^9)(0.05)} = 14.882 \times 10^{-9} \text{ m}^4$$

$$C = 11.045 \times 10^{-3} \text{ m} = 11.045 \text{ mm} \quad d = 2C = 22.8 \text{ mm}$$

Shaft diameter based on stress. $\tau = 80 \times 10^6 \text{ Pa} \quad \tau = \frac{TC}{J} = \frac{2T}{\pi C^3}$

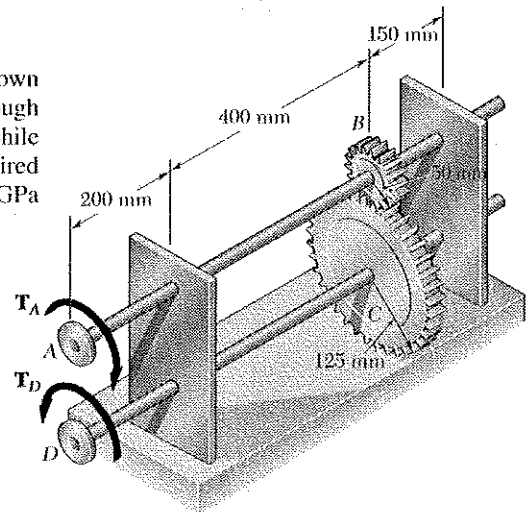
$$C^3 = \frac{2T}{\pi \tau} = \frac{(2)(180)}{\pi (80 \times 10^6)} = 1.43239 \times 10^{-6} \text{ m}^3$$

$$C = 11.273 \times 10^{-3} \text{ m} = 11.273 \text{ mm} \quad d = 2C = 22.5 \text{ mm}$$

Use the larger value to meet both limits. $d = 22.5 \text{ mm} \blacktriangleleft$

Problem 3.47

3.47 The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts and that the angle through which pulley A will rotate when subjected to a $200 \text{ N} \cdot \text{m}$ torque T_A while pulley D is held fixed will not exceed 7.5° . Determine the required diameter of the shafts if both shafts are made of a steel with $G = 77 \text{ GPa}$ and $\tau_{\text{all}} = 84 \text{ MPa}$.

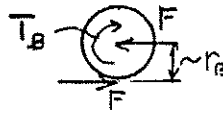


Statics, Gear B.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$r_B F - T_A = 0$$

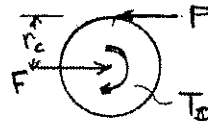
$$F = T_B / r_B$$



Gear C $+\circlearrowleft \Sigma M_C = 0:$

$$r_C F - T_D = 0$$

$$T_D = r_C F = \frac{r_C}{r_B} T_A = n T_B$$



$$n = \frac{r_C}{r_B} = \frac{125}{50} = 2.5$$

Torques in shafts.

$$T_{AB} = T_A = T_B$$

$$T_{CD} = T_C = n T_B = n T_A$$

Deformations:

$$\phi_{C/D} = \frac{T_{CD} L}{GJ} \curvearrowright = \frac{n T_A L}{GJ} \curvearrowright$$

$$\phi_{A/B} = \frac{T_{AB} L}{GJ} \curvearrowright = \frac{T_A L}{GJ} \curvearrowright$$

Kinematics:

$$\phi_D = 0 \quad \phi_C = \phi_D + \phi_{C/D} = 0 + \frac{n T_A L}{GJ} \curvearrowright$$

$$r_B \phi_B = -r_C \phi_C \quad \phi_B = -\frac{r_C}{r_B} \phi_C = -n \phi_C \quad \phi_B = \frac{n^2 T_A L}{GJ} \curvearrowright$$

$$\phi_A = \phi_C + \phi_{B/C} = \frac{n^2 T_A L}{GJ} + \frac{T_A L}{GJ} = \frac{(n^2 + 1) T_A L}{GJ} \curvearrowright$$

Diameter based on stress.

Longest torque: $T_m = T_{CD} = n T_A$

$$\tau_m = \frac{T_m c}{J} = \frac{2 n T_A}{\pi c^3}$$

$$\tau_m \leq \tau_{\text{all}} = 84 \text{ MPa},$$

$$T_A = 200 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{2 n T_A}{\pi \tau_m}} = \sqrt[3]{\frac{(2)(2.5)(2 \times 10^2)}{\pi (84000 \times 10^3)}} = 0.0156 \text{ m} \quad d = 2c = 0.0312 \text{ m}$$

Diameter based on rotation limit. $\phi = 7.5^\circ = 0.1309 \text{ rad}$

$$\phi = \frac{(n^2 + 1) T_A L}{GJ} = \frac{(2)(7.25) T_A L}{\pi c^4 G}$$

$$L = 0.2 + 0.4 = 0.6 \text{ m}$$

$$c = \sqrt[4]{\frac{(2)(7.25) T_A L}{\pi G \phi}} = \sqrt[4]{\frac{(2)(7.25)(2 \times 10^2)(0.6)}{\pi (77000 \times 10^3)(0.1309)}} = 0.01557 \text{ m} \quad d = 2c = 0.03114 \text{ m}$$

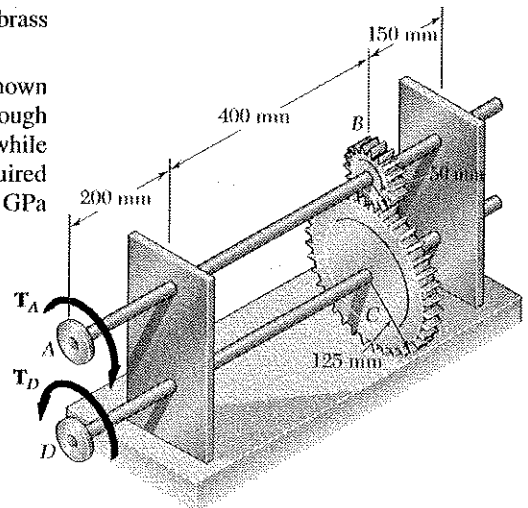
Choose the larger diameter.

$$d = 31.2 \text{ mm} \quad \blacktriangleleft$$

Problem 3.48

3.48 Solve Prob. 3.47, assuming that both shafts are made of a brass with $G = 39 \text{ GPa}$ and $\tau_{\text{all}} = 56 \text{ MPa}$.

3.47 The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts and that the angle through which pulley A will rotate when subjected to a $200 \text{ N} \cdot \text{m}$ torque T_A while pulley D is held fixed will not exceed 7.5° . Determine the required diameter of the shafts if both shafts are made of a steel with $G = 77 \text{ GPa}$ and $\tau_{\text{all}} = 84 \text{ MPa}$.

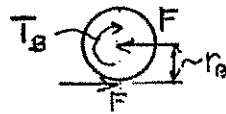


Statics, Gear B

$$+\circlearrowleft \Sigma M_B = 0:$$

$$r_B F = T_A = 0$$

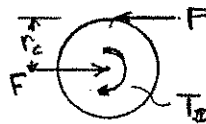
$$F = T_B / r_B$$



Gear C $+\circlearrowleft \Sigma M_C = 0:$

$$r_C F - T_D = 0$$

$$T_D = r_C F = \frac{r_C}{r_B} T_A = n T_B$$



$$n = \frac{r_C}{r_B} = \frac{125}{50} = 2.5$$

Torques in shafts.

$$T_{AB} = T_A = T_B$$

$$T_{CD} = T_C = n T_B = n T_A$$

Deformations:

$$\phi_{C/D} = \frac{T_{CD} L}{GJ} \curvearrowright = \frac{n T_A L}{GJ} \curvearrowright$$

$$\phi_{A/B} = \frac{T_{AB} L}{GJ} \curvearrowleft = \frac{T_A L}{GJ} \curvearrowleft$$

Kinematics:

$$\phi_D = 0 \quad \phi_C = \phi_D + \phi_{C/D} = 0 + \frac{n T_A L}{GJ} \curvearrowright$$

$$r_B \phi_B = -r_C \phi_C \quad \phi_B = -\frac{r_C}{r_B} \phi_C = -n \phi_C \quad \phi_B = \frac{n^2 T_A L}{GJ} \curvearrowleft$$

$$\phi_A = \phi_C + \phi_{B/C} = \frac{n^2 T_A L}{GJ} + \frac{T_A L}{GJ} = \frac{(n^2 + 1) T_A L}{GJ} \curvearrowleft$$

Diameter based on stress.

Longest torque $T_m = T_{CD} = n T_A$

$$\tau_m = \frac{T_m C}{J} = \frac{2 n T_A}{\pi C^3}$$

$$\tau_m = \tau_{\text{all}} = 56 \text{ MPa}, \quad T_A = 200 \text{ N} \cdot \text{m}$$

$$C = \sqrt[3]{\frac{2 n T_A}{\pi \tau_m}} = \sqrt[3]{\frac{(2)(2.5)(2 \times 10^2)}{\pi (56000 \times 10^3)}} = 0.0178 \text{ m} \quad d = 2C = 0.0356 \text{ m}$$

Diameter based on rotation limit $\phi = 7.5^\circ = 0.1309 \text{ rad}$

$$\phi = \frac{(n^2 + 1) T_A L}{GJ} = \frac{(2)(7.25) T_A L}{\pi C^4 G}$$

$$L = 0.2 + 0.4 = 0.6 \text{ m}$$

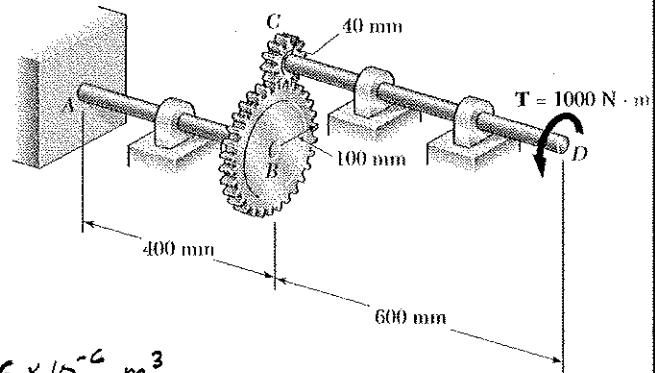
$$C = \sqrt[4]{\frac{(2)(7.25) T_A L}{\pi G \phi}} = \sqrt[4]{\frac{(2)(7.25)(2 \times 10^2)(0.6)}{\pi (39000 \times 10^9)(0.1309)}} = 0.01815 \text{ m} \quad d = 2C = 0.0363 \text{ m}$$

Choose the larger diameter,

$$d = 36.3 \text{ mm}$$

Problem 3.49

3.49 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD . It is further required that $\tau_{\max} \leq 60 \text{ MPa}$ and that the angle ϕ_D through which end D of shaft CD rotates not exceed 1.5° . Knowing that $G = 77 \text{ GPa}$, determine the required diameter of the shafts.



$$T_{CD} = T_D = 1000 \text{ N}\cdot\text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40} (1000) = 2500 \text{ N}\cdot\text{m}$$

For design based on stress, use larger torque. $T_{AB} = 2500 \text{ N}\cdot\text{m}$.

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi (60 \times 10^6)} = 26.526 \times 10^{-6} \text{ m}^3$$

$$c = 29.82 \times 10^{-3} \text{ m} = 29.82 \text{ mm}, \quad d = 2c = 59.6 \text{ mm}$$

Design based on rotation angle $\phi_D = 1.5^\circ = 26.18 \times 10^{-3} \text{ rad}$

Shaft AB : $T_{AB} = 2500 \text{ N}\cdot\text{m}$, $L = 0.4 \text{ m}$

$$\phi_{AB} = \frac{TL}{GJ} = \frac{(2500)(0.4)}{GJ} = \frac{1000}{GJ}$$

$$\text{Gears} \begin{cases} \phi_B = \phi_{AB} = \frac{1000}{GJ} \\ \phi_C = \frac{r_B}{r_C} \phi_B = \frac{100}{40} \cdot \frac{1000}{GJ} = \frac{2500}{GJ} \end{cases}$$

Shaft CD : $T_{CD} = 1000 \text{ N}\cdot\text{m}$, $L = 0.6 \text{ m}$

$$\phi_{CD} = \frac{TL}{GJ} = \frac{(1000)(0.6)}{GJ} = \frac{600}{GJ}$$

$$\phi_D = \phi_C + \phi_{CD} = \frac{2500}{GJ} + \frac{600}{GJ} = \frac{3100}{GJ} = \frac{3100}{G \frac{\pi}{2} c^4}$$

$$c^4 = \frac{(2)(3100)}{\pi G \phi_D} = \frac{(2)(3100)}{\pi (77 \times 10^9) (26.18 \times 10^{-3})} = 979.06 \times 10^{-9} \text{ m}^4$$

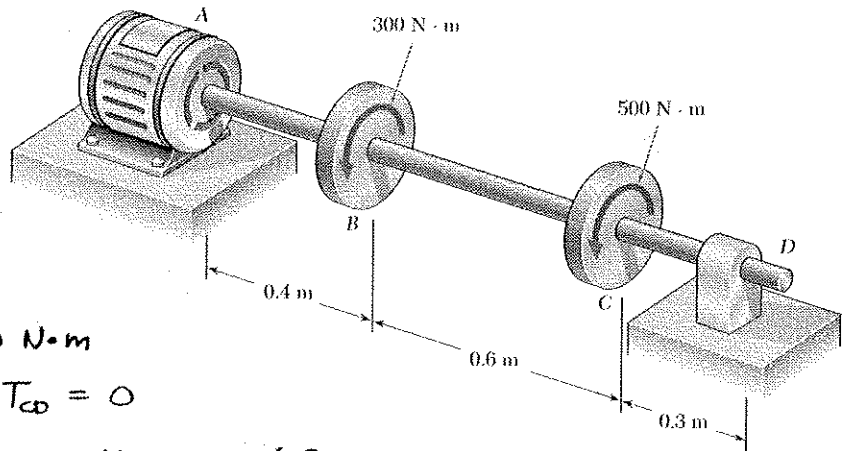
$$c = 31.46 \times 10^{-3} \text{ m} = 31.46 \text{ mm}, \quad d = 2c = 62.9 \text{ mm}$$

Design must use larger value for d .

$$d = 62.9 \text{ mm}$$

Problem 3.50

3.50 The electric motor exerts a torque of 800 N·m on the steel shaft *ABCD* when it is rotating at constant speed. Design specifications require that the diameter of the shaft be uniform from *A* to *D* and that the angle of twist between *A* and *D* not exceed 1.5° . Knowing that $\tau_{\max} \leq 60$ MPa and $G = 77$ GPa, determine the minimum diameter shaft that may be used.



Torques

$$T_{AB} = 300 + 500 = 800 \text{ N}\cdot\text{m}$$

$$T_{BC} = 500 \text{ N}\cdot\text{m}, \quad T_{CD} = 0$$

Design based on stress $\tau = 60 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(800)}{\pi(60 \times 10^6)} = 8.488 \times 10^{-6} \text{ m}^3$$

$$c = 20.40 \times 10^{-3} \text{ m} = 20.40 \text{ mm}, \quad d = 2c = 40.8 \text{ mm}$$

Design based on deformation $\phi_{D/A} = 1.5^\circ = 26.18 \times 10^{-3} \text{ rad}$

$$\phi_{D/C} = 0$$

$$\phi_{C/B} = \frac{T_{BC} L_{BC}}{GJ} = \frac{(500)(0.6)}{GJ} = \frac{600}{GJ}$$

$$\phi_{B/A} = \frac{T_{AB} L_{AB}}{GJ} = \frac{(800)(0.4)}{GJ} = \frac{320}{GJ}$$

$$\phi_{D/A} = \phi_{D/C} + \phi_{C/B} + \phi_{B/A} = \frac{620}{GJ} = \frac{620}{G \frac{\pi}{2} c^4} = \frac{(2)(620)}{\pi G c^4}$$

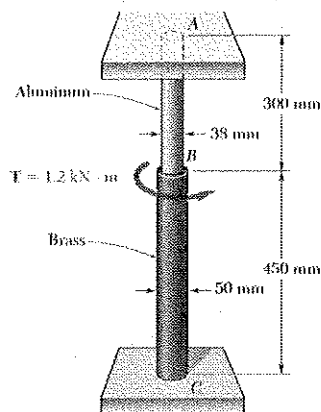
$$c^4 = \frac{(2)(620)}{\pi G \phi_{D/A}} = \frac{(2)(620)}{\pi(77 \times 10^9)(26.18 \times 10^{-3})} = 195.80 \times 10^{-9} \text{ m}^4$$

$$c = 21.04 \times 10^{-3} \text{ m} = 21.04 \text{ mm}, \quad d = 2c = 42.1 \text{ mm}$$

Design must use larger value of d .

$$d = 42.1 \text{ mm}$$

Problem 3.51



3.51 The solid cylinders AB and BC are bonded together at B and are attached to fixed supports at A and C. Knowing that the modulus of rigidity is 26 GPa for aluminum and 39 GPa for brass, determine the maximum shearing stress (a) in cylinder AB, (b) in cylinder BC.

The torques in cylinders AB and BC are statically indeterminate. Match the rotation ϕ_B for each cylinder.

Cylinder AB $c = \frac{1}{2}d = 0.019 \text{ m}$ $L = 0.3 \text{ m}$

$$J = \frac{\pi}{2} c^4 = 204.7 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L}{GJ} = \frac{T_{AB} (0.3)}{(26 \times 10^9)(204.7 \times 10^{-9})} = 56.37 \times 10^{-6} T_{AB}$$

Cylinder BC: $c = \frac{1}{2}d = 0.025 \text{ m}$ $L = 0.45 \text{ m}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.025)^4 = 613.6 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{BC} L}{GJ} = \frac{T_{BC} (0.45)}{(39 \times 10^9)(613.6 \times 10^{-9})} = 18.8 \times 10^{-6} T_{BC}$$

Matching expressions for ϕ_B . $56.37 \times 10^{-6} T_{AB} = 18.8 \times 10^{-6} T_{BC}$

$$T_{BC} = 2.9984 T_{AB} \quad (1)$$

Equilibrium of connection at B: $T_{AB} + T_{BC} - T = 0$ $T = 1200 \text{ Nm}$

$$T_{AB} + T_{BC} = 1.2 \times 10^3 \quad (2)$$

Substituting (1) into (2), $3.9984 T_{AB} = 1.2 \times 10^3$

$$T_{AB} = 300 \text{ Nm}$$

$$T_{BC} = 900 \text{ Nm}$$

(a) Maximum stress in cylinder AB.

$$\tau_{AB} = \frac{T_{AB} c}{J} = \frac{(300)(0.019)}{204.7 \times 10^{-9}} = 27.85 \text{ MPa}$$

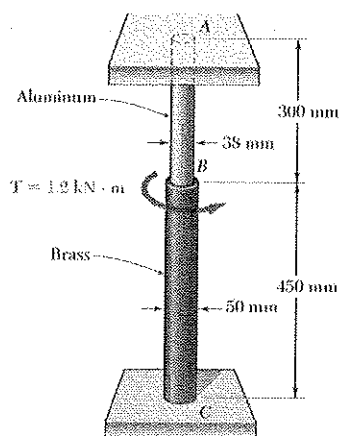
$$\tau_{AB} = 27.9 \text{ MPa} \quad \blacktriangleleft$$

(b) Maximum stress in cylinder BC.

$$\tau_{BC} = \frac{T_{BC} c}{J} = \frac{(900)(0.025)}{613.6 \times 10^{-9}} = 36.67 \text{ MPa}$$

$$\tau_{BC} = 36.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.52



3.52 Solve Prob. 3.51, assuming that cylinder AB is made of steel, for which $G = 77 \text{ GPa}$.

3.51 The solid cylinders AB and BC are bonded together at B and are attached to fixed supports at A and C. Knowing that the modulus of rigidity is 26 GPa for aluminum and 39 GPa for brass, determine the maximum shearing stress (a) in cylinder AB, (b) in cylinder BC.

The torques in cylinders AB and BC are statically indeterminate. Match the rotation ϕ_B for each cylinder.

Cylinder AB $c = \frac{1}{2}d = 0.019 \text{ m}$ $L = 0.3 \text{ m}$

$$J = \frac{\pi}{2} c^4 = 204.7 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L}{GJ} = \frac{T_{AB} (0.3)}{(77 \times 10^9)(204.7 \times 10^{-9})} = 19.03 \times 10^{-6} T_{AB}$$

Cylinder BC $c = \frac{1}{2}d = 0.025 \text{ m}$ $L = 0.45 \text{ m}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.025)^4 = 613.6 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{BC} L}{GJ} = \frac{T_{BC} (0.45)}{(39 \times 10^9)(613.6 \times 10^{-9})} = 18.8 \times 10^{-6} T_{BC}$$

Matching expressions for ϕ_B , $19.03 \times 10^{-6} T_{AB} = 18.8 \times 10^{-6} T_{BC}$

$$T_{BC} = 1.01223 T_{AB} \quad (1)$$

Equilibrium of connection at B: $T_{AB} + T_{BC} - T = 0$ $T = 1200 \text{ Nm}$

$$T_{AB} + T_{BC} = 1200 \quad (2)$$

Substituting (1) into (2) $2.01223 T_{AB} = 1200$

$$T_{AB} = 596 \text{ Nm}$$

$$T_{BC} = 603 \text{ Nm}$$

(a) Maximum stress in cylinder AB.

$$\tau_{AB} = \frac{T_{AB} c}{J} = \frac{(596)(0.019)}{204.7 \times 10^{-9}} = 55.32 \text{ MPa}$$

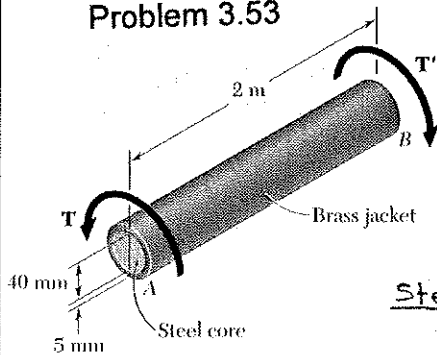
$$\tau_{AB} = 55.3 \text{ MPa} \quad \blacktriangleleft$$

(b) Maximum stress in cylinder BC.

$$\tau_{BC} = \frac{T_{BC} c}{J} = \frac{(603)(0.025)}{613.6 \times 10^{-9}} = 24.57 \text{ MPa}$$

$$\tau_{BC} = 24.6 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.53



3.53 The composite shaft shown consists of a 5-mm-thick brass jacket ($G_{\text{brass}} = 39$ GPa) bonded to a 40-mm-diameter steel core ($G_{\text{steel}} = 77.2$ GPa). Knowing that the shaft is subjected to a 600 N·m torque, determine (a) the maximum shearing stress in the brass jacket, (b) the maximum shearing stress in the steel core, (c) the angle of twist of B relative to A.

Steel core: $c = \frac{1}{2}d = 20 \text{ mm} = 0.020 \text{ m}$

$$J_1 = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

$$G_1 J_1 = (77.2 \times 10^9)(251.33 \times 10^{-9}) = 19.4025 \times 10^3 \text{ N·m}^2$$

Torque carried by steel core $T_1 = G_1 J_1 \frac{\phi}{L}$

Brass jacket: $c_1 = \frac{1}{2}d_1 = 20 \text{ mm} = 0.020 \text{ m}$ $c_2 = 20 + 5 = 25 \text{ mm} = 0.025 \text{ m}$

$$J_2 = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (0.025^4 - 0.020^4) = 362.265 \times 10^{-9} \text{ m}^4$$

$$G_2 J_2 = (39 \times 10^9)(362.265 \times 10^{-9}) = 14.1283 \times 10^3 \text{ N·m}^2$$

Torque carried by brass jacket $T_2 = G_2 J_2 \frac{\phi}{L}$

Total torque, $T = T_1 + T_2 = (G_1 J_1 + G_2 J_2) \frac{\phi}{L}$

$$\frac{\phi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{600}{19.4025 \times 10^3 + 14.1283 \times 10^3} = 17.894 \times 10^{-3} \text{ rad/m}$$

(a) Maximum shearing stress in brass jacket,

$$\tau_{\text{max}} = G \gamma_{\text{max}} = G_2 c_2 \frac{\phi}{L} = (39 \times 10^9)(0.025)(17.894 \times 10^{-3})$$

$$= 17.45 \times 10^6 \text{ Pa}$$

$$\tau_{\text{brass}} = 17.45 \text{ MPa} \blacktriangleleft$$

(b) Maximum shearing stress in steel core,

$$\tau_{\text{max}} = G \gamma_{\text{max}} = G_1 c \frac{\phi}{L} = (77.2 \times 10^9)(0.020)(17.894 \times 10^{-3})$$

$$= 27.6 \times 10^6 \text{ Pa}$$

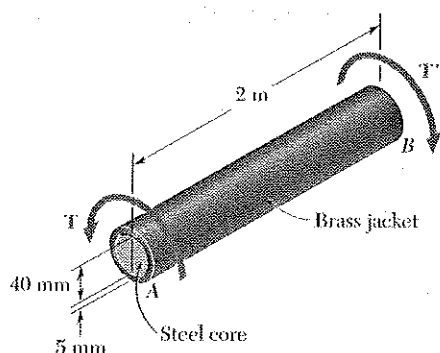
$$\tau_{\text{steel}} = 27.6 \text{ MPa} \blacktriangleleft$$

(c) Angle of twist,

$$\phi = L \frac{\phi}{L} = (2)(17.894 \times 10^{-3}) = 35.788 \times 10^{-3} \text{ rad}$$

$$\phi = 2.05^\circ \blacktriangleleft$$

Problem 3.54



3.54 For the composite shaft of Prob. 3.53 the allowable shearing stress in the brass jacket is 20 MPa and 45 MPa in the steel core. Determine (a) the largest torque which can be applied to the shaft, (b) the corresponding angle of twist of B relative to A.

3.53 The composite shaft shown consists of a 5-mm-thick brass jacket ($G_{\text{brass}} = 39 \text{ GPa}$) bonded to a 40-mm-diameter steel core ($G_{\text{steel}} = 77.2 \text{ GPa}$). Knowing that the shaft is subjected to a 600 N·m torque, determine (a) the maximum shearing stress in the brass jacket, (b) the maximum shearing stress in the steel core, (c) the angle of twist of B relative to A.

$$\tau_{\text{max}} = G \gamma_{\text{max}} = G C_{\text{max}} \frac{\phi}{L}$$

$$\frac{\phi_{\text{all}}}{L} = \frac{\tau_{\text{all}}}{G C_{\text{max}}} \text{ for each material}$$

$$T_i = G_i J_i \frac{\phi}{L} \text{ for each material}$$

Brass jacket: $\tau_{\text{all}} = 20 \times 10^6 \text{ Pa}$, $C_1 = 20 \text{ mm} = 0.020 \text{ m}$, $C_2 = 20 + 5 = 25 \text{ mm} = 0.025 \text{ m}$

$$\frac{\phi_{\text{all}}}{L} = \frac{20 \times 10^6}{(39 \times 10^9)(0.025)} = 20.513 \times 10^{-3} \text{ rad/m}$$

$$J_2 = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.025^4 - 0.020^4) = 362.265 \times 10^{-9} \text{ m}^4$$

Steel core: $\tau_{\text{all}} = 45 \times 10^6 \text{ Pa}$, $C = 0.020 \text{ m}$

$$\frac{\phi_{\text{all}}}{L} = \frac{45 \times 10^6}{(77.2 \times 10^9)(0.020)} = 29.145 \times 10^{-3} \text{ rad/m}$$

$$J_1 = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Smaller value of $\frac{\phi_{\text{all}}}{L}$ governs $\frac{\phi_{\text{all}}}{L} = 20.513 \times 10^{-3} \text{ rad/m}$

Torque carried by brass sleeve

$$T_2 = G_2 J_2 \frac{\phi_{\text{all}}}{L} = (39 \times 10^9)(362.265 \times 10^{-9})(20.513 \times 10^{-3}) = 287.8 \text{ N}\cdot\text{m}$$

Torque carried by steel core

$$T_1 = G_1 J_1 \frac{\phi_{\text{all}}}{L} = (77.2 \times 10^9)(251.33 \times 10^{-9})(20.513 \times 10^{-3}) = 398.5 \text{ N}\cdot\text{m}$$

(a) Allowable torque. $T = T_1 + T_2 = 687.8 \text{ N}\cdot\text{m}$

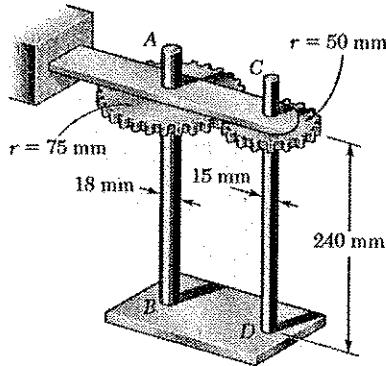
$$T_{\text{all}} = 688 \text{ N}\cdot\text{m}$$

(b) Allowable twist angle.

$$\phi_{\text{all}} = L \frac{\phi_{\text{all}}}{L} = (2)(20.513 \times 10^{-3}) = 41.026 \times 10^{-3} \text{ rad}$$

$$\phi_{\text{all}} = 2.35^\circ$$

Problem 3.55



3.55 At a time when rotation is prevented at the lower end of each shaft, a 80-N·m torque is applied to end A of shaft AB. Knowing that $G = 77.2$ GPa for both shafts, determine (a) the maximum shearing stress in shaft CD, (b) the angle of rotation at A.

Let $T_A =$ torque applied at A = 50 N·m.

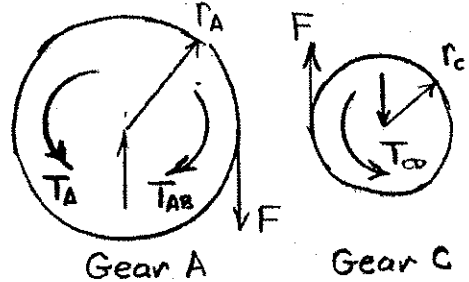
$T_{AB} =$ torque in shaft AB

$T_{CD} =$ torque in shaft CD

Statics:

$$T_A - T_{AB} - Fr_A = 0$$

$$T_{CD} - Fr_c = 0$$



$$T_{CD} = \frac{r_c}{r_A} (T_A - T_{AB}) = \frac{2}{3} (T_A - T_{AB})$$

Kinematics: $r_A \phi_A = r_c \phi_c$ $\phi_A = \frac{r_c}{r_A} \phi_c = \frac{2}{3} \phi_c$

Angles of twist: $\phi_A = \frac{T_{AB} L}{G J_{AB}}$ $\phi_c = \frac{T_{CD} L}{G J_{CD}} = \frac{2}{3} \frac{(T_A - T_{AB}) L}{G J_{CD}}$

$$\frac{T_{AB} L}{G J_{AB}} = \frac{2}{3} \cdot \frac{2}{3} \frac{(T_A - T_{AB}) L}{G J_{CD}}$$

$$\left(\frac{4}{9} + \frac{J_{CD}}{J_{AB}} \right) T_{AB} = \left(\frac{4}{9} + \left(\frac{15}{18} \right)^4 \right) T_{AB} = \frac{4}{9} T_A$$

$$T_{AB} = 0. \quad T_A = (0.47960)(80) = 38.368 \text{ N·m}$$

$$T_{CD} = \frac{2}{3} (80 - 38.368) = 27.755 \text{ N·m}$$

(a) Maximum shearing stress in shaft CD.

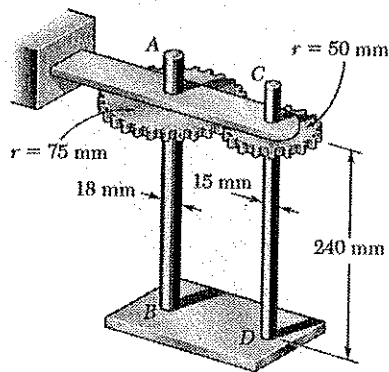
$$\tau_{CD} = \frac{T_{CD} C}{J_{CD}} = \frac{2 T_{CD}}{\pi C^3} = \frac{(2)(27.755)}{\pi (0.0075)^3} = 41.9 \times 10^6 \text{ Pa} \quad \tau_{CD} = 41.9 \text{ MPa} \quad \blacktriangleleft$$

(b) Angle of rotation at A.

$$\phi_A = \frac{T_{AB} L}{G J_A} = \frac{2 T_{AB} L}{\pi G C^4} = \frac{(2)(38.368)(0.240)}{\pi (77.2 \times 10^9)(0.009)^4} = 11.57 \times 10^{-3} \text{ rad}$$

$$\phi_A = 0.663^\circ \quad \blacktriangleleft$$

Problem 3.56



3.56 Solve Prob. 3.55, assuming that the 80-N · m torque is applied to end C of shaft CD.

3.55 At a time when rotation is prevented at the lower end of each shaft, a 50-N · m torque is applied to end A of shaft AB. Knowing that $G = 77.2$ GPa for both shafts, determine (a) the maximum shearing stress in shaft CD, (b) the angle of rotation at A.

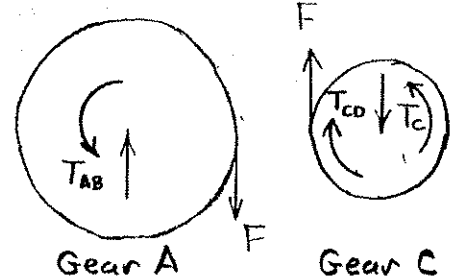
Let T_c = torque applied at C = 50 N · m
 T_{cd} = torque in shaft CD.
 T_{AB} = torque in shaft AB.

Statics:

$$T_{AB} - r_A F = 0$$

$$T_c - T_{cd} - r_c F = 0$$

$$T_{AB} = \frac{r_A}{r_c} (T_c - T_{cd}) = \frac{3}{2} (T_c - T_{cd})$$



Kinematics: $r_A \phi_A = r_c \phi_c$ $\phi_c = \frac{r_A}{r_c} \phi_A = \frac{3}{2} \phi_A$

Angles of twist: $\phi_c = \frac{T_{cd} L}{G J_{cd}}$ $\phi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{3}{2} \frac{(T_c - T_{cd}) L}{G J_{AB}}$

$$\frac{T_{cd} L}{G J_{cd}} = \frac{3}{2} \cdot \frac{3}{2} \frac{T_c - T_{cd}}{G J_{AB}}$$

$$\left(\frac{J_{AB}}{J_{cd}} + \frac{9}{4} \right) T_{cd} = \left(\left(\frac{18}{15} \right)^4 + \frac{9}{4} \right) T_{cd} = \frac{9}{4} T_c$$

$$T_{cd} = 0.4796 T_c = (0.52040)(80) = 41.632 \text{ N} \cdot \text{m}$$

$$T_{AB} = \frac{3}{2} (80 - 41.632) = 57.552 \text{ N} \cdot \text{m}$$

(a) Maximum shearing stress in CD.

$$\tau_{cd} = \frac{T_{cd} c}{J_{cd}} = \frac{2 T_{cd}}{\pi c^3} = \frac{(2)(41.632)}{\pi (0.0075)^3} = 62.8 \times 10^6 \text{ Pa} \quad \tau_{cd} = 62.8 \text{ MPa}$$

(b) Angle of rotation at A.

$$\phi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{2 T_{AB} L}{\pi G c^4} = \frac{(2)(57.552)(0.240)}{\pi (77.2 \times 10^9)(0.009)^4} = 17.36 \times 10^{-3} \text{ rad}$$

$$\phi_A = 0.995^\circ$$

Problem 3.57

3.57 and 3.58 Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77 \text{ GPa}$, determine the maximum shearing stress in each shaft when a torque of T of magnitude $570 \text{ N} \cdot \text{m}$ is applied to the flange indicated.

3.57 The torque T is applied to flange B.

Shaft AB

$$T = T_{AB}, L_{AB} = 0.6 \text{ m}, \quad c = \frac{1}{2} d = 0.015 \text{ m}$$

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.015)^4 = 79.5 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L_{AB}}{G J_{AB}}$$

$$T_{AB} = \frac{G J_{AB} \phi_B}{L_{AB}} = \frac{(77 \times 10^9)(79.5 \times 10^{-9})}{0.6} \phi_B = 10202.5 \phi_B$$

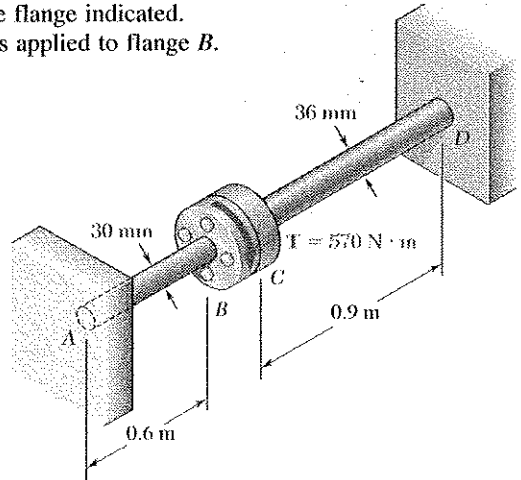
Shaft CD

$$T = T_{CD}, L_{CD} = 0.9 \text{ m}, \quad c = \frac{1}{2} d = 0.018 \text{ m}$$

$$J_{CD} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.018)^4 = 164.9 \times 10^{-9} \text{ m}^4$$

$$\phi_C = \frac{T_{CD} L_{CD}}{G J_{CD}}$$

$$T_{CD} = \frac{G J_{CD} \phi_C}{L_{CD}} = \frac{(77 \times 10^9)(164.9 \times 10^{-9})}{0.9} \phi_C = 14108.1 \phi_C$$



Applied torque
 $T = 570 \text{ N} \cdot \text{m}$

Clearance rotation for flange B: $\phi_B' = 1.5^\circ = 26.18 \times 10^{-3} \text{ rad}$

Torque to remove clearance: $T_{AB}' = (10202.5)(26.18 \times 10^{-3}) = 267.1 \text{ Nm}$

Torque T'' to cause additional rotation ϕ'' : $T'' = 570 - 267.1 = 302.9 \text{ Nm}$

$$T'' = T_{AB}'' + T_{CD}''$$

$$302.9 = (10202.5) \phi'' + (14108.1) \phi''$$

$$\therefore \phi'' = 12.46 \times 10^{-3} \text{ rad}$$

$$T_{AB}'' = (10202.5)(12.46 \times 10^{-3}) = 127.1 \text{ Nm}$$

$$T_{CD}'' = (14108.1)(12.46 \times 10^{-3}) = 175.8 \text{ Nm}$$

Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB} c}{J_{AB}} = \frac{(267.1 + 127.1)(0.015)}{79.5 \times 10^{-9}} = 74.38 \text{ MPa}$$

$$\tau_{AB} = 74.4 \text{ MPa} \quad \blacktriangleleft$$

Maximum shearing stress in CD.

$$\tau_{CD} = \frac{T_{CD} c}{J_{CD}} = \frac{(175.8)(0.018)}{164.9 \times 10^{-9}} = 19.19 \text{ MPa}$$

$$\tau_{CD} = 19.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.58

3.57 and 3.58 Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77 \text{ GPa}$, determine the maximum shearing stress in each shaft when a torque of T of magnitude $570 \text{ N} \cdot \text{m}$ is applied to the flange indicated.

3.58 The torque T is applied to flange C.

Shaft AB

$$T = T_{AB}, L_{AB} = 0.6 \text{ m}, c = \frac{1}{2}d = 0.015 \text{ m}$$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.5 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L_{AB}}{G J_{AB}}$$

$$T_{AB} = \frac{G J_{AB} \phi_B}{L_{AB}} = \frac{(77 \times 10^9)(79.5 \times 10^{-9})}{0.6} \phi_B$$

$$= 10202.5 \phi_B$$

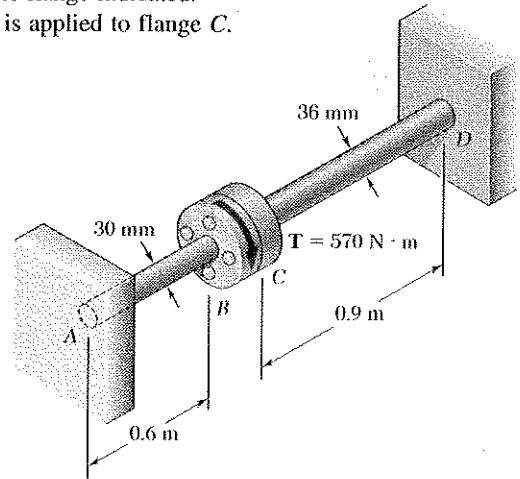
Shaft CD

$$T = T_{CD}, L_{CD} = 0.9 \text{ m}, c = \frac{1}{2}d = 0.018 \text{ m}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.9 \times 10^{-9} \text{ m}^4$$

$$\phi_C = \frac{T_{CD} L_{CD}}{G J_{CD}}$$

$$T_{CD} = \frac{G J_{CD} \phi_C}{L_{CD}} = \frac{(77 \times 10^9)(164.9 \times 10^{-9})}{0.9} \phi_C = 14108.1 \phi_C$$



Applied torque

$$T = 570 \text{ N} \cdot \text{m}$$

Clearance rotation for flange C: $\phi'_C = 1.5^\circ = 26.18 \times 10^{-3} \text{ rad}$

Torque to remove clearance: $T'_{CD} = (14108.1)(26.18 \times 10^{-3}) = 369.4 \text{ N} \cdot \text{m}$

Torque T'' to cause additional rotation ϕ'' : $T'' = 570 - 369.4 = 200.6 \text{ N} \cdot \text{m}$

$$T'' = T''_{AB} + T''_{CD}$$

$$200.6 = (10202.5)\phi'' + (14108.1)\phi''$$

$$\therefore \phi'' = 8.2515 \times 10^{-3} \text{ rad}$$

$$T''_{AB} = (10202.5)(8.2515 \times 10^{-3}) = 84.2 \text{ N} \cdot \text{m}$$

$$T''_{CD} = (14108.1)(8.2515 \times 10^{-3}) = 116.4 \text{ N} \cdot \text{m}$$

Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB} c}{J_{AB}} = \frac{(84.2)(0.015)}{79.5 \times 10^{-9}} = 15.89 \text{ MPa}$$

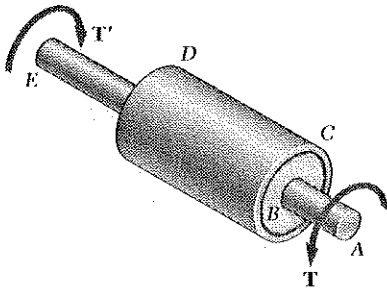
$$\tau_{AB} = 15.9 \text{ MPa}$$

Maximum shearing stress in CD.

$$\tau_{CD} = \frac{T_{CD} c}{J_{CD}} = \frac{(369.4 + 116.4)(0.018)}{164.9 \times 10^{-9}} = 53.03 \text{ MPa}$$

$$\tau_{CD} = 53 \text{ MPa}$$

Problem 3.59



3.59 The steel jacket CD has been attached to the 40-mm-diameter steel shaft AE by means of rigid flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If $500 \text{ N} \cdot \text{m}$ torques are applied as shown, determine the maximum shearing stress in the jacket.

Solid shaft: $c = \frac{1}{2}d = 0.020 \text{ m}$

$$J_s = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Jacket: $c_2 = \frac{1}{2}d = 0.040 \text{ m}$

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.040^4 - 0.036^4) = 1.3829 \times 10^{-6} \text{ m}^4$$

Torque carried by shaft, $T_s = GJ_s \phi / L$

Torque carried by jacket, $T_J = GJ_J \phi / L$

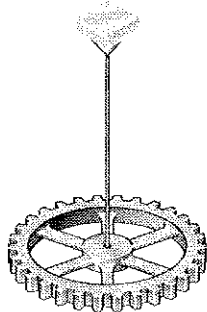
Total torque, $T = T_s + T_J = (J_s + J_J)G\phi / L \therefore \frac{G\phi}{L} = \frac{T}{J_s + J_J}$

$$T_J = \frac{J_J}{J_s + J_J} T = \frac{(1.3829 \times 10^{-6})(500)}{1.3829 \times 10^{-6} + 251.33 \times 10^{-9}} = 423.1 \text{ N} \cdot \text{m}$$

Maximum shearing stress in jacket.

$$\tau = \frac{T_J c}{J_J} = \frac{(423.1)(0.040)}{1.3829 \times 10^{-6}} = 12.24 \times 10^6 \text{ Pa} \quad 12.24 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.60



3.60 The mass moment of inertia of a gear is to be determined experimentally by using a torsional pendulum consisting of a 1.8-m steel wire. Knowing that $G = 77 \text{ GPa}$, determine the diameter of the wire for which the torsional spring constant will be $6 \text{ N} \cdot \text{m/rad}$.

Torsion spring constant $K = 6 \text{ N} \cdot \text{m/rad}$

$$K = \frac{T}{\phi} = \frac{T}{TL/GJ} = \frac{GJ}{L} = \frac{\pi G c^4}{2L}$$

$$c^4 = \frac{2LK}{\pi G} = \frac{(2)(1.8)(6)}{\pi(77 \times 10^9)} = 89.292 \times 10^{-12} \text{ m}^4$$

$$c = 0.00307 \text{ m}$$

$$d = 2c = 6 \text{ mm} \quad \blacktriangleleft$$

Problem 3.61

3.61 An annular plate of thickness t and modulus of rigidity G is used to connect shaft AB of radius r_1 to tube CD of inner radius r_2 . Knowing that a torque T is applied to end A of shaft AB and that end D of tube CD is fixed, (a) determine the magnitude and location of the maximum shearing stress in the annular plate, (b) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

$$\phi_{B/C} = \frac{T}{4\pi Gt} \left(\frac{1}{r_1^2} - \frac{1}{r_2^2} \right)$$

Use a free body consisting of shaft AB and an inner portion of the plate BC , the outer radius of this portion being ρ

The force per unit length of circumference is τt .

$$\Sigma M = 0$$

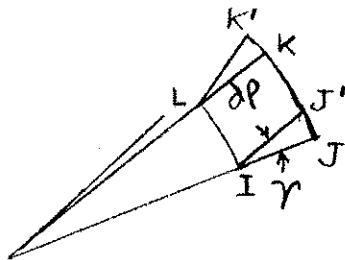
$$\tau t (2\pi \rho) \rho - T = 0$$

$$\tau = \frac{T}{2\pi t \rho^2}$$

(a) Maximum shearing stress occurs at $\rho = r_1$

$$\tau_{\max} = \frac{T}{2\pi t r_1^2}$$

Shearing strain $\gamma = \frac{\tau}{G} = \frac{T}{2\pi G t \rho^2}$



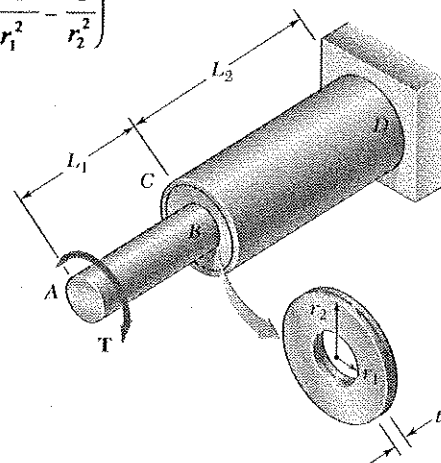
The relative circumferential displacement in radial length dp is

$$ds = \gamma dp = \rho d\phi$$

$$d\phi = \gamma \frac{dp}{\rho}$$

$$d\phi = \frac{T}{2\pi G t \rho^2} \frac{dp}{\rho} = \frac{T}{2\pi G t} \frac{dp}{\rho^3}$$

$$\begin{aligned} (b) \phi_{B/C} &= \int_{r_1}^{r_2} \frac{T}{2\pi G t} \frac{dp}{\rho^3} = \frac{T}{2\pi G t} \int_{r_1}^{r_2} \frac{dp}{\rho^3} = \frac{T}{2\pi G t} \left\{ -\frac{1}{2\rho^2} \right\} \bigg|_{r_1}^{r_2} \\ &= \frac{T}{2\pi G t} \left\{ -\frac{1}{2r_2^2} + \frac{1}{2r_1^2} \right\} = \frac{T}{4\pi G t} \left\{ \frac{1}{r_1^2} - \frac{1}{r_2^2} \right\} \end{aligned}$$



Problem 3.62

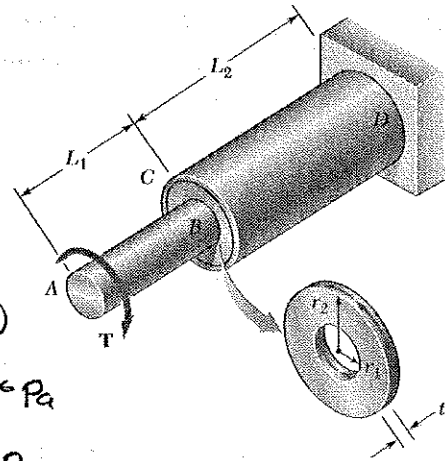
3.62 An annular brass plate ($G = 39 \text{ GPa}$), of thickness $t = 6 \text{ mm}$, is used to connect the brass shaft AB , of length $L_1 = 50 \text{ mm}$ and radius $r_1 = 30 \text{ mm}$, to the brass tube CD , of length $L_2 = 125 \text{ mm}$, inner radius $r_2 = 75 \text{ mm}$, and thickness $t = 3 \text{ mm}$. Knowing that a $2.8 \text{ kN} \cdot \text{m}$ torque T is applied to end A of shaft AB and that end D of tube CD is fixed, determine (a) the maximum shearing stress in the shaft-plate-tube system, (b) the angle through which end A rotates. (Hint: Use the formula derived in Prob. 3.61 to solve part b.)

Shaft AB .

$$\begin{aligned}\tau &= \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2T}{\pi r_1^3} \\ &= \frac{(2)(2800)}{\pi(0.030)^3} = 66.0 \times 10^6 \text{ Pa} \\ &= 66.0 \text{ MPa}\end{aligned}$$

Plate BC . (See PROBLEM 3.61 for derivation)

$$\begin{aligned}\tau &= \frac{T}{2\pi t r_1^2} = \frac{2800}{2\pi(0.006)(0.030)^2} = 82.5 \times 10^6 \text{ Pa} \\ &= 82.5 \text{ MPa}\end{aligned}$$



Shaft CD . $C_1 = r_2 = 0.075 \text{ m}$, $C_2 = r_2 + t = 0.075 + 0.003 = 0.078 \text{ m}$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.078^4 - 0.075^4) = 8.4421 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc_2}{J} = \frac{(2800)(0.078)}{8.4421 \times 10^{-6}} = 25.9 \times 10^6 \text{ Pa} = 25.9 \text{ MPa}$$

(a) Largest stress.

$$\tau_{\max} = 82.5 \text{ MPa} \quad \blacktriangleleft$$

Shaft AB .
$$\begin{aligned}\phi_{AB} &= \frac{T L_1}{GJ} = \frac{2 T L_1}{\pi G C^4} = \frac{(2)(2800)(0.050)}{\pi(39 \times 10^9)(0.030)^4} \\ &= 2.821 \times 10^{-3} \text{ rad}\end{aligned}$$

Plate BC . (See Problem 3.61 for derivation of equation below.)

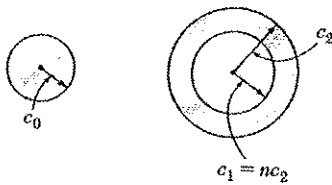
$$\begin{aligned}\phi_{BC} &= \frac{T}{4\pi G t} \left\{ \frac{1}{r_1^2} - \frac{1}{r_2^2} \right\} = \frac{2800}{4\pi(39 \times 10^9)(0.006)} \left\{ \frac{1}{0.030^2} - \frac{1}{0.075^2} \right\} \\ &= 0.889 \times 10^{-3} \text{ rad.}\end{aligned}$$

Shaft CD .
$$\phi_{CD} = \frac{T L_2}{GJ} = \frac{(2800)(0.125)}{(39 \times 10^9)(8.4421 \times 10^{-6})} = 1.063 \times 10^{-3} \text{ rad}$$

(b) Total rotation angle. $\phi_A = \phi_{AB} + \phi_{BC} + \phi_{CD} = 4.773 \times 10^{-3} \text{ rad}$

$$\phi_A = 0.273^\circ \quad \blacktriangleleft$$

Problem 3.63



3.63 A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by n the ratio c_1/c_2 , show that the ratio T_s/T_h of the torque T_s in the solid shaft to the torque T_h in the hollow shaft is (a) $\sqrt{(1-n^2)/(1+n^2)}$ if the maximum shearing stress is the same in each shaft, (b) $(1-n)/(1+n^2)$ if the angle of twist is the same for each shaft.

For equal weight and length, the areas are equal.

$$\pi c_0^2 = \pi(c_2^2 - c_1^2) = \pi c_2^2(1 - n^2) \therefore c_0 = c_2(1 - n^2)^{1/2}$$

$$J_s = \frac{\pi}{2} c_0^4 = \frac{\pi}{2} c_2^4 (1 - n^2)^2 \quad J_h = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} c_2^4 (1 - n^4)$$

(a) For equal stresses,

$$\tau = \frac{T_s c_0}{J_s} = \frac{T_h c_2}{J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s c_2}{J_h c_0} = \frac{\frac{\pi}{2} c_2^4 (1 - n^2)^2 c_2}{\frac{\pi}{2} c_2^4 (1 - n^4) c_2 (1 - n^2)^{1/2}} = \frac{1 - n^2}{(1 + n^2)(1 - n^2)^{1/2}} = \frac{(1 - n^2)^{1/2}}{1 + n^2}$$

(b) For equal angles of twist,

$$\phi = \frac{T_s L}{G J_s} = \frac{T_h L}{G J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s}{J_h} = \frac{\frac{\pi}{2} c_2^4 (1 - n^2)^2}{\frac{\pi}{2} c_2^4 (1 - n^4)} = \frac{(1 - n^2)^2}{1 - n^4} = \frac{1 - n^2}{1 + n^2}$$

Problem 3.64

3.64 Determine the maximum shearing stress in a solid shaft of 38-mm diameter as it transmits 55 kW at a speed of (a) 750 rpm, (b) 1500 rpm.

$$c = \frac{1}{2}d = 19 \text{ mm} \quad P = 55 \text{ kW}$$

$$(a) f = \frac{750}{60} = 12.5 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{55 \times 10^3}{2\pi (12.5)} = 700 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi (0.019)^3} = 65 \text{ MPa}$$

$$\tau = 65 \text{ MPa}$$

$$(b) f = \frac{1500}{60} = 25 \text{ Hz}$$

$$T = \frac{55 \times 10^3}{2\pi (25)} = 350 \text{ N}\cdot\text{m}$$

$$\tau = \frac{(2)(350)}{\pi (0.019)^3} = 32.5 \text{ MPa}$$

$$\tau = 32.5 \text{ MPa}$$

Problem 3.65

3.65 Determine the maximum shearing stress in a solid shaft of 12-mm diameter as it transmits 2.5 kW at a frequency of (a) 25 Hz, (b) 50 Hz.

$$c = \frac{1}{2}d = 6 \text{ mm} = 0.006 \text{ m} \quad P = 2.5 \text{ kW} = 2500 \text{ W}$$

$$(a) \quad f = 25 \text{ Hz}, \quad T = \frac{P}{2\pi f} = \frac{2500}{2\pi(25)} = 15.9155 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(15.9155)}{\pi(0.006)^3} = 46.9 \times 10^6 \text{ Pa} \quad \tau = 46.9 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad f = 50 \text{ Hz} \quad T = \frac{2500}{2\pi(50)} = 7.9577 \text{ N}\cdot\text{m}$$

$$\tau = \frac{2(7.9577)}{\pi(0.006)^3} = 23.5 \times 10^6 \text{ Pa} \quad \tau = 23.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.66

3.66 Using an allowable shearing stress of 50 MPa, design a solid steel shaft to transmit 15 kW at a frequency of (a) 30 Hz, (b) 60 Hz.

$$\tau_{\text{all}} = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa} \quad P = 15 \text{ kW} = 15 \times 10^3 \text{ W}$$

$$(a) \quad f = 30 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{15 \times 10^3}{2\pi(30)} = 79.577 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(79.577)}{\pi(50 \times 10^6)}} = 10.044 \times 10^{-3} \text{ m} = 10.044 \text{ mm}$$

$$d = 2c = 20.1 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad f = 60 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{15 \times 10^3}{2\pi(60)} = 39.789 \text{ N}\cdot\text{m}$$

$$c = \sqrt[3]{\frac{(2)(39.789)}{\pi(50 \times 10^6)}} = 7.972 \times 10^{-3} \text{ m} = 7.972 \text{ mm}$$

$$d = 2c = 15.94 \text{ mm} \quad \blacktriangleleft$$

Problem 3.67

3.67 Using an allowable shearing stress of 30 MPa, design a solid steel shaft to transmit 9 kW at speed of (a) 1200 rpm, (b) 2400 rpm.

$$\tau_{all} = 30 \text{ MPa}$$

$$P = 9 \text{ kW}$$

$$(a) f = \frac{1200}{60} = 20 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{9 \times 10^3}{2\pi(20)} = 71.62 \text{ Nm}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{2(71.62)}{\pi(30 \times 10^6)}} = 0.0115 \text{ m}$$

$$d = 2c = 23 \text{ mm}$$

$$(b) f = \frac{2400}{60} = 40 \text{ Hz}$$

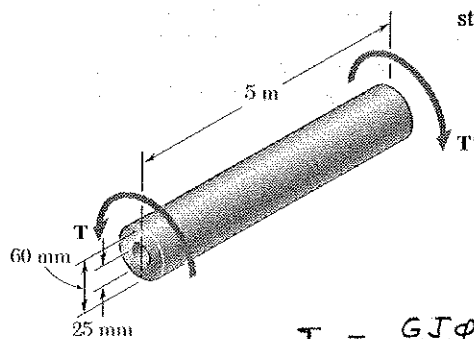
$$T = \frac{P}{2\pi f} = \frac{9 \times 10^3}{2\pi(40)} = 35.81 \text{ Nm}$$

$$c = \sqrt[3]{\frac{2(35.81)}{\pi(30 \times 10^6)}} = 0.009125 \text{ m}$$

$$d = 2c = 18.25 \text{ mm}$$

Problem 3.68

3.68 As the hollow steel shaft shown rotates at 180 rpm, a stroboscopic measurement indicates that the angle of twist of the shaft is 3° . Knowing that $G = 77.2 \text{ GPa}$, determine (a) the power being transmitted, (b) the maximum shearing stress in the shaft.



$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm} \quad c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(30)^4 - (12.5)^4]$$

$$= 1.234 \times 10^6 \text{ mm}^4 = 1.234 \times 10^{-6} \text{ m}^4$$

$$\phi = 3^\circ = 0.05236 \text{ rad}$$

$$\phi = \frac{TL}{GJ}$$

$$T = \frac{GJ\phi}{L} = \frac{(77.2 \times 10^9)(1.234 \times 10^{-6})(0.05236)}{5} = 997.61 \text{ N}\cdot\text{m}$$

Angular speed $f = 180 \text{ rpm} = 3 \text{ rev/sec} = 3 \text{ Hz}$

(a) Power being transmitted. $P = 2\pi fT = 2\pi(3)(997.61) = 18.80 \times 10^3 \text{ W}$

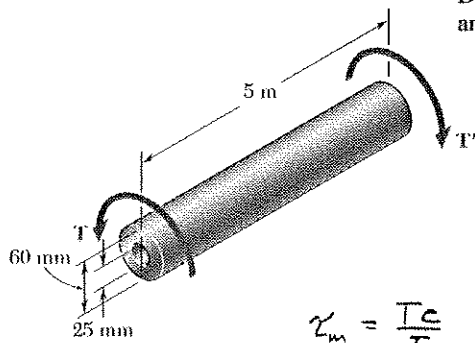
$$P = 18.80 \text{ kW}$$

(b) Maximum shearing stress. $\tau_m = \frac{Tc_2}{J} = \frac{(997.61)(30 \times 10^{-3})}{1.234 \times 10^{-6}}$

$$= 24.3 \times 10^6 \text{ Pa}$$

$$\tau_m = 24.3 \text{ MPa}$$

Problem 3.69



3.69 The hollow steel shaft shown ($G = 77.2 \text{ GPa}$, $\tau_{\text{all}} = 50 \text{ MPa}$) rotates at 240 rpm. Determine (a) the maximum power that can be transmitted, (b) the corresponding angle of twist of the shaft.

$$C_2 = \frac{1}{2} d_2 = 30 \text{ mm} \quad C_1 = \frac{1}{2} d_1 = 12.5 \text{ mm}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} [(30)^4 - (12.5)^4] \\ = 1.234 \times 10^6 \text{ mm}^4 = 1.234 \times 10^{-6} \text{ m}^4$$

$$\tau_m = 50 \times 10^6 \text{ Pa}$$

$$\tau_m = \frac{T C}{J}$$

$$T = \frac{\tau_m J}{C} = \frac{(50 \times 10^6)(1.234 \times 10^{-6})}{30 \times 10^{-3}} = 2056.7 \text{ N}\cdot\text{m}$$

Angular speed $f = 240 \text{ rpm} = 4 \text{ rev/sec} = 4 \text{ Hz}$

(a) Power being transmitted. $P = 2\pi f T = 2\pi(4)(2056.7) = 51.7 \times 10^3 \text{ W}$

$$P = 51.7 \text{ kW}$$

(b) Angle of twist. $\phi = \frac{TL}{GJ} = \frac{(2056.7)(5)}{(77.2 \times 10^9)(1.234 \times 10^{-6})} = 0.1078 \text{ rad}$

$$\phi = 6.17^\circ$$

Problem 3.70

3.70 One of two hollow drive shafts of a cruise ship is 40 m long, and its outer and inner diameters are 400 mm and 200 mm, respectively. The shaft is made of a steel for which $\tau_{\text{all}} = 60 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. Knowing that the maximum speed of rotation of the shaft is 160 rpm, determine (a) the maximum power that can be transmitted by one shaft to its propeller, (b) the corresponding angle of twist of the shaft.

$$L = 40 \text{ m}$$

$$C_2 = \frac{1}{2} d_o = 200 \text{ mm} = 0.200 \text{ m} \quad C_1 = \frac{1}{2} d_i = 100 \text{ mm} = 0.100 \text{ m}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.200^4 - 0.100^4) = 2.3562 \times 10^{-3} \text{ m}^4$$

$$\tau = \frac{T C_{\text{max}}}{J} \quad T = \frac{J \tau}{C_{\text{max}}} = \frac{(2.3562 \times 10^{-3})(60 \times 10^6)}{0.200} = 706.86 \times 10^3 \text{ N}\cdot\text{m}$$

$$f = \frac{160}{60} = 2.6667 \text{ Hz}$$

(a) Maximum power $P = 2\pi f T$

$$P = 2\pi(2.6667)(706.86 \times 10^3) = 11.844 \times 10^6 \text{ W} \quad P = 11.84 \text{ MW}$$

(b) Angle of twist $\phi = \frac{TL}{GJ}$

$$\phi = \frac{(706.86 \times 10^3)(40)}{(77.2 \times 10^9)(2.3562 \times 10^{-3})} = 155.44 \times 10^{-3} \text{ rad} \quad \phi = 8.91^\circ$$

Problem 3.71

3.71 A steel drive shaft is 1.8 m long and its outer and inner diameters are respectively equal to 56 mm and 42 mm. Knowing that the shaft transmits 180 kW while rotating at 1800 rpm, determine (a) the maximum shearing stress, (b) the angle of twist of the shaft ($G = 77 \text{ GPa}$).

$$L = 1.8 \text{ m}$$

$$C_2 = \frac{1}{2} d_o = 28 \text{ mm}$$

$$C_1 = \frac{1}{2} d_i = 21 \text{ mm}$$

$$P = 180 \text{ kW}$$

$$f = \frac{1800}{60} = 30 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{180 \times 10^3}{2\pi(30)} = 955.10 \text{ Nm}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.028^4 - 0.021^4) = 660 \times 10^{-9} \text{ m}^4$$

(a) Maximum shearing stress.

$$\tau = \frac{TC_2}{J}$$

$$\tau = \frac{(955)(0.028)}{660 \times 10^{-9}} = 40.5 \text{ MPa}$$

$$\tau = 40.5 \text{ MPa} \quad \blacktriangleleft$$

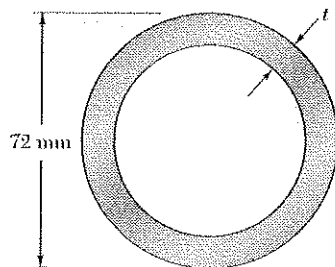
(b) Angle of twist.

$$\phi = \frac{TL}{GJ}$$

$$\phi = \frac{(955)(1.8)}{(77 \times 10^9)(660 \times 10^{-9})} = 0.0338 \text{ rad}$$

$$\phi = 1.94^\circ \quad \blacktriangleleft$$

Problem 3.72



3.72 A steel pipe of 72-mm outer diameter is to be used to transmit a torque of 2500 N · m without exceeding an allowable shearing stress of 55 MPa. A series of 72-mm-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 4 mm to 10 mm in 2-mm increments, choose the lightest pipe that can be used.

$$C_2 = \frac{1}{2} d_o = 36 \text{ mm} = 0.036 \text{ m}$$

$$\tau = \frac{TC_2}{J} = \frac{2TC_2}{\pi(C_2^4 - C_1^4)}$$

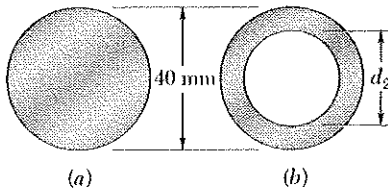
$$C_1^4 = C_2^4 - \frac{2TC_2}{\pi\tau} = 0.036^4 - \frac{(2)(2500)(0.036)}{\pi(55 \times 10^6)} = 637.875 \times 10^{-9}$$

$$C_1 = 28.26 \times 10^{-3} \text{ m} = 28.26 \text{ mm}$$

$$t = C_2 - C_1 = 36 \text{ mm} - 28.26 \text{ mm} = 7.74 \text{ mm}$$

$$\text{Use } t = 8 \text{ mm} \quad \blacktriangleleft$$

Problem 3.73



3.73 The design of a machine element calls for a 40-mm-outer-diameter shaft to transmit 45 kW. (a) If the speed of rotation is 720 rpm, determine the maximum shearing stress in shaft a. (b) If the speed of rotation can be increased 50% to 1080 rpm, determine the largest inner diameter of shaft b for which the maximum shearing stress will be the same in each shaft.

$$(a) \quad f = \frac{720}{60} = 12 \text{ Hz} \quad P = 45 \text{ kW} = 45 \times 10^3 \text{ W}$$

$$T = \frac{P}{2\pi f} = \frac{45 \times 10^3}{2\pi(12)} = 596.83 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 20 \text{ mm} = 0.020 \text{ m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(596.83)}{\pi(0.020)^3} = 47.494 \times 10^6 \text{ Pa} \quad \tau_{\max} = 47.5 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad f = \frac{1080}{60} = 18 \text{ Hz} \quad T = \frac{45 \times 10^3}{2\pi(18)} = 397.89 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)} \quad c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau}$$

$$c_1^4 = 0.020^4 - \frac{(2)(397.89)(0.020)}{\pi(47.494 \times 10^6)} = 53.333 \times 10^{-9} \quad c_1 = 15.20 \times 10^{-3} \text{ m}$$

$$c_1 = 15.20 \times 10^{-3} \text{ m} = 15.20 \text{ mm}$$

$$d_2 = 2c_1 = 30.4 \text{ mm} \quad \blacktriangleleft$$

Problem 3.74

3.74 A 1.5-m-long solid steel shaft of 22-mm diameter is to transmit 12 kW. Determine the minimum frequency at which the shaft can rotate, knowing that $G = 77.2 \text{ GPa}$, that the allowable shearing stress is 30 MPa, and that the angle of twist must not exceed 3.5° .

$$L = 1.5 \text{ m} \quad c = \frac{1}{2}d = 11 \text{ mm} = 0.011 \text{ m} \quad P = 12 \text{ kW} = 12 \times 10^3 \text{ W}$$

$$\phi = 3.5^\circ = 61.087 \times 10^{-3} \text{ rad} \quad G = 77.2 \times 10^9 \text{ Pa} \quad \tau = 30 \times 10^6 \text{ Pa}$$

Stress requirement, $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T = \frac{\pi}{2} \tau c^3 = \frac{\pi}{2} (30 \times 10^6) (0.011)^3 = 62.722 \text{ N}\cdot\text{m}$$

Twist angle requirement, $\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$

$$T = \frac{\pi}{2} \frac{G c^4 \phi}{L} = \frac{\pi (77.2 \times 10^9) (0.011)^4 (61.087 \times 10^{-3})}{(2)(1.5)} = 72.305 \text{ N}\cdot\text{m}$$

Maximum allowable torque is the smaller value $T = 62.722 \text{ N}\cdot\text{m}$

Minimum frequency to transmit a power of 12 kW.

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{12 \times 10^3}{2\pi(62.722)} = 30.4 \text{ Hz}$$

$$f = 30.4 \text{ Hz} \quad \blacktriangleleft$$

Problem 3.75

3.75 A 2.5-m-long solid steel shaft is to transmit 10 kW at a frequency of 25 Hz. Determine the required diameter of the shaft, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 30 MPa, and that the angle of twist must not exceed 4° .

$$P = 10 \text{ kW} = 10 \times 10^3 \text{ W}$$

$$f = 25 \text{ Hz}$$

$$\phi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{2\pi(25)} = 63.662 \text{ N}\cdot\text{m}$$

Stress requirement, $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(63.662)}{\pi(30 \times 10^6)}} = 11.055 \times 10^{-3} \text{ m} = 11.055 \text{ mm}$$

Twist angle requirement, $\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$

$$c = \sqrt[4]{\frac{2TL}{\pi G \phi}} = \sqrt[4]{\frac{(2)(63.662)(2.5)}{\pi(77.2 \times 10^9)(69.813 \times 10^{-3})}} = 11.709 \text{ mm}$$

Use the larger value, $c = 11.709 \text{ mm}$

$$d = 2c = 23.4 \text{ mm} \leftarrow$$

Problem 3.76

3.76 The two solid shafts and gears shown are used to transmit 12 kW from the motor at A operating at a speed of 1260 rpm, to a machine tool at D. Knowing that the maximum allowable shearing stress is 55 MPa, determine the required diameter (a) of shaft AB, (b) of shaft CD.

(a) Shaft AB: $P = 12 \text{ kW}$

$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$\tau = 55 \text{ MPa}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{12 \times 10^3}{2\pi(21)} = 91 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c = \sqrt[3]{\frac{2T}{\pi \tau}}$$

$$c = \sqrt[3]{\frac{(2)(91)}{\pi(55 \times 10^6)}} = 0.01017 \text{ m}$$

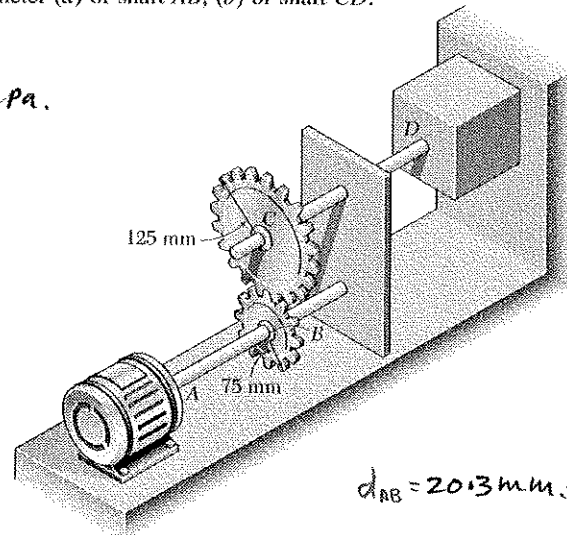
$$d_{AB} = 2c = 20.3 \text{ mm}$$

(b) Shaft CD

$$T_{CD} = \frac{r_A}{r_B} T_{AB} = \frac{5}{3}(91) = 151.7 \text{ N}\cdot\text{m}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(151.7)}{\pi(55 \times 10^6)}} = 0.01206 \text{ m}$$

$$d_{CD} = 2c = 0.024 \text{ m}$$



$$d_{AB} = 20.3 \text{ mm} \leftarrow$$

$$d_{CD} = 24 \text{ mm} \leftarrow$$

Problem 3.77

3.77 The two solid shafts and gears shown are used to transmit 12 kW from the motor at A operating at a speed of 1260 rpm, to a machine tool at D. Knowing that each shaft has a diameter of 25 mm, determine the maximum shearing stress (a) in shaft AB, (b) in shaft CD.

(a) Shaft AB: $P = 12 \text{ kW}$

$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{12 \times 10^3}{2\pi(21)} = 91 \text{ Nm}$$

$$c = \frac{1}{2}d = 12.5 \text{ mm}$$

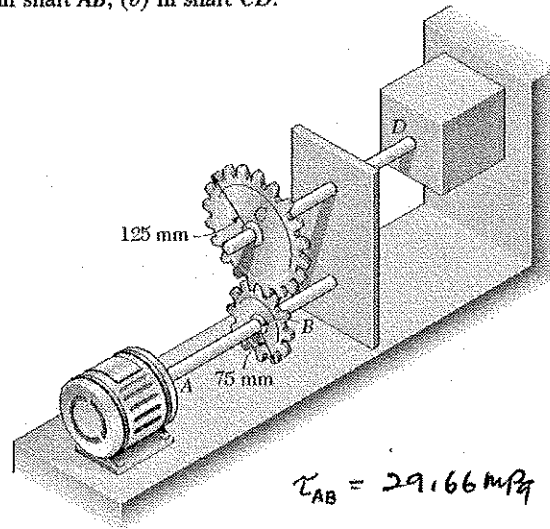
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$= \frac{(2)(91)}{\pi (0.0125)^3} = 29.66 \text{ MPa.}$$

(b) Shaft CD

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(91) = 151.7 \text{ Nm.}$$

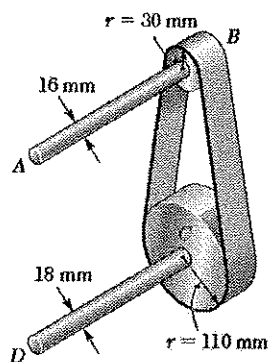
$$\tau = \frac{2T}{\pi c^3} = \frac{(2)(151.7)}{\pi (0.0125)^3} = 49.44 \text{ MPa.}$$



$$\tau_{AB} = 29.66 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{CD} = 49.44 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.78



3.78 The shaft-disk-belt arrangement shown is used to transmit 2 kW from point A to point D. (a) Using an allowable shearing stress of 66 MPa, determine the required speed of shaft AB. (b) Solve part a, assuming that the diameters of shafts AB and CD are, respectively, 18 mm and 15 mm.

SOLUTION

$$\tau = 66 \text{ MPa}, \quad P = 2 \text{ kW/s}$$

$$\tau = \frac{T_C}{J} = \frac{2T}{\pi C^3} \quad T = \frac{\pi}{2} C^3 \tau$$

Allowable torques

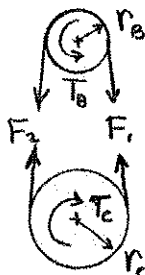
16 mm diameter shaft

$$C = 8 \text{ mm}, \quad T_{\text{all}} = \frac{\pi}{2} (0.008)^3 (66 \times 10^6) = 53 \text{ Nm}$$

18 mm diameter shaft

$$C = 9 \text{ mm}, \quad T_{\text{all}} = \frac{\pi}{2} (0.009)^3 (66 \times 10^6) = 75.6 \text{ Nm}$$

Statics:



$$T_B = r_B (F_1 - F_2) \quad T_C = r_C (F_1 - F_2)$$

$$T_B = \frac{r_B}{r_C} T_C = \frac{30}{110} T_C = 0.2727 T_C$$

(a) Allowable torques

$$T_{B,\text{all}} = 53 \text{ Nm}, \quad T_{C,\text{all}} = 75.6 \text{ Nm}$$

Assume $T_C = 75.6 \text{ Nm}$

$$\text{Then } T_B = (0.2727)(75.6) = 20.6 \text{ Nm}$$

$< 53 \text{ Nm}$ (okay)

$$P = 2\pi f T \quad f_{AB} = \frac{P}{2\pi T_B} = \frac{2000}{2\pi(20.6)}$$

$$f_{AB} = 15.45 \text{ Hz}$$

(b) Allowable torques

$$T_{B,\text{all}} = 75.6 \text{ Nm}, \quad T_{C,\text{all}} = 53 \text{ Nm}$$

Assume $T_C = 53 \text{ Nm}$

$$\text{Then } T_B = (0.2727)(53) = 14.5 \text{ Nm}$$

$< 75.6 \text{ Nm}$

$$P = 2\pi f T \quad f_{AB} = \frac{P}{2\pi T_B} = \frac{2000}{2\pi(14.5)}$$

$$f_{AB} = 21.95 \text{ Hz}$$

Problem 3.79

3.79 A 1.5-m-long solid steel shaft of 22-mm diameter is to transmit 13.5 kW. Determine the minimum speed at which the shaft can rotate, knowing that $G = 77$ GPa, that the allowable shearing stress is 30 MPa, and that the angle of twist must not exceed 3.5° .

$$L = 1.5 \text{ m} \quad c = \frac{1}{2}d = 11 \text{ mm} \quad P = 13.5 \text{ kW}$$

Stress requirement $\tau = 30 \text{ MPa}$ $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T = \frac{\pi}{2} \tau c^3 = \frac{\pi}{2} (30 \times 10^6) (0.011)^3 = 62.7 \text{ Nm}$$

Twist angle requirement $\phi = 3.5^\circ = 61.087 \times 10^{-3} \text{ rad}$ $\phi = \frac{TL}{GJ}$

$$T = \frac{GJ\phi}{L} = \frac{\pi G c^4 \phi}{2L} = \frac{\pi (77 \times 10^9) (0.011)^4 (61.087 \times 10^{-3})}{(2)(1.5)} = 7201 \text{ Nm}$$

Maximum allowable torque is the smaller value $T = 62.7 \text{ Nm}$

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{13.5 \times 10^3}{2\pi (62.7)} = 34.27 \text{ Hz}$$

$$f = 34.3 \text{ Hz} \quad \blacktriangleleft$$

$$= 2,058 \text{ rpm}$$

Problem 3.80

3.80 A 2.5-m-long steel shaft of 30-mm diameter rotates at a frequency of 30 Hz. Determine the maximum power that the shaft can transmit, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 50 MPa, and that the angle of twist must not exceed 7.5° .

$$c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m} \quad L = 2.5 \text{ m}$$

Stress requirement $\tau = 50 \times 10^6 \text{ Pa}$ $\tau = \frac{Tc}{J}$

$$T = \frac{\tau J}{c} = \frac{\pi}{2} \tau c^3 = \frac{\pi}{2} (50 \times 10^6) (0.015)^3 = 265.07 \text{ N}\cdot\text{m}$$

Twist angle requirement $\phi = 7.5^\circ = 130.90 \times 10^{-3} \text{ rad}$ $G = 77.2 \times 10^9 \text{ Pa}$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi}{2} G c^4 \phi = \frac{\pi}{2} (77.2 \times 10^9) (0.015)^4 (130.90 \times 10^{-3}) = 803.60 \text{ N}\cdot\text{m}$$

Smaller value of T is the maximum allowable torque. $T = 265.07 \text{ N}\cdot\text{m}$

Power transmitted at $f = 30 \text{ Hz}$

$$P = 2\pi f T = 2\pi (30) (265.07) = 49.96 \times 10^3 \text{ W}$$

$$P = 50.0 \text{ kW} \quad \blacktriangleleft$$

Problem 3.81

3.81 A steel shaft must transmit 150 kW at speed of 360 rpm. Knowing that $G = 77.2$ GPa, design a solid shaft so that the maximum shearing stress will not exceed 50 MPa and the angle of twist in a 2.5-m length must not exceed 3° .

$$P = 150 \text{ kW} = 150 \times 10^3 \text{ W}$$

$$f = \frac{360}{60} = 6 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{150 \times 10^3}{2\pi (6)} = 3.9789 \times 10^3 \text{ N}\cdot\text{m}$$

Stress requirement, $\tau = 50 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{2(3.9789 \times 10^3)}{\pi (50 \times 10^6)}} = 37.00 \times 10^{-3} \text{ m} = 37.00 \text{ mm}$$

Angle of twist requirement, $\phi = 3^\circ = 52.36 \times 10^{-3} \text{ rad}$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

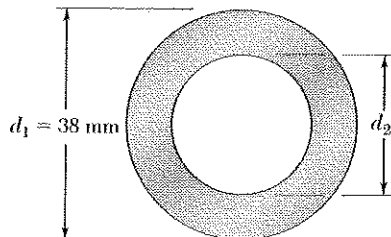
$$G = 77.2 \times 10^9 \text{ Pa}, L = 2.5 \text{ m}$$

$$c = \sqrt[4]{\frac{2TL}{\pi G \phi}} = \sqrt[4]{\frac{2(3.9789 \times 10^3)(2.5)}{\pi (77.2 \times 10^9)(52.36 \times 10^{-3})}} = 35.38 \times 10^{-3} \text{ m} = 35.38 \text{ mm}$$

Use larger value, $c = 37.00 \text{ mm}$

$$d = 2c = 74.0 \text{ mm}$$

Problem 3.82



3.82 A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 100 kW between a turbine and a generator. Determine the minimum frequency at which the shaft can rotate, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 60 MPa, and that the angle of twist must not exceed 3° .

$$L = 1.5 \text{ m}, \phi = 3^\circ = 52.360 \times 10^{-3} \text{ rad}$$

$$c_2 = \frac{1}{2} d_1 = 19 \text{ mm} = 0.019 \text{ m}, c_1 = \frac{1}{2} d_2 = 15 \text{ mm} = 0.015 \text{ m}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (0.019^4 - 0.015^4) = 125.186 \times 10^{-9} \text{ m}^4$$

Stress requirement, $\tau = 60 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc_2}{J}$$

$$T = \frac{J\tau}{c_2} = \frac{(125.186 \times 10^{-9})(60 \times 10^6)}{0.019} = 395.32 \text{ N}\cdot\text{m}$$

Twist angle requirement, $\phi = \frac{TL}{GJ}$

$$T = \frac{GJ\phi}{L} = \frac{(77.2 \times 10^9)(125.186 \times 10^{-9})(52.360 \times 10^{-3})}{1.5} = 337.35 \text{ N}\cdot\text{m}$$

Maximum allowable torque is the smaller value, $T = 337.35 \text{ N}\cdot\text{m}$

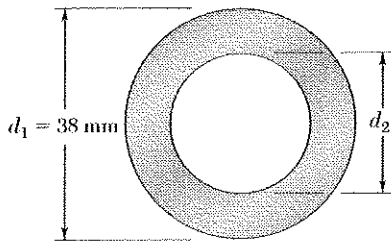
Power transmitted $P = 100 \text{ kW} = 100 \times 10^3 \text{ W}$

$$P = 2\pi f T$$

Frequency $f = \frac{P}{2\pi T} = \frac{100 \times 10^3}{2\pi (337.35)} = 47.2 \text{ Hz}$

$$f = 47.2 \text{ Hz}$$

Problem 3.83



3.83 A 1.5-m-long tubular steel shaft of 38-mm outer diameter d_1 is to be made of a steel for which $\tau_{all} = 65 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. Knowing that the angle of twist must not exceed 4° when the shaft is subjected to a torque of $600 \text{ N} \cdot \text{m}$, determine the largest inner diameter d_2 that can be specified in the design.

$$L = 1.5 \text{ m} \quad c_2 = \frac{1}{2} d_2 = 19 \text{ mm} = 0.019 \text{ m}$$

$$\tau = 65 \times 10^6 \text{ Pa} \quad \phi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4)$$

Stress requirement,

$$\tau = \frac{T c_2}{J} = \frac{2 T c_2}{\pi (c_2^4 - c_1^4)}$$

$$c_1 = \sqrt[4]{c_2^4 - \frac{2 T c_2}{\pi \tau}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(0.019)}{\pi (65 \times 10^6)}} = 11.689 \times 10^{-3} \text{ m} = 11.689 \text{ mm}$$

Twist angle requirement,

$$\phi = \frac{T L}{G J} = \frac{2 T L}{\pi G (c_2^4 - c_1^4)}$$

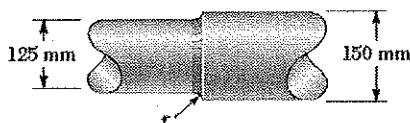
$$c_1 = \sqrt[4]{c_2^4 - \frac{2 T L}{\pi G \phi}} = \sqrt[4]{0.019^4 - \frac{(2)(600)(1.5)}{\pi (77.2 \times 10^9)(69.813 \times 10^{-3})}}$$

$$c_1 = 12.448 \times 10^{-3} \text{ m} = 12.448 \text{ mm}$$

Use smaller value of c_1 , $c_1 = 11.689 \text{ mm}$

$$d_2 = 2 c_1 = 23.4 \text{ mm} \quad \blacktriangleleft$$

Problem 3.84



3.84 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 12 \text{ mm}$, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 50 MPa .

$$d = 125 \text{ mm} \quad D = 150 \text{ mm} \quad r = 12 \text{ mm}$$

$$\frac{D}{d} = \frac{150}{125} = 1.20 \quad \frac{r}{d} = \frac{12}{125} = 0.096 \quad \text{From Fig 3.32 } K = 1.35$$

For the smaller side $c = \frac{1}{2} d = 62.5 \text{ mm}$

$$\tau = \frac{K T c}{J} = \frac{2 K T}{\pi c^3}$$

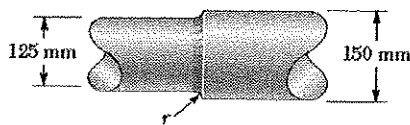
$$T = \frac{\pi c^3 \tau}{2 K} = \frac{\pi (0.0625)^3 (50 \times 10^6)}{(2)(1.35)} = 14.2 \text{ kNm}$$

$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

$$\text{Power } P = 2 \pi f T = 2 \pi (7.5)(14.2 \times 10^3) = 669 \text{ kW/s}$$

$$P = 669 \text{ kW} \quad \blacktriangleleft$$

Problem 3.85



3.85 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 5$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 50 MPa.

$$d = 125 \text{ mm} \quad D = 150 \text{ mm} \quad r = 5 \text{ mm}$$

$$\frac{D}{d} = \frac{150}{125} = 1.20 \quad \frac{r}{d} = \frac{5}{125} = 0.04 \quad \text{From Fig. 3.32} \quad K = 1.55$$

$$\text{For smaller side} \quad c = \frac{1}{2}d = 62.5 \text{ mm} \quad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

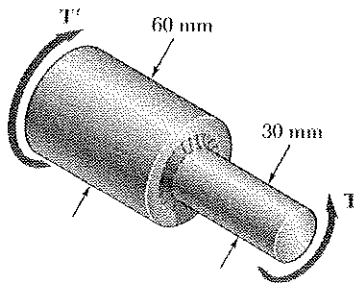
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.0625)^3 (50 \times 10^6)}{(2)(1.55)} = 12.4 \text{ kNm}$$

$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

$$\text{Power } P = 2\pi f T = 2\pi (7.5)(12.4 \times 10^3) = 584 \text{ kW/s}$$

$$P = 584 \text{ kW}$$

Problem 3.86



3.86 The stepped shaft shown must rotate at a frequency of 50 Hz. Knowing that the radius of the fillet is $r = 8$ mm and the allowable shearing stress is 45 MPa, determine the maximum power that can be transmitted.

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} \quad T = \frac{\pi c^3 \tau}{2K}$$

$$d = 30 \text{ mm} \quad c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$D = 60 \text{ mm}, \quad r = 8 \text{ mm}$$

$$\frac{D}{d} = \frac{60}{30} = 2, \quad \frac{r}{d} = \frac{8}{30} = 0.26667$$

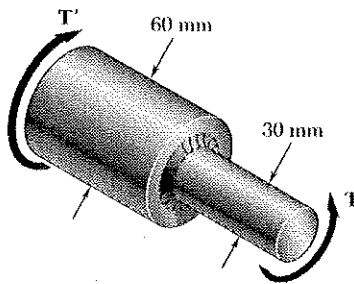
$$\text{From Fig. 3.32} \quad K = 1.18$$

$$\text{Allowable torque.} \quad T = \frac{\pi (15 \times 10^{-3})^3 (45 \times 10^6)}{(2)(1.18)} = 202.17 \text{ N}\cdot\text{m}$$

$$\text{Maximum power.} \quad P = 2\pi f T = (2\pi)(50)(202.17) = 63.5 \times 10^3 \text{ W}$$

$$P = 63.5 \text{ kW}$$

Problem 3.87



3.87 Knowing that the stepped shaft shown must transmit 45 kW at a speed of 2100 rpm, determine the minimum radius r of the fillet if an allowable shearing stress of 50 MPa is not to be exceeded.

$$f = \frac{2100}{60} = 35 \text{ Hz}$$

$$P = 45 \times 10^3 \text{ W}$$

$$T = \frac{P}{2\pi f} = \frac{45 \times 10^3}{2\pi (35)} = 204.63 \text{ N}\cdot\text{m}$$

For smaller side $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$ $\gamma = \frac{K T c}{J} = \frac{2 K T}{\pi c^3}$

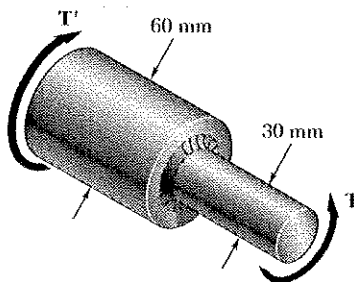
$$K = \frac{\pi \gamma c^3}{2 T} = \frac{\pi (50 \times 10^6) (0.015)^3}{(2)(204.63)} = 1.295$$

$$\frac{D}{d} = \frac{60}{30} = 2 \quad \text{From Fig. 3.32} \quad \frac{r}{d} = 0.17$$

$$r = 0.17 d = (0.17)(30) = 5.1 \text{ mm}$$

$$r = 5.1 \text{ mm} \blacktriangleleft$$

Problem 3.88



3.88 The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6 \text{ mm}$, determine the smallest permissible speed of the shaft.

$$\frac{r}{d} = \frac{6}{30} = 0.2 \quad \frac{D}{d} = \frac{60}{30} = 2$$

$$\text{From Fig. 3.32} \quad K = 1.26$$

For smaller side $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$

$$\gamma = \frac{K T c}{J} = \frac{2 K T}{\pi c^3}$$

$$T = \frac{\pi c^3 \gamma}{2 K} = \frac{\pi (0.015)^3 (40 \times 10^6)}{(2)(1.26)} = 168.30 \times 10^3 \text{ N}\cdot\text{m}$$

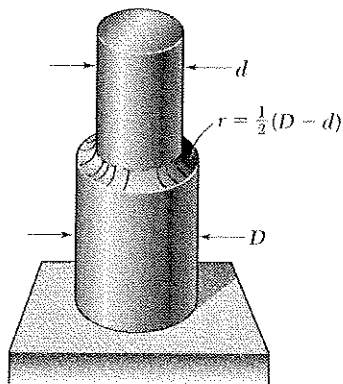
$$P = 45 \text{ kW} = 45 \times 10^3$$

$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{45 \times 10^3}{2\pi (168.30 \times 10^3)} = 42.6 \text{ Hz}$$

$$f = 42.6 \text{ Hz} \blacktriangleleft$$

Problem 3.89



Full quarter-circular fillet extends to edge of larger shaft

3.89 In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 80 MPa. Knowing that $D = 30$ mm, determine the largest allowable torque that can be applied to the shaft if (a) $d = 26$ mm, (b) $d = 24$ mm.

$$\tau = 80 \times 10^6 \text{ Pa}$$

$$(a) \quad \frac{D}{d} = \frac{30}{26} = 1.154 \quad r = \frac{1}{2}(D - d) = 2 \text{ mm}$$

$$\frac{r}{d} = \frac{2}{26} = 0.0768$$

From Fig. 3.32, $K = 1.36$

$$\text{Smaller side, } c = \frac{1}{2}d = 13 \text{ mm} = 0.013 \text{ m}$$

$$\tau = \frac{K T c}{J} = \frac{2 K T}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2 K} = \frac{\pi (0.013)^3 (80 \times 10^6)}{(2)(1.36)}$$

$$= 203 \text{ N}\cdot\text{m}$$

$$T = 203 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$(b) \quad \frac{D}{d} = \frac{30}{24} = 1.25 \quad r = \frac{1}{2}(D - d) = 3 \text{ mm} \quad \frac{r}{d} = \frac{3}{24} = 0.125$$

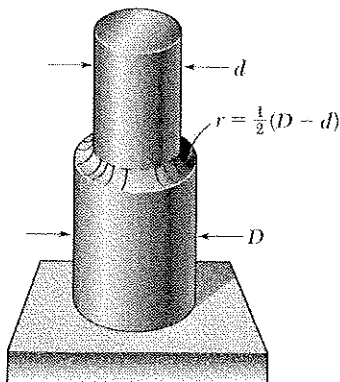
From Fig. 3.32, $K = 1.31$

$$c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$$

$$T = \frac{\pi c^3 \tau}{2 K} = \frac{\pi (0.012)^3 (80 \times 10^6)}{(2)(1.31)} = 165.8 \text{ N}\cdot\text{m}$$

$$T = 165.8 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 3.90



Full quarter-circular fillet extends to edge of larger shaft

3.90 In the stepped shaft shown, which has a full quarter-circular fillet, $D = 30$ mm and $d = 25$ mm. Knowing that the speed of the shaft is 2400 rpm and that the allowable shearing stress is 50 MPa, determine the maximum power that can be transmitted by the shaft.

$$\frac{D}{d} = \frac{30}{25} = 1.2 \quad r = \frac{1}{2}(D - d) = 2.5 \text{ mm}$$

$$\frac{r}{d} = \frac{2.5}{25} = 0.1$$

From Fig. 3.32 $K = 1.34$

$$\text{For smaller side } c = \frac{1}{2}d = 12.5 \text{ mm}$$

$$\tau = \frac{K T c}{J}$$

$$T = \frac{J \tau}{K c} = \frac{\pi c^3 \tau}{2 K}$$

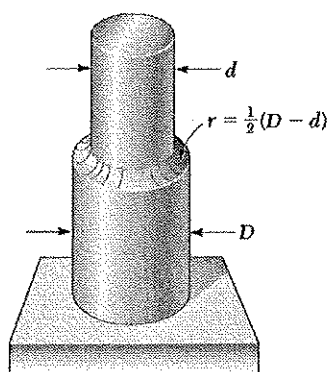
$$T = \frac{\pi (0.0125)^3 (50 \times 10^6)}{(2)(1.34)} = 114.5 \text{ N}\cdot\text{m}$$

$$f = 2400 \text{ rpm} = 40 \text{ Hz}$$

$$P = 2\pi f T = 2\pi(40)(114.5) = 28.8 \text{ kW}$$

$$P = 28.8 \text{ kW} \quad \blacktriangleleft$$

Problem 3.91



Full quarter-circular fillet extends to edge of larger shaft

3.91 A torque of magnitude $T = 22 \text{ N} \cdot \text{m}$ is applied to the stepped shaft shown, which has a full quarter-circular fillet. Knowing that $D = 25 \text{ mm}$, determine the maximum shearing stress in the shaft when (a) $d = 20 \text{ mm}$, (b) $d = 23 \text{ mm}$.

$$(a) \quad \frac{D}{d} = \frac{25}{20} = 1.25 \quad r = \frac{1}{2}(D - d) = 2.5 \text{ mm}$$

$$\frac{r}{d} = \frac{2.5}{20} = 0.125$$

$$\text{From Fig. 3.32} \quad K = 1.31$$

$$\text{For smaller side} \quad c = \frac{1}{2}d = 10 \text{ mm}$$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} = \frac{(2)(1.31)(22)}{\pi (0.01)^3} = 18.3 \text{ MPa}$$

$$\tau = 18.3 \text{ MPa}$$

$$(b) \quad \frac{D}{d} = \frac{25}{23} = 1.09 \quad r = \frac{1}{2}(D - d) = 1 \text{ mm}$$

$$\frac{r}{d} = \frac{1}{23} = 0.0435$$

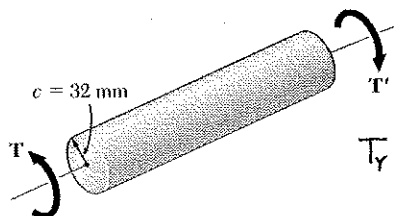
$$\text{From Fig. 3.32} \quad K = 1.48$$

$$\text{For smaller side} \quad c = \frac{1}{2}d = 11.5 \text{ mm}$$

$$\tau = \frac{2KT}{\pi c^3} = \frac{(2)(1.48)(22)}{\pi (11.5 \times 10^{-3})^3} = 13.6 \text{ MPa}$$

$$\tau = 13.6 \text{ MPa}$$

Problem 3.92



3.92 The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$. Determine the magnitude T of the applied torque when the plastic zone is (a) 16 mm deep, (b) 24 mm deep.

$$c = 32 \text{ mm} = 0.032 \text{ m}$$

$$\tau_Y = 145 \times 10^6 \text{ Pa}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2} (0.032)^3 (145 \times 10^6) = 7.4634 \times 10^3 \text{ N} \cdot \text{m}$$

$$(a) \quad t_p = 16 \text{ mm} = 0.016 \text{ m} \quad p_Y = c - t_p = 0.032 - 0.016 = 0.016 \text{ m}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{p_Y^3}{c^3} \right) = \frac{4}{3} (7.4634 \times 10^3) \left(1 - \frac{1}{4} \frac{0.016^3}{0.032^3} \right)$$

$$= 9.6402 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 9.64 \text{ kN} \cdot \text{m}$$

$$(b) \quad t_p = 24 \text{ mm} = 0.024 \text{ m} \quad p_Y = c - t_p = 0.032 - 0.024 = 0.008 \text{ m}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{p_Y^3}{c^3} \right) = \frac{4}{3} (7.4634 \times 10^3) \left(1 - \frac{1}{4} \frac{0.008^3}{0.032^3} \right)$$

$$= 9.9123 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 9.91 \text{ kN} \cdot \text{m}$$

Problem 3.93

3.93 A 30-mm-diameter solid rod is made of an elastoplastic material with $\tau_Y = 35$ MPa. Knowing that the elastic core of the rod is of diameter 25 mm, determine the magnitude of the torque applied to the rod.

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$\tau_Y = 35 \text{ MPa}$$

$$\rho_Y = \frac{1}{2}d_Y = 0.0125 \text{ m}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.015)^3(35 \times 10^6) = 185.6 \text{ N}\cdot\text{m}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(185.6)\left(1 - \frac{1}{4}\frac{0.0125^3}{0.015^3}\right) = 211.7 \text{ N}\cdot\text{m}$$

$$T = 211.7 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 3.94

3.94 A 50-mm-diameter solid shaft is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 140$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) 3 kN·m, (b) 4 kN·m.

$$c = \frac{1}{2}d = 25 \text{ mm}$$

$$\tau_Y = 140 \text{ MPa}$$

$$\text{Compute } T_Y \quad T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.025)^3(140 \times 10^6) = 3436 \text{ N}\cdot\text{m}$$

$$(a) \quad T = 3000 < T_Y \quad \text{elastic}$$

$$\rho = c = 25 \text{ mm} \quad \blacktriangleleft$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(3000)}{\pi(0.025)^3} = 122.2 \text{ MPa}$$

$$\tau_{\max} = 122.2 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad T = 4000 > T_Y \quad \text{plastic region with elastic core}$$

The maximum shearing stress is

$$\tau_{\max} = \tau_Y = 140 \text{ MPa} \quad \blacktriangleleft$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right)$$

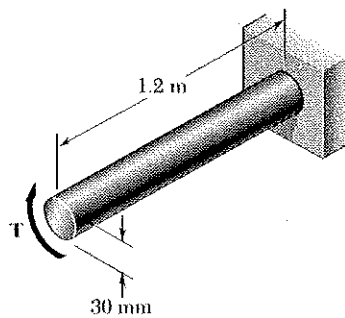
$$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(4000)}{3436} = 0.5076$$

$$\frac{\rho_Y}{c} = 0.8$$

$$\rho_Y = 0.8 = (0.8)(25)$$

$$\rho_Y = 20 \text{ mm} \quad \blacktriangleleft$$

Problem 3.95



3.95 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2$ GPa and $\tau_y = 145$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) $T = 600$ N·m, (b) $T = 1000$ N·m.

$$c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

at onset of yielding. $\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$\frac{\pi c^3 \tau_y}{2} = \frac{\pi (15 \times 10^{-3})^3 (145 \times 10^6)}{2} = 768.71 \text{ N·m}$$

(a) $T = 600$ N·m. Since $T < T_y$, the shaft is elastic.

Maximum shearing stress. $\frac{\tau_m}{\tau_y} = \frac{T}{T_y}$

$$\tau_m = \frac{\tau_y T}{T_y} = \frac{(145 \times 10^6)(600)}{768.71} = 113.3 \times 10^6 \text{ Pa} \quad \tau_m = 113.3 \text{ MPa} \leftarrow$$

Radius of elastic core $\rho = c \quad \rho_E = 15.00 \text{ mm} \leftarrow$

(b) $T = 1000$ N·m. Since $T > T_y$, there is a plastic zone.

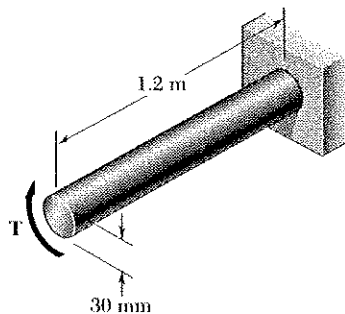
Maximum shearing stress. $\tau_m = \tau_y \quad \tau_m = 145.0 \text{ MPa} \leftarrow$

Radius of elastic core. $T = \frac{4}{3} T_y \left[1 - \left(\frac{\rho_y}{c} \right)^3 \right]$

$$\frac{\rho_y}{c} = \sqrt[3]{4 - 3 \frac{T}{T_y}} = \sqrt[3]{4 - \frac{(3)(1000)}{768.71}} = 0.46003$$

$$\rho_y = 0.46003 c = (0.46003)(15 \text{ mm}) \quad \rho_y = 6.90 \text{ mm} \leftarrow$$

Problem 3.96



3.96 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $\tau_y = 145$ MPa. Determine the radius of the elastic core caused by the application of a torque equal to $1.1 T_y$, where T_y is the magnitude of the torque at the onset of yield.

$$c = \frac{1}{2}d = 15 \text{ mm} \quad T = \frac{4}{3} T_y \left[1 - \left(\frac{\rho_y}{c} \right)^3 \right]$$

$$\frac{\rho_y}{c} = \sqrt[3]{4 - 3 \frac{T}{T_y}} = \sqrt[3]{4 - (3)(1.1)} = 0.88790$$

$$\rho_y = 0.88790 c = (0.88790)(15 \text{ mm}) \quad \rho_y = 13.32 \text{ mm} \leftarrow$$

Problem 3.97

3.97 It is observed that a straightened paper clip can be twisted through several revolutions by the application of a torque of approximately $60 \text{ mN} \cdot \text{m}$. Knowing that the diameter of the wire in the paper clip is 0.9 mm , determine the approximate value of the yield stress of the steel.

$$c = \frac{1}{2}d = 0.45 \text{ mm} = 0.45 \times 10^{-3} \text{ m}$$

$$T_P = 60 \text{ mN} \cdot \text{m} = 60 \times 10^{-3} \text{ N} \cdot \text{m}$$

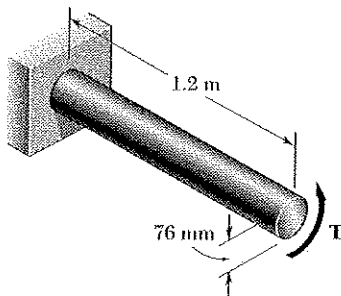
$$T_P = \frac{4}{3}T_Y = \frac{4}{3} \frac{J\tau_Y}{c} = \frac{4}{3} \cdot \frac{\pi}{2} c^3 T_Y = \frac{2\pi}{3} c^3 \tau_Y$$

$$\tau_Y = \frac{3T_P}{2\pi c^3} = \frac{(3)(60 \times 10^{-3})}{2\pi (0.45 \times 10^{-3})^3} = 314 \times 10^6 \text{ Pa}$$

$$\tau_Y = 314 \text{ MPa}$$

Problem 3.98

3.98 The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$ and $G = 77 \text{ GPa}$. Determine the angle of twist caused by the application of a torque of magnitude (a) $T = 8 \text{ kN} \cdot \text{m}$, (b) $T = 13 \text{ kN} \cdot \text{m}$.



$$c = \frac{1}{2}d = \frac{1}{2}(0.076) = 0.038 \text{ m} \quad \tau_Y = 145 \text{ MPa}$$

$$L = 1.2 \text{ m}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.038)^4 = 3275 \times 10^{-9} \text{ m}^4$$

$$\text{Torque at onset of yielding} \quad \tau = \frac{Tc}{J} \quad T = \frac{\tau J}{c}$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(145 \times 10^6)(3275 \times 10^{-9})}{0.038} = 15.829 \text{ kNm}$$

$$(a) \quad T = 8 \times 10^3 \text{ N} \cdot \text{m}$$

Since $T < T_Y$, the shaft is fully elastic.

$$\phi = \frac{TL}{GJ}$$

$$\phi = \frac{(8 \times 10^3)(1.2)}{(77 \times 10^9)(3275 \times 10^{-9})} = 38.069 \times 10^{-3} \text{ rad}$$

$$\phi = 2.18^\circ$$

$$(b) \quad T = 13 \times 10^3 \text{ N} \cdot \text{m}$$

$$T > T_Y$$

$$T = \frac{4}{3}T_Y \left[1 - \left(\frac{\phi_Y}{\phi} \right)^3 \right]$$

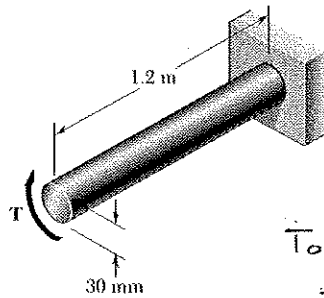
$$\phi_Y = \frac{T_Y L}{GJ} = \frac{(15.829 \times 10^3)(1.2)}{(77 \times 10^9)(3275 \times 10^{-9})} = 75.324 \times 10^{-3} \text{ rad}$$

$$\frac{\phi_Y}{\phi} = \sqrt[3]{4 - 3 \frac{T}{T_Y}} = \sqrt[3]{4 - \frac{(3)(13 \times 10^3)}{15.829 \times 10^3}} = 1.1538$$

$$\phi = \frac{\phi_Y}{1.1538} = \frac{75.324 \times 10^{-3}}{1.1538} = 65.283 \times 10^{-3} \text{ rad}$$

$$\phi = 3.74^\circ$$

Problem 3.99



3.99 For the solid circular shaft of Prob. 3.95, determine the angle of twist caused by the application of a torque of magnitude (a) $T = 600 \text{ N} \cdot \text{m}$, (b) $T = 1000 \text{ N} \cdot \text{m}$.

3.95 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $\tau_y = 145 \text{ MPa}$. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) $T = 600 \text{ N} \cdot \text{m}$, (b) $T = 1000 \text{ N} \cdot \text{m}$. ($G = 77.2 \text{ GPa}$).

$$c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

Torque at onset of yielding. $\phi = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T_y = \frac{\pi c^3 \tau_y}{2} = \frac{\pi (15 \times 10^{-3})^3 (145 \times 10^6)}{2} = 768.71 \text{ N} \cdot \text{m}$$

(a) $T = 600 \text{ N} \cdot \text{m}$. Since $T \leq T_y$ the shaft is elastic.

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G} = \frac{(2)(600)(1.2)}{\pi (15 \times 10^{-3})^4 (77.2 \times 10^9)} = 0.11728 \text{ rad} \quad \phi = 6.72^\circ$$

(b) $T = 1000 \text{ N} \cdot \text{m}$. $T > T_y$. A plastic zone has developed.

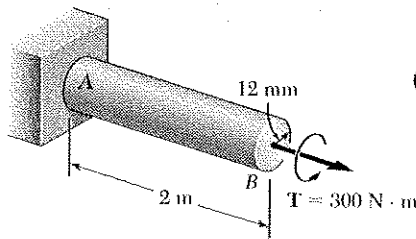
$$T = \frac{4}{3} T_y \left[1 - \left(\frac{\phi_y}{\phi} \right)^3 \right] \quad \frac{\phi_y}{\phi} = \sqrt[3]{4 - 3 \left(\frac{T}{T_y} \right)}$$

$$\phi_y = \frac{T_y L}{GJ} = \frac{2T_y L}{\pi c^4 G} = \frac{(2)(768.71)(1.2)}{\pi (15 \times 10^{-3})^4 (77.2 \times 10^9)} = 0.15026 \text{ rad}$$

$$\frac{\phi_y}{\phi} = \sqrt[3]{4 - \frac{(3)(1000)}{768.71}} = 0.46003$$

$$\phi = \frac{\phi_y}{0.46003} = \frac{0.15026}{0.46003} = 0.32663 \text{ rad} \quad \phi = 18.71^\circ$$

Problem 3.100



3.100 The shaft AB is made of a material that is elastoplastic with $\tau_Y = 90$ MPa and $G = 30$ GPa. For the loading shown, determine (a) the radius of the elastic core of the shaft, (b) the angle of twist at end B .

(a) $c = 12 \text{ mm} = 0.012 \text{ m}$

$\tau_Y = 90 \times 10^6 \text{ Pa}$

$$T_Y = \frac{J \tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y$$

$$= \frac{\pi}{2} (0.012)^3 (90 \times 10^6) = 244.29 \text{ N}\cdot\text{m}$$

$T = 300 \text{ N}\cdot\text{m} > T_Y$

plastic region with elastic core

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_Y^3}{c^3}\right) \quad \frac{\rho_Y^3}{c^3} = 1 - \frac{3T}{T_Y} = 1 - \frac{(3)(300)}{244.29} = 0.31585$$

$$\frac{\rho_Y}{c} = 0.68102 \quad \rho_Y = (0.68102)(0.012) = 8.17 \times 10^{-3} \text{ m} \quad \rho_Y = 8.17 \text{ mm} \quad \blacktriangleleft$$

(b) $L = 2 \text{ m} \quad G = 30 \times 10^9 \text{ Pa}$

$$\phi_Y = \frac{T_Y L}{J G} = \frac{2 T_Y L}{\pi c^4 G} = \frac{(2)(244.29)(2)}{\pi (0.012)^4 (30 \times 10^9)} = 0.5000 \text{ rad}$$

$$\frac{\phi_Y}{\phi} = \frac{\rho_Y}{c} \quad \phi = \frac{\phi_Y}{\rho_Y/c} = \frac{0.5000}{0.68102} = 0.734 \text{ rad} \quad \phi = 42.1^\circ \quad \blacktriangleleft$$

Problem 3.101

3.101 A 30-mm-diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_Y = 125$ MPa and $G = 77$ GPa. For an 2.4-m length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 850 N·m torque.

$c = \frac{1}{2} d = 15 \text{ mm}, \quad G = 77 \text{ GPa}, \quad \tau_Y = 125 \text{ MPa}$

$L = 2.4 \text{ m} \quad T = 850 \text{ N}\cdot\text{m}$

$$T_Y = \frac{J \tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2} (0.015)^3 (125 \times 10^6) = 662.7 \text{ N}\cdot\text{m}$$

$T > T_Y$ plastic region with elastic core $\therefore \tau_{\max} = \tau_Y = 125 \text{ MPa} \quad \blacktriangleleft$

$$\tau_Y = \frac{c \phi_Y}{L} \quad \therefore \phi_Y = \frac{L \tau_Y}{c G} = \frac{(2.4)(125 \times 10^6)}{(0.015)(77 \times 10^9)} = 0.26 \text{ rad}$$

$$T = T_Y \left(1 - \frac{1}{4} \frac{\phi^3}{\phi_Y^3}\right)$$

$$\frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}} = \frac{1}{\sqrt[3]{4 - \frac{(3)(850)}{662.7}}} = 1.873$$

$$\phi = 1.873 \phi_Y = (1.873)(0.26) = 0.487 \text{ rad} \quad \phi = 27.9^\circ \quad \blacktriangleleft$$

Problem 3.102

3.102 A 18-mm-diameter solid circular shaft is made of material that is assumed to be elastoplastic with $\tau_Y = 140$ MPa and $G = 77$ GPa. For a 1.2-m length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 200 N·m torque.

$$\tau_Y = 140 \text{ MPa}, \quad c = \frac{d}{2} = 9 \text{ mm}, \quad L = 1.2 \text{ m}, \quad T = 200 \text{ N}\cdot\text{m}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2} (0.009)^3 (140 \times 10^6) = 160 \text{ N}\cdot\text{m}$$

$$T > T_Y \quad \text{Plastic region with elastic core} \quad \tau_{\max} = \tau_Y = 140 \text{ MPa} \quad \blacktriangleleft$$

$$\phi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{2(160)(1.2)}{\pi (0.009)^4 (77 \times 10^9)} = 0.242 \text{ rad}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\phi^3}{\phi_Y^3}\right)$$

$$\left(\frac{\phi_Y}{\phi}\right)^3 = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(200)}{160} = 0.25 \quad \frac{\phi_Y}{\phi} = 0.63$$

$$\phi = \frac{\phi_Y}{0.63} = \frac{0.242}{0.63} = 0.384 \text{ rad} \quad \phi_Y = 22^\circ \quad \blacktriangleleft$$

Problem 3.103

3.103 A solid circular rod is made of a material that is assumed to be elastoplastic. Denoting by T_Y and ϕ_Y , respectively, the torque and the angle of twist at the onset of yield, determine the angle of twist if the torque is increased to (a) $T = 1.1 T_Y$, (b) $T = 1.25 T_Y$, (c) $T = 1.3 T_Y$.

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\phi^3}{\phi_Y^3}\right)$$

$$\frac{\phi_Y}{\phi} = \sqrt[3]{4 - \frac{3T}{T_Y}} \quad \text{or} \quad \frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}}$$

$$(a) \quad \frac{T}{T_Y} = 1.10 \quad \frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - (3)(1.10)}} = 1.126 \quad \phi = 1.126 \phi_Y \quad \blacktriangleleft$$

$$(b) \quad \frac{T}{T_Y} = 1.25 \quad \frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - (3)(1.25)}} = 1.587 \quad \phi = 1.587 \phi_Y \quad \blacktriangleleft$$

$$(c) \quad \frac{T}{T_Y} = 1.3 \quad \frac{\phi}{\phi_Y} = \frac{1}{\sqrt[3]{4 - (3)(1.3)}} = 2.15 \quad \phi = 2.15 \phi_Y \quad \blacktriangleleft$$

Problem 3.104

3.104 A 0.9-m-long solid shaft has a diameter of 62 mm and is made of a mild steel that is assumed to be elastoplastic with $\tau_y = 147$ MPa and $G = 77$ GPa. Determine the torque required to twist the shaft through an angle of (a) 2.5° , (b) 5° .

$$L = 0.9 \text{ m}$$

$$C = \frac{1}{2}d = 0.031 \text{ m}$$

$$\tau_y = 147 \text{ MPa}$$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.031)^4 = 1.45 \times 10^{-6} \text{ m}^4$$

$$\tau_y = \frac{T_y C}{J}$$

$$T_y = \frac{J \tau_y}{C} = \frac{(1.45 \times 10^{-6})(147 \times 10^6)}{0.031} = 6.876 \times 10^3 \text{ N.m}$$

$$\phi_y = \frac{T_y L}{G J} = \frac{(6.876 \times 10^3)(0.9)}{(77 \times 10^9)(1.45 \times 10^{-6})} = 55.427 \times 10^{-3} \text{ rad} = 3.176^\circ$$

(a) $\phi = 2.5^\circ = 43.633 \times 10^{-3} \text{ rad}$ $\phi < \phi_y$ The shaft remains elastic.

$$\phi = \frac{T L}{G J}$$

$$T = \frac{G J \phi}{L} = \frac{(77 \times 10^9)(1.45 \times 10^{-6})(43.633 \times 10^{-3})}{0.9}$$

$$= 5.413 \times 10^3 \text{ N.m}$$

$$T = 5.4 \text{ kN.m}$$

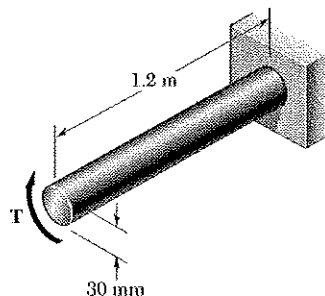
(b) $\phi = 5^\circ = 87.266 \times 10^{-3} \text{ rad}$ $\phi > \phi_y$ A plastic zone occurs.

$$T = \frac{4}{3} T_y \left[1 - \frac{1}{4} \left(\frac{\phi_y}{\phi} \right)^3 \right] = \frac{4}{3} (6.876 \times 10^3) \left[1 - \frac{1}{4} \left(\frac{55.427 \times 10^{-3}}{87.266 \times 10^{-3}} \right)^3 \right]$$

$$= 8.58 \times 10^3 \text{ N.m}$$

$$T = 8.6 \text{ kN.m}$$

Problem 3.105



3.105 For the solid shaft of Prob. 3.95, determine (a) the magnitude of the torque T required to twist the shaft through an angle of 15° , (b) the radius of the corresponding elastic core.

3.95 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) $T = 600 \text{ N} \cdot \text{m}$, (b) $T = 1000 \text{ N} \cdot \text{m}$. ($G = 77.2 \text{ GPa}$).

$$c = \frac{1}{2} d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$\phi = 15^\circ = 0.2618 \text{ rad.}$$

$$\phi_Y = \frac{L \gamma_Y}{c} = \frac{L \tau_Y}{c G} = \frac{(1.2)(145 \times 10^6)}{(15 \times 10^{-3})(77.2 \times 10^9)} = 0.15026 \text{ rad}$$

(a) Since $\phi > \phi_Y$, there is a plastic zone.

$$\tau_Y = \frac{T_Y c}{J} = \frac{2 T_Y}{\pi c^3}$$

$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi (15 \times 10^{-3})^3 (145 \times 10^6)}{2} = 768.71 \text{ N} \cdot \text{m}$$

$$T = \frac{4}{3} T_Y \left[1 - \frac{1}{4} \left(\frac{\phi_Y}{\phi} \right)^3 \right] = \frac{4}{3} (768.71) \left[1 - \frac{1}{4} \left(\frac{0.15026}{0.2618} \right)^3 \right]$$

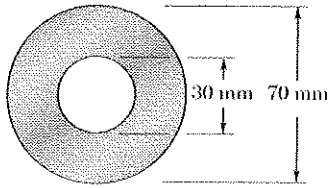
$$= 976.5 \text{ N} \cdot \text{m} \quad T = 977 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

(b) $L \gamma_Y = \rho_Y \phi = c \phi_Y$

$$\rho_Y = \frac{c \phi_Y}{\phi} = \frac{(15 \times 10^{-3})(0.15026)}{0.2618} = 8.63 \times 10^{-3} \text{ m}$$

$$\rho_Y = 8.61 \text{ mm} \quad \blacktriangleleft$$

Problem 3.106



3.106 A hollow shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with $\tau_Y = 180$ MPa and $G = 77.2$ GPa. Determine the applied torque and the corresponding angle of twist (a) at the onset of yield, (b) when the plastic zone is 10 mm deep.

(a) At the onset of yield, the stress distribution is the elastic distribution with $\tau_{max} = \tau_Y$

$$c_2 = \frac{1}{2}d_2 = 0.035 \text{ m}, \quad c_1 = \frac{1}{2}d_1 = 0.015 \text{ m}.$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.035^4 - 0.015^4) = 2.2777 \times 10^{-6} \text{ m}^4$$

$$\tau_{max} = \tau_Y = \frac{T_Y c_2}{J} \therefore T_Y = \frac{J \tau_Y}{c_2} = \frac{(2.2777 \times 10^{-6})(180 \times 10^6)}{0.035} = 11.714 \times 10^3 \text{ N}\cdot\text{m} \\ = 11.71 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi_Y = \frac{T_Y L}{GJ} = \frac{(11.714 \times 10^3)(0.9)}{(77.2 \times 10^9)(2.2777 \times 10^{-6})} = 59.96 \times 10^{-3} \text{ rad} \quad \phi_Y = 3.44^\circ \quad \blacktriangleleft$$

(b) $t = 0.010 \text{ m} \quad \rho_r = c_2 - t = 0.035 - 0.010 = 0.025 \text{ m}$

$$\gamma = \frac{\rho \phi}{L} = \frac{\rho_r \phi}{L} = \tau_Y = \frac{\tau_Y}{G}$$

$$\phi = \frac{\tau_Y L}{G \rho_r} = \frac{(180 \times 10^6)(0.9)}{(77.2 \times 10^9)(0.025)} = 83.94 \times 10^{-3} \text{ rad} \quad \phi = 4.81^\circ \quad \blacktriangleleft$$

Torque T_1 carried by elastic portion $c_1 \leq \rho \leq \rho_r$

$$\tau = \tau_Y \text{ at } \rho = \rho_r. \quad \tau_Y = \frac{T_1 \rho_r}{J_1} \quad \text{where } J_1 = \frac{\pi}{2}(\rho_r^4 - c_1^4)$$

$$J_1 = \frac{\pi}{2}(0.025^4 - 0.015^4) = 534.07 \times 10^{-9} \text{ m}^4$$

$$T_1 = \frac{J_1 \tau_Y}{\rho_r} = \frac{(534.07 \times 10^{-9})(180 \times 10^6)}{0.025} = 3.845 \times 10^3 \text{ N}\cdot\text{m}$$

Torque T_2 carried by plastic portion

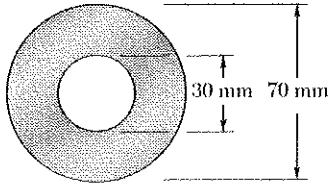
$$T_2 = 2\pi \int_{\rho_r}^{c_2} \tau_Y \rho^2 d\rho = 2\pi \tau_Y \left[\frac{\rho^3}{3} \right]_{\rho_r}^{c_2} = \frac{2\pi}{3} \tau_Y (c_2^3 - \rho_r^3) \\ = \frac{2\pi}{3} (180 \times 10^6)(0.035^3 - 0.025^3) = 10.273 \times 10^3 \text{ N}\cdot\text{m}$$

Total torque

$$T = T_1 + T_2 = 3.845 \times 10^3 + 10.273 \times 10^3 = 14.12 \times 10^3 \text{ N}\cdot\text{m}$$

$$14.12 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 3.107



3.107 A hollow shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with $\tau_y = 180$ MPa and $G = 77.2$ GPa. Determine the (a) angle of twist at which the section first becomes fully plastic, (b) the corresponding magnitude of the applied torque.

$$c_1 = \frac{1}{2} d_1 = 0.015 \text{ m} \quad c_2 = \frac{1}{2} d_2 = 0.035 \text{ m}$$

(a) For onset of fully plastic yielding, $\rho_r = c_1$

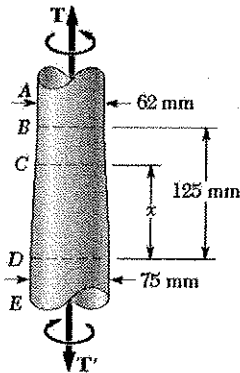
$$\tau = \tau_y \Rightarrow \gamma = \frac{\tau_y}{G} = \frac{\rho_r \phi}{L} = \frac{c_1 \phi}{L}$$

$$\phi = \frac{L \tau_y}{c_1 G} = \frac{(0.9)(180 \times 10^6)}{(0.015)(77.2 \times 10^9)} = 139.90 \times 10^{-3} \text{ rad} \quad \phi = 8.02^\circ$$

$$(b) T_p = 2\pi \int_{c_1}^{c_2} \tau_y \rho^2 d\rho = 2\pi \tau_y \left[\frac{\rho^3}{3} \right]_{c_1}^{c_2} = \frac{2\pi}{3} \tau_y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (180 \times 10^6) (0.035^3 - 0.015^3) = 14.89 \times 10^3 \text{ N}\cdot\text{m} \\ = 14.89 \text{ kN}\cdot\text{m}$$

Problem 3.108



3.108 A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 8500 \text{ N} \cdot \text{m}$ are applied. Assuming the steel to be elastoplastic with $\tau_y = 145 \text{ MPa}$ and $G = 77 \text{ GPa}$, determine (a) the radius of the elastic core in portion AB of the shaft, (b) the length of portion CD that remains fully elastic.

(a) In portion AB $c = \frac{1}{2}d = 31 \text{ mm}$.

$$T_y = \frac{J_{AB} \tau_y}{c} = \frac{\pi}{2} c^3 \tau_y = \frac{\pi}{2} (0.031)^3 (145 \times 10^6) = 6785 \text{ N} \cdot \text{m}$$

$$T = \frac{4}{3} T_y \left(1 - \frac{p_y^3}{c^3}\right)$$

$$\frac{p_y}{c} = \sqrt[3]{4 - \frac{3T}{T_y}} = \sqrt[3]{4 - \frac{(3)(8.5 \times 10^3)}{6.785 \times 10^3}} = 0.623$$

$$p_y = 0.623 \quad c = (0.623)(31) = 19.3 \text{ mm}$$

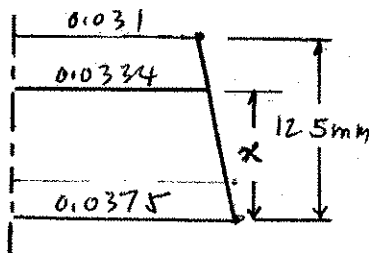
$$p_y = 19.3 \text{ mm}$$

(b) For yielding at point C

$$T = \frac{J_c \tau_y}{c_c} = \frac{\pi}{2} c_c^3 \tau_y$$

$$\tau = \tau_y, \quad c = c_c, \quad T = 8500 \text{ N} \cdot \text{m}$$

$$c_c = \sqrt[3]{\frac{2T}{\pi \tau_y}} = \sqrt[3]{\frac{(2)(8.5 \times 10^3)}{\pi (145 \times 10^6)}} = 0.0334 \text{ m}$$



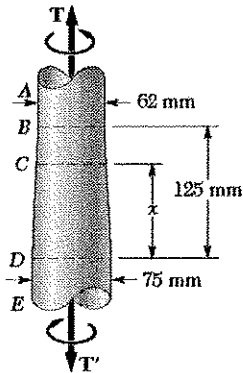
Using proportions from the sketch

$$\frac{37.5 - 33.4}{37.5 - 31} = \frac{x}{125}$$

$$x = 78.85 \text{ mm}$$

$$= 78.9 \text{ mm}$$

Problem 3.109



3.109 If the torques applied to the tapered shaft of Prob. 3.108 are slowly increased, determine (a) the magnitude T of the largest torques that can be applied to the shaft, (b) the length of the portion CD that remains fully elastic.

3.108 A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 8500 \text{ N} \cdot \text{m}$ are applied. Assuming the steel to be elastoplastic with $\tau_y = 145 \text{ MPa}$ and $G = 77 \text{ GPa}$, determine (a) the radius of the elastic core in portion AB of the shaft, (b) the length of portion CD that remains fully elastic.

(a) The largest torque that may be applied is that which makes portion AB fully plastic.

In portion AB $c = \frac{1}{2}d = 31 \text{ mm}$

$$T_y = \frac{J\tau_y}{c} = \frac{\pi c^3 \tau_y}{2} = \frac{\pi (0.031)^3 (145 \times 10^6)}{2} = 6785 \text{ N} \cdot \text{m}$$

For fully plastic shaft $p_r = 0$ $T = \frac{4}{3} T_y \left(1 - \frac{1}{4} \frac{p_r^2}{c^2}\right) = \frac{4}{3} T_y$

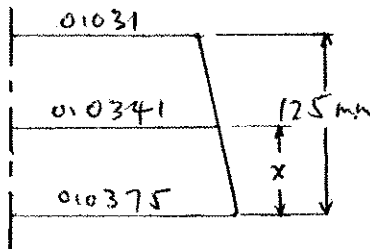
$$T = \frac{4}{3} (6785 \times 10^3) = 9046.7 \text{ N} \cdot \text{m}$$

$$T = 9.05 \text{ kN} \cdot \text{m}$$

(b) For yielding at point C , $\tau = \tau_y$, $c = c_x$, $T = 9046.7 \text{ N} \cdot \text{m}$

$$\tau_y = \frac{T c_x}{J_x} = \frac{2T}{\pi c_x^3}$$

$$c_x = \sqrt[3]{\frac{2T}{\pi \tau_y}} = \sqrt[3]{\frac{(2)(9046.7)}{\pi (145 \times 10^6)}} = 0.0341 \text{ m}$$

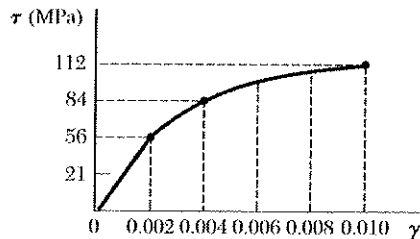


Using proportions from the sketch

$$\frac{37.5 - 34.1}{37.5 - 31} = \frac{x}{125}$$

$$x = 65.4 \text{ mm}$$

Problem 3.110



3.110 Using the stress-strain diagram shown, determine (a) the torque that causes a maximum shearing stress of 105 MPa in a 20-mm-diameter solid rod, (b) the corresponding angle of twist in a 0.6-m length of the rod.

(a) $\tau_{\max} = 105 \text{ MPa}$ $c = \frac{1}{2}d = 0.01 \text{ m}$
 From the stress-strain diagram $\gamma_{\max} = 0.008$
 Let $z = \frac{r}{\gamma_{\max}} = \frac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c^3 \int_0^1 z^2 \tau dz = 2\pi c^3 I$$

where the integral I is given by $I = \int_0^1 z^2 \tau dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below.

z	γ	τ, MPa	$z^2 \tau, \text{MPa}$	w	$w z^2 \tau, \text{MPa}$
0	0.000	0	0.000	1	0.00
0.25	0.002	56	3.5	4	14
0.5	0.004	84	21	2	42
0.75	0.006	98	55.125	4	110.25
1.0	0.008	105	105	1	105
					271.25 ← $\sum w z^2 \tau$

$$I = \frac{(0.25)(271.25)}{3} = 22.6 \text{ MPa}$$

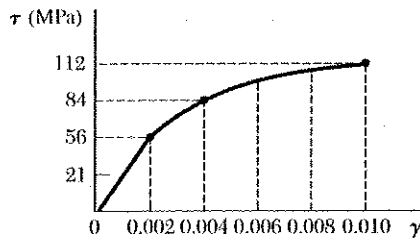
(a) $T = 2\pi c^3 I = 2\pi (0.01)^3 (22.6 \times 10^6) = 142 \text{ Nm}$

(b) $\gamma_{\max} = \frac{c \phi}{L}$

$$\phi = \frac{L \gamma_{\max}}{c} = \frac{(0.6)(0.008)}{0.01} = 480 \times 10^{-3} \text{ rad} = 27.5^\circ$$

Note: Answers may differ slightly due to differences of opinion in reading the stress-strain curve.

Problem 3.111



3.111 A hollow shaft of outer and inner diameters respectively equal to 15 mm and 5 mm is fabricated from an aluminum alloy for which the stress-strain diagram is given in the diagram shown. Determine the torque required to twist a 0.225-m length of the shaft through 10° .

$$\phi = 10^\circ = 174.53 \times 10^{-3} \text{ rad}$$

$$C_1 = \frac{1}{2}d_1 = 2.5 \text{ mm}, \quad C_2 = \frac{1}{2}d_2 = 7.5 \text{ mm}$$

$$\tau_{\max} = \frac{C_2 \phi}{L} = \frac{(7.5)(174.53 \times 10^{-3})}{0.225}$$

$$\tau_{\min} = \frac{C_1 \phi}{L} = \frac{(2.5)(174.53 \times 10^{-3})}{0.225}$$

$$\text{Let } z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{C_2} \quad z_1 = \frac{C_1}{C_2} = \frac{1}{3}$$

$$T = 2\pi \int_{C_1}^{C_2} \rho^2 \tau d\rho = 2\pi C_2^3 \int_{z_1}^1 z^2 \tau dz = 2\pi C_2^3 I$$

$$\text{where the integral } I \text{ is given by } I = \int_{z_1}^1 z^2 \tau dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = \frac{1}{6}$ we get the values given in the table below.

z	γ	τ, MPa	$z^2 \tau, \text{MPa}$	w	$w z^2 \tau, \text{MPa}$
1/3	0.00194	56	6.22	1	6.22
1/2	0.00291	70	17.5	4	70
2/3	0.00383	80.5	35.78	2	71.56
5/6	0.00485	91	63.19	4	252.76
1	0.00582	98	98	1	98
					498.54 ← $\sum w z^2 \tau$

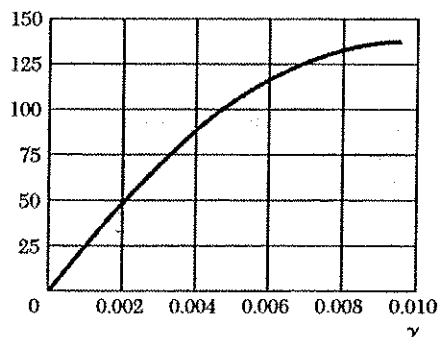
$$I = \frac{(1/6)(498.54)}{3} = 27.7 \text{ MPa}$$

$$T = 2\pi C_2^3 I = 2\pi (0.0075)^3 (27.7 \times 10^6) = 73.4 \text{ Nm}$$

Note: Answer may differ slightly due to differences of opinion in reading the stress-strain curve.

Problem 3.112

τ (MPa)



3.112 A solid aluminum rod of 40-mm diameter is subjected to a torque that produces in the rod a maximum shearing strain of 0.008. Using the τ - γ diagram shown for the aluminum alloy used, determine (a) the magnitude of the torque applied to the rod, (b) the angle of twist in a 750-mm length of the rod.

$$\gamma_{max} = 0.008 \quad c = \frac{1}{2}d = 0.020 \text{ m}$$

$$L = 750 \text{ mm} = 0.750 \text{ m}$$

(a) Let $z = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c^3 \int_0^1 z^2 \tau dz$$

$$= 2\pi c^3 I$$

where the integral I is given by

$$I = \int_0^1 z^2 \tau dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$ we get the values given in the table below.

z	γ	τ , MPa	$z^2 \tau$, MPa	w	$w z^2 \tau$, MPa
0	0	0	0	1	0
0.25	0.002	48	3.0	4	12.0
0.5	0.004	88	22.0	2	44.0
0.75	0.006	115	64.7	4	258.8
1	0.008	133	133.0	1	133.0
					447.8

← $\sum w z^2 \tau$

$$I = \frac{(0.25)(447.8)}{3} = 37.3 \text{ MPa} = 37.3 \times 10^6 \text{ Pa}$$

$$T = 2\pi (0.020)^3 (37.3 \times 10^6) = 1.876 \times 10^3 \text{ N}\cdot\text{m}$$

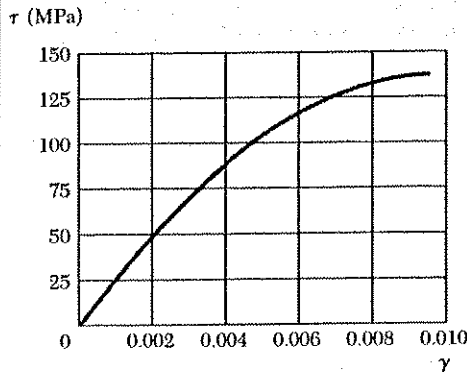
$$T = 1.876 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

(b) $\gamma_{max} = \frac{c\phi}{L}$

$$\phi = \frac{L \gamma_{max}}{c} = \frac{(0.750)(0.008)}{0.020} = 300 \times 10^{-3} \text{ rad}$$

$$\phi = 17.19^\circ \quad \blacktriangleleft$$

Problem 3.113



3.113 The curve shown in Fig. P3.112 can be approximated by the relation

$$\tau = 27.8 \times 10^9 \gamma - 1.390 \times 10^{12} \gamma^2$$

Using this relation and Eqs. (3.2) and (3.26), solve Prob. 3.112.

3.112 A solid aluminum rod of 40-mm diameter is subjected to a torque which produces in the rod a maximum shearing strain of 0.008. Using the τ - γ diagram shown for the aluminum alloy used, determine (a) the magnitude of the torque applied to the rod, (b) the angle of twist in a 750-mm length of the rod.

$$\gamma_{\max} = 0.008$$

$$c = \frac{1}{2}d = 0.020 \text{ m}$$

$$L = 750 \text{ mm} = 0.750 \text{ m}$$

$$(a) \text{ Let } z = \frac{\gamma}{\gamma_{\max}} = \frac{r}{c}$$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c^3 \int_0^1 z^2 \tau dz$$

$$\text{The given stress-strain curve is } \tau = B\gamma + C\gamma^2 = B\gamma_{\max}z + C\gamma_{\max}^2 z^2$$

$$\text{where } B = 27.8 \times 10^9 \text{ and } C = -1.390 \times 10^{12}$$

$$\begin{aligned} T &= 2\pi c^3 \int_0^1 z^2 (B\gamma_{\max}z + C\gamma_{\max}^2 z^2) dz \\ &= 2\pi c^3 \left\{ B\gamma_{\max} \int_0^1 z^3 dz + C\gamma_{\max}^2 \int_0^1 z^4 dz \right\} \\ &= 2\pi c^3 \left\{ \frac{1}{4} B\gamma_{\max} + \frac{1}{5} C\gamma_{\max}^2 \right\} \end{aligned}$$

$$= 2\pi (0.020)^3 \left\{ \frac{1}{4} (27.8 \times 10^9) (0.008) + \frac{1}{5} (-1.390 \times 10^{12}) (0.008)^2 \right\}$$

$$= 1.900 \times 10^3 \text{ N}\cdot\text{m}$$

$$T = 1.900 \text{ kN}\cdot\text{m}$$

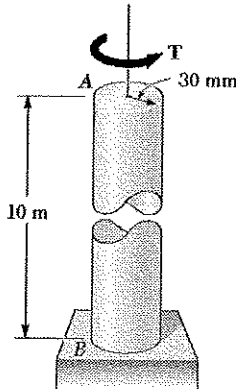
$$(b) \gamma_{\max} = \frac{c\phi}{L}$$

$$\phi = \frac{L\gamma_{\max}}{c} = \frac{(0.750)(0.008)}{0.020} = 300 \times 10^{-3} \text{ rad}$$

$$\phi = 17.19^\circ$$

Problem 3.114

3.114 The solid circular drill rod AB is made of a steel that is assumed to be elastoplastic with $\tau_Y = 154 \text{ MPa}$ and $G = 77 \text{ GPa}$. Knowing that a torque $T = 8475 \text{ N} \cdot \text{m}$ is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.



SOLUTION

$$c = 30 \text{ mm} \quad L = 10 \text{ m}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.03)^4 = 1.272 \times 10^{-6} \text{ m}^4$$

$$T_Y = \frac{J \tau_Y}{c} = \frac{(1.272 \times 10^{-6})(154 \times 10^6)}{0.03} = 6530 \text{ Nm}$$

Loading: $T = 8475$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_r^3}{c^3}\right)$$

$$\frac{\rho_r^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(8475)}{6530} = 0.1064$$

$$\frac{\rho_r}{c} = 0.4739 \quad \rho_r = 0.4739 \times 30 = 14.2 \text{ mm}$$

Unloading: $\gamma' = \frac{T \rho}{J}$ where $T = 8475 \text{ Nm}$

At $\rho = c$ $\gamma' = \frac{(8475)(0.03)}{1.272 \times 10^{-6}} = 200 \text{ MPa}$

At $\rho = \rho_r$ $\gamma' = \frac{(8475)(0.0142)}{1.272 \times 10^{-6}} = 95 \text{ MPa}$

Residual: $\gamma_{\text{res}} = \gamma_{\text{load}} - \gamma'$

At $\rho = c$ $\gamma_{\text{res}} = 154 - 200 = -46 \text{ MPa}$

At $\rho = \rho_r$ $\gamma_{\text{res}} = 154 - 95 = 59 \text{ MPa}$

maximum $\gamma_{\text{res}} = 59 \text{ MPa}$ 

Problem 3.115**3.115** In Prob. 3.114, determine the permanent angle of twist of the rod.**3.114** The solid circular drill rod AB is made of a steel that is assumed to be elastoplastic with $\tau_y = 154 \text{ MPa}$ and $G = 77 \text{ GPa}$. Knowing that a torque $T = 8475 \text{ N} \cdot \text{m}$ is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

From the solution to PROBLEM 3.114

$$C = 30 \text{ mm} \quad J = 1.272 \times 10^{-6} \text{ m}^4 \quad \frac{\rho_r}{C} = 0.4739, \quad \rho_r = 14.2 \text{ mm}$$

After loading $\gamma = \frac{\rho \phi}{L} \therefore \phi = \frac{L \gamma}{\rho} = \frac{L \gamma_r}{\rho_r} = \frac{L \tau_r}{\rho_r G} \quad L = 10 \text{ m}$

$$\phi_{\text{load}} = \frac{(10)(154 \times 10^6)}{(0.0142)(77 \times 10^9)} = 1.4085 \text{ rad} = 80.7^\circ$$

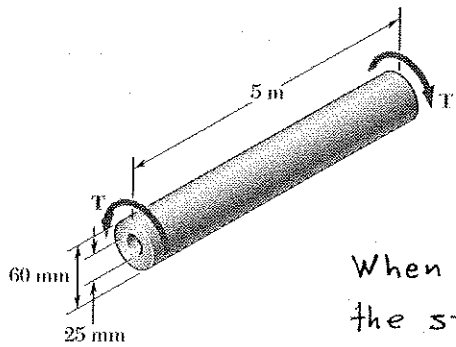
During unloading $\phi' = \frac{TL}{GJ} \quad (\text{elastic}) \quad T = 8475 \text{ N} \cdot \text{m}$

$$\phi' = \frac{(8475 \times 10^3)(10)}{(77 \times 10^9)(1.272 \times 10^{-6})} = 0.8653 \text{ rad} = 49.6^\circ$$

Permanent twist angle

$$\phi_{\text{perm}} = \phi_{\text{load}} - \phi = 1.4085 - 0.8653 = 0.5432 \quad \phi = 31.1^\circ$$

Problem 3.116



3.116 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

$$c_1 = \frac{1}{2} d_1 = 12.5 \text{ mm}$$

$$c_2 = \frac{1}{2} d_2 = 30 \text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y . The corresponding torque is calculated by integration.

$$dT = \rho \tau dA = \rho \tau_Y (2\pi \rho d\rho) = 2\pi \tau_Y \rho^2 d\rho$$

$$T = 2\pi \tau_Y \int_{c_1}^{c_2} \rho^2 d\rho = \frac{2\pi}{3} \tau_Y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (145 \times 10^6) [(30 \times 10^{-3})^3 - (12.5 \times 10^{-3})^3] = 7.6064 \times 10^3 \text{ N}\cdot\text{m}$$

Unloading, $T' = 7.6064 \times 10^3 \text{ N}\cdot\text{m}$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} [(30)^4 - (12.5)^4] = 1.234 \times 10^6 \text{ mm}^4 = 1.234 \times 10^{-6} \text{ m}^4$$

$$\tau'_1 = \frac{T' c_1}{J} = \frac{(7.6064 \times 10^3)(12.5 \times 10^{-3})}{1.234 \times 10^{-6}} = 77.05 \times 10^6 \text{ Pa} = 77.05 \text{ MPa}$$

$$\tau'_2 = \frac{T' c_2}{J} = \frac{(7.6064 \times 10^3)(30 \times 10^{-3})}{1.234 \times 10^{-6}} = 192.63 \times 10^6 \text{ Pa} = 192.63 \text{ MPa}$$

Residual stress.

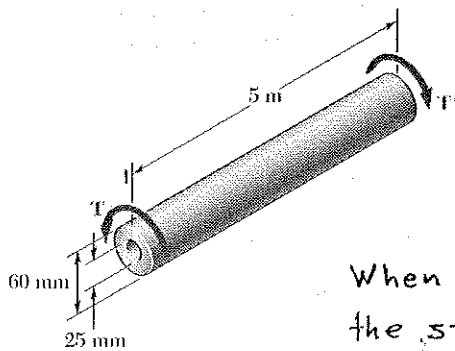
Inner surface: $\tau_{\text{res}} = \tau_Y - \tau'_1 = 145 - 77.05 = 67.95 \text{ MPa}$

Outer surface: $\tau_{\text{res}} = \tau_Y - \tau'_2 = 145 - 192.63 = -47.63 \text{ MPa}$

Maximum residual stress:

68.0 MPa at inner surface. $\blacktriangleleft$

Problem 3.117



3.117 In Prob. 3.116, determine the permanent angle of twist of the rod.

3.116 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

$$c_1 = \frac{1}{2} d_1 = 12.5 \text{ mm}$$

$$c_2 = \frac{1}{2} d_2 = 30 \text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y . The corresponding torque is calculated by integration.

$$dT = \rho \tau dA = \rho \tau_Y (2\pi \rho d\rho) = 2\pi \tau_Y \rho^2 d\rho$$

$$T = 2\pi \tau_Y \int_{c_1}^{c_2} \rho^2 d\rho = \frac{2\pi}{3} \tau_Y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (145 \times 10^6) [(30 \times 10^{-3})^3 - (12.5 \times 10^{-3})^3] = 7.6064 \times 10^3 \text{ N}\cdot\text{m}$$

Rotation angle at maximum torque.

$$\frac{c_1 \phi_{\max}}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\phi_{\max} = \frac{\tau_Y L}{G c_1} = \frac{(145 \times 10^6)(5)}{(77.2 \times 10^9)(12.5 \times 10^{-3})} = 0.75130 \text{ rad}$$

Unloading

$$T' = 7.6064 \times 10^3 \text{ N}\cdot\text{m}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} [(30)^4 - (12.5)^4] = 1.234 \times 10^6 \text{ mm}^4 = 1.234 \times 10^{-6} \text{ m}^4$$

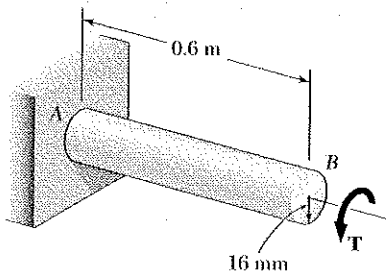
$$\phi' = \frac{T' L}{G J} = \frac{(7.6064 \times 10^3)(5)}{(77.2 \times 10^9)(1.234 \times 10^{-6})} = 0.39922 \text{ rad}$$

Permanent angle of twist.

$$\phi_{\text{perm}} = \phi_{\max} - \phi' = 0.75130 - 0.39922 = 0.35208 \text{ rad}$$

$$\phi_{\text{perm}} = 20.2^\circ$$

Problem 3.118



3.118 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The torque is increased in magnitude until the shaft has been twisted through 6° ; the torque is then removed. Determine (a) the magnitude and location of the maximum residual shearing stress, (b) the permanent angle of twist.

$$C = 0.016 \text{ m} \quad \phi = 6^\circ = 104.72 \times 10^{-3} \text{ rad}$$

$$\gamma_{\max} = \frac{C\phi}{L} = \frac{(0.016)(104.72 \times 10^{-3})}{0.6} = 0.0027925$$

$$\gamma_r = \frac{\tau_r}{G} = \frac{145 \times 10^6}{77.2 \times 10^9} = 0.0018782$$

$$\frac{\rho_r}{C} = \frac{\gamma_r}{\gamma_{\max}} = \frac{0.0018}{0.0027925} = 0.67260$$

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.016)^4 = 102.944 \times 10^{-9} \text{ m}^4$$

$$T_r = \frac{J\tau_r}{C} = \frac{\pi}{2} C^3 \tau_r = \frac{\pi}{2} (0.016)^3 (145 \times 10^6) = 932.93 \text{ N}\cdot\text{m}$$

$$\begin{aligned} \text{At end of loading, } T_{\text{load}} &= \frac{4}{3} T_r \left(1 - \frac{1}{4} \left(\frac{\rho_r}{C} \right)^3 \right) = \frac{4}{3} (932.93) \left[1 - \frac{1}{4} (0.67433)^3 \right] \\ &= 1.14855 \times 10^3 \text{ N}\cdot\text{m} \end{aligned}$$

Unloading: elastic $T' = -1.14855 \times 10^3 \text{ N}\cdot\text{m}$

$$\text{At } \rho = C \quad \tau' = \frac{T' C}{J} = \frac{(-1.14855 \times 10^3)(0.016)}{102.944 \times 10^{-9}} = -178.52 \times 10^6 \text{ Pa}$$

$$\text{At } \rho = \rho_r \quad \tau' = \frac{T' C}{J} \frac{\rho_r}{C} = -178.52 \times 10^6 (0.67433) = -120.38 \times 10^6 \text{ Pa}$$

$$\phi' = \frac{T' L}{GJ} = \frac{(-1.14855 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = -86.71 \times 10^{-3} \text{ rad} = -4.97^\circ$$

Residual: $\tau_{\text{res}} = \tau_{\text{load}} - \tau'$ $\phi_{\text{perm}} = \phi_{\text{load}} - \phi'$

$$\begin{aligned} \text{(a) At } \rho = C \quad \tau_{\text{res}} &= 145 \times 10^6 - 178.52 \times 10^6 = -33.52 \times 10^6 \text{ Pa} \\ &= -33.5 \text{ MPa} \end{aligned}$$

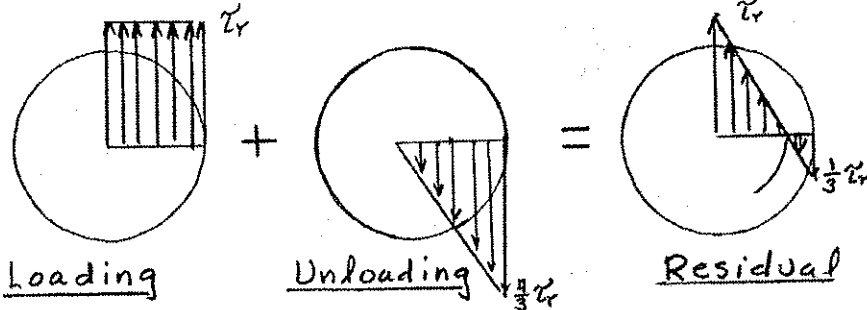
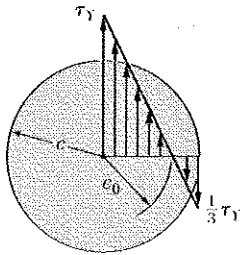
$$\begin{aligned} \text{At } \rho = \rho_r \quad \tau_{\text{res}} &= 145 \times 10^6 - 120.38 \times 10^6 = 24.62 \times 10^6 \text{ Pa} \\ &= 24.6 \text{ MPa} \end{aligned}$$

Maximum residual stress: 33.5 MPa at $\rho_i = 16 \text{ mm}$ $\blacktriangleleft$

$$\begin{aligned} \text{(b) } \phi_{\text{perm}} &= 104.72 \times 10^{-3} - 86.71 \times 10^{-3} = 17.78 \times 10^{-3} \text{ rad} \\ \phi_{\text{perm}} &= 1.032^\circ \quad \blacktriangleleft \end{aligned}$$

Problem 3.119

3.119 A torque T applied to a solid rod made of an elastoplastic material is increased until the rod is fully plastic and then removed. (a) Show that the distribution of residual shearing stresses is as represented in the figure. (b) Determine the magnitude of the torque due to the stresses acting on the portion of the rod located within a circle of radius c_0 .



(a)

After loading $p_r = 0$, $T_{\text{load}} = \frac{4}{3} T_r = \frac{4}{3} \frac{\pi}{2} c^3 \tau_y = \frac{2\pi}{3} c^3 \tau_y$

Unloading $\tau' = \frac{T_c}{J} = \frac{2 T_r}{\pi c^3} = \frac{2(T_{\text{load}})}{\pi c^3} = \frac{4}{3} \tau_y$ at $p = c$
 $\tau' = \frac{4}{3} \tau_y \frac{\rho}{c}$

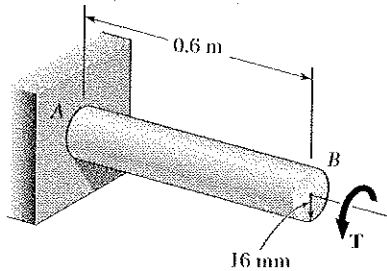
Residual $\tau_{\text{res}} = \tau_y - \frac{4}{3} \tau_y \frac{\rho}{c} = \tau_y \left(1 - \frac{4\rho}{3c}\right)$

To find c_0 set $\tau_{\text{res}} = 0$ and $\rho = c_0$

$$0 = 1 - \frac{4c_0}{3c} \quad \therefore \quad c_0 = \frac{3}{4} c$$

(b) $T = 2\pi \int_0^{c_0} \rho^2 \tau d\rho = 2\pi \int_0^{\frac{3}{4}c} \rho^2 \tau_y \left(1 - \frac{4\rho}{3c}\right) d\rho$
 $= 2\pi \tau_y \left(\frac{\rho^3}{3} - \frac{4}{3} \frac{\rho^4}{4c} \right) \Big|_0^{\frac{3}{4}c} = 2\pi \tau_y c^3 \left\{ \frac{1}{3} \left(\frac{3}{4}\right)^3 - \left(\frac{4}{3}\right) \frac{1}{4} \left(\frac{3}{4}\right)^4 \right\}$
 $= 2\pi \tau_y c^3 \left\{ \frac{9}{64} - \frac{27}{256} \right\} = \frac{9\pi}{128} \tau_y c^3 = 0.2209 \tau_y c^3$

Problem 3.120



3.120 After the solid shaft of Prob. 3.118 has been loaded and unloaded as described in that problem, a torque T_1 of sense opposite to the original torque T is applied to the shaft. Assuming no change in the value of ϕ_Y , determine the angle of twist ϕ_1 for which yield is initiated in this second loading and compare it with the angle ϕ_Y for which the shaft started to yield in the original loading.

3.118 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The torque is increased in magnitude until the shaft has been twisted through 6° ; the torque is then removed. Determine (a) the magnitude and location of the maximum residual shearing stress, (b) the permanent angle of twist.

From the solution to PROBLEM 3.118 $c = 0.016 \text{ m}$, $L = 0.6 \text{ m}$

$$\tau_Y = 145 \times 10^6 \text{ Pa}, \quad J = 102.944 \times 10^{-9} \text{ m}^4$$

The residual stress at $\rho = c$ is $\tau_{\text{res}} = 33.5 \text{ MPa}$

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_Y - \tau_{\text{res}} = 145 \times 10^6 - 33.5 \times 10^6 = 111.5 \times 10^6 \text{ Pa}$$

$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J \tau_1}{c} = \frac{(102.944 \times 10^{-9})(111.5 \times 10^6)}{0.016} = 717 \text{ N}\cdot\text{m}$$

Angle of twist at yielding under reversed torque.

$$\phi_1 = \frac{T_1 L}{G J} = \frac{(717 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 54.16 \times 10^{-3} \text{ rad}$$

$$\phi_1 = 3.10^\circ \blacktriangleleft$$

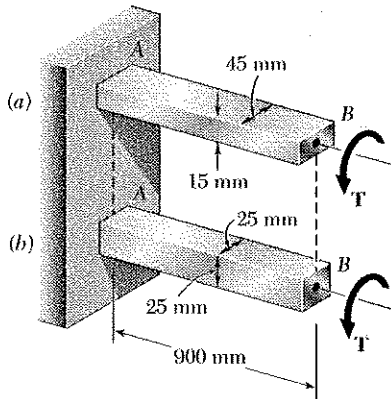
Angle of twist for yielding in original loading.

$$\tau = \frac{\tau_Y}{G} = \frac{c \phi_Y}{L}$$

$$\phi_Y = \frac{L \tau_Y}{c G} = \frac{(0.6)(145 \times 10^6)}{(0.016)(77.2 \times 10^9)} = 70.434 \times 10^{-3} \text{ rad}$$

$$\phi_Y = 4.04^\circ \blacktriangleleft$$

Problem 3.121



3.121 Using $\tau_{all} = 70 \text{ MPa}$ and $G = 27 \text{ GPa}$, determine for each of the aluminum bars shown the largest torque T that can be applied and the corresponding angle of twist at end B .

$$\tau_{all} = 70 \times 10^6 \text{ Pa} \quad G = 27 \times 10^9 \text{ Pa} \quad L = 0.900 \text{ m}$$

$$(a) \quad a = 45 \text{ mm} \quad b = 15 \text{ mm} \quad \frac{a}{b} = 3.0$$

$$\text{From Table 3.1, } c_1 = 0.267, c_2 = 0.263$$

$$\tau_{max} = \frac{T}{c_1 a b^2} \quad T = c_1 a b^2 \tau_{max}$$

$$T = (0.267)(0.045)(0.015)^2(70 \times 10^6) = 189.2 \text{ N}\cdot\text{m}$$

$$T = 189.2 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(189.2)(0.900)}{(0.263)(0.045)(0.015)^3(27 \times 10^9)} = 157.9 \times 10^{-3} \text{ rad} \quad \phi = 9.05^\circ \quad \blacktriangleleft$$

$$(b) \quad a = 25 \text{ mm} \quad b = 25 \text{ mm}, \quad \frac{a}{b} = 1.0. \quad \text{From Table 3.1, } c_1 = 0.208, c_2 = 0.1406$$

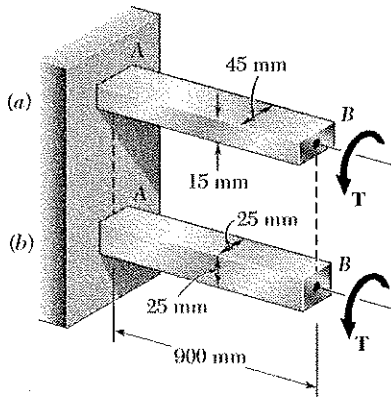
$$\tau_{max} = \frac{T}{c_1 a b^2} \quad T = c_1 a b^2 \tau_{max} = (0.208)(0.025)(0.025)^2(70 \times 10^6)$$

$$= 227.5 \text{ N}\cdot\text{m}$$

$$T = 227.5 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(227.5)(0.900)}{(0.1406)(0.025)(0.025)^3(27 \times 10^9)} = 138.1 \times 10^{-3} \text{ rad} \quad \phi = 7.91^\circ \quad \blacktriangleleft$$

Problem 3.122



3.122 Knowing that the magnitude of the torque T is $200 \text{ N}\cdot\text{m}$ and that $G = 27 \text{ GPa}$, determine for each of the aluminum bars shown the maximum shearing stress and the angle of twist at end B .

$$T = 200 \text{ N}\cdot\text{m} \quad L = 0.900 \text{ m} \quad G = 27 \times 10^9 \text{ Pa}$$

$$(a) \quad a = 45 \text{ mm}, \quad b = 15 \text{ mm}, \quad \frac{a}{b} = 3.0.$$

$$\text{From Table 3.1, } c_1 = 0.267 \quad c_2 = 0.263$$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{200}{(0.267)(0.045)(0.015)^2}$$

$$= 74.0 \times 10^6 \text{ Pa}$$

$$\tau_{max} = 74.0 \text{ MPa} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(200)(0.900)}{(0.263)(0.045)(0.015)^3(27 \times 10^9)}$$

$$= 166.9 \times 10^{-3} \text{ rad}$$

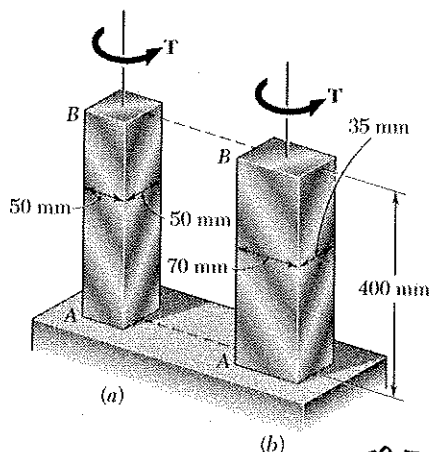
$$\phi = 9.56^\circ \quad \blacktriangleleft$$

$$(b) \quad a = 25 \text{ mm}, \quad b = 25 \text{ mm}, \quad \frac{a}{b} = 1.0. \quad \text{From Table 3.1, } c_1 = 0.208, c_2 = 0.1406$$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{200}{(0.208)(0.025)(0.025)^2} = 61.5 \times 10^6 \text{ Pa} \quad \tau_{max} = 61.5 \text{ MPa} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(200)(0.900)}{(0.1406)(0.025)(0.025)^3(27 \times 10^9)} = 121.4 \times 10^{-3} \text{ rad} \quad \phi = 6.95^\circ \quad \blacktriangleleft$$

Problem 3.123



3.123 Using $\tau_{all} = 50 \text{ MPa}$ and knowing that $G = 39 \text{ GPa}$, determine for each of the cold-rolled yellow brass bars shown the largest torque T that can be applied and the corresponding angle of twist at end B.

$$\tau_{all} = 50 \text{ MPa} \quad L = 0.4 \text{ m}$$

$$(a) \quad a = 50 \text{ mm} \quad b = 50 \text{ mm} \quad \frac{a}{b} = 1.0$$

$$\text{From Table 3.1} \quad c_1 = 0.208, \quad c_2 = 0.1406$$

$$\tau_{max} = \frac{T}{c_1 a b^2} \quad T = c_1 a b^2 \tau_{max}$$

$$T = (0.208)(0.05)(0.05)^2(50 \times 10^6) = 1300 \text{ N}\cdot\text{m}$$

$$T = 1.3 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(1.3 \times 10^3)(0.4)}{(0.1406)(0.05)(0.05)^3(39 \times 10^9)} \\ = 15.173 \times 10^{-3} \text{ rad}$$

$$\phi = 0.87^\circ \quad \blacktriangleleft$$

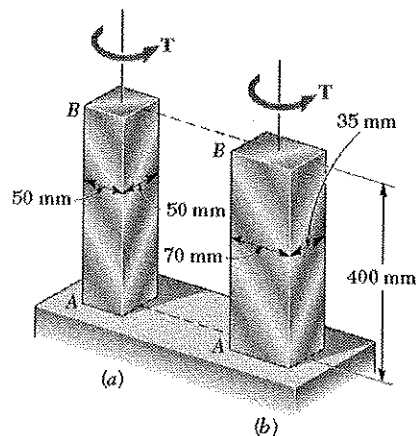
$$(b) \quad a = 70 \text{ mm} \quad b = 35 \text{ mm} \quad \frac{a}{b} = 2.0 \quad \text{From Table 3.1} \quad c_1 = 0.246, \quad c_2 = 0.229$$

$$\tau_{max} = \frac{T}{c_1 a b^2} \quad T = c_1 a b^2 \tau_{max} = (0.246)(0.07)(0.035)^2(50 \times 10^6) = 1.055 \text{ N}\cdot\text{m}$$

$$T = 1.055 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(1.055 \times 10^3)(0.4)}{(0.229)(0.07)(0.035)^3(39 \times 10^9)} = 15.74 \times 10^{-3} \text{ rad} \quad \phi = 0.902^\circ \quad \blacktriangleleft$$

Problem 3.124



3.124 Knowing that $T = 800 \text{ N}\cdot\text{m}$ and that $G = 39 \text{ GPa}$, determine for each of the cold-rolled yellow brass bars shown the maximum shearing stress and the angle of twist of end B.

$$T = 800 \text{ N}\cdot\text{m} \quad L = 0.4 \text{ m}$$

$$(a) \quad a = 50 \text{ mm} \quad b = 50 \text{ mm} \quad \frac{a}{b} = 1.0$$

$$\text{From Table 3.1,} \quad c_1 = 0.208, \quad c_2 = 0.1406$$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{800}{(0.208)(0.05)(0.05)^2} = 30.77 \text{ MPa}$$

$$\tau_{max} = 30.77 \text{ MPa} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(800)(0.4)}{(0.1406)(0.05)(0.05)^3(39000 \times 10^6)} \\ = 9.34 \times 10^{-3} \text{ rad}$$

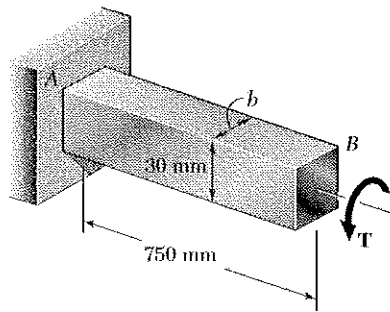
$$\phi = 0.535^\circ \quad \blacktriangleleft$$

$$(b) \quad a = 70 \text{ mm} \quad b = 35 \text{ mm} \quad \frac{a}{b} = 2.0 \quad \text{From Table 3.1} \quad c_1 = 0.246, \quad c_2 = 0.229$$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{800}{(0.246)(0.07)(0.035)^2} = 37.9 \text{ MPa} \quad \tau_{max} = 37.9 \text{ MPa} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(800)(0.4)}{(0.229)(0.07)(0.035)^3(39 \times 10^9)} = 0.01194 \text{ rad} \quad \phi = 0.684^\circ \quad \blacktriangleleft$$

Problem 3.125



3.125 The torque T causes a rotation of 2° at end B of the stainless steel bar shown. Knowing that $b = 20$ mm and $G = 75$ GPa, determine the maximum shearing stress in the bar.

$$a = 30 \text{ mm} = 0.030 \text{ m}, \quad b = 20 \text{ mm} = 0.020 \text{ m}$$

$$\phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad}$$

$$\phi = \frac{TL}{c_2 ab^3 G} \quad \therefore T = \frac{c_2 a b^3 G \phi}{L}$$

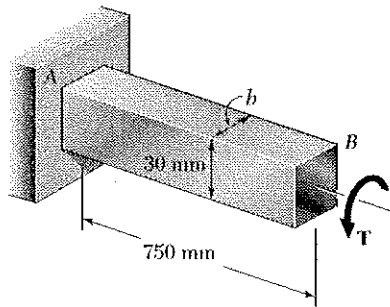
$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{c_2 a b^3 G \phi}{c_1 a b^2 L} = \frac{c_2 b G \phi}{c_1 L}$$

$$\frac{a}{b} = \frac{30}{20} = 1.5. \quad \text{From Table 3.1, } c_1 = 0.231, \quad c_2 = 0.1955$$

$$\tau_{\max} = \frac{(0.1955)(20 \times 10^{-3})(75 \times 10^9)(34.907 \times 10^{-3})}{(0.231)(750 \times 10^{-3})} = 59.2 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 59.2 \text{ MPa} \quad \leftarrow$$

Problem 3.126



3.126 The torque T causes a rotation of 0.6° at end B of the aluminum bar shown. Knowing that $b = 15$ mm and $G = 26$ GPa, determine the maximum shearing stress in the bar.

$$a = 30 \text{ mm} = 0.030 \text{ m}, \quad b = 15 \text{ mm} = 0.015 \text{ m}$$

$$\phi = 0.6^\circ = 10.472 \times 10^{-3} \text{ rad}$$

$$\phi = \frac{TL}{c_2 ab^3 G} \quad \therefore T = \frac{c_2 a b^3 G \phi}{c_1 L}$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{c_2 a b^3 G \phi}{c_1 a b^2 L} = \frac{c_2 b G \phi}{c_1 L}$$

$$\frac{a}{b} = \frac{30}{15} = 2.0. \quad \text{From Table 3.1, } c_1 = 0.246, \quad c_2 = 0.229$$

$$\tau_{\max} = \frac{(0.229)(15 \times 10^{-3})(26 \times 10^9)(10.472 \times 10^{-3})}{(0.246)(750 \times 10^{-3})} = 5.07 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 5.07 \text{ MPa} \quad \leftarrow$$

Problem 3.127

3.127 Determine the largest allowable square cross section of a steel shaft of length 6 m if the maximum shearing stress is not to exceed 120 MPa when the shaft is twisted through one complete revolution. Use $G = 77.2$ GPa.

$$L = 6 \text{ m}, \quad \tau = 120 \times 10^6 \text{ Pa}, \quad G = 77.2 \times 10^9 \text{ Pa} \quad \phi = 1 \text{ rev} = 2\pi \text{ rad}$$

$$\tau = \frac{T}{C_1 a b^2} \quad \phi = \frac{TL}{C_2 a b^3 G}$$

Divide to eliminate T ; then solve for b . $\frac{\phi}{\tau} = \frac{C_1 a b^2 L}{C_2 a b^3 G} = \frac{C_1 L}{C_2 b G}$

$$b = \frac{C_1 L \tau}{C_2 G \phi} \quad \text{For a square cross section } \frac{a}{b} = 1.0$$

From Table 3.1 $C_1 = 0.208, C_2 = 0.1406$

$$b = \frac{(0.208)(6)(120 \times 10^6)}{(0.1406)(77.2 \times 10^9)(2\pi)} = 2.20 \times 10^{-3} \text{ m} \quad b = 2.20 \text{ mm} \quad \blacktriangleleft$$

Problem 3.128

3.128 Determine the largest allowable length of a stainless steel shaft of 9×18 mm cross section if the shearing stress is not to exceed 105 MPa when the shaft is twisted through 15° . Use $G = 77$ GPa.

$$a = 18 \text{ mm}, \quad b = 9 \text{ mm}, \quad \tau_{\max} = 105 \text{ MPa}$$

$$\phi = 15^\circ = \frac{15\pi}{180} \text{ rad} = 0.26180 \text{ rad}$$

$$\tau_{\max} = \frac{T}{C_1 a b^2} \quad (1)$$

$$\phi = \frac{TL}{C_2 a b^3 G} \quad (2)$$

Divide (2) by (1) to eliminate T . $\frac{\phi}{\tau_{\max}} = \frac{C_1 a b^3 L}{C_2 a b^3 G} = \frac{C_1 L}{C_2 b G}$

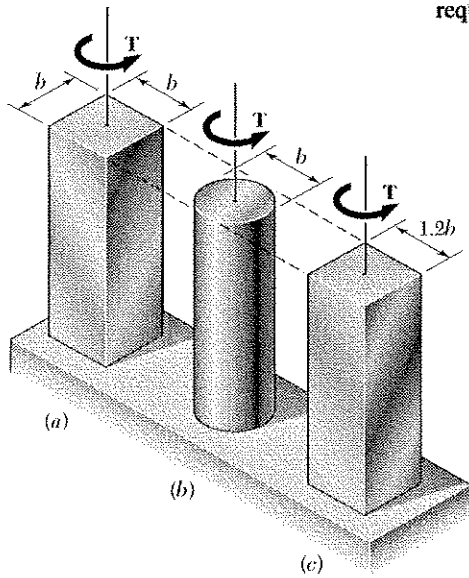
Solve for L . $L = \frac{C_2 b G \phi}{C_1 \tau_{\max}}$

$$\frac{a}{b} = \frac{18}{9} = 2 \quad \text{Table 3.1 gives } C_1 = 0.246, C_2 = 0.229$$

$$L = \frac{(0.229)(0.009)(77 \times 10^9)(0.26180)}{(0.246)(105 \times 10^6)} = 1.609 \text{ m} \quad L = 1.6 \text{ m} \quad \blacktriangleleft$$

Problem 3.129

3.129 Each of the three steel bars shown is subjected to a torque of magnitude $T = 275 \text{ N} \cdot \text{m}$. Knowing that the allowable shearing stress is 50 MPa , determine the required dimension b for each bar.



$$T = 275 \text{ N} \cdot \text{m}, \quad \tau_{\max} = 50 \times 10^6 \text{ Pa}$$

(a) Square: $a = b \quad \frac{a}{b} = 1.0$

From Table 3.1 $C_1 = 0.208$

$$\tau_{\max} = \frac{T}{C_1 a b^2} = \frac{T}{C_1 b^3}$$

$$b = \sqrt[3]{\frac{T}{C_1 \tau_{\max}}} = \sqrt[3]{\frac{275}{(0.208)(50 \times 10^6)}}$$

$$= 29.8 \times 10^{-3} \text{ m}$$

$$b = 29.8 \text{ mm} \quad \blacktriangleleft$$

(b) Circle: $c = \frac{1}{2} b \quad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(275)}{\pi(50 \times 10^6)}} = 15.18 \times 10^{-3} \text{ m}$$

$$c = 15.18 \text{ mm}$$

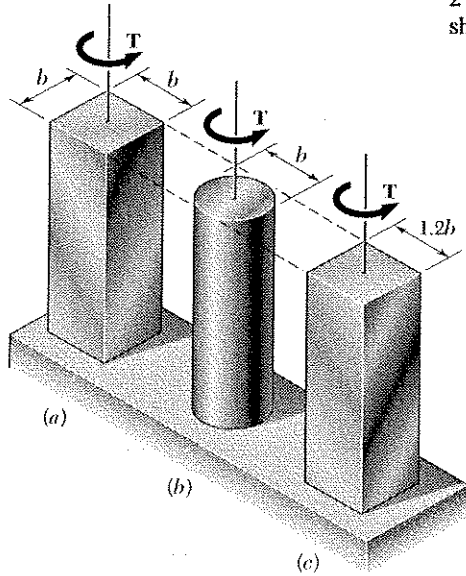
$$b = 2c = 30.4 \text{ mm} \quad \blacktriangleleft$$

(c) Rectangle: $a = 1.2b$ From Table 3.1 $C_1 = 0.219 \quad \tau_{\max} = \frac{T}{C_1 a b^2} = \frac{T}{1.2 C_1 b^3}$

$$b = \sqrt[3]{\frac{T}{1.2 C_1 \tau_{\max}}} = \sqrt[3]{\frac{275}{(1.2)(0.219)(50 \times 10^6)}} = 27.6 \times 10^{-3} \text{ m}$$

$$b = 27.6 \text{ mm} \quad \blacktriangleleft$$

Problem 3.130



3.130 Each of the three aluminum bars shown is to be twisted through an angle of 2° . Knowing that $b = 30 \text{ mm}$, $\tau_{\text{all}} = 50 \text{ MPa}$, and $G = 27 \text{ GPa}$, determine the shortest allowable length of each bar.

$$\phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad}, \quad \tau = 50 \times 10^6 \text{ Pa}$$

$$G = 27 \times 10^9 \text{ Pa}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$$

For square and rectangle

$$\tau = \frac{T}{c_1 a b^2}, \quad \phi = \frac{T L}{c_2 a b^3 G}$$

Divide to eliminate T ; then solve for L

$$\frac{\phi}{\tau} = \frac{c_1 a b^2 L}{c_2 a b^3 G}, \quad L = \frac{c_2 b G \phi}{c_1 \tau}$$

(a) Square: $\frac{a}{b} = 1.0$

From Table 3.1 $c_1 = 0.208, c_2 = 0.1406$

$$L = \frac{(0.1406)(0.030)(27 \times 10^9)(34.907 \times 10^{-3})}{(0.208)(50 \times 10^6)} = 382 \times 10^{-3} \text{ m} \quad L = 382 \text{ mm} \blacktriangleleft$$

(b) Circle: $c = \frac{1}{2} b = 0.015 \text{ m}$ $\tau = \frac{T c}{J}$ $\phi = \frac{T L}{G J}$

Divide to eliminate T ; then solve for L $\frac{\phi}{\tau} = \frac{J L}{c G J} = \frac{L}{c G}$

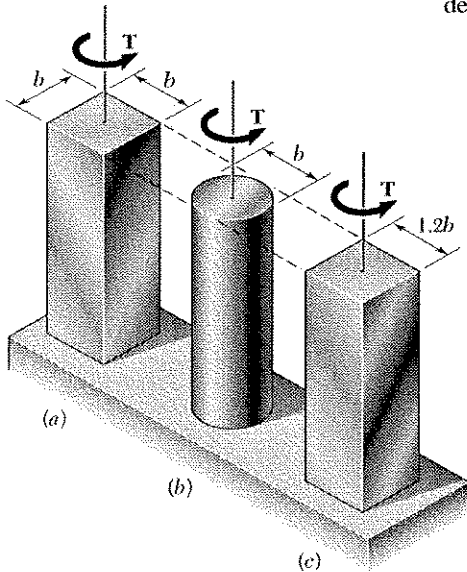
$$L = \frac{c G \phi}{\tau} = \frac{(0.015)(27 \times 10^9)(34.907 \times 10^{-3})}{50 \times 10^6} = 283 \times 10^{-3} \text{ m} \quad L = 283 \text{ mm} \blacktriangleleft$$

(c) Rectangle: $a = 1.2 b$ $\frac{a}{b} = 1.2$ From Table 3.1 $c_1 = 0.219, c_2 = 0.1661$

$$L = \frac{(0.1661)(0.030)(27 \times 10^9)(34.907 \times 10^{-3})}{(0.219)(50 \times 10^6)} = 429 \times 10^{-3} \text{ m} \quad L = 429 \text{ mm} \blacktriangleleft$$

Problem 3.131

3.131 Each of the three steel bars is subjected to a torque as shown. Knowing that the allowable shearing stress is 56 MPa and that $b = 38$ mm, determine the maximum torque T that can be applied to each bar.



$$\tau_{max} = 56 \text{ MPa}, \quad b = 38 \text{ mm}$$

(a) Square: $a = b = 0.038 \text{ m}$ $\frac{a}{b} = 1.0$

From Table 3.1 $C_1 = 0.208$

$$\tau_{max} = \frac{T}{C_1 a b^2} \quad T = C_1 a b^2 \tau_{max}$$

$$T = (0.208)(0.038)(0.038)^2(56 \times 10^6)$$

$$T = 639 \text{ Nm}$$

(b) Circle: $c = \frac{1}{2} b = 19 \text{ mm}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad T = \frac{\pi}{2} c^3 \tau_{max}$$

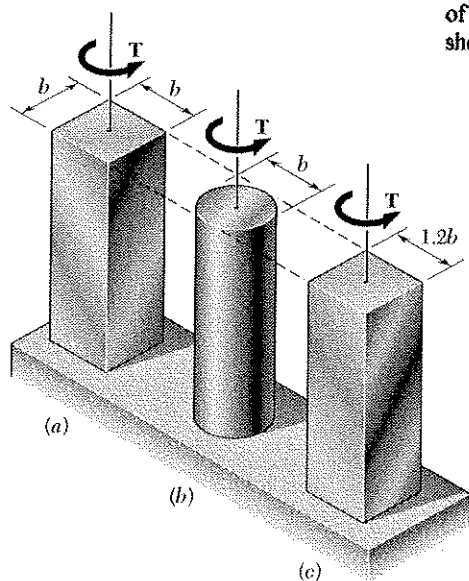
$$T = \frac{\pi}{2} (0.019)^3 (56 \times 10^6) \quad T = 603 \text{ Nm}$$

(c) Rectangle: $a = (1.2)(38) = 45.6 \text{ mm}$ $\frac{a}{b} = 1.2$

From Table 3.1 $C_1 = 0.219$

$$T = C_1 a b^2 \tau_{max} = (0.219)(0.0456)(0.038)^2(56 \times 10^6) \quad T = 807 \text{ Nm}$$

Problem 3.132



3.132 Each of the three aluminum bars shown is to be twisted through an angle of 2° . Knowing that $b = 30 \text{ mm}$, $\tau_{\text{all}} = 50 \text{ MPa}$, and $G = 27 \text{ GPa}$, determine the shortest allowable length of each bar.

$$\phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad} \quad \tau = 50 \times 10^6 \text{ Pa}$$

$$G = 27 \times 10^9 \text{ Pa} \quad b = 30 \text{ mm} = 0.030 \text{ m}$$

For square and rectangle

$$\tau = \frac{T}{c_1 b^2} \quad \phi = \frac{T L}{c_2 a b^3 G}$$

Divide to eliminate T ; then solve for L

$$\frac{\phi}{\tau} = \frac{c_1 a b^2 L}{c_2 a b^3 G} \quad L = \frac{c_2 b G \phi}{c_1 \tau}$$

(a) Square: $\frac{a}{b} = 1.0$

From Table 3.1 $c_1 = 0.208$, $c_2 = 0.1406$

$$L = \frac{(0.1406)(0.030)(27 \times 10^9)(34.907 \times 10^{-3})}{(0.208)(50 \times 10^6)} = 382 \times 10^{-3} \text{ m} \quad L = 382 \text{ mm} \blacktriangleleft$$

(b) Circle $c = \frac{1}{2} b = 0.015 \text{ m} \quad \tau = \frac{T c}{J} \quad \phi = \frac{T L}{G J}$

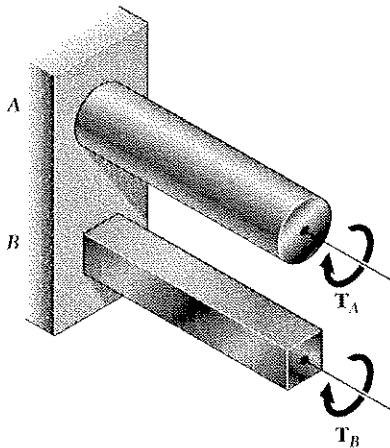
Divide to eliminate T ; then solve for L $\frac{\phi}{\tau} = \frac{J L}{c G J} = \frac{L}{c G}$

$$L = \frac{c G \phi}{\tau} = \frac{(0.015)(27 \times 10^9)(34.907 \times 10^{-3})}{50 \times 10^6} = 283 \times 10^{-3} \text{ m} \quad L = 283 \text{ mm} \blacktriangleleft$$

(c) Rectangle: $a = 1.2 b \quad \frac{a}{b} = 1.2$ From Table 3.1 $c_1 = 0.219$, $c_2 = 0.1661$

$$L = \frac{(0.1661)(0.030)(27 \times 10^9)(34.907 \times 10^{-3})}{(0.219)(50 \times 10^6)} = 429 \times 10^{-3} \text{ m} \quad L = 429 \text{ mm} \blacktriangleleft$$

Problem 3.133



3.133 Shafts A and B are made of the same material and have the same cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum torques T_A and T_B that can be safely applied to A and B , respectively.

Let c = radius of circular section A and b = side of square section B .

For equal areas $\pi c^2 = b^2$

$$c = \frac{b}{\sqrt{\pi}}$$

Circle: $\tau_A = \frac{T_A c}{J} = \frac{2 T_A}{\pi c^3} \therefore T_A = \frac{\pi}{2} c^3 \tau_A$

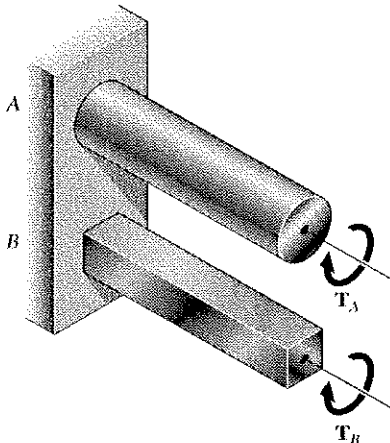
Square: $C_1 = 0.208$ from Table 3.1.

$$\tau_B = \frac{T_B}{C_1 a b^2} = \frac{T_B}{C_1 b^3} \therefore T_B = C_1 b^3 \tau_B$$

$$\text{Ratio: } \frac{T_A}{T_B} = \frac{\frac{\pi}{2} c^3 \tau_B}{C_1 b^3 \tau_B} = \frac{\frac{\pi}{2} \cdot \frac{b^3}{\pi^{3/2}} \tau_A}{C_1 b^3 \tau_B} = \frac{1}{2 C_1 \sqrt{\pi}} \frac{\tau_A}{\tau_B}$$

For the same stresses $\tau_B = \tau_A \therefore \frac{T_A}{T_B} = \frac{1}{(2)(0.208)\sqrt{\pi}} = 1.356 \quad \blacktriangleleft$

Problem 3.134



3.134 Shafts A and B are made of the same material and have the same length and cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum values of the angles ϕ_A and ϕ_B through which shafts A and B , respectively, can be twisted.

Let c = radius of circular section A and b = side of square section B .

For equal areas $\pi c^2 = b^2 \therefore b = \sqrt{\pi} c$

Circle: $\gamma_{max} = \frac{\tau_A}{G} = \frac{C \phi_A}{L} \therefore \phi_A = \frac{L \tau_A}{C G}$

Square: Table 3.1, $C_1 = 0.208$, $C_2 = 0.1406$

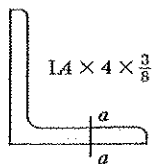
$$\tau_B = \frac{T_B}{C_1 a b^2} = \frac{T_B}{0.208 b^3} \therefore T_B = 0.208 b^3 \tau_B$$

$$\phi_B = \frac{T_B L}{C_2 a b^3 G} = \frac{0.208 b^3 \tau_B L}{0.1406 b^4 G} = \frac{1.4794 L \tau_B}{b G}$$

$$\text{Ratio } \frac{\phi_A}{\phi_B} = \frac{L \tau_A}{C G} \cdot \frac{b G}{1.4794 L \tau_B} = 0.676 \frac{b \tau_A}{C \tau_B} = 0.676 \sqrt{\pi} \frac{\tau_A}{\tau_B}$$

For equal stresses $\tau_A = \tau_B \quad \frac{\phi_B}{\phi_A} = 0.676 \sqrt{\pi} = 1.198 \quad \blacktriangleleft$

Problem 3.135



3.135 A 340 N · m torque is applied to a 1.8-m-long steel angle with an $L102 \times 102 \times 9.5$ cross section. From Appendix C, we find that the thickness of the section is 9.5 mm and that its area is 1850 mm^2 . Knowing that $G = 77 \text{ GPa}$, determine (a) the maximum shearing stress along line $a-a$, (b) the angle of twist.

$$A = 1850 \text{ mm}^2, \quad b = 9.5 \text{ mm}, \quad a = \frac{A}{b} = \frac{1850}{9.5} = 194.7 \text{ mm}$$

$$\frac{a}{b} = \frac{194.7}{9.5} = 20.5 \quad C_1 = C_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{a} \right) = 0.3230$$

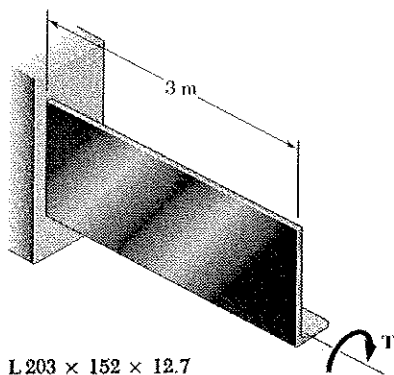
$$(a) \quad \tau_{\max} = \frac{T}{C_1 a b^2} = \frac{340}{(0.3230)(0.1947)(0.0095)^2} = 59.9 \text{ MPa}$$

$$(b) \quad \phi = \frac{TL}{C_2 a b^3 G} = \frac{(340)(1.8)}{(0.323)(0.1947)(0.0095)^3 77 \times 10^9} = 147.4 \times 10^{-3} \text{ rad}$$

$$\phi = 8.45^\circ$$

Note: $L = 1.8 \text{ m}$.

Problem 3.136



3.136 A 3-m-long steel angle has an $L203 \times 152 \times 12.7$ cross section. From Appendix C we find that the thickness of the section is 12.7 mm and that its area is 4350 mm^2 . Knowing that $\tau_{\text{all}} = 50 \text{ MPa}$ and that $G = 77.2 \text{ GPa}$, and ignoring the effect of stress concentration, determine (a) the largest torque T that can be applied, (b) the corresponding angle of twist.

$$A = 4350 \text{ mm}^2 \quad b = 12.7 \text{ mm} \quad a = ?$$

$$\text{Equivalent rectangle, } a = \frac{A}{b} = \frac{4350}{12.7} = 342.52 \text{ mm}$$

$$\frac{a}{b} = 26.97$$

$$C_1 = C_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{a} \right) = 0.32555$$

$$(a) \quad \tau_{\max} = \frac{T}{C_1 a b^2} \quad \tau_{\max} = 50 \times 10^6 \text{ Pa}$$

$$T = C_1 a b^2 \tau_{\max} = (0.32555)(26.97 \times 10^{-3})(12.7 \times 10^{-3})^2 (50 \times 10^6)$$

$$= 70.807 \text{ N} \cdot \text{m}$$

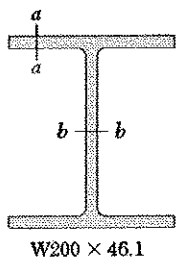
$$T = 70.8 \text{ N} \cdot \text{m}$$

$$(b) \quad \phi = \frac{TL}{C_2 a b^3 G} = \frac{(70.807)(3)}{(0.32555)(26.97 \times 10^{-3})(12.7 \times 10^{-3})^3 (77.2 \times 10^9)}$$

$$= 0.15299 \text{ rad}$$

$$\phi = 8.77^\circ$$

Problem 3.137



SOLUTION

3.137 An 2.4-m steel member with a W200 × 46.1 cross section is subjected to a 560 N · m torque. The properties of the rolled-steel section are given in Appendix C. Knowing that $G = 77$ GPa, determine (a) the maximum shearing stress along line $a-a$, (b) the maximum shearing stress along line $b-b$, (c) the angle of twist. (Hint: consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

Flange: $a = 203 \text{ mm}$, $b = 11 \text{ mm}$, $\frac{a}{b} = \frac{203}{11} = 18.45$

$$c_1 = c_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{a} \right) = 0.322 \quad \phi_F = \frac{T_F L}{c_2 a b^3 G}$$

$$T_F = c_2 a b^3 \frac{G \phi_F}{L} = K_F \frac{G \phi}{L} \quad \text{where } K_F = c_2 a b^3$$

$$K_F = (0.322)(0.203)(0.011)^3 = 87 \times 10^{-9} \text{ m}^3$$

Web: $a = 203 - (2)(11) = 181 \text{ mm}$, $b = 7.2 \text{ mm}$, $\frac{a}{b} = \frac{181}{7.2} = 25.14$

$$c_1 = c_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{a} \right) = 0.3249 \quad \phi_w = \frac{T_w L}{c_2 a b^3 G}$$

$$T_w = c_2 a b^3 \frac{G \phi_w}{L} = K_w \frac{G \phi}{L} \quad \text{where } K_w = c_2 a b^3$$

$$K_w = (0.3249)(0.181)(0.0072)^3 = 21.95 \times 10^{-9} \text{ m}^3$$

For matching twist angles $\phi_F = \phi_w = \phi$

Total torque $T = 2T_F + T_w = (2K_F + K_w) \frac{G \phi}{L}$

$$\frac{G \phi}{L} = \frac{T}{2K_F + K_w}, \quad T_F = \frac{K_F T}{2K_F + K_w}, \quad T_w = \frac{K_w T}{2K_F + K_w}$$

$$T_F = \frac{(87 \times 10^{-9})(560)}{(2)(87 \times 10^{-9}) + 21.95 \times 10^{-9}} = 248.6 \text{ N} \cdot \text{m}, \quad T_w = \frac{(21.95 \times 10^{-9})(560)}{(2)(87 \times 10^{-9}) + 21.95 \times 10^{-9}} = 62.7 \text{ N} \cdot \text{m}$$

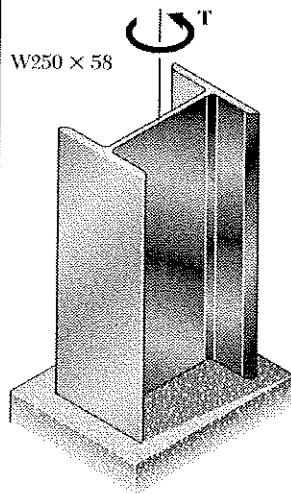
(a) $\tau_F = \frac{T_F}{c_1 a b^2} = \frac{248.6}{(0.3219)(0.203)(0.011)^2} = 31.44 \text{ MPa}$

(b) $\tau_w = \frac{T_w}{c_1 a b^2} = \frac{62.7}{(0.3249)(0.181)(0.0072)^2} = 20.57 \text{ MPa}$

(c) $\frac{G \phi}{L} = \frac{T}{2K_F + K_w} \therefore \phi = \frac{TL}{G(2K_F + K_w)} \quad \text{where } L = 2.4 \text{ m}$

$$\phi = \frac{(560)(2.4)}{(77 \times 10^9)[(2)(87 \times 10^{-9}) + 21.95 \times 10^{-9}]} = 89.1 \times 10^{-3} \text{ rad} = 5.1^\circ$$

Problem 3.138



3.138 A 3-m-long steel member has a W250 x 58 cross section. Knowing that $G = 77.2$ GPa and that the allowable shearing stress is 35 MPa, determine (a) the largest torque T that can be applied, (b) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (See hint of Prob. 3.137.)

Flange: $a = 203$ mm, $b = 13.5$ mm, $\frac{a}{b} = 15.04$

$$C_1 = C_2 = \frac{1}{3} \left(1 - 0.630 \frac{b}{a} \right) = 0.3194$$

$$\phi_F = \frac{T_F L}{C_2 a b^3 G} \quad \therefore T_F = C_2 a b^3 \frac{G \phi}{L} = K_F \frac{G \phi}{L}$$

$$K_F = (0.3194)(0.203)(0.0135)^3 = 159.53 \times 10^{-9} \text{ m}^4$$

Web: $a = 252 - (2)(13.5) = 225$ mm, $b = 8$ mm

$$\frac{a}{b} = 28.13, \quad C_1 = C_2 = \frac{1}{3} \left(1 - 0.63 \frac{b}{a} \right) = 0.3259$$

$$\phi_W = \frac{T_W L}{C_2 a b^3 G} \quad \therefore T_W = C_2 a b^3 \frac{G \phi}{L} = K_W \frac{G \phi}{L}$$

$$K_W = (0.3259)(0.225)(0.008)^3 = 37.54 \times 10^{-9} \text{ m}^4$$

For matching twist angles $\phi_F = \phi_W = \phi$

Total torque: $T = 2T_F + T_W = (2K_F + K_W) \frac{G \phi}{L}$

$$\frac{G \phi}{L} = \frac{T}{2K_F + K_W}, \quad T_F = \frac{K_F T}{2K_F + K_W} \quad \therefore T = \frac{2K_F + K_W}{K_F} T_F$$

$$T_W = \frac{K_W T}{2K_F + K_W} \quad \therefore T = \frac{2K_F + K_W}{K_W} T_W$$

Allowable value for T based on allowable value for T_F :

$$T_F = C_1 a b^2 \tau = (0.3194)(0.203)(0.0135)^2 (35 \times 10^6) = 413.6 \text{ N}\cdot\text{m}$$

$$T = \frac{(2)(159.53) + (37.54)}{159.53} (413.6) = 924.5 \text{ N}\cdot\text{m}$$

Allowable value for T based on allowable value for T_W :

$$T_W = C_1 a b^2 \tau = (0.3259)(0.225)(0.008)^2 (35 \times 10^6) = 164.25 \text{ N}\cdot\text{m}$$

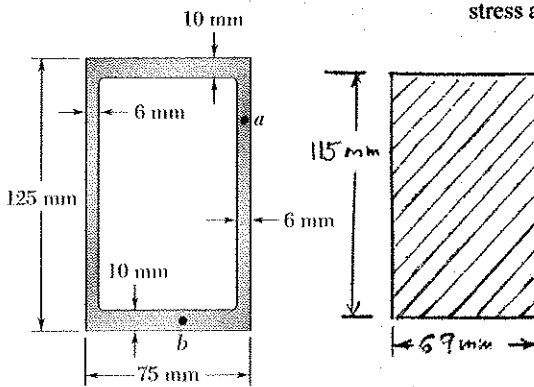
$$T = \frac{(2)(159.53) + 37.54}{37.54} (164.25) = 1560 \text{ N}\cdot\text{m}$$

Choose smaller value

$$T = 925 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{(2K_F + K_W)G} = \frac{(924.5)(3.00)}{(356.6 \times 10^{-9})(77.2 \times 10^9)} = 100.7 \times 10^{-3} \text{ rad} \quad \phi = 5.77^\circ \quad \blacktriangleleft$$

Problem 3.139



3.139 A torque $T = 5 \text{ kN} \cdot \text{m}$ is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b .

$$T = 5 \times 10^3 \text{ N} \cdot \text{m}$$

Area bounded by center line

$$Q = bh = (69)(115) = 7.935 \times 10^3 \text{ mm}^2 \\ = 7.935 \times 10^{-3} \text{ m}^2$$

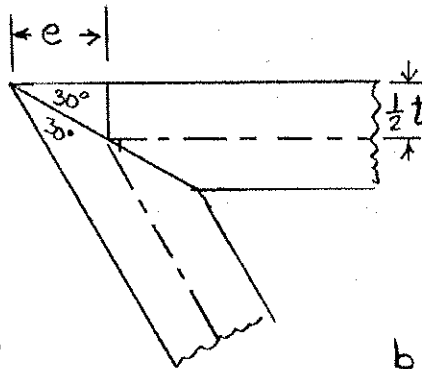
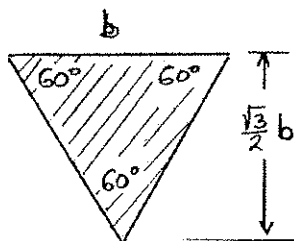
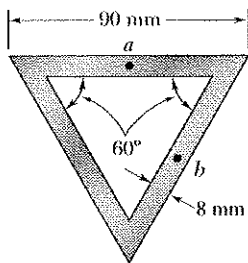
$$\text{At point } a \quad t = 6 \text{ mm} = 0.006 \text{ m}$$

$$\tau = \frac{T}{2tQ} = \frac{5 \times 10^3}{(2)(0.006)(7.935 \times 10^{-3})} \\ = 52.5 \times 10^6 \text{ Pa} \quad \tau = 52.5 \text{ MPa}$$

$$\text{At point } b \quad t = 10 \text{ mm} = 0.010 \text{ m}$$

$$\tau = \frac{T}{2tQ} = \frac{5 \times 10^3}{(2)(0.010)(7.935 \times 10^{-3})} = 31.5 \times 10^6 \text{ Pa} \quad \tau = 31.5 \text{ MPa}$$

Problem 3.140



Detail of corner.

$$\frac{1}{2}t = e \tan 30^\circ$$

$$e = \frac{t}{2 \tan 30^\circ} \\ = \frac{8}{2 \tan 30^\circ} = 6.928 \text{ mm}$$

$$b = 90 - 2e = 76.144 \text{ mm}$$

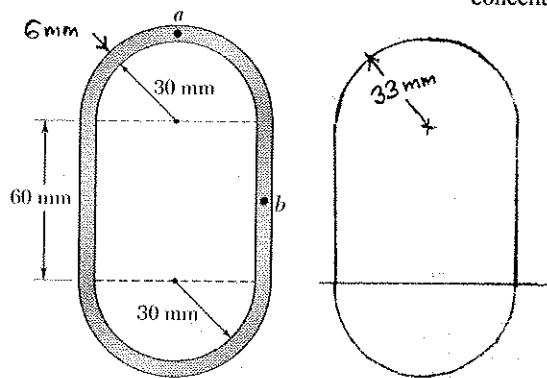
Area bounded by center line

$$Q = \frac{1}{2}b \frac{\sqrt{3}}{2}b = \frac{\sqrt{3}}{4}b^2 = \frac{\sqrt{3}}{4}(76.144)^2 \\ = 2510.6 \text{ mm}^2 = 2510.6 \times 10^{-6} \text{ m}^2$$

$$t = 0.008 \text{ m}$$

$$\tau = \frac{T}{2tQ} = \frac{750}{(2)(0.008)(2510.6 \times 10^{-6})} = 18.67 \times 10^6 \text{ Pa} \quad \tau = 18.67 \text{ MPa}$$

Problem 3.141



3.141 A 750-N · m torque is applied to a hollow shaft having the cross section shown and a uniform 6-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

Area bounded by center line.

$$Q = 2 \frac{\pi}{2} (33)^2 + (60)(66) = 7381 \text{ mm}^2$$

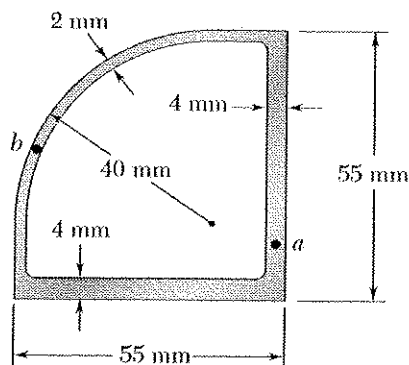
$$= 7381 \times 10^{-6} \text{ m}^2$$

$$t = 0.006 \text{ m at both } a \text{ and } b.$$

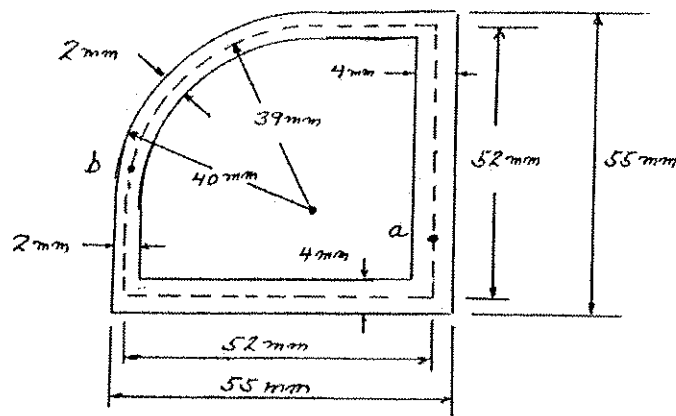
Then at points *a* and *b*,

$$\tau = \frac{T}{2tQ} = \frac{750}{(2)(0.006)(7381 \times 10^{-6})} = 8.47 \times 10^6 \text{ Pa} = 8.47 \text{ MPa} \blacktriangleleft$$

Problem 3.142



3.142 A 90-N · m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.



AREA BOUNDED BY CENTERLINE

$$52 \left[\begin{array}{c} 52 \\ \square \\ 52 \end{array} \right] - \begin{array}{c} 39 \\ \square \\ 39 \end{array} + \begin{array}{c} 39 \\ \text{quarter circle} \\ 39 \end{array} = \begin{array}{c} \text{final shape} \end{array}$$

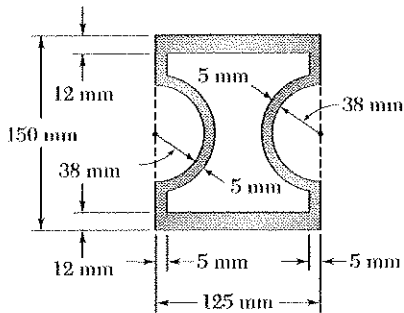
$$Q = 52 \times 52 - 39 \times 39 + \frac{\pi}{4} (39)^2 = 2378 \text{ mm}^2 = 2.378 \times 10^{-3} \text{ m}^2$$

$$T = 90 \text{ N} \cdot \text{m}$$

$$\tau_a = \frac{T}{2tQ} = \frac{90 \text{ N} \cdot \text{m}}{2(2 \times 10^{-3} \text{ m})(2.378 \times 10^{-3} \text{ m}^2)} \quad \tau_a = 9.46 \text{ MPa} \blacktriangleleft$$

$$\tau_b = \frac{T}{2tQ} = \frac{90 \text{ N} \cdot \text{m}}{2(2 \times 10^{-3} \text{ m})(2.378 \times 10^{-3} \text{ m}^2)} \quad \tau_b = 9.46 \text{ MPa} \blacktriangleleft$$

Problem 3.143



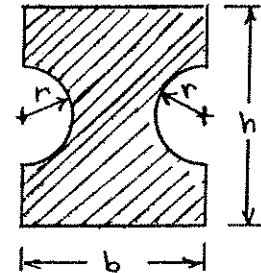
3.143 A hollow brass shaft has the cross section shown. Knowing that the shearing stress must not exceed 84 MPa and neglecting the effect of stress concentrations, determine the largest torque that can be applied to the shaft.

Calculate the area bounded by the center line of the wall cross section. The area is a rectangle with two semi-circular cutouts.

$$b = 125 - 5 = 120 \text{ mm}$$

$$h = 150 - 12 = 138 \text{ mm}$$

$$r = 38 - 2.5 = 35.5 \text{ mm.}$$



$$Q = bh - 2\left(\frac{\pi}{2}r^2\right) = (120)(138) - \pi(35.5)^2 = 12600 \text{ mm}^2 = 12.6 \times 10^{-3} \text{ m}^2$$

$$\tau_{\max} = \frac{T}{2Q t_{\min}}$$

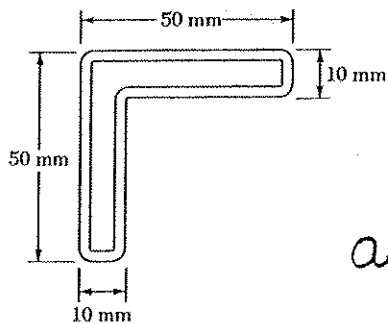
$$\tau_{\max} = 84 \text{ MPa}$$

$$t_{\min} = 0.005 \text{ m}$$

$$T = 2Q t_{\min} \tau_{\max} = (2)(12.6 \times 10^{-3})(0.005)(84 \times 10^6) = 10584 \text{ Nm}$$

$$T = 10.58 \text{ kNm}$$

Problem 3.144



Area bounded by centerline.

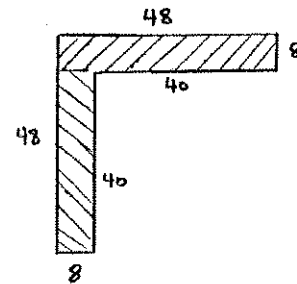
$$A = (48)(8) + (40)(8) = 704 \text{ mm}^2 = 704 \times 10^{-6} \text{ m}^2$$

$$t = 0.002 \text{ m}$$

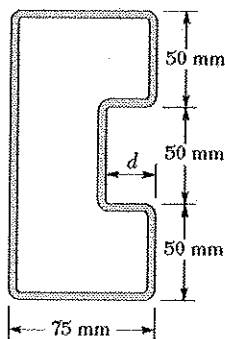
$$\tau = \frac{T}{2tA} \therefore T = 2tA\tau = (2)(0.002)(704 \times 10^{-6})(3 \times 10^6)$$

$$= 8.45 \text{ N}\cdot\text{m}$$

$$T = 8.45 \text{ N}\cdot\text{m}$$



Problem 3.145



Area bounded by centerline

$$A = (0.1485)(0.10735) - (0.0515)d = 0.01091 - 0.0515d$$

$$t = 1.5 \text{ mm}, \quad \tau = 5 \text{ MPa}, \quad T = 140 \text{ N}\cdot\text{m}$$

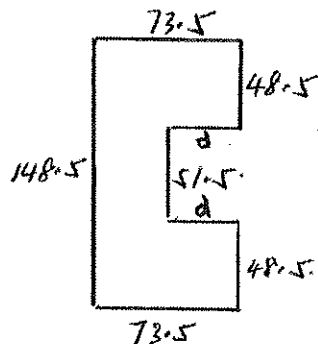
$$\tau = \frac{T}{2tA}$$

$$A = \frac{T}{2t\tau}$$

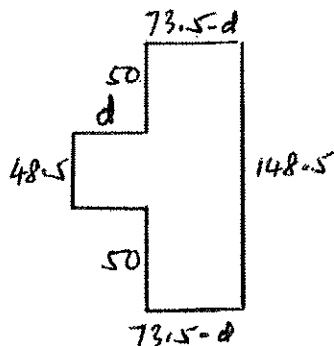
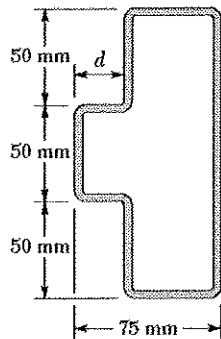
$$0.01091 - 0.0515d = \frac{140}{(2)(0.0015)(5 \times 10^6)} = 0.00933$$

$$d = \frac{0.00158}{0.0515} = 0.0306 \text{ m}$$

$$d = 30.6 \text{ mm}$$



Problem 3.146



3.145 and 3.146 A hollow member having the cross section shown is to be formed from sheet metal of 1.5 mm thickness. Knowing that a 140 N · m torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 5 MPa.

Area bounded by center

$$A = (0.1485)(0.0735 - d) + 0.0485d = 0.01091 - 0.1d$$

$$t = 1.5 \text{ mm}, \quad \tau = 5 \text{ MPa}, \quad T = 140 \text{ N} \cdot \text{m}$$

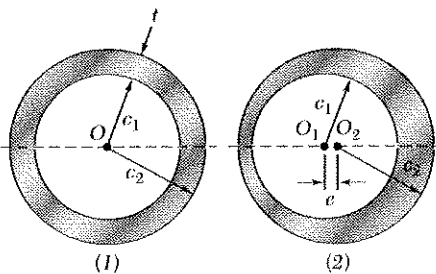
$$\tau = \frac{T}{2tA}$$

$$A = \frac{T}{2t\tau}$$

$$0.01091 - 0.1d = \frac{140}{(2)(0.0015)(5 \times 10^6)} = 0.00933$$

$$d = \frac{0.00158}{0.1} = 0.0158 \text{ m} \quad d = 15.8 \text{ mm} \blacktriangleleft$$

Problem 3.147



3.147 A hollow cylindrical shaft was designed with the cross section shown in Fig. (1) to withstand a maximum torque T_0 . Defective fabrication, however, resulted in a slight eccentricity e between the inner and outer cylindrical surfaces of the shaft as shown in Fig. (2). (a) Express the maximum torque T that can be safely applied to the defective shaft in terms of T_0 , e , and t . (b) Calculate the percent decrease in the allowable torque for values of the ratio e/t equal to 0.1, 0.5, and 0.9.

$$(a) \text{ For both configurations } A = \pi \left(\frac{c_1 + c_2}{2} \right)^2$$

$$\text{Let } t = c_2 - c_1 \text{ be the thickness of (1)}$$

$$\text{The minimum thickness of (2) is } t - e.$$

$$\text{For (1)} \quad \tau_{all} = \frac{T_0}{2tA}$$

$$\text{For (2)} \quad \tau_{all} = \frac{T}{2(t-e)A}$$

Forming the ratio

$$\frac{\tau_{all}}{\tau_{all}} = 1 = \frac{T}{2(t-e)A} \cdot \frac{2tA}{T_0} = \frac{Tt}{T_0(t-e)}$$

$$T = T_0 \left(1 - \frac{e}{t} \right) \blacktriangleleft$$

(b) Reduction in torque.

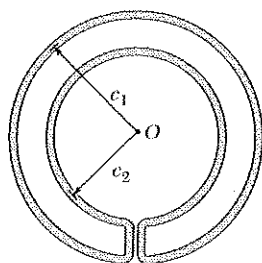
$$\frac{T_0 - T}{T_0} = \frac{e}{t}$$

Expressed as a %

$$\% \text{ reduction} = \frac{e}{t} \times 100\%$$

$\frac{e}{t}$	% red	
0.1	10%	
0.5	50%	
0.9	90%	

Problem 3.148



3.148 A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3-mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the center line of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN · m is applied to the tube, determine (a) the maximum shearing stress in the tube, (b) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

Area bounded by centerline,

$$A = \pi(c_1^2 - c_2^2) = \pi(150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2$$

$$= 39.27 \times 10^{-3} \text{ m}^2$$

$$t = 0.003 \text{ m}$$

$$(a) \tau = \frac{T}{2\pi A} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa} \quad \tau = 12.76 \text{ MPa} \leftarrow$$

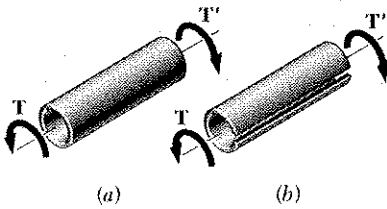
$$(b) T_1 = (2\pi c_1 t \tau c_1) = 2\pi c_1^2 t \tau$$

$$= 2\pi (0.150)^2 (0.003) (12.73 \times 10^6) = 5.46 \times 10^3 \text{ N} \cdot \text{m}$$

$$T_1 = 5.40 \text{ kN} \cdot \text{m} \leftarrow$$

Problem 3.149

3.149 Equal torques are applied to thin-walled tubes of the same length L , same thickness t , and same radius c . One of the tubes has been slit lengthwise as shown. Determine (a) the ratio τ_b/τ_a of the maximum shearing stresses in the tubes, (b) the ratio ϕ_b/ϕ_a of the angles of twist of the shafts.



Without slit

Area bounded by centerline: $A = \pi c^2$

$$\tau_a = \frac{T}{2tA} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t$$

$$\phi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t G}$$

With slit: $a = 2\pi c$, $b = t$, $\frac{a}{b} = \frac{2\pi c}{t} \gg 1$

$$C_1 = C_2 = \frac{1}{3}$$

$$\tau_b = \frac{T}{C_1 a b^2} = \frac{3T}{2\pi c t^2}$$

$$\phi_b = \frac{T}{C_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

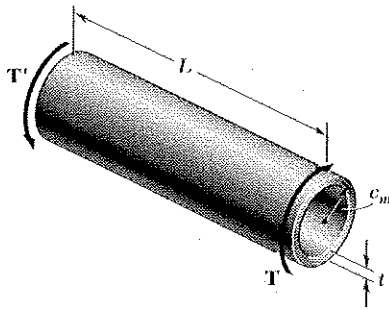
(a) Stress ratio: $\frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t}$

$$\frac{\tau_b}{\tau_a} = \frac{3c}{t} \quad \blacktriangle$$

(b) Twist ratio: $\frac{\phi_b}{\phi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t G}{TL} = \frac{3c^2}{t^2}$

$$\frac{\phi_b}{\phi_a} = \frac{3c^2}{t^2} \quad \blacktriangle$$

Problem 3.150



3.150 A hollow cylindrical shaft of length L , mean radius c_m , and uniform thickness t is subjected to a torque of magnitude T . Consider, on the one hand, the values of the average shearing stress τ_{ave} and the angle of twist ϕ obtained from the elastic torsion formulas developed in Secs. 3.4 and 3.5 and, on the other hand, the corresponding values obtained from the formulas developed in Sec. 3.13 for thin-walled shafts. (a) Show that the relative error introduced by using the thin-walled-shaft formulas rather than the elastic torsion formulas is the same for τ_{ave} and ϕ and that the relative error is positive and proportional to the ratio t/c_m . (b) Compare the percent error corresponding to values of the ratio t/c_m of 0.1, 0.2, and 0.4.

Let $c_2 = \text{outer radius} = c_m + \frac{1}{2}t$ and $c_1 = \text{inner radius} = c_m - \frac{1}{2}t$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(c_2^2 + c_1^2)(c_2 + c_1)(c_2 - c_1)$$

$$= \frac{\pi}{2}(c_m^2 + c_m t + \frac{1}{4}t^2 + c_m^2 - c_m t + \frac{1}{4}t^2)(2c_m)t$$

$$= 2\pi(c_m^2 + \frac{1}{4}t^2)c_m t$$

$$\tau_m = \frac{T c_m}{J} = \frac{T}{2\pi(c_m^2 + \frac{1}{4}t^2)t}$$

$$\phi_1 = \frac{TL}{JG} = \frac{TL}{2\pi(c_m^2 + \frac{1}{4}t^2)c_m t G}$$

Area bounded by centerline. $A = \pi c_m^2$

$$\tau_{ave} = \frac{T}{2tA} = \frac{T}{2\pi c_m^2 t}$$

$$\phi_2 = \frac{TL}{4Q^2 G} \oint \frac{ds}{t} = \frac{TL(2\pi c_m/t)}{4(\pi c_m^2)^2 G} = \frac{TL}{2\pi c_m^3 t G}$$

(a) Ratios: $\frac{\tau_{ave}}{\tau_m} = \frac{T}{2\pi c_m^2 t} \cdot \frac{2\pi(c_m^2 + \frac{1}{4}t^2)t}{T} = 1 + \frac{1}{4} \frac{t^2}{c_m^2}$

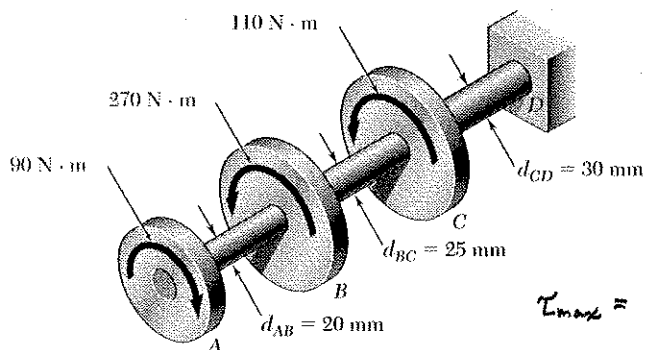
$$\frac{\phi_2}{\phi_1} = \frac{TL}{2\pi c_m^3 t G} \cdot \frac{2\pi(c_m^2 + \frac{1}{4}t^2)c_m t G}{TL} = 1 + \frac{1}{4} \frac{t^2}{c_m^2}$$

(b) $\frac{\tau_{ave}}{\tau_m} - 1 = \frac{\phi_2}{\phi_1} - 1 = \frac{1}{4} \frac{t^2}{c_m^2}$

$\frac{t}{c_m}$	0.1	0.2	0.4
$\frac{1}{4} \frac{t^2}{c_m^2}$	0.0025	0.01	0.04
%	0.25%	1%	4%

Problem 3.151

3.151 Knowing that a 10 mm-diameter hole has been drilled through each of the shafts AB, BC, and CD, determine (a) the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress.



$$\text{Hole: } c_1 = \frac{1}{2}d_1 = \frac{1}{2}(10) = 5 \text{ mm}$$

$$\text{Shaft AB: } T = 90 \text{ N}\cdot\text{m}$$

$$c_2 = \frac{1}{2}d_2 = 10 \text{ mm}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.01^4 - 0.005^4) = 14.73 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(90)(0.01)}{14.73 \times 10^{-9}} = 61.1 \text{ MPa (largest)}$$

$$c_2 = \frac{1}{2}d_2 = 12.5 \text{ mm}$$

$$\text{Shaft BC: } T = -90 + 270 = 180 \text{ N}\cdot\text{m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0125^4 - 0.005^4) = 37.37 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(180)(0.0125)}{37.37 \times 10^{-9}} = 60.2 \text{ MPa}$$

$$\text{Shaft CD: } T = -90 + 270 + 110 = 290 \text{ N}\cdot\text{m}$$

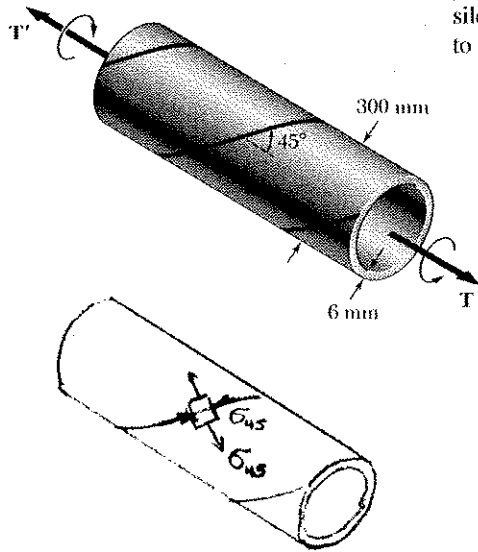
$$c_2 = \frac{1}{2}d_2 = 15 \text{ mm}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.015^4 - 0.005^4) = 78.54 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(290)(0.015)}{78.54 \times 10^{-9}} = 55.39 \text{ MPa}$$

Answers: (a) Shaft AB (b) 61.1 MPa

Problem 3.152



3.152 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 45° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable tensile stress in the weld is 84 MPa, determine the largest torque that can be applied to the pipe.

From equation (3.14) of the textbook,

$$\sigma_{45} = \tau_{max}$$

$$\text{hence, } \tau_{max} = 84 \text{ MPa}$$

$$C_2 = \frac{1}{2} d_o = \frac{1}{2} (300) = 150 \text{ mm}$$

$$C_1 = C_2 - t = 150 - 6 = 144 \text{ mm}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} [(0.15)^4 - (0.144)^4] = 119.8 \times 10^{-6} \text{ m}^4$$

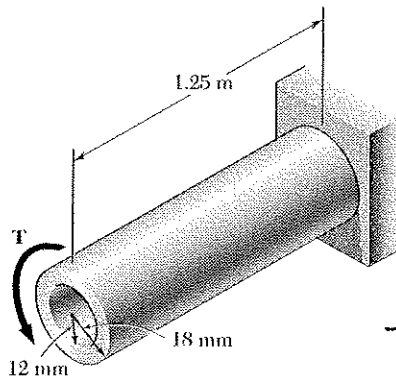
$$\tau_{max} = \frac{T C}{J}$$

$$T = \frac{\tau_{max} J}{C}$$

$$T = \frac{(84 \times 10^6)(119.8 \times 10^{-6})}{0.15} = 67.088$$

$$T = 67 \text{ kN}$$

Problem 3.153



3.153 For the aluminum shaft shown ($G = 27 \text{ GPa}$), determine (a) the torque T that causes an angle of twist of 4° , (b) the angle of twist caused by the same torque T in a solid cylindrical shaft of the same length and cross-sectional area.

$$(a) \quad \phi = \frac{T L}{G J}, \quad T = \frac{G J \phi}{L}$$

$$\phi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}, \quad L = 1.25 \text{ m}$$

$$G = 27 \text{ GPa} = 27 \times 10^9 \text{ Pa}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.018^4 - 0.012^4) = 132.324 \times 10^{-9} \text{ m}^4$$

$$T = \frac{(27 \times 10^9)(132.324 \times 10^{-9})(69.813 \times 10^{-3})}{1.25}$$

$$= 199.539 \text{ N}\cdot\text{m}$$

$$T = 199.5 \text{ N}\cdot\text{m}$$

$$(b) \text{ Matching areas } A = \pi c^2 = \pi (C_2^2 - C_1^2)$$

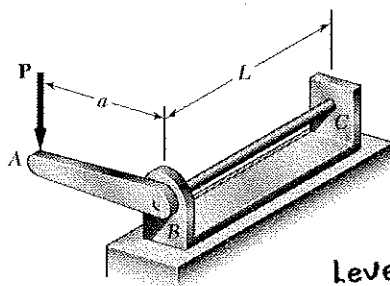
$$c = \sqrt{C_2^2 - C_1^2} = \sqrt{0.018^2 - 0.012^2} = 0.013416 \text{ m}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.013416)^4 = 50.894 \times 10^{-9} \text{ m}^4$$

$$\phi = \frac{T L}{G J} = \frac{(195.539)(1.25)}{(27 \times 10^9)(50.894 \times 10^{-9})} = 181.514 \times 10^{-3} \text{ rad}$$

$$\phi = 10.40^\circ$$

Problem 3.154



Lever AB turns through angle ϕ to position A'B as shown in the auxiliary figure.

Vertical displacement is $\Delta = a \sin \phi$
from which $\phi = \arcsin \frac{\Delta}{a}$

The maximum shearing stress in rod BC is

$$\tau_{\max} = G\gamma_{\max} = G \frac{C\phi}{L} = G \frac{d\phi}{2L} = \frac{Gd}{2L} \arcsin \frac{\Delta}{a}$$

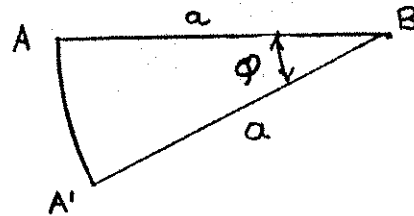
For small $\frac{\Delta}{a}$, $\arcsin \frac{\Delta}{a} \approx \frac{\Delta}{a}$

$$\tau_{\max} = \frac{Gd\Delta}{2La}$$

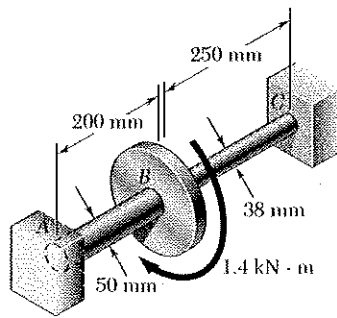
3.154 The solid cylindrical rod BC is attached to the rigid lever AB and to the fixed support at C . The vertical force P applied at A causes a small displacement Δ at point A . Show that the corresponding maximum shearing stress in the rod is

$$\tau = \frac{Gd}{2La} \Delta$$

where d is the diameter of the rod and G its modulus of rigidity.



Problem 3.155



3.155 Two solid steel shafts ($G = 77.2 \text{ GPa}$) are connected to a coupling disk B and to fixed supports at A and C. For the loading shown, determine (a) the reaction at each support, (b) the maximum shearing stress in shaft AB, (c) the maximum shearing stress in shaft BC.

Shaft AB

$$T = T_{AB}, \quad L_{AB} = 0.200 \text{ m}, \quad c = \frac{1}{2}d = 25 \text{ mm} = 0.025 \text{ m}$$

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.025)^4 = 613.59 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L_{AB}}{G J_{AB}}$$

$$T_{AB} = \frac{G J_{AB}}{L_{AB}} \phi_B = \frac{(77.2 \times 10^9)(613.59 \times 10^{-9})}{0.200} \phi_B = 236.847 \times 10^3 \phi_B$$

Shaft BC

$$T = T_{BC}, \quad L_{BC} = 0.250 \text{ m}, \quad c = \frac{1}{2}d = 19 \text{ mm} = 0.019 \text{ m}$$

$$J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.019)^4 = 204.71 \times 10^{-9} \text{ m}^4 \quad \phi_B = \frac{T_{BC} L_{BC}}{G J_{BC}}$$

$$T_{BC} = \frac{G J_{BC}}{L_{BC}} \phi_B = \frac{(77.2 \times 10^9)(204.71 \times 10^{-9})}{0.250} = 63.214 \times 10^3 \phi_B$$

Equilibrium of coupling disk. $T = T_{AB} + T_{BC}$

$$1.4 \times 10^3 = 236.847 \times 10^3 \phi_B + 63.214 \times 10^3 \phi_B$$

$$\phi_B = 4.6657 \times 10^{-3} \text{ rad.}$$

$$T_{AB} = (236.847 \times 10^3)(4.6657 \times 10^{-3}) = 1.10506 \times 10^3 \text{ N}\cdot\text{m}$$

$$T_{BC} = (63.214 \times 10^3)(4.6657 \times 10^{-3}) = 294.94 \text{ N}\cdot\text{m}$$

(a) Reactions at supports.

$$T_A = T_{AB} = 1105 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$T_C = T_{BC} = 295 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

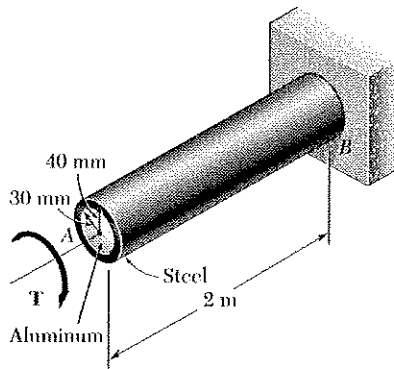
(b) Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB} c}{J_{AB}} = \frac{(1.10506 \times 10^3)(0.025)}{613.59 \times 10^{-9}} = 45.0 \times 10^6 \text{ Pa} \quad \tau_{AB} = 45.0 \text{ MPa} \quad \blacktriangleleft$$

(c) Maximum shearing stress in BC.

$$\tau_{BC} = \frac{T_{BC} c}{J_{BC}} = \frac{(294.94)(0.019)}{204.71 \times 10^{-9}} = 27.4 \times 10^6 \text{ Pa} \quad \tau_{BC} = 27.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.156



3.156 The composite shaft shown is twisted by applying a torque T at end A . Knowing that the maximum shearing stress in the steel shell is 150 MPa, determine the corresponding maximum shearing stress in the aluminum core. Use $G = 77.2$ GPa for steel and $G = 27$ GPa for aluminum.

SOLUTION

Let G_1 , J_2 , and τ_1 refer to the aluminum core.
and G_2 , J_2 , and τ_2 refer to the steel shell.

At the outer surface on the steel shell,

$$\gamma_2 = \frac{C_2 \phi}{L} = \frac{\phi}{L} = \frac{\tau_2}{G_2} = \frac{\tau_2}{C_2 G_2}$$

At the outer surface of the aluminum core,

$$\gamma_1 = \frac{C_1 \phi}{L} = \frac{\phi}{L} = \frac{\tau_1}{G_1} = \frac{\tau_1}{C_1 G_1}$$

Matching $\frac{\phi}{L}$ for both components,

$$\frac{\tau_2}{C_2 G_2} = \frac{\tau_1}{C_1 G_1}$$

$$\text{Solving for } \tau_2, \quad \tau_2 = \frac{C_2}{C_1} \cdot \frac{G_2}{G_1} \tau_1 = \frac{0.030}{0.040} \cdot \frac{27 \times 10^9}{77.2 \times 10^9} \cdot 150 \times 10^6 = 39.3 \times 10^6 \text{ Pa}$$

$$\tau_2 = 39.3 \text{ MPa} \quad \blacktriangleleft$$

Problem 3.157

3.157 In the gear-and-shaft system shown, the shaft diameters are $d_{AB} = 50$ mm and $d_{CD} = 38$ mm. Knowing that $G = 77$ GPa, determine the angle through which end D of shaft CD rotates.

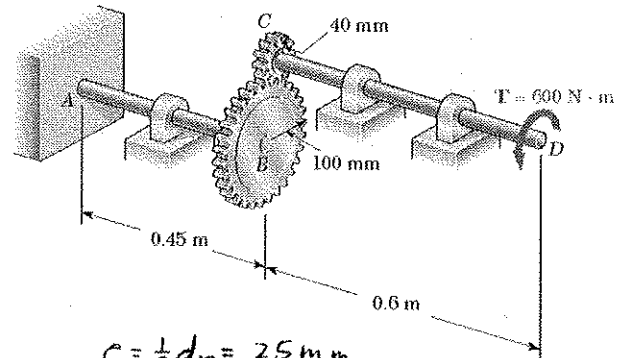
Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad T_{AB} = \frac{r_A}{r_C} T_{CD}$$

$$T_{CD} = 600 \text{ N}\cdot\text{m}$$

$$T_{AB} = \frac{10}{4}(600) = 1500 \text{ N}\cdot\text{m}$$



Twist in shaft AB: $L = 0.45$ m

$$c = \frac{1}{2}d_{AB} = 25 \text{ mm}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.6 \times 10^{-9} \text{ m}^4$$

$$\phi_{B/A} = \frac{TL}{GJ} = \frac{(1500)(0.45)}{(77 \times 10^9)(613.6 \times 10^{-9})} = 0.0143 \text{ rad}$$

Rotation at B. $\phi_B = \phi_{B/A} = 0.0143 \text{ rad}$

Tangential displacement at gear circle

$$s = r_B \phi_B = r_C \phi_C$$

Rotation at C $\phi_C = \frac{r_B}{r_C} \phi_B = \frac{10}{4}(0.0143) = 0.0357 \text{ rad}$

Twist in shaft CD: $L = 0.6$ m $c = \frac{1}{2}d_{CD} = 19 \text{ mm}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.019)^4 = 204.7 \times 10^{-9} \text{ m}^4$$

$$\phi_{D/C} = \frac{TL}{GJ} = \frac{(600)(0.6)}{(77 \times 10^9)(204.7 \times 10^{-9})} = 0.0228 \text{ rad}$$

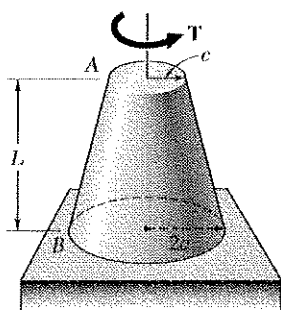
Rotation at D. $\phi_D = \phi_C + \phi_{D/C} = 0.0585 \text{ rad}$

$$\phi_D = 3.35^\circ$$

Problem 3.158

3.158 A torque T is applied as shown to a solid tapered shaft AB . Show by integration that the angle of twist at A is

$$\phi = \frac{7TL}{12\pi Gc^4}$$



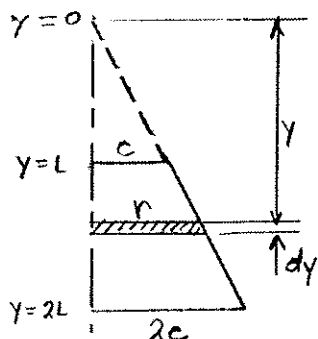
Introduce coordinate y as shown.

$$r = \frac{cy}{L}$$

Twist in length dy

$$d\phi = \frac{T dy}{GJ} = \frac{T dy}{G \frac{\pi}{2} r^4} = \frac{2TL^4 dy}{\pi Gc^4 y^4}$$

$$\begin{aligned} \phi &= \int_L^{2L} \frac{2TL^4}{\pi Gc^4} \frac{dy}{y^4} = \frac{2TL}{\pi Gc^4} \int_L^{2L} \frac{dy}{y^4} \\ &= \frac{2TL}{\pi Gc^4} \left\{ -\frac{1}{3y^3} \right\}_L^{2L} = \frac{2TL}{\pi Gc^4} \left\{ -\frac{1}{24L^3} + \frac{1}{3L^3} \right\} \\ &= \frac{2TL}{\pi Gc^4} \left\{ \frac{7}{24L^3} \right\} = \frac{7TL}{12\pi Gc^4} \end{aligned}$$



Problem 3.159

3.159 A 38-mm-diameter steel shaft of length 1.2 m will be used to transmit 45 kW between a motor and a pump. Knowing that $G = 77$ GPa, determine the lowest speed of rotation at which the stress does not exceed 60 MPa and the angle of twist does not exceed 2° .

$$c = \frac{1}{2}d = 19 \text{ mm}$$

$$L = 1.2 \text{ m}$$

$$P = 45 \text{ kW/s}$$

$$\text{Stress requirement } \tau = 60 \text{ MPa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$T = \frac{\pi}{2} \tau c^3 = \frac{\pi}{2} (60 \times 10^6) (0.019)^3 = 646 \text{ Nm}$$

$$\text{Twist angle requirement } \phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad}$$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi Gc^4}$$

$$T = \frac{\pi Gc^4 \phi}{2L} = \frac{\pi (77 \times 10^9) (0.019)^4 (34.907 \times 10^{-3})}{2(1.2)} = 458 \text{ Nm}$$

Maximum allowable torque is the smaller value. $T = 458 \text{ Nm}$

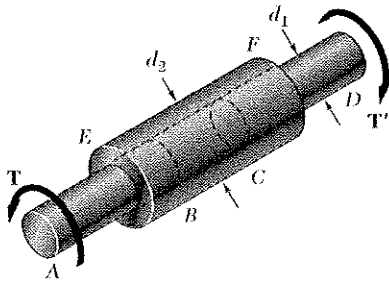
$$P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{45 \times 10^3}{2\pi (458)} = 15.6 \text{ Hz}$$

$$f = 936 \text{ rpm}$$

Problem 3.160

3.160 Two solid brass rods AB and CD are brazed to a brass sleeve EF . Determine the ratio d_2/d_1 for which the same maximum shearing stress occurs in the rods and in the sleeve.



$$\text{Let } c_1 = \frac{1}{2}d_1 \text{ and } c_2 = \frac{1}{2}d_2$$

$$\text{Shaft } AB. \quad \tau_1 = \frac{Tc_1}{J_1} = \frac{2T}{\pi c_1^3}$$

$$\text{Sleeve } EF. \quad \tau_2 = \frac{Tc_2}{J_2} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

$$\text{For equal stresses, } \frac{2T}{\pi c_1^3} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

$$c_2^4 - c_1^4 = c_1^3 c_2$$

$$\text{Let } x = \frac{c_2}{c_1} \quad x^4 - 1 = x \quad \text{or} \quad x = \sqrt[4]{1+x}$$

Solve by successive approximations starting with $x_0 = 1.0$.

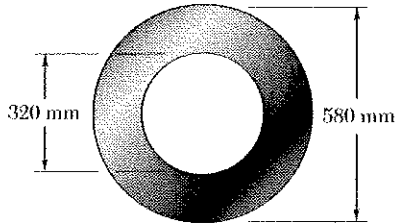
$$x_1 = \sqrt[4]{2} = 1.189, \quad x_2 = \sqrt[4]{2.189} = 1.216, \quad x_3 = \sqrt[4]{2.216} = 1.220$$

$$x_4 = \sqrt[4]{2.220} = 1.221, \quad x_5 = \sqrt[4]{2.221} = 1.221 \text{ (converged).}$$

$$x = 1.221 \quad \frac{c_2}{c_1} = 1.221$$

$$\frac{d_2}{d_1} = 1.221$$

Problem 3.161



3.161 One of the two hollow steel drive shafts of an ocean liner is 75 m long and has the cross section shown. Knowing that $G = 77.2$ GPa and that the shaft transmits 44 MW to its propeller when rotating at 144 rpm, determine (a) the maximum shearing stress in the shaft, (b) the angle of twist of the shaft.

$$L = 75 \text{ m}, \quad f = 144 \text{ rpm} = \frac{144}{60} = 2.4 \text{ Hz}$$

$$P = 44 \text{ MW} = 44 \times 10^6 \text{ W}$$

$$P = 2\pi f T \therefore T = \frac{P}{2\pi f} = \frac{44 \times 10^6}{2\pi(2.4)} = 2.9178 \times 10^6 \text{ N}\cdot\text{m}$$

$$c_1 = \frac{d_1}{2} = \frac{320}{2} = 160 \text{ mm} = 0.160 \text{ m}$$

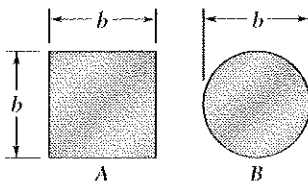
$$c_2 = \frac{d_2}{2} = \frac{580}{2} = 290 \text{ mm} = 0.290 \text{ m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.290^4 - 0.160^4) = 10.08 \times 10^{-3} \text{ m}^4$$

$$(a) \quad \tau = \frac{T c_2}{J} = \frac{(2.9178 \times 10^6)(0.290)}{10.08 \times 10^{-3}} = 83.9 \times 10^6 \text{ Pa} = 83.9 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \phi = \frac{TL}{GJ} = \frac{(2.9178 \times 10^6)(75)}{(77 \times 10^9)(10.08 \times 10^{-3})} = 281.9 \times 10^{-3} \text{ rad} = 16.15^\circ \quad \blacktriangleleft$$

Problem 3.162



3.162 Two shafts are made of the same material. The cross section of shaft A is a square of side b and that of shaft B is a circle of diameter b . Knowing that the shafts are subjected to the same torque, determine the ratio τ_A/τ_B of the maximum shearing stresses occurring in the shafts.

A. Square: $\frac{a}{b} = 1, \quad c_1 = 0.208$ (Table 3.1)

$$\tau_A = \frac{T}{c_1 a b^2} = \frac{T}{0.208 b^3}$$

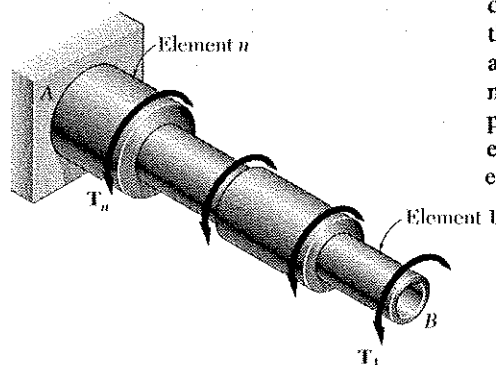
B. Circle: $c = \frac{1}{2}b$

$$\tau_B = \frac{T c}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$$

$$\text{Ratio } \frac{\tau_A}{\tau_B} = \frac{1}{0.208} \cdot \frac{\pi}{16} = 0.3005 \pi$$

$$\frac{\tau_A}{\tau_B} = 0.944 \quad \blacktriangleleft$$

PROBLEM 3.C1



3.C1 Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Its end A is fixed, while its end B is free, and it is subjected to the loading shown. The length of element i is denoted by L_i , its outer diameter by OD_i , its inner diameter by ID_i , its modulus of rigidity by G_i , and the torque applied to its right end by T_i , the magnitude T_i of this torque being assumed to be positive if T_i is observed as counterclockwise from end B and negative otherwise. (Note that $ID_i = 0$ if the element is solid.) (a) Write a computer program that can be used to determine the maximum shearing stress in each element, the angle of twist of each element, and the angle of twist of the entire shaft. (b) Use this program to solve Probs. 3.35 and 3.38.

SOLUTION

FOR EACH CYLINDRICAL ELEMENT, ENTER

$L_i, OD_i, ID_i, G_i, T_i$

AND COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

OUTLINE OF PROGRAM

UPDATE TORQUE $T = T + T_i$

AND COMPUTE

$$\tau_{\max,i} = T(OD_i/2)/J_i$$

$$\phi_{i2} = TL_i/G_i J_i$$

ANGLE OF TWIST OF ENTIRE SHAFT, STARTING

WITH $\Theta = 0$, UPDATE THROUGH n^{th} ELEMENT

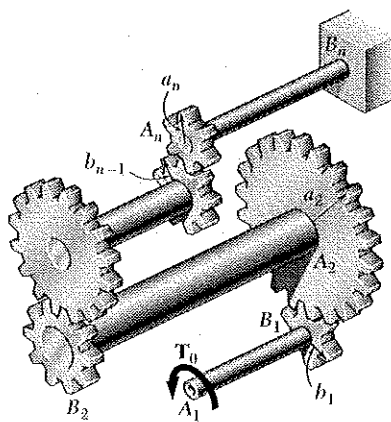
$$\Theta = \Theta + \phi_{i2}$$

PROGRAM OUTPUT

Problem 3.35		
Element	Maximum Stress (MPa)	Angle of Twist (degrees)
1.0000	11.9575	1.3841
2.0000	23.0259	1.8323
Angle of twist for entire shaft =		3.2164 °

Problem 3.38		
Element	Maximum Stress (MPa)	Angle of Twist (degrees)
1.0000	87.3278	4.1181
2.0000	56.5884	1.0392
3.0000	70.5179	0.8633
Angle of twist for entire shaft =		6.0206 °

PROBLEM 3.C2



3.C2 The assembly shown consists of n cylindrical shafts, which can be solid or hollow, connected by gears and supported by brackets (not shown). End A_1 of the first shaft is free and is subjected to a torque T_0 , while end B_n of the last shaft is fixed. The length of shaft $A_i B_i$ is denoted by L_i , its outer diameter by OD_i , its inner diameter by ID_i , and its modulus of rigidity by G_i . (Note that $ID_i = 0$ if the element is solid.) The radius of gear A_i is denoted by a_i , and the radius of gear B_i by b_i . (a) Write a computer program that can be used to determine the maximum shearing stress in each shaft, the angle of twist of each shaft, and the angle through which end A_1 rotates. (b) Use this program to solve Prob. 3.40 and 3.44.

SOLUTION

TORQUE IN SHAFTS. ENTER $T_L = T_0$

$$T_{L+1} = T_L (A_{L+1} / B_L)$$

FOR EACH SHAFT, ENTER

$$L_i \quad OD_i \quad ID_i \quad G_i$$

COMPUTE: $J_i = (\pi/32)(OD_i^4 - ID_i^4)$

$$\tau_{Li} = T_L (OD_i/2) / J_i$$

$$\phi_{Li} = T_L L_i / G_i J_i$$

ANGLE OF ROTATION AT END A_1

COMPUTE ROTATION AT THE "A" END OF EACH SHAFT

START WITH ANGLE = ϕ_{Ln} AND UPDATE

FROM n TO 1, AND ADD ϕ_{Li}

$$\text{ANGLE} = \text{ANGLE}(A_i) / B_{i-1} + \phi_{Li-1}$$

PROGRAM OUTPUT

Shaft No.	Max Stress (MPa)	Angle of Twist (degrees)
1	62.17	1.472
2	84.80	1.719

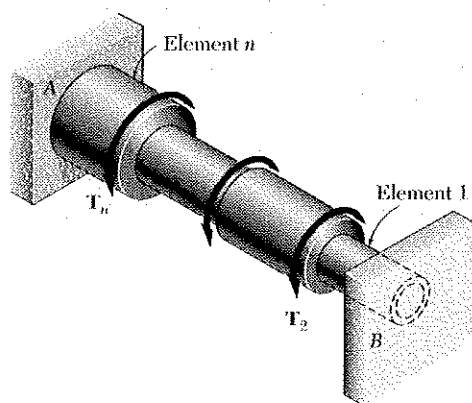
Angle through which A_1 rotates = 3.816°

PROGRAM OUTPUT

Shaft No.	Max Stress (MPa)	Angle of Twist (degrees)
1	382.17	22.403
2	191.09	11.202
3	95.55	5.601

Angle through which A_1 rotates = 22.403°

PROBLEM 3.C3



3.C3 Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Both of its ends are fixed, and it is subjected to the loading shown. The length of element i is denoted by L_i , its outer diameter by OD_i , its inner diameter by ID_i , its modulus of rigidity by G_i , and the torque applied to its right end by T_i , the magnitude T_i of this torque being assumed to be positive if T_i is observed as counterclockwise from end B and negative otherwise. Note that $ID_i = 0$ if the element is solid and also that $T_1 = 0$. Write a computer program that can be used to determine the reactions at A and B, the maximum shearing stress in each element, and the angle of twist of each element. Use this program (a) to solve Prob. 3.155, (b) to determine the maximum shearing stress in the shaft of Example 3.05.

SOLUTION WE CONSIDER THE REACTION AT B AS
REDUNDANT AND RELEASE THE SHAFT AT B.
COMPUTE Θ_B WITH $T_B = 0$:

FOR EACH ELEMENT, ENTER
 $L_i, OD_i, ID_i, G_i, T_i$ (NOTE $T_1 = T_B = 0$)

COMPUTE
 $J_i = (\pi/32)(OD_i^4 - ID_i^4)$

UPDATE TORQUE

$$T = T + T_i$$

AND COMPUTE FOR EACH ELEMENT

$$\tau_{i1} = T(OD_i/2)/J_i$$

$$\phi_{i1} = T L_i / G_i J_i$$

COMPUTE Θ_B : STARTING WITH $\Theta = 0$ AND
UPDATING THROUGH n ELEMENTS

$$\Theta_i = \Theta_i + \phi_{i1} \quad ; \quad \Theta_B = \Theta_n$$

COMPUTE Θ_B DUE TO UNIT TORQUE AT B

$$\text{UNIT } \tau_{i1} = OD_i/2J_i$$

$$\text{UNIT } \phi_{i1} = L_i/G_i J_i$$

FOR n ELEMENTS:

$$\text{UNIT } \Theta_B(L) = \text{UNIT } \Theta_B(L) + \text{UNIT } \phi_{i1}$$

SUPERPOSITION:

$$\text{FOR TOTAL ANGLE AT B TO BE ZERO.} \quad \Theta_B + T_B(\text{UNIT } \Theta_B(n)) = 0$$

$$T_B = -\Theta_B / (\text{UNIT } \Theta_B(n))$$

$$\text{THEN} \quad T_A = \sum T(L) + T_B$$

FOR EACH ELEMENT: MAX STRESS: TOTAL $\tau_{i1} = \tau_{i1} + T_B(\text{UNIT } \tau_{i1})$
ANGLE OF TWIST: TOTAL $\phi_{i1} = \phi_{i1} + T_B(\text{UNIT } \phi_{i1})$

PROGRAM OUTPUT

Problem 3.155

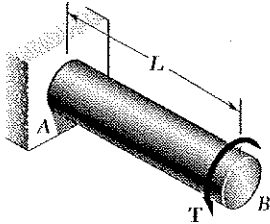
TA = -0.295 kN*m
TB = -1.105 kN*m

Element	tau max (MPa)	Angle of Twist (degrees)
1	-45.024	-0.267
2	27.375	-.267

Problem 3.05

TA = -69.767 Nm
TB = -50.232 Nm

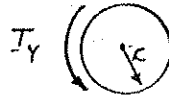
PROBLEM 3.C4



3.C4 The homogeneous, solid cylindrical shaft AB has a length L , a diameter d , a modulus of rigidity G , and a yield strength τ_Y . It is subjected to a torque T that is gradually increased from zero until the angle of twist of the shaft has reached a maximum value ϕ_m and then decreased back to zero. (a) Write a computer program that, for each of 16 values of ϕ_m equally spaced over a range extending from 0 to a value 3 times as large as the angle of twist at the onset of yield, can be used to determine the maximum value T_m of the torque, the radius of the elastic core, the maximum shearing stress, the permanent twist, and the residual shearing stress both at the surface of the shaft and at the interface of the elastic core and the plastic region. (b) Use this program to obtain approximate answers to Probs. 3.114, 3.115, and 3.118.

SOLUTION

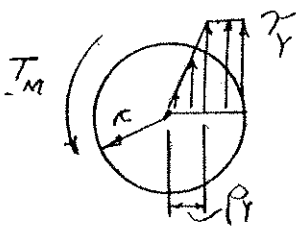
AT ONSET OF YIELD



$$T_Y = \tau_Y \frac{J}{r} = \frac{\pi}{2} \tau_Y r^3$$

$$\phi_Y = \frac{T_Y L}{GJ} = \left(\frac{\tau_Y J}{r} \right) \frac{L}{GJ} = \frac{\tau_Y L}{r G}$$

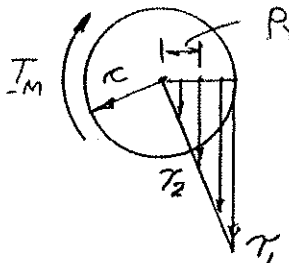
LOADING: $T_m > T_Y$



$$T_m = \frac{4}{3} T_Y \left[1 - \frac{1}{4} \left(\frac{\phi_Y}{\phi_m} \right)^3 \right] \quad \text{EQ. (1)}$$

$$r_Y = r \frac{\phi_Y}{\phi_m} \quad \text{EQ. (2)}$$

UNLOADING (ELASTIC)



$$\phi_U = \frac{T_m L}{GJ}$$

ϕ_U = ANGLE OF TWIST FOR UNLOADING

$$\tau_1 = T_m \frac{r}{J}$$

τ_1 = TAU AT $r = r$

$$\tau_2 = \tau_1 \frac{r_Y}{r}$$

τ_2 = TAU AT $r = r_Y$

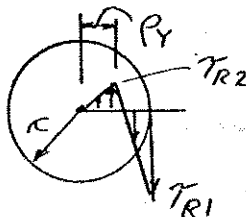
SUPERPOSE LOADING AND UNLOADING

FOR $\phi = 0$ TO $\phi = 3\phi_Y$ USING $0.2\phi_Y$ INCREMENTS

$$\text{WHEN } \phi < \phi_Y: T_m = \tau_Y \frac{\phi}{\phi_Y} \quad r_Y = \frac{1}{2} d \quad \phi_m = \phi_Y \frac{\phi}{\phi_Y}$$

$$\text{WHEN } \phi > \phi_Y: T_m \text{ USE EQ. (1)} \quad r_Y \text{ USE EQ. (2)}$$

$$\text{RESIDUAL: } \phi_R = \phi_m - \phi_U \quad \tau_{R2} = \tau_1 - \tau_Y \quad \tau_{R2} = \tau_2 - \tau_Y$$



CONTINUED

PROBLEM 3.C4 - CONTINUED

Interpolate between values at the values of T_{max} or ϕ_{max} indicated

Problem 3.114 and 3.115

PHIM deg	TM kNm	RY mm	TAUM MPa	PHIP deg	TAUR1 MPa	TAUR2 MPa
0	0	30.480	0	0	0	0
23.635	4.049	30.480	91.014	0	0	0
55.147	8.177	21.768	151.690	7.411	20.389	-32.138
86.660	8.786	13.843	151.690	35.372	61.917	-45.817
118.175	8.914	10.160	151.690	66.138	84.898	-48.693

← T_{max}
= 8.475
kNm.

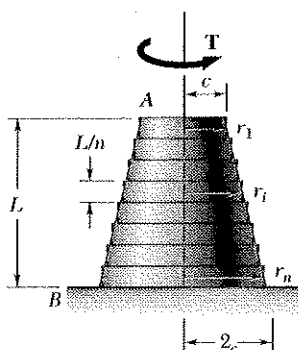
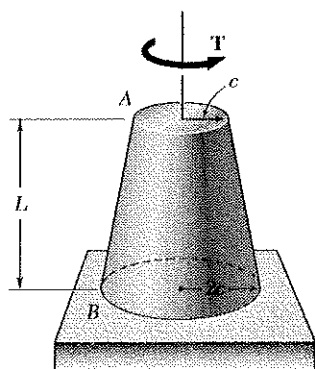
Problem 3.118

PHIM deg	TM kN*m	RY mm	TAUM MPa	PHIP deg	TAUR1 MPa	TAUR2 MPa
0.000	0.000	16.000	0.000	0.000	0.000	0.000
0.807	0.187	16.000	29.000	0.000	0.000	0.000
1.614	0.373	16.000	58.000	0.000	0.000	0.000
2.421	0.560	16.000	87.000	0.000	0.000	0.000
3.228	0.746	16.000	116.000	0.000	0.000	0.000
4.036	0.933	16.000	145.000	0.000	0.000	0.000
4.843	1.064	13.333	145.000	0.240	7.198	-20.363
5.650	1.131	11.429	145.000	0.759	19.486	-30.719
6.457	1.168	10.000	145.000	1.405	31.542	-36.533
7.264	1.191	8.889	145.000	2.114	42.197	-40.046
8.071	1.205	8.000	145.000	2.859	51.354	-42.292
8.878	1.215	7.273	145.000	3.624	59.184	-43.794
9.685	1.221	6.667	145.000	4.402	65.901	-44.837
10.492	1.226	6.154	145.000	5.188	71.699	-45.583
11.300	1.230	5.714	145.000	5.980	76.739	-46.132
12.107	1.232	5.333	145.000	6.776	81.152	-46.543

← $\phi_{max} = 6^\circ$

Proprietary Material. © 2009 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.C5



3.C5 The exact expression is given in Prob. 3.158 for the angle of twist of the solid tapered shaft AB when a torque T is applied as shown. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical shafts of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \dots, n$. Using for T, L, G , and c values of your choice, determine the percentage error in the approximate expression when (a) $n = 4$, (b) $n = 8$, (c) $n = 20$, (d) $n = 100$.

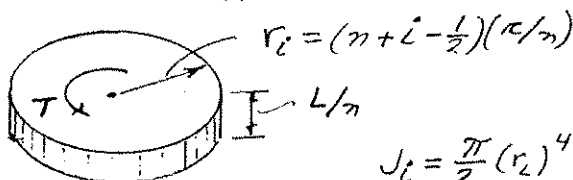
SOLUTION

FROM PROB. 3.158 EXACT EXPRESSION:

$$\phi = \frac{7TL}{12\pi Gc^4}$$

$$\text{OR, } \phi = \left(\frac{7}{12\pi}\right) \frac{TL}{Gc^4} = 0.18568 \frac{TL}{Gc^4}$$

CONSIDER TYPICAL i th SHAFT



$$r_i = (n + i - \frac{1}{2})(c/n)$$

$$J_i = \frac{\pi}{2} (r_i)^4$$

$$\Delta\phi = \frac{T(L/n)}{GJ_i}$$

ENTER UNIT VALUES OF T, L, G , AND c .
(NOTE: SPECIFIC VALUES CAN BE ENTERED)

ENTER INITIAL VALUE OF ZERO FOR ϕ

ENTER n = NUMBER CYLINDRICAL SHAFTS

FOR $i = 1$ TO n , UPDATE ϕ

$$\phi = \phi + \Delta\phi$$

OUTPUT OF PROGRAM

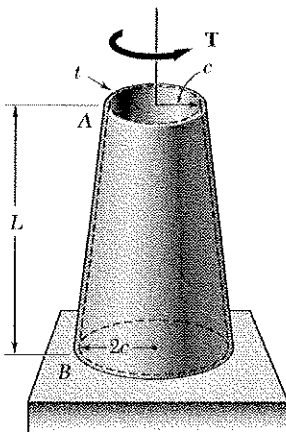
COEFFICIENT of TL/Gc^4

Exact coefficient from Prob. 3.158 is 0.18568

Number of elemental disks = n

n	approximate	exact	percent error
4	0.17959	0.18568	-3.28185
8	0.18410	0.18568	-0.85311
20	0.18542	0.18568	-0.13810
100	0.18567	0.18568	-0.00554

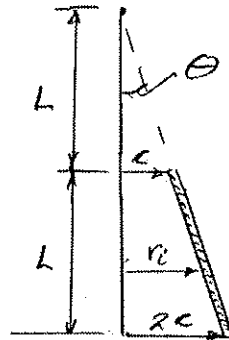
PROBLEM 3.C6



3.C6 A torque T is applied as shown to the long, hollow, tapered shaft AB of uniform thickness t . The exact expression for the angle of twist of the shaft can be obtained from the expression given in Prob. 3.153. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical rings of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \dots, n$. Using for T, L, G, c and t values of your choice, determine the percentage error in the approximate expression when (a) $n = 4$, (b) $n = 8$, (c) $n = 20$, (d) $n = 100$.

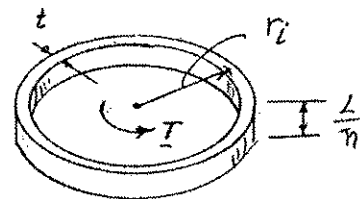
SOLUTION

SINCE THE SHAFT IS LONG $c \ll L$, THE ANGLE Θ IS SMALL AND WE CAN USE t AS THE THICKNESS OF THE n CYLINDRICAL RINGS.



FOR $c \ll L$

$$\Theta = \tan \Theta = \frac{2c - c}{L} = \frac{c}{L}$$



$$r_i = (n + i - \frac{1}{2})(\frac{c}{n})$$

$$J_i \approx (\text{AREA}) r_i^2 = (2\pi r_i t) r_i^2 = 2\pi t r_i^3$$

$$\Delta\phi = \frac{T(L/n)}{GJ_i}$$

ENTER UNIT VALUES FOR T, L, G, t , AND n

(NOTE: SPECIFIC VALUES CAN BE ENTERED IF DESIRED)

ENTER INITIAL VALUE OF ZERO FOR ϕ

ENTER n = NUMBER OF CYLINDRICAL RINGS

FOR $i = 1$ TO n , UPDATE ϕ

$$\phi = \phi + \Delta\phi$$

OUTPUT OF PROGRAM

COEFFICIENT of TL/Gtc^3

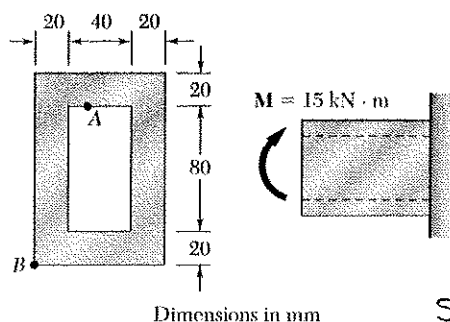
Exact coefficient from Prob. 3.153 is 0.05968
Number of elemental disks = n

n	approximate	exact	percent error
4	0.058559	0.059683	-1.883078
8	0.059394	0.059683	-0.483688
20	0.059637	0.059683	-0.078022
100	0.059681	0.059683	-0.003127

Chapter 4

Problem 4.1

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.



For rectangle $I = \frac{1}{12} b h^3$

Outside rectangle: $I_1 = \frac{1}{12} (80)(120)^3$

$$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$$

Cutout: $I_2 = \frac{1}{12} (40)(80)^3$

$$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

(a) $y_A = 40 \text{ mm} = 0.040 \text{ m}$ $\sigma_A = -\frac{M y_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$

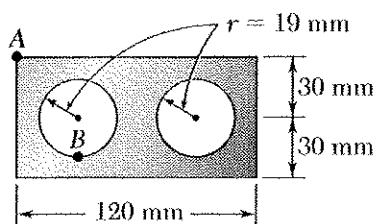
$\sigma_A = -61.6 \text{ MPa}$ ◀

(b) $y_B = -60 \text{ mm} = -0.060 \text{ m}$ $\sigma_B = -\frac{M y_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$

$\sigma_B = 91.7 \text{ MPa}$ ▶

Problem 4.2

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.



For the solid rectangle,

$$I_1 = \frac{1}{12} b h^3 = \frac{1}{12} (0.12)(0.06)^3 = 2.16 \times 10^{-6} \text{ m}^4$$

For one circular cutout, $I_2 = \frac{\pi}{4} r^4 = \frac{\pi}{4} (0.019)^4 = 102.35 \times 10^{-9} \text{ m}^4$

For the section, $I = I_1 - 2 I_2 = 2.16 \times 10^{-6} - (2)(102.35 \times 10^{-9}) = 1.955 \times 10^{-6} \text{ m}^4$

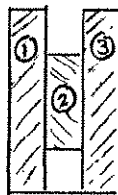
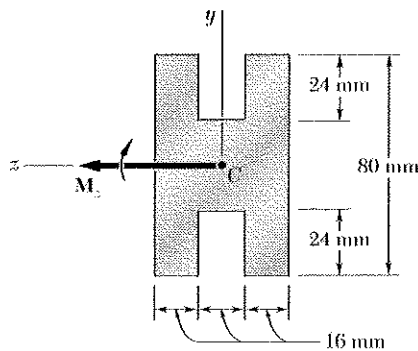
(a) $y_A = 0.03 \text{ m}$ $\sigma_A = -\frac{M y_A}{I} = -\frac{(2000)(0.03)}{1.955 \times 10^{-6}} = -30.69 \text{ MPa}$

$\sigma_A = -30.7 \text{ MPa}$ ◀

(b) $y_B = -0.019 \text{ m}$ $\sigma_B = -\frac{M y_B}{I} = -\frac{(2000)(-0.019)}{1.955 \times 10^{-6}} = 19.44 \text{ MPa}$

$\sigma_B = 19.4 \text{ MPa}$ ▶

Problem 4.3



4.3 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.

$$\text{Allowable stress} = \frac{\sigma_U}{F.S.} = \frac{450}{3} = 150 \text{ MPa} = 150 \times 10^6 \text{ Pa}$$

Moment of inertia about z axis.

$$I_1 = \frac{1}{12} (16)(80)^3 = 682.67 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} (16)(32)^3 = 43.69 \times 10^3 \text{ mm}^4$$

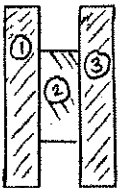
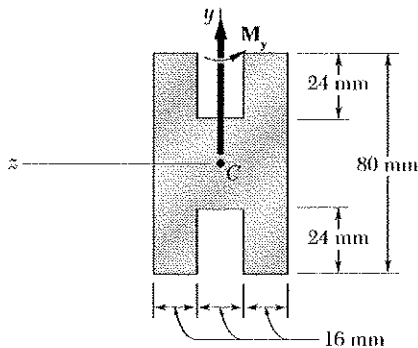
$$I_3 = I_1 = 682.67 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 1.40902 \times 10^6 \text{ mm}^4 = 1.40902 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with } c = \frac{1}{2}(80) = 40 \text{ mm} = 0.040 \text{ m}$$

$$M = \frac{I\sigma}{c} = \frac{(1.40902 \times 10^{-6})(150 \times 10^6)}{0.040} = 5.28 \times 10^3 \text{ N}\cdot\text{m} \quad M = 5.28 \text{ kN}\cdot\text{m}$$

Problem 4.4



4.4 Solve Prob. 4.3, assuming that the beam is bent about the y axis.

4.3 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z axis.

$$\text{Allowable stress} = \frac{\sigma_U}{F.S.} = \frac{450}{3.00} = 150 \text{ MPa} = 150 \times 10^6 \text{ Pa}$$

Moment of inertia about y-axis.

$$I_1 = \frac{1}{12} (80)(16)^3 + (80)(16)(16)^2 = 354.987 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} (32)(16)^3 = 10.923 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 354.987 \times 10^3 \text{ mm}^4$$

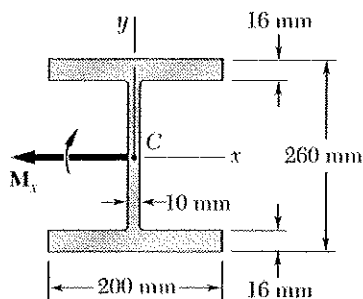
$$I = I_1 + I_2 + I_3 = 720.9 \times 10^3 \text{ mm}^4 = 720.9 \times 10^{-9} \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with } c = \frac{1}{2}(48) = 24 \text{ mm} = 0.024 \text{ m}$$

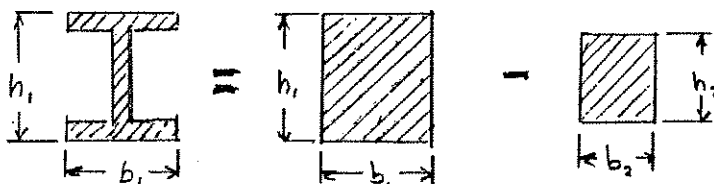
$$M = \frac{I\sigma}{c} = \frac{(720.9 \times 10^{-9})(150 \times 10^6)}{0.024} = 4.51 \times 10^3 \text{ N}\cdot\text{m} \quad M = 4.51 \text{ kN}\cdot\text{m}$$

Problem 4.5

4.5 The steel beam shown is made of a grade of steel for which $\sigma_Y = 250$ MPa and $\sigma_U = 400$ MPa. Using a factor of safety of 2.50, determine the largest couple that can be applied to the beam when it is bent about the x axis.



The moment of inertia I_x is equivalent to that of a rectangle with a cutout



Larger rectangle: $b_1 = 200$ mm $h_1 = 260$ mm $I_1 = \frac{1}{12} b_1 h_1^3$

$$I_1 = \frac{1}{12} (200)(260)^3 = 292.933 \times 10^6 \text{ mm}^4$$

Smaller rectangle: $b_2 = 200 - 10 = 190$ mm $h_2 = 260 - (2)(16) = 228$ mm

$$I_2 = \frac{1}{12} (190)(228)^3 = 187.662 \times 10^6 \text{ mm}^4$$

Section: $I_x = I_1 - I_2 = 105.271 \times 10^6 \text{ mm}^4 = 105.271 \times 10^{-6} \text{ m}^4$

$$c = \frac{260}{2} = 130 \text{ mm} = 0.130 \text{ m}$$

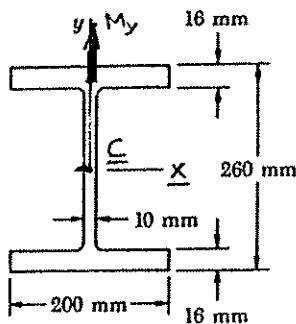
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{400}{2.50} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$\sigma_{all} = \frac{M_x c}{I_x}$$

$$M_x = \frac{I_x \sigma_{all}}{c} = \frac{(105.271 \times 10^{-6})(160 \times 10^6)}{0.130} = 129.564 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_x = 129.6 \text{ kN}\cdot\text{m}$$

Problem 4.6



4.6 Solve Prob. 4.5, assuming that the steel beam is bent about the y axis by a couple of moment M_y .

4.5 The steel beam shown is made of a grade of steel for which $\sigma_Y = 250$ MPa and $\sigma_U = 400$ MPa. Using a factor of safety of 2.50, determine the largest couple that can be applied to the beam when it is bent about the x axis.

For one flange, $I_f = \frac{1}{12} b_f h_f^3$

$b_f = 16 \text{ mm}$ $h_f = 200 \text{ mm}$

$I_f = \frac{1}{12} (16)(200)^3 = 10.6667 \times 10^6 \text{ mm}^4$

For the web $I_w = \frac{1}{12} b_w h_w^3$

$b_w = 260 - (2)(16) = 228 \text{ mm}$ $h_w = 10 \text{ mm}$

$I_w = \frac{1}{12} (228)(10)^3 = 19 \times 10^3 \text{ mm}^4$

Section: $I_y = 2 I_f + I_w = 21.352 \times 10^6 \text{ mm}^4 = 21.352 \times 10^{-6} \text{ m}^4$

$c = \frac{260}{2} = 130 \text{ mm} = 0.130 \text{ m}$

$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{400}{2.50} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

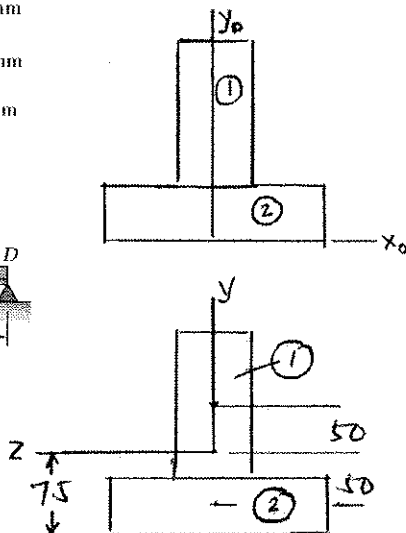
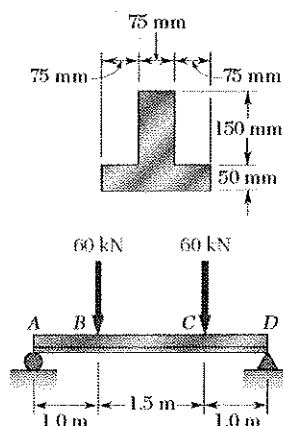
$\sigma_{all} = \frac{M_y c}{I_y}$

$M_y = \frac{I_y \sigma_{all}}{c} = \frac{(21.352 \times 10^{-6})(160 \times 10^6)}{0.130}$
 $= 26.24 \times 10^3 \text{ N}\cdot\text{m}$

$M_y = 34.2 \text{ kN}\cdot\text{m}$

Problem 4.7

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.



	A	$\bar{y}_0$	$A\bar{y}_0$
①	11250	125	1406250
②	11250	25	281250
Σ	22500		1687500

$$\bar{y}_0 = \frac{1687500}{22500} = 75 \text{ mm}$$

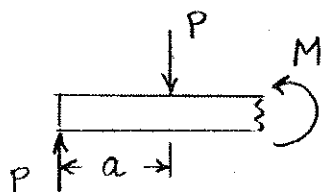
Neutral axis lies 75 mm. above the base.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (0.075)(0.15)^3 + (0.01125)(0.05)^2 = 49.22 \times 10^{-6} \text{ m}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (0.225)(0.105)^3 + (0.01125)(0.05)^2 = 30.47 \times 10^{-6} \text{ m}^4$$

$$I = I_1 + I_2 = 79.7 \times 10^{-6} \text{ m}^4$$

$$y_{\text{top}} = 125 \text{ mm} \quad y_{\text{bot}} = -75 \text{ mm}$$



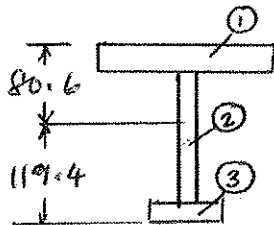
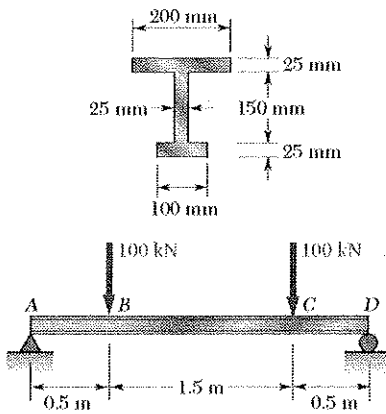
$$M - Pa = 0$$

$$M = Pa = (60)(1) = 60 \text{ kN}\cdot\text{m}$$

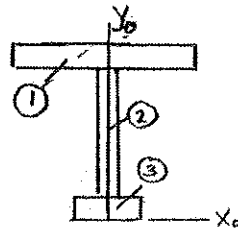
$$\sigma_{\text{top}} = -\frac{M y_{\text{top}}}{I} = -\frac{(60000)(0.125)}{79.7 \times 10^{-6}} = -94.1 \text{ MPa (compression)}$$

$$\sigma_{\text{bot}} = -\frac{M y_{\text{bot}}}{I} = -\frac{(60000)(-0.075)}{79.7 \times 10^{-6}} = 56.5 \text{ MPa (tension)}$$

Problem 4.8



4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.



	$(\times 10^3)$ A	$\bar{y}_0$	$(\times 10^3)$ $A\bar{y}_0$
①	5	187.5	937
②	3.75	100	375
③	2.5	12.5	31.25
Σ	11.25		1343.25

$$\bar{Y}_0 = \frac{1343.25}{11.25} = 119.4 \text{ mm}$$

Neutral axis lies 119.4 mm above the base.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (200)(25)^3 + (5000)(68.1)^2$$

$$= 23.448 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (25)(150)^3 + (3750)(19.4)^2$$

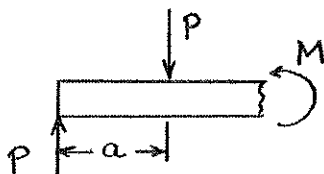
$$= 8.443 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12} (100)(25)^3 + (2500)(106.9)^2 = 28.699 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 60.59 \times 10^6 \text{ mm}^4$$

$$y_{\text{top}} = 80.6 \text{ mm}$$

$$y_{\text{bot}} = -119.4 \text{ mm}$$



$$M - Pa = 0$$

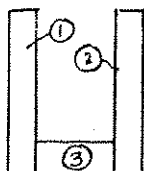
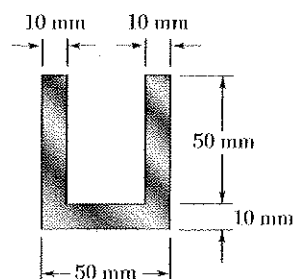
$$M = Pa = (60)(1) = 60 \text{ kN}\cdot\text{m}$$

$$\sigma_{\text{top}} = - \frac{M y_{\text{top}}}{I} = - \frac{(60000)(0.0806)}{60.59 \times 10^{-6}} = -79.8 \text{ MPa (compression)}$$

$$\sigma_{\text{bot}} = - \frac{M y_{\text{bot}}}{I} = - \frac{(60000)(-0.1194)}{60.59 \times 10^{-6}} = 118.2 \text{ MPa (tension)}$$

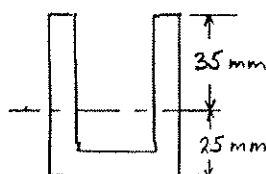
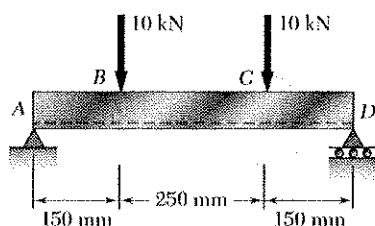
Problem 4.9

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.



	A, mm^2	$\bar{y}_o, \text{mm}$	$A\bar{y}_o, \text{mm}^3$
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3

$$\bar{Y}_o = \frac{37.5 \times 10^3}{1500} = 25 \text{ mm}$$



Neutral axis lies 25 mm above the base.

$$I_1 = \frac{1}{12} (10)(60)^3 + (600)(5)^2 = 195 \times 10^3 \text{ mm}^4$$

$$I_2 = I_1 = 195 \text{ mm}^4$$

$$I_3 = \frac{1}{12} (30)(10)^3 + (300)(20)^2 = 122.5 \times 10^3 \text{ mm}^4$$

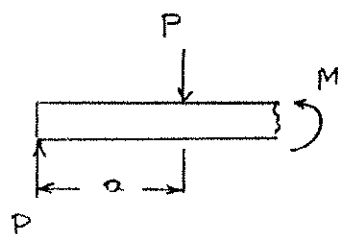
$$I = I_1 + I_2 + I_3 = 512.5 \times 10^3 \text{ mm}^4 = 512.5 \times 10^{-9} \text{ m}^4$$

$$y_{\text{top}} = 35 \text{ mm} = 0.035 \text{ m}$$

$$y_{\text{bot}} = -25 \text{ mm} = -0.025 \text{ m}$$

$$a = 150 \text{ mm} = 0.150 \text{ m} \quad P = 10 \times 10^3 \text{ N}$$

$$M = Pa = (10 \times 10^3)(0.150) = 1.5 \times 10^3 \text{ N}\cdot\text{m}$$



$$\sigma_{\text{top}} = - \frac{M y_{\text{top}}}{I} = - \frac{(1.5 \times 10^3)(0.035)}{512.5 \times 10^{-9}} = -102.4 \times 10^6 \text{ Pa}$$

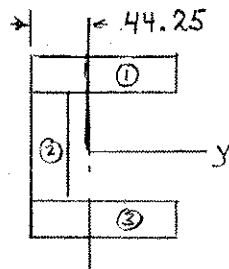
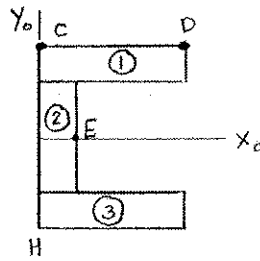
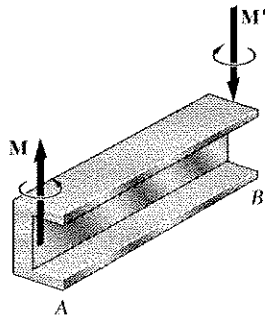
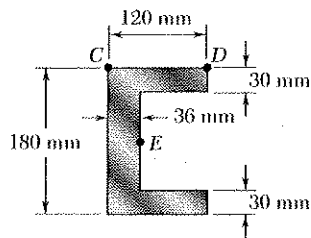
$$\sigma_{\text{top}} = -102.4 \text{ MPa} \quad \leftarrow \text{(compression)}$$

$$\sigma_{\text{bot}} = - \frac{M y_{\text{bot}}}{I} = - \frac{(1.5 \times 10^3)(-0.025)}{512.5 \times 10^{-9}} = 73.2 \times 10^6 \text{ Pa}$$

$$\sigma_{\text{bot}} = 73.2 \text{ MPa} \quad \leftarrow \text{(tension)}$$

Problem 4.10

4.10 Two equal and opposite couples of magnitude $M = 25 \text{ kN} \cdot \text{m}$ are applied to the channel-shaped beam AB . Observing that the couples cause the beam to bend in a horizontal plane, determine the stress at (a) point C , (b) point D , (c) point E .



	A, mm^2	$\bar{x}_0, \text{mm}$	$A\bar{x}_0, \text{mm}^3$
①	3600	60	216×10^3
②	4320	18	77.76×10^3
③	3600	60	216×10^3
Σ	11520		509.76×10^3

$$\bar{X} = \frac{509.76 \times 10^3}{11520} = 44.25 \text{ mm}$$

$$y_c = -44.25 \text{ mm} = -0.04425 \text{ m}$$

$$y_D = 120 - 44.25 = 75.75 \text{ mm} = 0.07575 \text{ m}$$

$$y_E = 36 - 44.25 = -8.25 \text{ mm} = -0.00825 \text{ m}$$

$$d_1 = 60 - 44.25 = 15.75 \text{ mm}$$

$$d_2 = 44.25 - 18 = 26.25 \text{ mm}$$

$$d_3 = d_1$$

$$I_1 = I_3 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (30)(120)^3 + (3600)(15.75)^2 = 5.2130 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (120)(36)^3 + (4320)(26.25)^2 = 3.4433 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187 \times 10^6 \text{ mm}^4 = 13.8694 \times 10^{-6} \text{ m}^4$$

$$M = 15 \times 10^3 \text{ N} \cdot \text{m}$$

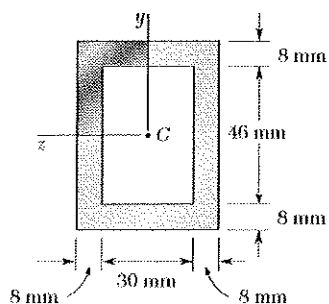
(a) Point C: $\sigma_c = -\frac{My_c}{I} = -\frac{(25 \times 10^3)(-0.04425)}{13.8694 \times 10^{-6}} = 79.8 \times 10^6 \text{ Pa}$
 $\sigma_c = 79.8 \text{ MPa}$

(b) Point D: $\sigma_D = -\frac{My_D}{I} = -\frac{(25 \times 10^3)(0.07575)}{13.8694 \times 10^{-6}} = -136.5 \times 10^6 \text{ Pa}$
 $\sigma_D = -136.5 \text{ MPa}$

(c) Point E: $\sigma_E = -\frac{My_E}{I} = -\frac{(25 \times 10^3)(-0.00825)}{13.8694 \times 10^{-6}} = 14.87 \times 10^6 \text{ Pa}$
 $\sigma_E = 14.87 \text{ MPa}$

Problem 4.11

4.11 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 900 N · m, determine the total force acting on the shaded portion of the beam.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

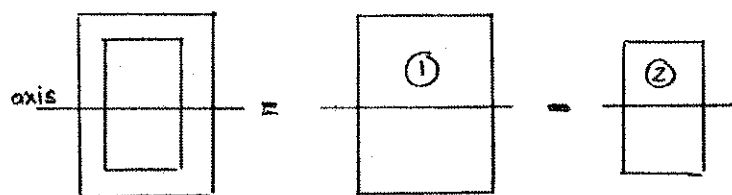
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

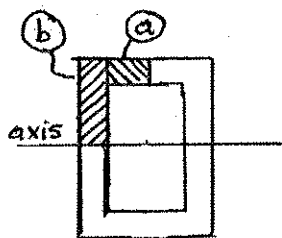
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



$$\begin{aligned} I &= I_1 - I_2 \\ &= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3 \\ &= \frac{1}{12} (46)(62)^3 - \frac{1}{12} (30)(46)^3 \\ &= 0.67025 \times 10^6 \text{ mm}^4 \end{aligned}$$



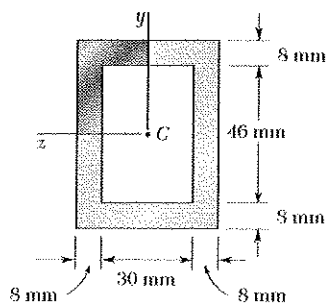
$$\begin{aligned} \bar{y}^* A^* &= \bar{y}_a A_a + \bar{y}_b A_b \\ &= (27)(23)(8) + (11.5)(23)(8) = 7084 \text{ mm}^3 \end{aligned}$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(900)(7084 \times 10^{-9})}{0.67025 \times 10^{-6}} = 9.51 \text{ kN}$$

Problem 4.12

4.12 Solve Prob. 4.11, assuming that the beam is bent about a vertical axis and that the bending moment is $900 \text{ N} \cdot \text{m}$.

4.11 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is $900 \text{ N} \cdot \text{m}$, determine the total force acting on the shaded portion of the beam.



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

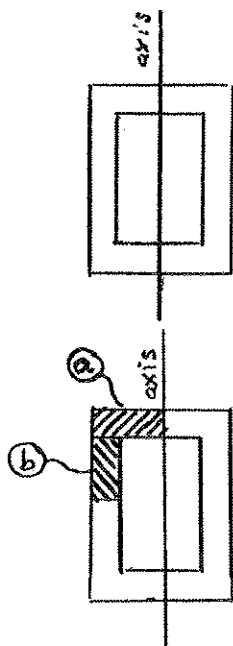
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



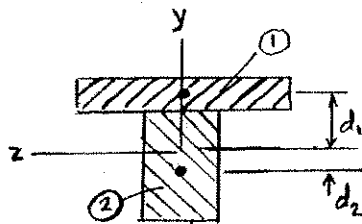
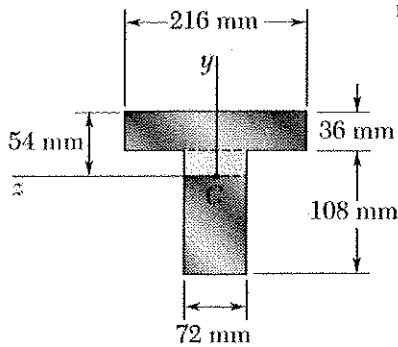
$$\begin{aligned} I &= I_1 - I_2 \\ &= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3 \\ &= \frac{1}{12} (62)(46)^3 - \frac{1}{12} (46)(30)^3 \\ &= 0.399403 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} \bar{y}^* A^* &= \bar{y}_a A_a + \bar{y}_b A_b \\ &= (11.5)(8)(23) + (18)(23)(8) \\ &= 5428 \text{ mm}^3 \end{aligned}$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(900)(5428 \times 10^{-9})}{0.399403 \times 10^{-6}} = 12.2 \text{ kN}$$

Problem 4.13

4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is $6 \text{ kN} \cdot \text{m}$, determine the total force acting on the top flange.



$$d_1 = 54 - 18 = 36 \text{ mm}$$

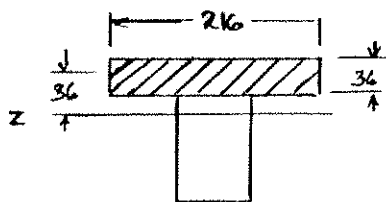
$$d_2 = 54 - 36 - 54 = 86 \text{ mm}$$

Moment of inertia of entire cross section.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (216)(36)^3 + (216)(36)(36)^2 = 10.9175 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (72)(108)^3 + (72)(108)(36)^2 = 17.6360 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 28.5535 \times 10^6 \text{ mm}^4 = 28.5535 \times 10^{-6} \text{ m}^4$$



For the shade area

$$A^* = (216)(36) = 7776 \text{ mm}^2$$

$$\bar{y}^* = 36 \text{ mm}$$

$$A^* \bar{y}^* = 279.936 \times 10^3 \text{ mm}^3 = 279.936 \times 10^{-6} \text{ m}^3$$

$$F = - \left| \frac{M A^* \bar{y}^*}{I} \right| = \frac{(6 \times 10^3)(279.936 \times 10^{-6})}{28.5535 \times 10^{-6}}$$

$$= 58.8 \times 10^3 \text{ N}$$

$$F = 58.8 \text{ kN} \quad \blacktriangleleft$$

The stress distribution over the entire cross-section is given by the bending stress formula

$$\sigma_x = - \frac{M y}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x dA = - \frac{M y}{I} dA$$

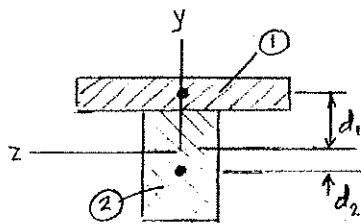
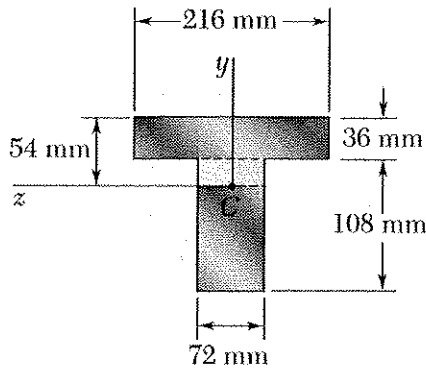
The total force on the shaded area is then

$$F = \int dF = - \int \frac{M y}{I} dA = - \frac{M}{I} \int y dA = - \frac{M}{I} \bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

Problem 4.14

4.14 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is $6 \text{ kN} \cdot \text{m}$, determine the total force acting on the shaded portion of the web.



$$d_1 = 54 - 18 = 36 \text{ mm}$$

$$d_2 = 54 + 36 - 54 = 36 \text{ mm}$$

Moment of inertia of entire cross section.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (216)(36)^3 + (216)(36)(36)^2 = 10.9175 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (72)(108)^3 + (72)(108)(36)^2 = 17.6360 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 28.5535 \times 10^6 \text{ mm}^4 = 28.5535 \times 10^{-6} \text{ m}^4$$

For the shaded area

$$A^* = (72)(90) = 6480 \text{ mm}^2$$

$$\bar{y}^* = 45 \text{ mm}$$

$$A^* \bar{y}^* = 291.6 \times 10^3 \text{ mm}^3 = 291.6 \times 10^{-6} \text{ m}^3$$

$$F = \left| \frac{M A^* \bar{y}^*}{I} \right| = \frac{(6 \times 10^3)(291.6 \times 10^{-6})}{28.5535 \times 10^{-6}}$$

$$= 61.3 \times 10^3 \text{ N}$$

$$F = 61.3 \text{ kN}$$

The stress distribution over the entire cross-section is given by the bending stress formula

$$\sigma_x = - \frac{M y}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x dA = - \frac{M y}{I} dA$$

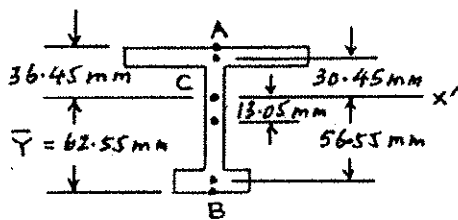
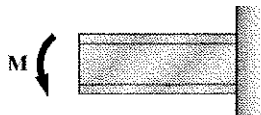
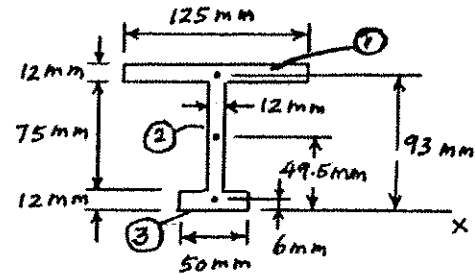
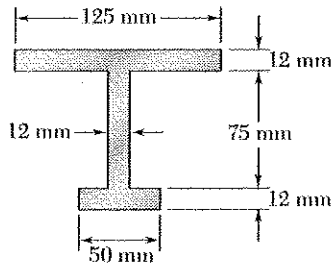
The total force on the shaded area is then

$$F = \int dF = - \int \frac{M y}{I} dA = - \frac{M}{I} \int y dA = - \frac{M}{I} \bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

Problem 4.15

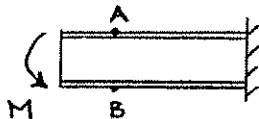
4.15 Knowing that for the casting shown the allowable stress is 42 MPa in tension and 105 MPa in compression, determine the largest couple M that can be applied.



	A, mm^2	$\bar{y}, \text{mm}$	$A\bar{y}, \text{mm}^3$
①	1500	93	139500
②	900	49.5	44550
③	600	6	3600
Σ	3000		187650

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{187650}{3000} = 62.55 \text{ mm}$$

$$\begin{aligned} I_{x'} &= \Sigma \left(\frac{1}{12} b h^3 + A d^2 \right) \\ &= \frac{1}{12} (125)(12)^3 + (125)(12)(30.45)^2 \\ &\quad + \frac{1}{12} (12)(75)^3 + (12)(75)(13.05)^2 \\ &\quad + \frac{1}{12} (50)(12)^3 + (50)(12)(56.55)^2 \\ &= 3909900 \text{ mm}^4 = 3.91 \times 10^{-6} \text{ m}^4 \end{aligned}$$



ASSUME: $\sigma_A = +\sigma_{\text{ALL}} = +42 \text{ MPa}$

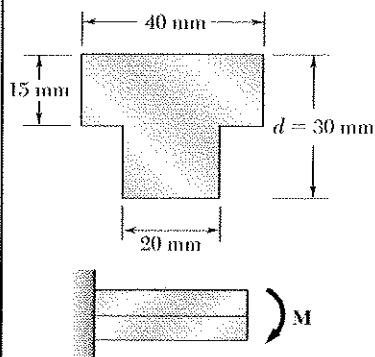
$$M = \sigma_A \frac{I_{x'}}{c} = (42 \times 10^6) \frac{3.91 \times 10^{-6}}{0.03645} = 4505 \text{ Nm}$$

ASSUME: $\sigma_B = -\sigma_{\text{ALL}} = -105 \text{ MPa}$

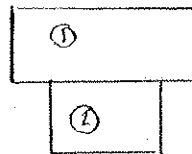
$$M = \sigma_B \frac{I_{x'}}{c} = (105 \times 10^6) \frac{3.91 \times 10^{-6}}{0.06255} = 6563.5 \text{ Nm}$$

CHOOSE THE SMALLER M : $M_{\text{ALL}} = 4.5 \text{ kNm}$

Problem 4.16



4.16 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple M that can be applied to the beam.



	A, mm^2	$\bar{y}_o, \text{mm}$	$A\bar{y}_o, \text{mm}^3$
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3

$$\bar{Y}_o = \frac{15.75 \times 10^3}{900} = 17.5 \text{ mm}$$

The neutral axis lies 17.5 mm above the bottom.

$$y_{\text{top}} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}, \quad y_{\text{bot}} = -17.5 \text{ mm} = -0.0175 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (40)(15)^3 + (600)(5)^2 = 26.25 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (20)(15)^3 + (300)(10)^2 = 35.625 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 61.875 \times 10^3 \text{ mm}^4 = 61.875 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{My}{I} \right| \quad M = \left| \frac{\sigma I}{y} \right|$$

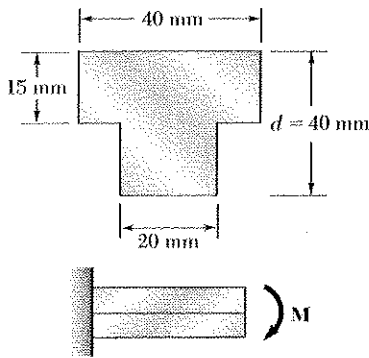
$$\text{Top: tension side. } M = \frac{(24 \times 10^6)(61.875 \times 10^{-9})}{0.0125} = 118.8 \text{ N}\cdot\text{m}$$

$$\text{Bottom: compression. } M = \frac{(30 \times 10^6)(61.875 \times 10^{-9})}{0.0175} = 106.1 \text{ N}\cdot\text{m}$$

Choose smaller value.

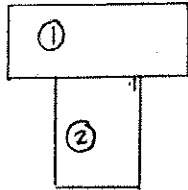
$$M = 106.1 \text{ N}\cdot\text{m}$$

Problem 4.17



4.17 Solve Prob. 4.16, assuming that $d = 40$ mm.

4.16 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple M that can be applied to the beam.



	A, mm^2	$\bar{y}_o, \text{mm}$	$A\bar{y}_o, \text{mm}^3$
①	600	32.5	19.5×10^3
②	500	12.5	6.25×10^3
Σ	1100		25.75×10^3

$$\bar{Y}_o = \frac{25.75 \times 10^3}{1100} = 23.41 \text{ mm}$$

The neutral axis lies 23.41 mm above the bottom.

$$y_{\text{top}} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$$

$$y_{\text{bot}} = -23.41 \text{ mm} = -0.02341 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (40)(15)^3 + (600)(9.09)^2 = 60.827 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (20)(40)^3 + (500)(10.91)^2 = 85.556 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 146.383 \times 10^3 \text{ mm}^4 = 146.383 \times 10^{-9} \text{ m}^4$$

$$161 = \left| \frac{My}{I} \right| \quad M = \left| \frac{\sigma I}{y} \right|$$

$$\text{Top: tension side.} \quad M = \frac{(24 \times 10^6)(146.383 \times 10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$$

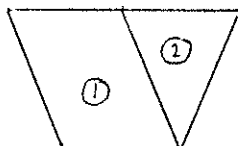
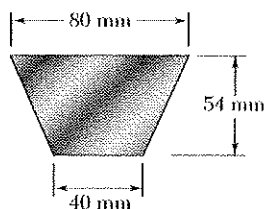
$$\text{Bottom: compression.} \quad M = \frac{(30 \times 10^6)(146.383 \times 10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$$

Choose smaller value.

$$M = 187.6 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.18

4.18 and 4.19 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple M that can be applied.



	A, mm^2	$\bar{y}_o, \text{mm}$	$A\bar{y}_o, \text{mm}^3$	d, mm
①	2160	27	58320	3
②	1080	36	38880	3
Σ	3240		97200	

$$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$$

The neutral axis lies 30 mm above the bottom.

$$y_{\text{top}} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m}$$

$$y_{\text{bot}} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{36} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{36} (40)(54)^3 + \frac{1}{2} (40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{My}{I} \right|$$

$$|M| = \left| \frac{\sigma I}{y} \right|$$

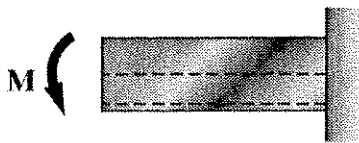
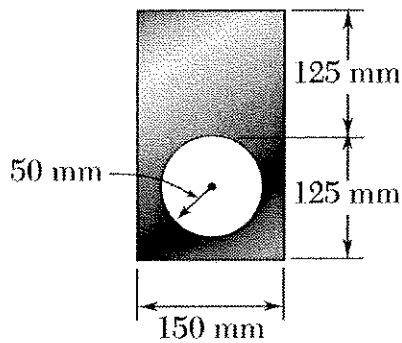
$$\text{top: tension side} \quad M = \frac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{bottom: compression} \quad M = \frac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller as M_{all} . $M_{\text{all}} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

$$M_{\text{all}} = 3.79 \text{ kN}\cdot\text{m}$$

Problem 4.19



4.18 and 4.19 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple M that can be applied.

① rectangle ② circular cutout

$$A_1 = (150)(250) = 37.5 \times 10^3 \text{ mm}^2$$

$$A_2 = -\pi(50)^2 = -7.85398 \times 10^3 \text{ mm}^2$$

$$A = A_1 - A_2 = 29.64602 \times 10^3 \text{ mm}^2$$

$$\bar{y}_1 = 0 \text{ mm}$$

$$\bar{y}_2 = -50 \text{ mm}$$

$$\bar{Y} = \frac{\sum A \bar{y}}{\sum A}$$

$$\bar{Y} = \frac{(37.5 \times 10^3)(0) + (-7.85398 \times 10^3)(-50)}{29.64602 \times 10^3}$$

$$= 13.2463 \text{ mm}$$

$$I_{x'} = \sum (I + Ad^2) = I_1 - I_2$$

$$= \left[\frac{1}{12} (150)(250)^3 + (37.5 \times 10^3)(13.2463)^2 \right]$$

$$- \left[\frac{\pi}{4} (50)^4 + (7.85398 \times 10^3)(50 + 13.2463)^2 \right]$$

$$= 201.892 \times 10^6 - 36.3254 \times 10^6 = 165.567 \times 10^6 \text{ mm}^4 = 165.567 \times 10^{-6} \text{ m}^4$$

Top: (Tension side) $C = 125 - 13.2463 = 111.7537 \text{ mm} = 0.11175 \text{ m}$

$$\sigma = \frac{Mc}{I} \quad M = \frac{I\sigma}{c} = \frac{(165.567 \times 10^{-6})(120 \times 10^6)}{0.11175}$$

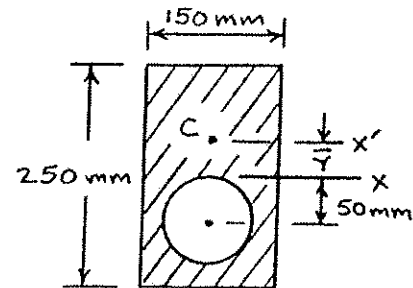
$$= 177.79 \times 10^3 \text{ N}\cdot\text{m}$$

Bottom: (Compression side) $C = 125 + 13.2463 = 138.2463 \text{ mm}$
 $= 0.13825 \text{ m}$

$$\sigma = \frac{Mc}{I} \quad M = \frac{I\sigma}{c} = \frac{(165.567 \times 10^{-6})(150 \times 10^6)}{0.13825}$$

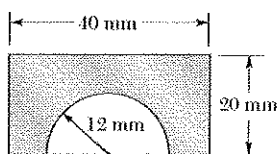
$$= 179.64 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller, $M = 177.8 \times 10^3 \text{ N}\cdot\text{m}$ $M = 177.8 \text{ kN}\cdot\text{m}$



Problem 4.20

4.20 Knowing that for the beam shown the allowable stress is 84 MPa in tension and 110 MPa in compression, determine the largest couple M that can be applied.



① = rectangle ② = semi-circular cutout

$$A_1 = (40)(20) = 800 \text{ mm}^2$$

$$A_2 = \frac{\pi}{2}(12)^2 = 226.2 \text{ mm}^2$$

$$A = 800 - 226.2 = 573.8 \text{ mm}^2$$

$$\bar{y}_1 = 10 \text{ mm}$$

$$\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(12)}{3\pi} = 5.1 \text{ mm}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{(800)(10) - (226.2)(5.1)}{573.8} = 11.9 \text{ mm}$$

Neutral axis lies 11.9 mm above the bottom

Moment of inertia about the base

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(40)(20)^3 - \frac{\pi}{8}(12)^4 = 104075 \text{ mm}^4$$

Centroidal moment of inertia

$$\begin{aligned} \bar{I} &= I_b - A\bar{Y}^2 = 104075 - (573.8)(11.9)^2 \\ &= 22819 \text{ mm}^4 \end{aligned}$$

$$y_{\text{top}} = 20 - 11.9 = 8.1 \text{ mm}, \quad y_{\text{bot}} = -11.9 \text{ mm}$$

$$|\sigma| = \left| \frac{My}{I} \right| \quad M = \left| \frac{\sigma I}{y} \right|$$

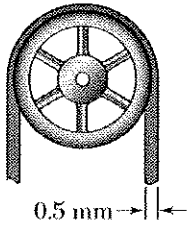
Top: tension side $M = \frac{(84 \times 10^6)(22819 \times 10^{-12})}{0.0081} = 236.6 \text{ Nm}$

Bottom: compression $M = \frac{(110 \times 10^6)(22819 \times 10^{-12})}{0.0119} = 210.9 \text{ Nm}$

Choose the smaller value

$$M = 210.9 \text{ Nm}$$

Problem 4.21



4.21 A steel band blade, that was originally straight, passes over 200 mm diameter pulleys when mounted on a band saw. Determine the maximum stress in the blade, knowing that it is 0.5 mm thick and 16 mm wide. Use $E = 200$ GPa.

Band blade thickness: $t = 0.5 \text{ mm}$

Radius of pulley: $r = \frac{1}{2}d = 100 \text{ mm}$

Radius of curvature of center line of blade:

$$\rho = r + \frac{1}{2}t = 100.25 \text{ mm}$$

$$c = \frac{1}{2}t = 0.25 \text{ mm}$$

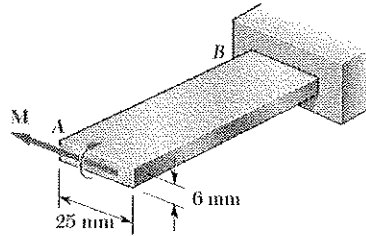
Maximum strain: $\epsilon_m = \frac{c}{\rho} = \frac{0.25}{100.25} = 0.002494$

Maximum stress: $\sigma_m = E\epsilon_m = (200 \times 10^9)(0.002494)$

$$\sigma_m = 498.8 \text{ MPa}$$

$$\sigma_m = 498.8 \text{ MPa}$$

Problem 4.22



4.22 Knowing that $\sigma_{\text{all}} = 165 \text{ MPa}$ for the steel strip AB , determine (a) the largest couple M that can be applied, (b) the corresponding radius of curvature. Use $E = 200 \text{ GPa}$.

$$I = \frac{1}{12} b h^3 = \left(\frac{1}{12}\right)(25)(6)^3 = 450 \text{ mm}^4$$

$$\sigma = \frac{Mc}{I} \quad c = \left(\frac{1}{2}\right)(6) = 3 \text{ mm}$$

$$(a) \quad M = \frac{\sigma I}{c} = \frac{(165 \times 10^6)(450 \times 10^{-12})}{0.003}$$

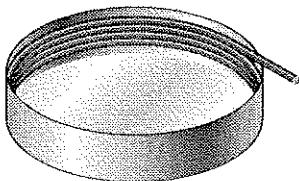
$$M = 24.75 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$(b) \quad \frac{c}{\rho} = \frac{\sigma_{\text{max}}}{E}$$

$$\rho = \frac{Ec}{\sigma_{\text{max}}} = \frac{(200 \times 10^9)(0.003)}{165 \times 10^6}$$

$$\rho = 4 \text{ m} \quad \blacktriangleleft$$

Problem 4.23



4.23 Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine (a) the maximum stress in a coiled rod, (b) the corresponding bending moment in the rod. Use $E = 200 \text{ GPa}$.

Let D = inside diameter of the drum.
 d = diameter of rod, $c = \frac{1}{2}d$,
 ρ = radius of curvature of center line of rods when bent.

$$\rho = \frac{1}{2}D - \frac{1}{2}d = \frac{1}{2}(1.25) - \frac{1}{2}(6 \times 10^{-3}) = 0.622 \text{ m}$$

$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (0.003)^4 = 63.617 \times 10^{-12} \text{ m}^4$$

$$(a) \quad \sigma_{\text{max}} = \frac{Ec}{\rho} = \frac{(200 \times 10^9)(0.003)}{0.622} = 965 \times 10^6 \text{ Pa}$$

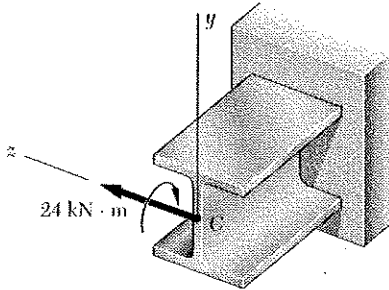
$$\sigma = 965 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad M = \frac{EI}{\rho} = \frac{(200 \times 10^9)(63.617 \times 10^{-12})}{0.622} = 20.5 \text{ N}\cdot\text{m}$$

$$M = 20.5 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.24

4.24 A 24 kN · m couple is applied to the W200 × 46.1 rolled-steel beam shown. (a) Assuming that the couple is applied about the z axis as shown, determine the maximum stress and the radius of curvature of the beam. (b) Solve part a, assuming that the couple is applied about the y axis. Use $E = 200$ GPa.



For W 200 × 46.1 rolled steel section

$$I_x = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$$

$$S_x = 448 \times 10^3 \text{ mm}^3 = 448 \times 10^{-6} \text{ m}^3$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$$

$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

$$(a) \quad M_z = 24 \text{ kN} \cdot \text{m} = 24 \times 10^3 \text{ N} \cdot \text{m}$$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{448 \times 10^{-6}} = 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(45.5 \times 10^{-6})} = 2.637 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 379 \text{ m}$$

$$(b) \quad M_y = 24 \text{ kN} \cdot \text{m} = 24 \times 10^3 \text{ N} \cdot \text{m}$$

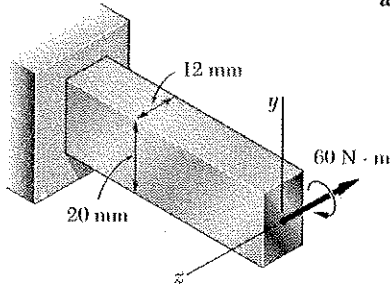
$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{151 \times 10^{-6}} = 158.9 \times 10^6 \text{ Pa} = 158.9 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(15.3 \times 10^{-6})} = 7.84 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 127.5 \text{ m}$$

Problem 4.25

4.25 A 60 N · m couple is applied to the steel bar shown. (a) Assuming that the couple is applied about the z axis as shown, determine the maximum stress and the radius of curvature of the bar. (b) Solve part a, assuming that the couple is applied about the y axis. Use $E = 200$ GPa.



(a) Bending about z-axis.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (12)(20)^3 = 8 \times 10^3 \text{ mm}^4 = 8 \times 10^{-9} \text{ m}^4$$

$$c = \frac{20}{2} = 10 \text{ mm} = 0.010 \text{ m}$$

$$\sigma = \frac{M c}{I} = \frac{(60)(0.010)}{8 \times 10^{-9}} = 75.0 \times 10^6 \text{ Pa} \quad \sigma = 75.0 \text{ MPa} \quad \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{E I} = \frac{60}{(200 \times 10^9)(8 \times 10^{-9})} = 37.5 \times 10^{-3} \text{ m}^{-1} \quad \rho = 26.7 \text{ m} \quad \blacktriangleleft$$

(b) Bending about y-axis.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (20)(12)^3 = 2.88 \times 10^3 \text{ mm}^4 = 2.88 \times 10^{-9} \text{ m}^4$$

$$c = \frac{12}{2} = 6 \text{ mm} = 0.006 \text{ m}$$

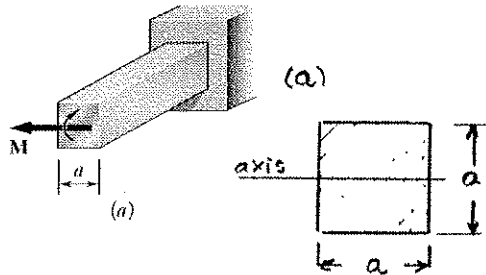
$$\sigma = \frac{M c}{I} = \frac{(60)(0.006)}{2.88 \times 10^{-9}} = 125.0 \times 10^6 \text{ Pa} \quad \sigma = 125.0 \text{ MPa} \quad \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{E I} = \frac{60}{(200 \times 10^9)(2.88 \times 10^{-9})} = 104.17 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 9.60 \text{ m} \quad \blacktriangleleft$$

Problem 4.26

4.26 A couple of magnitude M is applied to a square bar of side a . For each of the orientations shown, determine the maximum stress and the curvature of the bar.



$$I = \frac{1}{12} b h^3 = \frac{1}{12} a a^3 = \frac{a^4}{12}$$

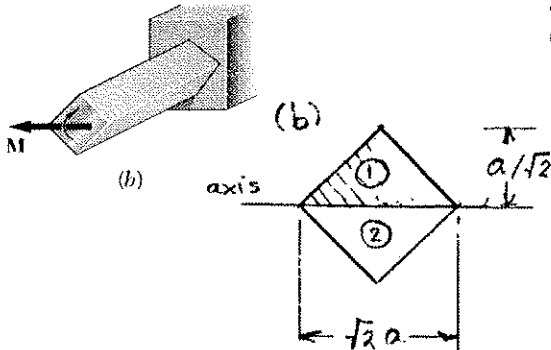
$$c = \frac{a}{2}$$

$$\sigma_{max} = \frac{M c}{I} = \frac{M \frac{a}{2}}{\frac{a^4}{12}}$$

$$\sigma_{max} = \frac{6M}{a^3} \quad \blacktriangle$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E \frac{a^4}{12}}$$

$$\frac{1}{\rho} = \frac{12M}{Ea^4} \quad \blacktriangle$$



For one triangle, the moment of inertia about its base is

$$I_1 = \frac{1}{12} b h^3 = \frac{1}{12} (\sqrt{2}a) \left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

$$I_2 = I_1 = \frac{a^4}{24}$$

$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}}$$

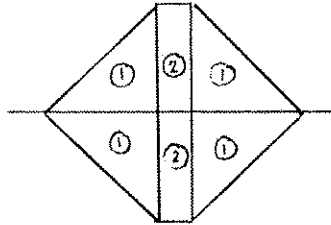
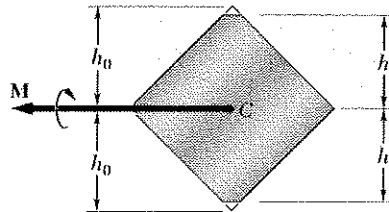
$$\sigma_{max} = \frac{M c}{I} = \frac{M \frac{a}{\sqrt{2}}}{\frac{a^4}{12}} = \frac{6\sqrt{2}M}{a^3}$$

$$\sigma_{max} = \frac{8.49M}{a^3} \quad \blacktriangle$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E \frac{a^4}{12}}$$

$$\frac{1}{\rho} = \frac{12M}{Ea^4} \quad \blacktriangle$$

Problem 4.27



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple M . Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple M , and determine the value of k .

$$\begin{aligned} I &= 4I_1 + 2I_2 \\ &= (4)\left(\frac{1}{12}\right)h h^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)(h^3) \\ &= \frac{1}{3}h^4 + \frac{4}{3}h_0 h^3 - \frac{4}{3}h h^3 = \frac{4}{3}h_0 h^3 - h^4 \end{aligned}$$

$$c = h$$

$$\sigma = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0 h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$

For the original square, $h = h_0$, $c = h_0$

$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

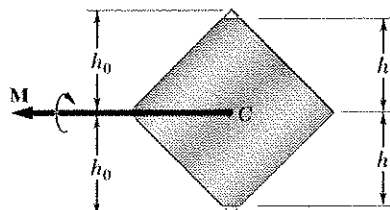
$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - 3(0.9)h_0)(0.9h_0^2)} = 0.950$$

$$\sigma = 0.950 \sigma_0$$

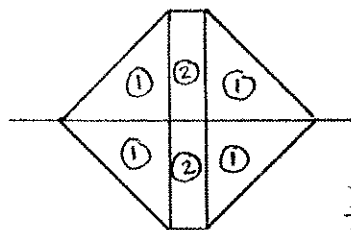
$$k = 0.950$$

Problem 4.28

4.28 In Prob. 4.27, determine (a) the value of h for which the maximum stress σ_m is as small as possible, (b) the corresponding value of k .



4.27 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple M . Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$ where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple M , and determine the value of k .



$$I = 4I_1 + 2I_2$$

$$= (4)(\frac{1}{12})h h^3 + (2)(\frac{1}{3})(2h_0 - 2h)h^3$$

$$= \frac{1}{3}h^4 - \frac{4}{3}h_0 h^3 + \frac{4}{3}h^3 = \frac{4}{3}h_0 h^3 - h^4$$

$$c = h \quad \frac{I}{c} = \frac{4}{3}h_0 h^2 - h^3$$

$$\frac{I}{c} \text{ is maximum at } \frac{d}{dh}[\frac{4}{3}h_0 h^2 - h^3] = 0$$

$$\frac{8}{3}h_0 h - 3h^2 = 0$$

$$h = \frac{8}{9}h_0$$

$$\frac{I}{c} = \frac{4}{3}h_0(\frac{8}{9}h_0)^2 - (\frac{8}{9}h_0)^3 = \frac{256}{729}h_0^3$$

$$\sigma = \frac{Mc}{I} = \frac{729}{256} \frac{M}{h_0^3}$$

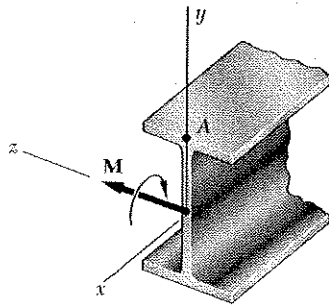
$$\text{For the original square, } h = h_0 \quad c = h_0 \quad \frac{I_0}{c_0} = \frac{1}{3}h_0^3$$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256} \cdot \frac{1}{3} = \frac{729}{768} = 0.949$$

$$k = 0.949$$

Problem 4.29



4.29 A W200 x 31.3 rolled-steel beam is subjected to a couple M of moment 45 kN·m. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine (a) the radius of curvature ρ , (b) the radius of curvature ρ' of a transverse cross section.

For W 200 x 31.3 rolled steel section

$$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$$

$$(a) \quad \frac{1}{\rho} = \frac{M}{EI} = \frac{45 \times 10^3}{(200 \times 10^9)(31.4 \times 10^{-6})} = 7.17 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 139.6 \text{ m}$$

$$(b) \quad \frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(7.17 \times 10^{-3}) = 2.07 \times 10^{-3} \text{ m}^{-1} \quad \rho' = 481 \text{ m}$$

Problem 4.30

4.30 For the bar and loading of Example 4.01, determine (a) the radius of curvature ρ , (b) the radius of curvature ρ' of a transverse cross section, (c) the angle between the sides of the bar that were originally vertical. Use $E = 200$ GPa and $\nu = 0.29$.

From Example 4.01 $M = 3 \text{ kNm}$, $I = 360000 \text{ mm}^4$

$$(a) \quad \frac{1}{\rho} = \frac{M}{EI} = \frac{(3 \times 10^3)}{(200 \times 10^9)(36 \times 10^{-8})} = 0.04167 \text{ m}^{-1} \quad \rho = 24 \text{ m}$$

$$(b) \quad \varepsilon' = \nu \varepsilon = \nu \frac{C}{\rho} = \nu \frac{C}{\rho'}$$

$$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(0.04167) \text{ m}^{-1} = 0.01208 \text{ m}^{-1} \quad \rho' = 82.75 \text{ m}$$

$$(c) \quad \theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.02}{82.75} = 241.7 \times 10^{-6} \text{ rad} = 0.01380^\circ$$

Problem 4.31

4.31 For the aluminum bar and loading of Sample Problem 4.1, determine (a) the radius of curvature ρ' of a transverse cross section, (b) the angle between the sides of the bar that were originally vertical. Use $E = 73$ GPa and $\nu = 0.33$.

From Sample Problem 4.1 $I = 4.38 \times 10^6 \text{ mm}^4$ $M = 10.1 \text{ kN}$

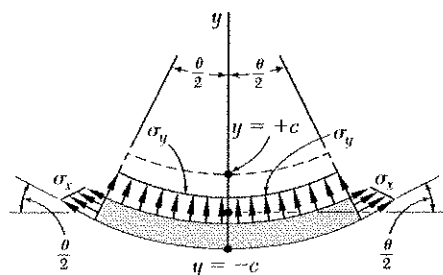
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{10.1 \times 10^3}{(73 \times 10^9)(4.38 \times 10^{-6})} = 0.03159 \text{ m}^{-1}$$

$$(a) \quad \frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.33)(0.03159) = 0.010424 \text{ m}^{-1}$$

$$\rho' = 95.9 \text{ mm}$$

$$(b) \quad \theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.08}{95.9} = 834.2 \times 10^{-6} \text{ rad} = 0.0478^\circ$$

Problem 4.32



4.32 It was assumed in Sec. 4.3 that the normal stresses σ_x in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (a) derive an approximate expression for σ_y as a function of y , (b) show that $(\sigma_y)_{\max} = -(c/2\rho)(\sigma_x)_{\max}$ and, thus, that σ_y can be neglected in all practical situations. (Hint: Consider the free-body diagram of the portion of beam located below the surface of ordinate y and assume that the distribution of the stress σ_x is still linear.)

Denote the width of the beam by b and the length by L .

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos \frac{\theta}{2} \approx 1$

$$\sum F_y = 0 \quad \sigma_y b L + 2 \int_{-c}^y \sigma_x b dy \sin \frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L} \sin \frac{\theta}{2} \int_{-c}^y \sigma_x dy \approx -\frac{\theta}{L} \int_{-c}^y \sigma_x dy = -\frac{1}{\rho} \int_{-c}^y \sigma_x dy$$

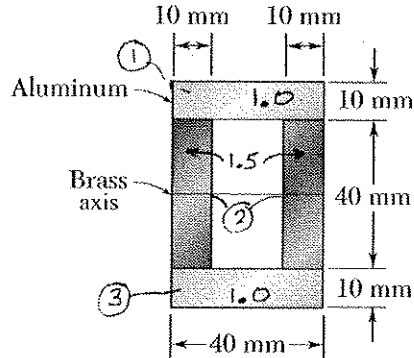
$$\text{But } \sigma_x = -(\sigma_x)_{\max} \frac{y}{c}$$

$$(a) \quad \sigma_y = \frac{(\sigma_x)_{\max}}{\rho c} \int_{-c}^y y dy = \frac{(\sigma_x)_{\max}}{\rho c} \left. \frac{y^2}{2} \right|_{-c}^y = \frac{(\sigma_x)_{\max}}{2\rho c} (y^2 - c^2)$$

The maximum value σ_y occurs at $y = 0$

$$(b) \quad (\sigma_y)_{\max} = -\frac{(\sigma_x)_{\max} c^2}{2\rho c} = -\frac{(\sigma_x)_{\max} c}{2\rho}$$

Problem 4.33



4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12} (40)(10)^3 + (1.0)(40)(10)(25)^2 = 253.333 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12} (20)(40)^3 = 160 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 253.333 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 666.67 \times 10^3 \text{ mm}^4 = 666.67 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

Aluminum: $n = 1.0$, $|y| = 30 \text{ mm} = 0.030 \text{ m}$, $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(666.67 \times 10^{-9})}{(1.0)(0.030)} = 2.2222 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 20 \text{ mm} = 0.020 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(666.67 \times 10^{-9})}{(1.5)(0.020)} = 3.5556 \times 10^3 \text{ N}\cdot\text{m}$$

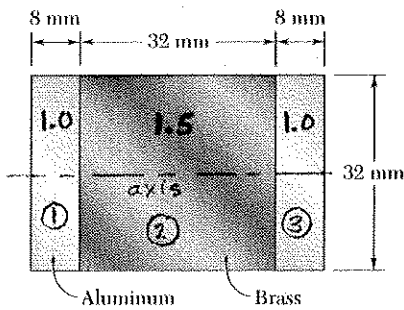
Choose the smaller value.

$$M = 2.22 \times 10^3 \text{ N}\cdot\text{m}$$

$$M = 2.22 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.34

4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.



	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12} (8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12} (32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 174.7626 \times 10^3 \text{ mm}^4 = 174.7626 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(174.7626 \times 10^{-9})}{(1.0)(0.016)} = 1.0923 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(174.7626 \times 10^{-9})}{(1.5)(0.016)} = 1.1651 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.092 \times 10^3 \text{ N}\cdot\text{m}$

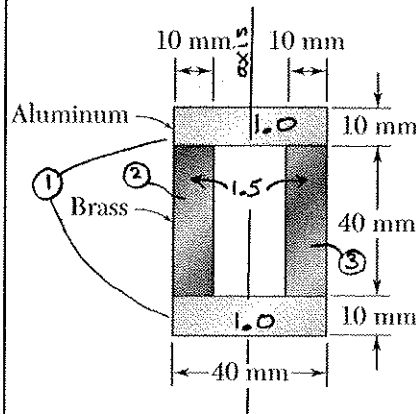
$M = 1.092 \text{ kN}\cdot\text{m}$

Problem 4.35

4.35 and 4.36 For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

4.35 Bar of Prob. 4.33.

4.33 and 4.34 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.



	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12} (20)(40^3) = 106.6667 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12} (40)(10)^3 + (1.5)(40)(10)(15)^2 = 140 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 140 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 386.667 \times 10^3 \text{ mm}^4 = 386.667 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

Aluminum: $n = 1.0$, $|y| = 20 \text{ mm} = 0.020$, $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(386.667 \times 10^{-9})}{(1.0)(0.020)} = 1933 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 20 \text{ mm} = 0.020 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

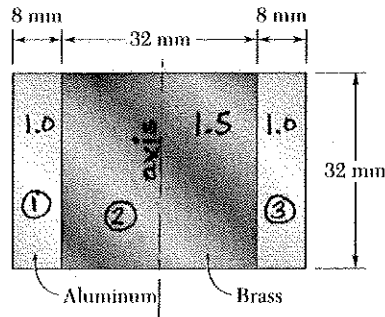
$$M = \frac{(160 \times 10^6)(386.667 \times 10^{-9})}{(1.5)(0.020)} = 2062 \text{ N}\cdot\text{m}$$

Choose the smaller value.

$$M = 1933 \text{ N}\cdot\text{m}$$

$$M = 1.933 \text{ kN}\cdot\text{m}$$

Problem 4.36



4.35 and 4.36 For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

4.36 Bar of Prob. 4.34.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material
 For aluminum, $n = 1.0$
 For brass, $n = E_b/E_a = 105/70 = 1.5$
 Values of n are shown on the figure.

For the transformed section

$$I_1 = \frac{n_1}{12} h_1 b_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12} (32)(8)^3 + (1.0)[(32)(8)](20)^2 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.5}{12} (32)(32)^3 = 131.072 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 103.7653 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 338.58 \times 10^3 \text{ mm}^4 = 338.58 \times 10^{-9} \text{ m}^4$$

$$161 = \frac{n M y}{I} \quad M = \left| \frac{\sigma I}{n y} \right|$$

Aluminum: $n = 1.0$, $|y| = 24 \text{ mm} = 0.024 \text{ m}$, $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(338.58 \times 10^{-9})}{(1.0)(0.024)} = 1.411 \times 10^3 \text{ N}\cdot\text{m}$$

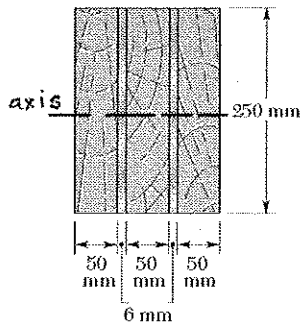
Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(338.58 \times 10^{-9})}{(1.5)(0.016)} = 2.257 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.411 \times 10^3 \text{ N}\cdot\text{m}$

$M = 1.411 \text{ kN}\cdot\text{m}$ ◀

Problem 4.37



4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity	14 GPa	200 GPa
Allowable stress	14 MPa	150 MPa

Use wood as the reference material

For wood $n_w = 1$

For steel $n = E_s / E_w = 200 / 14 = 14.29$

Properties of the geometric section.

$$\text{Steel: } I_s = \frac{1}{12} (6 + 6) (250)^3 = 15.625 \times 10^6 \text{ mm}^4$$

$$\text{Wood: } I_w = \frac{1}{12} (50 + 50 + 50) (250)^3 = 195.3125 \times 10^6 \text{ mm}^4$$

Transformed section

$$I_{\text{trans}} = n_s I_s + n_w I_w = (14.29) (15.625 \times 10^6) + (1) (195.3125 \times 10^6) \\ = 418.6 \times 10^6 \text{ mm}^4$$

$$|\sigma| = \left| \frac{n M y}{I} \right| \quad M = \left| \frac{\sigma I}{n y} \right|$$

Wood: $n = 1$ $|y| = 125 \text{ mm}$ $\sigma = 14 \text{ MPa}$

$$M = \frac{(14 \times 10^6) (418.6 \times 10^6)}{(1) (0.125)} = 46.9 \times 10^3 \text{ Nm}$$

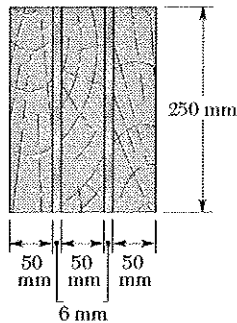
Steel: $n = 14.29$ $|y| = 125 \text{ mm}$ $\sigma = 150 \text{ MPa}$

$$M = \frac{(150 \times 10^6) (418.6 \times 10^6)}{(14.29) (0.125)} = 35.2 \times 10^3 \text{ Nm}$$

Choose the smaller value $M = 35.2 \times 10^3 \text{ Nm}$

$$M = 35.2 \text{ kN.m} \quad \blacktriangleleft$$

Problem 4.38



4.38 For the composite member of Prob. 4.37, determine the largest permissible bending moment when the member is bent about a vertical axis.

4.37 Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity	14 GPa	200 GPa
Allowable stress	14 MPa	150 MPa

Use wood as the reference material

For wood $n_w = 1$

For steel $n_s = E_s/E_w = 200/14 = 14.29$

Properties of the geometric section

$$\text{Total: } I_t = \frac{1}{12} h b^3 = \frac{1}{12} (250)(162)^3 = 88573500 \text{ mm}^4$$

$$\text{Steel: } I_s = \frac{1}{12} h (B^3 - b^3) = \frac{1}{12} (250)(62^3 - 50^3) = 2361000 \text{ mm}^4$$

$$\text{Wood: } I_w = I_t - I_s = 86212500 \text{ mm}^4$$

Transformed section

$$I_{\text{trans}} = n_s I_s + n_w I_w = (14.29)(2361000) + (1)(86212500) = 120 \times 10^6 \text{ mm}^4$$

$$|\sigma| = \left| \frac{n M y}{I} \right| \quad M = \left| \frac{\sigma I}{n y} \right|$$

$$\text{Wood: } n = 1 \quad |y| = 81 \text{ mm} \quad \sigma = 14 \text{ MPa}$$

$$M = \frac{(14 \times 10^6)(120 \times 10^6)}{(1)(0.081)} = 20.74 \times 10^3 \text{ N}\cdot\text{m}$$

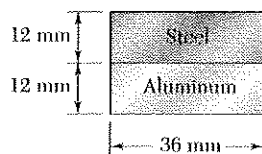
$$\text{Steel: } n = 14.29 \quad |y| = 31 \text{ mm} \quad \sigma = 150 \text{ MPa}$$

$$M = \frac{(150 \times 10^6)(120 \times 10^6)}{(14.29)(0.031)} = 40.6 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 20.74 \text{ kN}\cdot\text{m}$

$$M = 20.74 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.39



4.39 and 4.40 A steel bar ($E_s = 210$ GPa) and an aluminum bar ($E_a = 70$ GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (a) the aluminum, (b) the steel, when the bar is bent about a horizontal axis, with $M = 200$ N·m.

Use aluminum as the reference material.

For aluminum $n = 1$

For steel $n = E_s/E_a = 210/70 = 3$

Transformed section

①	$n = 3$
②	$n = 1$

	A, mm^2	nA, mm^2	$\bar{y}_0, \text{mm}$	$nA\bar{y}_0, \text{mm}^3$
①	432	1296	18	23328
②	432	432	6	2592
Σ		1728		25920

$$\bar{y}_0 = \frac{25920}{1728} = 15 \text{ mm}$$

The neutral axis lies 15 mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{3}{12} (36)(12)^3 + (1296)(3)^2 = 27.216 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12} (36)(12)^3 + (432)(9)^2 = 40.176 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 67.392 \times 10^3 \text{ mm}^4 = 67.392 \times 10^{-9} \text{ m}^4$$

$$M = 200 \text{ N}\cdot\text{m}$$

$$\sigma = -\frac{nMy}{I}$$

(a) Aluminum: $n = 1$, $y = -15 \text{ mm} = -0.015 \text{ m}$

$$\sigma_a = -\frac{(1)(200)(-0.015)}{67.392 \times 10^{-9}} = 44.516 \times 10^6 \text{ Pa}$$

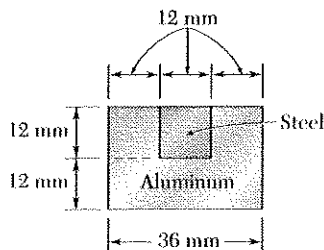
$$\sigma_a = 44.5 \text{ MPa} \quad \blacktriangleleft$$

(b) Steel: $n = 3$, $y = 9 \text{ mm} = 0.009 \text{ m}$

$$\sigma_s = -\frac{(3)(200)(0.009)}{67.392 \times 10^{-9}} = -80.128 \times 10^6 \text{ Pa}$$

$$\sigma_s = -80.1 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.40



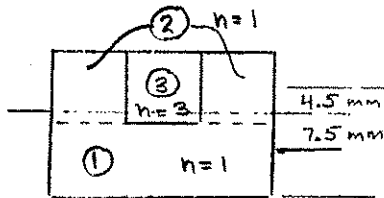
4.39 and 4.40 A steel bar ($E_s = 210 \text{ GPa}$) and an aluminum bar ($E_a = 70 \text{ GPa}$) are bonded together to form the composite bar shown. Determine the maximum stress in (a) the aluminum, (b) the steel, when the bar is bent about a horizontal axis, with $M = 200 \text{ N} \cdot \text{m}$.

Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 210/70 = 3$

Transformed section.



	A, mm^2	nA, mm^2	$\bar{y}_o, \text{mm}$	$nA\bar{y}_o, \text{mm}^3$
①	432	432	6	2592
②	288	288	18	5184
③	144	432	18	7776
		1152		15552

$$\bar{Y}_o = \frac{15552}{1152} = 13.5 \text{ mm} \quad \text{The neutral axis lies 13.5 mm above the bottom.}$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12} (36)(12)^3 + (432)(7.5)^2 = 29.484 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12} (24)(12)^3 + (288)(4.5)^2 = 9.288 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 + n_3 A_3 d_3^2 = \frac{3}{12} (12)(12)^3 + (432)(4.5)^2 = 13.932 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 52.704 \times 10^3 \text{ mm}^4 = 52.704 \times 10^{-9} \text{ m}^4$$

$$M = 60 \text{ N} \cdot \text{m}$$

$$\sigma = - \frac{nMy}{I}$$

(a) Aluminum: $n = 1, y = -13.5 \text{ mm} = -0.0135 \text{ m}$

$$\sigma_a = - \frac{(1)(200)(-0.0135)}{52.704 \times 10^{-9}} = 51.2 \times 10^6 \text{ Pa}$$

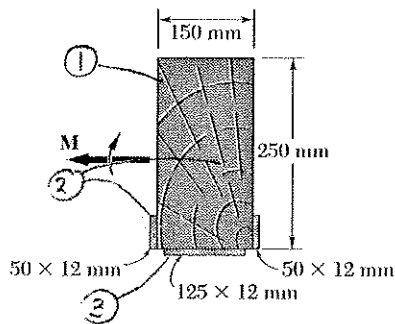
$$\sigma_a = 51.2 \text{ MPa} \quad \blacktriangleleft$$

(b) Steel: $n = 3, y = 10.5 \text{ mm} = 0.0105 \text{ m}$

$$\sigma_s = - \frac{(3)(200)(0.0105)}{52.704 \times 10^{-9}} = -119.5 \times 10^6 \text{ Pa}$$

$$\sigma_s = -119.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.41



4.41 and 4.42 The 150×250 -mm timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 12 GPa and for steel 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 50 \text{ kN} \cdot \text{m}$, determine the maximum stress in (a) the wood, (b) the steel.

Use wood as the reference material.

For wood, $n = 1$

For steel, $n = \frac{E_s}{E_w} = \frac{200}{12} = 16.667$

Transformed section.

$$\bar{Y} = \frac{6027517.8}{82500.9} = 73 \text{ mm}$$

	A, mm^2	$n A, \text{mm}^2$	$\bar{y}, \text{mm}$	$n A \bar{y}, \text{mm}^3$
①	37500	37500	137	5137500
②	1200	20000.4	37	740014.8
③	1500	25000.5	6	150003
		82500.9		6027517.8

The neutral axis lies 73 mm above the bottom.

Distances from neutral axis.

$$d_1 = 137 - 73 = 64 \text{ mm}$$

$$d_2 = |37 - 73| = 36 \text{ mm}$$

$$d_3 = |6 - 73| = 67 \text{ mm}$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12} (150)(250)^3 + (37500)(64)^2 = 348.91 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{16.667}{12} (24)(50)^3 + (20000.4)(36)^2 = 30.09 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 + n_3 A_3 d_3^2 = \frac{16.667}{12} (125)(12)^3 + (25000.5)(67)^2 = 112.53 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 491.53 \times 10^6 \text{ mm}^4 = 491.53 \times 10^{-6} \text{ m}^4$$

$$M = 50 \text{ kNm}$$

(a) Wood: $n = 1$, $y = 262 - 73 = 189 \text{ mm} = 0.189 \text{ m}$

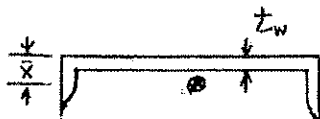
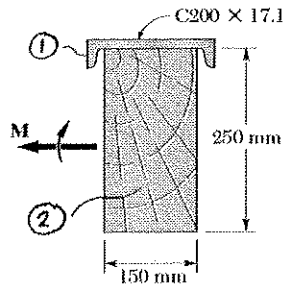
$$\sigma_w = -\frac{nMy}{I} = -\frac{(1)(50 \times 10^3)(0.189)}{491.53 \times 10^{-6}} = -19.225 \text{ MPa} \quad \sigma_w = -19.2 \text{ MPa} \quad \blacktriangleleft$$

(b) Steel: $n = 16.667$, $y = -0.073 \text{ m}$

$$\sigma_s = -\frac{nMy}{I} = -\frac{(16.667)(50 \times 10^3)(-0.073)}{491.53 \times 10^{-6}} = 123.77 \text{ MPa}$$

$$\sigma_s = 123.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.42



4.41 and 4.42 The 150×250 mm timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 12 GPa and for steel 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 50 \text{ kN} \cdot \text{m}$, determine the maximum stress in (a) the wood, (b) the steel.

Use wood as the reference material.

For wood $n = 1$

For steel $n = E_s/E_w = 200/12 = 16.67$

For C8 x 11.5 channel section, $A = 2170 \text{ mm}^2$

$t_w = 5.6 \text{ mm}$, $\bar{x} = 14.4 \text{ mm}$, $I_y = 538000 \text{ mm}^4$

For the composite section the centroid of the channel lies $250 + 5.6 - 14.4 = 241.2 \text{ mm}$ above the base. $\bar{y}_0 = 241.2 \text{ mm}$ for channel.

Transformed section

$$\bar{y}_0 = \frac{13412669}{73674} = 182.1 \text{ mm}$$

The neutral axis lies 182.1 mm above the bottom.

	A, mm^2	nA, mm^2	$\bar{y}_0, \text{mm}$	$nA\bar{y}_0, \text{mm}^3$
①	2170	36174	241.2	8725169
②	37500	37500	125	4687500
		73674		13412669

$$I_1 = n_1 \bar{I}_x + n_1 A_1 d_1^2 = (16.67)(538000) + (36174)(241.2 - 182.1)^2 = 135.32 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12} (150)(250)^3 + (37500)(182.1 - 125)^2 = 317.58 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 452.9 \times 10^6 \text{ mm}^4$$

$$M = 50 \text{ kNm} \quad \sigma = - \frac{nMy}{I}$$

(a) Wood: $n = 1$, $y = -8.433 \text{ in.}$

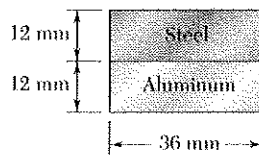
$$\sigma_w = - \frac{(50 \times 10^3)(-182.1 \times 10^{-3})}{452.9 \times 10^6} = 20.1 \text{ MPa}$$

$$\sigma_w = 20.1 \text{ MPa}$$

(b) Steel: $n = 16.67$, $y = 250 + 5.6 - 182.1 = 73.5 \text{ mm}$

$$\sigma_s = - \frac{(16.67)(50 \times 10^3)(73.5 \times 10^{-3})}{452.9 \times 10^6} = -135.3 \text{ MPa} \quad \sigma_s = -135.3 \text{ MPa}$$

Problem 4.43



4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment $200 \text{ N} \cdot \text{m}$.

4.43 Bar of Prob. 4.39

See solution to Problem 4.39 for calculation of I .

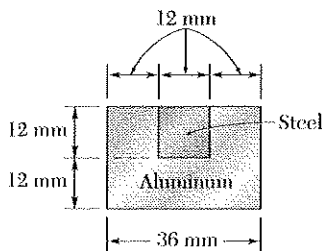
$$I = 67.392 \times 10^{-9} \text{ m}^4 \quad E_a = 70 \times 10^9 \text{ Pa}$$

$$M = 200 \text{ N} \cdot \text{m}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{200}{(70 \times 10^9)(67.392 \times 10^{-9})} = 42.396 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 23.6 \text{ m} \blacktriangleleft$$

Problem 4.44



4.43 and 4.44 For the composite bar indicated, determine the radius of curvature caused by the couple of moment $200 \text{ N} \cdot \text{m}$.

4.44 Bar of Prob. 4.40

See solution to Problem 4.40 for calculation of I .

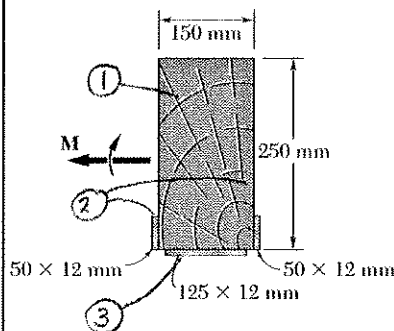
$$I = 52.704 \times 10^{-9} \text{ m}^4 \quad E_a = 70 \times 10^9 \text{ Pa}$$

$$M = 200 \text{ N} \cdot \text{m}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{200}{(70 \times 10^9)(52.704 \times 10^{-9})} = 54.211 \text{ m}^{-1}$$

$$\rho = 18.45 \text{ m} \blacktriangleleft$$

Problem 4.45



4.45 and 4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment $50 \text{ kN} \cdot \text{m}$.

4.45 Beam of Prob. 4.41

See solution to Problem 4.41 for calculation of I .

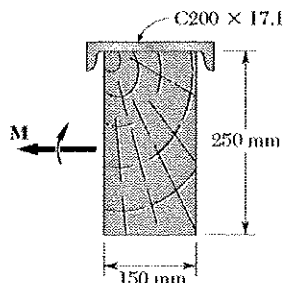
$$I = 491.53 \times 10^{-6} \text{ m}^4 \quad E_w = 12 \times 10^9 \text{ Pa}$$

$$M = 50 \times 10^3 \text{ Nm}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{50 \times 10^3}{(12 \times 10^9)(491.53 \times 10^{-6})} = 8.4769 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 117.96 \text{ m} = 118 \text{ m} \blacktriangleleft$$

Problem 4.46



4.45 and 4.46 For the composite beam indicated, determine the radius of curvature caused by the couple of moment $50 \text{ kN} \cdot \text{m}$.

4.45 Bar of Prob. 4.41.

4.46 Bar of Prob. 4.42.

See solution to Problem 4.42 for calculation of I .

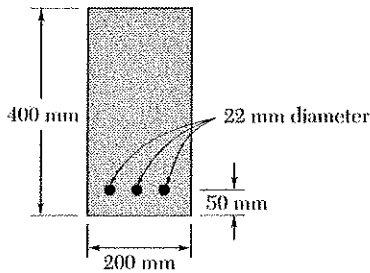
$$I = 452.9 \times 10^6 \text{ mm}^4 \quad E_w = 126 \text{ Pa}$$

$$M = 50 \text{ kNm}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{50 \times 10^3}{(12 \times 10^9)(452.9 \times 10^{-6})} = 0.0092 \text{ m}^{-1}$$

$$\rho = 108.7 \text{ m} \blacktriangleleft$$

Problem 4.47



4.47 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress of 9.45 MPa for the concrete and 140 MPa for the steel, determine the largest allowable positive bending moment in the beam.

$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{20 \times 10^9} = 10$$

$$A_s = 3 \left(\frac{\pi}{4} d^2 \right) = 3 \left(\frac{\pi}{4} \right) (22)^2 = 1140.4 \text{ mm}^2$$

$$nA_s = 11404 \text{ mm}^2$$

Locate neutral axis.

$$200 \times \frac{x}{2} - (11404)(400 - x) = 0$$

$$100x^2 + 11404x - 4561600 = 0$$

$$\text{Solve for } x \quad x = \frac{-11404 + [11404^2 + (4)(100)(4561600)]^{1/2}}{2(100)} = 164 \text{ mm}$$

$$400 - x = 236 \text{ mm}$$

$$I = \frac{1}{3} 200 x^3 + nA_s (400 - x)^2 = \frac{1}{3} (200)(164)^3 + (11404)(236)^2$$

$$= 296.75 \times 10^6 \text{ mm}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \therefore M = \frac{\sigma I}{ny}$$

Concrete: $n = 1.0$, $|y| = 164 \text{ mm}$, $|\sigma| = 9.45 \text{ MPa}$

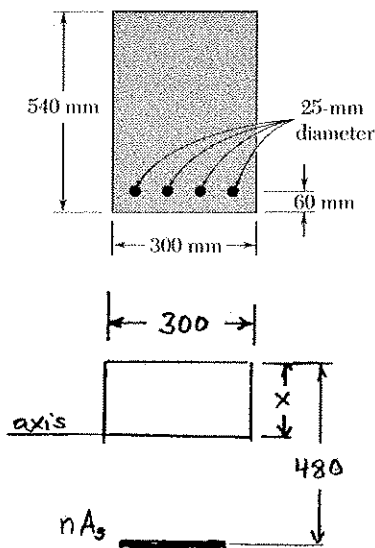
$$M = \frac{(9.45 \times 10^6)(296.75 \times 10^6)}{(1.0)(0.164)} = 17.1 \text{ kNm}$$

Steel: $n = 10$, $|y| = 236 \text{ mm}$, $\sigma = 140 \text{ MPa}$

$$M = \frac{(140000 \times 10^3)(296.75 \times 10^6)}{(10)(0.236)} = 17.6 \text{ kNm}$$

Choose the smaller value $M = 17.1 \text{ kNm}$

Problem 4.48



4.48 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4) \left(\frac{\pi}{4} \right) (25)^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$

Locate the neutral axis.

$$300 \times \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$150x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x.

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(150)(7.5398 \times 10^6)}}{(2)(150)}$$

$$x = 177.87 \text{ mm}, \quad 480 - x = 302.13 \text{ mm}$$

$$\begin{aligned} I &= \frac{1}{3} 300 x^3 + (15.708 \times 10^3)(480 - x)^2 \\ &= \frac{1}{3} (300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2 \\ &= 1.9966 \times 10^9 \text{ mm}^4 = 1.9966 \times 10^{-3} \text{ m}^4 \end{aligned}$$

$$\sigma = -\frac{nMy}{I}$$

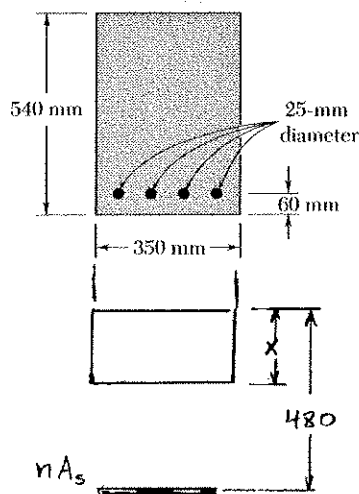
(a) Steel: $y = -302.45 \text{ mm} = -0.30245 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6 \text{ Pa} = 212 \text{ MPa} \quad \blacktriangleleft$$

(b) Concrete: $y = 177.87 \text{ mm} = 0.17787 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6 \text{ Pa} = -15.59 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.49



4.49 Solve Prob. 4.48, assuming that the 300-mm width is increased to 350 mm.

4.48 The reinforced concrete beam shown is subjected to a positive bending moment of $175 \text{ kN} \cdot \text{m}$. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \frac{\pi}{4} d^2 = (4) \left(\frac{\pi}{4} \right) (25)^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$

locate the neutral axis

$$350 \times \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$175x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x .

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(175)(7.5398 \times 10^6)}}{(2)(175)}$$

$$x = 167.48 \text{ mm}, \quad 480 - x = 312.52 \text{ mm}$$

$$\begin{aligned} I &= \frac{1}{3}(350)x^3 + (15.708 \times 10^3)(480 - x)^2 \\ &= \frac{1}{3}(350)(167.48)^3 + (15.708 \times 10^3)(312.52)^2 \\ &= 2.0823 \times 10^9 \text{ mm}^4 = 2.0823 \times 10^{-3} \text{ m}^4 \end{aligned}$$

$$\sigma = -\frac{nMy}{I}$$

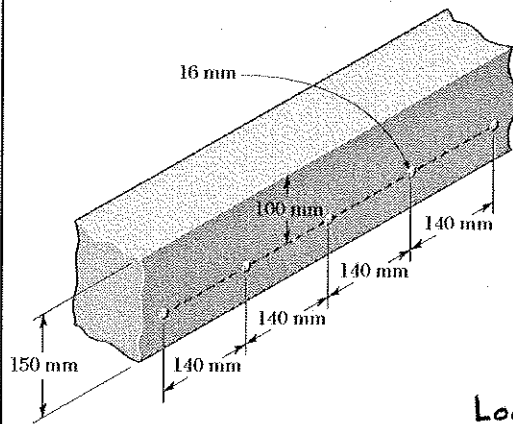
(a) Steel: $y = -312.52 \text{ mm} = -0.31252 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.31252)}{2.0823 \times 10^{-3}} = 210 \times 10^6 \text{ Pa} = 210 \text{ MPa} \blacktriangleleft$$

(b) Concrete: $y = 167.48 \text{ mm} = 0.16748 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.16748)}{2.0823 \times 10^{-3}} = -14.08 \times 10^6 \text{ Pa} = -14.08 \text{ MPa} \blacktriangleleft$$

Problem 4.50



4.50 A concrete slab is reinforced by 16-mm-diameter rods placed on 140-mm centers as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress of 9 MPa for the concrete and 140 MPa for the steel, determine the largest bending moment per foot of width that can be safely applied to the slab.

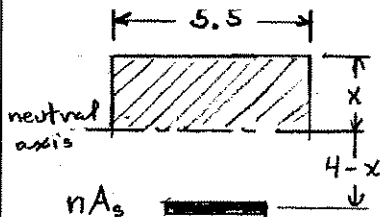
$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{20 \times 10^9} = 10$$

Consider a section 140 mm wide.

$$A_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4} (16)^2 = 201 \text{ mm}^2$$

$$nA_s = 2010 \text{ mm}^2$$

Locate the neutral axis.



$$140 \times \frac{x}{2} - (100 - x)(2010) = 0$$

$$70x^2 + 2010x - 201000 = 0$$

Solving for x

$$x = 41.1 \text{ mm}$$

$$100 - x = 58.9 \text{ mm}$$

$$I = \frac{1}{3}(140)x^3 + (2010)(100 - x)^2$$

$$= \frac{1}{3}(140)(41.1)^3 + (2010)(58.9)^2$$

$$= 10.213 \times 10^6 \text{ mm}^4$$

$$|s| = \left| \frac{nMy}{I} \right|$$

$$M = \left| \frac{Is}{ny} \right|$$

Concrete: $n = 1$, $y = 41.1 \text{ mm}$, $s = 9 \text{ MPa}$.

$$M = \frac{(10.213 \times 10^6)(9 \times 10^6)}{(1.0)(0.0411)} = 2236 \text{ Nm}$$

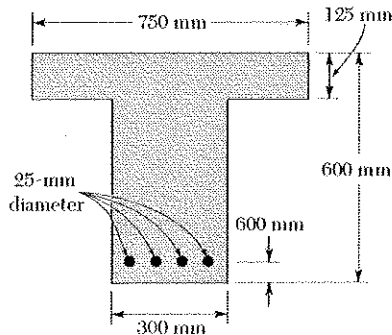
Steel: $n = 10$, $y = 58.9 \text{ mm}$, $s = 140 \text{ MPa}$.

$$M = \frac{(10.213 \times 10^6)(140 \times 10^6)}{(10)(0.0589)} = 2428 \text{ Nm}$$

Choose the smaller value as the allowable moment for a 140 mm width.

$$M = 2236 \text{ Nm} = 2.24 \text{ kNm}$$

Problem 4.51



4.51 Knowing that the bending moment in the reinforced concrete beam is $+200 \text{ kN} \cdot \text{m}$ and that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{25 \times 10^9} = 8$$

$$A_s = 4 \frac{\pi}{4} d^2 = 4 \left(\frac{\pi}{4} \right) (25)^2 = 1963.5 \text{ mm}^2$$

$$nA_s = 15708 \text{ mm}^2$$

Locate the neutral axis

$$(750)(125)(x + 62.5) + 300x^2/2$$

$$- (15708)(415 - x) = 0$$

$$93750x + 5859375 + 150x^2 - 6518820 + 15708x = 0$$

$$150x^2 + 109458x - 659445 = 0$$

Solve for x
$$x = \frac{-109458 + [(109458)^2 + (4)(150)(659445)]^{1/2}}{2(150)} = 6 \text{ mm}$$

$$415 - x = 409 \text{ mm}$$

$$I_1 = \frac{1}{12} b h^3 + A_1 d_1^2 = \frac{1}{12} (750)(125)^3 + (750)(125)(62.5)^2 = 557.166 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{3} b_2 x^3 = \frac{1}{3} (12)(6)^3 = 864 \text{ mm}^4$$

$$I_3 = nA_s d_3^2 = (15708)(409)^2 = 2627.65 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 3.1848 \times 10^9 \text{ mm}^4$$

$$\sigma = -\frac{nMy}{I} \quad \text{where} \quad M = 200 \text{ kNm}$$

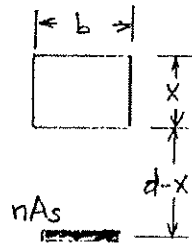
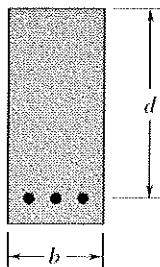
(a) Steel $n = 8.0$, $y = -409 \text{ mm}$

$$\sigma = -\frac{(8.0)(200 \times 10^3)(409 \times 10^{-3})}{3.1848 \times 10^{-3}} = 205.5 \text{ MPa}$$

(b) Concrete $n = 1.0$, $y = 131 \text{ mm}$

$$\sigma = -\frac{(1.0)(200 \times 10^3)(0.131)}{3.1848 \times 10^{-3}} = -8.2 \text{ MPa}$$

Problem 4.52



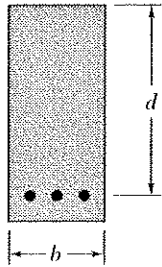
4.52 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c . Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \frac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

$$\begin{aligned} \sigma_s &= \frac{n M (d-x)}{I} & \sigma_c &= \frac{M x}{I} \\ \frac{\sigma_s}{\sigma_c} &= \frac{n (d-x)}{x} = n \frac{d}{x} - n \\ \frac{d}{x} &= 1 + \frac{1}{n} \frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c} \\ x &= \frac{d}{1 + \frac{E_c \sigma_s}{E_s \sigma_c}} \end{aligned}$$

Problem 4.53



4.53 For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that $b = 200$ mm and $d = 450$ mm, and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine (a) the required area A_s of the steel reinforcement if the beam is to be balanced, (b) the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)

$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{25 \times 10^9} = 8.0$$

$$\sigma_s = \frac{n M (d-x)}{I} \quad \sigma_c = \frac{M x}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n (d-x)}{x} = n \frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n} \frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0} \cdot \frac{140 \times 10^6}{12.5 \times 10^6} = 2.40$$

$$x = 0.41667 d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis.

$$b x \frac{x}{2} - n A_s (d-x)$$

$$(a) \quad A_s = \frac{b x^2}{2 n (d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2 \quad (a) \quad A_s = 1674 \text{ mm}^2$$

$$I = \frac{1}{3} b x^3 + n A_s (d-x)^2 = \frac{1}{3} (200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4$$

$$\sigma = \frac{n M y}{I} \quad M = \frac{I \sigma}{n y}$$

Concrete: $n = 1.0$ $y = 187.5 \text{ mm} = 0.1875 \text{ m}$ $\sigma = 12.5 \times 10^6 \text{ Pa}$

$$M = \frac{(1.3623 \times 10^{-3})(12.5 \times 10^6)}{(1.0)(0.1875)} = 90.8 \times 10^3 \text{ N}\cdot\text{m}$$

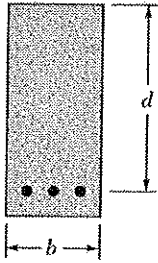
Steel: $n = 8.0$ $y = 262.5 \text{ mm} = 0.2625 \text{ m}$ $\sigma = 140 \times 10^6 \text{ Pa}$

$$M = \frac{(1.3623 \times 10^{-3})(140 \times 10^6)}{(8.0)(0.2625)} = 90.8 \times 10^3 \text{ N}\cdot\text{m}$$

Note that both values are the same for balanced design.

$$(b) \quad M = 90.8 \text{ kN}\cdot\text{m}$$

Problem 4.54



4.54 For the concrete beam shown, the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel. Knowing that $b = 200$ mm and $d = 450$ mm and using an allowable stress of 12.5 MPa for the concrete and 140 MPa for the steel, determine (a) the required area A_s of the steel reinforcement if the beam is to be balanced, (b) the largest allowable bending moment. (See Prob. 4.52 for definition of a balanced beam.)

$$n = \frac{E_s}{E_c} = \frac{200 \times 10^9}{25 \times 10^9} = 8.0$$

$$\sigma_s = \frac{n M (d-x)}{I} \quad \sigma_c = \frac{M x}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n (d-x)}{x} = n \frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n} \frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.0} \cdot \frac{140 \times 10^6}{12.5 \times 10^6} = 2.40$$

$$x = 0.41667 d = (0.41667)(450) = 187.5 \text{ mm}$$

Locate neutral axis

$$b x \frac{x}{2} - n A_s (d-x)$$

$$(a) \quad A_s = \frac{b x^2}{2 n (d-x)} = \frac{(200)(187.5)^2}{(2)(8.0)(262.5)} = 1674 \text{ mm}^2 \quad (a) \quad A_s = 1674 \text{ mm}^2$$

$$I = \frac{1}{3} b x^3 + n A_s (d-x)^2 = \frac{1}{3} (200)(187.5)^3 + (8.0)(1674)(262.5)^2$$

$$= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4$$

$$\sigma = \frac{n M y}{I} \quad M = \frac{I \sigma}{n y}$$

Concrete: $n = 1.0$ $y = 187.5 \text{ mm} = 0.1875 \text{ m}$ $\sigma = 12.5 \times 10^6 \text{ Pa}$

$$M = \frac{(1.3623 \times 10^{-3})(12.5 \times 10^6)}{(1.0)(0.1875)} = 90.8 \times 10^3 \text{ N}\cdot\text{m}$$

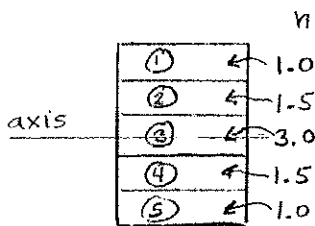
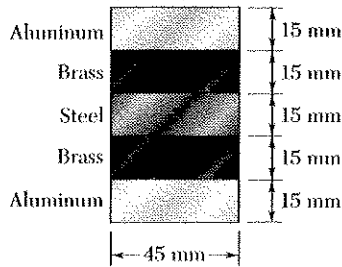
Steel: $n = 8.0$ $y = 262.5 \text{ mm} = 0.2625 \text{ m}$ $\sigma = 140 \times 10^6 \text{ Pa}$

$$M = \frac{(1.3623 \times 10^{-3})(140 \times 10^6)}{(8.0)(0.2625)} = 90.8 \times 10^3 \text{ N}\cdot\text{m}$$

Note that both values are the same for balanced design.

$$(b) \quad M = 90.8 \text{ kN}\cdot\text{m}$$

Problem 4.55



4.55 and 4.56 Five metal strips, each of 15×45 -mm cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment $1400 \text{ N} \cdot \text{m}$, determine (a) the maximum stress in each of the three metals, (b) the radius of curvature of the composite beam.

Use aluminum as the reference material.

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel.}$$

$$n = \frac{E_b}{E_a} = \frac{105}{70} = 1.5 \text{ in brass.}$$

$$n = 1.0 \text{ in aluminum.}$$

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12} (45)(15)^3 + (1.0)(45)(15)(30)^2 = 620.16 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12} (45)(15)^3 + (1.5)(45)(15)(15)^2 = 246.80 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3}{12} (45)(15)^3 = 37.97 \times 10^3 \text{ mm}^4$$

$$I_4 = I_2 = 246.80 \times 10^3 \text{ mm}^4 \quad I_5 = I_1 = 620.16 \times 10^3 \text{ mm}^4$$

$$I = \sum_1^5 I_i = 1.77189 \times 10^6 \text{ mm}^4 = 1.77189 \times 10^{-6} \text{ m}^4$$

(a) Aluminum: $y = 37.5 \text{ mm} = 0.0375 \text{ m} \quad n = 1.0$

$$\sigma = \frac{nMy}{I} = \frac{(1.0)(1400)(0.0375)}{1.77189 \times 10^{-6}} = 29.6 \times 10^6 \text{ Pa} \quad \sigma_a = 29.6 \text{ MPa} \quad \blacktriangleleft$$

Brass: $y = 22.5 \text{ mm} = 0.0225 \text{ m} \quad n = 1.5$

$$\sigma = \frac{nMy}{I} = \frac{(1.5)(1400)(0.0225)}{1.77189 \times 10^{-6}} = 26.7 \times 10^6 \text{ Pa} \quad \sigma_b = 26.7 \text{ MPa} \quad \blacktriangleleft$$

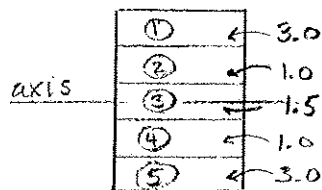
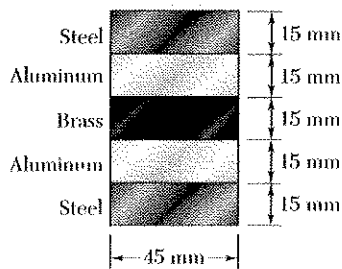
Steel: $y = 7.5 \text{ mm} = 0.0075 \text{ m} \quad n = 3.0$

$$\sigma = \frac{nMy}{I} = \frac{(3.0)(1400)(0.0075)}{1.77189 \times 10^{-6}} = 17.78 \times 10^6 \text{ Pa} \quad \sigma_s = 17.78 \text{ MPa} \quad \blacktriangleleft$$

(b) $\frac{1}{\rho} = \frac{M}{E_a I} = \frac{1400}{(70 \times 10^9)(1.77189 \times 10^{-6})} = 11.287 \times 10^{-3} \text{ m}^{-1}$

$$\rho = 88.6 \text{ m} \quad \blacktriangleleft$$

Problem 4.56



4.55 and 4.56 Five metal strips, each of 15×45 -mm cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by couple of moment $1400 \text{ N} \cdot \text{m}$, determine (a) the maximum stress in each of the three metals, (b) the radius of curvature of the composite beam.

Use aluminum as the reference material.

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{210}{105} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum}$$

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{3.0}{12} (45)(15)^3 + (3.0)(45)(15)(30)^2 = 1.860469 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12} (45)(15)^3 + (1.0)(45)(15)(15)^2 = 164.53 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12} (45)(15)^3 = 18.98 \times 10^3 \text{ mm}^4$$

$$I_4 = I_2 = 164.53 \times 10^3 \text{ mm}^4 \quad I_5 = I_1 = 1.860469 \times 10^6 \text{ mm}^4$$

$$I = \sum_1^5 I_i = 4.0690 \times 10^6 \text{ mm}^4 = 4.0690 \times 10^{-6} \text{ m}^4$$

(a) Steel: $y = 37.5 \text{ mm} = 0.0375 \text{ m} \quad n = 3.0$

$$\sigma = \frac{nMy}{I} = \frac{(3.0)(1400)(0.0375)}{4.0690 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa} \quad \sigma_s = 38.7 \text{ MPa} \quad \blacktriangleleft$$

Aluminum: $y = 22.5 \text{ mm} = 0.0225 \text{ m} \quad n = 1.0$

$$\sigma = \frac{nMy}{I} = \frac{(1.0)(1400)(0.0225)}{4.0690 \times 10^{-6}} = 7.74 \times 10^6 \text{ Pa} \quad \sigma_a = 7.74 \text{ MPa} \quad \blacktriangleleft$$

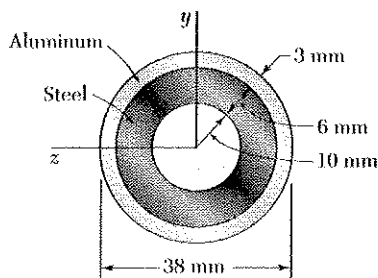
Brass: $y = 7.5 \text{ mm} = 0.0075 \text{ m} \quad n = 1.5$

$$\sigma = \frac{nMy}{I} = \frac{(1.5)(1400)(0.0075)}{4.0690 \times 10^{-6}} = 3.87 \times 10^6 \text{ Pa} \quad \sigma_b = 3.87 \text{ MPa} \quad \blacktriangleleft$$

(b) $\frac{1}{\rho} = \frac{M}{E_a I} = \frac{1400}{(70 \times 10^9)(4.0690 \times 10^{-6})} = 4.9152 \times 10^{-3} \text{ m}^{-1}$

$$\rho = 203 \text{ m} \quad \blacktriangleleft$$

Problem 4.57



4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (a) in the aluminum, (b) in the steel.

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel}$$

$$\text{Steel: } I_1 = n_1 \frac{\pi}{4} (r_o^4 - r_i^4) = (3.0) \frac{\pi}{4} (16^4 - 10^4) = 130.85 \times 10^3 \text{ mm}^4$$

$$\text{Aluminum: } I_2 = n_2 \frac{\pi}{4} (r_o^4 - r_i^4) = (1.0) \frac{\pi}{4} (19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 181.73 \times 10^3 \text{ mm}^4 = 181.73 \times 10^{-9} \text{ m}^4$$

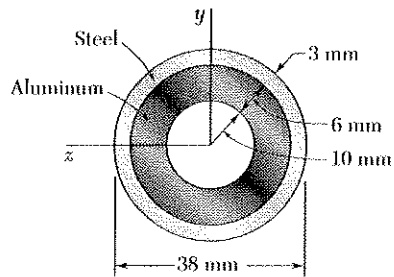
$$(a) \text{ Aluminum: } c = 19 \text{ mm} = 0.019 \text{ m}$$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.019)}{181.73 \times 10^{-9}} = 52.3 \times 10^6 \text{ Pa} = 52.3 \text{ MPa}$$

$$(b) \text{ Steel: } c = 16 \text{ mm} = 0.016 \text{ m}$$

$$\frac{nMc}{I} = \frac{(3.0)(500)(0.016)}{181.73 \times 10^{-9}} = 132.1 \times 10^6 \text{ Pa} = 132.1 \text{ MPa}$$

Problem 4.58



4.58 Solve Prob. 4.57, assuming that the 6-mm-thick inner pipe is made of aluminum and that the 3-mm-thick outer pipe is made of steel.

4.57 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couple of moment 500 N · m, determine the maximum stress (a) in the aluminum, (b) in the steel.

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel.}$$

$$\text{Steel: } I_1 = n_1 \frac{\pi}{4} (r_o^4 - r_i^4) = (3.0) \frac{\pi}{4} (19^4 - 16^4) = 152.65 \times 10^3 \text{ mm}^4$$

$$\text{Aluminum: } I_2 = n_2 \frac{\pi}{4} (r_o^4 - r_i^4) = (1.0) \frac{\pi}{4} (16^4 - 10^4) = 43.62 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 196.27 \times 10^3 \text{ mm}^4 = 196.27 \times 10^{-9} \text{ m}^4$$

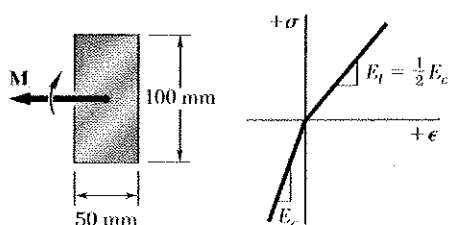
$$(a) \text{ Aluminum: } c = 16 \text{ mm} = 0.016 \text{ m}$$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.016)}{196.27 \times 10^{-9}} = 40.8 \times 10^6 \text{ Pa} = 40.8 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \text{ Steel: } c = 19 \text{ mm} = 0.019 \text{ m}$$

$$\sigma = \frac{nMc}{I} = \frac{(3.0)(500)(0.019)}{196.27 \times 10^{-9}} = 145.2 \times 10^6 \text{ Pa} = 145.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.59

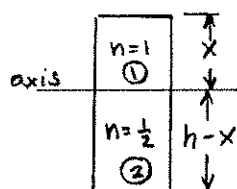


4.59 The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600 \text{ N} \cdot \text{m}$, determine the maximum (a) tensile stress, (b) compressive stress.

$n = \frac{1}{2}$ on the tension side of neutral axis.

$n = 1$ on the compression side.

Locate neutral axis.



$$n_1 b x \frac{x}{2} - n_2 b (h-x) \frac{h-x}{2} = 0$$

$$\frac{1}{2} b x^2 - \frac{1}{4} b (h-x)^2 = 0$$

$$x^2 = \frac{1}{2} (h-x)^2 \quad x = \frac{1}{\sqrt{2}} (h-x)$$

$$x = \frac{1}{\sqrt{2}+1} h = 0.41421 h = 41.421 \text{ mm}$$

$$h-x = 58.579 \text{ mm}$$

$$I_1 = n_1 \frac{1}{3} b x^3 = (1) \left(\frac{1}{3} \right) (50) (41.421)^3 = 1.1844 \times 10^6 \text{ mm}^4$$

$$I_2 = n_2 \frac{1}{3} b (h-x)^3 = \left(\frac{1}{2} \right) \left(\frac{1}{3} \right) (50) (58.579)^3 = 1.6751 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.8595 \times 10^6 \text{ mm}^4 = 2.8595 \times 10^{-6} \text{ m}^4$$

(a) tensile stress: $n = \frac{1}{2}$, $y = -58.579 \text{ mm} = -0.058579 \text{ m}$

$$\sigma = - \frac{n M y}{I} = - \frac{(0.5)(600)(-0.058579)}{2.8595 \times 10^{-6}} = 6.15 \times 10^6 \text{ Pa}$$

$$\sigma_T = 6.15 \text{ MPa}$$

(b) compressive stress: $n = 1$, $y = 41.421 \text{ mm} = 0.041421 \text{ m}$

$$\sigma = - \frac{n M y}{I} = - \frac{(1.0)(600)(0.041421)}{2.8595 \times 10^{-6}} = -8.69 \times 10^6 \text{ Pa}$$

$$\sigma_C = -8.69 \text{ MPa}$$

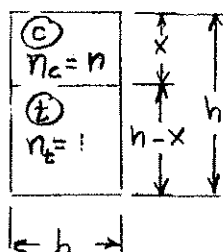
Problem 4.60

*4.60 A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$



Use E_t as the reference modulus.

Then $E_c = n E_t$

Locate neutral axis.

$$n b x \frac{x}{2} - b (h - x) \frac{h - x}{2} = 0$$

$$n x^2 - (h - x)^2 = 0 \quad \sqrt{n} x = (h - x)$$

$$x = \frac{h}{\sqrt{n} + 1} \quad h - x = \frac{\sqrt{n} h}{\sqrt{n} + 1}$$

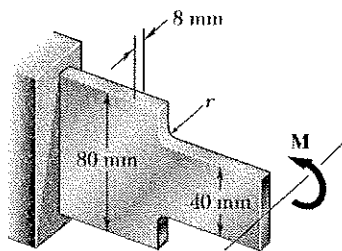
$$\begin{aligned} I_{trans} &= \frac{n}{3} b x^3 + \frac{1}{3} b (h - x)^3 = \left[\frac{n}{3} \left(\frac{1}{\sqrt{n} + 1} \right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n} + 1} \right)^3 \right] b h^3 \\ &= \frac{1}{3} \frac{n + n^{3/2}}{(\sqrt{n} + 1)^3} b h^3 = \frac{1}{3} \frac{n(1 + \sqrt{n})}{(\sqrt{n} + 1)^3} b h^3 = \frac{1}{3} \cdot \frac{n}{(\sqrt{n} + 1)^2} b h^3 \end{aligned}$$

$$\frac{1}{\rho} = \frac{M}{E_t I_{trans}} = \frac{M}{E_r I} \quad \text{where } I = \frac{1}{12} b h^3$$

$$E_r I = E_t I_{trans}$$

$$\begin{aligned} E_r &= \frac{E_t I_{trans}}{I} = \frac{12}{b h^3} \cdot E_t \cdot \frac{n}{3(\sqrt{n} + 1)^2} b h^3 \\ &= \frac{4 E_t E_c / E_t}{(\sqrt{E_c / E_t} + 1)^2} = \frac{4 E_t E_c}{(\sqrt{E_c} + \sqrt{E_t})^2} \end{aligned}$$

Problem 4.61



4.61 Knowing that the allowable stress for the beam shown is 90 MPa, determine (a) the allowable bending moment M when the radius r of the fillets is (a) 8 mm, (b) 12 mm.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

$$(a) \frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.2$$

$$\text{From Fig. 4.31 } K = 1.50$$

$$\sigma_{\max} = K \frac{Mc}{I}$$

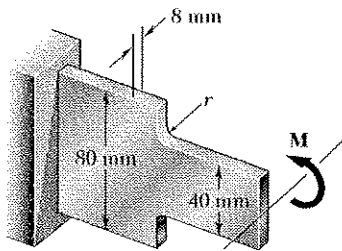
$$M = \frac{\sigma_{\max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)} = 128 \text{ N}\cdot\text{m}$$

$$(b) \frac{r}{d} = \frac{12 \text{ mm}}{40 \text{ mm}} = 0.3$$

$$\text{From Fig. 4.31 } K = 1.35$$

$$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)} = 142 \text{ N}\cdot\text{m}$$

Problem 4.62



4.62 Knowing that $M = 250 \text{ N}\cdot\text{m}$, determine the maximum stress in the beam shown when the radius r of the fillets is (a) 4 mm, (b) 8 mm.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

$$(a) \frac{r}{d} = \frac{4 \text{ mm}}{40 \text{ mm}} = 0.10$$

$$\text{From Fig. 4.31 } K = 1.87$$

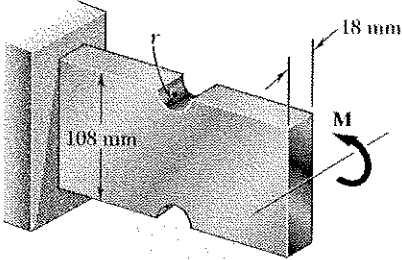
$$\sigma_{\max} = K \frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667 \times 10^{-9}} = 219 \times 10^6 \text{ Pa} = 219 \text{ MPa}$$

$$(b) \frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.20$$

$$\text{From Fig. 4.31 } K = 1.50$$

$$\sigma_{\max} = K \frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667 \times 10^{-9}} = 176 \times 10^6 \text{ Pa} = 176 \text{ MPa}$$

Problem 4.63



4.63 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 450 \text{ N} \cdot \text{m}$, determine the maximum stress in the member when the radius r of the semicircular grooves is (a) $r = 9 \text{ mm}$, (b) $r = 18 \text{ mm}$.

$$(a) \quad d = D - 2r = 108 - (2)(9) = 90 \text{ mm}$$

$$\frac{D}{d} = \frac{108}{90} = 1.20 \quad \frac{r}{d} = \frac{9}{90} = 0.1$$

From Fig. 4.32, $K = 2.07$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (18)(90)^3 = 1.0935 \times 10^6 \text{ mm}^4 = 1.0935 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 45 \text{ mm} = 0.045 \text{ m}$$

$$\sigma_{\max} = \frac{K M c}{I} = \frac{(2.07)(450)(0.045)}{1.0935 \times 10^{-6}} = 38.3 \times 10^6 \text{ Pa} \quad \sigma_{\max} = 38.3 \text{ MPa} \blacktriangleleft$$

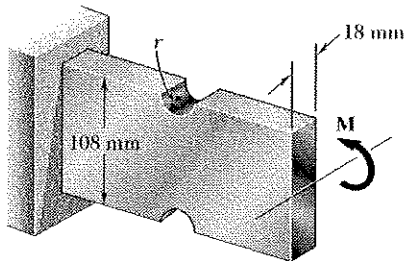
$$(b) \quad d = D - 2r = 108 - (2)(18) = 72 \text{ mm} \quad \frac{D}{d} = \frac{108}{72} = 1.5 \quad \frac{r}{d} = \frac{18}{72} = 0.25$$

From Fig. 4.32, $K = 1.61 \quad c = \frac{1}{2} d = 36 \text{ mm} = 0.036 \text{ m}$

$$I = \frac{1}{12} (18)(72)^3 = 559.87 \times 10^3 \text{ mm}^4 = 559.87 \times 10^{-9} \text{ m}^4$$

$$\sigma_{\max} = \frac{K M c}{I} = \frac{(1.61)(450)(0.036)}{559.87 \times 10^{-9}} = 46.6 \times 10^6 \text{ Pa} \quad \sigma_{\max} = 46.6 \text{ MPa} \blacktriangleleft$$

Problem 4.64



4.64 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 60 MPa, determine the largest bending moment that can be applied to the member when (a) $r = 9 \text{ mm}$, (b) $r = 18 \text{ mm}$.

$$(a) \quad d = D - 2r = 108 - (2)(9) = 90 \text{ mm}$$

$$\frac{D}{d} = \frac{108}{90} = 1.20 \quad \frac{r}{d} = \frac{9}{90} = 0.1$$

From Fig. 4.32, $K = 2.07$

$$I = \frac{1}{12} (18)(90)^3 = 1.0935 \times 10^6 \text{ mm}^4 = 1.0935 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 45 \text{ mm} = 0.045 \text{ m}$$

$$\sigma = \frac{K M c}{I} \quad M = \frac{\sigma I}{K c} = \frac{(60 \times 10^6)(1.0935 \times 10^{-6})}{(2.07)(0.045)} = 704 \quad M = 704 \text{ N} \cdot \text{m} \blacktriangleleft$$

$$(b) \quad d = 108 - (2)(18) = 72 \text{ mm} \quad \frac{D}{d} = \frac{108}{72} = 1.5 \quad \frac{r}{d} = \frac{18}{72} = 0.25$$

$c = \frac{1}{2} d = 36 \text{ mm} = 0.036 \text{ m}$ From Fig. 4.32, $K = 1.61$

$$I = \frac{1}{12} (18)(72)^3 = 559.87 \times 10^3 \text{ mm}^4 = 559.87 \times 10^{-9} \text{ m}^4$$

$$M = \frac{\sigma I}{K c} = \frac{(60 \times 10^6)(559.87 \times 10^{-9})}{(1.61)(0.036)} = 580 \quad M = 580 \text{ N} \cdot \text{m} \blacktriangleleft$$

Problem 4.65

4.65 A couple of moment $M = 2 \text{ kN} \cdot \text{m}$ is to be applied to the end of a steel bar. Determine the maximum stress in the bar (a) if the bar is designed with grooves having semicircular portions of radius $r = 10 \text{ mm}$, as shown in Fig. 4.65a, (b) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65b.

For both configurations

$$D = 150 \text{ mm}, \quad d = 100 \text{ mm}$$

$$r = 10 \text{ mm}$$

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{10}{100} = 0.10$$

For configuration (a),

Fig 4.32 gives $K_a = 2.21$.

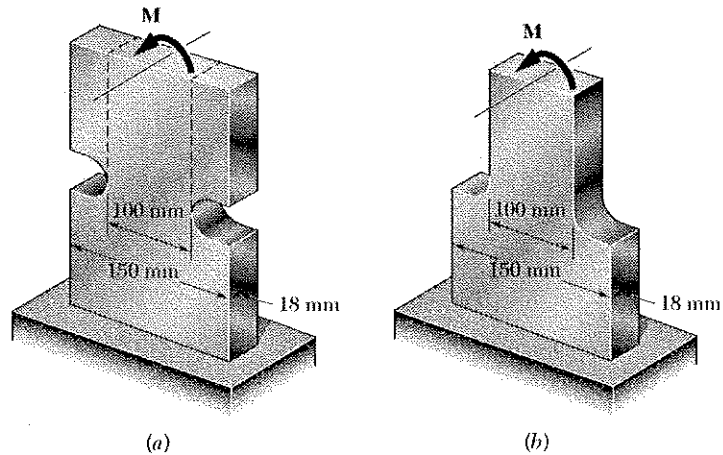
For configuration (b), Fig. 4.31 gives $K_b = 1.79$.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 50 \text{ mm} = 0.05 \text{ m}$$

$$(a) \quad \sigma = \frac{K M c}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147 \times 10^6 \text{ Pa} = 147 \text{ MPa}$$

$$(b) \quad \sigma = \frac{K M c}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119 \times 10^6 \text{ Pa} = 119 \text{ MPa}$$



Problem 4.66

4.66 The allowable stress used in the design of a steel bar is 80 MPa. Determine the largest couple M that can be applied to the bar (a) if the bar is designed with grooves having semicircular portions of radius $r = 15$ mm, as shown in Fig. 4.65a, (b) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65b.

For both configurations

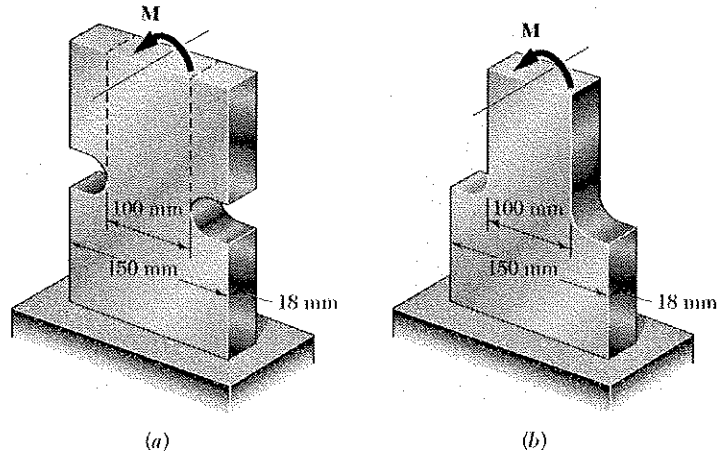
$$D = 150 \text{ mm}, d = 100 \text{ mm}, \\ r = 15 \text{ mm}.$$

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (a), Fig 4.32 gives $K_a = 1.92$.

For configuration (b) Fig 4.31 gives $K_b = 1.57$.



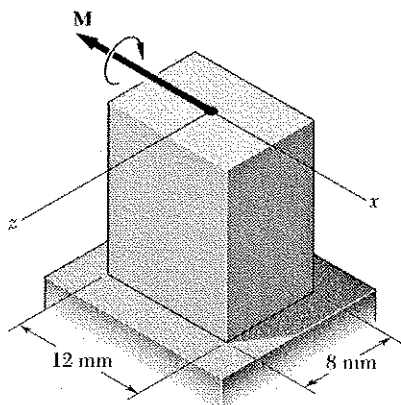
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 50 \text{ mm} = 0.050 \text{ m}$$

$$(a) \quad \sigma = \frac{K M c}{I} \therefore M = \frac{\sigma I}{K c} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92)(0.05)} = 1.25 \times 10^3 \text{ N}\cdot\text{m} \\ = 1.25 \text{ kN}\cdot\text{m}$$

$$(b) \quad M = \frac{\sigma I}{K c} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.53 \times 10^3 \text{ N}\cdot\text{m} = 1.53 \text{ kN}\cdot\text{m}$$

Problem 4.67



4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple M parallel to the x axis. Determine the moment M of the couple for which (a) yield first occurs, (b) the elastic core of the bar is 4 mm thick.

$$(a) \quad I = \frac{1}{12} b h^3 = \frac{1}{12} (12)(8)^3 = 512 \text{ mm}^4 = 512 \times 10^{-12} \text{ m}^4$$

$$c = \frac{1}{2} h = 4 \text{ mm} = 0.004 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(512 \times 10^{-12})}{0.004}$$

$$= 38.4 \text{ N}\cdot\text{m}$$

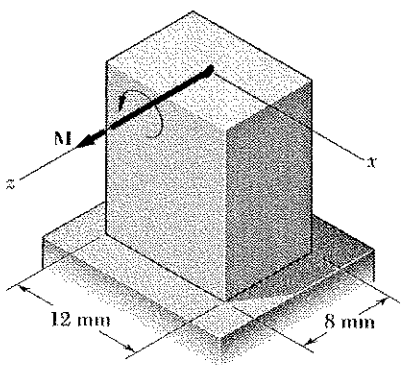
$$M_Y = 38.4 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$(b) \quad y_r = \frac{1}{2}(4) = 2 \text{ mm} \quad \frac{y_r}{c} = \frac{2}{4} = 0.5$$

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3} \left(\frac{y_r}{c} \right)^2 \right] = \frac{3}{2} (38.4) \left[1 - \frac{1}{3} (0.5)^2 \right] = 52.8 \text{ N}\cdot\text{m}$$

$$M = 52.8 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.68



4.68 Solve Prob. 4.67, assuming that the couple M is parallel to the z axis.

4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple M parallel to the x axis. Determine the moment M of the couple for which (a) yield first occurs, (b) the elastic core of the bar is 4 mm thick.

$$(a) \quad I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(12)^3 = 1.152 \times 10^3 \text{ mm}^4 = 1.152 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} h = 6 \text{ mm} = 0.006 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(1.152 \times 10^{-9})}{0.006}$$

$$= 57.6 \text{ N}\cdot\text{m}$$

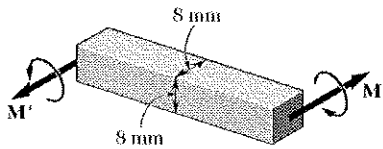
$$M_Y = 57.6 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$(b) \quad y_r = \frac{1}{2}(4) = 2 \text{ mm} \quad \frac{y_r}{c} = \frac{2}{6} = \frac{1}{3}$$

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3} \left(\frac{y_r}{c} \right)^2 \right] = \frac{3}{2} (57.6) \left[1 - \frac{1}{3} \left(\frac{1}{3} \right)^2 \right] = 83.2 \text{ N}\cdot\text{m}$$

$$M = 83.2 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.69



4.69 For the steel bar of Prob. 4.70, determine the thickness of the plastic zones at the top and bottom of the bar when (a) $M = 30 \text{ N}\cdot\text{m}$, (b) $M = 35 \text{ N}\cdot\text{m}$.

4.70 A bar having the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 330 \text{ MPa}$. Determine the bending moment M at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 2 mm thick.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(8)^3 = 341.3 \text{ mm}^4.$$

$$c = \frac{1}{2} h = 4 \text{ mm}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(330 \times 10^6)(341.3 \times 10^{-12})}{0.004} = 28.16 \text{ N}\cdot\text{m}.$$

$$M_y = 28.16 \text{ N}\cdot\text{m}.$$

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3} \left(\frac{y_Y}{c} \right)^2 \right]$$

$$\frac{y_Y}{c} = \sqrt{3 - 2 \frac{M}{M_Y}}$$

(a) $M = 30 \text{ N}\cdot\text{m}$

$$\frac{y_Y}{c} = \sqrt{3 - 2 \left(\frac{30}{28.16} \right)} = 0.9324$$

$$y_Y = (0.9324)(4) = 3.73 \text{ mm}$$

$$t_p = 4 - 3.73 = 0.27 \text{ mm} \quad \blacktriangleleft$$

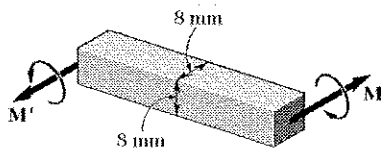
(b) $M = 35 \text{ N}\cdot\text{m}$

$$\frac{y_Y}{c} = \sqrt{3 - 2 \left(\frac{35}{28.16} \right)} = 0.7171$$

$$y_Y = (0.7171)(4) = 2.87 \text{ mm}$$

$$t_p = c - y_Y = 1.13 \text{ mm} \quad \blacktriangleleft$$

Problem 4.70



4.70 A bar having the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 330 \text{ MPa}$. Determine the bending moment M at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 2 mm thick.

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(8)^3 = 341.3 \text{ mm}^4.$

$$c = \frac{1}{2} h = 4 \text{ mm}.$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(330 \times 10^6)(341.3 \times 10^{-12})}{0.004} = 28.16 \text{ N}\cdot\text{m}.$$

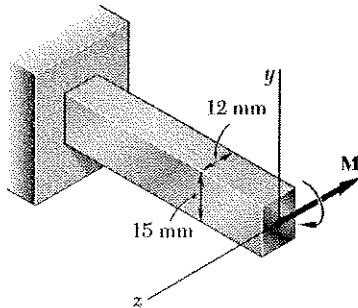
$$M_Y = 28.16 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

(b) $y_Y = c - t_p = 4 - 2 = 2 \text{ mm}$

$$M_P = \frac{3}{2} M_Y \left[1 - \frac{1}{3} \left(\frac{y_Y}{c} \right)^2 \right] = \frac{3}{2} (28.16) \left[1 - \frac{1}{3} \left(\frac{2}{4} \right)^2 \right]$$

$$M_P = 38.72 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.71



4.71 The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 300 \text{ MPa}$, is subjected to a couple of $162 \text{ N} \cdot \text{m}$ parallel to the z axis. Determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

$$(a) \quad I = \frac{1}{12} (0.012)(0.015)^3 = 3.375 \times 10^{-9} \text{ m}^4$$

$$C = \frac{1}{2} (0.015) = 0.0075 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{C} = \frac{(300 \times 10^6)(3.375 \times 10^{-9})}{0.0075} = 135 \text{ Nm}$$

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3} \left(\frac{y_r}{C} \right)^3 \right]$$

$$\frac{y_r}{C} = \sqrt{3 - 2 \frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(162)}{135}} = 0.7746$$

$$y_r = (0.7746)(0.0075) = 5.81 \times 10^{-3} \text{ m}$$

Thickness of elastic core =

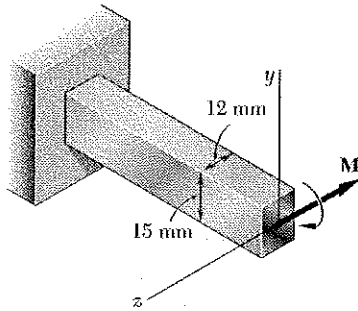
$$2y_r = 11.6 \text{ mm}$$

$$(b) \quad y_r = \epsilon_Y \rho = \frac{\sigma_Y}{E} \rho$$

$$\rho = \frac{y_r E}{\sigma_Y} = \frac{(5.81 \times 10^{-3})(200 \times 10^9)}{300 \times 10^6} = 3.873 \text{ m}$$

$$\rho = 3.9 \text{ m}$$

Problem 4.72



4.72 Solve Prob. 4.71, assuming that the $162 \text{ N} \cdot \text{m}$ couple is parallel to the y axis.

4.71 The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 300 \text{ MPa}$, is subjected to a couple of $162 \text{ N} \cdot \text{m}$ parallel to the z axis. Determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

$$(a) \quad I = \frac{1}{12} (0.015) (0.012)^3 = 2.18 \times 10^{-9} \text{ m}^4$$

$$c = \left(\frac{1}{2}\right) (0.012) = 0.006 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6) (2.18 \times 10^{-9})}{0.006} = 109 \text{ Nm}$$

$$M = \frac{3}{2} \left[1 - \frac{1}{3} \left(\frac{y_Y}{c} \right)^2 \right]$$

$$\frac{y_Y}{c} = \sqrt{3 - 2 \frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(162)}{109}} = 0.1659$$

$$y_Y = (0.1659) (0.006) = 9.954 \times 10^{-4}$$

Thickness of elastic core

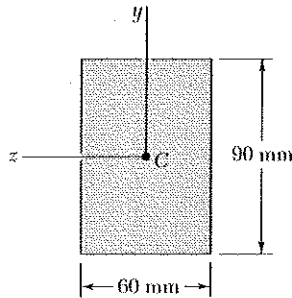
$$2y_Y = 2 \text{ mm}$$

$$(b) \quad y_Y = \epsilon_Y \rho = \frac{\sigma_Y}{E} \rho$$

$$\rho = \frac{y_Y E}{\sigma_Y} = \frac{(9.954 \times 10^{-4}) (200 \times 10^9)}{300 \times 10^6} = 0.6636 \text{ m}$$

$$\rho = 0.67 \text{ m}$$

Problem 4.73



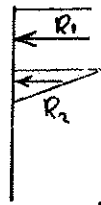
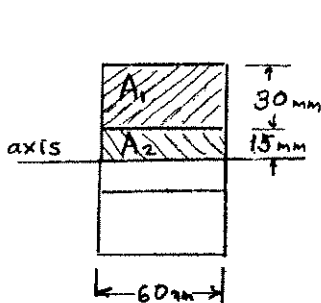
4.73 and 4.74 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic $E = 200 \text{ GPa}$ and $\sigma_y = 240 \text{ MPa}$. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 30-mm thick.

$$(a) \quad I = \frac{1}{12} b h^3 = \frac{1}{12} (60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} h = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_Y = \frac{\sigma_y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10 \text{ N}\cdot\text{m}$$

$$M_Y = 19.44 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$



$$R_1 = \sigma_y A_1 = (240 \times 10^6)(0.060)(0.030)$$

$$= 432 \times 10^3 \text{ N}$$

$$y_1 = 15 \text{ mm} + 15 \text{ mm} = 0.030 \text{ m}$$

$$R_2 = \frac{1}{2} \sigma_y A_2 = \left(\frac{1}{2}\right)(240 \times 10^6)(0.060)(0.015)$$

$$= 108 \times 10^3 \text{ N}$$

$$y_2 = \frac{2}{3}(15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$$

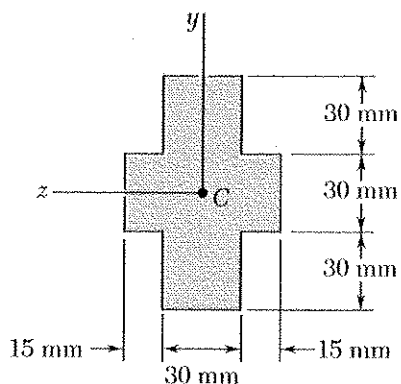
$$(b) \quad M = 2(R_1 y_1 + R_2 y_2) = 2[(432 \times 10^3)(0.030) + (108 \times 10^3)(0.010)]$$

$$= 28.08 \times 10^3 \text{ N}\cdot\text{m}$$

$$M = 28.1 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

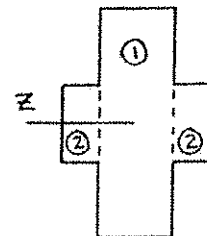
Problem 4.74

4.73 and 4.74 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 30 mm thick.



$$\begin{aligned} (a) \quad I_{\textcircled{1}} &= \frac{1}{12} b_1 h_1^3 \\ &= \frac{1}{12} (30)(90)^3 \\ &= 1.8225 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_{\textcircled{2}} &= \frac{1}{12} b_2 h_2^3 \\ &= \frac{1}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4 \end{aligned}$$

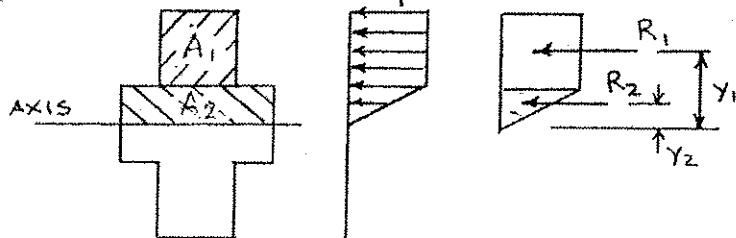


$$I = I_{\textcircled{1}} + I_{\textcircled{2}} = 1.89 \times 10^6 \text{ mm}^4 = 1.89 \times 10^{-6} \text{ m}^4 \quad C = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{C} = \frac{(240 \times 10^6)(1.89 \times 10^{-6})}{0.045} = 10.08 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_Y = 10.08 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

(b)



$$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.030)(0.030) = 216 \times 10^3 \text{ N}$$

$$y_1 = 15 + 15 = 30 \text{ mm} = 0.030 \text{ m}$$

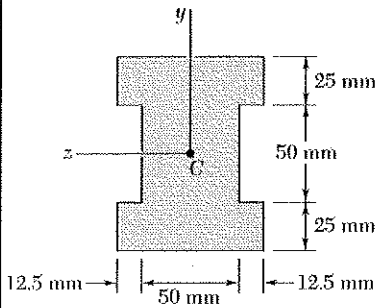
$$R_2 = \frac{1}{2} \sigma_Y A_2 = \frac{1}{2} (240 \times 10^6)(0.060)(0.015) = 108 \times 10^3 \text{ N}$$

$$y_2 = \frac{2}{3} (15) = 10 \text{ mm} = 0.010 \text{ m}$$

$$\begin{aligned} M &= 2(R_1 y_1 + R_2 y_2) \\ &= 2[(216 \times 10^3)(0.030) + (108 \times 10^3)(0.015)] \\ &= 16.12 \times 10^3 \text{ N}\cdot\text{m} \end{aligned}$$

$$M = 16.12 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.75



4.75 and 4.76 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic $E = 200 \text{ GPa}$ and $\sigma_y = 300 \text{ MPa}$. For bending about the z axis, determine the bending moment and radius of curvature at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 25 mm thick.

$$(a) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (75)(25)^3 + (75)(25)(37.5)^2 = 2734375 \text{ mm}^4$$

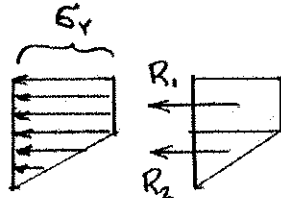
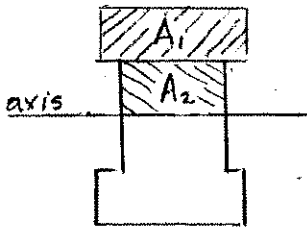
$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (50)(50)^3 = 520833 \text{ mm}^4$$

$$I_3 = I_1 = 2734375 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 5989583 \text{ mm}^4$$

$$c = 50 \text{ mm}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(300 \times 10^6)(5989583 \times 10^{-12})}{0.050} = 35.9 \text{ kNm} \quad \blacktriangleleft$$



$$R_1 = \sigma_y A_1 = (300 \times 10^6)(0.075)(0.025) = 562.5 \text{ kN}$$

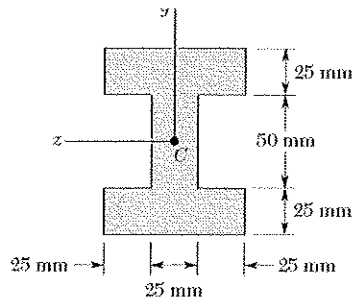
$$y_1 = 25 + 12.5 = 37.5 \text{ mm}$$

$$R_2 = \frac{1}{2} \sigma_y A_2 = \frac{1}{2} (300 \times 10^6)(0.05)(0.025) = 187.5 \text{ kN}$$

$$y_2 = \frac{2}{3} (25) = 16.67 \text{ mm}$$

$$(b) M = 2(R_1 y_1 + R_2 y_2) = 2[(562.5)(0.0375) + (187.5)(0.01667)] = 48.5 \text{ kNm} \quad \blacktriangleleft$$

Problem 4.76



4.75 and 4.76 A beam of the cross section shown is made of a steel that is assumed to be elastoplastic $E = 200$ GPa and $\sigma_Y = 300$ MPa. For bending about the z axis, determine the bending moment and radius of curvature at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 25 mm thick.

$$(a) \quad I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (75)(25)^3 + (75)(25)(37.5)^2 = 2734375 \text{ mm}^4$$

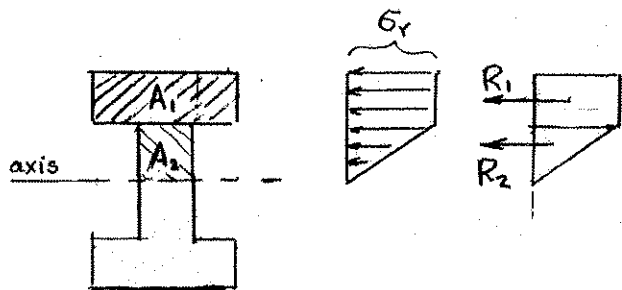
$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (25)(50)^3 = 260417 \text{ mm}^4$$

$$I_3 = I_1 = 2734375 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 5729167 \text{ mm}^4$$

$$c = 50 \text{ mm}$$

$$M_y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(5729167 \times 10^{-12})}{0.05} = 34.4 \text{ kNm} \quad \blacktriangleleft$$



$$R_1 = \sigma_Y A_1 = (300 \times 10^6)(0.075)(0.025) = 562.5 \text{ kN}$$

$$y_1 = 25 + 12.5 = 37.5 \text{ mm}$$

$$R_2 = \frac{1}{2} \sigma_Y A_2 = \frac{1}{2} (300 \times 10^6)(25 \times 10^{-3})^2 = 93.75 \text{ kN}$$

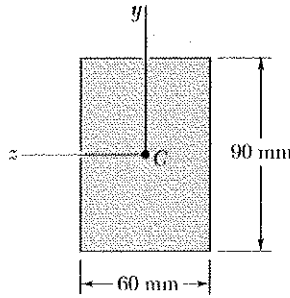
$$y_2 = \frac{2}{3} (25) = 16.67 \text{ mm}$$

$$(b) \quad M = 2(R_1 y_1 + R_2 y_2) = 2[(562.5)(0.0375) + (93.75)(0.01667)] = 45.3 \text{ kNm} \quad \blacktriangleleft$$

Problem 4.77

4.77 through 4.80 For the beam indicated, determine (a) the fully plastic moment M_p , (b) the shape factor of the cross section.

4.77 Beam of Prob. 4.73.



From PROBLEM 4.73 $E = 200 \text{ GPa}$ and $\sigma_Y = 240 \text{ MPa}$.

$$A_1 = (60)(45) = 2700 \text{ mm}^2$$

$$= 2700 \times 10^{-6} \text{ m}^2$$

$$R = \sigma_Y A_1$$

$$= (240 \times 10^6)(2700 \times 10^{-6})$$

$$= 648 \times 10^3 \text{ N}$$

$$d = 45 \text{ mm} = 0.045 \text{ m}$$

$$(a) \quad M_p = R d = (648 \times 10^3)(0.045) = 29.16 \times 10^3 \text{ N}\cdot\text{m} \quad M_p = 29.2 \text{ kN}\cdot\text{m} \blacktriangleleft$$

$$(b) \quad I = \frac{1}{12} b h^3 = \frac{1}{12} (60)(90)^3 = 3.645 \times 10^6 \text{ mm}^4 = 3.645 \times 10^{-6} \text{ m}^4$$

$$c = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(3.645 \times 10^{-6})}{0.045} = 19.44 \times 10^3 \text{ N}\cdot\text{m}$$

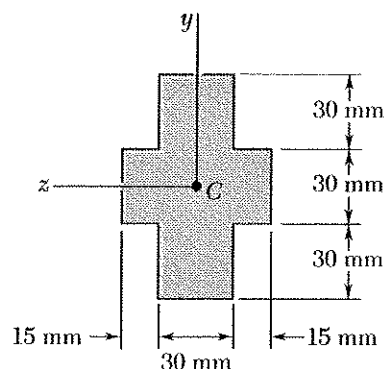
$$k = \frac{M_p}{M_Y} = \frac{29.16}{19.44}$$

$$k = 1.500 \blacktriangleleft$$

Problem 4.78

4.77 through 4.80 For the beam indicated, determine (a) the fully plastic moment M_p , (b) the shape factor of the cross section.

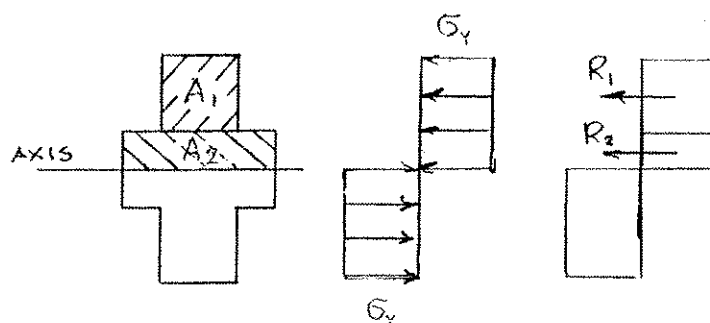
4.78 Beam of Prob. 4.74.



From PROBLEM 4.74 $E = 200 \text{ GPa}$ and $\sigma_Y = 240 \text{ MPa}$

$$\begin{aligned} (a) \quad R_1 &= \sigma_Y A_1 \\ &= (240 \times 10^6)(0.030)(0.030) \\ &= 216 \times 10^3 \text{ N} \end{aligned}$$

$$\bar{y}_1 = 30 \text{ mm} = 0.030 \text{ m}$$



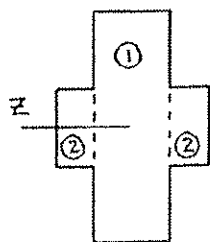
$$\begin{aligned} R_2 &= \sigma_Y A_2 \\ &= (240 \times 10^6)(0.060)(0.015) \\ &= 216 \times 10^3 \text{ N} \end{aligned}$$

$$\bar{y}_2 = 7.5 \text{ mm} = 0.0075 \text{ m}$$

$$\begin{aligned} M_p &= 2(R_1 \bar{y}_1 + R_2 \bar{y}_2) = 2[(216 \times 10^3)(0.030) + (216 \times 10^3)(0.0075)] \\ &= 16.20 \times 10^3 \text{ N}\cdot\text{m} \end{aligned}$$

$$M_p = 16.20 \text{ kN}\cdot\text{m}$$

(b)



$$I_1 = \frac{1}{12} b_1 h_1^3 = \frac{1}{12} (30)(90)^3 = 1.8225 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.89 \times 10^6 \text{ mm}^4 = 1.89 \times 10^{-6} \text{ m}^4$$

$$c = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(1.89 \times 10^{-6})}{0.045} = 10.08 \times 10^3 \text{ N}\cdot\text{m}$$

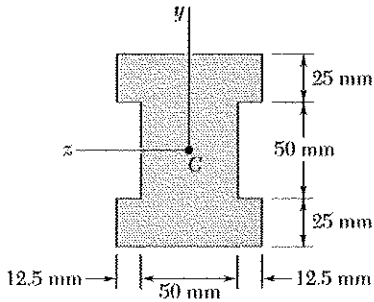
$$k = \frac{M_p}{M_Y} = \frac{16.20 \times 10^3}{10.08 \times 10^3}$$

$$k = 1.607$$

Problem 4.79

4.77 through 4.80 For the beam indicated, determine (a) the fully plastic moment M_p , (b) the shape factor of the cross section.

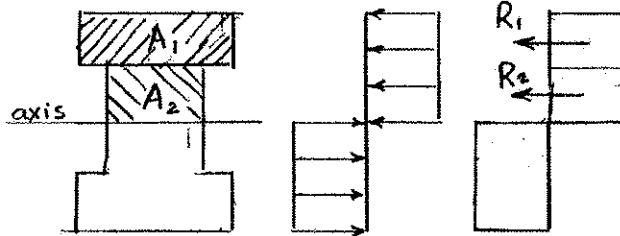
4.79 Beam of Prob. 4.75.



From PROBLEM 4.76 $E = 29 \times 10^6$ and $\sigma_r = 300 \text{ MPa}$.

$$R_1 = \sigma_r A_1 = (300 \times 10^6)(0.075)(0.025) = 562.5 \text{ kN}$$

$$y_1 = 25 + 12.5 = 37.5 \text{ mm}$$



$$R_2 = \sigma_r A_2 = (300 \times 10^6)(0.05)(0.025) = 375 \text{ kN}$$

$$y_2 = \frac{1}{2}(25) = 12.5 \text{ mm}$$

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(562.5)(0.0375) + (375)(0.0125)] = 51.6 \text{ kN}$$

$$(b) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(25)^3 + (75)(25)(37.5)^2 = 2734375 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(50)(50) = 520833 \text{ mm}^4$$

$$I_3 = I_1 = 2734375 \text{ mm}^4$$

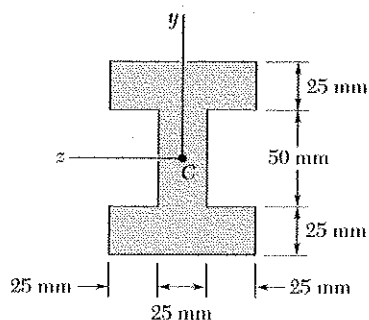
$$I = I_1 + I_2 + I_3 = 5989583$$

$$c = 50 \text{ mm}$$

$$M_y = \frac{\sigma_r I}{c} = \frac{(300 \times 10^6)(5989583 \times 10^{-12})}{0.05} = 35.9 \text{ kNm}$$

$$k = \frac{M_p}{M_y} = \frac{51.6}{35.9} = 1.437$$

Problem 4.80



4.77 through 4.80 For the beam indicated, determine (a) the fully plastic moment M_p , (b) the shape factor of the cross section.

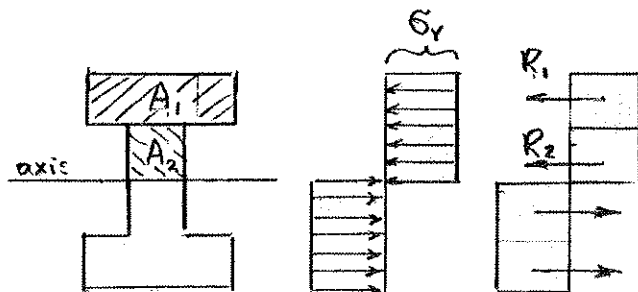
4.77 Beam of Prob. 4.73.

4.78 Beam of Prob. 4.74.

4.79 Beam of Prob. 4.75.

4.80 Beam of Prob. 4.76.

From PROBLEM 4.75 $E = 200 \times 10^9 \text{ Pa}$ and $\sigma_Y = 300 \text{ MPa}$



$$R_1 = \sigma_Y A_1 = (300 \times 10^6)(0.075)(0.025) = 562.5 \text{ kN}$$

$$\bar{y}_1 = 25 + 12.5 = 37.5 \text{ mm}$$

$$R_2 = \sigma_Y A_2 = (300 \times 10^6)(0.025)(0.025) = 187.5 \text{ kN}$$

$$\bar{y}_2 = \frac{1}{2}(25) = 12.5 \text{ mm}$$

$$M_p = 2(R_1 \bar{y}_1 + R_2 \bar{y}_2) = 2[(562.5)(0.0375) + (187.5)(0.0125)] = 46.9 \text{ kNm}$$

$$(b) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (75)(25)^3 + (75)(25)(37.5)^2 = 2734375 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (25)(50)^3 = 260417 \text{ mm}^4$$

$$I_3 = I_1 = 2734375 \text{ mm}^4$$

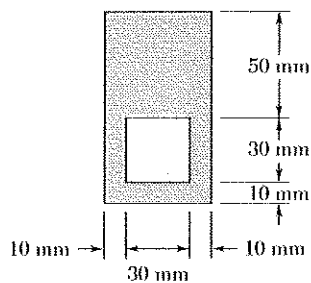
$$I = I_1 + I_2 + I_3 = 5729167 \text{ mm}^4$$

$$c = 50 \text{ mm}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(5729167 \text{ mm}^4)}{50} = 34.4 \text{ kNm}$$

$$k = \frac{M_p}{M_Y} = \frac{46.9}{34.4} = 1.363$$

Problem 4.81

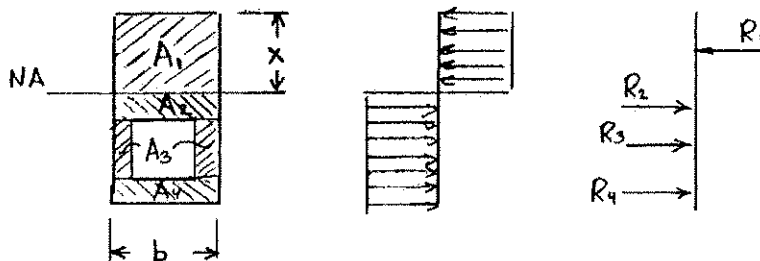


4.81 and 4.82 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

$$\text{Total area: } A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$$

$$\frac{1}{2} A = 1800 \text{ mm}^2$$

$$x = \frac{\frac{1}{2} A}{b} = \frac{1800}{50} = 36 \text{ mm}$$



$$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm}, \quad A_1 \bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$$

$$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm}, \quad A_2 \bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$$

$$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm}, \quad A_3 \bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$$

$$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm}, \quad A_4 \bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$$

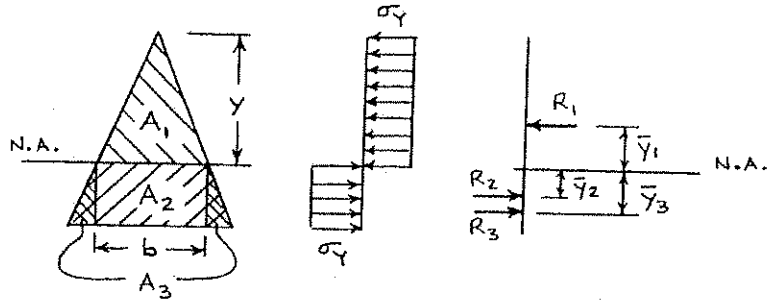
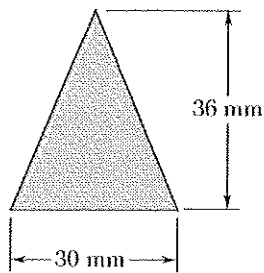
$$A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3 + A_4 \bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$$

$$M_p = G_y \sum A_i \bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_p = 19.01 \text{ kN} \cdot \text{m}$$

Problem 4.82

4.81 and 4.82 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.



$$\text{Total area: } A = \frac{1}{2}(30)(36) = 540 \text{ mm}^2$$

$$\frac{1}{2}A = 270 \text{ mm}^2 = A_1$$

$$\text{By similar triangles, } \frac{b}{y} = \frac{30}{36} \quad b = \frac{5}{6}y$$

$$\text{Since } A_1 = \frac{1}{2}by = \frac{5}{12}y^2, \quad y^2 = \frac{12}{5}A_1$$

$$y = \sqrt{\frac{12}{5}(270)} = 25.4558 \text{ mm} \quad b = 21.2132 \text{ mm}$$

$$A_1 = \frac{1}{2}(21.2132)(25.4558) = 270 \text{ mm}^2 = 270 \times 10^{-6} \text{ m}^2$$

$$A_2 = (21.2132)(36 - 25.4558) = 223.676 \text{ mm}^2 = 223.676 \times 10^{-6} \text{ m}^2$$

$$A_3 = A - A_1 - A_2 = 46.324 = 46.324 \times 10^{-6} \text{ m}^2$$

$$R_2 = \sigma_y A_2 = 240 \times 10^6 A_2$$

$$R_1 = 64.8 \times 10^3 \text{ N}, \quad R_2 = 53.6822 \times 10^3 \text{ N}, \quad R_3 = 11.1178 \times 10^3 \text{ N}$$

$$\bar{y}_1 = \frac{1}{3}y = 8.4853 \text{ mm} = 8.4853 \times 10^{-3} \text{ m}$$

$$\bar{y}_2 = \frac{1}{2}(36 - 25.4558) = 5.2721 \text{ mm} = 5.2721 \times 10^{-3} \text{ m}$$

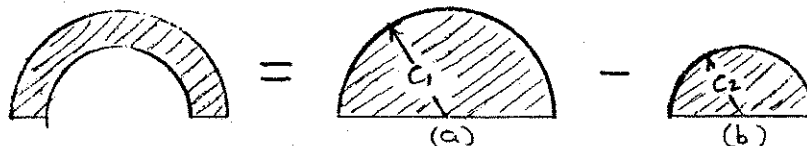
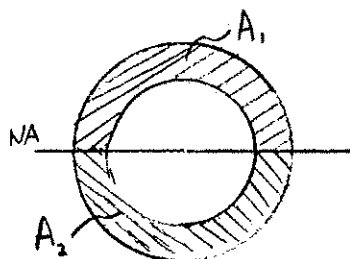
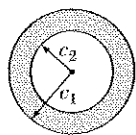
$$\bar{y}_3 = \frac{2}{3}(36 - 25.4558) = 7.0295 \text{ mm} = 7.0295 \times 10^{-3} \text{ m}$$

$$M_p = R_1 \bar{y}_1 + R_2 \bar{y}_2 + R_3 \bar{y}_3 = 911 \text{ N}\cdot\text{m}$$

$$M_p = 911 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Proprietary Material. © 2009 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

Problem 4.83



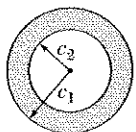
$$\begin{aligned} A_1 \bar{y}_1 &= A_a \bar{y}_a - A_b \bar{y}_b \\ &= \left(\frac{\pi}{2} c_1^2 \right) \left(\frac{4c_1}{3\pi} \right) - \left(\frac{\pi}{2} c_2^2 \right) \left(\frac{4c_2}{3\pi} \right) \\ &= \frac{2}{3} (c_1^3 - c_2^3) \end{aligned}$$

$$A_2 \bar{y}_2 = A_1 \bar{y}_1 = \frac{2}{3} (c_1^3 - c_2^3)$$

$$M_p = \sigma_Y (A_1 \bar{y}_1 + A_2 \bar{y}_2) = \frac{4}{3} \sigma_Y (c_1^3 - c_2^3) \quad \blacktriangleleft$$

Problem 4.84

4.84 Determine the plastic moment M_p of a thick-walled pipe of the cross section shown, knowing that $c_1 = 60$ mm, $c_2 = 40$ mm, and $\sigma_Y = 240$ MPa.



See the solution to PROBLEM 4.83 for derivation of the following expression for M_p :

$$M_p = \frac{4}{3} \sigma_Y (c_1^3 - c_2^3)$$

Data: $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}$

$$c_1 = 60 \text{ mm} = 0.060 \text{ m}$$

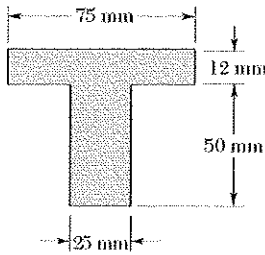
$$c_2 = 40 \text{ mm} = 0.040 \text{ m}$$

$$M_p = \frac{4}{3} (240 \times 10^6) (0.060^3 - 0.040^3) = 48.67 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_p = 48.6 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.85

4.85 Determine the plastic moment M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 330 MPa.



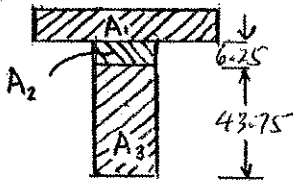
$$\text{Total area: } A = (75)(12) + (50)(25) = 2150 \text{ mm}^2$$

$$\frac{1}{2} A = 1075 \text{ mm}^2 \quad y_n = \frac{\frac{1}{2} A}{b} = \frac{1075}{25} = 43 \text{ mm}$$

$$A_1 = (75)(12) = 900 \text{ mm}^2, \quad \bar{y}_1 = 12.25 \text{ mm}, \quad A_1 \bar{y}_1 = 11025 \text{ mm}^3$$

$$A_2 = (6.25)(25) = 156.25 \text{ mm}^2, \quad \bar{y}_2 = 3.125 \text{ mm}, \quad A_2 \bar{y}_2 = 488.3 \text{ mm}^3$$

$$A_3 = (43.75)(25) = 1093.75 \text{ mm}^2, \quad \bar{y}_3 = 21.875 \text{ mm}, \quad A_3 \bar{y}_3 = 23925.8$$

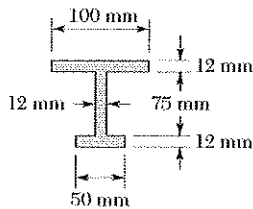


$$M_p = \sigma_y (A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3)$$

$$= 330 \times 10^6 (11025 + 488.3 + 23925.8) (1 \times 10^{-9}) = 11.7 \text{ kNm}$$

Problem 4.86

4.86 Determine the plastic moment M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 250 MPa.



$$\text{Total area: } A = (100)(12) + (75)(12) + (50)(12) = 2700 \text{ mm}^2$$

$$\frac{1}{2} A = 1350 \text{ mm}^2$$

$$A_1 = 1200 \text{ mm}^2, \quad \bar{y}_1 = 18.5, \quad A_1 \bar{y}_1 = 22200 \text{ mm}^3$$

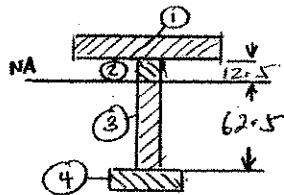
$$A_2 = 150 \text{ mm}^2, \quad \bar{y}_2 = 6.25, \quad A_2 \bar{y}_2 = 937.5 \text{ mm}^3$$

$$A_3 = 750 \text{ mm}^2, \quad \bar{y}_3 = 31.25, \quad A_3 \bar{y}_3 = 23437.5 \text{ mm}^3$$

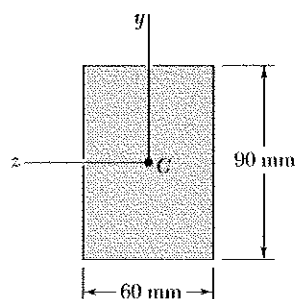
$$A_4 = 600 \text{ mm}^2, \quad \bar{y}_4 = 68.5, \quad A_4 \bar{y}_4 = 41100 \text{ mm}^3$$

$$M_p = \sigma_y (A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3 + A_4 \bar{y}_4)$$

$$= (250 \times 10^6) (22200 + 937.5 + 23437.5 + 41100) (1 \times 10^{-9}) = 21.9 \text{ kNm}$$



Problem 4.87



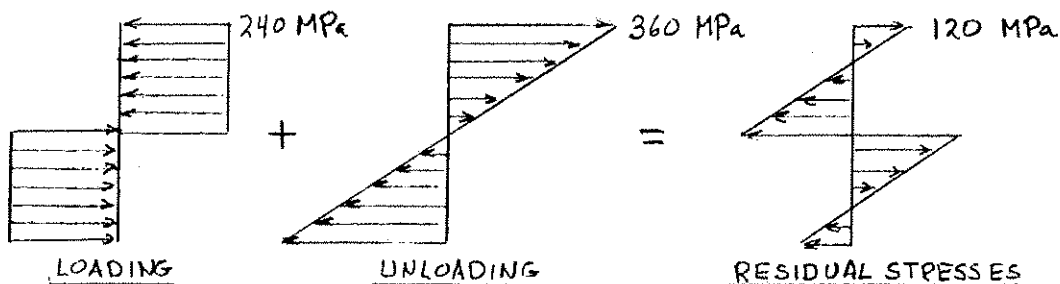
4.87 and 4.88 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

4.87 Beam of Prob. 4.73.

$$M_p = 29.16 \times 10^3 \text{ N}\cdot\text{m} \quad (\text{See solutions to Problems 4.73 and 4.77})$$

$$I = 3.645 \times 10^{-6} \text{ m}^4, \quad c = 0.045 \text{ m}$$

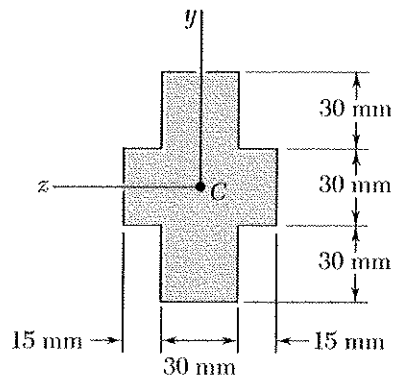
$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{at } y = c = 45 \text{ mm.}$$



$$\sigma' = \frac{(29.16 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 360 \times 10^6 \text{ Pa}$$

$$\sigma_{res} = \sigma' - \sigma_y = 360 \times 10^6 - 240 \times 10^6 = 120 \times 10^6 \text{ Pa} \quad \sigma_{res} = 120.0 \text{ MPa}$$

Problem 4.88



4.87 and 4.88 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45$ mm.

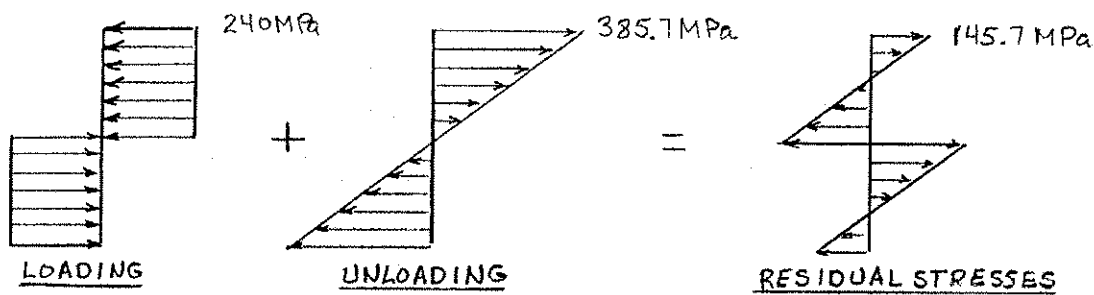
4.88 Beam of Prob. 4.74.

$$M_p = 16.20 \times 10^3 \text{ N}\cdot\text{m} \text{ from the solution to PROBLEM 4.78}$$

$$I = 1.86 \times 10^{-6} \text{ m}^4$$

$$c = 0.045 \text{ m}$$

$$\begin{aligned} \sigma' &= \frac{M_{\max} c}{I} = \frac{M_p c}{I} = \frac{(16.20 \times 10^3)(0.045)}{1.89 \times 10^{-6}} \\ &= 385.7 \times 10^6 \text{ Pa} = 385.7 \text{ MPa} \end{aligned}$$



$$\text{At } y = 45$$

$$\sigma = 240 \text{ MPa}$$

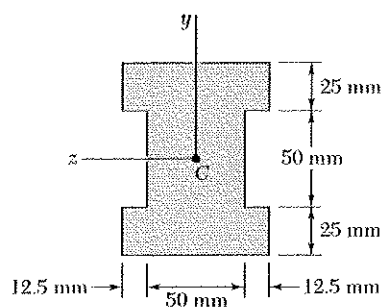
$$\sigma' = 385.7 \text{ MPa}$$

Residual stress

$$\sigma_{\text{res}} = \sigma' - \sigma_y$$

$$\sigma_{\text{res}} = 145.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.89



4.89 and 4.90 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 290 MPa, determine the residual stress at (a) $y = 25$ mm, (b) $y = 50$ mm.

4.89 Beam of Prob. 4.75.

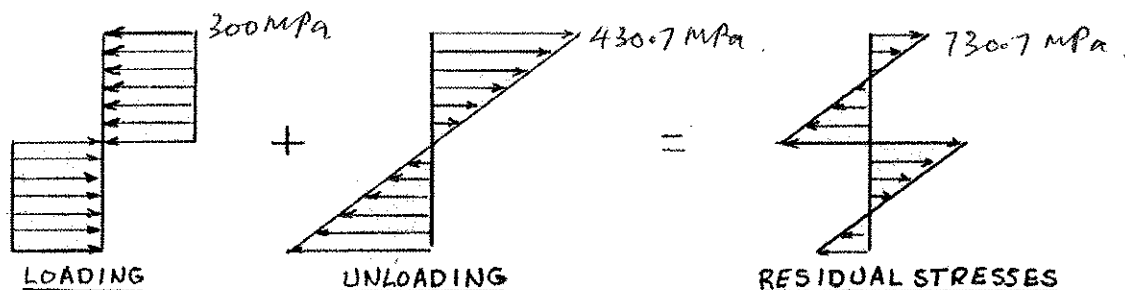
4.90 Beam of Prob. 4.76.

$$M_p = 51.6 \text{ kN} \cdot \text{m} \quad (\text{See solutions to Problems 4.76 and 4.80})$$

$$I = 5989583 \text{ mm}^4, \quad c = 50 \text{ mm}.$$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{for } y = c$$

$$\sigma' = \frac{(51.6 \times 10^3)(0.05)}{5989583 \times 10^{-12}} = 430.7 \text{ MPa}.$$



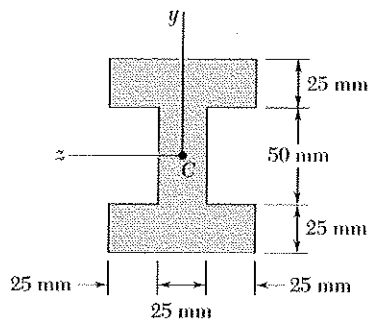
$$(a) \text{ At } y = 25 \text{ mm} = \frac{1}{2}c \quad \sigma' = \frac{1}{2}(430.7) = 215.35 \text{ MPa}.$$

$$\sigma_{res} = -300 + 215.35 = -84.65 \text{ MPa}.$$

$$(b) \text{ At } y = 50 \text{ mm} = c \quad \sigma' = 430.7 \text{ MPa}$$

$$\sigma_{res} = -300 + 430.7 = 130.7 \text{ MPa}.$$

Problem 4.90



4.89 and 4.90 For the beam indicated, a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 290 MPa, determine the residual stress at (a) $y = 25$ mm, (b) $y = 50$ mm.

4.89 Beam of Prob. 4.75.

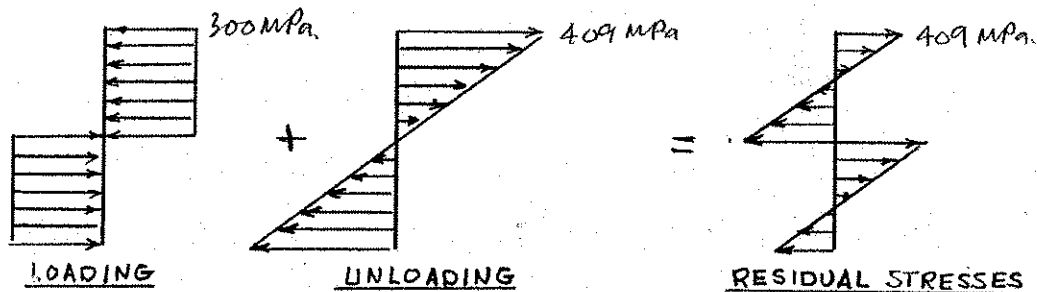
4.90 Beam of Prob. 4.76.

$$M_p = 46.9 \text{ kNm} \quad (\text{See solutions to Problems 4.75 and 4.79})$$

$$I = 5729167 \text{ mm}^4 \quad c = 50 \text{ mm}$$

$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \quad \text{at } y = c.$$

$$\sigma' = \frac{(46.9 \times 10^3)(0.05)}{5729167 \times 10^{-12}} = 409 \text{ MPa}$$



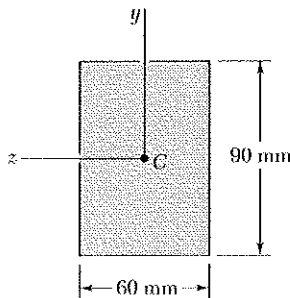
(a) At $y = 25 \text{ mm} = \frac{1}{2}c$ $\sigma' = \frac{1}{2}(409) = 204.5 \text{ MPa}$

$$\sigma_{\text{res}} = -300 + 204.5 = -95.5 \text{ MPa}$$

(b) At $y = 50 \text{ mm} = c$ $\sigma' = 409 \text{ MPa}$

$$\sigma_{\text{res}} = -300 + 409 = 109 \text{ MPa}$$

Problem 4.91



4.91 A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (a) the residual stress at $y = 45$ mm, (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the beam.

See SOLUTION to PROBLEM 4.73 for bending couple and stress distribution during loading.

$$M = 28.08 \times 10^3 \text{ N}\cdot\text{m} \quad y_r = 15 \text{ mm} = 0.015 \text{ m}$$

$$E = 200 \text{ GPa} \quad \sigma_r = 240 \text{ MPa}$$

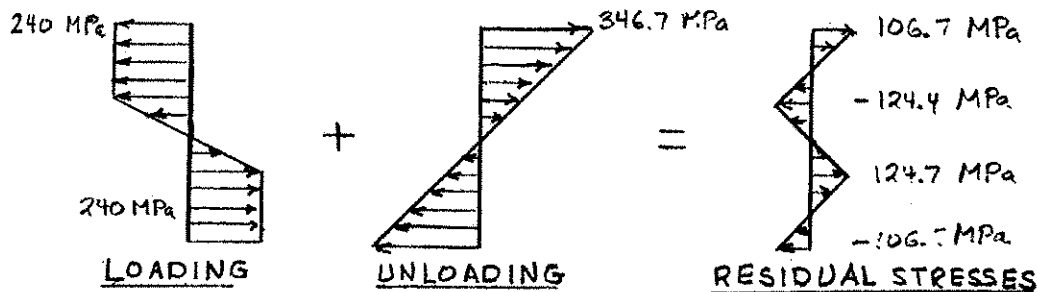
$$I = 3.645 \times 10^{-6} \text{ m}^4 \quad c = 0.045 \text{ m}$$

$$(a) \quad \sigma' = \frac{Mc}{I} = \frac{(28.08 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 346.7 \times 10^6 \text{ Pa} = 346.7 \text{ MPa}$$

$$\sigma'' = \frac{My_r}{I} = \frac{(28.08 \times 10^3)(0.015)}{3.645 \times 10^{-6}} = 115.6 \times 10^6 \text{ Pa} = 115.6 \text{ MPa}$$

$$\text{At } y = c, \quad \sigma_{res} = \sigma' - \sigma_r = 346.7 - 240 \quad \sigma_{res} = 106.7 \text{ MPa} \quad \blacktriangleleft$$

$$\text{At } y = y_r, \quad \sigma_{res} = \sigma'' - \sigma_r = 115.6 - 240 \quad \sigma_{res} = -124.4 \text{ MPa}$$



$$(b) \quad \sigma_{res} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_r = 0$$

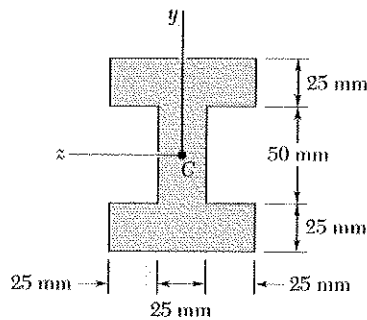
$$y_0 = \frac{I \sigma_r}{M} = \frac{(3.645 \times 10^{-6})(240 \times 10^6)}{28.08 \times 10^3} = 31.15 \times 10^{-3} \text{ m} = 31.15 \text{ mm}$$

$$\text{ans. } y_0 = -31.15 \text{ mm}, 0, 31.15 \text{ mm} \quad \blacktriangleleft$$

$$(c) \quad \text{At } y = y_r, \quad \sigma_{res} = -124.4 \times 10^6 \text{ Pa}$$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.015)}{-124.4 \times 10^6} \quad \rho = 24.1 \text{ m} \quad \blacktriangleleft$$

Problem 4.92



4.92 A bending couple is applied to the beam of Prob. 4.76, causing plastic zones 50 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (a) the residual stress at $y = 50$ mm, (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the beam.

See SOLUTION to PROBLEM 4.75 for bending couple and stress distribution

$$M = 45.3 \text{ kNm}$$

$$y_r = 25 \text{ mm}$$

$$E = 200 \text{ GPa}$$

$$\sigma_r = 300 \text{ MPa}$$

$$I = 5729167 \text{ mm}^4$$

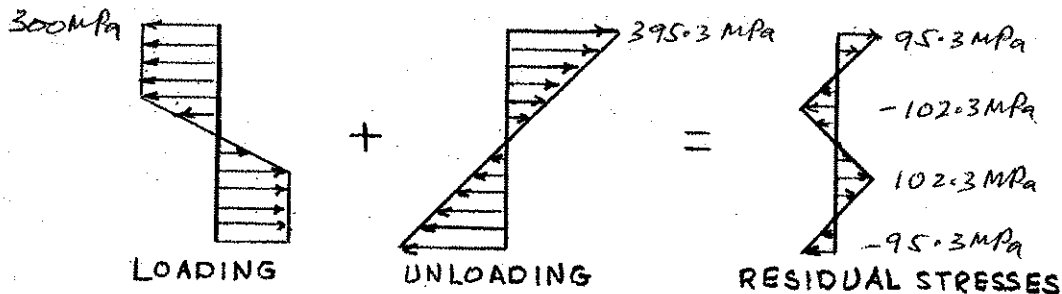
$$c = 50 \text{ mm}$$

$$(a) \quad \sigma' = \frac{Mc}{I} = \frac{(45.3 \times 10^3)(0.05)}{5729167 \times 10^{-12}} = 395.3 \text{ MPa}$$

$$\sigma'' = \frac{My_r}{I} = \frac{(45.3 \times 10^3)(0.025)}{5729167 \times 10^{-12}} = 197.7 \text{ MPa}$$

$$\text{At } y = c \quad \sigma_{\text{res}} = \sigma' - \sigma_r = 395.3 - 300 = 95.3 \text{ MPa}$$

$$\text{At } y = y_r \quad \sigma_{\text{res}} = \sigma'' - \sigma_r = 197.7 - 300 = -102.3 \text{ MPa}$$



$$(b) \quad \sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_o}{I} - \sigma_r = 0$$

$$y_o = \frac{I\sigma_r}{M} = \frac{(5729167 \times 10^{-12})(300 \times 10^6)}{45300} = 0.0379 \text{ m}$$

$$\text{ans. } y_o = -37.9 \text{ mm}, 0, 37.9 \text{ mm}$$

$$(c) \quad \text{At } y = y_r, \quad \sigma_{\text{res}} = -102.3 \text{ MPa}$$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.025)}{102.3 \times 10^6} = 48.9 \text{ m}$$

Problem 4.93

***4.93** A rectangular bar that is straight and unstressed is bent into an arc of circle of radius ρ by two couples of moment M . After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R . Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation:

$$\frac{1}{\rho_R} = \frac{1}{\rho} \left\{ 1 - \frac{3}{2} \frac{\rho}{\rho_Y} \left[1 - \frac{1}{3} \left(\frac{\rho}{\rho_Y} \right)^2 \right] \right\}$$

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \quad M = \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \quad \text{Let } m \text{ denote } \frac{M}{M_Y}$$

$$m = \frac{M}{M_Y} = \frac{3}{2} \left(1 - \frac{\rho^2}{\rho_Y^2} \right) \quad \therefore \quad \frac{\rho^2}{\rho_Y^2} = 3 - 2m$$

$$\begin{aligned} \frac{1}{\rho_R} &= \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{m M_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} \\ &= \frac{1}{\rho} \left\{ 1 - \frac{\rho}{\rho_Y} m \right\} = \frac{1}{\rho} \left\{ 1 - \frac{3}{2} \frac{\rho}{\rho_Y} \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \right\} \end{aligned}$$

Problem 4.94

4.94 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y , respectively, the bending moment and radius of curvature at the onset of yield, determine (a) the radius of curvature when a couple of moment $M = 1.25 M_Y$ is applied to the bar, (b) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.93.

$$(a) \quad \frac{1}{\rho_Y} = \frac{M_Y}{EI}, \quad M = \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \quad \text{Let } m = \frac{M}{M_Y} = 1.25$$

$$m = \frac{M}{M_Y} = \frac{3}{2} \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \quad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

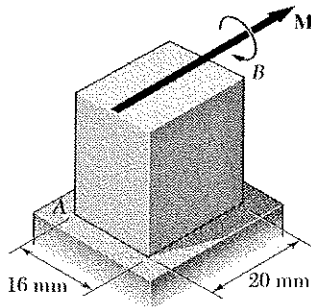
$$\rho = 0.70711 \rho_Y$$

$$(b) \quad \frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{m M_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711 \rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y}$$

$$\rho_R = 6.09 \rho_Y$$

Problem 4.95



4.95 The prismatic bar AB is made of a steel that is assumed to be elastoplastic and for which $E = 200 \text{ GPa}$. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350 \text{ N} \cdot \text{m}$ is applied as shown, determine (a) the yield strength of the steel, (b) the thickness of the elastic core of the bar.

$$\begin{aligned} M &= \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \\ &= \frac{3}{2} \frac{\sigma_Y I}{c} \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right) \\ &= \frac{3}{2} \frac{\sigma_Y b (2c)^3}{12 c} \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right) \\ &= \sigma_Y b c^2 \left(1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right) \end{aligned}$$

(a) $b c^2 \sigma_Y \left(1 - \frac{\rho^2 \sigma_Y^2}{3 E^2 c^2} \right) = M$ Cubic equation for σ_Y

Data: $E = 200 \times 10^9 \text{ Pa}$, $M = 420 \text{ N} \cdot \text{m}$, $\rho = 2.4 \text{ m}$

$b = 20 \text{ mm} = 0.020 \text{ m}$, $c = \frac{1}{2} h = 8 \text{ mm} = 0.008 \text{ m}$

$(1.28 \times 10^{-6}) \sigma_Y [1 - 750 \times 10^{-21} \sigma_Y^2] = 350$

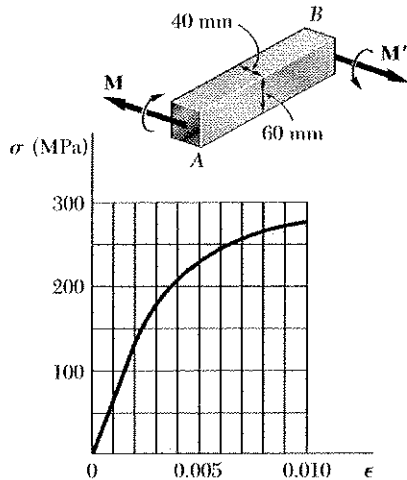
$\sigma_Y [1 - 750 \times 10^{-21} \sigma_Y^2] = 273.44 \times 10^6$

Solving by trial $\sigma_Y = 292 \times 10^6 \text{ Pa}$, $\sigma_Y = 292 \text{ MPa}$ $\rightarrow$

(b) $y_r = \frac{\sigma_Y \rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3} \text{ m} = 3.504 \text{ mm}$

thickness of elastic core $= 2y_r = 7.01 \text{ mm}$ $\leftarrow$

Problem 4.96



4.96 The prismatic bar AB is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ - ϵ diagram is the same in compression as in tension, determine (a) the radius of curvature of the bar when the maximum stress is 250 MPa, (b) the corresponding value of the bending moment. (Hint: For part b, plot σ versus y and use an approximate method of integration.)

$$(a) \quad \sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$$

$$\epsilon_m = 0.0064 \quad \text{from curve}$$

$$c = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$$

$$b = 40 \text{ mm} = 0.040 \text{ m}$$

$$\frac{1}{\rho} = \frac{\epsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1}$$

$$\rho = 4.69 \text{ m} \quad \blacktriangleleft$$

(b) Strain distribution. $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m u$ where $u = \frac{y}{c}$

Bending couple,

$$M = -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 u |\sigma| du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u |\sigma| du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3} \sum w u |\sigma|$$

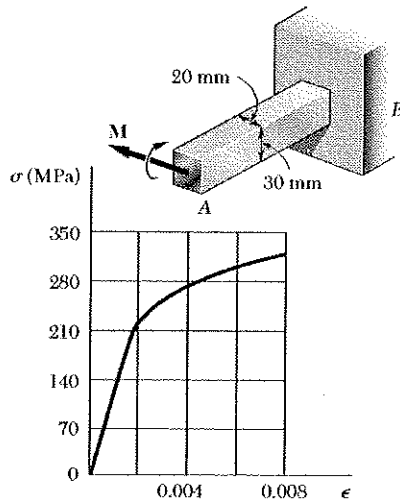
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$ \epsilon $	$ \sigma , (\text{MPa})$	$u \sigma , (\text{MPa})$	w	$w u \sigma , (\text{MPa})$
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215 $\leftarrow \sum w u \sigma $

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(101.25 \times 10^6) = 7.29 \times 10^3 \text{ N}\cdot\text{m} \quad M = 7.29 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.97



4.97 The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ - ϵ diagram is the same in compression as in tension, determine (a) the maximum stress in the bar when the radius of curvature of the bar is 2.5 m, (b) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)

(a) $\rho = 2500 \text{ mm}$, $b = 20 \text{ mm}$, $c = 15 \text{ mm}$.

$$\epsilon_m = \frac{c}{\rho} = \frac{15}{2500} = 0.006$$

From the curve $\sigma_m = 300 \text{ MPa}$.

(b) Strain distribution $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m U$ where $U = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 U |\sigma| dU = 2bc^2 J$$

where the integral J is given by $\int_0^1 U |\sigma| dU$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta U}{3} \sum w U |\sigma|$$

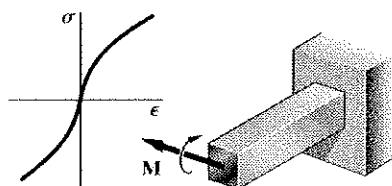
where w is a weighting factor. Using $\Delta U = 0.25$ we get the values given in the table below:

U	$ U $	$ \sigma , \text{MPa}$	$U \sigma , \text{MPa}$	w	$w U \sigma , \text{MPa}$
0	0	0	0	1	0
0.25	0.0015	175	43.8	4	175
0.5	0.003	262	126	2	252
0.75	0.0045	280	210	4	840
1.00	0.006	300	300	1	300
					1567 $\leftarrow \sum w U \sigma $

$$J = \frac{(0.25)(1567)}{3} = 130.6 \text{ MPa}$$

$$M = (2)(0.02)(0.015)^2 (130.6 \times 10^6) = 1.175 \text{ kN}\cdot\text{m}$$

Problem 4.98



4.98 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\epsilon = k\sigma^n$ for $\sigma > 0$ and $\epsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple M is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n} \frac{Mc}{I}$$

Strain distribution: $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m U$ where $U = \frac{y}{c}$

Bending couple.

$$\begin{aligned} M &= -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 \frac{y}{c} |\sigma| \frac{dy}{c} \\ &= 2bc^2 \int_0^1 U |\sigma| dU \end{aligned}$$

For $\epsilon = K\sigma^n$, $\epsilon_m = K\sigma_m^n$

$$\frac{\epsilon}{\epsilon_m} = U = \left(\frac{\sigma}{\sigma_m}\right)^n \quad \therefore |\sigma| = \sigma_m U^{\frac{1}{n}}$$

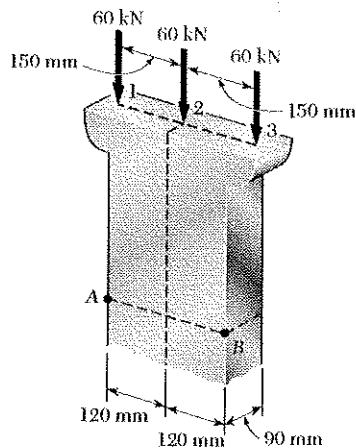
$$\begin{aligned} \text{Then, } M &= 2bc^2 \int_0^1 U \sigma_m U^{\frac{1}{n}} dU = 2bc^2 \sigma_m \int_0^1 U^{1+\frac{1}{n}} dU \\ &= 2bc^2 \sigma_m \left. \frac{U^{2+\frac{1}{n}}}{2+\frac{1}{n}} \right|_0^1 = \frac{2n}{2n+1} bc^2 \sigma_m \end{aligned}$$

$$\sigma_m = \frac{2n+1}{2} \frac{M}{bc^2}$$

$$\text{Recall } \frac{I}{C} = \frac{1}{12} \frac{b(2c)^3}{c} = \frac{2}{3} bc^2 \quad \frac{1}{bc^2} = \frac{2}{3} \frac{c}{I}$$

$$\sigma_m = \frac{2n+1}{3n} \frac{Mc}{I}$$

Problem 4.99



4.99 Determine the stress at points A and B, (a) for the loading shown, (b) if the 60-kN loads are applied at points 1 and 2 only.

(a) Loading is centric.

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-6} \text{ m}^2$$

$$\text{At A and B } \sigma = -\frac{P}{A} = -\frac{180 \times 10^3}{21.6 \times 10^{-3}} = -8.33 \times 10^6 \text{ Pa} = -8.33 \text{ MPa}$$

(b) Eccentric loading

$$P = 120 \text{ kN} = 120 \times 10^3 \text{ N}$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N}\cdot\text{m}$$

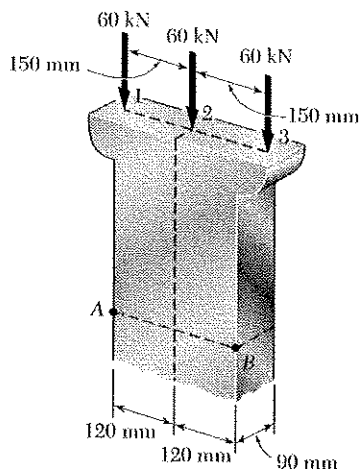
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

$$c = 120 \text{ mm} = 0.120 \text{ m}$$

$$\text{At A } \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -15.97 \times 10^6 \text{ Pa} = -15.97 \text{ MPa}$$

$$\text{At B } \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 4.86 \times 10^6 \text{ Pa} = 4.86 \text{ MPa}$$

Problem 4.100



4.100 Determine the stress at points A and B, (a) for the loading shown, (b) if the 60-kN loads applied at points 2 and 3 are removed.

(a) Loading is centric.

$$P = 180 \text{ kN}$$

$$A = (0.09)(0.24) = 0.0216 \text{ m}^2$$

$$\text{At A and B } \sigma = -\frac{P}{A}$$

$$\sigma = -\frac{180 \times 10^3}{0.0216}$$

$$\sigma = -8.3 \text{ MPa}$$

(b) Eccentric loading.

$$P = 60 \text{ kN}$$

$$M = (60)(0.15) = 9 \text{ kNm}$$

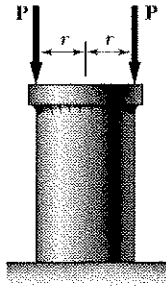
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.09)(0.24)^3 = 103.68 \times 10^{-6} \text{ m}^4$$

$$c = 0.12 \text{ m}$$

$$\text{At A } \sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{60 \times 10^3}{0.0216} - \frac{(9 \times 10^3)(0.12)}{103.68 \times 10^{-6}} = -13.19 \text{ MPa} \quad \sigma_A = -13.2 \text{ MPa}$$

$$\text{At B } \sigma = -\frac{P}{A} + \frac{Mc}{I} = -\frac{60 \times 10^3}{0.0216} + \frac{(9 \times 10^3)(0.12)}{103.68 \times 10^{-6}} = 7.64 \text{ MPa} \quad \sigma_B = 7.6 \text{ MPa}$$

Problem 4.101



4.101 Two forces P can be applied separately or at the same time to a plate that is welded to a solid circular bar of radius r . Determine the largest compressive stress in the circular bar, (a) when both forces are applied, (b) when only one of the forces is applied.

For a solid circular section $A = \pi r^2$, $I = \frac{\pi}{4} r^4$, $c = r$

$$\begin{aligned} \text{Compressive stress } \sigma &= -\frac{F}{A} - \frac{Mc}{I} \\ &= -\frac{F}{\pi r^2} - \frac{4M}{\pi r^3} \end{aligned}$$

(a) Both forces applied. $F = 2P$, $M = 0$

$$\sigma = -2P/\pi r^2 \quad \blacktriangleleft$$

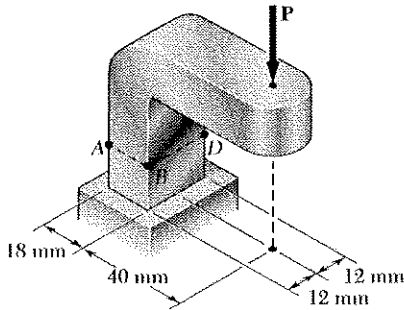
(b) One force applied. $F = P$, $M = Pr$

$$\sigma = -\frac{F}{\pi r^2} - \frac{4Pr}{\pi r^3}$$

$$\sigma = -5P/\pi r^2 \quad \blacktriangleleft$$

Problem 4.102

4.102 Knowing that the magnitude of the vertical force P is 2 kN, determine the stress at (a) point A, (b) point B.



$$A = (24)(18) = 432 \text{ mm}^2 = 432 \times 10^{-6} \text{ m}^2$$

$$\begin{aligned} I &= \frac{1}{12}(24)(18)^3 = 11.664 \times 10^3 \text{ mm}^4 \\ &= 11.664 \times 10^{-9} \text{ m}^4 \end{aligned}$$

$$c = \frac{1}{2}(18) = 9 \text{ mm} = 0.009 \text{ m}$$

$$e = \frac{1}{2}(18) + 40 = 49 \text{ mm} = 0.049 \text{ m}$$

$$M = Pe = (2 \times 10^3)(0.049) = 98 \text{ N}\cdot\text{m}$$

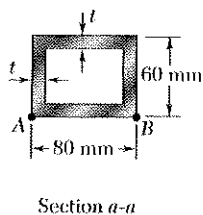
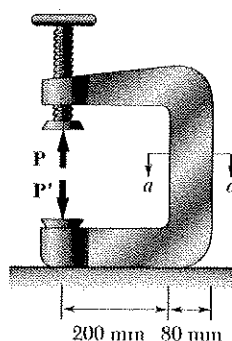
$$(a) \quad \sigma_A = -\frac{P}{A} + \frac{Mc}{I} = -\frac{2 \times 10^3}{432 \times 10^{-6}} + \frac{(98)(0.009)}{11.664 \times 10^{-9}} = 71.0 \times 10^6 \text{ Pa}$$

$$\sigma_A = 71.0 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2 \times 10^3}{432 \times 10^{-6}} - \frac{(98)(0.009)}{11.664 \times 10^{-9}} = -80.2 \times 10^6 \text{ Pa}$$

$$\sigma_B = -80.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.103



4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (a) point A, (b) point B.

Rectangular cutout is $60 \text{ mm} \times 40 \text{ mm}$.

$$A = (80)(60) - (60)(40) = 2.4 \times 10^3 \text{ mm}^2 = 2.4 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84 \times 10^6 \text{ mm}^4 = 1.84 \times 10^{-6} \text{ m}^4$$

$$c = 40 \text{ mm} = 0.040 \text{ m} \quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$

$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N}\cdot\text{m}$$

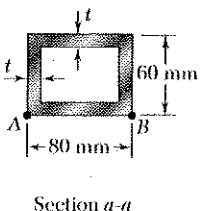
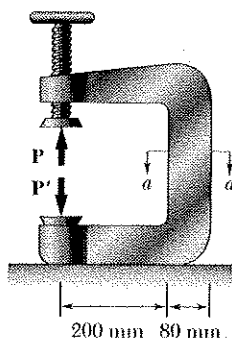
$$(a) \quad \sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{2.4 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = 112.7 \times 10^6 \text{ Pa}$$

$$\sigma_A = 112.8 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{2.4 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = 96.0 \times 10^6 \text{ Pa}$$

$$\sigma_B = -96.0 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.104



4.104 Solve Prob. 4.103, assuming that $t = 8$ mm.

4.103 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (a) point A, (b) point B.

Rectangular cutout is $64 \text{ mm} \times 44 \text{ mm}$.

$$A = (80)(60) - (64)(44) = 1.984 \times 10^3 \text{ mm}^2 = 1.984 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(44)(64)^3 = 1.59881 \times 10^6 \text{ mm}^4 = 1.59881 \times 10^{-6} \text{ m}^4$$

$$c = 40 \text{ mm} = 0.040 \text{ m} \quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$

$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \quad \sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = 130.2 \times 10^6 \text{ Pa}$$

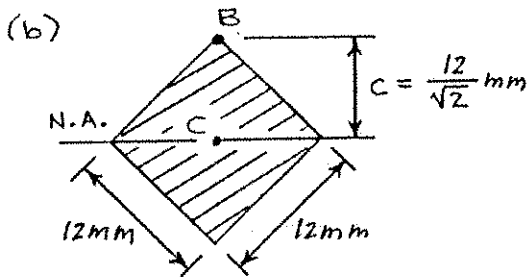
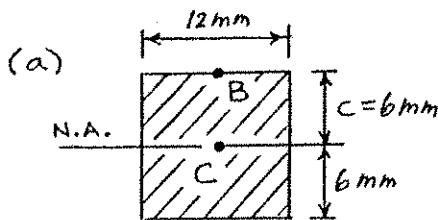
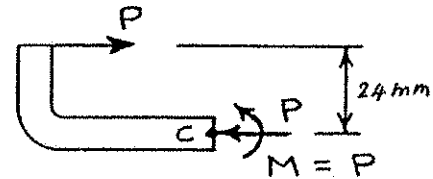
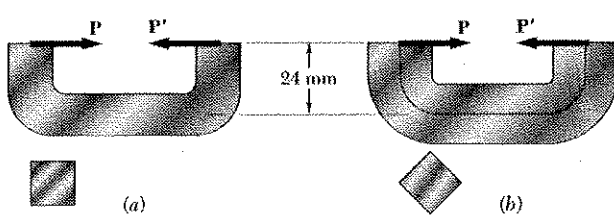
$$\sigma_A = 130.2 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = -110.0 \times 10^6 \text{ Pa}$$

$$\sigma_B = -110.0 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.105

4.105 Portions of a 12 × 12 mm square bar have been bent to form the two machine components shown. Knowing that the allowable stress is 105 MPa, determine the maximum load that can be applied to each component.



The maximum stress occurs at point B

$$\sigma_B = -105 \text{ MPa}$$

$$\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP$$

$$\text{where } K = \frac{1}{A} + \frac{ec}{I}$$

$$A = (0.012)(0.012) = 144 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (0.012)(0.012)^3 = 1.728 \times 10^{-9} \text{ m}^4$$

$$e = 0.024 \text{ m}$$

$$(a) \quad c = 0.006 \text{ m}$$

$$K = \frac{1}{144 \times 10^{-6}} + \frac{(0.024)(0.006)}{1.728 \times 10^{-9}} = 90277.8 \text{ m}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-105 \times 10^6)}{90277.8}$$

$$P = 1163 \text{ N}$$

$$(b) \quad c = \frac{0.012}{\sqrt{2}} = 0.00848 \text{ m}$$

For a square I is the same for all centroidal axes.

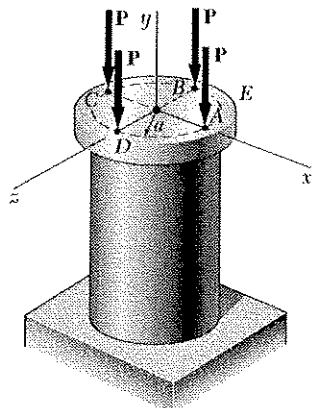
$$I = 1.728 \times 10^{-9} \text{ m}^4$$

$$K = \frac{1}{144 \times 10^{-6}} + \frac{(0.024)(0.00848)}{1.728 \times 10^{-9}} = 124722.2 \text{ m}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-105 \times 10^6)}{124722.2}$$

$$P = 842 \text{ N}$$

Problem 4.106



4.106 The four forces shown are applied to a rigid plate supported by a solid steel post of radius a . Knowing that $P = 100 \text{ kN}$ and $a = 40 \text{ mm}$, determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.

For a solid circular section of radius a

$$A = \pi a^2 \quad I = \frac{\pi}{4} a^4$$

(a) Centric force. $F = 4P$, $M_x = M_z = 0$

$$\sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2}$$

(b) Force at D is removed.

$$F = 3P \quad M_x = -Pa, \quad M_z = 0$$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4} a^4} = -\frac{7P}{\pi a^2}$$

(c) Forces at C and D are removed.

$$F = 2P \quad M_x = -Pa \quad M_z = -Pa$$

$$\text{Resultant bending couple} \quad M = \sqrt{M_x^2 + M_z^2} = \sqrt{2} Pa$$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2} Pa a}{\frac{\pi}{4} a^4} = -\frac{2+4\sqrt{2}}{\pi} \frac{P}{a^2} = -2.437 P/a^2$$

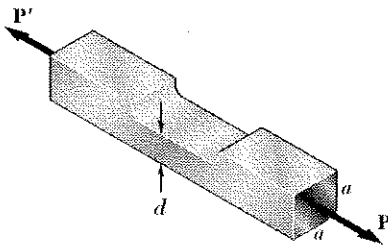
$$\text{Numerical data: } P = 100 \times 10^3 \text{ N} \quad a = 0.040 \text{ m}$$

$$\text{Answers: (a) } \sigma = -\frac{(4)(100 \times 10^3)}{\pi (0.040)^2} = -79.6 \times 10^6 \text{ Pa} \quad -79.6 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \sigma = -\frac{(7)(100 \times 10^3)}{\pi (0.040)^2} = -139.3 \times 10^6 \text{ Pa} \quad -139.3 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \sigma = -\frac{(2.437)(100 \times 10^3)}{(0.040)^2} = -152.3 \times 10^6 \text{ Pa} \quad -152.3 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.107



4.107 A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30$ mm, $d = 20$ mm, and $\sigma_{all} = 60$ MPa, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \quad \text{where } K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

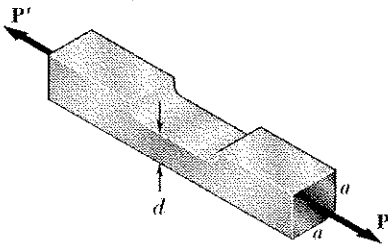
$$\text{Data: } a = 30 \text{ mm} = 0.030 \text{ m} \quad d = 20 \text{ mm} = 0.020 \text{ m}$$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60 \times 10^6}{4.1667 \times 10^3} = 14.40 \times 10^3 \text{ N}$$

$$P = 14.40 \text{ kN} \quad \blacktriangleleft$$

Problem 4.108



4.108 A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude $P = 18$ kN are applied at the centers of the ends of the bar. Knowing that $a = 30$ mm and $\sigma_{all} = 135$ MPa, determine the smallest allowable depth d of the milled portion of the bar.

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P \left(\frac{a-d}{2} \right) \left(\frac{1}{2}d \right)}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \quad \text{or} \quad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

$$\text{Solving for } d, \quad d = \frac{1}{2\sigma} \left\{ \sqrt{\left(\frac{2P}{a} \right)^2 + 12P\sigma} - \frac{2P}{a} \right\}$$

$$\text{Data: } a = 0.030 \text{ m}, \quad P = 18 \times 10^3 \text{ N}, \quad \sigma = 135 \times 10^6 \text{ Pa}$$

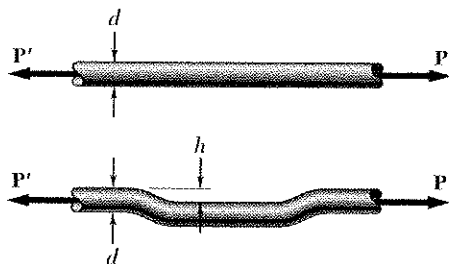
$$d = \frac{1}{(2)(135 \times 10^6)} \left\{ \sqrt{\left[\frac{(2)(18 \times 10^3)}{0.030} \right]^2 + 12(18 \times 10^3)(135 \times 10^6)} - \frac{(2)(18 \times 10^3)}{0.030} \right\}$$

$$= 16.04 \times 10^{-3}$$

$$d = 16.04 \text{ mm} \quad \blacktriangleleft$$

Problem 4.109

4.109 An offset h must be introduced into a solid circular rod of diameter d . Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.



For centric loading $\sigma_c = \frac{P}{A}$

For eccentric loading $\sigma_e = \frac{P}{A} + \frac{Phc}{I}$

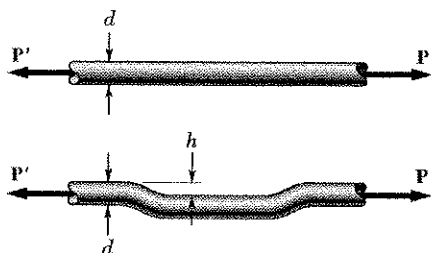
Given $\sigma_e = 5\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 5\frac{P}{A}$$

$$\frac{Phc}{I} = 4\frac{P}{A} \quad \therefore h = \frac{4I}{cA} = \frac{(4)(\frac{\pi}{64}d^4)}{(\frac{d}{2})(\frac{\pi}{4}d^2)} = \frac{1}{2}d = 0.500d \quad \blacktriangleleft$$

Problem 4.110

4.110 An offset h must be introduced into a metal tube of 18-mm outer diameter and 2-mm wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the rod when it is straight, determine the largest offset that can be used.



$$c = \frac{1}{2}d = 9 \text{ mm}$$

$$c_1 = c - t = 9 - 2 = 7 \text{ mm}$$

$$A = \pi(c^2 - c_1^2) = \pi(9^2 - 7^2) = 100.5 \text{ mm}^2$$

$$I = \frac{\pi}{4}(c^4 - c_1^4) = \frac{\pi}{4}(9^4 - 7^4) = 3267.3 \text{ mm}^4$$

For centric loading $\sigma_{cen} = \frac{P}{A}$

For eccentric loading $\sigma_{ecc} = \frac{P}{A} + \frac{Phc}{I}$

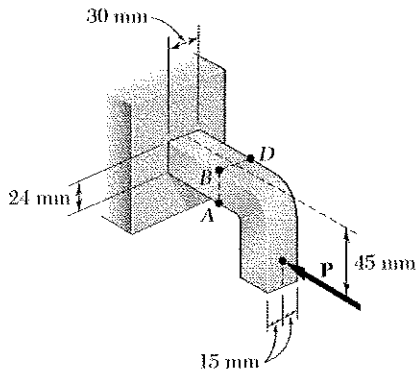
$$\sigma_{ecc} = 4\sigma_{cen} \quad \text{or} \quad \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

$$\frac{hc}{I} = \frac{3}{A} \quad h = \frac{3I}{Ac} = \frac{(3)(3267.3 \times 10^{-12})}{(100.5 \times 10^{-6})(0.009)}$$

$$h = 10.8 \text{ mm} \quad \blacktriangleleft$$

Problem 4.111

4.111 Knowing that the allowable stress in section ABD is 70 MPa, determine the largest force P that can be applied to the bracket shown.



$$A = (0.03)(0.024) = 72 \times 10^{-5} \text{ m}^2$$

$$I = \frac{1}{12} (0.03)(0.024)^3 = 34.56 \times 10^{-9} \text{ m}^4$$

$$e = \frac{1}{2} (0.024) = 0.012 \text{ m}$$

$$e = 0.045 - \frac{0.024}{2} = 0.021 \text{ m}$$

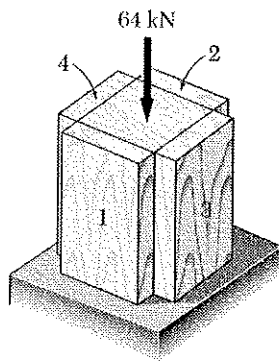
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = PK$$

$$\text{where } K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{72 \times 10^{-5}} + \frac{(0.021)(0.012)}{34.56 \times 10^{-9}} = 8680.6 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{70 \times 10^6}{8680.6}$$

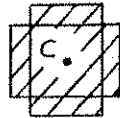
$$P = 8.06 \text{ kN}$$

Problem 4.112



4.112 A short column is made by nailing four 25×100 mm planks to a 100×100 mm timber. Determine the largest compressive stress created in the timber if (a) the column is as described, (b) plank 1 is removed, (c) planks 1 and 2 are removed, (d) planks 1, 2, and 3 are removed, (e) all planks are removed.

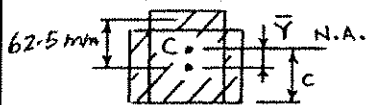
(a) Centric loading. $M = 0$ $\sigma = -\frac{P}{A}$



$$A = (100)(100) + (4)(25)(100) = 2 \times 10^4 \text{ mm}^2$$

$$\sigma = -\frac{64 \times 10^3}{0.02} = \sigma = -3.2 \text{ MPa}$$

(b) Eccentric loading. $M = Pe$ $\sigma = -\frac{P}{A} - \frac{Pec}{I}$



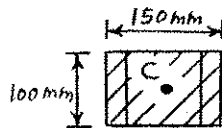
$$A = (100)(100) + (3)(25)(100) = 17500 \text{ mm}^2 \quad e = \bar{y}$$

$$\bar{y} = \frac{\sum A \bar{y}}{A} = \frac{(25)(100)(62.5)}{17500} = 8.93 \text{ mm}$$

$$I = \sum (\bar{I} + Ad^2) = \frac{1}{12}(100)(100)^3 + (100)(100)(8.93)^2 + \frac{1}{12}(100)(25)^3 + (100)(25)(53.75)^2$$

$$= 21.05 \times 10^6 \text{ mm}^4$$

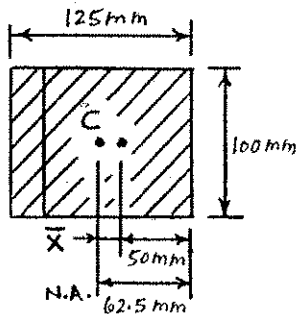
$$\sigma = -\frac{64 \times 10^3}{17500 \times 10^{-6}} - \frac{(64 \times 10^3)(8.93 \times 10^{-3})(58.93 \times 10^{-3})}{21.05 \times 10^{-6}} \quad \sigma = -5.26 \text{ MPa}$$



(c) Centric loading. $M = 0$ $\sigma = -\frac{P}{A}$

$$A = (150)(100) = 15000 \text{ mm}^2$$

$$\sigma = -\frac{64 \times 10^3}{0.015} \quad \sigma = -4.3 \text{ MPa}$$



(d) Eccentric loading. $M = Pe$ $\sigma = -\frac{P}{A} - \frac{Pec}{I}$

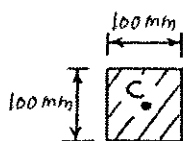
$$A = (100)(100) + (1)(100)(25) = 12.5 \times 10^3 \text{ mm}^2 \quad e = \bar{x}$$

$$\bar{x} = 62.5 - 50 = 12.5 \text{ mm}$$

$$I = \frac{1}{12}(100)(125)^3 = 16.276 \times 10^6 \text{ mm}^4$$

$$\sigma = -\frac{64 \times 10^3}{12.5 \times 10^3} - \frac{(64 \times 10^3)(0.0125)(0.0625)}{16.276 \times 10^{-6}}$$

$$\sigma = -8.2 \text{ MPa}$$



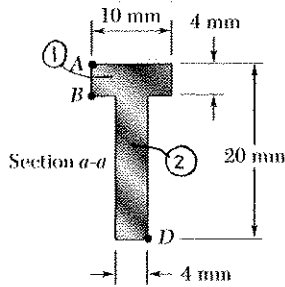
(e) Centric loading. $M = 0$ $\sigma = -\frac{P}{A}$

$$A = (100)(100) = 1 \times 10^4 \text{ mm}^2$$

$$\sigma = -\frac{64 \times 10^3}{0.01} \quad \sigma = -6.4 \text{ MPa}$$

Problem 4.113

4.113 Knowing that the clamp shown has been tightened on wooden planks being glued together until $P = 400$ N, determine in section $a-a$ (a) the stress at point A, (b) the stress at point D, (c) the location of the neutral axis.



Locate centroid.

Part	A, mm^2	$\bar{y}_o, \text{mm}$	$A\bar{y}_o, \text{mm}^3$
①	40	18	720
②	64	8	512
	104		1232

$$\bar{y}_o = \frac{1232}{104} = 11.846 \text{ mm}$$

The centroid lies 11.846 mm above point D.

Eccentricity: $e = (50 + 20 - 11.846) = -58.154 \text{ mm}$

Bending couple: $M = Pe = (400)(-58.154 \times 10^{-3}) = -23.262 \text{ N}\cdot\text{m}$

$$A = 104 \text{ mm}^2 = 104 \times 10^{-6} \text{ m}^2$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (10)(4)^3 + (40)(6.154)^2 = 1.5632 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (4)(16)^3 + (64)(3.846)^2 = 2.3120 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 3.8802 \times 10^3 \text{ mm}^4 = 3.8802 \times 10^{-9} \text{ m}^4$$

(a) Stress at point A. $y = 20 - 11.846 = 8.154 \text{ mm} = 8.154 \times 10^{-3} \text{ m}$

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{400}{104 \times 10^{-6}} - \frac{(-23.262)(8.154 \times 10^{-3})}{3.8802 \times 10^{-9}} = 52.7 \times 10^6 \text{ Pa}$$

$$\sigma_A = 52.7 \text{ MPa} \quad \blacktriangleleft$$

(b) Stress at point D. $y = -11.846 \text{ mm} = -11.846 \times 10^{-3} \text{ m}$

$$\sigma_D = \frac{P}{A} + \frac{My}{I} = \frac{400}{104 \times 10^{-6}} - \frac{(-23.262)(-11.846 \times 10^{-3})}{3.8802 \times 10^{-9}} = -67.2 \times 10^6 \text{ Pa}$$

$$\sigma_D = -67.2 \text{ MPa} \quad \blacktriangleleft$$

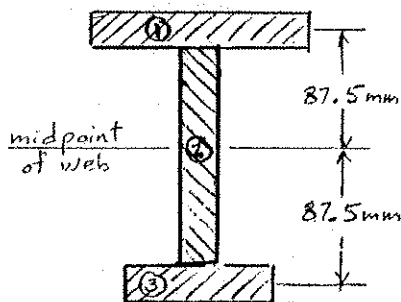
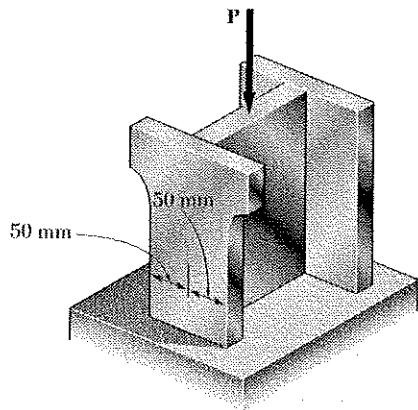
(c) Location of neutral axis. $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pe y}{I} = 0$$

$$y = \frac{I}{Ae} = \frac{3.8802 \times 10^{-9}}{(104 \times 10^{-6})(-58.154 \times 10^{-3})} = -0.642 \times 10^{-6} \text{ m} = -0.642 \text{ mm}$$

Neutral axis lies $11.846 - 0.642 = 11.204 \text{ mm}$ 11.20 mm above D $\blacktriangleleft$

Problem 4.114



4.114 Three steel plates, each of 25×150 -mm cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 25-mm strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 100 MPa, determine the largest force P (a) that could be applied to the original column, (b) that can be applied to the modified column.

(a) Centric loading $\sigma = -\frac{P}{A}$

$$A = (3)(150)(25) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$P = -\sigma A = -(-100 \times 10^6)(11.25 \times 10^{-3})$$

$$= 1.125 \times 10^6 \text{ N} \quad P = 1125 \text{ kN}$$

(b) Eccentric loading (reduced cross section)

	$A, 10^3 \text{ mm}^2$	$\bar{y}, \text{ mm}$	$A\bar{y} (10^3 \text{ mm}^3)$	$d, \text{ mm}$
①	3.75	87.5	328.125	76.5625
②	3.75	0	0	10.9375
③	2.50	-87.5	-218.75	98.4375
Σ	10.00		109.375	

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{109.375 \times 10^3}{10.00 \times 10^3} = 10.9375 \text{ mm}$$

The centroid lies 10.9375 mm from the midpoint of the web.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (150)(25)^3 + (3.75 \times 10^3)(76.5625)^2 = 22.177 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (25)(150)^3 + (3.75 \times 10^3)(10.9375)^2 = 7.480 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12} (100)(25)^3 + (2.50 \times 10^3)(98.4375)^2 = 24.355 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 54.012 \times 10^6 \text{ mm}^4 = 54.012 \times 10^{-6} \text{ m}^4$$

$$c = 10.9375 + 75 + 25 = 110.9375 \text{ mm} = 0.1109375 \text{ m}$$

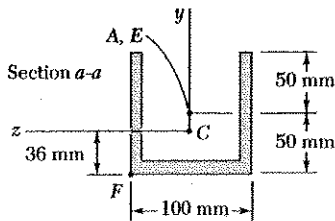
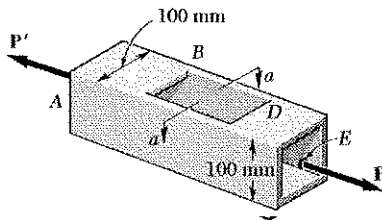
$$M = Pe \quad \text{where } e = 10.4375 \text{ mm} = 10.4375 \times 10^{-3} \text{ m}$$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pe c}{I} = -KP \quad A = 10.00 \times 10^{-3} \text{ m}^2$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{10.00 \times 10^{-3}} + \frac{(10.4375 \times 10^{-3})(0.1109375)}{54.012 \times 10^{-6}} = 122.465 \text{ m}^{-2}$$

$$P = -\frac{\sigma}{K} = -\frac{(-100 \times 10^6)}{122.465} = 817 \times 10^3 \text{ N} \quad P = 817 \text{ kN}$$

Problem 4.115



4.115 In order to provide access to the interior of a hollow square tube of 6-mm wall thickness, the portion CD of one side of the tube has been removed. Knowing that the loading of the tube is equivalent to two equal and opposite 60-kN forces acting at the geometric centers A and E of the ends of the tube, determine (a) the maximum stress in section $a-a$, (b) the stress at point F . Given: the centroid of the cross section is at C and $I_z = 2 \times 10^6 \text{ mm}^4$.

$$\text{Area } A = (2)(100)(6) + (88)(6) = 1728 \text{ mm}^2$$

$$M = Pe = (60)(0.05 - 0.036) = 0.84 \text{ kNm}$$

(a) Maximum stress occurs at top of section where $y = 100 - 36 = 64 \text{ mm}$.

$$\sigma_{\max} = \frac{P}{A} + \frac{My}{I} = \frac{60 \times 10^3}{1728 \times 10^{-6}} + \frac{(840)(0.064)}{2 \times 10^{-6}}$$

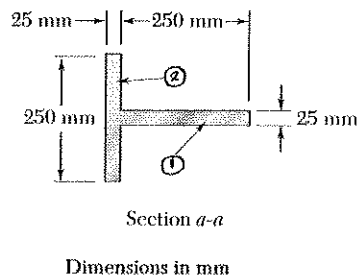
$$\sigma_{\max} = 61.6 \text{ MPa} \quad \blacktriangleleft$$

(b) At point F $y = -36 \text{ mm}$.

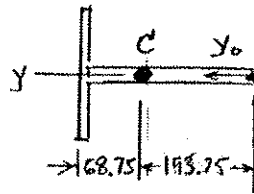
$$\sigma_F = \frac{P}{A} + \frac{My}{I} = \frac{60 \times 10^3}{1728 \times 10^{-6}} + \frac{(840)(-0.036)}{2 \times 10^{-6}}$$

$$\sigma_F = 19.6 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.116



4.116 Knowing that the allowable stress in section $a-a$ of the hydraulic press shown is 40 MPa in tension and 80 MPa in compression, determine the largest force P that can be exerted by the press.

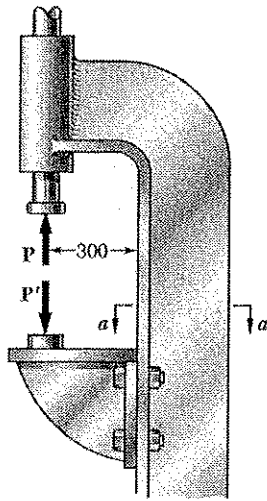


Locate centroid of cross section.

	A, mm^2	$\bar{y}_0, \text{mm}$	$A\bar{y}_0, \text{mm}^3$
①	6250	18.5	781.25×10^3
②	6250	262.5	1.640625×10^6
Σ	12500		2.421875×10^6

$$\bar{y}_0 = \frac{2.421875 \times 10^6}{12500} = 193.75 \text{ mm}$$

The centroid lies at point C.



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (25)(250)^3 + (6250)(68.75)^2$$

$$= 62.093 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (250)(25)^3 + (6250)(262.5)^2$$

$$= 29.867 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 82.194 \times 10^6 \text{ mm}^4 = 91.960 \times 10^{-6} \text{ m}^4$$

$$A = 12500 \text{ mm}^2 = 12.5 \times 10^{-3} \text{ m}^2$$

$$\text{Eccentricity } e = 300 + 25 + 250 - 193.75 = 381.25 \text{ mm}$$

The bending moment about the centroid is $M = -Pe$.

$$\text{Stress: } \sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} + \frac{Pe y}{I} = KP \quad \text{where } K = \frac{1}{A} + \frac{ey}{I}$$

$$\text{Then } P = \frac{\sigma}{K}$$

$$\text{Tensile stress: } \sigma = 40 \times 10^6 \text{ Pa}, \quad y = 81.25 \times 10^{-3} \text{ m}$$

$$K = \frac{1}{12.5 \times 10^{-3}} + \frac{(381.25 \times 10^{-3})(81.25 \times 10^{-3})}{91.960 \times 10^{-6}} = 416.85 \text{ m}^{-2}$$

$$P = \frac{40 \times 10^6}{416.85} = 96.0 \times 10^3 \text{ N}$$

$$\text{Compressive stress: } \sigma = -80 \times 10^6 \text{ Pa}, \quad y = -193.75 \times 10^{-3} \text{ m}$$

$$K = \frac{1}{12.5 \times 10^{-3}} + \frac{(381.25 \times 10^{-3})(-193.75 \times 10^{-3})}{91.960 \times 10^{-6}} = -723.25 \text{ m}^{-2}$$

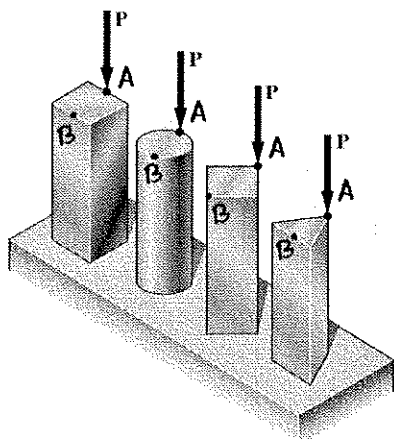
$$P = \frac{-80 \times 10^6}{-723.25} = 110.6 \times 10^3 \text{ N}$$

Choose the smaller value $P = 96.0 \times 10^3 \text{ N}$

$P = 96.0 \text{ kN}$ $\blacktriangleleft$

Problem 4.117

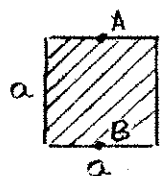
4.117 The four bars shown have the same cross-sectional area. For the given loadings, show that (a) the maximum compressive stresses are in the ratio 4:5:7:9, (b) the maximum tensile stresses are in the ratio 2:3:5:3. (Note: the cross section of the triangular bar is an equilateral triangle.)



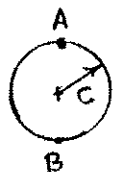
Stresses

$$\begin{aligned} \text{At A} \quad \sigma_A &= -\frac{P}{A} - \frac{Pec_A}{I} \\ &= -\frac{P}{A} \left(1 + \frac{Aec_A}{I} \right) \end{aligned}$$

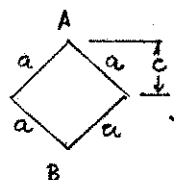
$$\begin{aligned} \text{At B} \quad \sigma_B &= -\frac{P}{A} + \frac{Pec_B}{I} \\ &= \frac{P}{A} \left(\frac{Aec_B}{I} - 1 \right) \end{aligned}$$



$$\begin{cases} A_1 = a^2, \quad I_1 = \frac{1}{12} a^4, \quad c_A = c_B = \frac{1}{2} a, \quad e = \frac{1}{2} a \\ \sigma_A = -\frac{P}{A} \left(1 + \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^4} \right) = -4 \frac{P}{A_1} \\ \sigma_B = \frac{P}{A} \left(\frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^4} - 1 \right) = 2 \frac{P}{A_1} \end{cases}$$



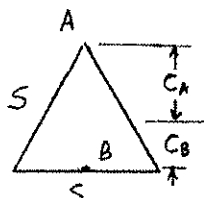
$$\begin{cases} A_2 = \pi c^2 = a^2 \quad \therefore c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4} c^4 \\ \sigma_A = -\frac{P}{A_2} \left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4} c^4} \right) = -5 \frac{P}{A_2} \\ \sigma_B = \frac{P}{A_2} \left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4} c^4} - 1 \right) = 3 \frac{P}{A_2} \end{cases}$$



$$\begin{cases} A_3 = a^2, \quad c = \frac{\sqrt{2}}{2} a, \quad I_3 = \frac{1}{12} a^4, \quad e = c \\ \sigma_A = -\frac{P}{A_3} \left(1 + \frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4} \right) = -7 \frac{P}{A_3} \\ \sigma_B = \frac{P}{A_3} \left(\frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4} - 1 \right) = 5 \frac{P}{A_3} \end{cases}$$

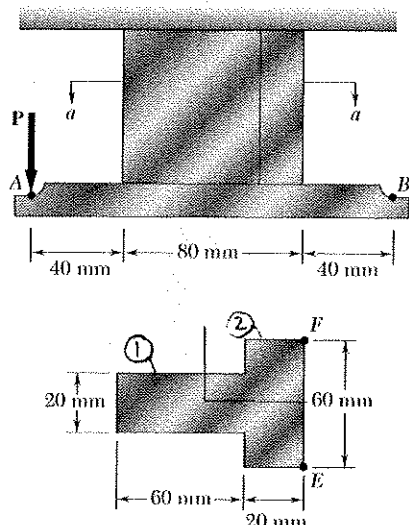
$$A_4 = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{\sqrt{3}}{4} s^2 \quad I_4 = \frac{1}{36} s \left(\frac{\sqrt{3}}{2}s\right)^3 = \frac{\sqrt{3}}{96} s^4$$

$$c_A = \frac{2}{3} \frac{\sqrt{3}}{2} s = \frac{s}{\sqrt{3}} = e \quad c_B = \frac{s}{2\sqrt{3}}$$



$$\begin{aligned} \sigma_A &= -\frac{P}{A_4} \left(1 + \frac{(\frac{\sqrt{3}}{4} s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96} s^4} \right) = -9 \frac{P}{A_4} \\ \sigma_B &= \frac{P}{A_4} \left(\frac{(\frac{\sqrt{3}}{4} s^2)(\frac{s}{\sqrt{3}})(\frac{s}{2\sqrt{3}})}{\frac{\sqrt{3}}{96} s^4} - 1 \right) = 3 \frac{P}{A_4} \end{aligned}$$

Problem 4.118



4.118 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (a) the largest vertical force *P* that can be applied at point *A*, (b) the corresponding location of the neutral axis of section *a-a*.

Locate centroid.

	A, mm^2	$\bar{y}_0, \text{mm}$	$A\bar{y}_0, \text{mm}^3$
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\begin{aligned}\bar{y}_0 &= \frac{\Sigma A\bar{y}_0}{\Sigma A} \\ &= \frac{120 \times 10^3}{2400} \\ &= 50 \text{ mm}\end{aligned}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple: $M = Pe$

$$e = 40 + 50 = 90 \text{ mm} = 0.090 \text{ m}$$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360 \times 10^6 \text{ mm}^4 = 1.360 \times 10^{-6} \text{ m}^4, \quad A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on tensile stress at left edge: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = K P$$

$$K = \frac{1}{A} - \frac{e y}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(0.090)(-0.050)}{1.360 \times 10^{-6}} = 3.7255 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150 \times 10^6}{3.7255 \times 10^3} = 40.3 \times 10^3 \text{ N} = 40.3 \text{ kN}$$

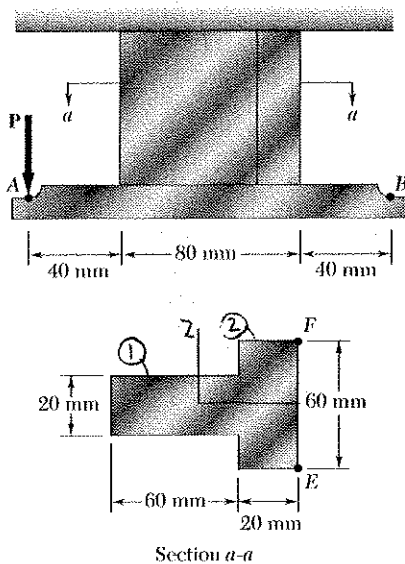
(b) Location of neutral axis: $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = 0 \quad \frac{e y}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(0.090)} = 6.30 \times 10^{-3} \text{ m} = 6.30 \text{ mm}$$

The neutral axis lies 6.30 mm to the right of the centroid or 56.30 mm from the left face.

Problem 4.119



4.119 Solve Prob 4.118, assuming that the vertical force P is applied at point B .

4.118 Knowing that the allowable stress is 150 MPa in section $a-a$ of the hanger shown, determine (a) the largest vertical force P that can be applied at point A , (b) the corresponding location of the neutral axis of section $a-a$.

Locate centroid.

	A, mm^2	$\bar{y}_0, \text{mm}$	$A\bar{y}_0, \text{mm}^3$
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\begin{aligned}\bar{y}_0 &= \frac{\Sigma A\bar{y}_0}{\Sigma A} \\ &= \frac{120 \times 10^3}{2400} \\ &= 50 \text{ mm}\end{aligned}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple: $M = Pe$

$$e = 50 - 120 = -70 \text{ mm} = -0.070 \text{ m}$$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360 \times 10^6 \text{ mm}^4 = 1.360 \times 10^{-6} \text{ m}^4, \quad A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on stress at left edge of section: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = K_L P$$

$$K_L = \frac{1}{A} - \frac{e y}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(-0.070)(-0.050)}{1.360 \times 10^{-6}} = -2.1569 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K_L} = \frac{-150 \times 10^6}{-2.1569 \times 10^3} = 69.6 \times 10^3 \text{ N}$$

Based on stress at right edge of section: $y = 30 \text{ mm} = 0.030 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = K_R P$$

$$K_R = \frac{1}{A} - \frac{e y}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(-0.070)(0.030)}{1.360 \times 10^{-6}} = 1.9608 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K_R} = \frac{150 \times 10^6}{1.9608 \times 10^3} = 76.5 \times 10^3 \text{ N}$$

Choose the smaller value $P = 69.6 \times 10^3 \text{ N} = 69.6 \text{ kN}$

(b) Location of neutral axis: $\sigma = 0$

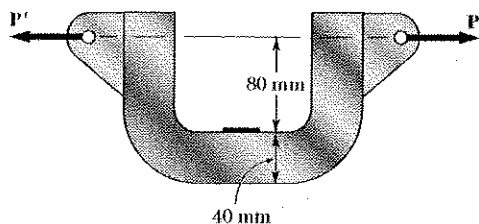
$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = 0 \quad \frac{e y}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(-0.070)} = -8.10 \times 10^{-3} \text{ m} = -8.10 \text{ mm}$$

Neutral axis lies $50 - 8.10 = 41.9 \text{ mm}$ from left face.

Problem 4.120

4.120 The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and the strain on the inner edge was measured and found to be 450μ , determine the magnitude P of the forces. Use $E = 200 \text{ GPa}$.



At the strain gage location

$$\sigma = E\epsilon = (200 \times 10^9)(450 \times 10^{-6})$$

$$= 90 \text{ MPa}$$

$$A = (40)(40) = 1600 \text{ mm}^2$$

$$I = \frac{1}{12}(40)(40)^3 = 213333.3 \text{ mm}^4$$

$$e = 80 + 20 = 100 \text{ mm.}$$

$$c = 20 \text{ mm.}$$

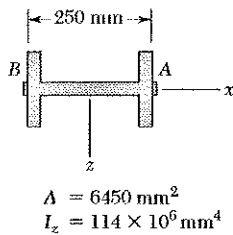
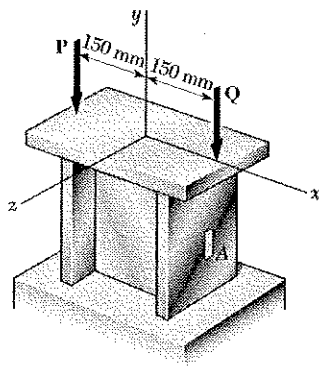
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600 \times 10^{-6}} + \frac{(0.1)(0.02)}{213333.3 \times 10^{-12}} = 10000 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90000 \times 10^3}{10000} = 9000 \text{ N}$$

$$P = 9 \text{ kN}$$

Problem 4.121



4.121 A short length of a rolled-steel column supports a rigid plate on which two loads P and Q are applied as shown. The strains at two points A and B on the center line of the outer faces of the flanges have been measured and found to be

$$\epsilon_A = -400 \times 10^{-6} \text{ mm/mm} \quad \epsilon_B = -300 \times 10^{-6} \text{ mm/mm}$$

Knowing that $E = 200 \text{ GPa}$, determine the magnitude of each load.

Stresses at A and B from strain gages

$$\sigma_A = E \epsilon_A = (200 \times 10^9)(-400 \times 10^{-6}) = -80 \text{ MPa}$$

$$\sigma_B = E \epsilon_B = (200 \times 10^9)(-300 \times 10^{-6}) = -60 \text{ MPa}$$

$$\text{Centric force} \quad F = P + Q$$

$$\text{Bending couple} \quad M = 6P - 6Q$$

$$c = 125 \text{ mm}$$

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{6450 \times 10^{-6}} + \frac{(0.15P - 0.15Q)(0.125)}{114 \times 10^{-6}}$$

$$-80 \times 10^6 = 9.5P - 319.5Q \quad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{6450 \times 10^{-6}} - \frac{(0.15P - 0.15Q)(0.125)}{114 \times 10^{-6}}$$

$$-60 \times 10^6 = -319.5P + 9.5Q \quad (2)$$

Solving (1) and (2) simultaneously

$$P = 195.4 \text{ kN}$$

$$Q = 256.2 \text{ kN}$$

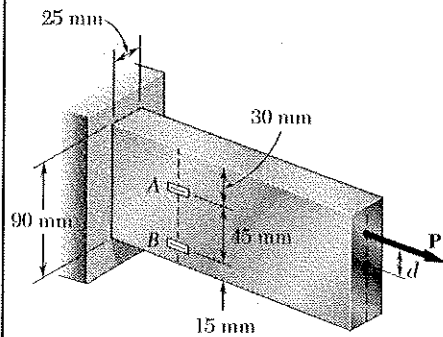
Problem 4.122

4.122 An eccentric force P is applied as shown to a steel bar of 25×90 -mm cross section. The strains at A and B have been measured and found to be

$$\epsilon_A = +350 \mu$$

$$\epsilon_B = -70 \mu$$

Knowing that $E = 200$ GPa, determine (a) the distance d , (b) the magnitude of the force P .



$$h = 15 + 45 + 30 = 90 \text{ mm}$$

$$b = 25 \text{ mm}$$

$$c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \quad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gauges at A and B :

$$\sigma_A = E\epsilon_A = (200 \times 10^9)(350 \times 10^{-6}) = 70 \times 10^6 \text{ Pa}$$

$$\sigma_B = E\epsilon_B = (200 \times 10^9)(-70 \times 10^{-6}) = -14 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \quad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \quad (2)$$

Subtracting, $\sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(84 \times 10^6)}{0.045} = -2835 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting,

$$y_A \sigma_B - y_B \sigma_A = (y_A - y_B) \frac{P}{A}$$

$$P = \frac{A(y_A \sigma_B - y_B \sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(-14 \times 10^6) - (-0.030)(70 \times 10^6)]}{0.045}$$

$$= 94.5 \times 10^3 \text{ N}$$

(a) $M = -Pd \therefore d = -\frac{M}{P} = -\frac{-2835}{94.5 \times 10^3} = 0.030 \text{ m} \quad d = 30.0 \text{ mm} \leftarrow$

(b)

$$P = 94.5 \text{ kN} \leftarrow$$

Problem 4.123

4.123 Solve prob. 4.122, assuming that the measured strains are

$$\epsilon_A = +600 \mu$$

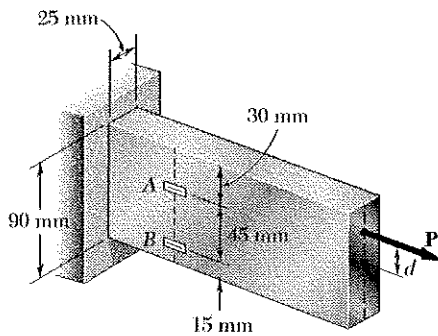
$$\epsilon_B = +420 \mu$$

4.122 An eccentric force P is applied as shown to a steel bar of 25×90 -mm cross section. The strains at A and B have been measured and found to be

$$\epsilon_A = +350 \mu$$

$$\epsilon_B = -70 \mu$$

Knowing that $E = 200$ GPa, determine (a) the distance d , (b) the magnitude of the force P .



$$h = 15 + 45 + 30 = 90 \text{ mm}$$

$$b = 25 \text{ mm} \quad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \quad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gauges at A and B :

$$\sigma_A = E\epsilon_A = (200 \times 10^9)(600 \times 10^{-6}) = 120 \times 10^6 \text{ Pa}$$

$$\sigma_B = E\epsilon_B = (200 \times 10^9)(420 \times 10^{-6}) = 84 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \quad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \quad (2)$$

Subtracting, $\sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(36 \times 10^6)}{0.045} = -1215 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting,

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

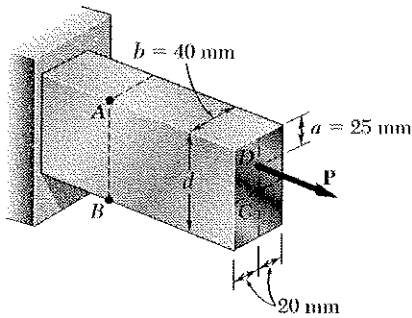
$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(84 \times 10^6) - (-0.030)(120 \times 10^6)]}{0.045} = 243 \times 10^3 \text{ N}$$

$$M = -Pd$$

$$(a) \quad \therefore d = -\frac{M}{P} = -\frac{-1215}{243 \times 10^3} = 5 \times 10^{-3} \text{ m} \quad d = 5.00 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad P = 243 \text{ kN} \quad \blacktriangleleft$$

Problem 4.124



4.124 The eccentric axial force P acts at point D , which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (a) the depth d of the bar for which the tensile stress at point A is maximum, (b) the corresponding stress at point A .

$$A = bd \quad I = \frac{1}{12} bd^3$$

$$c = \frac{1}{2}d \quad e = \frac{1}{2}d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

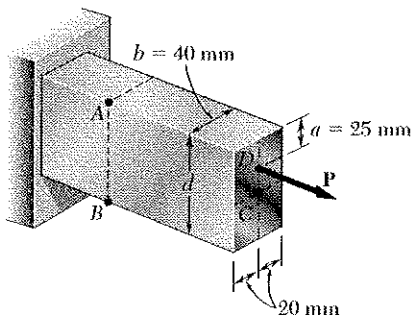
$$\sigma_A = \frac{P}{b} \left\{ \frac{1}{d} + \frac{12(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3} \right\} = \frac{P}{b} \left\{ \frac{4}{d} - \frac{6a}{d^2} \right\}$$

(a) Depth d for maximum σ_A . Differentiate with respect to d .

$$\frac{d\sigma_A}{dd} = \frac{P}{b} \left\{ -\frac{4}{d^2} + \frac{12a}{d^3} \right\} = 0 \quad d = 3a = 75 \text{ mm}$$

$$(b) \quad \sigma_A = \frac{60 \times 10^3}{40 \times 10^{-3}} \left\{ \frac{4}{75 \times 10^{-3}} - \frac{(6)(25 \times 10^{-3})}{(75 \times 10^{-3})^2} \right\} = 40 \times 10^6 \text{ Pa} = 40 \text{ MPa}$$

Problem 4.125



4.125 For the bar and loading of Prob. 4.124, determine (a) the depth d of the bar for which the compressive stress at point B is maximum, (b) the corresponding stress at point B .

4.124 The eccentric axial force P acts at point D , which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (a) the depth d of the bar for which the tensile stress at point A is maximum, (b) the corresponding stress at point A .

$$A = bd \quad I = \frac{1}{12} bd^3$$

$$c = \frac{1}{2}d \quad e = \frac{1}{2}d - a$$

$$\sigma_B = \frac{P}{A} - \frac{Pec}{I}$$

$$\sigma_B = \frac{P}{b} \left\{ \frac{1}{d} - \frac{12(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3} \right\} = \frac{P}{b} \left\{ -\frac{2}{d} + \frac{6a}{d^2} \right\}$$

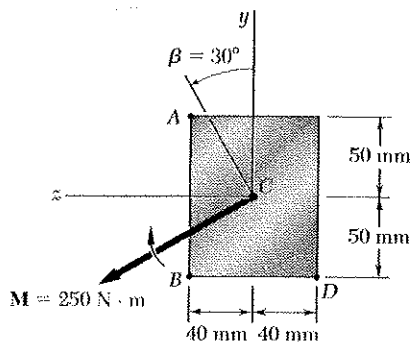
(a) Depth d for maximum σ_B : Differentiate with respect to d .

$$\frac{d\sigma_B}{dd} = \frac{P}{b} \left\{ \frac{2}{d^2} - \frac{12a}{d^3} \right\} = 0 \quad d = 6a = 150 \text{ mm}$$

$$(b) \quad \sigma_B = \frac{60 \times 10^3}{40 \times 10^{-3}} \left\{ -\frac{2}{150 \times 10^{-3}} + \frac{(6)(25 \times 10^{-3})}{(150 \times 10^{-3})^2} \right\} = -10 \times 10^6 \text{ Pa} = -10 \text{ MPa}$$

Problem 4.126

4.126 through 4.128 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.



$$I_z = \frac{1}{12}(80)(100)^3 = 6.6667 \times 10^6 \text{ mm}^4 = 6.6667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(100)(80)^3 = 4.2667 \times 10^6 \text{ mm}^4 = 4.2667 \times 10^{-6} \text{ m}^4$$

$$y_A = -y_B = -y_D = 50 \text{ mm}$$

$$z_A = z_B = -z_D = 40 \text{ mm}$$

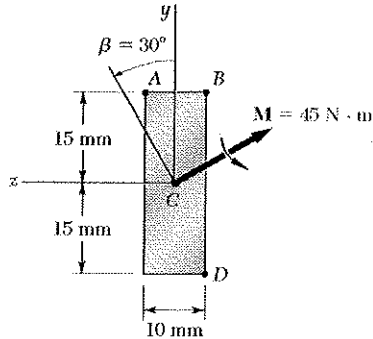
$$M_y = -250 \sin 30^\circ = -125 \text{ N}\cdot\text{m} \quad M_z = 250 \cos 30^\circ = 216.51 \text{ N}\cdot\text{m}$$

$$\begin{aligned} \text{(a)} \quad \sigma_A &= -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(0.040)}{4.2667 \times 10^{-6}} \\ &= -2.80 \times 10^6 \text{ Pa} \quad -2.80 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \sigma_B &= -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(-0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(0.040)}{4.2667 \times 10^{-6}} \\ &= 0.452 \times 10^6 \text{ Pa} \quad 0.452 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \sigma_D &= -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-0.050)}{6.6667 \times 10^{-6}} + \frac{(-125)(-0.040)}{4.2667 \times 10^{-6}} \\ &= 2.80 \times 10^6 \text{ Pa} \quad 2.80 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$

Problem 4.127



4.126 through 4.128 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.

$$I_z = \frac{1}{12}(10)(30)^3 = 22500 \text{ mm}^4$$

$$I_y = \frac{1}{12}(30)(10)^3 = 2500 \text{ mm}^4$$

$$y_A = y_B = -y_D = 15 \text{ mm}$$

$$z_A = -z_B = -z_D = \left(\frac{1}{2}\right)(10) = 5 \text{ mm}$$

$$M_y = 45 \cos 60^\circ = 22.5 \text{ N}\cdot\text{m}, \quad M_z = -45 \sin 60^\circ = -39 \text{ N}\cdot\text{m}$$

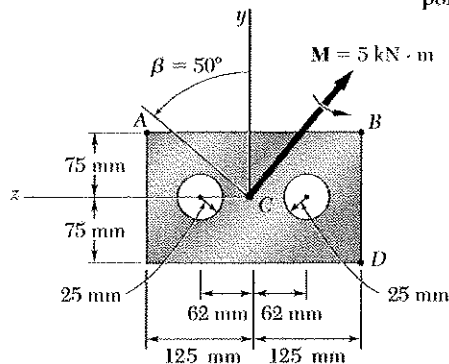
$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-39)(0.015)}{22500 \times 10^{-12}} + \frac{(22.5)(0.005)}{2500 \times 10^{-12}} = 71 \text{ MPa}$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-39)(0.015)}{22500 \times 10^{-12}} + \frac{(22.5)(-0.005)}{2500 \times 10^{-12}} = -19 \text{ MPa}$$

$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-39)(0.015)}{22500 \times 10^{-12}} + \frac{(22.5)(-0.005)}{2500 \times 10^{-12}} = -71 \text{ MPa}$$

Problem 4.128

4.126 through 4.128 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.



$$M_z = -5 \sin 40^\circ = -3.214 \text{ kNm}$$

$$M_y = 5 \cos 40^\circ = 3.83 \text{ kNm}$$

$$y_A = y_B = -y_D = 75 \text{ mm}$$

$$z_A = -z_B = -z_D = 125 \text{ mm}$$

$$I_z = \frac{1}{12} (250)(150)^3 - 2 \left[\frac{\pi}{4} (25)^4 \right] = 69.7 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12} (150)(250)^3 - 2 \left[\frac{\pi}{4} (25)^4 + \pi (25)^2 (62)^2 \right] = 187.2 \times 10^6 \text{ mm}^4$$

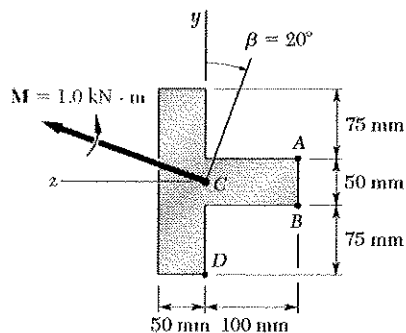
$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-3214)(0.075)}{69.7 \times 10^{-6}} + \frac{(3830)(0.125)}{187.2 \times 10^{-6}} = 6.016 \text{ MPa} \quad \sigma_A = 6.02 \text{ MPa} \quad \triangleleft$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-3214)(0.075)}{69.7 \times 10^{-6}} + \frac{(3830)(-0.125)}{187.2 \times 10^{-6}} = 0.9 \text{ MPa} \quad \sigma_B = 0.9 \text{ MPa} \quad \triangleleft$$

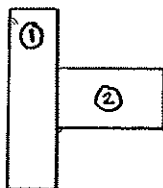
$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-3214)(-0.075)}{69.7 \times 10^{-6}} + \frac{(3830)(-0.125)}{187.2 \times 10^{-6}} = -6.016 \text{ MPa} \quad \sigma_D = -6.02 \text{ MPa} \quad \triangleleft$$

Problem 4.129

4.129 through 4.131 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.



Locate centroid



	A, mm^2	$\bar{Z}, \text{mm}$	$A\bar{Z}, \text{mm}^3$
①	10000	-25	-250000
②	5000	50	250000
Σ	15000		0

The centroid lies at point C

$$I_z = \frac{1}{12}(50)(200)^3 + \frac{1}{12}(100)(50)^3 = 34.375 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12}(200)(50)^3 + \frac{1}{12}(50)(100)^3 = 25 \times 10^6 \text{ mm}^4$$

$$y_A = -y_B = 25 \text{ mm}, \quad y_D = -100 \text{ mm}$$

$$z_A = z_B = -100 \text{ mm}, \quad z_D = 0$$

$$M_z = 1 \cos 20^\circ = 0.94 \text{ kNm}$$

$$M_y = 1 \sin 20^\circ = 0.342 \text{ kNm}$$

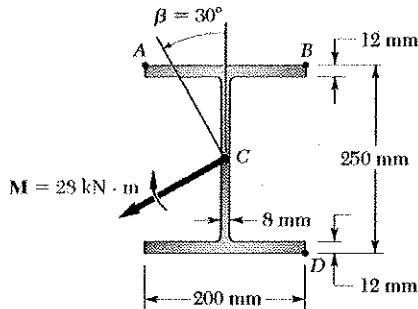
$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(940)(0.025)}{34.375 \times 10^{-6}} + \frac{(342)(-0.1)}{25 \times 10^{-6}} = 2.05 \text{ MPa}$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(940)(-0.025)}{34.375 \times 10^{-6}} + \frac{(342)(-0.1)}{25 \times 10^{-6}} = -0.684 \text{ MPa}$$

$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(940)(-0.1)}{34.375 \times 10^{-6}} + \frac{(342)(0)}{25 \times 10^{-6}} = 2.73 \text{ MPa}$$

Problem 4.130

4.129 through 4.131 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.



$$\text{Flange: } I_z = \frac{1}{12}(200)(12)^3 + (200)(12)(119)^2 = 34015200 \text{ mm}^4$$

$$I_y = \frac{1}{12}(12)(200)^3 = 8 \times 10^6 \text{ mm}^4$$

$$\text{Web: } I_z = \frac{1}{12}(8)(226)^3 = 7695451 \text{ mm}^4$$

$$I_y = \frac{1}{12}(226)(8)^3 = 9643 \text{ mm}^4$$

$$\text{Total: } I_z = (2)(34015200) + 7695451 = 75725851 \text{ mm}^4$$

$$I_y = (2)(8 \times 10^6) + 9643 = 16009643 \text{ mm}^4$$

$$y_A = y_B = -y_D = 125 \text{ mm}; \quad z_A = -z_B = -z_D = 100 \text{ mm}$$

$$M_z = 28 \cos 30^\circ = 24.25 \text{ kNm}$$

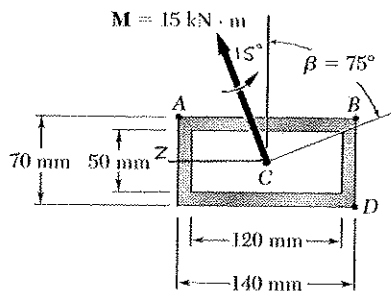
$$M_y = -28 \sin 30^\circ = -14 \text{ kNm}$$

$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(24.25 \times 10^3)(0.125)}{75725851 \times 10^{-12}} + \frac{(-14 \times 10^3)(0.1)}{16009643 \times 10^{-12}} = -127.5 \text{ MPa}$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(24.25)(0.125)}{75725851 \times 10^{-12}} + \frac{(-14000)(-0.1)}{16009643 \times 10^{-12}} = 47.4 \text{ MPa}$$

$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(24.25)(-0.125)}{75725851 \times 10^{-12}} + \frac{(-14000)(-0.1)}{16009643 \times 10^{-12}} = 127.5 \text{ MPa}$$

Problem 4.131



4.129 through 4.131 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.

$$I_z = \frac{1}{12}(140)(70)^3 - \frac{1}{12}(120)(50)^3 = 2.7517 \times 10^6 \text{ mm}^4 \\ = 2.7517 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(70)(140)^3 - \frac{1}{12}(50)(120)^3 = 8.8067 \times 10^6 \text{ mm}^4 \\ = 8.8067 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 35 \text{ mm} = 0.035 \text{ m}$$

$$z_A = -z_B = -z_D = 70 \text{ mm} = 0.070 \text{ m}$$

$$M_z = M \sin 15^\circ = (15 \times 10^3) \sin 15^\circ = 3.8823 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = M \cos 15^\circ = (15 \times 10^3) \cos 15^\circ = 14.4889 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(3.8823 \times 10^3)(0.035)}{2.7517 \times 10^{-6}} + \frac{(14.4889 \times 10^3)(0.070)}{8.8067 \times 10^{-6}} \\ = 65.8 \times 10^6 \text{ Pa}$$

$$\sigma_A = 65.8 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(3.8823 \times 10^3)(0.035)}{2.7517 \times 10^{-6}} + \frac{(14.4889 \times 10^3)(-0.070)}{8.8067 \times 10^{-6}} \\ = -164.5 \times 10^6 \text{ Pa}$$

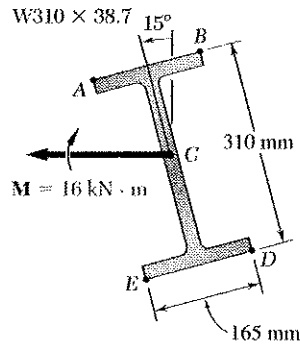
$$\sigma_B = -164.5 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(3.8823 \times 10^3)(-0.035)}{2.7517 \times 10^{-6}} + \frac{(14.4889 \times 10^3)(-0.070)}{8.8067 \times 10^{-6}} \\ = -65.8 \times 10^6 \text{ Pa}$$

$$\sigma_D = -65.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.132

4.132 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.



For W 310 x 38.7 rolled steel shape,

$$I_z = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4$$

$$I_y = 7.27 \times 10^6 \text{ mm}^4 = 7.27 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = -y_E = \left(\frac{1}{2}\right)(310) = 155 \text{ mm}$$

$$z_A = z_E = -z_B = -z_D = \left(\frac{1}{2}\right)(165) = 82.5 \text{ mm}$$

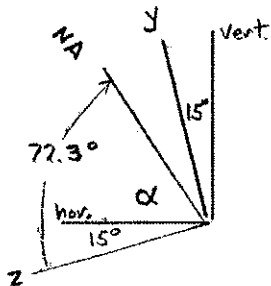
$$M_z = (16 \times 10^3) \cos 15^\circ = 15.455 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = (16 \times 10^3) \sin 15^\circ = 4.1411 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{85.1 \times 10^{-6}}{7.27 \times 10^{-6}} \tan 15^\circ = 3.1365$$

$$\phi = 72.3^\circ$$

$$\alpha = 72.3 - 15 = 57.3^\circ$$



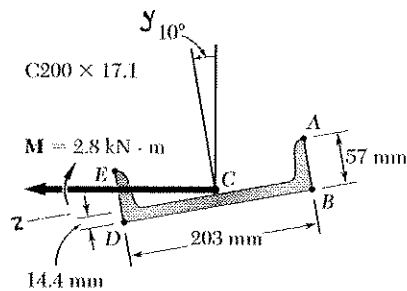
(b) Maximum tensile stress occurs at point E.

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{85.1 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.27 \times 10^{-6}}$$

$$= 75.1 \times 10^6 \text{ Pa}$$

$$\sigma_E = 75.1 \text{ MPa}$$

Problem 4.133



4.133 and 4.134 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.

For C200 x 17.1 rolled steel shape,

$$I_z = 0.538 \times 10^6 \text{ mm}^4 = 0.538 \times 10^{-6} \text{ m}^4$$

$$I_y = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$$

$$z_E = z_D = -z_A = -z_B = \frac{1}{2}(203) = 101.5 \text{ mm}$$

$$y_D = y_B = -14.4 \text{ mm}$$

$$y_E = y_A = 57 - 14.4 = 42.6 \text{ mm}$$

$$M_z = (2.8 \times 10^3) \cos 10^\circ = 2.7575 \times 10^3 \text{ N}\cdot\text{m}$$

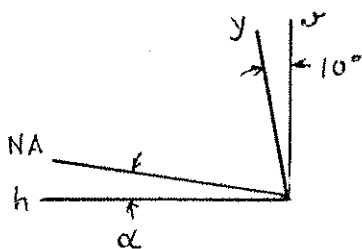
$$M_y = (2.8 \times 10^3) \sin 10^\circ = 486.21 \text{ N}\cdot\text{m}$$

$$(a) \tan \varphi = \frac{I_z}{I_y} \tan \theta = \frac{0.538}{13.4} \tan 10^\circ = 0.007079$$

$$\varphi = 0.4056^\circ$$

$$\alpha = 10^\circ - 0.4056^\circ$$

$$\alpha = 9.59^\circ \quad \blacktriangleleft$$

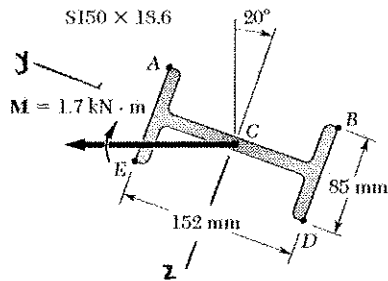


(b) Maximum tensile stress occurs at point D.

$$\begin{aligned} \sigma_D &= -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(2.7575 \times 10^3)(-14.4 \times 10^{-3})}{0.538 \times 10^{-6}} + \frac{(486.21)(0.1015)}{13.4 \times 10^{-6}} \\ &= 73.807 \times 10^6 + 3.682 \times 10^6 = 77.5 \times 10^6 \text{ Pa} \end{aligned}$$

$$\sigma_D = 77.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.134



4.133 and 4.134 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.

For SI 150 x 18.6 rolled steel shape

$$I_z = 9.11 \times 10^6 \text{ mm}^4$$

$$I_y = 0.782 \times 10^6 \text{ mm}^4$$

$$z_E = -z_A = -z_B = z_D = \frac{1}{2}(85) = 42.5 \text{ mm}$$

$$y_A = y_B = -y_D = -y_E = \frac{1}{2}(152) = 76 \text{ mm}$$

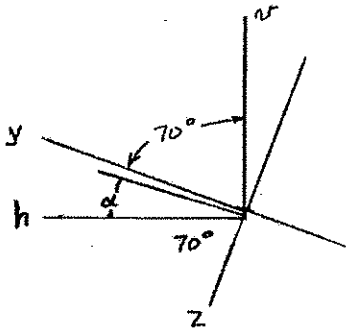
$$M_z = 1.7 \sin 20^\circ = 0.581 \text{ kNm}$$

$$M_y = 1.7 \cos 20^\circ = 1.6 \text{ kNm}$$

$$(a) \tan \varphi = \frac{I_z}{I_y} \tan \theta = \frac{9.11}{0.782} \tan (90^\circ - 20^\circ) = 32.10$$

$$\varphi = 88.2^\circ$$

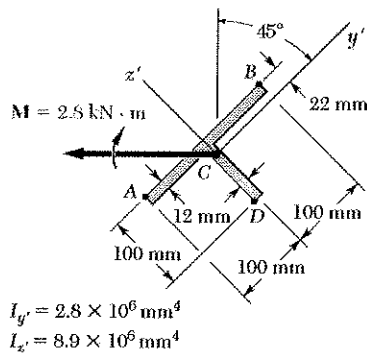
$$\alpha = 88.2^\circ - 70^\circ = 18.2^\circ$$



(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(581)(-0.076)}{9.11 \times 10^{-6}} + \frac{(1600)(0.0425)}{0.782 \times 10^{-6}} = 91.6 \text{ MPa}$$

Problem 4.135



4.135 and 4.136 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.

$$I_{z'} = 8.9 \times 10^6 \text{ mm}^4, \quad I_{y'} = 2.8 \times 10^6 \text{ mm}^4$$

$$z'_A = z'_B = 22 \text{ mm}$$

$$z_O = -100 + 22 = -78$$

$$y_A = -100 \text{ mm}, \quad y_B = 100 \text{ mm}, \quad y_D = -12 \text{ mm}$$

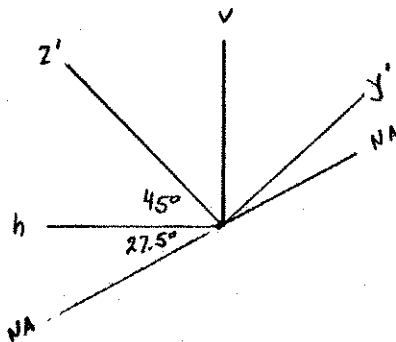
$$M_{y'} = -2.8 \sin 45^\circ = -1.98 \text{ kNm}$$

$$M_{z'} = 2.8 \cos 45^\circ = 1.98 \text{ kNm}$$

$$(a) \tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{8.9}{2.8} \tan (-45^\circ) = -3.1786$$

$$\phi = -72.5^\circ$$

$$\alpha = 72.5^\circ - 45^\circ = 27.5^\circ$$

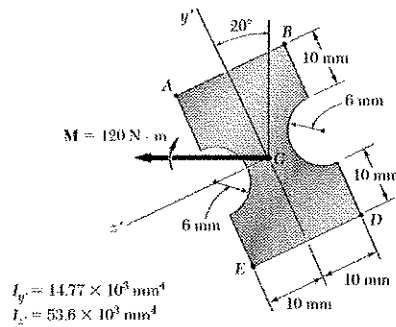


(b) Maximum tensile stress occurs at point D.

$$\begin{aligned} \sigma_D &= -\frac{M_{z'} y_D}{I_{z'}} + \frac{M_{y'} z_D}{I_{y'}} = -\frac{(1.980)(-0.012)}{8.9 \times 10^{-6}} + \frac{(-1.980)(-0.078)}{2.8 \times 10^{-6}} \\ &= 57.8 \text{ MPa} \end{aligned}$$

Problem 4.136

4.135 and 4.136 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal, (b) the maximum tensile stress in the beam.

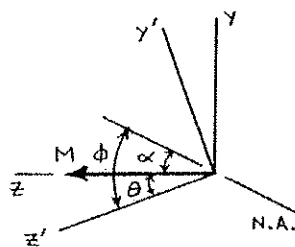


$$I_{z'} = 53.6 \times 10^3 \text{ mm}^4 = 53.6 \times 10^{-9} \text{ m}^4$$

$$I_{y'} = 14.77 \times 10^3 \text{ mm}^4 = 14.77 \times 10^{-9} \text{ m}^4$$

$$M_{z'} = 120 \sin 70^\circ = 112.763 \text{ N}\cdot\text{m}$$

$$M_{y'} = 120 \cos 70^\circ = 41.042 \text{ N}\cdot\text{m}$$



(a) $\theta = 20^\circ$

$$\tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{53.6 \times 10^{-9}}{14.77 \times 10^{-9}} \tan 20^\circ$$

$$= 1.32084$$

$$\phi = 52.871^\circ$$

$$\alpha = 52.871^\circ - 20^\circ \quad \alpha = 32.9^\circ$$

(b) The maximum tensile stress occurs at point E,

$$y_E' = -16 \text{ mm} = -0.016 \text{ m}, \quad z_E' = 10 \text{ mm} = 0.010 \text{ m}$$

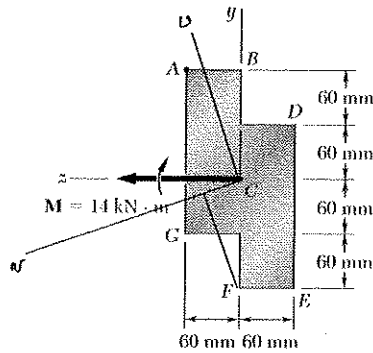
$$\sigma_E = -\frac{M_{z'} y_E'}{I_{z'}} + \frac{M_{y'} z_E'}{I_{y'}} = -\frac{(112.763)(-0.016)}{53.6 \times 10^{-9}} + \frac{(41.042)(0.010)}{14.77 \times 10^{-9}}$$

$$= 61.448 \times 10^6 \text{ Pa}$$

$$\sigma_E = 61.4 \text{ MPa}$$

Problem 4.137

*4.137 through *4.139 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A .



$$I_y = 2 \left\{ \frac{1}{12} (180)(60)^3 \right\} = 25.92 \times 10^6 \text{ mm}^4$$

$$I_z = 2 \left\{ \frac{1}{12} (60)(180)^3 + (60)(180)(30)^2 \right\} = 77.76 \times 10^6 \text{ mm}^4$$

$$I_{yz} = 2 \{ (60)(180)(30)(30) \} = 19.44 \times 10^6 \text{ mm}^4$$

Using Mohr's circle determine the principal axes and principal moments of inertia.

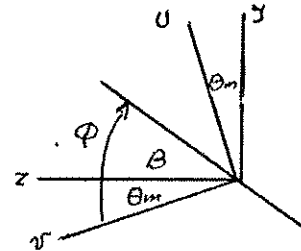
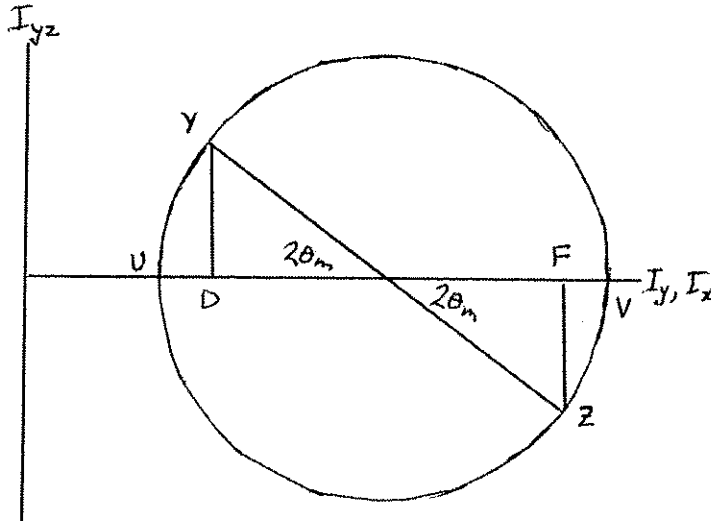
$$Y = (25.92 \times 10^6 \text{ mm}^4, 19.44 \times 10^6 \text{ mm}^4)$$

$$Z = (77.76 \times 10^6 \text{ mm}^4, -19.44 \times 10^6 \text{ mm}^4)$$

$$E = (51.84 \times 10^6 \text{ mm}^4, 0)$$

$$\tan 2\theta_m = \frac{DY}{DE} = \frac{19.44}{25.92}$$

$$2\theta_m = 36.87^\circ \quad \theta_m = 18.435^\circ$$



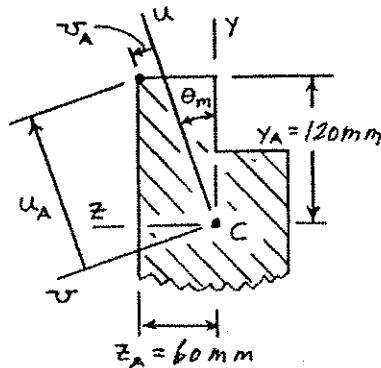
$$R = \sqrt{DE^2 + DY^2} = 32.4 \times 10^6 \text{ mm}^4$$

$$I_u = (51.84 - 32.4) \times 10^6 = 19.44 \times 10^6 \text{ mm}^4$$

$$I_v = (51.84 + 32.4) \times 10^6 = 84.24 \times 10^6 \text{ mm}^4$$

$$M_u = 14 \sin 18.435^\circ = 4.43 \text{ kNm}$$

$$M_v = 14 \cos 18.435^\circ = 13.3 \text{ kNm}$$



$$u_A = 0.12 \cos 18.435^\circ + 0.06 \sin 18.435^\circ = 0.1328 \text{ m}$$

$$v_A = -0.12 \sin 18.435^\circ + 0.06 \cos 18.435^\circ = 0.01897 \text{ m}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u}$$

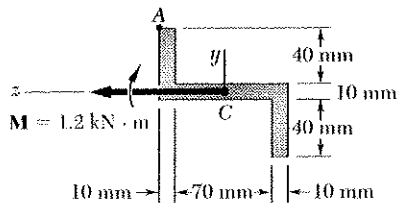
$$= -\frac{(13300)(0.1328)}{84.24 \times 10^{-6}} + \frac{(4430)(0.01897)}{19.44 \times 10^{-6}}$$

$$= -16.64 \text{ MPa}$$

$$\sigma_A = -16.6 \text{ MPa}$$

Problem 4.138

*4.137 through *4.139 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A .



$$I_y = 1.894 \times 10^6 \text{ mm}^4$$

$$I_z = 0.614 \times 10^6 \text{ mm}^4$$

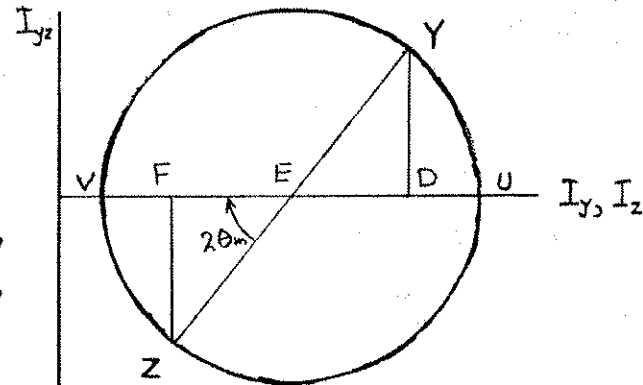
$$I_{yz} = +0.800 \times 10^6 \text{ mm}^4$$

$$Y (1.894, 0.800) \times 10^6 \text{ mm}^4$$

$$Z (0.614, 0.800) \times 10^6 \text{ mm}^4$$

$$E (1.254, 0) \times 10^6 \text{ mm}^4$$

Using Mohr's circle, determine the principal axes and the principal moments of inertia.



$$R = \sqrt{EF^2 + FZ^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6 \text{ mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6 \text{ mm}^4 = 0.2295 \times 10^6 \text{ mm}^4 = 0.2295 \times 10^{-6} \text{ m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6 \text{ mm}^4 = 2.2785 \times 10^6 \text{ mm}^4 = 2.2785 \times 10^{-6} \text{ m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \quad \theta_m = 25.67^\circ$$

$$M_v = M \cos \theta_m = (1.2 \times 10^3) \cos 25.67^\circ = 1.0816 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_u = -M \sin \theta_m = -(1.2 \times 10^3) \sin 25.67^\circ = -0.5198 \times 10^3 \text{ N}\cdot\text{m}$$

$$u_A = y_A \cos \theta_m - z_A \sin \theta_m = 45 \cos 25.67^\circ - 45 \sin 25.67^\circ = 21.07 \text{ mm}$$

$$v_A = z_A \cos \theta_m + y_A \sin \theta_m = 45 \cos 25.67^\circ + 45 \sin 25.67^\circ = 60.05 \text{ mm}$$

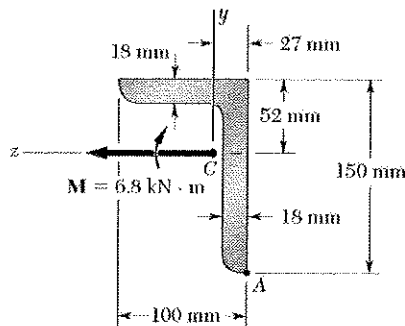
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6 \text{ Pa}$$

$$\sigma_A = 113.0 \text{ MPa}$$

Problem 4.139

*4.137 through *4.139 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.



$$I_y = 3.65 \times 10^6 \text{ mm}^4$$

$$I_z = 10.1 \times 10^6 \text{ mm}^4$$

$$I_{yz} = 3.45 \times 10^6 \text{ mm}^4$$

$$Y (3.65, 3.45) \times 10^6 \text{ mm}^4$$

$$Z (10.1, 3.45) \times 10^6 \text{ mm}^4$$

$$E (6.875, 0) \times 10^6 \text{ mm}^4$$

$$EF = 3.225 \times 10^6 \text{ mm}^4$$

$$FZ = 3.45 \times 10^6 \text{ mm}^4$$

$$R = \sqrt{3.225^2 + 3.45^2} = 4.72 \times 10^6 \text{ mm}^4 \quad \tan 2\theta_m = \frac{FZ}{EF} = \frac{3.45}{3.225} = 1.0698$$

$$\theta_m = 23.5^\circ \quad I_o = (6.875 - 4.72) \times 10^6 = 2.155 \times 10^6 \text{ mm}^4 \quad I_v = (6.875 + 4.72) \times 10^6 = 11.6 \times 10^6 \text{ mm}^4$$

$$M_u = (6.8) \sin 23.5^\circ = 2.71 \text{ kNm}$$

$$M_v = (6.8) \cos 23.5^\circ = 6.24 \text{ kNm}$$

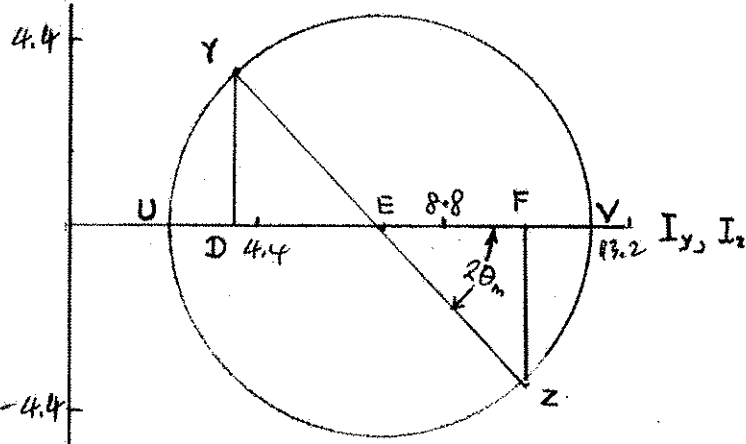
$$u_A = y_A \cos \theta_m + z_A \sin \theta_m = -98 \cos 23.5^\circ - 27 \sin 23.5^\circ = -100.6 \text{ mm}$$

$$v_A = z_A \cos \theta_m - y_A \sin \theta_m = -27 \cos 23.5^\circ + 98 \sin 23.5^\circ = 14.3 \text{ mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_o} = -\frac{(6240)(-0.1006)}{11.6 \times 10^6} + \frac{(2710)(0.0143)}{2.155 \times 10^6}$$

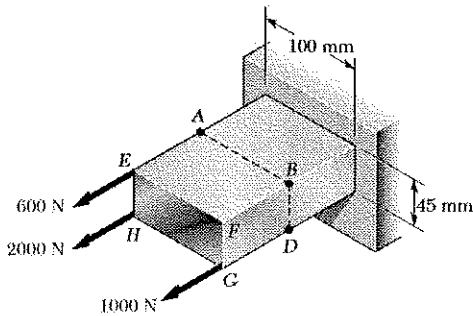
$$= 72.1 \text{ MPa}$$

Using Mohr's circle, determine the principal axes and principal moments of inertia $I_{yz} (10^6) \text{ mm}^4$.



Problem 4.140

4.140 For the loading shown, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.



Add y and z axes as shown

$$A = (100)(45) = 4500 \text{ mm}^2$$

$$I_z = \frac{1}{12}(100)(45)^3 = 759375 \text{ mm}^4$$

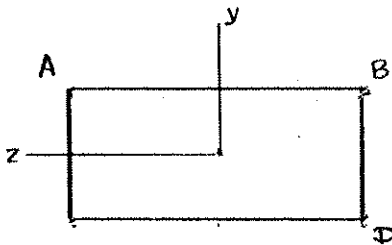
$$I_y = \frac{1}{12}(45)(100)^3 = 3.75 \times 10^6 \text{ mm}^4$$

Resultant force and bending couples

$$P = 600 + 2000 + 1000 = 3600 \text{ N}$$

$$M_z = [-(600)(0.225) + (2000)(0.225) + (1000)(0.225)](1 \times 10^{-3}) = 54 \text{ Nm}$$

$$M_y = (600)(0.05) + (2000)(0.05) - (1000)(0.05) = 80 \text{ Nm}$$



$$(a) \sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{759375 \times 10^{-12}} + \frac{(80)(0.05)}{3.75 \times 10^{-6}} = 266.7 \text{ kPa} \leftarrow$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{1.944} + \frac{(80)(-2)}{9.6} = -1.87 \text{ MPa} \leftarrow$$

(b) Intersection of neutral axis with line AB or its extension.

$$\sigma = 0, \quad y = 0.0225 \text{ m}, \quad z = ?$$

$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{(54)(0.0225)}{759375 \times 10^{-12}} + \frac{80 z}{3.75 \times 10^{-6}}$$

$$-0.8 \times 10^6 + 21.33 z = 0 \quad z = 0.0375 \text{ m}$$

Intersects AB at 12.5 mm from A $\leftarrow$

Intersection of neutral axis with line BD or its extension.

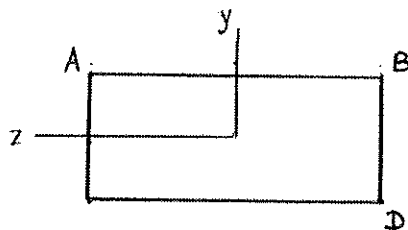
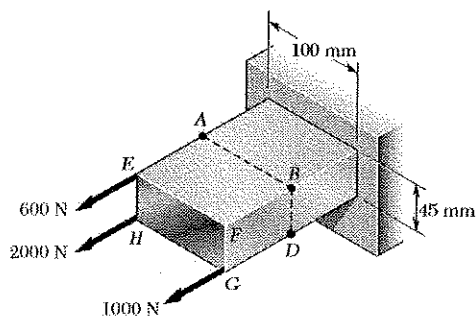
$$\sigma = 0 \quad z = -50 \text{ mm} \quad y = ?$$

$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{3600}{4500 \times 10^{-6}} - \frac{54 y}{759375 \times 10^{-12}} + \frac{(80)(-0.05)}{3.75 \times 10^{-6}}$$

$$-0.2667 \times 10^6 - 71.111 y = 0 \quad y = -0.00375 \text{ m}$$

Intersects BD at 18.75 mm from D $\leftarrow$

Problem 4.141



4.141 Solve Prob. 4.140, assuming that the magnitude of the force applied at G is increased from 1.0 kN to 1.6 kN.

4.140 For the loading shown, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.

Add y and z axes as shown.

$$A = (100 \times 45) = 4500 \text{ mm}^2$$

$$I_z = \frac{1}{12} (100) (45)^3 = 759375 \text{ mm}^4$$

$$I_y = \frac{1}{12} (45) (100)^3 = 3.75 \times 10^6 \text{ mm}^4$$

Resultant force and bending couples

$$P = 600 + 2000 + 1600 = 4200 \text{ N.}$$

$$M_z = -(600)(0.0225) + (2000)(0.0225) + (1600)(0.0225) = 67.5 \text{ Nm}$$

$$M_y = (600)(0.05) + (2000)(0.05) - (1600)(0.05) = 50 \text{ Nm}$$

$$(a) \quad \sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{(50)(0.05)}{3.75 \times 10^{-6}} = -400 \text{ kPa}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{(50)(-0.05)}{3.75 \times 10^{-6}} = -1.73 \text{ MPa}$$

(b) Intersection of neutral axis with line AB or its extension.

$$\sigma = 0, \quad y = 0.9 \text{ in.} \quad z = ?$$

$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{4200}{0.0045} - \frac{(67.5)(0.0225)}{759375 \times 10^{-12}} + \frac{50z}{3.75 \times 10^{-6}}$$

$$-1.0667 \times 10^6 + 13.3333 z = 0 \quad z = 0.08 \text{ m} = 80 \text{ mm}$$

Does not intersect AB

Intersection of neutral axis with line BD or its extension.

$$\sigma = 0, \quad z = -2 \text{ in.} \quad y = ?$$

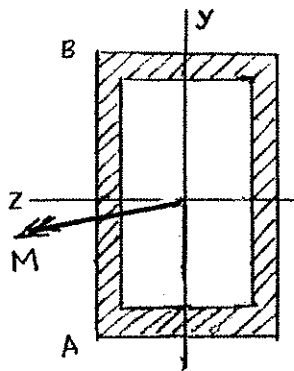
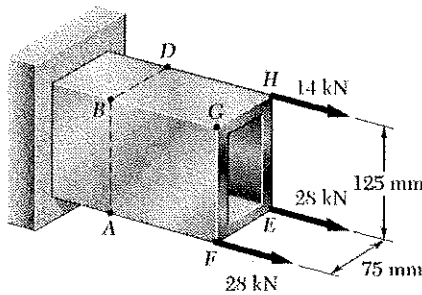
$$0 = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{4200}{0.0045} - \frac{67.5y}{759375 \times 10^{-12}} + \frac{(50)(-0.05)}{3.75 \times 10^{-6}}$$

$$10 \times 0.26667 - 88.8889 y = 0 \quad y = 0.003 \text{ m} = 3 \text{ mm}$$

Intersects BD at 19.5 mm from B

Problem 4.142

4.142 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.



Add y- and z- axes as shown. Cross section is a 75 mm $\times$ 125 mm rectangle with a 51 mm $\times$ 101 mm rectangular cutout.

$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283 \times 10^6 \text{ mm}^4 \\ = 7.8283 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781 \times 10^6 \text{ mm}^4 \\ = 3.2781 \times 10^{-6} \text{ m}^4$$

$$A = (75)(125) - (51)(101) = 4.224 \times 10^3 \text{ mm}^2 \\ = 4.224 \times 10^{-3} \text{ m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 + 28 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$M_z = -(62.5 \text{ mm})(14 \text{ kN}) + (62.5 \text{ mm})(28 \text{ kN}) + (62.5 \text{ mm})(28 \text{ kN}) \\ = 2625 \text{ N}\cdot\text{m}$$

$$M_y = -(37.5 \text{ mm})(14 \text{ kN}) + (37.5 \text{ mm})(28 \text{ kN}) + (37.5 \text{ mm})(28 \text{ kN}) \\ = -525 \text{ N}\cdot\text{m}$$

$$(a) \quad \sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{70 \times 10^3}{4.224 \times 10^{-3}} - \frac{(2625)(-0.0625)}{7.8283 \times 10^{-6}} + \frac{(-525)(0.0375)}{3.2781 \times 10^{-6}} \\ = 31.524 \times 10^6 \text{ Pa} \quad \sigma_A = 31.5 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{70 \times 10^3}{4.224 \times 10^{-3}} - \frac{(2625)(0.0625)}{7.8283 \times 10^{-6}} + \frac{(-525)(0.0375)}{3.2781 \times 10^{-6}} \\ = -10.39 \times 10^6 \text{ Pa} \quad \sigma_B = -10.39 \text{ MPa} \quad \blacktriangleleft$$

(b) Let point H be the point where the neutral axis intersects AB.

$$z_H = 0.0375 \text{ m}, \quad y_H = ?, \quad \sigma_H = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$y_H = \frac{I_z}{M_z} \left(\frac{P}{A} + \frac{M_y z_H}{I_y} \right) = \frac{7.8283 \times 10^6}{2625} \left[\frac{70 \times 10^3}{4.224 \times 10^{-3}} + \frac{(-525)(0.0375)}{3.2781 \times 10^{-6}} \right] \\ = 0.03151 \text{ m} = 31.51 \text{ mm}$$

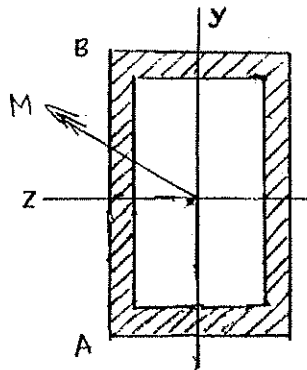
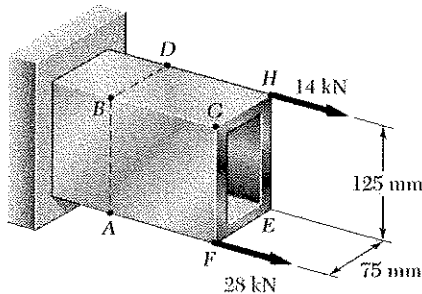
$$31.51 + 62.5 = 94.0 \text{ mm}$$

Answer: 94.0 mm above point A. $\blacktriangleleft$

Problem 4.143

4.143 Solve Prob. 4.142, assuming that the 28-kN force at point E is removed.

4.142 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.



Add y- and z-axes as shown. Cross section is a 75 mm x 125 mm rectangle with a 51 mm x 101 mm rectangular cutout.

$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283 \times 10^6 \text{ mm}^4$$

$$= 7.8283 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781 \times 10^6 \text{ mm}^4$$

$$= 3.2781 \times 10^{-6} \text{ m}^4$$

$$A = (75)(125) - (51)(101) = 4.224 \times 10^3 \text{ mm}^2$$

$$= 4.224 \times 10^{-3} \text{ m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 = 42 \text{ kN} = 42 \times 10^3 \text{ N}$$

$$M_z = -(62.5 \text{ mm})(14 \text{ kN}) + (62.5 \text{ mm})(28 \text{ kN})$$

$$= 875 \text{ N}\cdot\text{m}$$

$$M_y = -(37.5 \text{ mm})(14 \text{ kN}) - (37.5 \text{ mm})(28 \text{ kN})$$

$$= 525 \text{ N}\cdot\text{m}$$

$$(a) \sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{42 \times 10^3}{4.224 \times 10^{-3}} - \frac{(875)(-0.0625)}{7.8283 \times 10^{-6}} + \frac{(525)(0.0375)}{3.2781 \times 10^{-6}}$$

$$= 22.935 \times 10^6 \text{ Pa} \quad \sigma_A = 22.9 \text{ MPa}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{42 \times 10^3}{4.224 \times 10^{-3}} - \frac{(875)(0.0625)}{7.8283 \times 10^{-6}} + \frac{(525)(0.0375)}{3.2781 \times 10^{-6}}$$

$$= 8.9831 \times 10^6 \text{ Pa} \quad \sigma_B = 8.96 \text{ MPa}$$

(b) Let point K be the point where the neutral axis intersects BD.

$$z_K = ?, \quad y_K = 0.0625 \text{ m}, \quad \sigma_K = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_K}{I_z} + \frac{M_y z_K}{I_y}$$

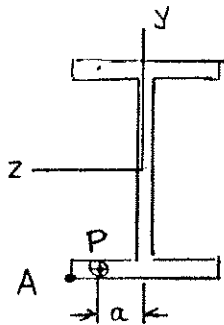
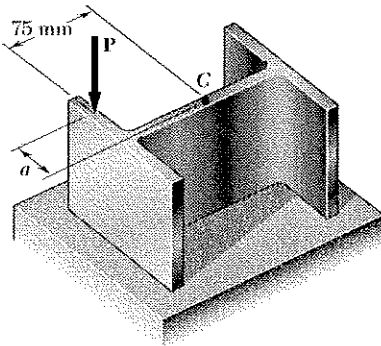
$$z_K = \frac{I_y}{M_y} \left(\frac{M_z y_K}{I_z} - \frac{P}{A} \right) = \frac{3.2781 \times 10^{-6}}{525} \left[\frac{(875)(0.0625)}{7.8283 \times 10^{-6}} - \frac{42 \times 10^3}{4.224 \times 10^{-3}} \right]$$

$$= -0.018465 \text{ m} = -18.465 \text{ mm}$$

$$37.5 + 18.465 = 56.0 \text{ mm} \quad \text{Answer: } 56.0 \text{ mm to the right of point B.}$$

Problem 4.144

4.144 An axial load P of magnitude 50 kN is applied as shown to a short section of a W150 × 24 rolled-steel member. Determine the largest distance a for which the maximum compressive stress does not exceed 90 MPa.



Add y - and z - axes.

For W 150 × 24 rolled-steel section

$$A = 3060 \text{ mm}^2 = 3060 \times 10^{-6} \text{ m}^2$$

$$I_z = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$$

$$I_y = 1.83 \times 10^6 \text{ mm}^4 = 1.83 \times 10^{-6} \text{ m}^4$$

$$d = 160 \text{ mm}, \quad b_f = 102 \text{ mm}$$

$$y_A = -\frac{d}{2} = -80 \text{ mm}, \quad z_A = \frac{b_f}{2} = 51 \text{ mm}.$$

$$P = 50 \times 10^3 \text{ N}$$

$$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = -Pa$$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

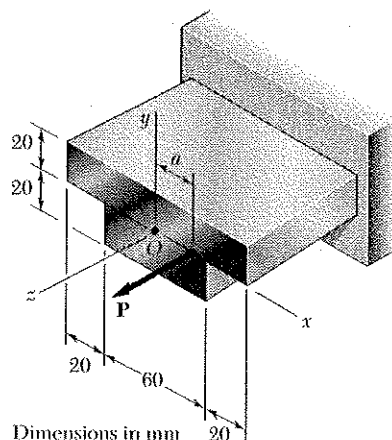
$$\sigma_A = -90 \times 10^6 \text{ Pa}$$

$$\begin{aligned} M_y &= \frac{I_y}{z_A} \left\{ \frac{M_z y_A}{I_z} + \frac{P}{A} + \sigma_A \right\} \\ &= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}} \left\{ \frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6) \right\} \\ &= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}} \left\{ +22.388 + 16.340 - 90 \right\} \times 10^6 \\ &= -1.8398 \times 10^3 \text{ N}\cdot\text{m} \end{aligned}$$

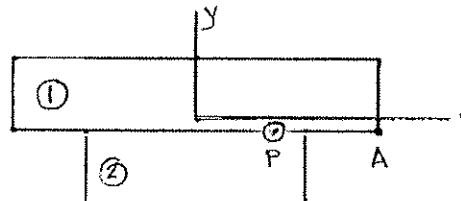
$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3} \text{ m} \quad a = 36.8 \text{ mm} \quad \blacktriangleleft$$

Problem 4.145

4.145 A horizontal load P of magnitude 100 kN is applied to the beam shown. Determine the largest distance a for which the maximum tensile stress in the beam does not exceed 75 MPa.



Locate the centroid.



	A, mm^2	$\bar{y}, \text{mm}$	$A\bar{y}, \text{mm}^3$
①	2000	10	20×10^3
②	1200	-10	-12×10^3
Σ	3200		8×10^3

$$\begin{aligned}\bar{Y} &= \frac{\Sigma A\bar{y}}{\Sigma A} \\ &= \frac{8 \times 10^3}{3200} \\ &= 2.5 \text{ mm}\end{aligned}$$

Move coordinate origin to the centroid.

Coordinates of load point: $x_P = a$, $y_P = -2.5 \text{ mm}$

Bending couples $M_x = y_P P$ $M_y = -aP$

$$I_x = \frac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \frac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4 = 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(20)(100)^3 + \frac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} \quad \sigma_A = 75 \times 10^6 \text{ Pa}, \quad P = 100 \times 10^3 \text{ N}$$

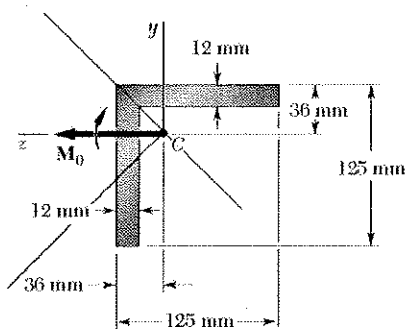
$$M_y = \frac{I_y}{x} \left\{ \frac{P}{A} + \frac{M_x y}{I_x} - \sigma \right\} \quad \text{For point A } x = 50 \text{ mm}, y = -2.5 \text{ mm}$$

$$\begin{aligned}M_y &= \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}} \left\{ \frac{100 \times 10^3}{3200 \times 10^{-6}} + \frac{(-2.5)(100 \times 10^3)(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} - 75 \times 10^6 \right\} \\ &= \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}} \{ 31.25 + 1.537 - 75 \} \times 10^6 = -1.7111 \times 10^3 \text{ N}\cdot\text{m}\end{aligned}$$

$$a = -\frac{M_y}{P} = -\frac{(-1.7111 \times 10^3)}{100 \times 10^3} = 17.11 \times 10^3 \text{ m}$$

$$a = 17.11 \text{ mm} \quad \blacktriangleleft$$

Problem 4.146



4.146 A beam having the cross section shown is subjected to a couple M_0 that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 84 MPa. Given: $I_y = I_z = 4.7 \times 10^6 \text{ mm}^4$, $A = 3064 \text{ mm}^2$, $k_{\min} = 25 \text{ mm}$. (Hint: By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak_{\min}^2$ and $I_{\min} + I_{\max} = I_y + I_z$.)

$$M_u = M_0 \sin 45^\circ = 0.70711 M_0$$

$$M_v = M_0 \cos 45^\circ = 0.7071 M_0$$

$$I_{\min} = Ak_{\min}^2 = (3064)(25)^2 = 1.915 \times 10^6 \text{ mm}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = (4.7 + 4.7 - 1.915) \times 10^6 = 7.485 \times 10^6 \text{ mm}^4$$

$$u_B = y_B \cos 45^\circ + z_B \sin 45^\circ = -89 \cos 45^\circ + 24 \sin 45^\circ = -46 \text{ mm}$$

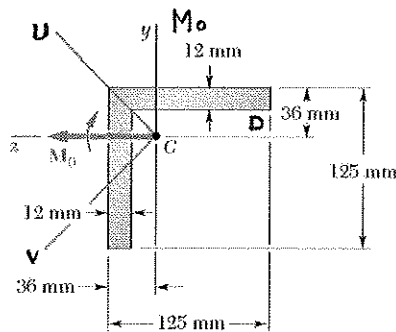
$$v_B = z_B \cos 45^\circ - y_B \sin 45^\circ = 24 \cos 45^\circ - (-89) \sin 45^\circ = 80 \text{ mm}$$

$$\sigma_B = -\frac{M_u u_B}{I_v} + \frac{M_v v_B}{I_u} = -0.70711 M_0 \left[-\frac{u_B}{I_{\min}} + \frac{v_B}{I_{\max}} \right]$$

$$= 0.70711 M_0 \left[-\frac{(-0.046)}{1.915 \times 10^6} + \frac{0.08}{7.485 \times 10^6} \right] = 24543 M_0$$

$$M_0 = \frac{\sigma_B}{24543} = \frac{84 \times 10^6}{24543} = 3.42 \text{ kN m}$$

Problem 4.147



4.147 Solve Prob. 4.146, assuming that the couple M_0 acts in a horizontal plane.

4.146 A beam having the cross section shown is subjected to a couple M_0 that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 84 MPa. Given: $I_y = I_z = 4.7 \times 10^6 \text{ mm}^4$, $A = 3064 \text{ mm}^2$, $k_{min} = 25 \text{ mm}$. (Hint: By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_y + I_z$.)

$$M_U = M_0 \cos 45^\circ = 0.70711 M_0$$

$$M_V = -M_0 \sin 45^\circ = -0.70711 M_0$$

$$I_{min} = A k_{min}^2 = (3064)(25)^2 = 1.915 \times 10^6 \text{ mm}^4$$

$$I_{max} = I_y + I_z - I_{min} = 10^6(4.7 + 4.7 + 1.915) = 7.485 \times 10^6 \text{ mm}^4$$

$$U_D = y_D \cos 45^\circ + z_D \sin 45^\circ = 24 \cos 45^\circ + (-89 \sin 45^\circ) = -46 \text{ mm}$$

$$V_D = z_D \cos 45^\circ - y_D \sin 45^\circ = (-89) \cos 45^\circ - (24) \sin 45^\circ = 80 \text{ mm}$$

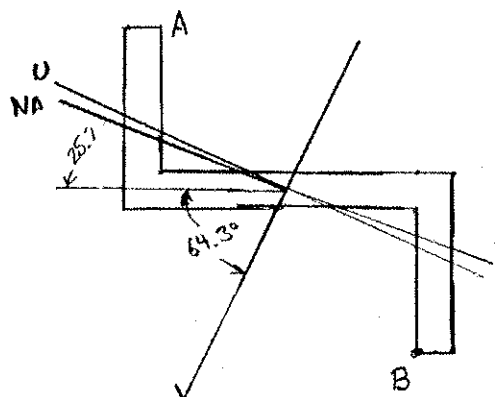
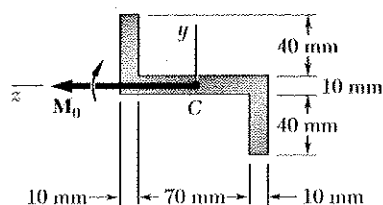
$$\sigma_D = -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = 0.70711 M_0 \left[-\frac{U_D}{I_{min}} + \frac{V_D}{I_{max}} \right]$$

$$= 0.70711 M_0 \left[-\frac{(-0.046)}{1.915 \times 10^6} + \frac{0.08}{7.485 \times 10^6} \right] = 24543 M_0$$

$$M_0 = \frac{\sigma_D}{24543} = \frac{12}{24543} = 3.42 \text{ kN m.}$$

Problem 4.148

4.148 The Z section shown is subjected to a couple M_0 acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. Given: $I_{max} = 2.28 \times 10^{-6} \text{ m}^4$, $I_{min} = 0.23 \times 10^{-6} \text{ m}^4$, principal axes 25.7° and 64.3° .



$$I_v = I_{max} = 2.28 \times 10^{-6} \text{ m}^4 = 2.28 \times 10^{-6} \text{ m}^4$$

$$I_u = I_{min} = 0.23 \times 10^{-6} \text{ m}^4 = 0.23 \times 10^{-6} \text{ m}^4$$

$$M_v = M_0 \cos 64.3^\circ$$

$$M_u = M_0 \sin 64.3^\circ$$

$$\theta = 64.3^\circ$$

$$\tan \phi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3^\circ = 20.597$$

$$\phi = 87.22^\circ$$

Points A and B are farthest from the neutral axis.

$$U_B = y_B \cos 64.3^\circ + z_B \sin 64.3^\circ = (-45) \cos 64.3^\circ + (-35) \sin 64.3^\circ$$

$$= -51.05 \text{ mm}$$

$$V_B = z_B \cos 64.3^\circ - y_B \sin 64.3^\circ = (-35) \cos 64.3^\circ - (-45) \sin 64.3^\circ$$

$$= +25.37 \text{ mm}$$

$$\sigma_B = -\frac{M_v U_B}{I_v} + \frac{M_u V_B}{I_u}$$

$$80 \times 10^6 = -\frac{(M_0 \cos 64.3^\circ)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_0 \sin 64.3^\circ)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

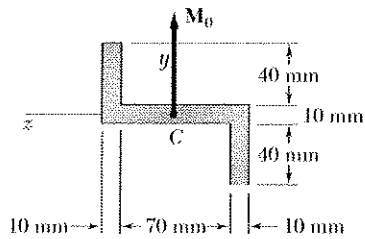
$$= 93.81 \times 10^3 M_0$$

$$M_0 = \frac{80 \times 10^6}{109.1 \times 10^3}$$

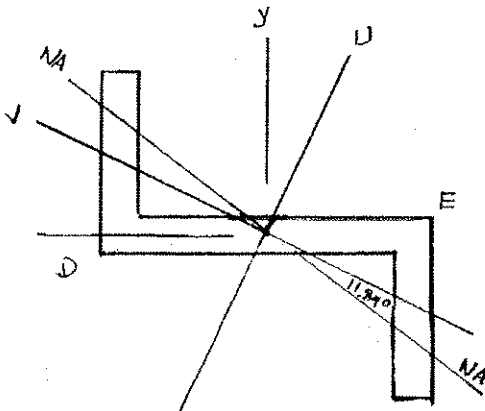
$$M_0 = 733 \text{ N}\cdot\text{m}$$

Problem 4.149

4.149 Solve Prob. 4.148 assuming that the couple M_0 acts in a horizontal plane.



4.148 The Z section shown is subjected to a couple M_0 acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. Given: $I_{\max} = 2.28 \times 10^{-6} \text{ mm}^4$, $I_{\min} = 0.23 \times 10^{-6} \text{ mm}^4$, principal axes 25.7° and 64.3° .



$$I_V = I_{\min} = 0.23 \times 10^{-6} \text{ mm}^4 = 0.23 \times 10^{-6} \text{ m}^4$$

$$I_U = I_{\max} = 2.28 \times 10^{-6} \text{ mm}^4 = 2.28 \times 10^{-6} \text{ m}^4$$

$$M_V = M_0 \cos 64.3^\circ$$

$$M_U = M_0 \sin 64.3^\circ$$

$$\theta = 64.3^\circ$$

$$\tan \phi = \frac{I_V}{I_U} \tan \theta$$

$$= \frac{0.23 \times 10^{-6}}{2.28 \times 10^{-6}} \tan 64.3^\circ = 0.20961$$

$$\phi = 11.84^\circ$$

Points D and E are farthest from the neutral axis.

$$U_D = y_D \cos 25.7^\circ - z_D \sin 25.7^\circ = (-5) \cos 25.7^\circ - 45 \sin 25.7^\circ$$

$$= -24.02 \text{ mm}$$

$$V_D = z_D \cos 25.7^\circ + y_D \sin 25.7^\circ = 45 \cos 25.7^\circ + (-5) \sin 25.7^\circ$$

$$= 38.38 \text{ mm}$$

$$\sigma_D = -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = -\frac{(M_0 \cos 64.3^\circ)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}} + \frac{(M_0 \sin 64.3^\circ)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$

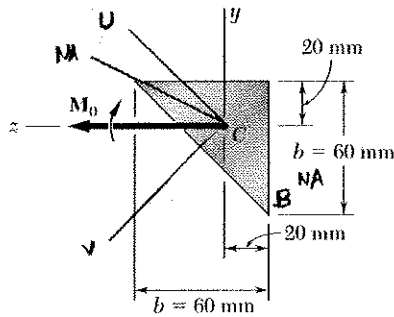
$$80 \times 10^6 = 60.48 \times 10^3 M_0$$

$$M_0 = 1.323 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_0 = 1.323 \text{ kN}\cdot\text{m}$$

Problem 4.150

4.150 A beam having the cross section shown is subjected to a couple M_0 acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 100 MPa. Given: $I_y = I_z = b^4/36$ and $I_{yz} = b^4/72$.

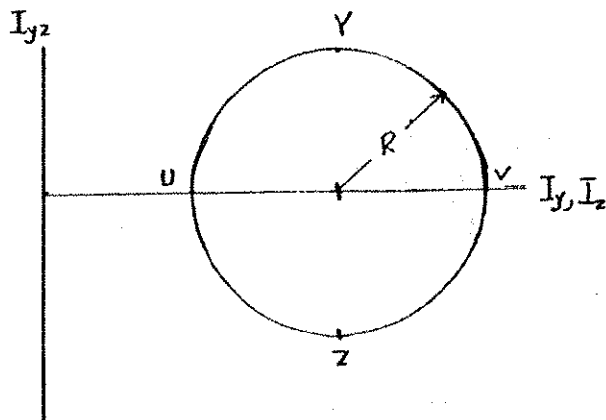


$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.

Using Mohr's circle determine the principal moments of inertia.



$$R = |I_{yz}| = 0.180 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_y + I_z}{2} + R$$

$$= 0.540 \times 10^6 \text{ mm}^4 = 0.540 \times 10^{-6} \text{ m}^4$$

$$I_u = \frac{I_y + I_z}{2} - R$$

$$= 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$$M_u = M_0 \sin 45^\circ = 0.70711 M_0, \quad M_v = M_0 \cos 45^\circ = 0.70711 M_0$$

$$\theta = 45^\circ \quad \tan \phi = \frac{I_v}{I_u} \tan \theta = \frac{0.540 \times 10^6}{0.180 \times 10^6} \tan 45^\circ = 3$$

$$\phi = 71.56^\circ$$

Point A: $u_A = 0, \quad v_A = -20\sqrt{2} \text{ mm}$

$$\sigma_A = -\frac{M_u u_A}{I_u} + \frac{M_v v_A}{I_v} = 0 + \frac{(0.70711 M_0)(-20\sqrt{2} \times 10^{-3})}{0.180 \times 10^{-6}} = -11.11 \times 10^3 M_0$$

$$M_0 = \frac{\sigma_A}{-11.11 \times 10^3} = \frac{-100 \times 10^6}{-11.11 \times 10^3} = 900 \text{ N}\cdot\text{m}$$

Point B: $u_B = -\frac{60}{\sqrt{2}} \text{ mm}, \quad v_B = \frac{20}{\sqrt{2}} \text{ mm}$

$$\sigma_B = -\frac{M_u u_B}{I_u} + \frac{M_v v_B}{I_v} = -\frac{(0.70711 M_0)(-\frac{60}{\sqrt{2}} \times 10^{-3})}{0.540 \times 10^{-6}} + \frac{(0.70711 M_0)(\frac{20}{\sqrt{2}} \times 10^{-3})}{0.180 \times 10^{-6}}$$

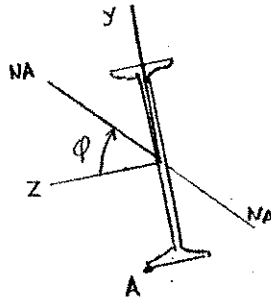
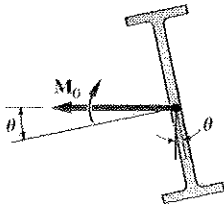
$$= 111.11 \times 10^3 M_0$$

$$M_0 = \frac{\sigma_B}{111.11 \times 10^3} = \frac{100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N}\cdot\text{m}$$

Choose the smaller value.

$$M_0 = 900 \text{ N}\cdot\text{m}$$

Problem 4.151



4.151 A couple M_0 acting in a vertical plane is applied to a W310 x 23.8 rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.

For W 310X23.6. rolled steel section

$$I_z = 42.7 \times 10^6 \text{ mm}^4, \quad I_y = 1.16 \times 10^6 \text{ mm}^4$$

$$d = 305 \text{ mm}, \quad b_f = 101 \text{ mm}$$

$$y_A = -\frac{d}{2}, \quad z_A = \frac{b_f}{2}$$

$$\tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{42.7}{1.16} \tan \theta = 36.81 \tan \theta$$

Point A is farthest from the neutral axis.

$$M_y = M_0 \sin \theta, \quad M_z = M_0 \cos \theta$$

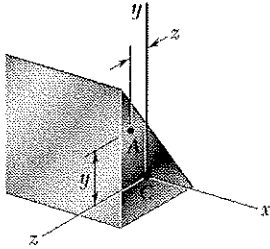
$$\begin{aligned} \sigma_A &= -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z} \cos \theta + \frac{M_0 b_f}{2I_y} \sin \theta \\ &= \frac{M_0 d}{2I_z} \left(1 + \frac{I_z b_f}{I_y d} \tan \theta \right) \end{aligned}$$

$$\text{For } \theta = 0, \quad \sigma_0 = \frac{M_0 d}{2I_z}$$

$$\sigma_A = \sigma_0 \left(1 + \frac{I_z b_f}{I_y d} \tan \theta \right) = 2\sigma_0$$

$$\tan \theta = \frac{I_y d}{I_z b_f} = \frac{(1.16)(305)}{(42.7)(101)} = 0.08204, \quad \theta = 4.70^\circ$$

Problem 4.152



4.152 A beam of unsymmetric cross section is subjected to a couple M_0 acting in the vertical plane xy . Show that the stress at point A , of coordinates y and z , is

$$\sigma_A = -\frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where I_y , I_z , and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_z the moment of the couple.

The stress σ_A varies linearly with the coordinates y and z . Since the axial force is zero, the y - and z -axes are centroidal axes.

$$\sigma_A = C_1 y + C_2 z \quad \text{where } C_1 \text{ and } C_2 \text{ are constants.}$$

$$\begin{aligned} M_y &= \int z \sigma_A dA = C_1 \int yz dA + C_2 \int z^2 dA \\ &= I_{yz} C_1 + I_y C_2 = 0 \\ C_2 &= -\frac{I_{yz}}{I_y} C_1 \end{aligned}$$

$$\begin{aligned} M_z &= -\int y \sigma_A dz = -C_1 \int y^2 dA + C_2 \int yz dA \\ &= -I_z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1 \end{aligned}$$

$$I_y M_z = -(I_y I_z - I_{yz}^2) C_1$$

$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2} \quad C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

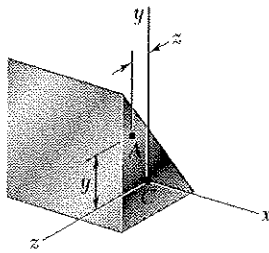
$$\sigma_A = -\frac{I_y y - I_{yz} z}{I_y I_z - I_{yz}^2} M_z$$

Problem 4.153

4.153 A beam of unsymmetric cross section is subjected to a couple M_0 acting in the horizontal plane xz . Show that the stress at point A , of coordinates y and z , is

$$\sigma_A = \frac{zI_z - yI_{yz}}{I_y I_z - I_{yz}^2} M_y$$

where I_y , I_z , and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.



The stress σ_A varies linearly with the coordinates y and z . Since the axial force is zero, the y - and z -axes are centroidal axes.

$$\sigma_A = C_1 y + C_2 z \quad \text{where } C_1 \text{ and } C_2 \text{ are constants.}$$

$$\begin{aligned} M_z &= -\int y \sigma_A dA = -C_1 \int y^2 dA - C_2 \int yz dA \\ &= -I_z C_1 - I_{yz} C_2 = 0 \\ C_1 &= -\frac{I_{yz}}{I_z} C_2 \end{aligned}$$

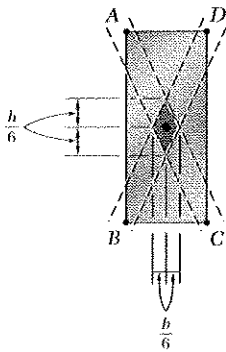
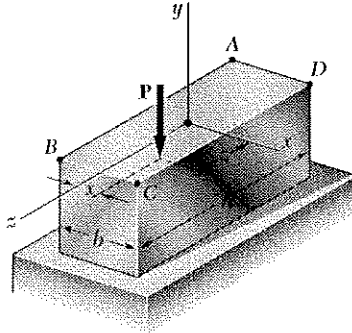
$$\begin{aligned} M_y &= \int z \sigma_A dA = C_1 \int yz dA + C_2 \int z^2 dA \\ &= I_{yz} C_1 + I_y C_2 \\ &\quad - I_{yz} \frac{I_{yz}}{I_z} C_2 + I_y C_2 \end{aligned}$$

$$I_z M_y = (I_y I_z - I_{yz}^2) C_2$$

$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2} \quad C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{I_z z - I_{yz} y}{I_y I_z - I_{yz}^2} M_y$$

Problem 4.154



4.154 (a) Show that the stress at corner A of the prismatic member shown in Fig. P4.154a will be zero if the vertical force P is applied at a point located on the line

$$\frac{x}{b/6} + \frac{z}{h/6} = 1$$

(b) Further show that, if no tensile stress is to occur in the member, the force P must be applied at a point located within the area bounded by the line found in part a and three similar lines corresponding to the condition of zero stress at B, C, and D, respectively. This area, shown in Fig. 4.154b, is known as the *kern* of the cross section.

$$I_z = \frac{1}{12} h b^3 \quad I_x = \frac{1}{12} b h^3 \quad A = bh$$

$$z_A = -\frac{h}{2} \quad x_A = -\frac{b}{2}$$

Let P be the load point.

$$M_z = -P x_p \quad M_x = P z_p$$

$$\begin{aligned} \sigma_A &= -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} \\ &= -\frac{P}{bh} + \frac{(-P x_p)(-\frac{b}{2})}{\frac{1}{12} h b^3} - \frac{P z_p(-\frac{h}{2})}{\frac{1}{12} b h^3} \\ &= -\frac{P}{bh} \left[1 - \frac{x_p}{b/6} - \frac{z_p}{h/6} \right] \end{aligned}$$

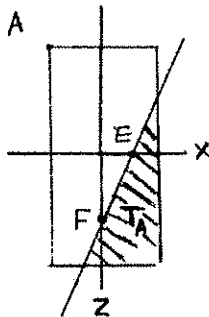
For $\sigma_A = 0$,

$$1 - \frac{x}{b/6} - \frac{z}{h/6} = 0, \quad \frac{x}{b/6} + \frac{z}{h/6} = 1$$

At point E: $z = 0 \quad \therefore x_E = b/6$

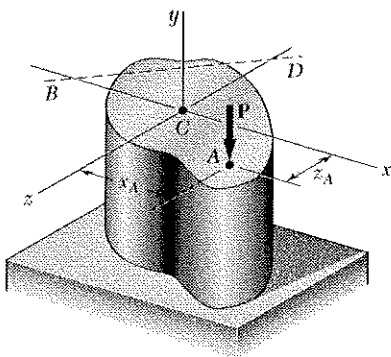
At point F: $x = 0 \quad \therefore z_F = h/6$

If the line of action (x_p, z_p) lies within the portion marked T_A , a tensile will occur at corner A.



By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

Problem 4.155



4.155 (a) Show that, if a vertical force P is applied at point A of the section shown, the equation of the neutral axis BD is

$$\left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

where k_z and k_x denote the radius of gyration of the cross section with respect to the z axis and the x axis, respectively. (b) Further show that, if a vertical force Q is applied at any point located on line BD , the stress at point A will be zero.

Definitions: $k_x^2 = \frac{I_x}{A}$, $k_z^2 = \frac{I_z}{A}$

(a) $M_x = Pz_A$, $M_z = -Px_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ak_z^2} - \frac{Pz_A z_E}{Ak_x^2}$$

$$= -\frac{P}{A} \left[1 + \left(\frac{x_A}{k_z^2}\right)x_E + \left(\frac{z_A}{k_x^2}\right)z_E \right] = 0 \text{ if } E \text{ lies on neutral axis.}$$

$$1 + \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = 0, \quad \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

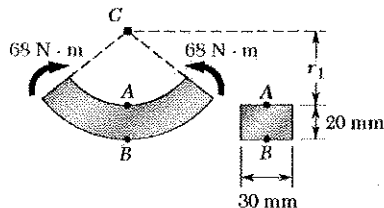
(b) $M_x = Pz_E$, $M_z = -Px_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} = -\frac{P}{A} - \frac{Px_E x_A}{Ak_z^2} - \frac{Pz_E z_A}{Ak_x^2}$$

$$= 0 \text{ by equation from Part (a).}$$

Problem 4.156

4.156 For the curved bar and loading shown, determine the stress point A when (a) $r_1 = 30$ mm, (b) $r_1 = 50$ mm.



$$h = 20 \text{ mm} \quad A = (30)(20) = 600 \text{ mm}^2$$

$$(a) \quad r_1 = 30 \text{ mm} \quad r_2 = 50 \text{ mm}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{5}{3}} = 39.15 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40 \text{ mm}$$

$$e = \bar{r} - R = 0.85 \text{ mm}$$

$$y_A = 39.15 - 30 = 9.15 \text{ mm}$$

$$r_A = r_1 = 30 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(68)(0.00915)}{(600 \times 10^6)(0.0085)(0.03)} = -4.07 \text{ MPa}$$

$$-4.07 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad r_1 = 50 \text{ mm} \quad r_2 = 70 \text{ mm}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{7}{5}} = 59.44 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 60 \text{ mm}$$

$$e = \bar{r} - R = 0.56 \text{ mm}$$

$$y_A = 59.44 - 50 = 9.44 \text{ mm}$$

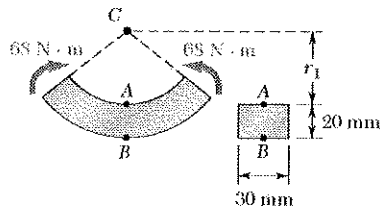
$$r_A = r_1 = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(68)(0.00944)}{(600 \times 10^6)(0.0056)(0.05)} = -38.2 \text{ MPa}$$

$$-38.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.157

4.157 For the curved bar and loading shown, determine the stress points A and B when $r_1 = 40$ mm.



$$h = 20 \text{ mm} \quad r_1 = 40 \text{ mm} \quad r_2 = 60 \text{ mm}$$

$$A = (30)(20) = 600 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{60}{40}} = 49.3261 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 50 \text{ mm}$$

$$e = \bar{r} - R = 0.6739 \text{ mm}$$

$$y_A = 49.3261 - 40 = 9.3261 \text{ mm} \quad r_A = 40 \text{ mm}$$

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(68)(0.0093261)}{(600 \times 10^{-6})(0.0006739)(0.04)} = -39.2 \text{ MPa}$$

$$-39.2 \text{ MPa} \leftarrow$$

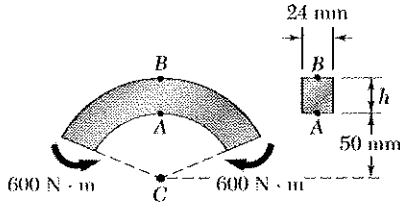
$$y_B = 49.3261 - 60 = -10.6739 \quad r_B = 60 \text{ mm}$$

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(68)(-10.6739)10^{-3}}{(600 \times 10^{-6})(0.0006739)(0.06)} = 29.9 \text{ MPa}$$

$$29.9 \text{ MPa} \leftarrow$$

Problem 4.158

4.158 For the curved bar and loading shown, determine the stress at points A and B when $h = 55$ mm.



$$h = 55 \text{ mm}, \quad r_1 = 50 \text{ mm}, \quad r_2 = 105 \text{ mm},$$

$$A = (24)(55) = 1.320 \times 10^3 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{55}{\ln \frac{105}{50}} = 74.13025 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 77.5 \text{ mm}$$

$$e = \bar{r} - R = 3.36975 \text{ mm}$$

$$y_A = 74.13025 - 50 = 24.13025 \text{ mm} \quad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(600)(24.13025 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(50 \times 10^{-3})} = -65.1 \times 10^6 \text{ Pa}$$

$$\sigma_A = -65.1 \text{ MPa} \quad \blacktriangleleft$$

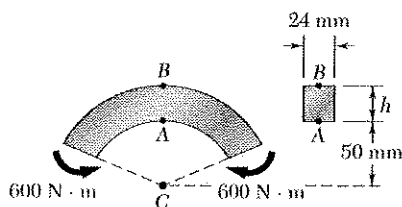
$$y_B = 74.13025 - 105 = -30.86975 \text{ mm} \quad r_B = 105 \text{ mm}$$

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(600)(-30.8697 \times 10^{-3})}{(1.320 \times 10^{-3})(3.36975 \times 10^{-3})(105 \times 10^{-3})} = 39.7 \times 10^6 \text{ Pa}$$

$$\sigma_B = 39.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.159

4.159 For the curved bar and loading shown, determine the stress at point A when (a) $h = 50 \text{ mm}$, (b) $h = 60 \text{ mm}$.



$$(a) \quad h = 50 \text{ mm}, \quad r_1 = 50 \text{ mm}, \quad r_2 = 100 \text{ mm}$$

$$A = (24)(50) = 1.200 \times 10^3 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{50}{\ln \frac{100}{50}} = 72.13475 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 75 \text{ mm}$$

$$e = \bar{r} - R = 2.8652 \text{ mm}$$

$$y_A = 72.13475 - 50 = 22.13475 \text{ mm} \quad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(22.13475 \times 10^{-3})}{(1.200 \times 10^{-3})(2.8652 \times 10^{-3})(50 \times 10^{-3})} = -77.3 \times 10^6 \text{ Pa}$$

$$\sigma_A = -77.3 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad h = 60 \text{ mm}, \quad r_1 = 50 \text{ mm}, \quad r_2 = 110 \text{ mm}, \quad A = (24)(60) = 1.440 \times 10^3 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{60}{\ln \frac{110}{50}} = 76.09796 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 80 \text{ mm} \quad e = \bar{r} - R = 3.90204 \text{ mm}$$

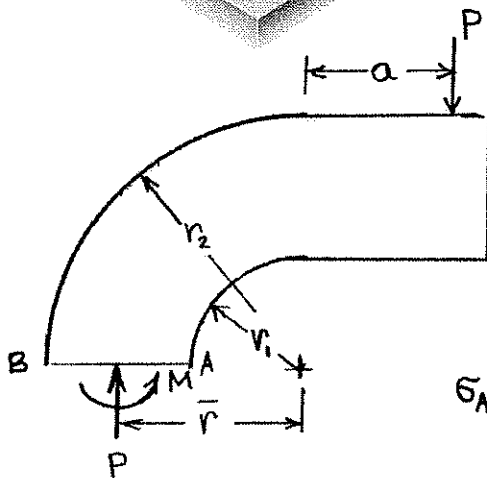
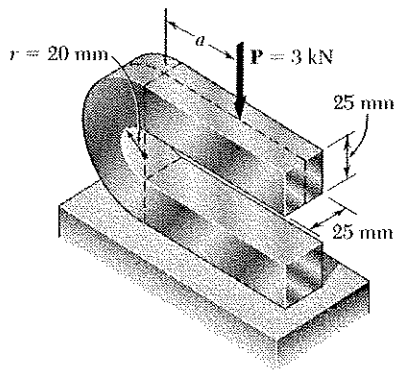
$$y_A = 76.09796 - 50 = 26.09796 \text{ mm} \quad r_A = 50 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(600)(26.09796 \times 10^{-3})}{(1.440 \times 10^{-3})(3.90204 \times 10^{-3})(50 \times 10^{-3})} = -55.7 \times 10^6 \text{ Pa}$$

$$\sigma_A = -55.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.160

4.160 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.



Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \quad \text{Also } e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\begin{aligned} \sigma_A &= -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{P}{A} - \frac{P(a + \bar{r}) y_A}{A e r_1} \\ &= -K \frac{P}{A} \quad \text{with } y_A = R - r_1 \end{aligned}$$

$$\text{Thus, } K = 1 + \frac{(a + \bar{r})(R - r_1)}{e r_1}$$

$$\text{Data: } h = 25 \text{ mm, } r_1 = 20 \text{ mm, } r_2 = 45 \text{ mm, } \bar{r} = 32.5 \text{ mm}$$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm, } e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm, } A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$$

$$R - r_1 = 10.8288 \text{ mm}$$

$$P = 3 \times 10^3 \text{ N} \quad \sigma_A = -150 \times 10^6 \text{ Pa}$$

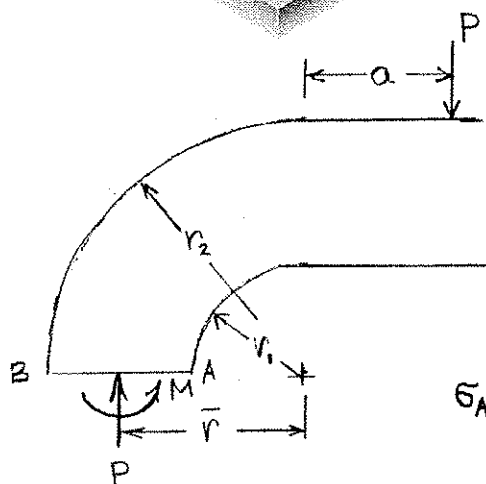
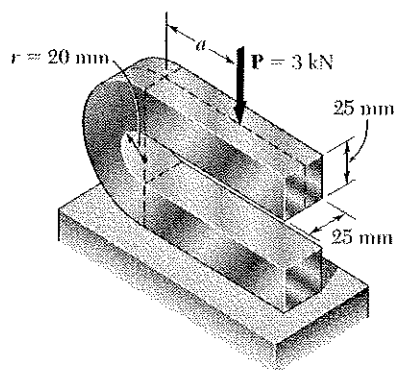
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)e r_1}{R - r_1} = \frac{(31.25-1)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$$a = 93.37 - 32.5$$

$$a = 60.9 \text{ mm} \quad \blacktriangleleft$$

Problem 4.161



4.161 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \quad \text{Also } e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\begin{aligned} \sigma_A &= -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{P}{A} - \frac{P(a + \bar{r}) y_A}{A e r_1} \\ &= -K \frac{P}{A} \quad \text{with } y_A = R - r_1 \end{aligned}$$

$$\text{Thus, } K = 1 + \frac{(a + \bar{r})(R - r_1)}{e r_1}$$

$$\text{Data: } h = 25 \text{ mm}, \quad r_1 = 20 \text{ mm}, \quad r_2 = 45 \text{ mm}, \quad \bar{r} = 32.5 \text{ mm}$$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$$

$$a = 60 \text{ mm}, \quad a + \bar{r} = 92.5 \text{ mm}, \quad R - r_1 = 10.8288 \text{ mm}$$

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968$$

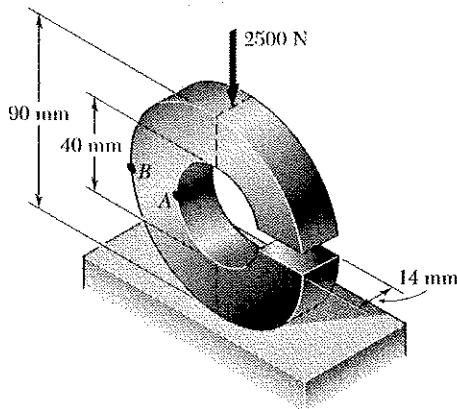
$$P = 3 \times 10^3 \text{ N}$$

$$\sigma_A = -\frac{KP}{A} = -\frac{(30.968)(3 \times 10^3)}{625 \times 10^{-6}} = -148.6 \times 10^6 \text{ Pa}$$

$$\sigma_A = -148.6 \text{ MPa}$$

Problem 4.162

4.162 For the split ring shown, determine the stress at (a) point A, (b) point B.



$$r_1 = \frac{1}{2}40 = 20 \text{ mm}, \quad r_2 = \frac{1}{2}(90) = 45 \text{ mm}$$

$$h = r_2 - r_1 = 25 \text{ mm}$$

$$A = (14)(25) = 350 \text{ mm}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}$$

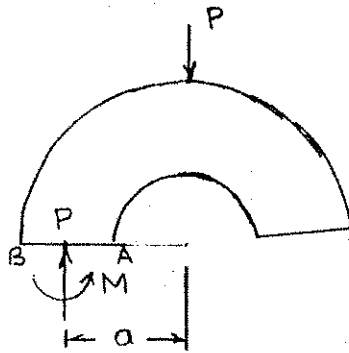
$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 32.5 \text{ mm}$$

$$e = \bar{r} - R = 1.6712 \text{ mm}$$

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the cross section. The bending couple is

$$M = Pa = P\bar{r}$$

$$= (2500)(32.5 \times 10^{-3}) = 81.25 \text{ N}\cdot\text{m}$$



(a) Point A: $r_A = 20 \text{ mm}$

$$y_A = 30.8288 - 20 = 10.8288 \text{ mm}$$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{AeR} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(10.8288 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -82.4 \times 10^6 \text{ Pa}$$

$$\sigma_A = -82.4 \text{ MPa} \quad \blacktriangleleft$$

(b) Point B: $r_B = 45 \text{ mm}$

$$y_B = 30.8288 - 45 = -14.1712 \text{ mm}$$

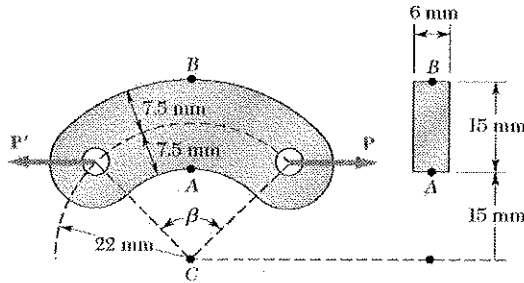
$$\sigma_B = -\frac{P}{A} - \frac{My_B}{AeR} = -\frac{2500}{350 \times 10^{-6}} - \frac{(81.25)(-14.1712 \times 10^{-3})}{(350 \times 10^{-6})(1.6712 \times 10^{-3})(45 \times 10^{-3})}$$

$$= 36.6 \times 10^6 \text{ Pa}$$

$$\sigma_B = 36.6 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.163

4.163 Steel links having the cross section shown are available with different central angles β . Knowing that the allowable stress is 100 MPa, determine the largest force P that can be applied to a link for which $\beta = 90^\circ$.



Reduce section force to a force-couple system at G , the centroid of the cross section AB .

$$a = \bar{r} \left(1 - \cos \frac{\beta}{2} \right)$$

The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}$$

$$\text{Also } e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aen_1} = \frac{P}{A} + \frac{Pa y_A}{Aen_1} = \frac{P}{A} \left(1 + \frac{a y_A}{en_1} \right) = K \frac{P}{A}$$

$$\text{where } K = 1 + \frac{a y_A}{en_1} \quad \text{and} \quad y_A = R - r_1$$

$$P = \frac{A \sigma_A}{K}$$

Data: $\bar{r} = 22 \text{ mm}$, $r_1 = 15 \text{ mm}$, $r_2 = 30 \text{ mm}$, $h = 15 \text{ mm}$, $b = 6 \text{ mm}$.

$$A = (15)(6) = 90 \text{ mm}^2, \quad R = \frac{15}{\ln \frac{30}{15}} = 21.64 \text{ mm}$$

$$e = 22 - 21.64 = 0.36 \text{ mm}, \quad y_A = 21.64 - 15 = 6.64 \text{ mm}$$

$$a = 22 (1 - \cos 45^\circ) = 6.44 \text{ mm}$$

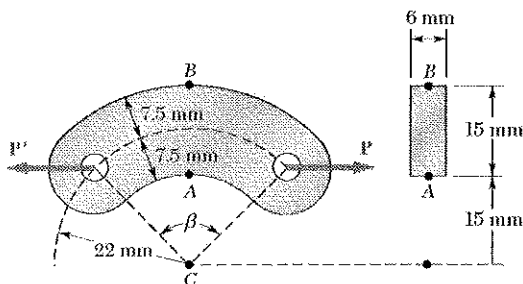
$$K = 1 + \frac{(6.44)(6.64)}{(0.36)(15)} = 8.92$$

$$P = \frac{(90 \times 10^{-6})(100 \times 10^6)}{8.92} = 1009 \text{ N} = 1 \text{ kN}$$

Problem 4.164

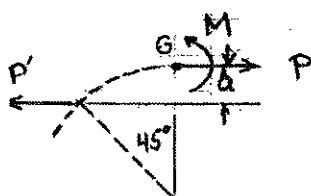
4.164 Solve Prob. 4.163, assuming that $\beta = 60^\circ$.

4.163 Steel links having the cross section shown are available with different central angles β . Knowing that the allowable stress is 100 MPa, determine the largest force $\mathbf{P}$ that can be applied to a link for which $\beta = 90^\circ$.



Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r} (1 - \cos \frac{\beta}{2})$$



The bending couple is: $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \quad \text{Also } e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} + \frac{M y_A}{A e r_1} = \frac{P}{A} + \frac{P a y_A}{A e r_1} = \frac{P}{A} \left(1 + \frac{a y_A}{e r_1} \right) = K \frac{P}{A}$$

where $K = 1 + \frac{\Delta y_A}{e r_i}$ and $y_A = R - r_i$

$$P = \frac{AG_A}{K}$$

Data: $\bar{r} = 22\text{ mm}$, $r_1 = 15\text{ mm}$, $r_2 = 30\text{ mm}$, $h = 15\text{ mm}$, $b = 6\text{ mm}$.

$$A = (6)(15) = 900 \text{ mm}^2, \quad R = \frac{15}{\ln \frac{30}{15}} = 21.64 \text{ mm}$$

$$e = 22 - 21.64 = 0.36 \text{ mm} \quad y_A = 21.64 - 15 = 6.64 \text{ mm}$$

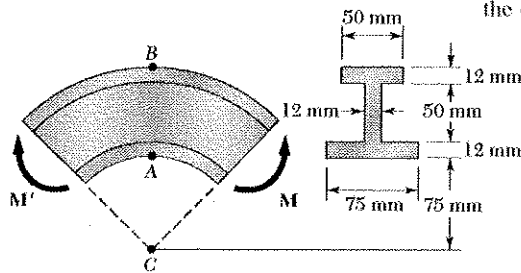
$$a = 22(1 - \cos 30^\circ) = 2.95 \text{ mm.}$$

$$K = 1 + \frac{(2.95)(6.64)}{(0.36)(15)} = 4.627$$

$$P = \frac{(900 \times 10^6)(100 \times 10^6)}{4.627} = 1945 \text{ N} = 1.945 \text{ kN}.$$

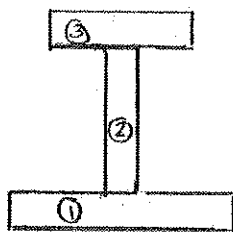
Problem 4.165

4.165 Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.



$$R = \frac{\sum A}{\sum \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A}{\sum b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\sum A \bar{r}_i}{\sum A}$$



r	part	b	h	A	$b \ln \frac{r}{r_i}$	$\bar{r}$	$A \bar{r}$
75	①	75	12	900	11.1315	81	72900
87	②	12	50	600	5.4489	112	67200
137	③	50	12	600	4.1983	143	85800
149	Σ			2100	20.7787		225900

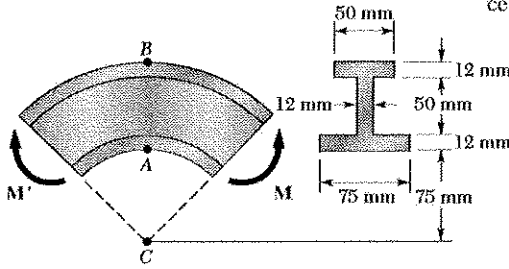
$$R = \frac{2100}{20.7787} = 101.07 \text{ mm}, \quad \bar{r} = \frac{225900}{2100} = 107.57$$

$$e = \bar{r} - R = 6.5 \text{ mm}$$

6.5 mm

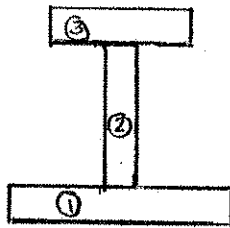
Problem 4.166

4.166 Three plates are welded together to form the curved beam shown. For $M = 900 \text{ N} \cdot \text{m}$, determine the stress at (a) point A, (b) point B, (c) the centroid of the cross section.



$$R = \frac{\sum A}{\sum \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A}{\sum b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\sum A \bar{r}_i}{\sum A}$$



r	part	b	h	A	$b \ln \frac{r_2}{r_1}$	$\bar{r}$	$A \bar{r}$
75	①	75	12	900	11.1315	81	72900
87	②	12	50	600	5.4489	112	67200
137	③	50	12	600	4.1983	143	85800
149	Σ			2100	20.7787		225900

$$R = \frac{2100}{20.7787} = 101.07 \text{ mm}, \quad \bar{r} = \frac{225900}{2100} = 107.57 \text{ mm}$$

$$e = \bar{r} - R = 6.5 \text{ mm}$$

$$M = -900 \text{ N} \cdot \text{m}$$

$$(a) \quad y_A = R - r_1 = 101.07 - 75 = 26.07 \text{ mm}$$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-900)(0.02607)}{(2100 \times 10^{-6})(0.0065)(0.075)} = 22.9 \text{ MPa}$$

$$(b) \quad y_B = R - r_2 = 101.07 - 149 = -47.93$$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(-900)(-0.04793)}{(2100 \times 10^{-6})(0.0065)(0.149)} = -21.2 \text{ MPa}$$

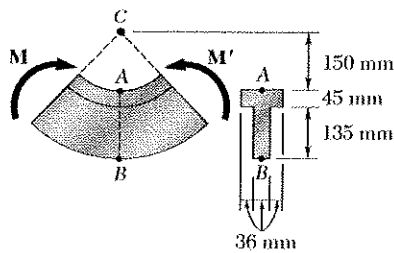
$$(c) \quad y_C = R - \bar{r} = -e =$$

$$\sigma_C = -\frac{M y_C}{A e \bar{r}} = -\frac{M e}{A \bar{r}} = -\frac{M}{A \bar{r}} = -\frac{900}{(2100 \times 10^{-6})(0.10757)}$$

$$= 4 \text{ MPa}$$

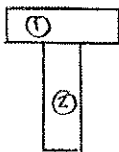
Problem 4.167

4.167 and 4.168 Knowing that $M = 20 \text{ kN} \cdot \text{m}$, determine the stress at (a) point A, (b) point B.



$$R = \frac{\sum A_i}{\sum \frac{1}{r_i}} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\sum A_i \bar{r}_i}{\sum A_i}$$



r, mm	Part	b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}, \text{mm}$	$\bar{r}_i, \text{mm}$	$A \bar{r}_i, \text{mm}^3$
150	①	108	45	4860	28.3353	172.5	838.35×10^3
195	②	36	135	4860	18.9394	262.5	1275.75×10^3
330	Σ			9720	47.2747		2114.1×10^3

$$R = \frac{9720}{47.2747} = 205.606 \text{ mm} \quad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5 \text{ mm}$$

$$e = \bar{r} - R = 11.894 \text{ mm} \quad M = 20 \times 10^3 \text{ N} \cdot \text{m}$$

(a) $y_A = R - r_1 = 205.606 - 150 = 55.606 \text{ mm}$

$$\sigma_A = - \frac{M y_A}{A e r_1} = - \frac{(20 \times 10^3)(55.606 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -64.1 \times 10^6 \text{ Pa} \quad \sigma_A = -64.1 \text{ MPa} \quad \blacktriangleleft$$

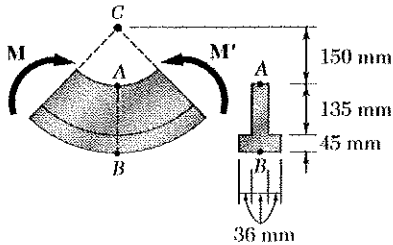
(b) $y_B = R - r_2 = 205.606 - 330 = -124.394 \text{ mm}$

$$\sigma_B = - \frac{M y_B}{A e r_2} = - \frac{(20 \times 10^3)(-124.394 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 65.2 \times 10^6 \text{ Pa} \quad \sigma_B = 65.2 \text{ MPa} \quad \blacktriangleleft$$

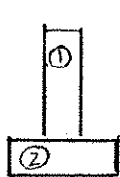
Problem 4.168

4.167 and 4.168 Knowing that $M = 20 \text{ kN} \cdot \text{m}$, determine the stress at (a) point A, (b) point B.



$$R = \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\sum A_i \bar{r}_i}{\sum A_i}$$



r, mm	b, mm	h, mm	A, mm^2	$b \ln \frac{r_{i+1}}{r_i}, \text{mm}$	$\bar{r}, \text{mm}$	$A \bar{r}, \text{mm}^3$
150	36	135	4860	23.1067	217.5	1.05705×10^6
285	108	45	4860	15.8332	307.5	1.49445×10^6
330						
Σ			9720	38.9399		2.5515×10^6

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \quad \bar{r} = \frac{2.5515 \times 10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \quad M = 20 \times 10^3 \text{ N} \cdot \text{m}$$

$$(a) \quad y_A = R - r_1 = 249.615 - 150 = 99.615 \text{ mm}$$

$$\sigma_A = - \frac{M y_A}{A e \bar{r}} = - \frac{(20 \times 10^3)(99.615 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -106.1 \times 10^6 \text{ Pa}$$

$$\sigma_A = -106.1 \text{ MPa} \leftarrow$$

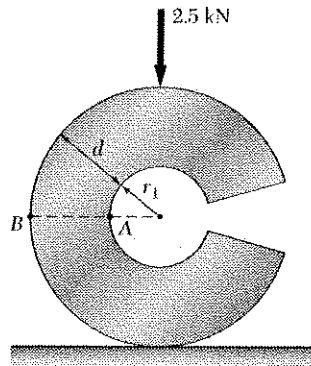
$$(b) \quad y_B = R - r_2 = 249.615 - 330 = -80.385 \text{ mm}$$

$$\sigma_B = - \frac{M y_B}{A e \bar{r}} = - \frac{(20 \times 10^3)(-80.385 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 38.9 \times 10^6 \text{ Pa}$$

$$\sigma_B = 38.9 \text{ MPa} \leftarrow$$

Problem 4.169



4.169 The split ring shown has an inner radius $r_1 = 20$ mm and a circular cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (a) point A, (b) point B.

$$c = \frac{1}{2}d = 16 \text{ mm} \quad r_1 = 20 \text{ mm}, \quad r_2 = r_1 + d = 52 \text{ mm}$$

$$\bar{r} = r_1 + c = 36 \text{ mm}$$

$$R = \frac{1}{2} \left[\bar{r} + \sqrt{\bar{r}^2 - c^2} \right] = \frac{1}{2} \left[36 + \sqrt{36^2 - 16^2} \right]$$

$$= 34.1245 \text{ mm}$$

$$e = \bar{r} - R = 1.8755 \text{ mm}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$P = 2.5 \times 10^3 \text{ N}$$

$$M = P \bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90 \text{ N}\cdot\text{m}$$

(a) Point A: $y_A = R - r_1 = 34.1245 - 20 = 14.1245 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -45.2 \times 10^6 \text{ Pa} \quad \sigma_A = -45.2 \text{ MPa} \quad \leftarrow$$

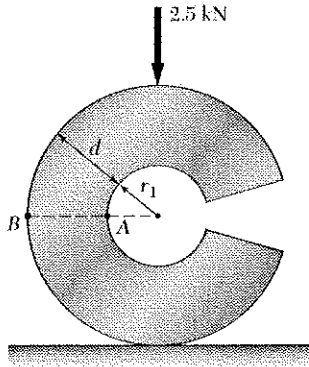
(b) Point B: $y_B = R - r_2 = 34.1245 - 52 = -17.8755 \text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})}$$

$$= 17.40 \times 10^6 \text{ Pa} \quad \sigma_B = 17.40 \text{ MPa} \quad \leftarrow$$

Problem 4.170

4.170 The split ring shown has an inner radius $r_1 = 16$ mm and a circular cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (a) point A, (b) point B.



$$c = \frac{1}{2}d = 16 \text{ mm}, \quad r_1 = 16 \text{ mm}, \quad r_2 = r_1 + d = 48 \text{ mm}$$

$$\bar{r} = r_1 + c = 32 \text{ mm}$$

$$R = \frac{1}{2} \left[\bar{r} + \sqrt{\bar{r}^2 - c^2} \right] = \frac{1}{2} \left[32 + \sqrt{32^2 - 16^2} \right]$$

$$= 29.8564 \text{ mm}$$

$$e = \bar{r} - R = 2.1436 \text{ mm}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$P = 2.5 \times 10^3 \text{ N}$$

$$M = P \bar{r} = (2.5 \times 10^3)(32 \times 10^{-3}) = 80 \text{ N}\cdot\text{m}$$

(a) Point A: $y_A = R - r_1 = 29.8564 - 16 = 13.8564 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(13.8564 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(16 \times 10^{-3})}$$

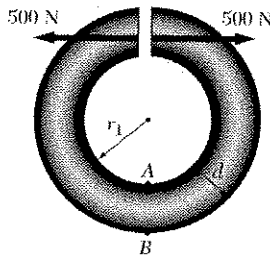
$$= -43.3 \times 10^6 \text{ Pa} \quad \sigma_A = -43.3 \text{ MPa} \quad \leftarrow$$

(b) Point B: $y_B = R - r_2 = 29.8564 - 48 = -18.1436 \text{ mm}$

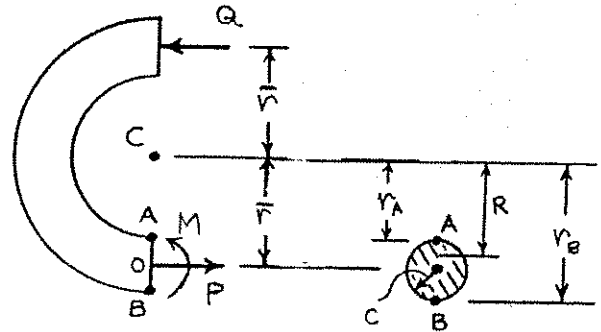
$$\sigma_B = -\frac{P}{A} - \frac{M y_B}{A e r_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(-18.1436 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(48 \times 10^{-3})}$$

$$= 14.43 \times 10^6 \text{ Pa} \quad \sigma_B = 14.43 \text{ MPa} \quad \leftarrow$$

Problem 4.171



4.171 The split ring shown has an inner radius $r_1 = 20$ mm and a circular cross section of diameter $d = 15$ mm. Knowing that each of the 500-N forces is applied at the centroid of the cross section, determine the stress at (a) point A, (b) point B.



$$r_A = r_1 = 20 \text{ mm}$$

$$r_B = r_A + d = 20 + 15 = 35 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_A + r_B) = 27.5 \text{ mm}$$

$$c = \frac{1}{2}d = 7.5 \text{ mm}$$

$$A = \pi c^2 = \pi (7.5)^2 \approx 176.7 \text{ mm}^2 \quad \text{for solid circular section}$$

$$R = \frac{1}{2} \left[\bar{r} + \sqrt{\bar{r}^2 - c^2} \right] = \frac{1}{2} \left[27.5 + \sqrt{(27.5)^2 - (7.5)^2} \right] = 27 \text{ mm}$$

$$e = \bar{r} - R = 27.5 - 27 = 0.5 \text{ mm}$$

$$Q = 500 \text{ N}$$

$$\sum F_x = 0: P - Q = 0$$

$$P = 500 \text{ N}$$

$$\sum M_O = 0: 2\bar{r}Q + M = 0$$

$$M = -2\bar{r}Q = -(2)(27.5)(500) = -27.5 \text{ Nm}$$

(a) $r = r_A = 20 \text{ mm}$

$$\begin{aligned} \sigma_A &= \frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = \frac{500}{176.7 \times 10^{-6}} + \frac{(-27.5)(0.02 - 0.027)}{(176.7 \times 10^{-6})(0.0005)(0.02)} \\ &= 111.77 \times 10^6 \text{ Pa} \end{aligned}$$

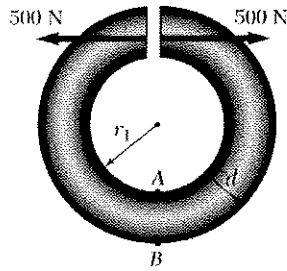
$$\sigma_A = 111.8 \text{ MPa} \quad \blacktriangleleft$$

(b) $r = r_B = 35 \text{ mm}$

$$\begin{aligned} \sigma_B &= \frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = \frac{500}{176.7 \times 10^{-6}} + \frac{(-27.5)(0.035 - 0.027)}{(176.7 \times 10^{-6})(0.0005)(0.035)} \\ &= -68.32 \times 10^6 \end{aligned}$$

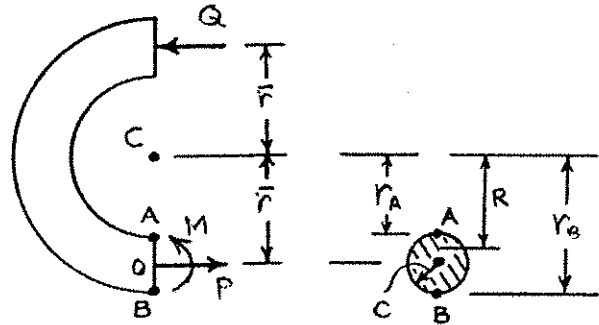
$$\sigma_B = -68.3 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.172



4.172 Solve Prob. 4.171, assuming that the ring has an inner radius $r_1 = 15 \text{ mm}$ and a cross-sectional diameter $d = 20 \text{ mm}$.

4.171 The split ring shown has an inner radius $r_1 = 20 \text{ mm}$ and a circular cross section of diameter $d = 15 \text{ mm}$. Knowing that each of the 500-N forces is applied at the centroid of the cross section, determine the stress at (a) point A, (b) point B.



$$r_A = r_1 = 15 \text{ mm}$$

$$r_B = r_A + d = 15 + 20 = 35 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_A + r_B) = 25 \text{ mm}$$

$$c = \frac{1}{2}d = 10 \text{ mm}$$

$$A = \pi c^2 = \pi (10)^2 = 314.16 \text{ mm}^2 \quad \text{for solid circular section.}$$

$$R = \frac{1}{2}[\bar{r} + \sqrt{\bar{r}^2 - c^2}] = \frac{1}{2}[25 + \sqrt{(25)^2 - (10)^2}] = 24 \text{ mm}$$

$$e = \bar{r} - R = 25 - 24 = 1 \text{ mm}$$

$$Q = 500 \text{ N}$$

$$\pm \sum F_x = 0$$

$$P - Q = 0$$

$$P = 500 \text{ N}$$

$$+\circlearrowleft \sum M_O = 0: 2\bar{r}Q + M = 0 \quad M = -2\bar{r}Q = -(2)(0.001)(500) = -1 \text{ Nm}$$

$$(a) \quad r = r_A = 15 \text{ mm}$$

$$\sigma_A = \frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = \frac{500}{314.16 \times 10^{-6}} + \frac{(-1)(0.015 - 0.024)}{(314.16 \times 10^{-6})(0.001)(0.015)}$$

$$= 3.501 \times 10^6 \text{ Pa}$$

$$\sigma_A = 3.5 \text{ MPa} \quad \leftarrow$$

$$(b) \quad r = r_B = 35 \text{ mm}$$

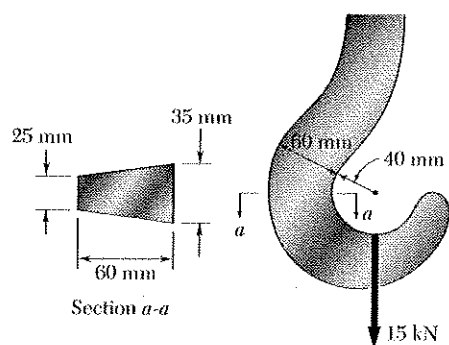
$$\sigma_B = \frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = \frac{500}{314.16 \times 10^{-6}} + \frac{(-1)(0.035 - 0.024)}{(314.16 \times 10^{-6})(0.001)(0.035)}$$

$$= -0.59 \times 10^6$$

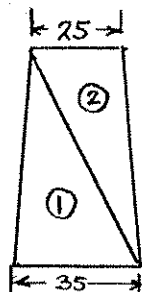
$$\sigma_B = -0.6 \text{ MPa} \quad \leftarrow$$

Problem 4.173

4.173 For the crane hook shown, determine the largest tensile stress in section a-a.



Locate centroid.



	A, mm^2	$\bar{r}, \text{mm}$	$A\bar{r}, \text{mm}^3$
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103 \times 10^3}{1800} = 68.333 \text{ mm.}$$

Force-couple system at centroid: $P = 15 \times 10^3 \text{ N}$

$$M = -P\bar{r} = -(15 \times 10^3)(68.333 \times 10^{-3}) = -1.025 \times 10^3 \text{ N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(60)^2 (35 + 25)}{[(35)(100) - (25)(40)] \ln \frac{100}{40} - (60)(35 - 25)} = 63.878 \text{ mm.}$$

$$e = \bar{r} - R = 4.452 \text{ mm.}$$

Maximum tensile stress occurs at point A.

$$y_A = R - r_1 = 23.878 \text{ mm.}$$

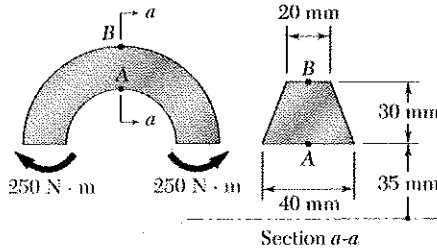
$$\sigma_A = \frac{P}{A} - \frac{My_A}{Ae\bar{r}} = \frac{15 \times 10^3}{1800 \times 10^{-6}} - \frac{-(1.025 \times 10^3)(23.878 \times 10^{-3})}{(1800 \times 10^{-6})(4.452 \times 10^{-3})(40 \times 10^{-3})}$$

$$= 84.7 \times 10^6 \text{ Pa}$$

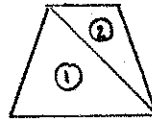
$$\sigma_A = 84.7 \text{ MPa} \leftarrow$$

Problem 4.174

4.174 For the curved beam and loading shown, determine the stress at (a) point A, (b) point B.



locate centroid.



	A, mm^2	$\bar{r}, \text{mm}$	$A\bar{r}, \text{mm}^3$
①	600	45	27×10^3
②	300	55	16.5×10^3
Σ	900		43.5×10^3

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40 + 20)}{[(40)(65) - (20)(35)]\ln\frac{65}{35} - (30)(40 - 20)}$$

$$\bar{r} = \frac{43.5 \times 10^3}{900} = 48.333 \text{ mm}$$

$$= 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm}$$

$$M = -250 \text{ N}\cdot\text{m}$$

$$(a) \quad y_A = R - r_1 = 11.8608 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa}$$

$$\sigma_A = 63.9 \text{ MPa} \rightarrow$$

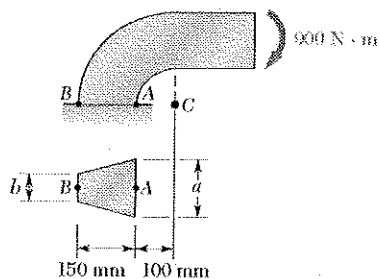
$$(b) \quad y_B = R - r_2 = -18.1392 \text{ mm}$$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa}$$

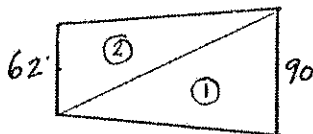
$$\sigma_B = -52.6 \text{ MPa} \rightarrow$$

Problem 4.175

4.175 Knowing that the machine component shown has a trapezoidal cross section with $a = 90$ mm and $b = 62$ mm, determine the stress at (a) point A, (b) point B.



Locate centroid



	A, mm^2	$\bar{r}, \text{mm}$	$A\bar{r}, \text{mm}^3$
①	6750	150	1.0125×10^6
②	4650	200	0.93×10^6
Σ	11400		1942500

$$\bar{r} = \frac{1942500}{11400} = 170.4 \text{ mm}$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h (b_1 - b_2)}$$

$$= \frac{(0.5)(150)^2(90 + 62)}{[(90)(250) - (62)(100)] \ln \frac{250}{100} - (150)(90 - 62)} = 159.3 \text{ mm}$$

$$e = \bar{r} - R = 11.1$$

$$M = 9 \text{ kN}$$

$$(a) \quad y_A = R - r_1 = 159.3 - 100 = 59.3 \text{ mm}$$

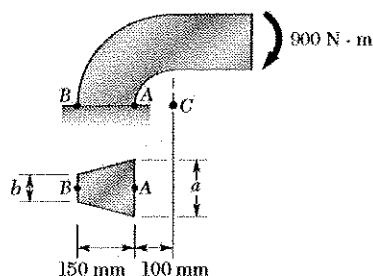
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(9000)(0.0593)}{(11400 \times 10^{-6})(0.0111)(0.1)} = -42.2 \text{ MPa}$$

$$(b) \quad y_B = R - r_2 = 159.3 - 250 = -90.7 \text{ mm}$$

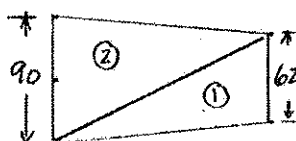
$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(9000)(-0.0907)}{(11400 \times 10^{-6})(0.0111)(0.25)} = 25.8 \text{ MPa}$$

Problem 4.176

4.176 Knowing that the machine component shown has a trapezoidal cross section with $a = 62 \text{ mm}$ and $b = 90 \text{ mm}$, determine the stress at (a) point A, (b) point B.



Locate centroid



	A, in^2	$\bar{r}, \text{in.}$	$A\bar{r}, \text{in}^3$
①	4650	150	697500
②	6750	200	1.35×10^6
Σ	11400		2047500

$$\bar{r} = \frac{2047500}{11400} = 179.6 \text{ mm}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1)\ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(150)^2(62 + 90)}{[(62)(250) - (90)(100)]\ln \frac{250}{100} - (150)(62 - 90)} = 168.4 \text{ mm}$$

$$e = \bar{r} - R = 11.2 \text{ mm}$$

$$M = 9 \text{ kN}$$

$$(a) \quad y_A = R - r_1 = 68.4 \text{ mm}$$

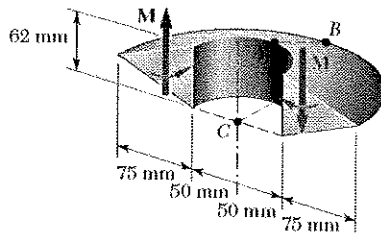
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(9000)(0.0684)}{(11400 \times 10^{-6})(0.0112)(0.1)} = -48.2 \text{ MPa}$$

$$(b) \quad y_B = R - r_2 = -81.6 \text{ mm}$$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(9000)(-0.0816)}{(11400 \times 10^{-6})(0.0112)(0.25)} = 23 \text{ MPa}$$

Problem 4.177

4.177 and 4.178 Knowing that $M = 560 \text{ N} \cdot \text{m}$, determine the stress at (a) point A, (b) point B.



$$A = \frac{1}{2} b h = \frac{1}{2} (62)(75) = 2325 \text{ mm}^2$$

$$\bar{r} = 50 + 25 = 75 \text{ mm}$$

$$b_1 = 62 \text{ mm}, r_1 = 50 \text{ mm}, b_2 = 0, r_2 = 125 \text{ mm}$$

Use formula for trapezoid with $b_2 = 0$.

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(75)^2 (62 + 0)}{[(62)(125) - (0)(50)] \ln \frac{125}{50} - 75(62 - 0)} = 71.14 \text{ mm}$$

$$e = \bar{r} - R = 3.86 \text{ mm}$$

$$M = 560 \text{ Nm}$$

$$(a) \quad y_A = R - r_1 = 21.14 \text{ mm}$$

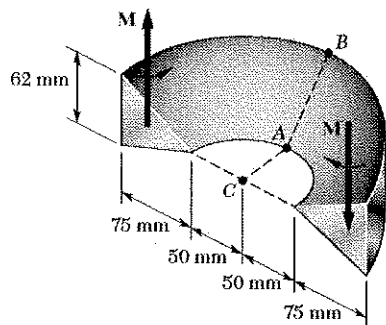
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(560)(0.02114)}{(2325 \times 10^{-6})(0.00386)(0.05)} = -26.4 \text{ MPa}$$

$$(b) \quad y_B = R - r_2 = -53.86 \text{ mm}$$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(560)(-0.05386)}{(2325 \times 10^{-6})(0.00386)(0.125)} = 26.9 \text{ MPa}$$

Problem 4.178

4.177 and 4.178 Knowing that $M = 560 \text{ N} \cdot \text{m}$, determine the stress at (a) point A, (b) point B.



$$A = \frac{1}{2} (62)(75) = 2325 \text{ mm}^2$$

$$\bar{r} = 50 + 50 = 100 \text{ mm}$$

$$b_1 = 0, r_1 = 50 \text{ mm}, b_2 = 62 \text{ mm}, r_2 = 125 \text{ mm}$$

Use formula for trapezoid with $b_1 = 0$.

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h (b_1 - b_2)}$$

$$= \frac{(0.5)(75)^2 (0 + 62)}{[(0)(125) - (62)(50)] \ln \frac{125}{50} - 75(0 - 62)} = 96.4 \text{ mm}$$

$$e = \bar{r} - R = 3.6 \text{ mm}$$

$$M = 560 \text{ N} \cdot \text{m}$$

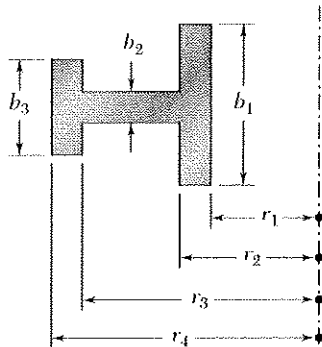
$$(a) y_A = R - r_1 = 46.4 \text{ mm}$$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(560)(0.0464)}{(2325 \times 10^{-6})(0.0036)(0.05)} = -62.1 \text{ MPa}$$

$$(b) y_B = R - r_2 = -28.6 \text{ mm}$$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(560)(-0.0286)}{(2325 \times 10^{-6})(0.0036)(0.125)} = 15.3 \text{ MPa}$$

Problem 4.179



4.179 Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln \left[\left(\frac{r_2}{r_1} \right)^{b_1} \left(\frac{r_3}{r_2} \right)^{b_2} \left(\frac{r_4}{r_3} \right)^{b_3} \right]}$$

where A is the total area of the cross section.

$$\begin{aligned} R &= \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{A}{\sum b_i \ln \frac{r_{i+1}}{r_i}} \\ &= \frac{A}{\sum \ln \left(\frac{r_{i+1}}{r_i} \right)^{b_i}} = \frac{A}{\ln \left[\left(\frac{r_2}{r_1} \right)^{b_1} \left(\frac{r_3}{r_2} \right)^{b_2} \left(\frac{r_4}{r_3} \right)^{b_3} \right]} \end{aligned}$$

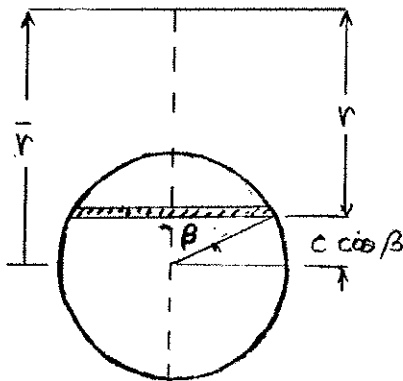
Note that for each rectangle $\int \frac{1}{r} dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$
 $= b_i \int_{r_i}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$

Problem 4.180

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

*4.180 A circular cross section.

Use polar coordinate β as shown.



$$\text{width: } w = 2c \sin \beta$$

$$r = \bar{r} - c \cos \beta$$

$$dr = c \sin \beta d\beta$$

$$dA = w dr = 2c^2 \sin^2 \beta d\beta$$

$$\int \frac{dA}{r} = \int_0^\pi \frac{2c^2 \sin^2 \beta}{\bar{r} - c \cos \beta} d\beta$$

$$\int \frac{dA}{r} = \int_0^\pi \frac{c^2(1 - \cos^2 \beta)}{\bar{r} - c \cos \beta} d\beta = 2 \int_0^\pi \frac{\bar{r}^2 - c^2 \cos^2 \beta - (\bar{r}^2 - c^2)}{\bar{r} - c \cos \beta} d\beta$$

$$= 2 \int_0^\pi (\bar{r} + c \cos \beta) d\beta - 2(\bar{r}^2 - c^2) \int_0^\pi \frac{dr}{\bar{r} - c \cos \beta}$$

$$= 2\bar{r} \beta \Big|_0^\pi + 2c \sin \beta \Big|_0^\pi$$

$$- 2(\bar{r}^2 - c^2) \frac{2}{\sqrt{\bar{r}^2 - c^2}} \tan^{-1} \frac{\sqrt{\bar{r}^2 - c^2} \tan(\frac{1}{2}\beta)}{\bar{r} + c} \Big|_0^\pi$$

$$= 2\bar{r}(\pi - 0) + 2c(0 - 0) - 4\sqrt{\bar{r}^2 - c^2} \cdot (\frac{\pi}{2} - 0)$$

$$2\pi \bar{r} - 2\pi \sqrt{\bar{r}^2 - c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\pi c^2}{2\pi \bar{r} - 2\pi \sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2} \frac{c^2}{\bar{r} - \sqrt{\bar{r}^2 - c^2}} \cdot \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2} \frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2} \frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{c^2}$$

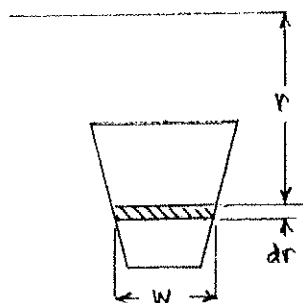
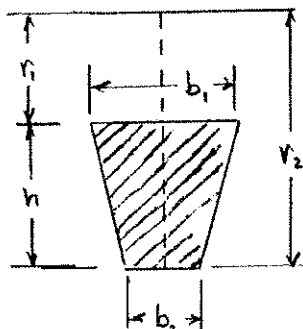
$$= \frac{1}{2} (\bar{r} + \sqrt{\bar{r}^2 - c^2})$$

Problem 4.181

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.181 A trapezoidal cross section.

The section width w varies linearly with r .



$$w = C_0 + C_1 r$$

$$w = b_1 \text{ at } r = r_1 \text{ and } w = b_2 \text{ at } r = r_2$$

$$b_1 = C_0 + C_1 r_1$$

$$b_2 = C_0 + C_1 r_2$$

$$b_1 - b_2 = C_1(r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1) C_0 = h C_0$$

$$C_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$

$$\begin{aligned} \int \frac{dA}{r} &= \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{C_0 + C_1 r}{r} dr \\ &= C_0 \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2} \\ &= C_0 \ln \frac{r_2}{r_1} + C_1 (r_2 - r_1) \\ &= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h \\ &= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2) \end{aligned}$$

$$A = \frac{1}{2}(b_1 + b_2) h$$

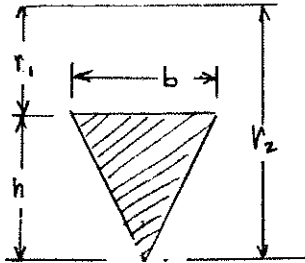
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

Problem 4.182

4.180 through 4.182 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.182 A triangular cross section.

The section width w varies linearly with r .



$$w = C_0 + C_1 r$$

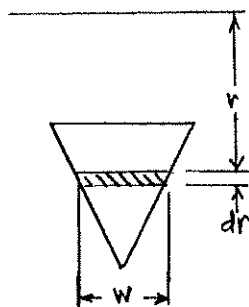
$$w = b \text{ at } r = r_1 \text{ and } w = 0 \text{ at } r = r_2$$

$$b = C_0 + C_1 r_1$$

$$0 = C_0 + C_1 r_2$$

$$b = C_1 (r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b}{h} \text{ and } C_0 = -C_1 r_2 = \frac{b r_2}{h}$$



$$\int \frac{dA}{V} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{C_0 + C_1 r}{r} dr$$

$$= C_0 \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2}$$

$$= C_0 \ln \frac{r_2}{r_1} + C_1 (r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b \left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)$$

$$A = \frac{1}{2} b h$$

$$R = \frac{A}{\int \frac{dA}{V}} = \frac{\frac{1}{2} b h}{b \left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

Problem 4.183

*4.183 For a curved bar of rectangular cross section subjected to a bending couple M , show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae} \left(1 - \frac{r_i}{R} - \ln \frac{R}{r_i} \right)$$

At radial distance r

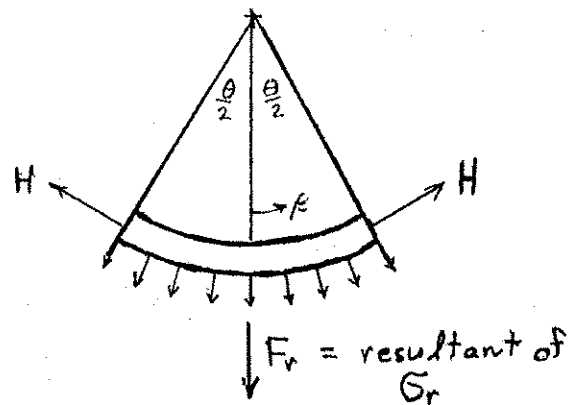
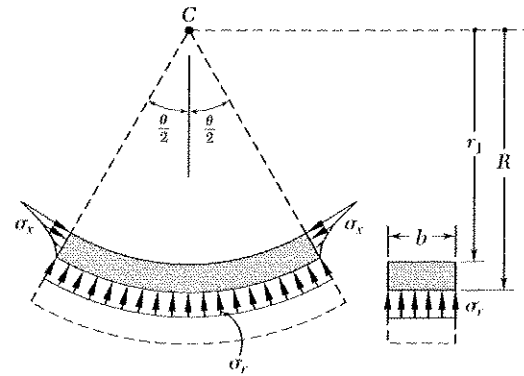
$$\begin{aligned} \sigma_a &= \frac{M(r-R)}{Aer} \\ &= \frac{M}{Ae} - \frac{MR}{Aer} \end{aligned}$$

For portion above the neutral axis, the resultant force is

$$\begin{aligned} H &= \int \sigma_a dA = \int_{r_i}^R \sigma_a b dr \\ &= \frac{Mb}{Ae} \int_{r_i}^R dr - \frac{MRb}{Ae} \int_{r_i}^R \frac{dr}{r} \\ &= \frac{Mb}{Ae} (R - r_i) - \frac{MRb}{Ae} \ln \frac{R}{r_i} = \frac{MbR}{Ae} \left(1 - \frac{r_i}{R} - \ln \frac{R}{r_i} \right) \end{aligned}$$

Resultant of σ_r :

$$\begin{aligned} F_r &= \int \sigma_r \cos \beta dA \\ &= \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \sigma_r \cos \beta b R d\beta \\ &= \sigma_r b R \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \cos \beta d\beta \\ &= \sigma_r b R \sin \beta \Big|_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \\ &= 2 \sigma_r b R \sin \frac{\theta}{2} \end{aligned}$$



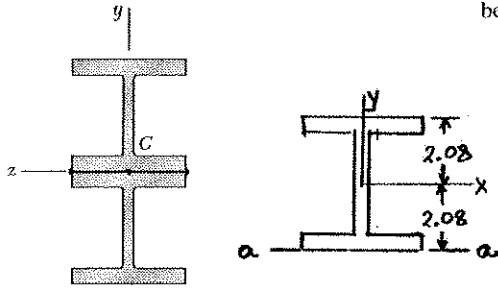
For equilibrium: $F_r - 2H \sin \frac{\theta}{2} = 0$

$$2 \sigma_r b R \sin \frac{\theta}{2} - 2 \frac{MbR}{Ae} \left(1 - \frac{r_i}{R} - \ln \frac{R}{r_i} \right) \sin \frac{\theta}{2} = 0$$

$$\sigma_r = \frac{M}{Ae} \left(1 - \frac{r_i}{R} - \ln \frac{R}{r_i} \right)$$

Problem 4.184

4.184 and 4.185 Two W100 × 19.3 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_y = 250$ MPa and $\sigma_u = 400$ MPa and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



Properties of W100 × 19.3 rolled section
See Appendix B

$$\text{Area} = 2480 \text{ mm}^2 \quad \text{Depth} = 106 \text{ mm.}$$

$$I_x = + 4.77 \times 10^6 \text{ mm}^4$$

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + A d^2 = 4.77 \times 10^6 + (2480)(52)^2 = 11.476 \times 10^6 \text{ mm}^4.$$

For both sections $I_2 = 2 I_a = 22.95 \times 10^6 \text{ mm}^4.$

$$c = \text{depth} = 106 \text{ mm.}$$

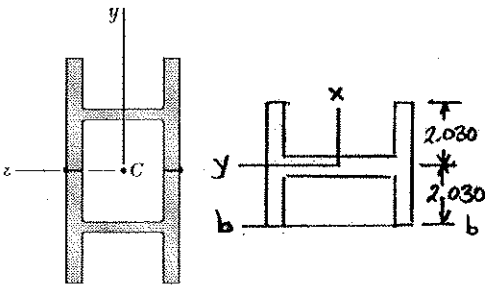
$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{400}{3.0} = 133.33 \text{ MPa}$$

$$\sigma = \frac{M c}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(133.33 \times 10^6)(22.95 \times 10^6)}{0.106} = 28.9 \text{ kNm}$$

Problem 4.185

4.184 and 4.185 Two W100 × 19.3 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_y = 250$ MPa and $\sigma_u = 400$ MPa and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



Properties of W100 × 19.3 rolled section
See Appendix B

$$\text{Area} = 2480 \text{ mm}^2 \quad \text{Width} = 103 \text{ mm}$$

$$I_y = 1.61 \times 10^6 \text{ mm}^4$$

For one rolled section, moment of inertia about axis b-b is

$$I_b = I_y + A d^2 = 1.61 \times 10^6 + (2480)(51.5)^2 = 8187580 \text{ mm}^4$$

For both sections $I_2 = 2 I_b = 16.375 \times 10^6 \text{ mm}^4.$

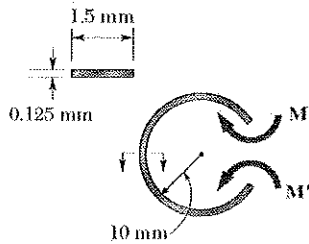
$$c = \text{width} = 103 \text{ mm.}$$

$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{400}{3.0} = 133.33 \text{ MPa}$$

$$\sigma = \frac{M c}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(133.33 \times 10^6)(16.375 \times 10^6)}{0.103} = 21.2 \text{ kNm.}$$

Problem 4.186



4.186 It is observed that a thin steel strip of 1.5 mm width can be bent into a circle of 10-mm diameter without any resulting permanent deformation. Knowing that $E = 200$ GPa, determine (a) the maximum stress in the bent strip, (b) the magnitude of the couples required to bend the strip.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (1.5) (0.125)^3 = 2.44 \times 10^{-4}$$

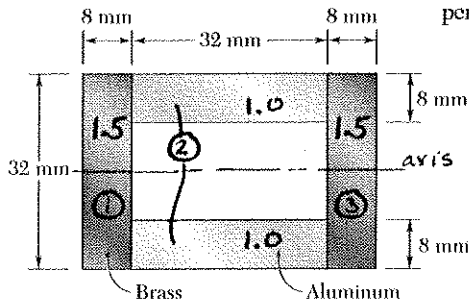
$$\rho = \frac{1}{2} D = \frac{1}{2} (20) = 10 \text{ mm}$$

$$c = \frac{1}{2} h = 0.0625 \text{ mm}$$

$$(a) \quad \sigma_{max} = \frac{E c}{\rho} = \frac{(200 \times 10^9) (0.0625 \times 10^{-3})}{0.01} = 1250 \text{ MPa}$$

$$(b) \quad M = \frac{E I}{\rho} = \frac{(200 \times 10^9) (2.44 \times 10^{-4})}{0.01} = 0.00488 \text{ Nm.}$$

Problem 4.187



4.187 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12} (8) (32)^3 = 32.768 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 (H_2^3 - h_2^3) = \frac{1.0}{12} (32) (32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N}\cdot\text{m}$$

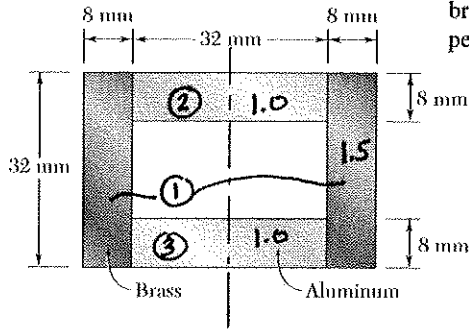
Brass: $n = 1.5$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N}\cdot\text{m}$$

Choose the smaller value.

$$M = 887 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 4.188



4.188 For the composite bar of Prob. 4.187, determine the largest permissible bending moment when the bar is bent about a vertical axis.

4.187 A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n and shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} h_1 (B_1^3 - b_1^3) = \frac{1.5}{12} (32)(48^3 - 32^3) = 311.296 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.0}{12} (8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 354.99 \times 10^3 \text{ mm}^4 = 354.99 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{n M y}{I} \right| \quad M = \left| \frac{\sigma I}{n y} \right|$$

Aluminum: $n = 1.0$, $|y| = 16 \text{ mm} = 0.016 \text{ m}$, $\sigma = 100 \times 10^6 \text{ Pa}$

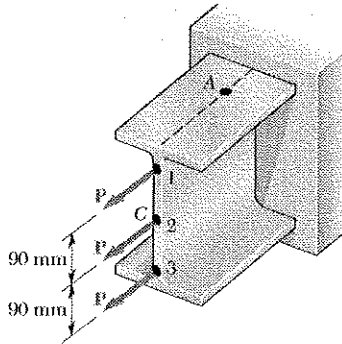
$$M = \frac{(100 \times 10^6)(354.99 \times 10^{-9})}{(1.0)(0.016)} = 2.2187 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $|y| = 24 \text{ mm} = 0.024 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(354.99 \times 10^{-9})}{(1.5)(0.024)} = 1.57773 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value. $M = 1.57773 \times 10^3 \text{ N}\cdot\text{m}$ $M = 1.578 \text{ kN}\cdot\text{m}$ ◀

Problem 4.189



4.189 As many as three axial loads each of magnitude $P = 40 \text{ kN}$ can be applied to the end of a $W200 \times 31.3$ rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 1 and 2 only.

For $W200 \times 31.3$ Appendix C gives

$$A = 4000 \text{ mm}^2 \quad d = 210 \text{ mm}, \quad I_x = 31.4 \times 10^6$$

At point A $y = \frac{1}{2}d = 105 \text{ mm}$.

$$\sigma = \frac{F}{A} - \frac{My}{I}$$

(a) Centric loading: $F = 120 \text{ kN}$, $M = 0$

$$\sigma = \frac{120000}{4000 \times 10^{-6}}$$

$$\sigma = 30 \text{ MPa} \quad \blacktriangleleft$$

(b) Eccentric loading: $F = 2P = 80 \text{ kN}$

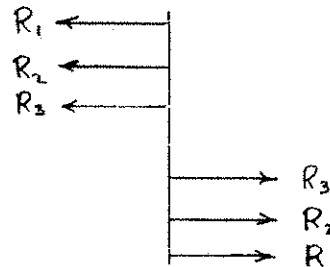
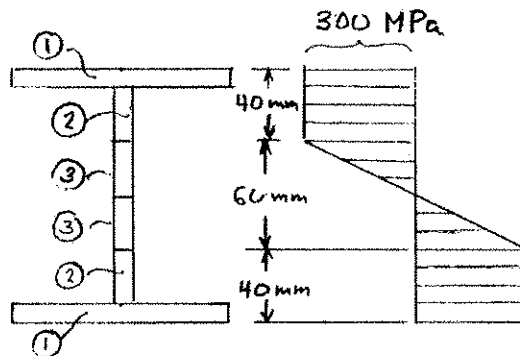
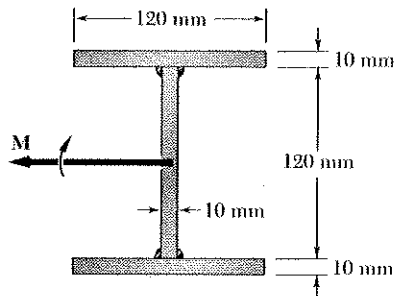
$$M = -(40)(0.09) = 3.6 \text{ kNm}$$

$$\sigma = \frac{80 \times 10^3}{4 \times 10^{-3}} - \frac{(-3.6 \times 10^3)(0.105)}{31.4 \times 10^{-6}}$$

$$\sigma = 32 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.190

4.190 Three 120×10 -mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (a) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (b) the corresponding radius of curvature of the beam.



$$A_1 = (120)(10) = 1200 \text{ mm}^2$$

$$R_1 = \sigma_Y A_1 = (300 \times 10^6)(1200 \times 10^{-6}) = 360 \times 10^3 \text{ N}$$

$$A_2 = (30)(10) = 300 \text{ mm}^2$$

$$R_2 = \sigma_Y A_2 = (300 \times 10^6)(300 \times 10^{-6}) = 90 \times 10^3 \text{ N}$$

$$A_3 = (30)(10) = 300 \text{ mm}^2$$

$$R_3 = \frac{1}{2} \sigma_Y A_2 = \frac{1}{2} (300 \times 10^6)(300 \times 10^{-6}) = 45 \times 10^3 \text{ N}$$

$$y_1 = 65 \text{ mm} = 65 \times 10^{-3} \text{ m}$$

$$y_2 = 45 \text{ mm} = 45 \times 10^{-3} \text{ m}$$

$$y_3 = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$$

$$(a) \quad M = 2(R_1 y_1 + R_2 y_2 + R_3 y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$$

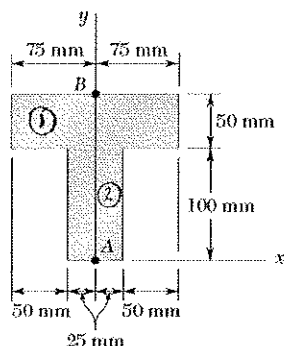
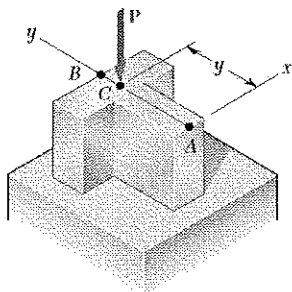
$$= 56.7 \times 10^3 \text{ N}\cdot\text{m} = 56.7 \text{ kN}\cdot\text{m}$$

$$(b) \quad \frac{y_r}{\rho} = \frac{\sigma_r}{E}$$

$$\rho = \frac{E y_r}{\sigma_r} = \frac{(200 \times 10^9)(30 \times 10^{-3})}{300 \times 10^6} = 20 \text{ m}$$

Problem 4.191

4.191 A vertical force P of magnitude 80 kN is applied at a point C located on the axis of symmetry of the cross section of a short column. Knowing that $y = 125$ mm, determine (a) the stress at point A , (b) the stress at point B , (c) the location of the neutral axis.



Locate centroid

Part	A, mm^2	$\bar{y}, \text{mm}$	$A\bar{y}, \text{mm}^3$
①	7500	125	937500
②	5000	50	250000
Σ	12500		1187500

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{1187500}{12500} = 95 \text{ mm.}$$

Eccentricity of load $e = 125 - 95 = 30 \text{ mm}$

$$I_1 = \frac{1}{12}(50)(50)^3 + (7500)(30)^2 = 8.3125 \times 10^6 \text{ mm}^4.$$

$$I_2 = \frac{1}{12}(50)(100)^3 + (5000)(45)^2 = 14.2917 \times 10^6 \text{ mm}^4.$$

$$I = I_1 + I_2 = 22.6 \times 10^6 \text{ mm}^4.$$

(a) Stress at A $c_A = 95 \text{ mm.}$

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = \frac{80000}{12500 \times 10^{-6}} + \frac{(80000)(0.03)(0.095)}{22.6 \times 10^{-6}} = 3.69 \text{ MPa.}$$

(b) Stress at B $c_B = 150 - 95 = 55 \text{ mm.}$

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = \frac{80000}{12500 \times 10^{-6}} - \frac{(80000)(0.03)(0.055)}{22.6 \times 10^{-6}} = -12.24 \text{ MPa.}$$

(c) Location of neutral axis: $\sigma = 0$

$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \frac{ea}{I} = \frac{1}{A}$$

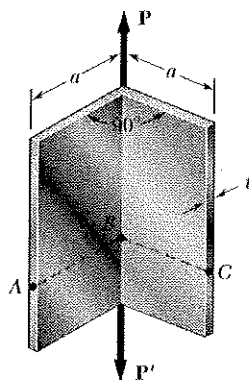
$$a = \frac{I}{Ae} = \frac{22.6 \times 10^6}{(12500)(30)} = 60.3 \text{ mm.}$$

Neutral axis lies 60.3 below centroid or $95 - 60.3$
 $= 34.7 \text{ mm}$ above point A .

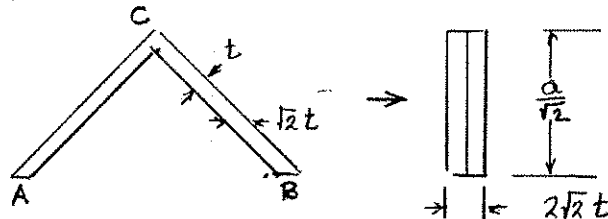
Answer 34.7 mm from point A

Problem 4.192

4.192 The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (σ) at A, (b) at B, (c) at C.



Moment of inertia about centroid



$$I = \frac{1}{12}(2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{1}{12}ta^3$$

$$\text{Area } A = (2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right) = 2at$$

$$c = \frac{a}{\sqrt{2}}$$

$$(a) \quad \sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3} =$$

$$\sigma_A = -\frac{P}{2at} \quad \leftarrow$$

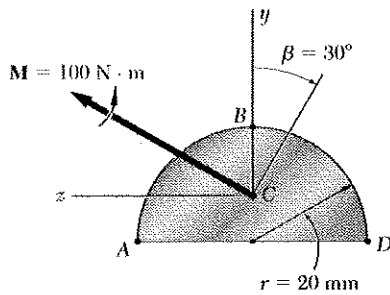
$$(b) \quad \sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3}$$

$$\sigma_B = \frac{2P}{at} \quad \leftarrow$$

$$(c) \quad \sigma_C = \sigma_A$$

$$\sigma_C = -\frac{P}{2at} \quad \leftarrow$$

Problem 4.193



4.193 The couple M is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.

$$I_z = \frac{\pi}{8} r^4 - \left(\frac{\pi}{2} r^2 \right) \left(\frac{4r}{3\pi} \right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) r^4$$

$$= (0.109757)(20)^4 = 17.5611 \times 10^{-3} \text{ mm}^4 = 17.5611 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{\pi}{8} r^4 = \frac{\pi (20)^4}{8} = 62.832 \times 10^{-3} \text{ mm}^4 = 62.832 \times 10^{-9} \text{ m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883 \text{ mm}$$

$$y_B = 20 - 8.4883 = 11.5117 \text{ mm}$$

$$z_A = -z_D = 20 \text{ mm} \quad z_B = 0$$

$$M_z = 100 \cos 30^\circ = 86.603 \text{ N}\cdot\text{m}$$

$$M_y = 100 \sin 30^\circ = 50 \text{ N}\cdot\text{m}$$

$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883 \times 10^{-3})}{17.5611 \times 10^{-9}} + \frac{(50)(20 \times 10^{-3})}{62.832 \times 10^{-9}}$$

$$= 57.8 \times 10^6 \text{ Pa} \quad \sigma_A = 57.8 \text{ MPa} \quad \blacktriangleleft$$

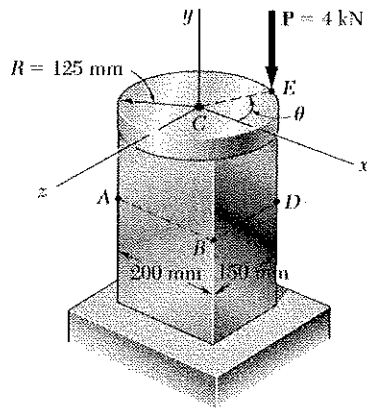
$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117 \times 10^{-3})}{17.5611 \times 10^{-9}} + \frac{(50)(0)}{62.832 \times 10^{-9}}$$

$$= -56.8 \times 10^6 \text{ Pa} \quad \sigma_B = -56.8 \text{ MPa} \quad \blacktriangleleft$$

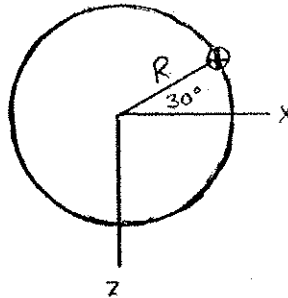
$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883 \times 10^{-3})}{17.5611 \times 10^{-9}} + \frac{(50)(-20 \times 10^{-3})}{62.832 \times 10^{-9}}$$

$$= 25.9 \times 10^6 \text{ Pa} \quad \sigma_D = 25.9 \text{ MPa} \quad \blacktriangleleft$$

Problem 4.194



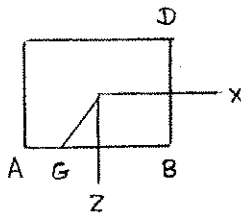
4.194 A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force P is applied at E with $\theta = 30^\circ$, determine (a) the stress at point A , (b) the stress at point B , (c) the point where the neutral axis intersects line ABD .



$$P = 4 \times 10^3 \text{ N (compression)}$$

$$\begin{aligned} M_x &= -PR \sin 30^\circ \\ &= -(4 \times 10^3)(125 \times 10^{-3}) \sin 30^\circ \\ &= -250 \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} M_z &= -PR \cos 30^\circ \\ &= -(4 \times 10^3)(125 \times 10^{-3}) \cos 30^\circ \\ &= -433 \text{ N}\cdot\text{m} \end{aligned}$$



$$I_x = \frac{1}{12} (200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12} (150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm}$$

$$z_A = z_B = 75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-6} \text{ m}^2$$

$$\begin{aligned} (a) \quad \sigma_A &= -\frac{P}{A} + \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-6}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(-100 \times 10^{-3})}{100 \times 10^{-6}} \\ &= 633 \times 10^3 \text{ Pa} = 633 \text{ kPa} \end{aligned}$$

$$\begin{aligned} (b) \quad \sigma_B &= -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-6}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(100 \times 10^{-3})}{100 \times 10^{-6}} \\ &= -233 \times 10^3 \text{ Pa} = -233 \text{ kPa} \end{aligned}$$

(c) Let G be the point on AB where neutral axis intersects.

$$\sigma_G = 0 \quad z_G = 75 \text{ mm} \quad x_G = ?$$

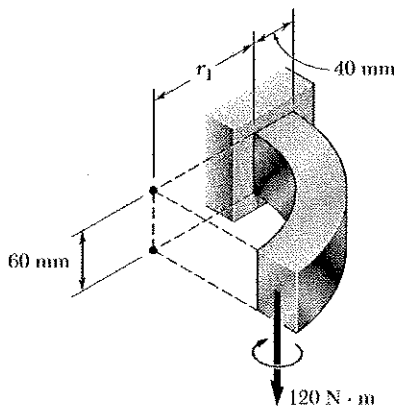
$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z} \left\{ \frac{P}{A} + \frac{M_x z_G}{I_x} \right\} = \frac{100 \times 10^{-6}}{-433} \left\{ \frac{4 \times 10^3}{30 \times 10^{-6}} + \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} \right\}$$

$$= 46.2 \times 10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 46.2 mm from point A .

Problem 4.195



4.195 The curved bar shown has a cross section of 40×60 mm and an inner radius $r_1 = 15$ mm. For the loading shown, determine the largest tensile and compressive stresses in the bar.

$$h = 40 \text{ mm}, \quad r_1 = 15 \text{ mm}, \quad r_2 = 55 \text{ mm}$$

$$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{55}{15}} = 30.786 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35 \text{ mm}$$

$$e = \bar{r} - R = 4.214 \text{ mm} \quad \sigma = -\frac{My}{Aer}$$

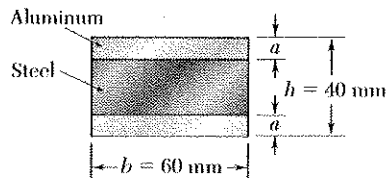
$$\text{At } r = 15 \text{ mm} \quad y = 30.786 - 15 = 15.786 \text{ mm}$$

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^6 \text{ Pa} \\ = -12.49 \text{ MPa} \quad \text{(compression)}$$

$$\text{At } r = 55 \text{ mm} \quad y = 30.786 - 55 = -24.214 \text{ mm}$$

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^6 \text{ Pa} \\ = 5.22 \text{ MPa} \quad \text{(tension)}$$

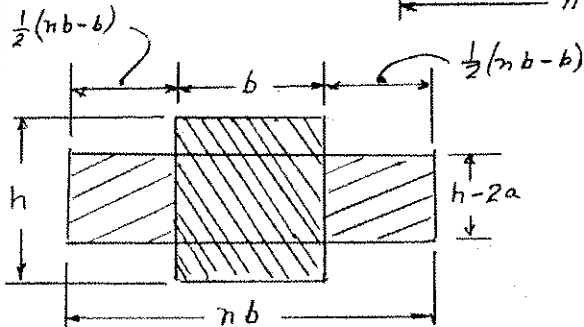
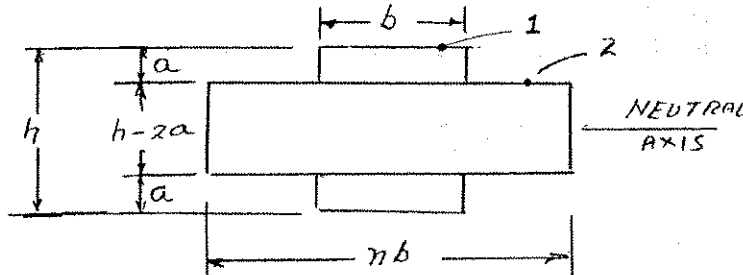
PROBLEM 4.C1



4.C1 Two aluminum strips and a steel strip are to be bonded together to form a composite member of width $b = 60 \text{ mm}$ and depth $h = 40 \text{ mm}$. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that $M = 1500 \text{ N} \cdot \text{m}$, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of a from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (a) the largest stress that can occur in the steel, (b) the corresponding value of a .

SOLUTION

TRANSFORMED SECTION (ALL STEEL) $n = \frac{E_{\text{STEEL}}}{E_{\text{ALUM}}}$



$$\bar{I} = \frac{1}{12} b h^3 + \frac{1}{12} \left[2 \left(\frac{1}{2} \right) (nb-b) \right] (h-2a)^3$$

AT POINT 1: $\sigma_{\text{ALUM}} = \frac{M \left(\frac{h}{2} \right)}{\bar{I}}$

AT POINT 2: $\sigma_{\text{STEEL}} = n \frac{M \left(\frac{h}{2} - a \right)}{\bar{I}}$

FOR $a = 0$ TO 20 mm USING 2-mm INTERVALS COMPUTE: $n, \bar{I}, \sigma_{\text{ALUM}}, \sigma_{\text{STEEL}}$.

$b = 60 \text{ mm}$ $h = 40 \text{ mm}$ $M = 1500 \text{ N} \cdot \text{m}$
Moduli of elasticity: Steel = 200 GPa Aluminum = 75 GPa

PROGRAM OUTPUT

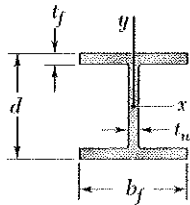
a	I	sigma aluminum	sigma steel
mm	$\text{m}^4/10^6$	MPa	MPa
0.000	0.8533	35.156	93.750
2.000	0.7088	42.325	101.580
4.000	0.5931	50.585	107.914
6.000	0.5029	59.650	111.347
8.000	0.4352	68.934	110.294
10.000	0.3867	77.586	103.448
12.000	0.3541	84.714	90.361
14.000	0.3344	89.713	71.770
16.000	0.3243	92.516	49.342
18.000	0.3205	93.594	24.958
20.000	0.3200	93.750	0.000

Find 'a' for max steel stress
and the corresponding aluminum stress

6.600	0.4804	62.447	111.572083
6.610	0.4800	62.494	111.572159
6.620	0.4797	62.540	111.572113

Max Steel Stress = 111.6 MPa occurs when $a = 6.61 \text{ mm}$
Corresponding Aluminum stress = 62.5 MPa

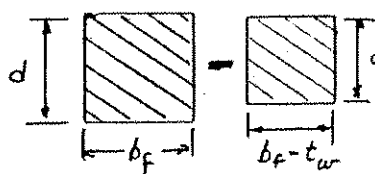
PROBLEM 4.C2



4.C2 A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_Y and a modulus of elasticity E , is bent about the x axis. (a) Denoting by y_Y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_Y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_f$. Neglect the effect of fillets. (b) Use this program to solve Prob. 4.190

SOLUTION

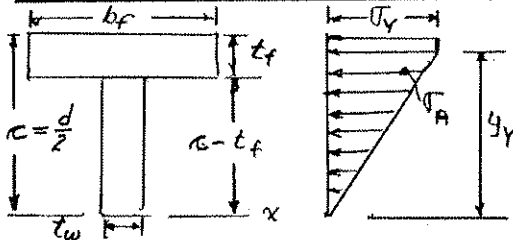
COMPUTE MOMENT OF INERTIA I_x



$$I_x = \frac{1}{12} b_f d^3 - \frac{1}{12} (b_f - t_w) (d - 2t_f)^3$$

MAXIMUM ELASTIC MOMENT: $M_Y = \sigma_Y \frac{I_x}{(d/2)}$

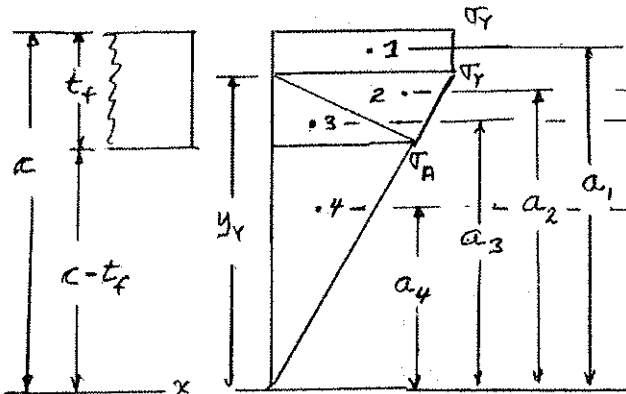
FOR YIELDING IN THE FLANGES: (CONSIDER UPPER HALF OF CROSS SECTION) $c = \frac{d}{2}$



STRESS AT JUNCTION OF WEB AND FLANGE

$$\sigma_A = \frac{(d/2) - t_f}{y_Y} \sigma_Y$$

DETAIL OF STRESS DIAGRAM



RESULTANT FORCES

$$\begin{aligned} R_1 &= \sigma_Y [b_f (c - y_Y)] \\ R_2 &= \frac{1}{2} \sigma_Y b_f [y_Y - (c - t_f)] \\ R_3 &= \frac{1}{2} \sigma_A b_f [y_Y - (c - t_f)] \\ R_4 &= \frac{1}{2} \sigma_A t_w (c - t_f) \end{aligned}$$

$$\begin{aligned} a_1 &= \frac{1}{2} (c + y_Y) \\ a_2 &= y_Y - \frac{1}{3} [y_Y - (c - t_f)] \\ a_3 &= y_Y - \frac{2}{3} [y_Y - (c - t_f)] \\ a_4 &= \frac{2}{3} (c - t_f) \end{aligned}$$

BENDING MOMENT

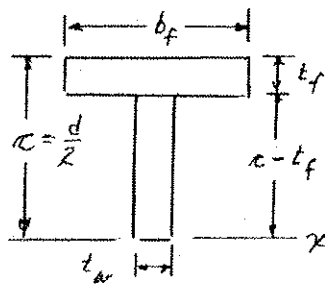
$$M = 2 \sum_{n=1}^4 R_n a_n$$

RADIUS OF CURVATURE

$$y_Y = E_Y \rho = \frac{\sigma_Y}{E} \rho; \quad \rho = \frac{y_Y E}{\sigma_Y}$$

CONTINUED

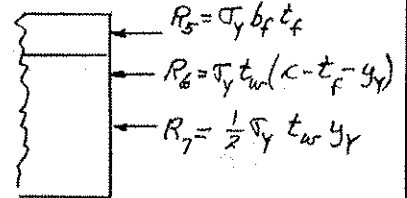
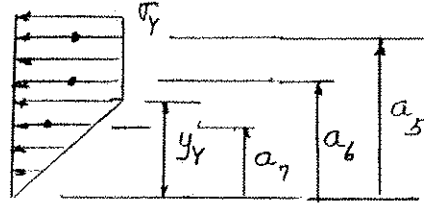
PROBLEM 4.C2 - CONTINUED



FOR YIELDING IN THE WEB

$$c = d/2$$

(CONSIDER UPPER HALF OF CROSS SECTION)



$$a_5 = c - \frac{1}{2}t_f$$

$$a_6 = \frac{1}{2}[y_Y + (c - t_f)]$$

$$a_7 = \frac{2}{3}y_Y$$

BENDING MOMENT

$$M = 2 \sum_{n=5}^7 R_n a_n$$

RADIUS OF CURVATURE

$$y_Y = \epsilon_Y \rho = \frac{\sigma_Y}{E} \rho$$

$$\rho = \frac{y_Y E}{\sigma_Y}$$

PROGRAM: KEY IN EXPRESSIONS FOR a_n AND R_n FOR $n=1$ TO 7 .

FOR $y_Y = c$ TO $(c - t_f)$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2 \sum R_n a_n$ FOR $n=1$ TO 4 AND $\rho = \frac{y_Y E}{\sigma_Y}$, THEN PRINT

FOR $y_Y = (c - t_w)$ TO $c/3$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2 \sum R_n a_n$ FOR $n=5$ TO 7 AND $\rho = \frac{y_Y E}{\sigma_Y}$, THEN PRINT

INPUT NUMERICAL VALUES AND RUN PROGRAM

PROGRAM OUTPUT

For a beam of Prob 4.190

Depth $d = 140.00$ mm

Thickness of flange $t_f = 10.00$ mm

Width of flange $b_f = 120.00$ mm

Thickness of web $t_w = 10.00$ mm

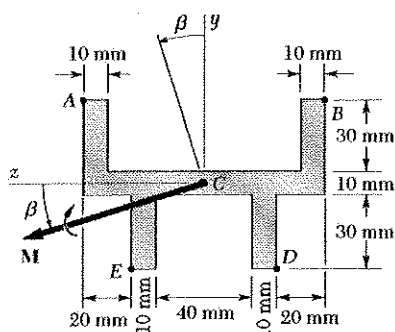
$I = 0.000011600$ m to the 4th

Yield strength of Steel $\sigma_{Y} = 300$ MPa

Yield Moment $M_Y = 49.71$ kip.in.

y_Y (mm)	M (kN.m)	ρ (m)
For yielding still in the flange.		
70.000	49.71	46.67
65.000	52.59	43.33
60.000	54.00	40.00
For yielding in the web		
60.000	54.00	40.00
55.000	54.58	36.67
50.000	55.10	33.33
45.000	55.58	30.00
40.000	56.00	26.67
35.000	56.38	23.33
30.000	56.70	20.00
25.000	56.97	16.67

PROBLEM 4.C3



4.C3 A 900 N · m couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at **C** and that the **y** and **z** axes are principal axes, write a computer program to calculate the stress at **A**, **B**, **C**, and **D** for values of β from 0 to 180° using 10° increments. (Given: $I_y = 2.59 \times 10^6 \text{ mm}^4$ and $I_z = 0.62 \times 10^6 \text{ mm}^4$.)

SOLUTION

INPUT COORDINATES OF A, B, C, D

$$\begin{aligned} z_A &= z(1) = 50 & y_A &= y(1) = 35 \\ z_B &= z(2) = -50 & y_B &= y(2) = 35 \\ z_D &= z(3) = -30 & y_D &= y(3) = -35 \\ z_E &= z(4) = 30 & y_E &= y(4) = -35 \end{aligned}$$

COMPONENTS OF M.

$$M_y = -M \sin \beta$$

$$M_z = M \cos \beta$$

Eq. 4.55 page 273:
$$\sigma(n) = - \frac{M_z y(n)}{I_z} + \frac{M_y z(n)}{I_y}$$

PROGRAM: FOR $\beta = 0$ TO 180° USING 10° INCREMENTS.

FOR $n = 1$ TO 4 USING UNIT INCREMENTS.

EVALUATE EQ 4.55 AND PRINT STRESSES

RETURN

ICE TURN

PROGRAM OUTPUT

Moment of couple $M = 900 \text{ N} \cdot \text{m}$

Moments of inertia: $I_y = 2.59 \times 10^6 \text{ mm}^4$

$I_z = 0.62 \times 10^6 \text{ mm}^4$

Coordinates of points A, B, D, and E ($\times 10^{-3}$)

Point A: $z(1) = 50$; $y(1) = 35$

Point B: $z(2) = -50$; $y(2) = 35$

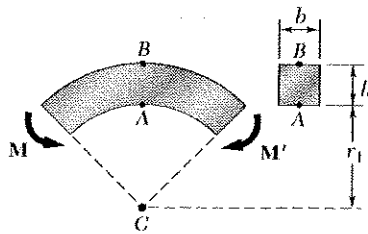
Point D: $z(3) = -30$; $y(3) = -35$

Point E: $z(4) = 30$; $y(4) = -35$

- - - Stress at Points - - -

beta deg	A MPa	B MPa	D MPa	E MPa
0	-52.161	-52.161	52.161	52.161
40	-51.340	-28.573	45.652	34.268
90	-17.713	17.713	8.853	-8.853
130	19.961	47.093	-26.746	-40.308
180	52.161	52.161	-52.161	-52.161

PROBLEM 4.C4



4.C4 Couples of moment $M = 2 \text{ kN} \cdot \text{m}$ are applied as shown to a curved bar having a rectangular cross section with $h = 100 \text{ mm}$ and $b = 25 \text{ mm}$. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50 percent larger than the maximum stress in a straight bar of the same cross section.

SOLUTION INPUT: $h = 100 \text{ mm}$, $b = 25 \text{ mm}$, $M = 2 \text{ kN} \cdot \text{m}$

FOR STRAIGHT BAR: $\sigma_{\text{STRAIGHT}} = \frac{M}{S} = \frac{6M}{h^2 b} = 48 \text{ MPa}$

FOLLOWING NOTATION OF SEC. 4.15, KEY IN THE FOLLOWING:

$$r_2 = h + r_1; \quad R = h / \ln(r_2/r_1); \quad \bar{r} = r_1 + r_2; \quad e = \bar{r} - R; \quad A = bh = 2500 \quad \text{I}$$

$$\text{STRESSES: } \sigma_A = \sigma_1 = M(r_1 - R)/(Aer_1) \quad \sigma_B = \sigma_2 = M(r_2 - R)/(Aer_2) \quad \text{II}$$

SINCE $h = 100 \text{ mm}$, FOR $r_1/h = 10$, $r_1 = 1000 \text{ mm}$. ALSO $r_1/h = 10$, $r_1 = 100$

PROGRAM: FOR $r_1 = 1000$ TO 100 AT -100 DECREMENTS

USING EQUATIONS OF LINES I AND II EVALUATE $r_2, R, \bar{r}, e, \sigma_1$ AND σ_2

ALSO EVALUATE: $\text{ratio} = \sigma_1 / \sigma_{\text{STRAIGHT}}$

RETURN AND REPEAT FOR $r_1 = 100$ TO 10 AT -10 DECREMENT

PROGRAM OUTPUT

M = Bending Moment = 2. kN.m h = 100.000 in. A = 2500.00 mm²
Stress in straight beam = 48.00 MPa

r1 mm	rbar mm	R mm	e mm	sigma1 MPa	sigma2 MPa	r1/h	ratio
1000	1050	1049	0.794	-49.57	46.51	10.000	-1.033
900	950	949	0.878	-49.74	46.36	9.000	-1.036
800	850	849	0.981	-49.95	46.18	8.000	-1.041
700	750	749	1.112	-50.22	45.95	7.000	-1.046
600	650	649	1.284	-50.59	45.64	6.000	-1.054
500	550	548	1.518	-51.08	45.24	5.000	-1.064
400	450	448	1.858	-51.82	44.66	4.000	-1.080
300	350	348	2.394	-53.03	43.77	3.000	-1.105
200	250	247	3.370	-55.35	42.24	2.000	-1.153
100	150	144	5.730	-61.80	38.90	1.000	-1.288
=====							
100	150	144	5.730	-61.80	38.90	1.000	-1.288
90	140	134	6.170	-63.15	38.33	0.900	-1.316
80	130	123	6.685	-64.80	37.69	0.800	-1.350
70	120	113	7.299	-66.86	36.94	0.700	-1.393
60	110	102	8.045	-69.53	36.07	0.600	-1.449
50	100	91	8.976	-73.13	35.04	0.500	-1.523
40	90	80	10.176	-78.27	33.79	0.400	-1.631
30	80	68	11.803	-86.30	32.22	0.300	-1.798
20	70	56	14.189	-100.95	30.16	0.200	-2.103
10	60	42	18.297	-138.62	27.15	0.100	-2.888

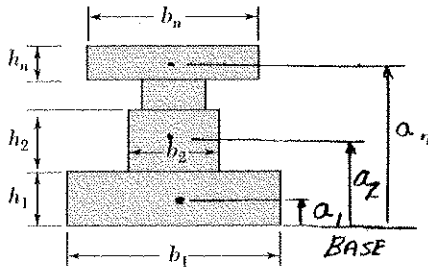
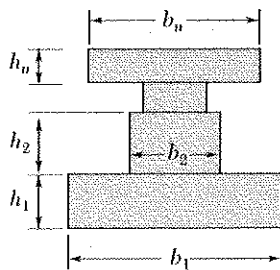
Find r1/h for (sigma max)/(sigma straight) = 1.5

52.70	103	94	8.703	-72.036	35.34	0.527	-1.501
52.80	103	94	8.693	-71.998	35.35	0.528	-1.500
52.90	103	94	8.683	-71.959	35.36	0.529	-1.499

Ratio of stresses is 1.5 for r1 = 52.8 mm or r1/h = 0.529

[Note: The desired ratio r1/h is valid for any beam having a rectangular cross section.]

PROBLEM 4.C5



4.C5 The couple M is applied to a beam of the cross section shown. (a) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (b) Use this program to solve Probs. 4.7, 4.8, and 4.9.

SOLUTION

INPUT: BENDING MOMENT M



```

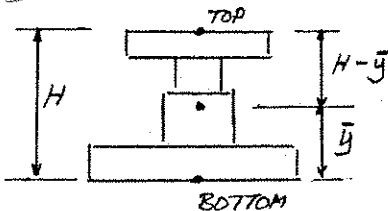
FOR n=1 TO n: ENTER b_n AND h_n
    ΔAREA = b_n h_n (PRINT)
    a_n = a_{n-1} + (h_{n-1})/2 + h_n/2
    [MOMENT OF RECTANGLE ABOUT BASE]
    Δm = (ΔAREA) a_n
    [FOR WHOLE CROSS SECTION]
    m = m + Δm ; AREA = AREA + ΔAREA
LOCATION OF CENTROID ABOVE BASE
    y-bar = m/AREA (PRINT)
    
```

MOMENT OF INERTIA ABOUT HORIZONTAL CENTROIDAL AXIS

```

FOR n=1 TO n:
    a_n = a_{n-1} + (h_{n-1})/2 + h_n/2
    ΔI = b_n h_n^3 / 12 + (b_n h_n) (y-bar - a_n)^2
    I = I + ΔI (PRINT)
    
```

COMPUTATION OF STRESSES



TOTAL HEIGHT: FOR $n=1$ TO n
 $H = H + h_n$

STRESS AT TOP

$$M_{TOP} = -M \frac{H - \bar{y}}{I} \quad (\text{PRINT})$$

STRESS AT BOTTOM

$$M_{BOTTOM} = M \frac{\bar{y}}{I} \quad (\text{PRINT})$$

SEE NEXT PAGE FOR PRINT OUTS

CONTINUED

PROBLEM 4.C5 - CONTINUED

Problem 4.7

Summary of Cross Section Dimensions

Width (m)	Height (m)
0.225	0.05
0.075	0.15

Bending Moment = 60,000 Nm

Centroid is 0.075 m above lower edge

Centroidal Moment of Inertia is $79. \times 10^{-6} \text{m}^4$

Stress at top of beam = -94.103 MPa

Stress at bottom of beam = 56.462 MPa

Problem 4.8

Summary of Cross Section Dimensions

Width (m)	Height (m)
0.1	0.025
0.025	0.15
0.2	0.025

Bending Moment = 60000 Nm

Centroid is 0.1194 m above lower edge

Centroidal Moment of Inertia is $60.59 \times 10^{-6} \text{m}^4$

Stress at top of beam = -79.815 MPa

Stress at bottom of beam = 118.237 MPa

PROBLEM 4.9

Summary of Cross Section Dimensions

Width (mm)	Height (mm)
50	10
20	50

Bending Moment = 1500.0000 N.m

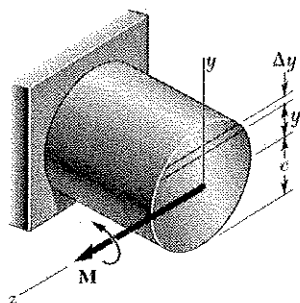
Centroid is 25.000 mm above lower edge

Centroidal Moment of Inertia is 512500 mm^4

Stress at top of beam = -102.439 MPa

Stress at bottom of beam = 73.171 MPa

PROBLEM 4.C6



4.C6 A solid rod of radius $c = 30$ mm is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 290$ MPa. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_Y and then to the plastic moment M_p . Denoting by y_Y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_Y from 30 mm to 0 using 5-mm decrements. (Hint: Divide the cross section into 80 horizontal elements of 1-mm height.)

SOLUTION

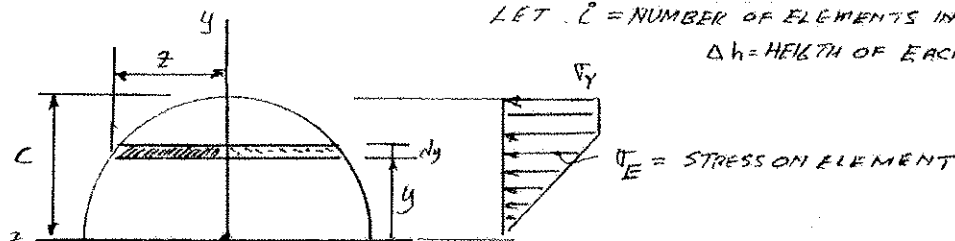
$$M_Y = \sigma_Y \frac{\pi}{4} c^3 = (290 \times 10^6) \frac{\pi}{4} (0.03)^3 = 615 \text{ MPa}$$

$$M_p = \sigma_Y \frac{4}{3} c^3 = (290 \times 10^6) \frac{4}{3} (0.03)^3 = 104.4 \text{ kNm}$$

CONSIDER TOP HALF OF ROD

LET L = NUMBER OF ELEMENTS IN TOP HALF

Δh = HEIGHT OF EACH ELEMENT: $\Delta h = \frac{c}{L}$



FOR $n = 0$ TO $L-1$ STEP 1

$$y = n(\Delta h)$$

$$z = [c^2 - \{(n+0.5)\Delta h\}^2]^{1/2}$$

← z AT MIDHEIGHT OF ELEMENT

IF $y \geq y_Y$ GO TO 100

$$\sigma_E = \sigma_Y \frac{(n+0.5)\Delta h}{y_Y}$$

← STRESS IN ELASTIC CORE

GO TO 200

$$\sigma_E = \sigma_Y$$

← STRESS IN PLASTIC ZONE

100

200

$$\Delta \text{AREA} = z \Delta h$$

$$\Delta \text{FORCE} = \sigma_E (\Delta \text{AREA})$$

$$\Delta \text{MOMENT} = \Delta \text{FORCE} (n+0.5) \Delta h$$

$$M = M + \Delta \text{MOMENT}$$

$$\rho = y_Y E / \sigma_Y$$

PRINT y_Y , M , AND ρ .

NEXT

REPEAT

FOR

$$y_Y = 30 \text{ mm}$$

TO

$$y_Y = 0$$

AT 5mm

DECREMENTS

PROGRAM OUTPUT

Radius of rod = 0.03 m

Yield point of steel = 290 MPa

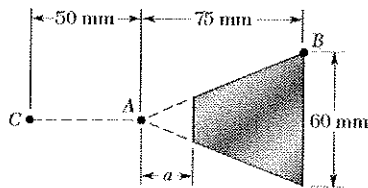
Yield moment = 615 MPa

Plastic moment = 104.4 kNm

Number of elements in half of the rod = 40

y_Y (mm)	P (mm $\times 10^3$)
30	20.690
25	17.241
20	13.793
15	10.345
10	6.897
5	3.448
0	0

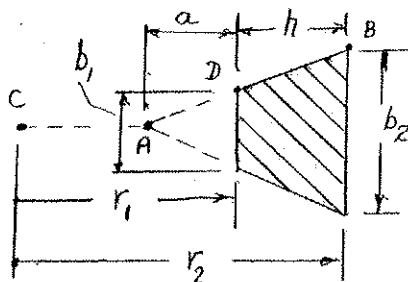
PROBLEM 4.C7



4.C7 The machine element of Prob. 4.178 is to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width a will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of a from 0 to 25 mm using 2.5-mm increments. Using appropriate smaller increments, determine the distance a for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION SEE FIG 4.79 PAGE 289

$$M = 560 \text{ Nm} \quad r_2 = 125 \text{ mm} \quad b_2 = 62 \text{ mm}$$



FOR $a = 0$ TO 1.0 AT 0.1 INTERVALS

$$h = 3 - a$$

$$r_1 = 2 + a$$

$$b_1 = b_2 (a / (h + a))$$

$$\text{AREA} = (b_1 + b_2) (h / 2)$$

$$\bar{x} = a + \left[\frac{1}{2} b_1 h \left(\frac{1}{3} \right) + \frac{1}{2} b_2 h \left(\frac{2}{3} \right) \right] / \text{AREA}$$

$$\bar{r} = r_2 - (h - \bar{x})$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h (b_1 - b_2)}$$

$$e = \bar{r} - R$$

$$\sigma_D = M (r_1 - R) / [\text{AREA} (e \times r_1)]$$

$$\sigma_B = M (r_2 - R) / [\text{AREA} (e \times r_2)]$$

PRINT AND RETURN

PROGRAM OUTPUT

a mm	R mm	σ_D MPa	σ_B MPa	b_1 mm	$\bar{r}$ mm	e mm
0	97.917	-58.656	14.489	0	101.600	3.683
5	98.273	-50.127	14.955	4.318	101.857	3.556
12.5	99.771	-45.288	16.990	10.668	102.870	3.023
20	102.057	-45.575	20.583	17.018	104.394	2.388
25	103.861	-48.022	23.782	21.082	105.918	1.981

Determination of the maximum compressive stress that is as small as possible

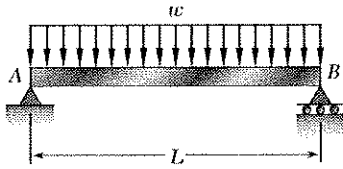
a mm	R mm	σ_D MPa	σ_B MPa	b_1 mm	$\bar{r}$ mm	e mm
15.5	100.609	-44.9001	18.2200	1.321	10.338	0.277
15.625	100.660	-44.8992	18.2766	1.321	10.338	0.277
15.75	100.686	-44.8994	18.3345	1.321	10.338	0.277

Answer: When $a = 15.625$ mm the compressive stress is 44.9 MPa

Chapter 5

Problem 5.1

5.1 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.

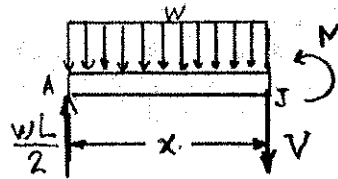
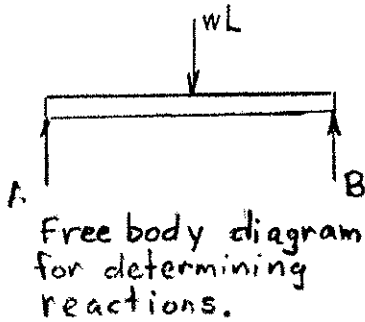


Reactions.

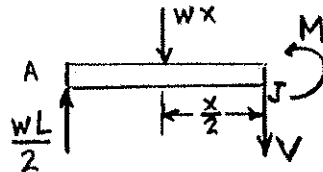
$$\sum M_B = 0: -AL + wL \cdot \frac{L}{2} = 0 \quad A = \frac{wL}{2}$$

$$\sum M_A = 0: BL - wL \cdot \frac{L}{2} = 0 \quad B = \frac{wL}{2}$$

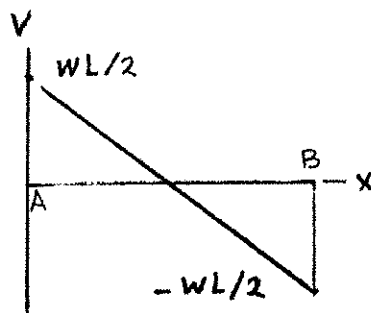
Over whole beam, $0 < x < L$



Place section at x.



Replace distributed load by equivalent concentrated load.



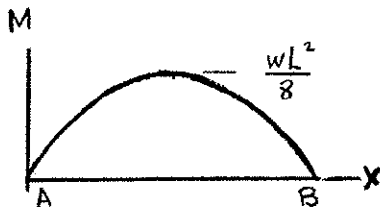
$$\sum F_y = 0: \frac{wL}{2} - wx - V = 0$$

$$V = w \left(\frac{L}{2} - x \right)$$

$$\sum M_x = 0: -\frac{wL}{2}x + wx \cdot \frac{x}{2} + M = 0$$

$$M = \frac{w}{2} (Lx - x^2)$$

$$M = \frac{w}{2} x (L - x)$$

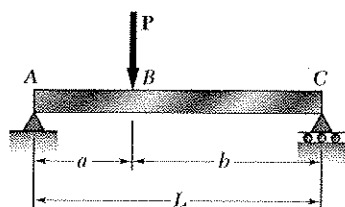


Maximum bending moment occurs at $x = \frac{L}{2}$.

$$M_{max} = \frac{wL^2}{8}$$

Problem 5.2

5.2 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



Reactions.

$$\sum M_C = 0: LA - bP = 0 \quad A = \frac{Pb}{L}$$

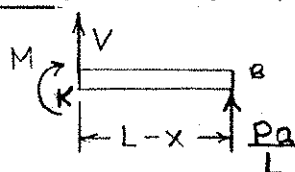
$$\sum M_A = 0: LC - aP = 0 \quad C = \frac{Pa}{L}$$

From A to B: $0 < x < a$

$$\begin{aligned} \uparrow \sum F_y = 0: \quad \frac{Pb}{L} - V &= 0 \\ V &= \frac{Pb}{L} \end{aligned}$$

$$\begin{aligned} \sum M_J = 0: \quad M - \frac{Pb}{L}x &= 0 \\ M &= \frac{Pbx}{L} \end{aligned}$$

From B to C: $a < x < L$



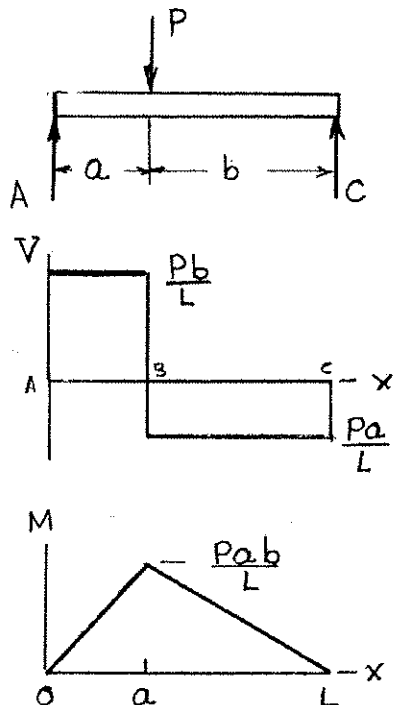
$$\uparrow \sum F_y = 0: \quad V + \frac{Pa}{L} = 0$$

$$V = -\frac{Pa}{L}$$

$$\sum M_K = 0: \quad -M + \frac{Pa}{L}(L-x) = 0$$

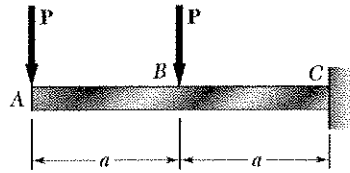
$$M = \frac{Pa(L-x)}{L}$$

At section B: $M = \frac{Pab}{L}$

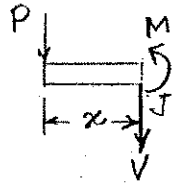


Problem 5.3

5.1 through 5.6 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



From A to B: $0 < x < a$

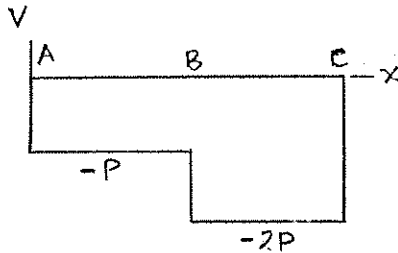


$$+\uparrow \sum F_y = 0: -P - V = 0$$

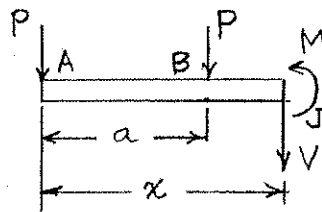
$$V = -P$$

$$+\circlearrowleft \sum M_J = 0: Px + M = 0$$

$$M = -Px$$



From B to C: $a < x \leq 2a$



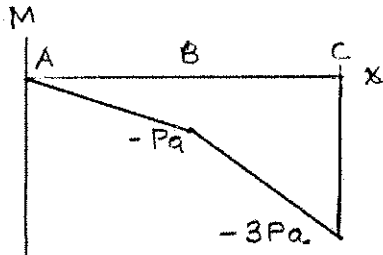
$$+\uparrow \sum F_y = 0:$$

$$-P - P - V = 0$$

$$V = -2P$$

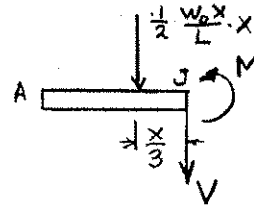
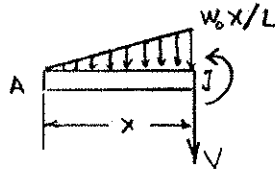
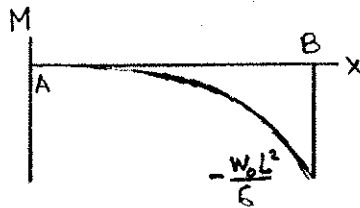
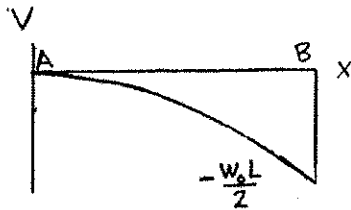
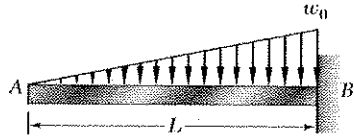
$$+\circlearrowleft \sum M_J = 0: Px + P(x-a) + M = 0$$

$$M = -2Px + Pa$$



Problem 5.4

5.4 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



$$+\uparrow \Sigma F_y = 0$$

$$-\frac{1}{2} \frac{w_0 x}{L} \cdot x - V = 0$$

$$V = -\frac{w_0 x^2}{2L}$$

$$\circlearrowleft \Sigma M_J = 0$$

$$\frac{1}{2} \frac{w_0 x}{L} \cdot x \cdot \frac{x}{3} + M = 0$$

$$M = -\frac{w_0 x^3}{6L}$$

$$\text{At } x = L,$$

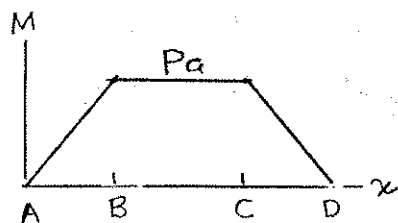
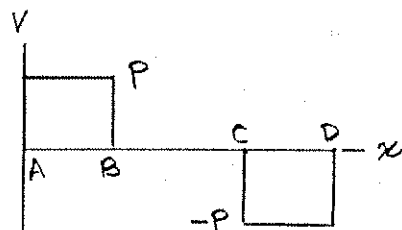
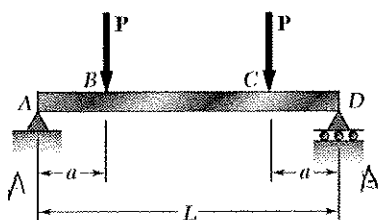
$$V = -\frac{w_0 L}{2}$$

$$|V|_{\max} = \frac{w_0 L}{2}$$

$$M = -\frac{w_0 L^2}{6}$$

$$|M|_{\max} = \frac{w_0 L^2}{6}$$

Problem 5.5



5.1 through 5.6 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.

Using entire beam ABCD as a free body,

$$+\circlearrowleft M_D = 0: -AL + P(L-a) + Pa = 0$$

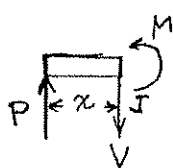
$$A = P$$

$$+\circlearrowleft M_A = 0: DL - Pa - P(L-a) = 0$$

$$D = P$$

$$\text{Check: } +\uparrow \Sigma F_y = P - P - P + P = 0$$

From A to B: $0 < x < a$



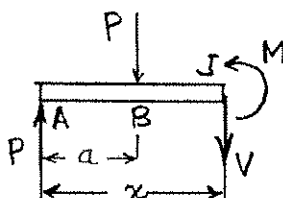
$$+\uparrow \Sigma F_y = 0: P - V = 0$$

$$V = P$$

$$+\circlearrowleft \Sigma M_J = 0: -Px + M = 0$$

$$M = Px$$

From B to C: $a < x < L-a$



$$+\uparrow \Sigma F_y = 0$$

$$P - P - V = 0$$

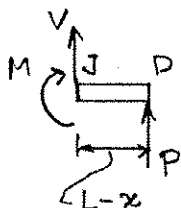
$$V = 0$$

$$+\circlearrowleft \Sigma M_J = 0$$

$$-Px + P(x-a) + M = 0$$

$$M = Pa$$

From C to D: $L-a < x < L$



$$+\uparrow \Sigma F_y = 0: V + P = 0$$

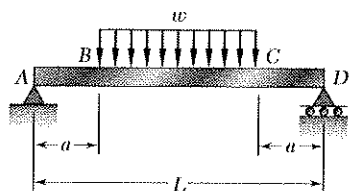
$$V = -P$$

$$+\circlearrowleft \Sigma M_J = 0: -M + P(L-x) = 0$$

$$M = P(L-x)$$

Problem 5.6

5.6 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.

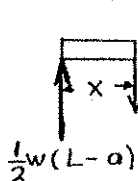


Calculate reactions after replacing distributed load by an equivalent concentrated load.

Reactions are

$$A = D = \frac{1}{2}w(L-2a)$$

From A to B: $0 < x < a$

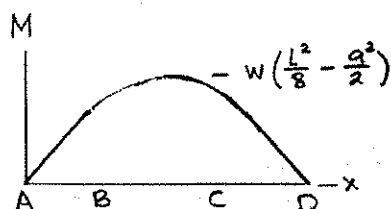
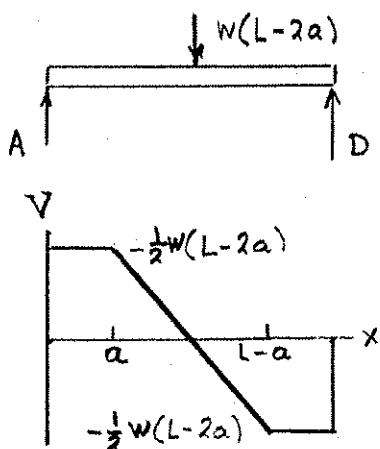


$$+\uparrow \sum F_y = 0: \frac{1}{2}w(L-2a) - V = 0$$

$$V = \frac{1}{2}w(L-2a) \quad \leftarrow$$

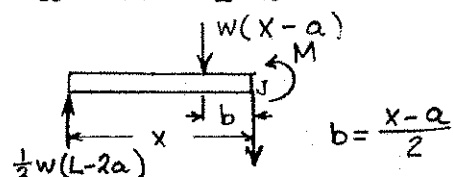
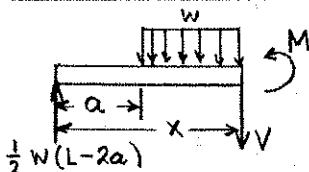
$$\circlearrowleft \sum M = 0: -\frac{1}{2}w(L-2a)x + M = 0$$

$$M = \frac{1}{2}w(L-2a)x \quad \leftarrow$$



From B to C:

$a < x < L-a$



Place section cut at x. Replace distributed load by equiv. conc. load.

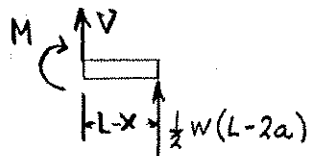
$$+\uparrow \sum F_y = 0: \frac{1}{2}w(L-2a) - w(x-a) - V = 0 \quad V = w\left(\frac{L}{2} - x\right) \quad \leftarrow$$

$$\circlearrowleft \sum M_s = 0: -\frac{1}{2}w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M = 0$$

$$M = \frac{1}{2}w[(L-2a)x - (x-a)^2] \quad \leftarrow$$

From C to D:

$L-a < x < L$



$$+\uparrow \sum F_y = 0: V + \frac{1}{2}w(L-2a) = 0$$

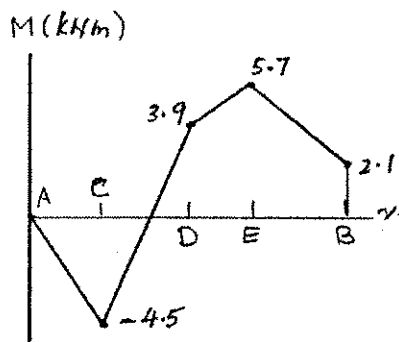
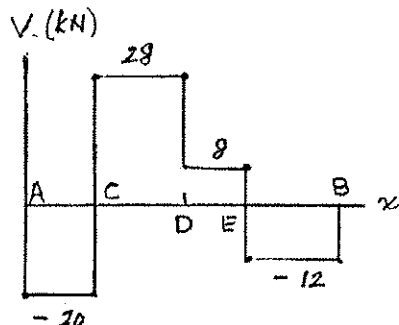
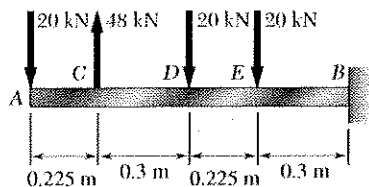
$$V = -\frac{w}{2}(L-2a)$$

$$\circlearrowleft \sum M_r = 0: -M + \frac{1}{2}w(L-2a)(L-x) = 0$$

$$M = \frac{1}{2}w(L-2a)(L-x) \quad \leftarrow$$

$$\text{At } x = \frac{L}{2} \quad M_{\max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right) \quad \leftarrow$$

Problem 5.7



5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

Shear Diagram:

$$\text{A to C. } +\uparrow \Sigma F_y = 0: \quad -20 - V = 0 \quad V = -20 \text{ kN}$$

$$\text{C to D. } +\uparrow \Sigma F_y = 0: \quad -20 + 48 - V = 0 \quad V = 28 \text{ kN}$$

$$\text{D to E. } +\uparrow \Sigma F_y = 0: \quad -20 + 48 - 20 - V = 0 \quad V = 8 \text{ kN}$$

$$\text{E to B. } +\uparrow \Sigma F_y = 0: \quad -20 + 48 - 20 - 20 - V = 0 \quad V = -12 \text{ kN}$$

Bending Moment Diagram.

$$\text{At A:} \quad M = 0$$

$$\begin{aligned} \text{At C. } +\circlearrowleft \Sigma M_C = 0: \\ (20)(0.225) + M = 0 \\ M = -4.5 \text{ kNm} \end{aligned}$$

$$\begin{aligned} \text{At D. } +\circlearrowleft \Sigma M_D = 0: \\ (20)(0.525) - (48)(0.3) + M = 0 \\ M = 3.9 \text{ kNm} \end{aligned}$$

$$\begin{aligned} +\circlearrowleft \Sigma M_E = 0: \quad (20)(0.75) - (48)(0.525) + (20)(0.225) + M = 0 \\ M = 5.7 \text{ kNm} \end{aligned}$$

$$+\circlearrowleft \Sigma M_B = 0:$$

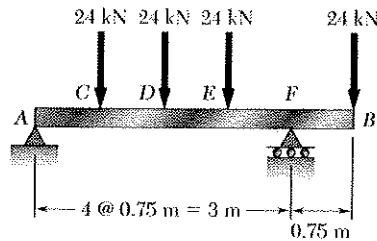
$$\begin{aligned} (20)(1.05) - (48)(0.825) + (20)(0.525) + (20)(0.3) + M = 0 \\ M = 2.1 \text{ kNm} \end{aligned}$$

From the diagrams,

$$(a) \quad |V|_{\max} = 28 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad |M|_{\max} = 5.7 \text{ kNm} \quad \blacktriangleleft$$

Problem 5.8



5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

Reactions at A and F.

$$\uparrow \Sigma M_F = 0:$$

$$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24) - (0.75)(24) = 0$$

$$R_A = 30 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0:$$

$$-(0.75)(24) - (1.50)(24) - (2.25)(24) + 3R_F - (3.75)(24) = 0$$

$$R_F = 66 \text{ kN} \uparrow$$

Shear diagram.

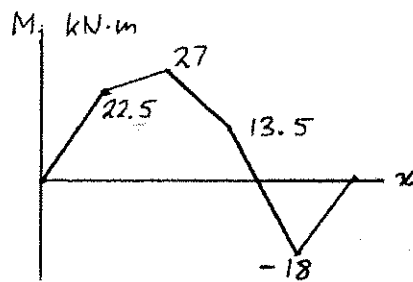
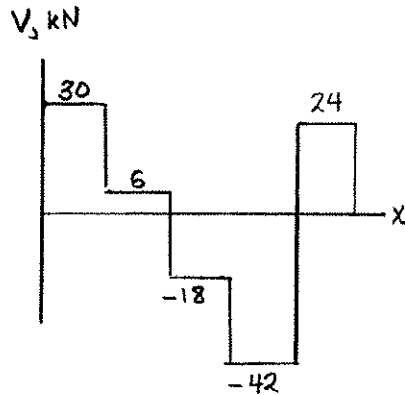
$$A \text{ to } C. \quad V = 30 \text{ kN}$$

$$C \text{ to } D. \quad V = 30 - 24 = 6 \text{ kN}$$

$$D \text{ to } E. \quad V = 6 - 24 = -18 \text{ kN}$$

$$E \text{ to } F. \quad V = -18 - 24 = -42 \text{ kN}$$

$$F \text{ to } B. \quad V = -42 + 66 = 24 \text{ kN}$$

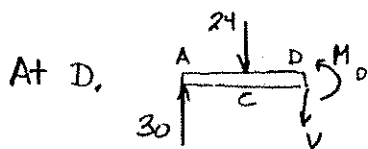


$$\text{At } A \text{ and } B. \quad M_A = M_B = 0$$

$$\text{At } C. \quad +\circlearrowleft \Sigma M_C = 0:$$

$$-(0.75)(30) + M_C = 0$$

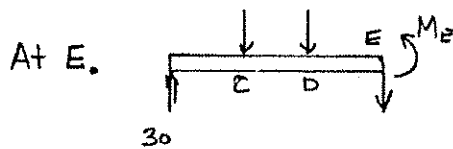
$$M_C = 22.5 \text{ kN}\cdot\text{m}$$



$$+\circlearrowleft \Sigma M_D = 0:$$

$$-(1.50)(30) + (0.75)(24) + M_D = 0$$

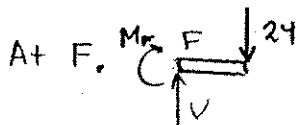
$$M_D = 27 \text{ kN}\cdot\text{m}$$



$$+\circlearrowleft \Sigma M_E = 0:$$

$$-(2.25)(30) + (1.50)(24) + (0.75)(24) + M_E = 0$$

$$M_E = 13.5 \text{ kN}\cdot\text{m}$$



$$\uparrow \Sigma M_F = 0:$$

$$-M_F - (0.75)(24) = 0$$

$$M_F = -18 \text{ kN}\cdot\text{m}$$

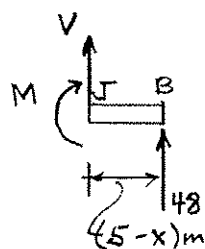
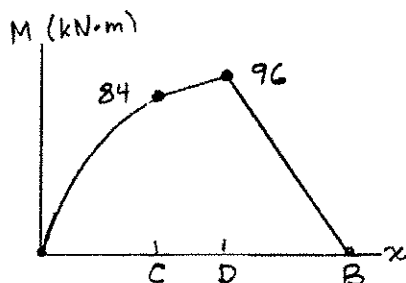
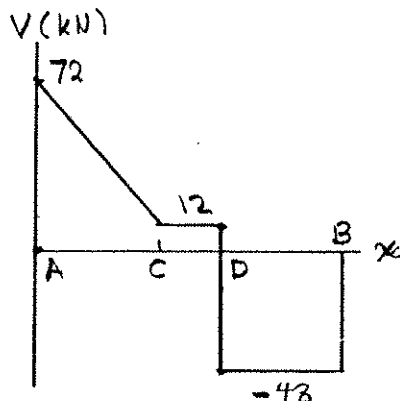
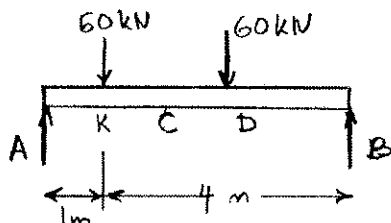
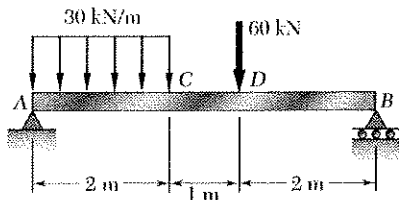
From the diagrams

$$(a) |V|_{\max} = 42.0 \text{ kN}$$

$$(b) |M|_{\max} = 27.0 \text{ kN}\cdot\text{m}$$

Problem 5.9

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



$$+\uparrow \Sigma F_y = 0: V + 48 = 0$$

$$V = -48 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 0: -M + 48(5-x)$$

$$M = 240 - 48x$$

Calculate reactions at A and B. Replace distributed load by an equivalent concentrated load after drawing the free body ACDB.

$$+\circlearrowleft \Sigma M_B = 0: -5A + (4)(60) + (2)(60) = 0$$

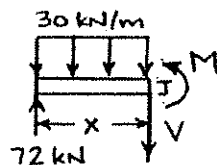
$$A = 72 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: 5B - (1)(60) - (3)(60) = 0$$

$$B = 48 \text{ kN} \uparrow$$

$$\text{Check } +\uparrow \Sigma F_y = 0: 72 - 60 - 60 + 48 = 0$$

A to C $0 < x \leq 2 \text{ m}$



$$+\uparrow \Sigma F_y = 0:$$

$$72 - 30x - V = 0$$

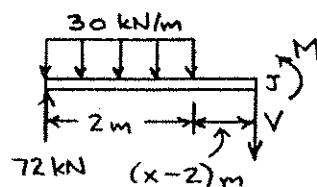
$$V = (72 - 30x) \text{ kN}$$

$$+\circlearrowleft \Sigma M_x = 0:$$

$$-72x + (30x)\frac{x}{2} + M = 0$$

$$M = (72 - 15x^2) \text{ kN} \cdot \text{m}$$

C to D. $2 \text{ m} \leq x < 3 \text{ m}$



$$+\uparrow \Sigma F_y = 0$$

$$72 - (30)(2) - V = 0$$

$$V = 12 \text{ kN}$$

$$+\circlearrowleft \Sigma M_x = 0:$$

$$-72x + (30)(2)(x-1) + M = 0$$

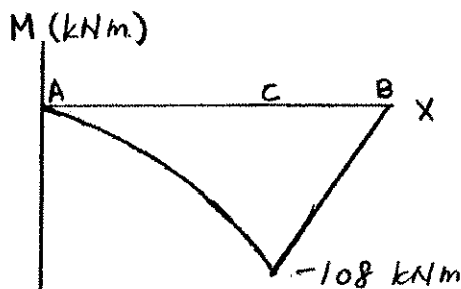
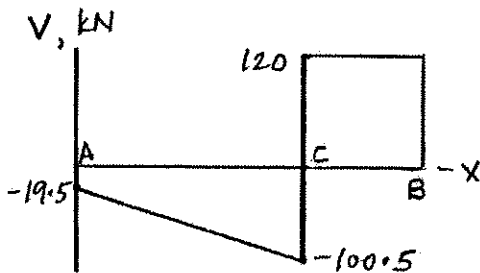
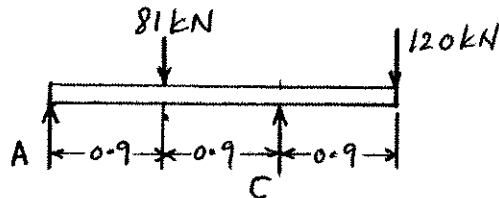
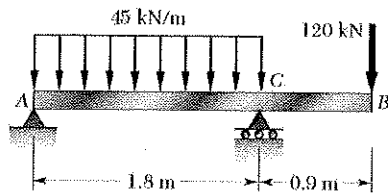
$$M = (12x + 60) \text{ kN} \cdot \text{m}$$

From the diagrams

(a) $|V|_{\max} = 72.0 \text{ kN}$

(b) $|M|_{\max} = 96.0 \text{ kN} \cdot \text{m}$

Problem 5.10



Reactions

$$\sum M_C = 0 \quad -1.8A + (0.9)(81) - (0.9)(120) = 0$$

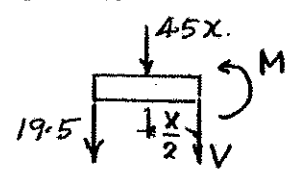
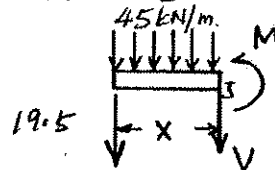
$$A = -19.5 \text{ kN} \text{ i.e. } 19.5 \downarrow$$

$$\sum M_A = 0 \quad 1.8C - (0.9)(81) - (2.7)(120) = 0$$

$$C = 220.5 \text{ kN} \uparrow$$

A to C

$$0 < x < 1.8 \text{ m}$$



$$\sum F_y = 0 \quad -19.5 - 45x - V = 0$$

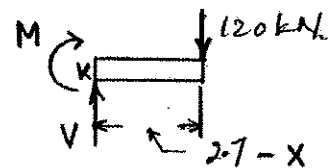
$$V = -19.5 - 45x \text{ kN}$$

$$\sum M_x = 0 \quad -19.5x - (45x)\left(\frac{x}{2}\right) - M = 0$$

$$M = -19.5x - 22.5x^2 \text{ kNm}$$

C to B

$$1.8 \text{ m} < x < 2.7 \text{ m}$$



$$\sum F_y = 0 \quad V - 120 = 0$$

$$V = 120 \text{ kN}$$

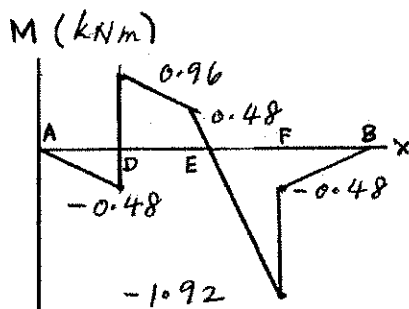
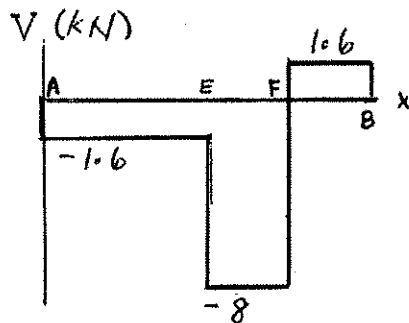
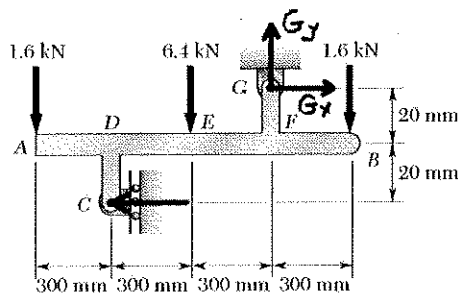
$$\sum M_x = 0 \quad -M - (2.7 - x)(120) = 0$$

$$M = 120x - 324 \text{ kNm}$$

From the diagrams (a) $|V|_{\max} = 120 \text{ kN}$

(b) $|M|_{\max} = 108 \text{ kNm}$

Problem 5.11



(a) Maximum $|V|$
 $= 8 \text{ kN}$

(b) Maximum $|M|$
 $= 1.92 \text{ kNm}$

5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

$$\begin{aligned} \sum M_G = 0 \\ -40C + (900)(1.6) + (300)(6.4) - (300)(1.6) &= 0 \\ C &= 72 \text{ kN} \\ \sum F_x = 0 \quad -C + G_x &= 0 \quad G_x = 72 \text{ kN} \\ \sum F_y = 0 \quad -1.6 - 6.4 + G_y - 1.6 &= 0 \\ G_y &= 9.6 \text{ kN} \end{aligned}$$

$$\begin{aligned} A \text{ to } E \quad V &= -1.6 \text{ kN} \\ E \text{ to } F \quad V &= -8 \text{ kN} \\ F \text{ to } B \quad V &= 1.6 \text{ kN} \end{aligned}$$

$$\text{At } A \text{ and } B \quad M = 0$$

$$\begin{aligned} \text{At } D^- \quad \sum M_D = 0 \\ (0.3)(1.6) + M &= 0 \\ M &= -0.48 \text{ kNm} \end{aligned}$$

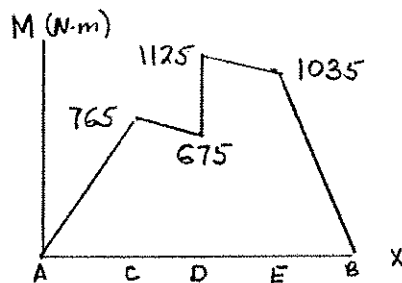
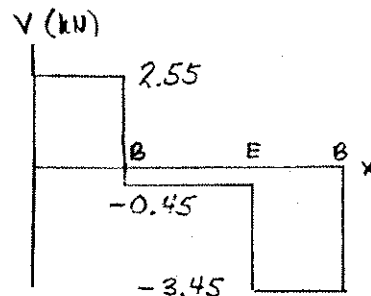
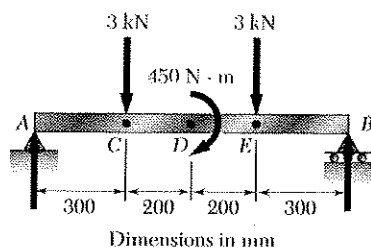
$$\begin{aligned} \text{At } D^+ \quad \sum M_D = 0 \\ (0.3)(1.6) - (0.02)(72) + M &= 0 \\ M &= 0.96 \text{ kNm} \end{aligned}$$

$$\begin{aligned} \text{At } E \quad \sum M_E = 0 \\ (0.6)(1.6) - (0.02)(72) + M &= 0 \\ M &= 0.48 \text{ kNm} \end{aligned}$$

$$\begin{aligned} \text{At } F^- \quad \sum M_F = 0 \\ -M - (0.02)(72) - (0.3)(1.6) &= 0 \\ M &= -1.92 \text{ kNm} \end{aligned}$$

$$\begin{aligned} \text{At } F^+ \quad \sum M_F = 0 \\ -M - (0.3)(1.6) &= 0 \\ M &= -0.48 \text{ kNm} \end{aligned}$$

Problem 5.12



5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

$$\sum M_B = 0: (700)(3) - 450 + (300)(3) - 1000A = 0$$

$$A = 2.55 \text{ kN} \uparrow$$

$$\sum M_A = 0: -(300)(3) - 450 - (700)(3) + 1000B = 0$$

$$B = 3.45 \text{ kN} \uparrow$$

At A. $V = 2.55 \text{ kN}$ $M = 0$

A to C. $V = 2.55 \text{ kN}$

At C.

$$\sum M_C = 0$$

$$-(300)(2.55) + M = 0$$

$$M = 765 \text{ N}\cdot\text{m}$$

C to E. $V = -0.45 \text{ N}\cdot\text{m}$

At D.

$$\sum M_D = 0:$$

$$-(500)(2.55) + (200)(3) + M = 0$$

$$M = 675 \text{ N}\cdot\text{m}$$

At D.

$$\sum M_E = 0:$$

$$-(500)(2.55) + (200)(3) - 450 + M = 0$$

$$M = 1125 \text{ N}\cdot\text{m}$$

E to B. $V = -3.45 \text{ kN}$

At E.

$$\sum M_E = 0:$$

$$-M + (300)(3.45) = 0$$

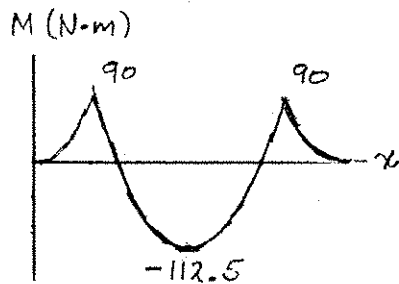
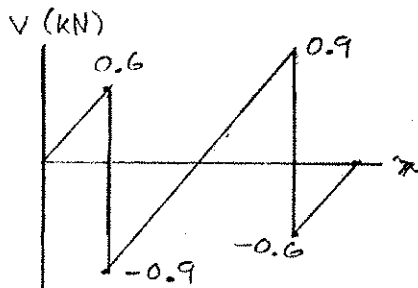
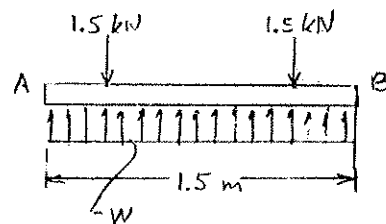
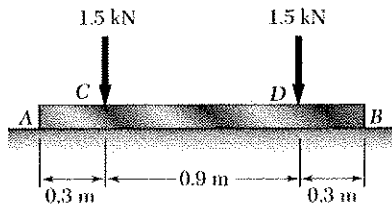
$$M = 1035 \text{ N}\cdot\text{m}$$

At B. $V = 3.45 \text{ kN}$, $M = 0$

(a) Maximum $|V| = 3.45 \text{ kN}$ ▶

(b) Maximum $|M| = 1125 \text{ N}\cdot\text{m}$ ▶

Problem 5.13



5.13 and 5.14 Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

Over the whole beam.

$$+\uparrow \sum F_y = 0: 1.5w - 1.5 - 1.5 = 0$$

$$w = 2 \text{ kN/m}$$

A to C. $0 \leq x < 0.3 \text{ m}$

$$+\uparrow \sum F_y = 0: 2x - V = 0$$

$$V = (2x) \text{ kN}$$

$$+\circlearrowleft \sum M_J = 0: -(2x)\left(\frac{x}{2}\right) + M = 0$$

$$M = (x^2) \text{ kN}\cdot\text{m}$$

At C^- , $x = 0.3 \text{ m}$

$$V = 0.6 \text{ kN}, M = 0.090 \text{ kN}\cdot\text{m}$$

$$= 90 \text{ N}\cdot\text{m}$$

C to D. $0.3 \text{ m} < x < 1.2 \text{ m}$

$$+\uparrow \sum F_y = 0: 2x - 1.5 - V = 0$$

$$V = (2x - 1.5) \text{ kN}$$

$$+\circlearrowleft \sum M_J = 0: -(2x)\left(\frac{x}{2}\right) + (1.5)(x - 0.3) + M = 0$$

$$M = (x^2 - 1.5x + 0.45) \text{ kN}\cdot\text{m}$$

At the center of the beam: $x = 0.75 \text{ m}$

$$V = 0$$

$$M = -0.1125 \text{ kN}\cdot\text{m}$$

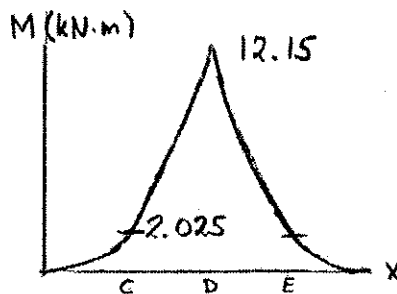
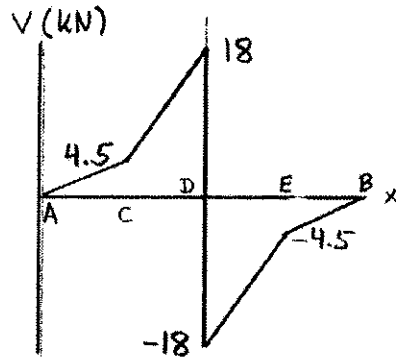
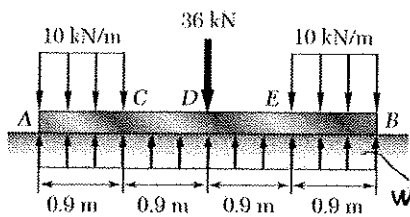
$$= -112.5 \text{ N}\cdot\text{m}$$

At C^+ , $x = 0.3 \text{ m}$, $V = -0.9 \text{ kN}$

(a) Maximum $|V| = 0.9 \text{ kN} = 900 \text{ N}$

(b) Maximum $|M| = 112.5 \text{ N}\cdot\text{m}$

Problem 5.14



(a) Maximum $|V|$
= 18 kN

(b) Maximum $|M|$
= 12.15 kN.m

5.13 and 5.14 Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value of (a) of the shear, (b) of the bending moment.

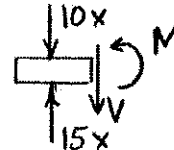
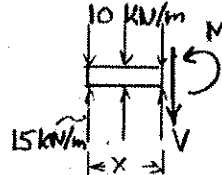
Over whole beam $\uparrow \sum F_y = 0$

$$3.6 w - (0.9)(10) - 36 - (0.9)(10) = 0$$

$$w = 15 \text{ kN/m}$$

A to C

$$0 < x < 0.9 \text{ m}$$



$$\uparrow \sum F_y = 0 \quad 15x - 10x - V = 0$$

$$V = 5x$$

$$\sum M_J = 0 \quad -(15x)\frac{x}{2} + (10x)\frac{x}{2} + M = 0$$

$$M = 2.5x^2$$

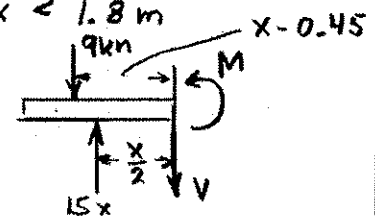
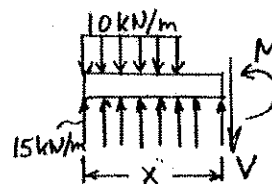
At $x = C$

$$V = 4.5 \text{ kN}$$

$$M = 2.025 \text{ kN.m}$$

C to D

$$0.9 \text{ m} < x < 1.8 \text{ m}$$



$$\uparrow \sum F_y = 0 \quad 15x - 9 - V = 0$$

$$V = 15x - 9$$

$$\sum M_J = 0 \quad -(15x)\left(\frac{x}{2}\right) + 9(x - 0.45) + M = 0$$

$$M = 7.5x^2 - 9x + 4.05 = 0$$

At D

$$V = 18 \text{ kN}$$

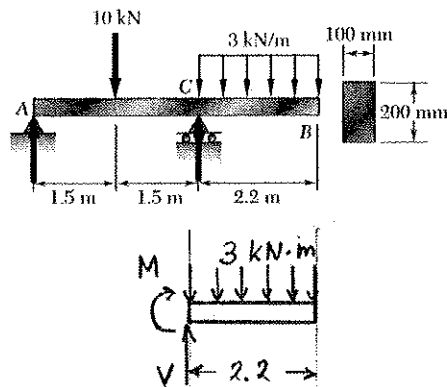
$$M = 12.15 \text{ kN.m}$$

D to B

Use symmetry to calculate the shear and bending moment.

Problem 5.15

5.15 and 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



Using CB as a free body

$$\sum M_C = 0$$

$$-M + (2.2)(3 \times 10^3)(1.1) = 0$$

$$M = 7.26 \times 10^3 \text{ N}\cdot\text{m}$$

Section modulus for rectangle

$$S = \frac{1}{6} b h^2$$

$$= \frac{1}{6} (100)(200)^2 = 666.7 \times 10^3 \text{ mm}^3$$

$$= 666.7 \times 10^{-6} \text{ m}^3$$

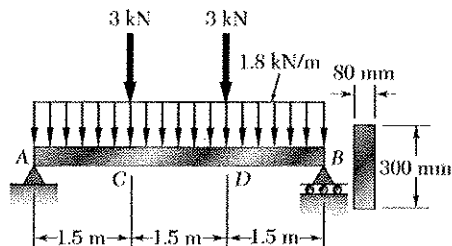
Normal stress

$$\sigma = \frac{M}{S} = \frac{7.26 \times 10^3}{666.7 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$$

$$\sigma = 10.89 \text{ MPa}$$

Problem 5.16

5.15 and 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



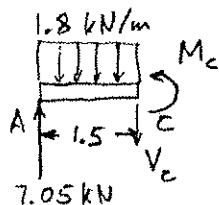
Reaction at A.

$$+\sum M_B = 0:$$

$$-4.5 A + (3.0)(3) + (1.5)(3) + (1.8)(4.5)(2.25) = 0$$

$$A = 7.05 \text{ kN} \uparrow$$

Use AC as free body.



$$+\sum M_C = 0: M_C - (7.05)(1.5) + (1.8)(1.5)(0.75) = 0$$

$$M_C = 8.55 \text{ kN}\cdot\text{m} = 8.55 \times 10^3 \text{ N}\cdot\text{m}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (80)(300)^3 = 180 \times 10^6 \text{ mm}^4 = 180 \times 10^{-6} \text{ m}^4$$

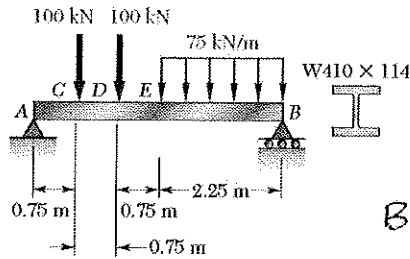
$$c = \frac{1}{2}(300) = 150 \text{ mm} = 0.150 \text{ m}$$

$$\sigma = \frac{M c}{I} = \frac{(8.55 \times 10^3)(0.150)}{180 \times 10^{-6}} = 7.125 \times 10^6 \text{ Pa}$$

$$\sigma = 7.13 \text{ MPa}$$

Problem 5.17

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



Reaction at A. $\rightarrow \sum M_B = 0$

$$-4.5 R_A + (3.75)(100) + (3)(100) + (75)(2.25)(1.125) = 0 \quad R_A = 192.2 \text{ kN}$$

Bending moment at C. $\rightarrow \sum M_C = 0$

$$-(0.75)(192.2) + M = 0$$

$$M = 144.15 \text{ kN}\cdot\text{m}$$

For W410 x 114, rolled steel section $S = 2200 \times 10^3 \text{ mm}^3$.

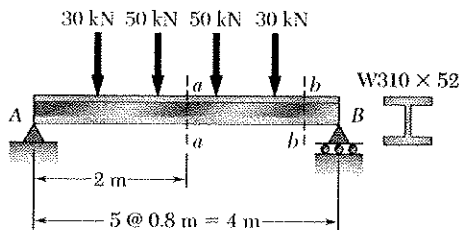
Normal stress at C

$$\sigma = \frac{M}{S} = \frac{144.15 \times 10^3}{2.2 \times 10^{-3}} = 65.5 \text{ MPa}$$



Problem 5.18

5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section a-a.



Reactions: By symmetry, $A = B$

$$+\uparrow \sum F_y = 0: \quad A = B = 80 \text{ kN}$$

Using left half of beam as free body,

$$\rightarrow \sum M_J = 0:$$

$$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$$

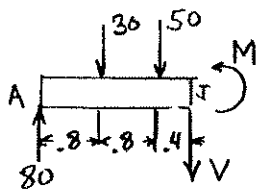
$$M = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$$

For W310 x 52,

$$S = 748 \times 10^3 \text{ mm}^3 = 748 \times 10^{-6} \text{ m}^3$$

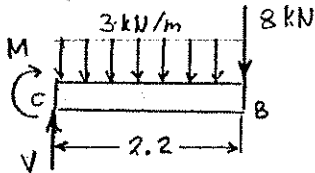
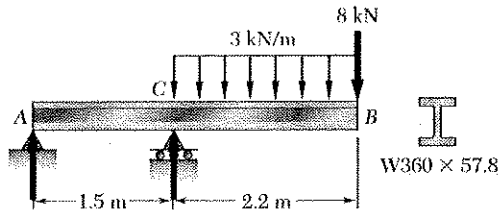
Normal stress. $\sigma = \frac{M}{S} = \frac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa}$

$$\sigma = 139.0 \text{ MPa}$$



Problem 5.19

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



Use portion CB as free body.

$$\oplus \sum M_C = 0:$$

$$-M - (1.1)(3)(2.2) - (2.2)(8) = 0$$

$$M = -24.86 \text{ kN}\cdot\text{m}$$

$$= -24.86 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{For } W360 \times 57.8, S = 899 \times 10^3 \text{ mm}^3 = 899 \times 10^{-6} \text{ m}^3$$

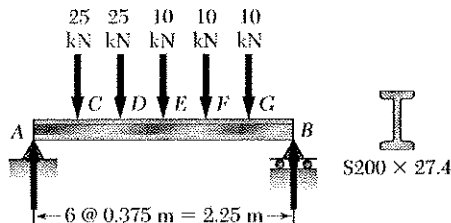
$$\text{Normal stress. } \sigma = \frac{|M|}{S}$$

$$\sigma = \frac{24.86 \times 10^3}{899 \times 10^{-6}} = 27.7 \times 10^6 \text{ Pa}$$

$$\sigma = 27.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 5.20

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



Use entire beam as free body.

$$\oplus \sum M_B = 0:$$

$$2.25 A - (1.875)(25) - (1.5)(25) - (1.125)(10) - (0.75)(10) - (0.375)(10) = 0$$

$$A = 47.5 \text{ kN}$$

Use portion AC as free body.

$$-(0.375)(47.5) + M = 0 \quad M = 17.8125 \text{ kN}\cdot\text{m}$$

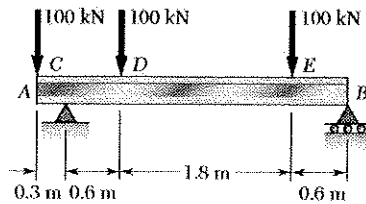
$$\text{For } S200 \times 27.4, S = 235 \times 10^3 \text{ mm}^3 = 235 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress. } \sigma = \frac{M}{S} = \frac{17.8125 \times 10^3}{235 \times 10^{-6}} = 75.8 \times 10^6 \text{ Pa}$$

$$\sigma = 75.8 \text{ MPa} \quad \blacktriangleleft$$

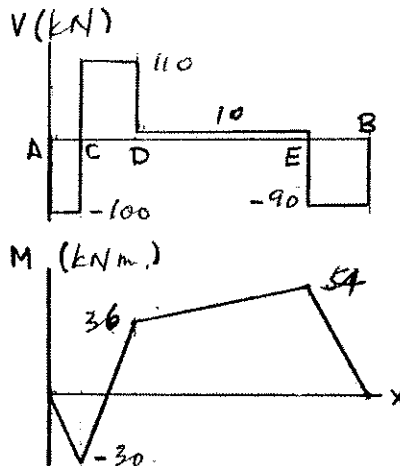
Problem 5.21

5.21 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending



$$\begin{aligned} \sum M_B = 0 \\ (3.3)(100) - 3C + (2.4)(100) + (0.6)(100) = 0 \\ C = 210 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_C = 0 \\ (100)(0.3) - (100)(0.6) - (2.4)(100) + 3B = 0 \\ B = 90 \text{ kN} \end{aligned}$$



Shear

$$\begin{aligned} A \text{ to } C^- & V = -100 \text{ kN} \\ C^+ \text{ to } D^- & V = 110 \text{ kN} \\ D^+ \text{ to } E^- & V = -90 \text{ kN} \\ E^+ \text{ to } B & V = 10 \text{ kN} \end{aligned}$$

Bending moments

$$\begin{aligned} \text{At C} \\ \sum M_C = 0 \\ (0.3)(100) + M = 0 \\ M = -30 \text{ kNm} \end{aligned}$$

$$\begin{aligned} \text{At D} \\ \sum M_D = 0 \\ (0.9)(100) - (0.6)(210) + M = 0 \\ M = 36 \text{ kNm} \end{aligned}$$

$$\begin{aligned} \text{At E} \\ \sum M_E = 0 \\ -M + (0.6)(90) = 0 \\ M = 54 \text{ kNm} \end{aligned}$$

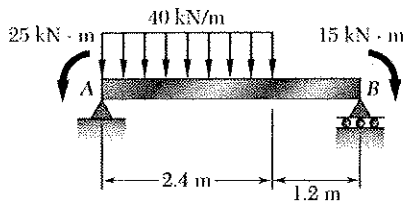
$$\max |M| = 54 \text{ kNm}$$

$$\text{For S310 X 52 rolled steel section} \quad S = 625 \times 10^3 \text{ mm}^3$$

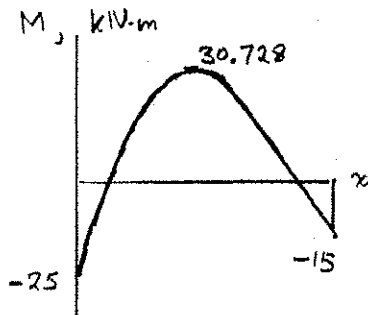
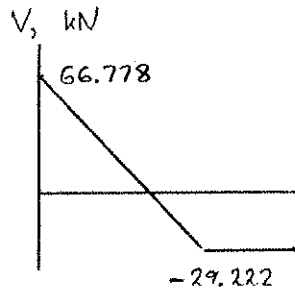
$$\text{Normal stress} \quad \sigma = \frac{M}{S} = \frac{54 \times 10^3}{625 \times 10^3} = 86.4 \text{ MPa}$$

Problem 5.22

5.22 and 5.23 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.




W310 × 38.7



Reaction at A. $\circlearrowleft \sum M_B = 0:$

$$-3.6 R_A + 25 + (2.4)(2.4)(40) - 15 = 0$$

$$R_A = 66.778 \text{ kN}$$

Reaction at B. $+ \circlearrowright \sum M_A = 0:$

$$25 - (1.2)(2.4)(40) - 15 + 3.6 R_B = 0$$

$$R_B = 29.222 \text{ kN}$$

$$0 \leq x \leq 2.4 \text{ m}$$

$$+\uparrow \sum F_y = 0:$$

$$66.778 - 40x - V = 0$$

$$V = 66.778 - 40x$$

$$V = 0 \text{ at } x = 1.6944 \text{ m}$$

$$+\circlearrowleft \sum M_y = 0:$$

$$25 + (40x)\frac{x}{2} - 66.778x + M = 0$$

$$M = -20x^2 + 66.778x - 25$$

$$M = -(20)(1.6944)^2 + (66.778)(1.6944) - 25$$

$$= 30.728 \text{ kN}\cdot\text{m at } x = 1.6944 \text{ m}$$

$$2.4 \leq x < 3.6$$

$$V = -29.222 \text{ kN}$$

$$M = 29.222(3.6 - x) - 15$$

$$M = 20.067 \text{ kN}\cdot\text{m at } x = 2.4 \text{ m}$$

$$M_{\max} = 30.728 \text{ kN}\cdot\text{m} = 30.728 \times 10^3 \text{ N}\cdot\text{m}$$

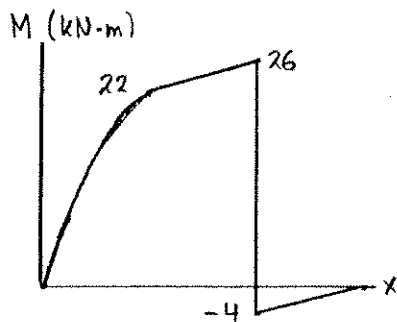
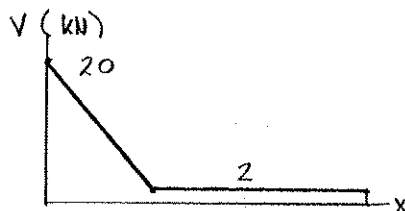
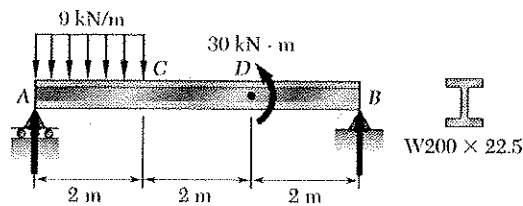
For W310 × 38.7 rolled steel section, $S = 549 \times 10^3 \text{ mm}^3$
 $= 549 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{M_{\max}}{S} = \frac{30.728 \times 10^3}{549 \times 10^{-6}} = 55.97 \times 10^6 \text{ Pa}$$

$$\sigma_m = 56.0 \text{ MPa} \quad \blacktriangleleft$$

Problem 5.23

5.22 and 5.23 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$\sum M_B = 0:$$

$$-6A + (2)(9)(5) + 30 = 0$$

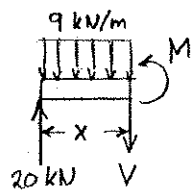
$$A = 20 \text{ kN}$$

$$\sum M_A = 0:$$

$$-(2)(9)(1) + 30 + 6B = 0$$

$$B = -2 \text{ kN} \quad \text{i.e. } 2 \text{ kN} \downarrow$$

A to C, $0 < x < 2 \text{ m}$



$$\sum F_y = 0 \quad 20 - 9x - V = 0$$

$$V = 20 - 9x$$

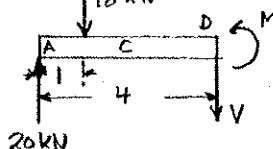
$$\sum M_x = 0:$$

$$-20x + (9x)\left(\frac{x}{2}\right) + M = 0$$

$$M = 20x - 4.5x^2$$

At C, $V = 2 \text{ kN}$, $M = 22 \text{ kN}\cdot\text{m}$

At D, $V = 18 \text{ kN}$



$$\sum F_y = 0:$$

$$20 - 18 - V = 0$$

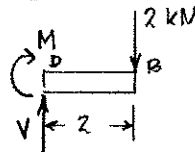
$$V = 2 \text{ kN}$$

$$\sum M_D = 0:$$

$$-(4)(20) + (3)(18) + M = 0$$

$$M = 26 \text{ kN}\cdot\text{m}$$

At D⁺



$$\sum F_y = 0: V - 2 = 0$$

$$V = 2 \text{ kN}$$

$$\sum M_D = 0:$$

$$-M - (2)(2) = 0$$

$$M = -4 \text{ kN}\cdot\text{m}$$

$$\max |M| = 26 \text{ kN}\cdot\text{m} = 26 \times 10^3 \text{ N}\cdot\text{m}$$

For rolled steel section W 200 x 22.5,

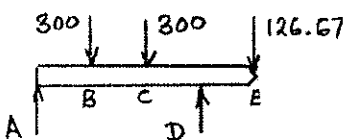
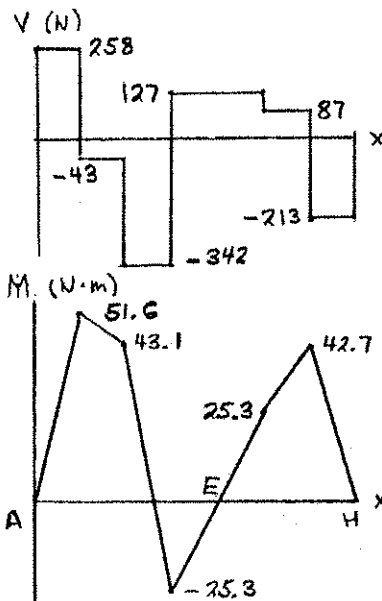
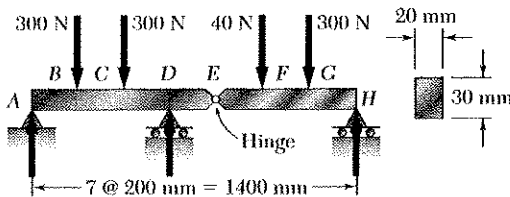
$$S = 194 \times 10^3 \text{ mm}^3$$

$$= 194 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress, } \sigma = \frac{|M|}{S} = \frac{26 \times 10^3}{194 \times 10^{-6}} = 134.0 \times 10^6 \text{ Pa} = 134.0 \text{ MPa} \quad \leftarrow$$

Problem 5.24

5.24 and 5.25 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Free body EFGH.
Note that $M_E = 0$ due to hinge.

$$\sum M_E = 0:$$

$$0.6 H - (0.2)(40) - (0.4)(300) = 0$$

$$H = 213.33 \text{ N}$$

$$\sum F_y = 0: V_E - 40 - 300 + 213.33 = 0$$

$$V_E = 126.67 \text{ N}$$

$$\text{Shear: E to F. } V = 126.67 \text{ N}$$

$$\text{F to G. } V = 86.67 \text{ N}$$

$$\text{G to H. } V = -213.33 \text{ N}$$

Bending moment at F.

$$\sum M_F = 0: M_F - (0.2)(126.67) = 0$$

$$M_F = 25.33 \text{ N}\cdot\text{m}$$

Bending moment at G.

$$\sum M_G = 0: -M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N}\cdot\text{m}$$

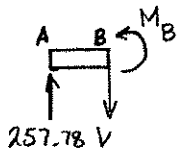
Free body ABCDE.

$$\sum M_B = 0: -0.6 A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$$

$$A = 257.78 \text{ N}$$

$$\sum M_A = 0: -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6 D = 0$$

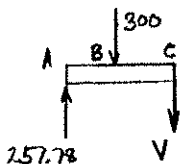
$$D = 468.89 \text{ N}$$



Bending moment at B.

$$\sum M_B = 0: -(0.2)(257.78) + M_B = 0$$

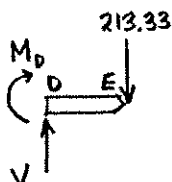
$$M_B = 51.56 \text{ N}\cdot\text{m}$$



Bending moment at C.

$$\sum M_C = 0: -(0.4)(257.78) + (0.2)(300) + M_C = 0$$

$$M_C = 43.11 \text{ N}\cdot\text{m}$$



Bending moment at D.

$$\sum M_D = 0: -M_D - (0.2)(213.33) = 0$$

$$M_D = -25.33 \text{ N}\cdot\text{m}$$

$$\max |M| = 51.56 \text{ N}\cdot\text{m}$$

$$S = \frac{1}{6} b h^2 = \frac{1}{6} (20)(30)^2$$

$$= 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$$

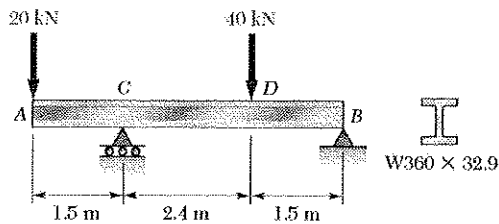
Normal stress.

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa}$$

$$\sigma = 17.19 \text{ MPa}$$

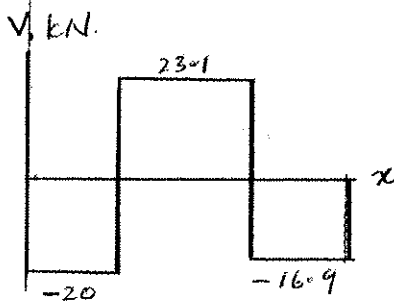
Problem 5.25

5.25 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



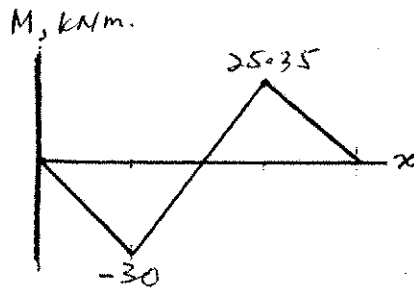
$$\begin{aligned} \text{Reaction at C} \quad \sum M_B = 0 \\ (5.4)(20) - 3.9C + (1.5)(40) = 0 \\ C = 43.1 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Reaction at B} \quad +\sum M_C = 0 \\ (1.5)(20) - (2.4)(40) + 3.9B = 0 \\ B = 16.9 \text{ kN} \end{aligned}$$



Shear diagram.

$$\begin{aligned} \text{A to C} \quad V &= -20 \text{ kN} \\ \text{C to D} \quad V &= -20 + 43.1 = 23.1 \text{ kN} \\ \text{D to B} \quad V &= 23.1 - 40 = -16.9 \text{ kN} \end{aligned}$$



$$\text{At A and B} \quad M = 0$$

$$\begin{aligned} \text{At C} \quad +\sum M_C = 0 \\ (20)(1.5) + M_C = 0 \\ M_C = -30 \text{ kNm} \end{aligned}$$

$$\begin{aligned} \text{At D} \quad +\sum M_D = 0 \\ -M_D + (1.5)(16.9) = 0 \\ M_D = 25.35 \text{ kNm} \end{aligned}$$

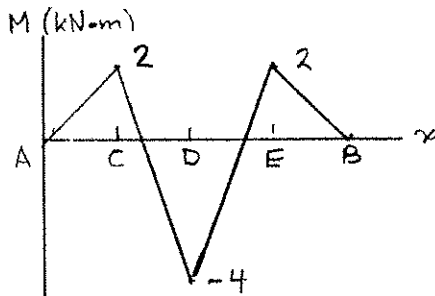
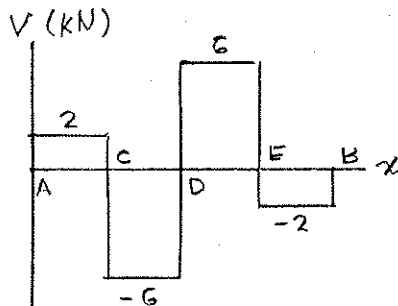
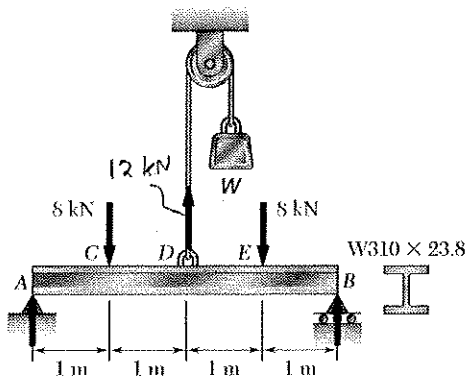
$$|M|_{\max} \text{ occurs at C} \quad |M|_{\max} = 30 \text{ kNm}$$

$$\text{For W360 x 32.9 rolled steel section} \quad S = 474 \times 10^3 \text{ mm}^3$$

$$\text{Normal stress} \quad \sigma = \frac{M}{S} = \frac{30 \text{ kNm}}{474 \times 10^{-6}} = 63.3 \text{ MPa}$$

Problem 5.26

5.26 Knowing that $W = 12 \text{ kN}$, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.



By symmetry, $A = B$

$$+\uparrow \sum F_y = 0: A - 8 + 12 - 8 + B = 0$$

$$A = B = 2 \text{ kN}$$

Shear:

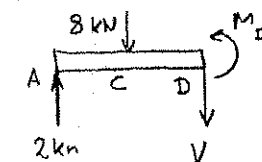
$$\begin{aligned} A \text{ to } C: & V = 2 \text{ kN} \\ C^+ \text{ to } D: & V = -6 \text{ kN} \\ D^+ \text{ to } E: & V = 6 \text{ kN} \\ E^+ \text{ to } B: & V = -2 \text{ kN} \end{aligned}$$

Bending moment:

$$A \text{ to } C: +\circlearrowleft \sum M_C = 0:$$

$$\begin{aligned} M_C - (1)(2) &= 0 \\ M_C &= 2 \text{ kN}\cdot\text{m} \end{aligned}$$

$$A \text{ to } D:$$



$$+\circlearrowleft \sum M_D = 0:$$

$$M_D - (2)(2) + (8)(1) = 0$$

$$M_D = -4 \text{ kN}\cdot\text{m}$$

$$M_E = 2 \text{ kN}\cdot\text{m}$$

By symmetry, $M = 2 \text{ kN}\cdot\text{m}$ at E

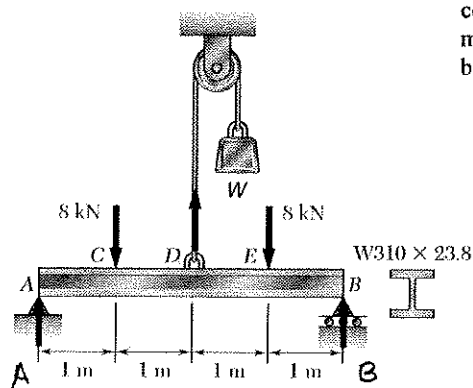
$\max |M| = 4 \text{ kN}\cdot\text{m}$ occurs at E

$$\text{For } W310 \times 23.8, S_x = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress: } \sigma_{\max} = \frac{|M|_{\max}}{S_x} = \frac{4 \times 10^3}{280 \times 10^{-6}} = 14.29 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = 14.29 \text{ MPa}$$

Problem 5.27



5.27 Determine (a) the magnitude of the counterweight W for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (Hint: Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)

By symmetry, $A = B$

$$+\uparrow \sum F_y = 0: A - 8 + W - 8 + B = 0$$

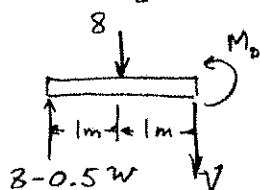
$$A = B = 8 - 0.5W$$

Bending moment at C. $+\circlearrowleft \sum M_C = 0:$

$$M_C - (8 - 0.5W)(1) + M_C = 0$$

$$M_C = (8 - 0.5W) \text{ kN}\cdot\text{m}$$

Bending moment at D.



$$-\circlearrowleft \sum M_D = 0: -(8 - 0.5W)(2) + (8)(1) + M_D = 0$$

$$M_D = (8 - W) \text{ kN}\cdot\text{m}$$

Equate. $-M_D = M_C$

$$W - 8 = 8 - 0.5W$$

(a)

$$W = 10.6667 \text{ kN} \quad W = 10.67 \text{ kN} \blacktriangleleft$$

$$M_C = -2.6667 \text{ kN}\cdot\text{m}$$

$$M_D = 2.6667 \text{ kN}\cdot\text{m} = 2.6667 \times 10^3 \text{ N}\cdot\text{m}$$

$$|M|_{\max} = 2.6667 \text{ kN}\cdot\text{m}$$

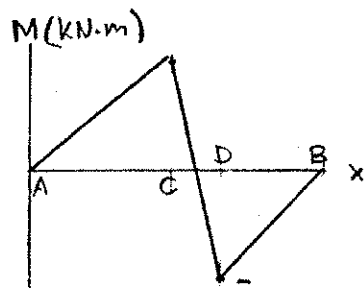
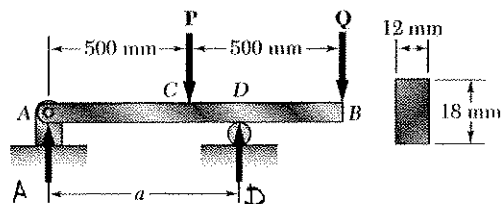
For W310 x 23.8 rolled steel shape,

$$S_x = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{ m}^3$$

(b) $\sigma_{\max} = \frac{|M|_{\max}}{S_x} = \frac{2.6667 \times 10^3}{280 \times 10^{-6}} = 9.52 \times 10^6 \text{ Pa} \quad \sigma_{\max} = 9.52 \text{ MPa} \blacktriangleleft$

Problem 5.28

5.28 Knowing that $P = Q = 480 \text{ N}$, determine (a) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)



$$P = 480 \text{ N} \quad Q = 480 \text{ N}$$

Reaction at A. $+\circlearrowleft \Sigma M_D = 0:$

$$-Aa + 480(a - 0.5) - 480(1 - a) = 0$$

$$A = (960 - \frac{720}{a}) \text{ N}$$

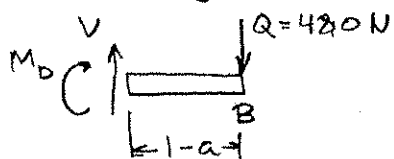
Bending moment at C. $+\circlearrowleft \Sigma M_C = 0:$

$$-0.5A + M_C = 0$$

$$M_C = 0.5A$$

$$= (480 - \frac{360}{a}) \text{ N}\cdot\text{m}$$

Bending moment at D. $+\circlearrowleft \Sigma M_D = 0:$



$$-M_D - 480(1 - a) = 0$$

$$M_D = -480(1 - a) \text{ N}\cdot\text{m}$$

(a) Equate, $-M_D = M_C$ $480(1 - a) = 480 - \frac{360}{a}$

$$a = 0.86603 \text{ m} \quad a = 866 \text{ mm} \quad \blacktriangleleft$$

$$A = 128.62 \text{ N} \quad M_C = 64.31 \text{ N}\cdot\text{m} \quad M_D = -64.31 \text{ N}\cdot\text{m}$$

(b) For rectangular section, $S = \frac{1}{6}bh^2$

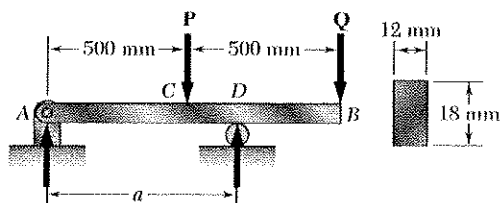
$$S = \frac{1}{6}(12)(18)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{ m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{64.31}{648 \times 10^{-9}} = 99.2 \times 10^6 \text{ Pa} \quad \sigma_{\max} = 99.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 5.29

5.29 Solve Prob. 5.28, assuming that $P = 480 \text{ N}$ and $Q = 320 \text{ N}$.

5.28 Knowing that $P = Q = 480 \text{ N}$, determine (a) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)



$$P = 480 \text{ N} \quad Q = 320 \text{ N}$$

$$\text{Reaction at A. } +\circlearrowleft \Sigma M_D = 0:$$

$$Aa + 480(a - 0.5) - 320(1 - a) = 0$$

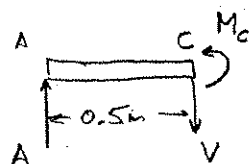
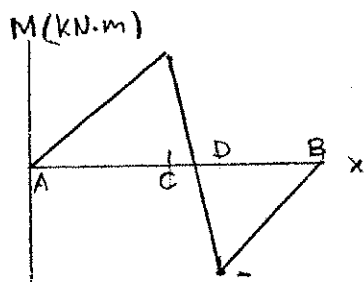
$$A = (800 - \frac{560}{a}) \text{ N}$$

$$\text{Bending moment at C. } +\circlearrowleft \Sigma M_C = 0:$$

$$-0.5A + M_C = 0$$

$$M_C = 0.5A$$

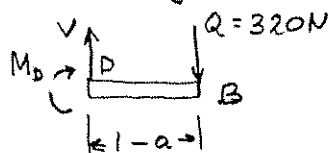
$$= (400 - \frac{280}{a}) \text{ N}\cdot\text{m}$$



$$\text{Bending moment at D. } +\circlearrowleft \Sigma M_D = 0:$$

$$-M_D - 320(1 - a) = 0$$

$$M_D = (-320 + 320a) \text{ N}\cdot\text{m}$$



$$(a) \text{ Equate. } -M_D = M_C$$

$$320 - 320a = 400 - \frac{280}{a}$$

$$320a^2 + 80a - 280 = 0$$

$$a = 0.81873 \text{ m}, -1.06873 \text{ m}$$

$$\text{Reject negative root. } a = 819 \text{ mm} \blacktriangleleft$$

$$A = 116.014 \text{ N}$$

$$M_C = 58.007 \text{ N}\cdot\text{m}$$

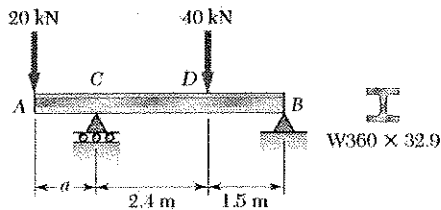
$$M_D = -58.006 \text{ N}\cdot\text{m}$$

$$(b) \text{ For rectangular section, } S = \frac{1}{6}bh^2$$

$$S = \frac{1}{6}(12)(18)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{ m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{58.0065}{648 \times 10^{-9}} = 89.5 \times 10^6 \text{ Pa} \quad \sigma_{\max} = 89.5 \text{ MPa} \blacktriangleleft$$

Problem 5.30



5.30 Determine (a) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)
(Hint: Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)

Reaction at B. $+\circlearrowright \Sigma M_C = 0$

$$20a - (24)(40) + 3.9 R_B = 0$$

$$R_B = \frac{1}{3.9}(96 - 20a)$$

Bending moment at D.

$$+\circlearrowright \Sigma M_D = 0$$

$$-M_D + 1.5 R_B = 0$$

$$M_D = 1.5 R_B = \frac{1.5}{3.9}(96 - 20a)$$

Bending moment at C

$$+\circlearrowright \Sigma M_C = 0$$

$$20a + M_C = 0$$

$$M_C = -20a$$

Equate $-M_C = M_D$

$$20a = \frac{1.5}{3.9}(96 - 20a)$$

$$a = 1.33 \text{ m}$$

$$(a) \quad a = 1.33 \text{ m} \quad \blacktriangleleft$$

Then $-M_C = M_D = (20)(1.33) = 26.7 \text{ kNm}$

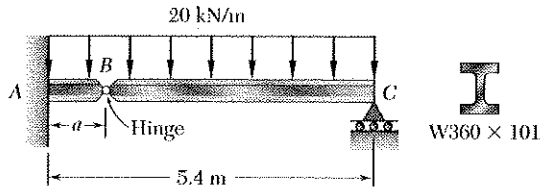
$$|M|_{\max} = 26.7 \text{ kNm}$$

For W360x32.9 rolled steel section $S = 474 \times 10^3 \text{ mm}^3$

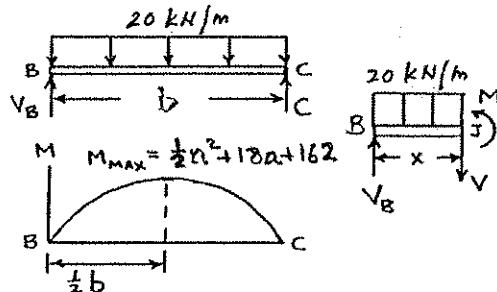
Normal stress $\sigma = \frac{M}{S} = \frac{26.7 \times 10^3}{474 \times 10^{-6}} = 56.3 \text{ MPa} \quad (b) \quad 56.3 \text{ MPa} \quad \blacktriangleleft$

Problem 5.31

5.31 Determine (a) the distance a for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)



SEGMENT BC:



For W360 x 101, $S_x = 1690 \times 10^3 \text{ mm}^3$

Let $b = (5.4 - a) \text{ m}$

By symmetry, $V_B = C$

$$+\uparrow \Sigma F_y = 0: V_B + C - 20b = 0$$

$$V_B = 10b$$

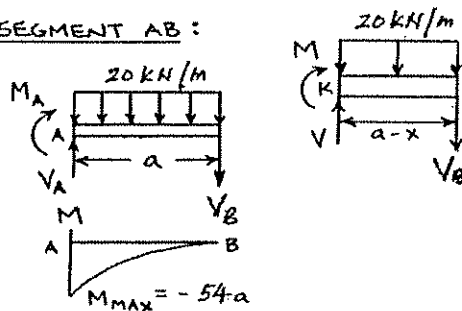
$$+\circlearrowleft \Sigma M_J = 0: -V_B x + (20x)(\frac{x}{2}) + M = 0$$

$$M = V_B x - 10x^2 = 10bx - 10x^2 \text{ kNm}$$

$$\frac{dM}{dx} = 10b - 20x_m = 0 \quad x_m = \frac{1}{2}b$$

$$M_{\max} = 5b^2 - \frac{5}{2}b^2 = \frac{5}{2}b^2$$

SEGMENT AB:



$$+\circlearrowleft \Sigma M_K = 0:$$

$$-20(a-x)(\frac{a-x}{2}) - V_B(a-x) - M = 0$$

$$M = -10(a-x)^2 + 10b(a-x)$$

$|M_{\max}|$ occurs at $x = 0$.

$$|M_{\max}| = -10a^2 - 10ab$$

$$= -10a^2 - 10a(5.4 - a) = 54a$$

(a) Equate the two values of $|M_{\max}|$.

$$54a = \frac{5}{2}b^2 = \frac{5}{2}(5.4 - a)^2 = 72.9 - 27a + \frac{5}{2}a^2$$

$$\frac{5}{2}a^2 - 81a + 72.9 = 0 \quad a = \left[81 \pm \sqrt{(81)^2 - (4)(\frac{5}{2})(72.9)} \right] / 5$$

$$a = (81 \pm 76.37) / 5 = 0.926 \text{ m}$$

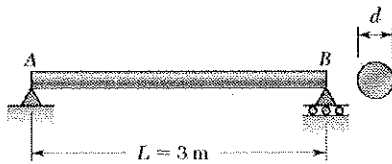
$$a = 0.93 \text{ m}$$

(b) $|M|_{\max} = 54a = 50 \text{ kNm}$

$$\sigma = \frac{|M|_{\max}}{S_x} = \frac{50 \times 10^3}{1690 \times 10^{-6}} = 29.58 \text{ MPa}$$

$$\sigma_m = 29.6 \text{ MPa}$$

Problem 5.32



5.32 A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 7860 \text{ kg/m}^3$, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 28 MPa.

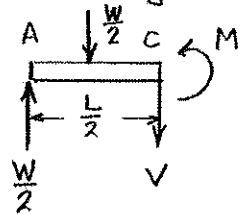
Let $W = \text{total weight}$

$$W = \gamma V = AL\gamma = \frac{\pi}{4} d^2 L \gamma$$

Reaction at A

$$A = \frac{1}{2} W$$

Bending moment at center of beam



$$\sum M_C = 0$$

$$-\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section ($c = \frac{1}{2}d$)

$$I = \frac{\pi}{4} c^4, \quad S = \frac{I}{c} = \frac{\pi}{4} c^3 = \frac{\pi}{32} d^3$$

Normal stress

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32} d^2 L^2 \gamma}{\frac{\pi}{32} d^3} = \frac{L^2 \gamma}{d}$$

Solving for d
$$d = \frac{L^2 \gamma}{\sigma}$$

Data: $L = 3 \text{ m}$

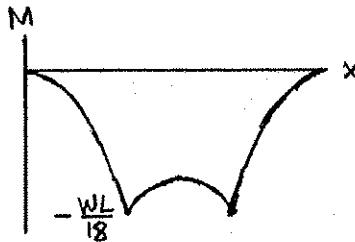
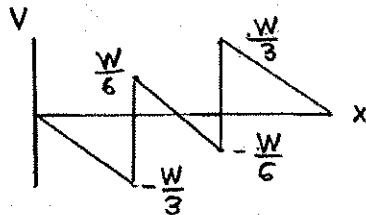
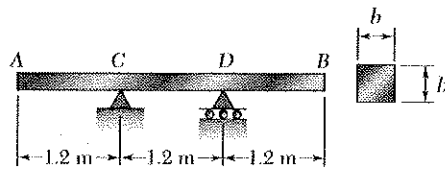
$$\gamma = (7860 \times 9.81) \text{ N/m}^3$$

$$\sigma = 28 \text{ MPa}$$

$$d = \frac{(3)^2 (7860 \times 9.81)}{28 \times 10^6} = 0.0248 \text{ m} = 24.8 \text{ mm}$$

Problem 5.33

5.33 A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860 \text{ kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (a) 10 MPa, (b) 50 MPa.



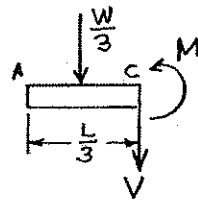
Weight density $\gamma = \rho g$

Let L = total length of beam.

$$W = \gamma V = AL\rho g = b^2 L\rho g$$

Reactions at C and D. $C = D = \frac{W}{2}$

Bending moment at C.

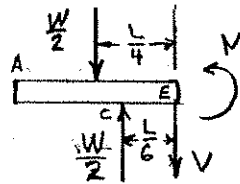


$$\sum M_C = 0:$$

$$\left(\frac{L}{3}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$

Bending moment at center of beam.



$$\sum M_E = 0:$$

$$\left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0$$

$$M = -\frac{WL}{24}$$

$$\max |M| = \frac{WL}{18} = \frac{b^2 L^2 \rho g}{18}$$

For a square section, $S = \frac{1}{6} b^3$

$$\text{Normal stress, } \sigma = \frac{|M|}{S} = \frac{b^2 L^2 \rho g / 18}{b^3 / 6} = \frac{L^2 \rho g}{3b}$$

$$\text{Solve for } b. \quad b = \frac{L^2 \rho g}{3\sigma}$$

Data: $L = 3.6 \text{ m}$ $\rho = 7860 \text{ kg/m}^3$ $g = 9.81 \text{ m/s}^2$

(a) $\sigma = 10 \times 10^6 \text{ Pa}$

(b) $\sigma = 50 \times 10^6 \text{ Pa}$

$$(a) \quad b = \frac{(3.6)^2 (7860) (9.81)}{(3)(10 \times 10^6)} = 33.3 \times 10^{-3} \text{ m}$$

$$b = 33.3 \text{ mm} \quad \blacktriangleleft$$

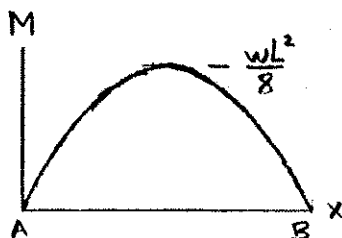
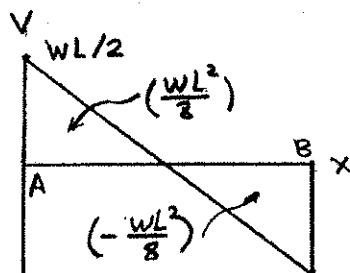
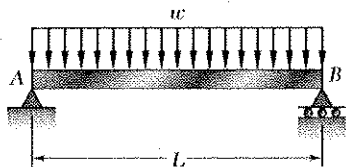
$$(b) \quad b = \frac{(3.6)^2 (7860) (9.81)}{(3)(50 \times 10^6)} = 6.66 \times 10^{-3} \text{ m}$$

$$b = 6.66 \text{ mm} \quad \blacktriangleleft$$

Problem 5.34

5.34 Using the method of Sec. 5.3, solve Prob. 5.1a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$\oplus \sum M_B = 0: -AL + wL \cdot \frac{L}{2} = 0 \quad A = \frac{wL}{2}$$

$$\oplus \sum M_A = 0: BL - wL \cdot \frac{L}{2} = 0 \quad B = \frac{wL}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = - \int_0^x w \, dx = -wx$$

$$V = V_A - wx = A - wx = \frac{wL}{2} - wx$$

$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V \, dx = \int_0^x \left(\frac{wL}{2} - wx \right) dx$$

$$= \frac{wLx}{2} - \frac{wx^2}{2}$$

$$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w}{2} (Lx - x^2)$$

Maximum M occurs at $x = \frac{L}{2}$, where

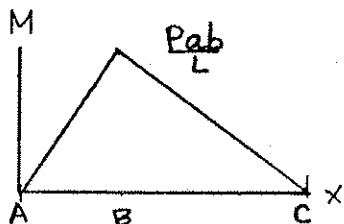
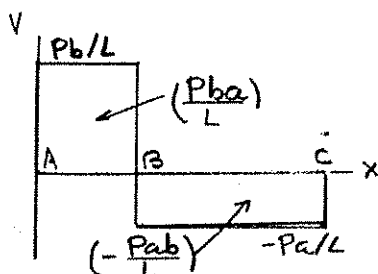
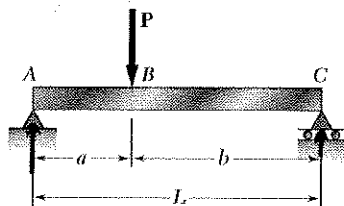
$$V = \frac{dM}{dx} = 0$$

$$M_{\max} = \frac{wL^2}{8}$$

Problem 5.35

5.35 Using the method of Sec. 5.3, solve Prob. 5.2a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$\sum M_C = 0: \quad LA - bP = 0 \quad A = \frac{Pb}{L}$$

$$\sum M_A = 0: \quad Lc - aP = 0 \quad C = \frac{Pa}{L}$$

$$\text{At } A^+: \quad V = A = \frac{Pb}{L} \quad M = 0$$

$$\text{A to B:} \quad 0 < x < a$$

$$w = 0 \quad \int_0^x w dx = 0$$

$$V - V_A = 0 \quad V = \frac{Pb}{L}$$

$$M_B - M_A = \int_0^a V dx = \int_0^a \frac{Pb}{L} dx = \frac{Pba}{L}$$

$$M_B = \frac{Pba}{L}$$

$$\text{At } B^+: \quad V = A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$$

$$\text{B to C:} \quad a < x < L$$

$$w = 0 \quad \int_a^x w dx = 0$$

$$V_C - V_B = 0 \quad V = -\frac{Pa}{L}$$

$$M_C - M_B = \int_a^L V dx = -\frac{Pa}{L}(L-a) = -\frac{Pab}{L}$$

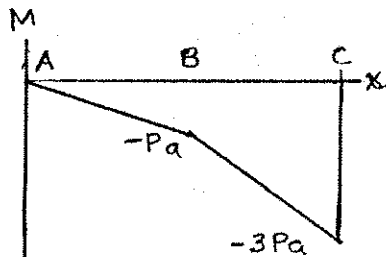
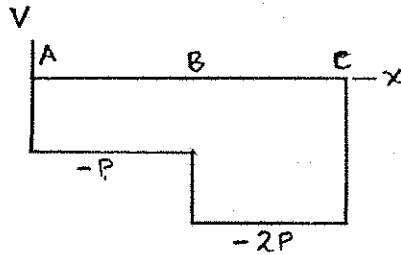
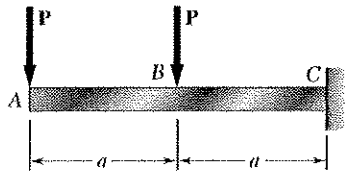
$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

$$|M|_{\max} = \frac{Pab}{L}$$

Problem 5.36

5.36 Using the method of Sec. 5.3, solve Prob. 5.3a.

5.1 through 5.6 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



$$\text{At } A^+ \quad V_A = -P$$

$$\text{Over } AB, \quad \frac{dV}{dx} = -w = 0$$

$$\frac{dM}{dx} = V = V_A = -P$$

$$M = -Px + C$$

$$M = 0 \text{ at } x = 0 \quad C_1 = 0$$

$$M = -Px$$

$$\text{At point B, } x = a \quad M = -Pa$$

$$\text{At point B}^+ \quad V = -P - P = -2P$$

$$\text{Over } BC, \quad \frac{dV}{dx} = -w = 0$$

$$\frac{dM}{dx} = V = -2P$$

$$M = -2Px + C_2$$

$$\text{At B, } x = a \quad M = -Pa$$

$$-Pa = -2Pa + C_2 \quad C_2 = Pa$$

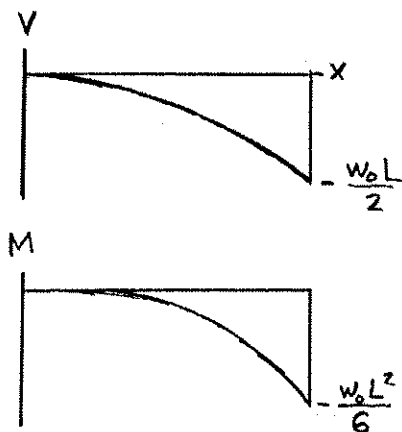
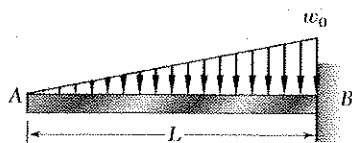
$$M = -2Px + Pa$$

$$\text{At C, } x = 2a \quad M = -3Pa$$

Problem 5.37

5.37 Using the method of Sec. 5.3, solve Prob. 5.4a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$w = w_0 \frac{x}{L}$$

$$V_A = 0, \quad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V - V_A = -\int_0^x \frac{w_0 x}{L} dx = -\frac{w_0 x^2}{2L}$$

$$V = -\frac{w_0 x^2}{2L}$$

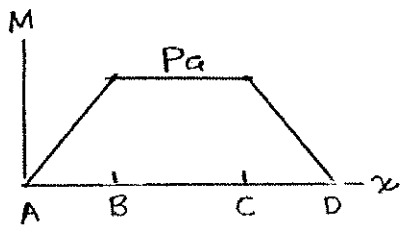
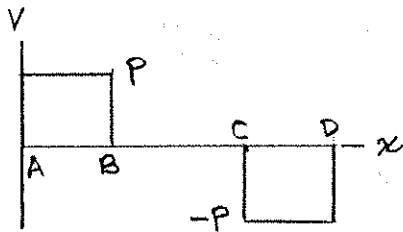
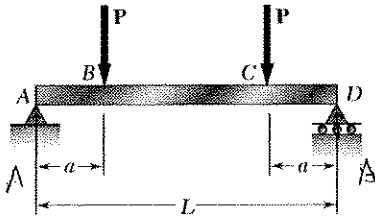
$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V dx = -\int_0^x \frac{w_0 x^2}{2L} dx = -\frac{w_0 x^3}{6L}$$

Problem 5.38

5.38 Using the method of Sec. 5.3, solve Prob. 5.5a.

5.1 through 5.6 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.



Reactions. By symmetry: $A = B$

$$+\uparrow \Sigma F_y = 0: A - P - P + B = 0$$

$$A = B = P$$

Over each portion AB, BC, and CD $w = 0$.

$$\frac{dV}{dx} = 0 \quad V = \text{constant.}$$

Shear diagram.

$$A \text{ to } B: V = P$$

$$B \text{ to } C: V = P - P = 0$$

$$C \text{ to } D: V = 0 - P = -P$$

Bending moment diagram.

$$A \text{ to } B: \frac{dM}{dx} = V = P$$

$$M = \int P dx = Px + C_1$$

$$M = 0 \text{ at } x = 0. \text{ Thus } C_1 = 0$$

$$M = Px$$

$$\text{At } B: M = Pa$$

$$B \text{ to } C: \frac{dM}{dx} = V = 0$$

$$M = Pa$$

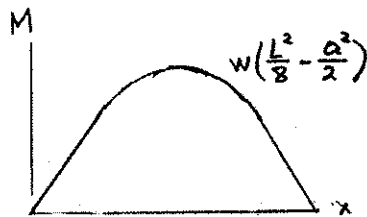
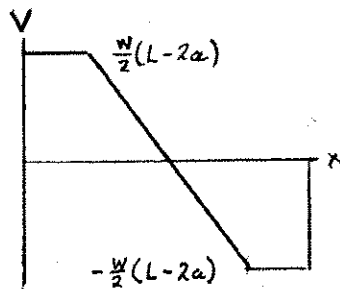
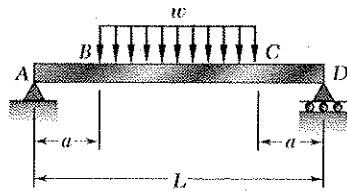
$$C \text{ to } D: \frac{dM}{dx} = V = -P$$

$$M = -\int P dx = -Px + C_2$$

$$M = 0 \text{ at } x = L. \text{ Thus } C_2 = PL$$

$$M = P(L - x)$$

Problem 5.39



5.39 Using the method of Sec. 5.3, solve Prob. 5.6a.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

Reactions. $A = D = \frac{1}{2} w(L - 2a)$

At A. $V_A = A = \frac{1}{2} w(L - 2a)$, $M_A = 0$

A to B. $0 < x < a$ $w = 0$

$$V_B - V_A = -\int_0^a w dx = 0$$

$$V_B = V_A = \frac{1}{2} w(L - 2a)$$

$$M_B - M_A = \int_0^a V dx = \int_0^a \frac{1}{2} w(L - 2a) dx$$

$$M_B = \frac{1}{2} w(L - 2a)a$$

B to C. $a < x < L - a$ $w = w$

$$V - V_B = -\int_a^x w dx = -w(x - a)$$

$$V = \frac{1}{2} w(L - 2a) - w(x - a) = \frac{1}{2} w(L - 2x)$$

$$\frac{dM}{dx} = V = \frac{1}{2} w(L - 2x)$$

$$M - M_B = \int_a^x V dx = \frac{1}{2} w(Lx - x^2) \Big|_a^x$$

$$= \frac{1}{2} w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2} w(L - 2a)a + \frac{1}{2} w(Lx - x^2 - La + a^2)$$

$$= \frac{1}{2} w(Lx - x^2 - a^2)$$

At C. $x = L - a$ $V_C = -\frac{1}{2} w(L - 2a)$ $M_C = \frac{1}{2} w(L - 2a)a$

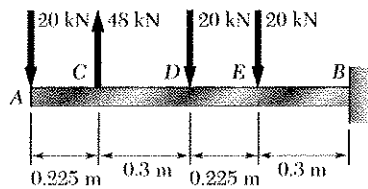
C to D. $V = V_C = -\frac{1}{2} w(L - 2a)$

$$M_D = 0$$

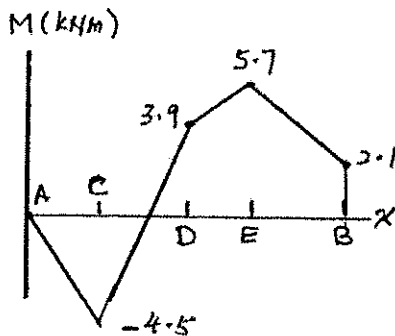
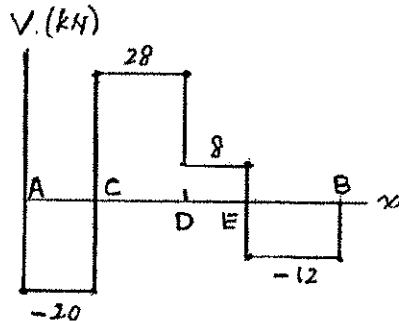
At $x = \frac{L}{2}$, $M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$

Problem 5.40

5.40 Using the method of Sec. 5.3, solve Prob. 5.7.



5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Over portions AC, CD, DE, and EB,

$$\frac{dV}{dx} = -w = 0 \quad V = \text{constant}$$

Shear diagram.

A to C: $V = -20 \text{ kN}$

C to D: $V = -20 + 48 = 28 \text{ kN}$

D to E: $V = 28 - 20 = 8 \text{ kN}$

E to B: $V = 8 - 20 = -12 \text{ kN}$

Draw the shear diagram as shown.

Areas of shear diagram.

A to C: $A_{AC} = (-20)(0.225) = -4.5 \text{ kNm}$

C to D: $A_{CD} = (28)(0.3) = 8.4 \text{ kNm}$

D to E: $A_{DE} = (8)(0.225) = 1.8 \text{ kNm}$

E to B: $A_{EB} = (-12)(0.3) = -3.6 \text{ kNm}$

Bending moments.

$$M_A = 0$$

$$M_C = 0 - 4.5 = -4.5 \text{ kNm}$$

$$M_D = -4.5 + 8.4 = 3.9 \text{ kNm}$$

$$M_E = 3.9 + 1.8 = 5.7 \text{ kNm}$$

$$M_B = 5.7 - 3.6 = 2.1 \text{ kNm}$$

Since V is constant over each of the portions AC, CD, DE and EB, draw the bending moment diagram with constant slopes over these portions.

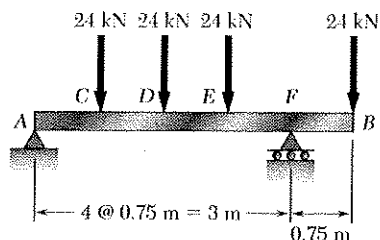
(a) $|V|_{\max} = 28 \text{ kN}$

(b) $|M|_{\max} = 5.7 \text{ kNm}$

Problem 5.41

5.41 Using the method of Sec. 5.3, solve Prob. 5.8.

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Reactions at A and F.

$$+\circlearrowleft \sum M_F = 0:$$

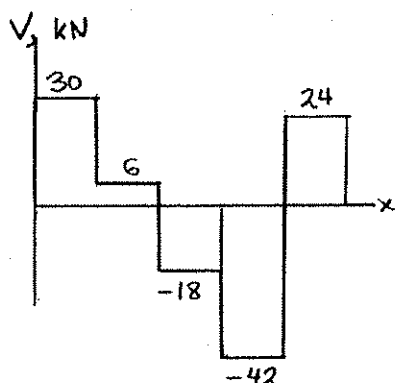
$$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24) - (0.75)(24) = 0$$

$$R_A = 30 \text{ kN} \uparrow$$

$$+\circlearrowleft \sum M_A = 0:$$

$$-(0.75)(24) - (1.50)(24) - (2.25)(24) + 3R_F - (3.75)(24) = 0$$

$$R_F = 66 \text{ kN} \uparrow$$



Shear diagram.

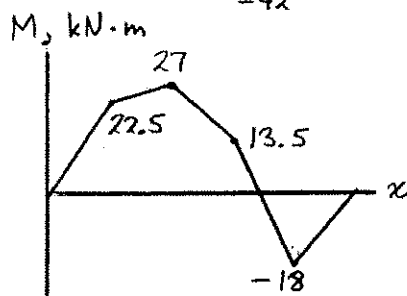
$$A \text{ to } C. \quad V = 30 \text{ kN}$$

$$C \text{ to } D. \quad V = 30 - 24 = 6 \text{ kN}$$

$$D \text{ to } E. \quad V = 6 - 24 = -18 \text{ kN}$$

$$E \text{ to } F. \quad V = -18 - 24 = -42 \text{ kN}$$

$$F \text{ to } B. \quad V = -42 + 66 = 24 \text{ kN}$$



Areas of shear diagram.

$$A \text{ to } C. \quad A_{AC} = (0.75)(30) = 22.5 \text{ kN}\cdot\text{m}$$

$$C \text{ to } D. \quad A_{CD} = (0.75)(6) = 4.5 \text{ kN}\cdot\text{m}$$

$$D \text{ to } E. \quad A_{DE} = (0.75)(-18) = -13.5 \text{ kN}\cdot\text{m}$$

$$E \text{ to } F. \quad A_{EF} = (0.75)(-42) = -31.5 \text{ kN}\cdot\text{m}$$

$$F \text{ to } B. \quad A_{FB} = (0.75)(24) = 18 \text{ kN}\cdot\text{m}$$

Bending moments.

$$M_A = 0$$

$$M_C = 0 + 22.5 = 22.5 \text{ kN}\cdot\text{m}$$

$$M_D = 22.5 + 4.5 = 27 \text{ kN}\cdot\text{m}$$

$$M_E = 27 - 13.5 = 13.5 \text{ kN}\cdot\text{m}$$

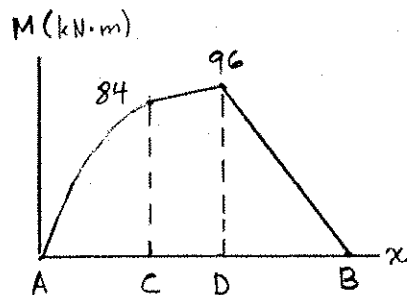
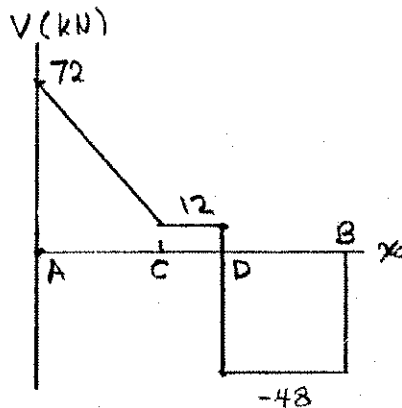
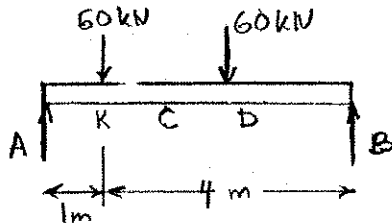
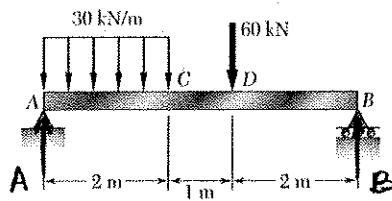
$$M_F = 13.5 - 31.5 = -18 \text{ kN}\cdot\text{m}$$

$$M_B = -18 + 18 = 0$$

$$(a) |V|_{\max} = 42.0 \text{ kN}$$

$$(b) |M|_{\max} = 27.0 \text{ kN}\cdot\text{m}$$

Problem 5.42



5.42 Using the method of Sec. 5.3, solve Prob. 5.9.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

Reactions A and B.

$$+\circlearrowleft M_B = 0: -5A + (2)(30)(4) + (60)(2) = 0$$

$$A = 72 \text{ kN} \uparrow$$

$$+\circlearrowleft M_A = 0: 5B - (3)(60) - (1)(30)(2) = 0$$

$$B = 48 \text{ kN} \uparrow$$

$$\text{Check: } +\uparrow \Sigma F_y = 72 - (2)(30) - 60 + 48 = 0$$

Shear diagram.

$$V_A = 72 \text{ kN}$$

$$\text{A to C. } 0 < x < 2 \text{ m} \quad w = 30 \text{ kN/m}$$

$$V_C - V_A = -\int_0^2 30 \, dx = -60 \text{ kN}$$

$$V_C = 72 - 60 = 12 \text{ kN}$$

$$\text{C to D. } V = V_C = 12 \text{ kN}$$

$$\text{D to B. } V = V_C - 60 = -48 \text{ kN}$$

Areas of shear diagram.

$$\text{A to C. } A_{AC} = \frac{1}{2}(72 + 12)(2) = 84 \text{ kN}\cdot\text{m}$$

$$\text{C to D. } A_{CD} = (12)(1) = 12 \text{ kN}\cdot\text{m}$$

$$\text{D to B. } A_{DB} = (-48)(2) = -96 \text{ kN}\cdot\text{m}$$

Bending moments: $M_A = 0$

$$M_C = 0 + 84 = 84 \text{ kN}$$

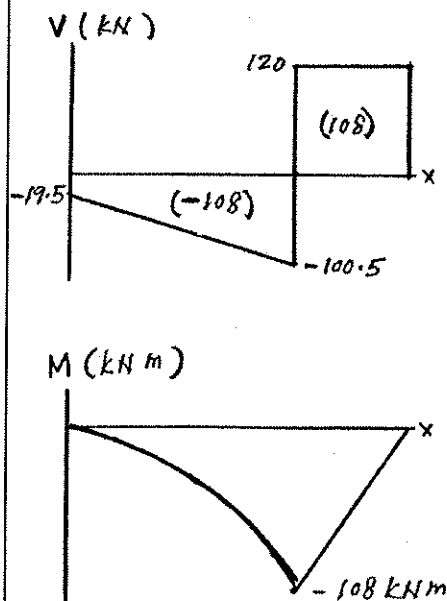
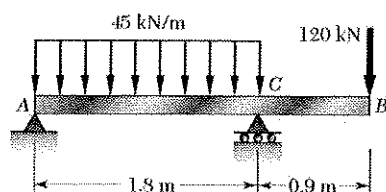
$$M_D = 84 + 12 = 96 \text{ kN}$$

$$M_B = 96 - 96 = 0 \text{ as expected.}$$

$$(a) |V|_{\max} = 72.0 \text{ kN} \quad \blacktriangleleft$$

$$(b) |M|_{\max} = 96.0 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 5.43



5.43 Using the method of Sec. 5.3, solve Prob. 5.10.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

$$+\circlearrowleft \sum M_C = 0: -1.8A + (0.9)(81) - (0.9)(120) = 0$$

$$A = -19.5 \text{ kN} \quad \text{i.e. } 19.5 \text{ kN} \downarrow$$

$$+\circlearrowleft \sum M_B = 0: 1.8C - (0.9)(81) - (2.7)(120) = 0$$

$$C = 220.5 \text{ kN} \uparrow$$

Shear diagram.

$$V_A = -19.5 \text{ kN}$$

$$A \text{ to } C. \quad 0 < x < 1.8 \text{ m}$$

$$w = -45 \text{ kN/m}$$

$$V_B - V_A = -\int_0^{1.8} w \, dx = -\int_0^{1.8} 45 \, dx = -81 \text{ kN}$$

$$V_B = -19.5 - 81 = -100.5 \text{ kN}$$

$$C \text{ to } B. \quad V = -100.5 + 220.5 = 120 \text{ kN}$$

Areas of shear diagram.

$$A \text{ to } C. \quad \int V \, dx = \left(\frac{1}{2}\right)(-19.5 - 100.5)(1.8) = -108 \text{ kNm}$$

$$C \text{ to } B. \quad \int V \, dx = (0.9)(120) = 108 \text{ kNm}$$

Bending moments.

$$M_A = 0$$

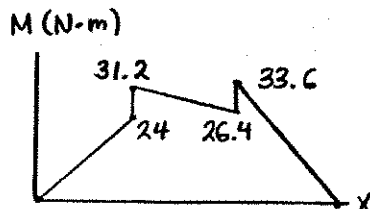
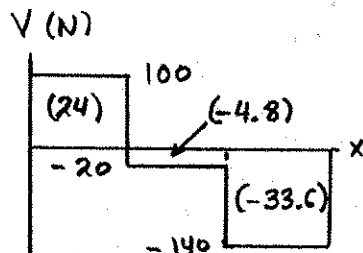
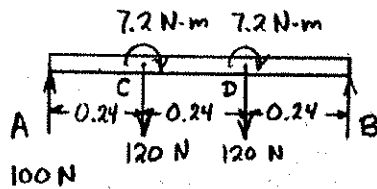
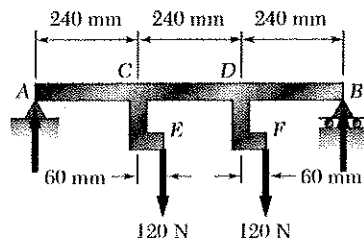
$$M_C = M_A + \int V \, dx = 0 - 108 = -108 \text{ kNm}$$

$$M_B = M_C + \int V \, dx = -108 + 108 = 0$$

$$\text{Maximum } |V| = 120 \text{ kN}$$

$$\text{Maximum } |M| = 108 \text{ kNm}$$

Problem 5.44



5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

$$+\circlearrowleft \sum M_B = 0: -0.72 A + (0.48)(120) + (0.24)(120) - 7.2 - 7.2 = 0$$

$$A = 100 \text{ N} \uparrow$$

$$+\circlearrowleft \sum M_A = 0: -(0.24)(120) - (0.48)(120) - 7.2 - 7.2 + 0.72 B = 0$$

$$B = 140 \text{ N} \uparrow$$

Shear diagram.

A to C. $V = 100 \text{ N}$

C to D. $V = 100 - 120 = -20 \text{ N}$

D to B. $V = -20 - 120 = -140 \text{ N}$

Areas of shear diagram.

A to C. $\int V dx = (0.24)(100) = 24 \text{ N}\cdot\text{m}$

C to D. $\int V dx = (0.24)(-20) = -4.8 \text{ N}\cdot\text{m}$

D to B. $\int V dx = (0.24)(-140) = -33.6 \text{ N}\cdot\text{m}$

Bending moments.

$M_A = 0$

$M_C^- = 0 + 24 = 24 \text{ N}\cdot\text{m}$

$M_C^+ = 24 + 7.2 = 31.2 \text{ N}\cdot\text{m}$

$M_D^- = 31.2 - 4.8 = 26.4 \text{ N}\cdot\text{m}$

$M_D^+ = 26.4 + 7.2 = 33.6 \text{ N}\cdot\text{m}$

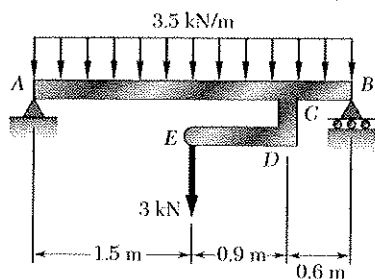
$M_B = 33.6 - 33.6 = 0$

(a) Maximum $|V| = 140 \text{ N}$ ▶

(b) Maximum $|M| = 33.6 \text{ N}\cdot\text{m}$ ▶

Problem 5.45

5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Reaction at A.

$$+\circlearrowleft \sum M_B = 0:$$

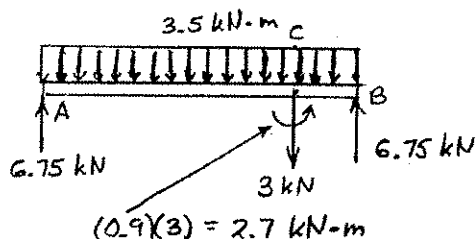
$$-3.0 A + (1.5)(3.0)(3.5) + (1.5)(3) = 0$$

$$A = 6.75 \text{ kN} \uparrow$$

Reaction at B.

$$B = 6.75 \text{ kN} \uparrow$$

Beam ACB and loading. See sketch.



Areas of load diagram.

$$A \text{ to } C, \quad (2.4)(3.5) = 8.4 \text{ kN}$$

$$C \text{ to } B, \quad (0.6)(3.5) = 2.1 \text{ kN}$$

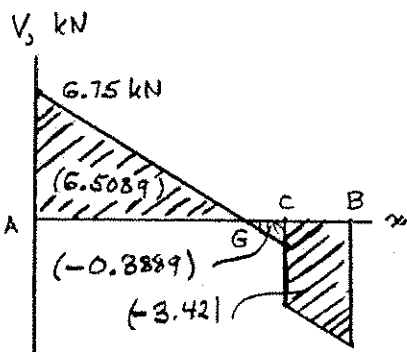
Shear diagram.

$$V_A = 6.75 \text{ kN}$$

$$V_C^- = 6.75 - 8.4 = -1.65 \text{ kN}$$

$$V_C^+ = -1.65 - 3 = -4.65 \text{ kN}$$

$$V_B = -4.65 - 2.1 = -6.75 \text{ kN}$$



Over A to C, $V = 6.75 - 3.5x$

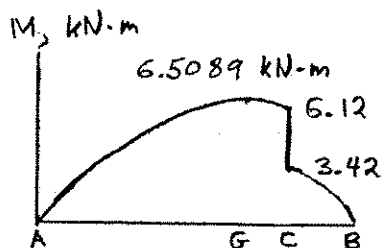
$$\text{At } G, \quad V = 6.75 - 3.5x_G = 0 \quad x_G = 1.9286 \text{ m}$$

Areas of shear diagram.

$$A \text{ to } G, \quad \frac{1}{2}(1.9286)(6.75) = 6.5089 \text{ kN}\cdot\text{m}$$

$$G \text{ to } C, \quad \frac{1}{2}(0.4714)(-1.65) = -0.3889 \text{ kN}\cdot\text{m}$$

$$C \text{ to } B, \quad \frac{1}{2}(0.6)(-4.65 - 6.75) = -3.42 \text{ kN}\cdot\text{m}$$



Bending moments.

$$M_A = 0$$

$$M_G = 0 + 6.5089 = 6.5089 \text{ kN}\cdot\text{m}$$

$$M_C^- = 6.5089 - 0.3889 = 6.12 \text{ kN}\cdot\text{m}$$

$$M_C^+ = 6.12 - 2.7 = 3.42 \text{ kN}\cdot\text{m}$$

$$M_B = 3.42 - 3.42 = 0$$

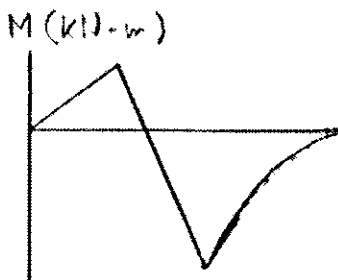
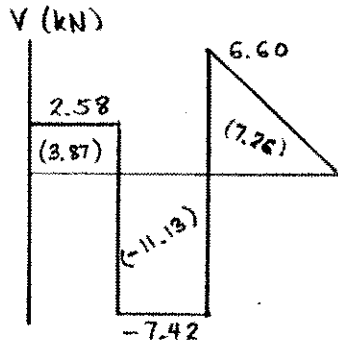
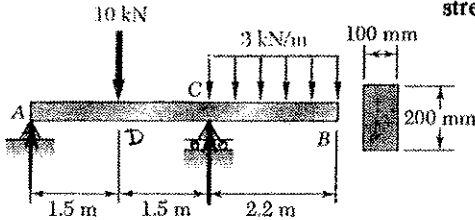
$$(a) |V|_{\max} = 6.75 \text{ kN} \quad \blacktriangleleft$$

$$(b) |M|_{\max} = 6.51 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 5.46

5.46 Using the method of Sec. 5.3, solve Prob. 5.15.

5.15 and 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



$$\begin{aligned} \sum M_C &= 0 \\ -3A + (1.5)(10) - (1.1)(2.2)(3) &= 0 \\ A &= 2.58 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_A &= 0 \\ -(1.5)(10) + 3C - (4.1)(2.2)(3) &= 0 \\ C &= 14.02 \text{ kN} \end{aligned}$$

Shear

$$\begin{aligned} A \text{ to } D^- & \quad V = 2.58 \text{ kN} \\ D^+ \text{ to } C^- & \quad V = 2.58 - 10 = -7.42 \text{ kN} \\ C^+ & \quad V = -7.42 + 14.02 = 6.60 \text{ kN} \\ B & \quad V = 6.60 - (2.2)(3) = 0 \end{aligned}$$

Areas under shear diagram

$$\begin{aligned} A \text{ to } D & \quad \int V dx = (1.5)(2.58) = 3.87 \text{ kN}\cdot\text{m} \\ D \text{ to } C & \quad \int V dx = (1.5)(-7.42) = -11.13 \text{ kN}\cdot\text{m} \\ C \text{ to } B & \quad \int V dx = \left(\frac{1}{2}\right)(2.2)(6.60) = 7.26 \text{ kN}\cdot\text{m} \end{aligned}$$

Bending moments

$$\begin{aligned} M_A &= 0 \\ M_D &= 0 + 3.87 = 3.87 \text{ kN}\cdot\text{m} \\ M_C &= 3.87 - 11.13 = -7.26 \text{ kN}\cdot\text{m} \\ M_B &= 7.26 - 7.26 = 0 \end{aligned}$$

$$|M_C| = 7.26 \text{ kN}\cdot\text{m} = 7.26 \times 10^3 \text{ N}\cdot\text{m}$$

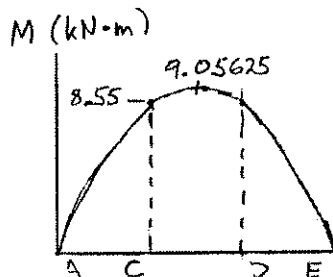
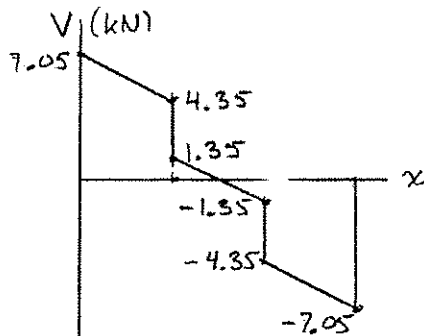
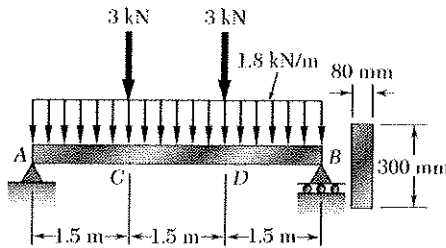
$$\begin{aligned} \text{For rectangular cross section} \quad S &= \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(100)(200)^2 \\ &= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Normal stress} \quad \sigma &= \frac{|M_C|}{S} = \frac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa} \\ &= 10.89 \text{ MPa} \end{aligned}$$

Problem 5.47

5.47 Using the method of Sec. 5.3, solve Prob. 5.16.

5.15 and 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



By symmetry: $A = B$

$$+\uparrow \sum F_y = 0: A + B - 3 - 3 - (4.5)(1.8) = 0$$

$$A = B = 7.05 \text{ kN}$$

Shear diagram: $V_A = 7.05 \text{ kN}$

A to C: $w = 1.8 \text{ kN/m}$

$$\text{At } C^-: V = 7.05 - (1.8)(1.5) = 4.35 \text{ kN}$$

$$\text{At } C^+: V = 4.35 - 3 = 1.35 \text{ kN}$$

C+ to D: $w = 1.8 \text{ kN/m}$

$$\text{At } D^-: V = 1.35 - (1.5)(1.8) = -1.35 \text{ kN}$$

$$\text{At } D^+: V = -1.35 - 3 = -4.35 \text{ kN}$$

D+ to B: $w = 1.8 \text{ kN/m}$

$$\text{At } B: V = -4.35 - (1.5)(1.8) = -7.05 \text{ kN}$$

Draw the shear diagram.

$V = 0$ at point E, the midpoint of CD.

Areas of the shear diagram.

$$\text{A to C: } \frac{1}{2}(7.05 + 4.35)(1.5) = 8.55 \text{ kN}\cdot\text{m}$$

$$\text{C to E: } \frac{1}{2}(1.35)(0.75) = 0.50625 \text{ kN}\cdot\text{m}$$

$$\text{E to D: } \frac{1}{2}(-1.35)(0.75) = -0.50625 \text{ kN}\cdot\text{m}$$

$$\text{D to B: } \frac{1}{2}(-4.35 - 7.05)(1.5) = -8.55 \text{ kN}\cdot\text{m}$$

Bending moments: $M_A = 0$

$$M_C = 0 + 8.55 = 8.55 \text{ kN}\cdot\text{m}$$

$$M_E = 8.55 + 0.50625 = 9.05625 \text{ kN}\cdot\text{m}$$

$$M_D = 9.05625 - 0.50625 = 8.55 \text{ kN}\cdot\text{m}$$

$$M_B = 8.55 - 8.55 = 0$$

$$M_C = 8.55 \times 10^3 \text{ N}\cdot\text{m}$$

For a rectangular section

$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(80)(300)^2$$

$$= 1.2 \times 10^6 \text{ mm}^3 = 1.2 \times 10^{-3} \text{ m}^3$$

Maximum normal stress at C.

$$\sigma = \frac{M_C}{S} = \frac{8.55 \times 10^3}{1.2 \times 10^{-3}}$$

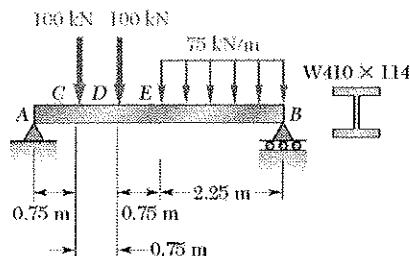
$$= 7.125 \times 10^6 \text{ Pa}$$

$$\sigma = 7.13 \text{ MPa}$$

Problem 5.48

5.48 Using the method of Sec. 5.3, solve Prob. 5.17.

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



$$+\circlearrowleft \sum M_B = 0$$

$$-4.5 A + (3-75)(100) + (3)(100) + (75)(2.25)(1.125) = 0$$

$$A = 192.2 \text{ kN.}$$

$$\text{Shear } A \text{ to } C \quad V = 192.2 \text{ kN.}$$

$$\text{Area under shear curve } A \text{ to } C \quad \int V dx = (0.75)(192.2) = 144.15 \text{ kNm.}$$

$$M_A = 0$$

$$M_C = 0 + 144.15 = 144.15 \text{ kNm.}$$

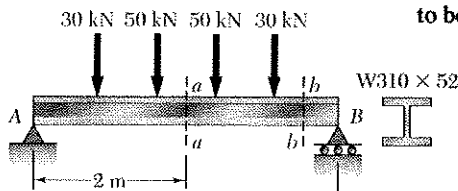
$$\text{For } W 410 \times 114 \text{ rolled steel section } S = 2200 \times 10^3 \text{ mm}^3.$$

$$\text{Normal stress } \sigma = \frac{M}{S} = \frac{144.15 \times 10^3}{2200 \times 10^3} = 65.5 \text{ MPa.}$$

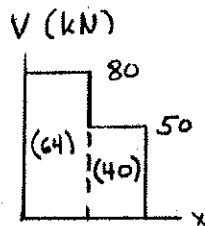
Problem 5.49

5.49 Using the method of Sec. 5.3, solve Prob. 5.18.

5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section $a-a$.



Reactions: By symmetry, $A = B$
 $\uparrow \sum F_y = 0: A = B = 80 \text{ kN}$

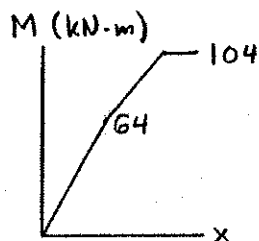


Shear diagram.

A to C. $V = 80 \text{ kN}$
 C to D. $V = 80 - 30 = 50 \text{ kN}$
 D to E. $V = 50 - 50 = 0$

Areas of shear diagram.

A to C. $\int V dx = (80)(0.8) = 64 \text{ kN}\cdot\text{m}$
 C to D. $\int V dx = (50)(0.8) = 40 \text{ kN}\cdot\text{m}$
 D to E. $\int V dx = 0$



Bending moments.

$M_A = 0$
 $M_C = 0 + 64 = 64 \text{ kN}\cdot\text{m}$
 $M_D = 64 + 40 = 104 \text{ kN}\cdot\text{m}$
 $M_E = 104 + 0 = 104 \text{ kN}\cdot\text{m}$

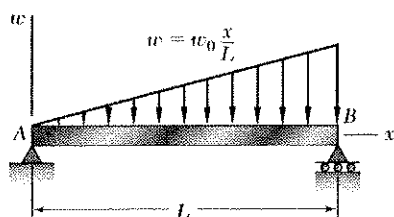
$M_E = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$

For W 310 x 52, $S = 748 \times 10^3 \text{ mm}^3 = 748 \times 10^{-6} \text{ m}^3$

Normal stress. $\sigma = \frac{M}{S} = \frac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa}$

$\sigma = 139.0 \text{ MPa}$

Problem 5.50



5.50 and 5.51 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.

$$\frac{dV}{dx} = -w = -w_0 \frac{x}{L}$$

$$V = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = 0 \text{ at } x = L$$

$$0 = -\frac{1}{6} w_0 L^2 + C_1 L \quad C_1 = \frac{1}{6} w_0 L$$

$$(a) \quad V = -\frac{1}{2} w_0 \frac{x^2}{L} + \frac{1}{6} w_0 L = \frac{1}{6} w_0 (L^2 - 3x^2)/L$$

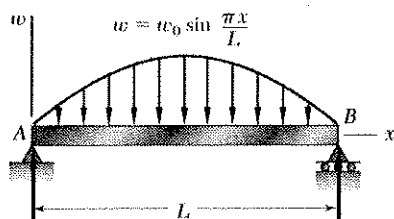
$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + \frac{1}{6} w_0 L x = \frac{1}{6} w_0 (Lx - x^3/L)$$

$$(b) \quad M_{\max} \text{ occurs when } \frac{dM}{dx} = V = 0 \quad L^2 - 3x_m^2 = 0$$

$$x_m = \frac{L}{\sqrt{3}}$$

$$M_{\max} = \frac{1}{6} w_0 \left(\frac{L^2}{\sqrt{3}} - \frac{L^2}{3\sqrt{3}} \right) \quad M_{\max} = 0.0642 w_0 L^2$$

Problem 5.51



5.50 and 5.51 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = 0 \text{ at } x = L$$

$$0 = 0 + C_1 L + 0 \quad C_1 = 0$$

(a)

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = 0 \text{ at } x = \frac{L}{2}$$

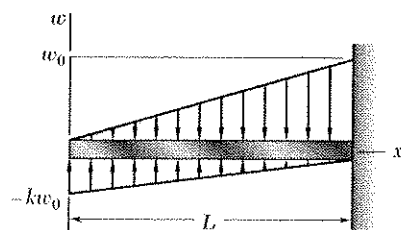
(b)

$$M_{\max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2}$$

$$M_{\max} = \frac{w_0 L^2}{\pi^2}$$

Problem 5.52

5.52 For the beam and loading shown, determine the equations of the shear and bending-moment curves and the maximum absolute value of the bending moment in the beam, knowing that (a) $k = 1$, (b) $k = 0.5$.



$$w = \frac{w_0 x}{L} - \frac{k w_0 (L-x)}{L} = (1+k) \frac{w_0 x}{L} - k w_0$$

$$\frac{dV}{dx} = -w = k w_0 - (1+k) \frac{w_0 x}{L}$$

$$V = k w_0 x - (1+k) \frac{w_0 x^2}{2L} + C_1$$

$$V = 0 \text{ at } x = 0$$

$$C_1 = 0$$

$$\frac{dM}{dx} = V = k w_0 x - (1+k) \frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1+k) \frac{w_0 x^3}{6L} + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = \frac{k w_0 x^2}{2} - \frac{(1+k) w_0 x^3}{6L}$$

(a) $k = 1$.

$$V = w_0 x - \frac{w_0 x^2}{L}$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L}$$

Maximum M occurs at $x = L$. $|M|_{\max} = \frac{w_0 L^2}{6}$

(b) $k = \frac{1}{2}$.

$$V = \frac{w_0 x}{2} - \frac{3 w_0 x^2}{4L}$$

$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L}$$

$$V = 0 \text{ at } x = \frac{2}{3} L$$

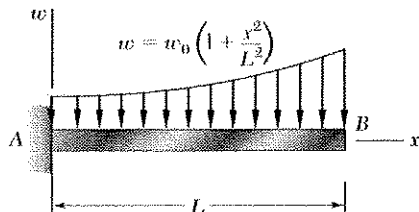
$$\text{At } x = \frac{2}{3} L \quad M = \frac{w_0 \left(\frac{2}{3} L\right)^2}{4} - \frac{w_0 \left(\frac{2}{3} L\right)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704 w_0 L^2$$

$$\text{At } x = L \quad M = 0$$

$$|M|_{\max} = \frac{w_0 L^2}{27}$$

Problem 5.53

5.53 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -w_0 \left(1 + \frac{x^2}{L^2} \right)$$

$$V = -w_0 \left(x + \frac{x^3}{3L^2} \right) + C_1$$

$$V = 0 \text{ at } x = L.$$

$$0 = -w_0 \left(L + \frac{1}{3}L \right) + C_1 \quad C_1 = \frac{4}{3}w_0L$$

$$\frac{dM}{dx} = V = -w_0 \left(\frac{4}{3}L - x - \frac{1}{3}\frac{x^3}{L^2} \right)$$

$$M = w_0 \left(\frac{4}{3}Lx - \frac{1}{2}x^2 - \frac{1}{12}\frac{x^4}{L^2} \right) + C_2$$

$$M = 0 \text{ at } x = L, \quad w_0 \left(\frac{4}{3}L^2 - \frac{1}{2}L^2 - \frac{1}{12}L^2 \right) + C_2 = 0$$

$$C_2 = -\frac{3}{4}w_0L^2$$

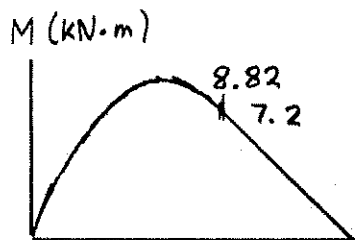
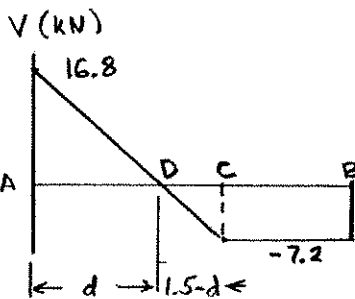
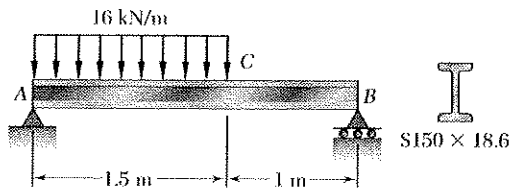
$$M = w_0 \left(\frac{4}{3}Lx - \frac{1}{2}x^2 - \frac{1}{12}\frac{x^4}{L^2} - \frac{3}{4}L^2 \right)$$

$$|M|_{\max} \text{ occurs at } x = 0.$$

$$|M|_{\max} = \frac{3}{4}w_0L^2$$

Problem 5.54

5.54 and 5.55 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$+\circlearrowleft \sum M_B = 0:$$

$$-2.5 A + (1.75)(1.5)(16) = 0$$

$$A = 16.8 \text{ kN}$$

$$+\circlearrowleft \sum M_A = 0:$$

$$-(0.75)(1.5)(16) + 2.5 B = 0$$

$$B = 7.2 \text{ kN}$$

Shear diagram.

$$V_A = 16.8 \text{ kN}$$

$$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$$

$$V_B = -7.2 \text{ kN}$$

Locate point D where $V = 0$.

$$\frac{d}{16.8} = \frac{1.5-d}{7.2} \quad 24d = 25.2$$

$$d = 1.05 \text{ m} \quad 1.5 - d = 0.45 \text{ m}$$

Areas of the shear diagram.

$$A \text{ to } D. \int V dx = \left(\frac{1}{2}\right)(1.05)(16.8) = 8.82 \text{ kN}\cdot\text{m}$$

$$D \text{ to } C. \int V dx = \left(\frac{1}{2}\right)(0.45)(-7.2) = -1.62 \text{ kN}\cdot\text{m}$$

$$C \text{ to } B. \int V dx = (1)(-7.2) = -7.2 \text{ kN}\cdot\text{m}$$

Bending moments.

$$M_A = 0$$

$$M_D = 0 + 8.82 = 8.82 \text{ kN}\cdot\text{m}$$

$$M_C = 8.82 - 1.62 = 7.2 \text{ kN}\cdot\text{m}$$

$$M_B = 7.2 - 7.2 = 0$$

$$\text{Maximum } |M| = 8.82 \text{ kN}\cdot\text{m} = 8.82 \times 10^3 \text{ N}\cdot\text{m}$$

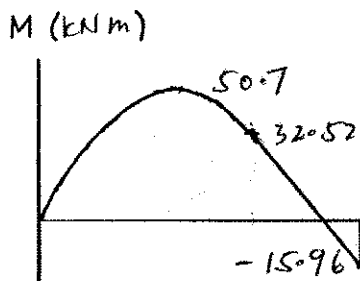
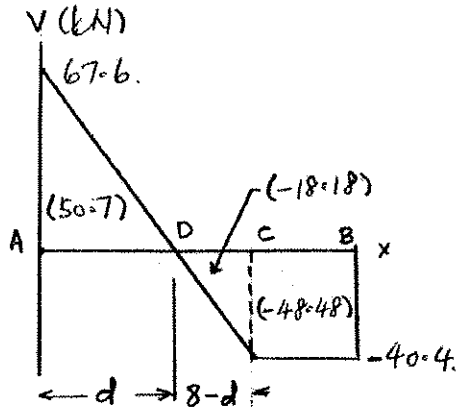
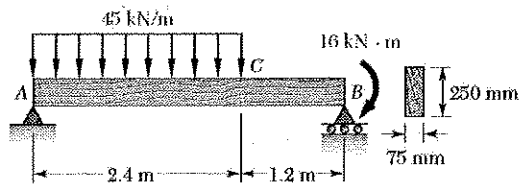
For S 150 x 18.6 rolled steel section, $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

$$\text{Normal stress. } \sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa}$$

$$\sigma = 73.5 \text{ MPa}$$

Problem 5.55

5.54 and 5.55 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$\begin{aligned} +\circlearrowleft \sum M_B &= 0 \\ -3.6A + (45)(2.4)(2.4) - 16 &= 0 \\ A &= 67.6 \text{ kN} \end{aligned}$$

$$\begin{aligned} \uparrow \sum M_A &= 0 \\ -(45)(2.4)(1.2) + 3.6B - 16 &= 0 \\ B &= 40.4 \text{ kN} \end{aligned}$$

Shear: $V_A = 67.6 \text{ kN}$

$$\begin{aligned} V_C &= 67.6 - (45)(2.4) = -40.4 \text{ kN} \\ \text{C to B} \quad V &= -40.4 \text{ kN} \end{aligned}$$

Locate point D where $V = 0$

$$\begin{aligned} \frac{d}{67.6} &= \frac{2.4 - d}{40.4} & 1.6d &= 2.4 \\ d &= 1.5 \text{ m} & 2.4 - d &= 0.9 \text{ m} \end{aligned}$$

Areas under shear diagram

$$\text{A to D} \quad \int V dx = \left(\frac{1}{2}\right)(1.5)(67.6) = 50.7 \text{ kN}\cdot\text{m}$$

$$\text{D to C} \quad \int V dx = \left(\frac{1}{2}\right)(0.9)(-40.4) = -18.18 \text{ kN}\cdot\text{m}$$

$$\text{C to B} \quad \int V dx = -(1.2)(40.4) = -48.48 \text{ kN}\cdot\text{m}$$

Bending moments: $M_A = 0$

$$M_D = 0 + 50.7 = 50.7 \text{ kN}\cdot\text{m}$$

$$M_C = 50.7 - 18.18 = 32.52 \text{ kN}\cdot\text{m}$$

$$M_B = 32.52 - 48.48 = -15.96 \text{ kN}\cdot\text{m}$$

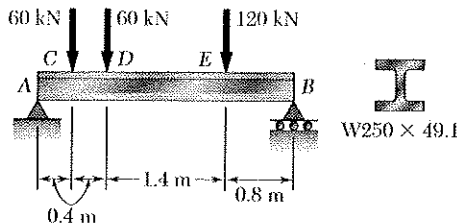
Maximum $|M| = 50.7 \text{ kN}\cdot\text{m}$.

For rectangular cross section $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(75)(250)^2 = 781250 \text{ mm}^3$.

$$\text{Normal stress} \quad \sigma = \frac{MI}{S} = \frac{50.7 \times 10^3}{781250 \times 10^{-9}} = 64.9 \text{ MPa}$$

Problem 5.56

5.56 and 5.57 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$+\circlearrowleft \sum M_B = 0:$$

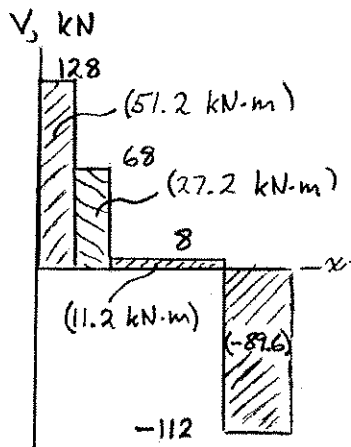
$$-3.0 A + (2.6)(60) + (2.2)(60) + (0.8)(120) = 0$$

$$A = 128 \text{ kN}$$

$$+\circlearrowleft \sum M_A = 0:$$

$$-(0.4)(60) - (0.8)(60) - (2.2)(120) + 3.0 B = 0$$

$$B = 112 \text{ kN}$$



Shear diagram.

$$A \text{ to } C^-: V = 128 \text{ kN}$$

$$C^+ \text{ to } D^-: V = 128 - 60 = 68 \text{ kN}$$

$$D^+ \text{ to } E^-: V = 68 - 60 = 8 \text{ kN}$$

$$E^+ \text{ to } B: V = 8 - 120 = -112 \text{ kN}$$

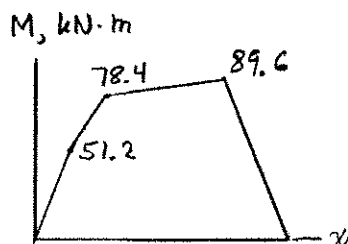
Areas of shear diagram.

$$A \text{ to } C: (0.4)(128) = 51.2 \text{ kN}\cdot\text{m}$$

$$C \text{ to } D: (0.4)(68) = 27.2 \text{ kN}\cdot\text{m}$$

$$D \text{ to } E: (1.4)(8) = 11.2 \text{ kN}\cdot\text{m}$$

$$E \text{ to } B: (0.8)(-112) = -89.6 \text{ kN}\cdot\text{m}$$



Bending moments.

$$M_A = 0$$

$$M_C = 0 + 51.2 = 51.2 \text{ kN}\cdot\text{m}$$

$$M_D = 51.2 + 27.2 = 78.4 \text{ kN}\cdot\text{m}$$

$$M_E = 78.4 + 11.2 = 89.6 \text{ kN}\cdot\text{m}$$

$$M_B = 89.6 - 89.6 = 0$$

$$|M|_{\max} = 89.6 \text{ kN}\cdot\text{m} = 89.6 \times 10^3 \text{ N}\cdot\text{m}$$

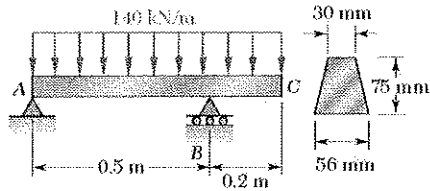
For W 250 x 49.1 rolled steel section $S = 572 \times 10^3 \text{ mm}^3 = 572 \times 10^{-6} \text{ m}^3$

$$\text{Normal stress: } \sigma = \frac{|M|}{S} = \frac{89.6 \times 10^3}{572 \times 10^{-6}} = 156.6 \times 10^6 \text{ Pa}$$

$$\sigma = 156.6 \text{ MPa}$$

Problem 5.57

5.57 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

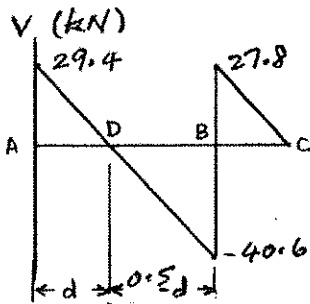


$$\sum M_B = 0 \quad -0.5A + (140)(0.7)(0.15) = 0$$

$$A = 29.4 \text{ kN}$$

$$\sum M_A = 0 \quad 0.5B - (140)(0.7)(0.35) = 0$$

$$B = 68.4 \text{ kN}$$



Shear: $V_A = 29.4 \text{ kN}$

$$B^- \quad V_B^- = 29.4 - (140)(0.5) = -40.6 \text{ kN}$$

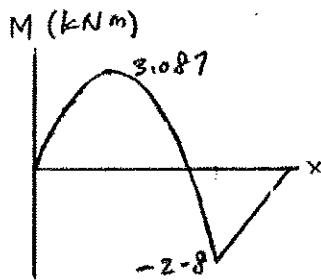
$$B^+ \quad V_B^+ = -40.6 + (68.4) = 27.8 \text{ kN}$$

$$C \quad V_C = 27.8 - (140)(0.2) \approx 0$$

Locate point D where $V = 0$

$$\frac{d}{29.4} = \frac{0.5-d}{40.6} \quad 70.3d = 14.7$$

$$d = 0.21 \text{ m} \quad 0.5 - d = 0.29 \text{ m}$$



Areas under shear diagram

$$A \text{ to } D \quad \int V dx = \left(\frac{1}{2}\right)(29.4)(0.21) = 3.087 \text{ kNm}$$

$$D \text{ to } B \quad \int V dx = \left(\frac{1}{2}\right)(-40.6)(0.29) = -5.887 \text{ kNm}$$

$$B \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(27.8)(0.2) = 2.78 \text{ kNm}$$

Bending moments: $M_A = 0$

$$M_D = 0 + 3.087 = 3.087 \text{ kNm}$$

$$M_B = 3.087 - 5.887 = -2.8 \text{ kNm}$$

$$M_C = -2.8 + 2.78 \approx 0$$

$$\text{Maximum } |M| = 3.087 \text{ kNm}$$

Locate centroid of cross section

$$\bar{Y} = \frac{108750}{3225} = 33.72 \text{ mm from bottom}$$

$$\text{For each triangle } \bar{I} = \frac{1}{36} b h^3$$

Moment of inertia

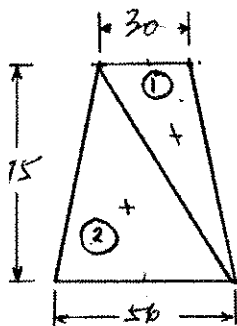
$$I = \sum \bar{I} + \sum A d^2$$

$$= 457849 + 1007813 = 1465662 \text{ mm}^4$$

Normal stress

$$\sigma = \frac{M_C}{I} = \frac{(3.087 \times 10^3)(41.28 \times 10^{-3})}{1465662 \times 10^{-12}}$$

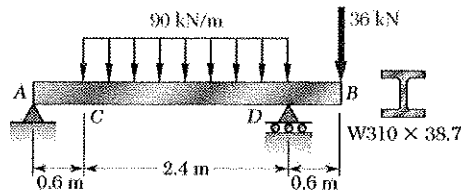
$$= 86.9 \text{ MPa}$$



Part	A, mm ²	$\bar{y}$, mm	$A\bar{y}$, mm ³	d, mm	$A d^2$, mm ⁴	$\bar{I}$, mm ⁴
①	1125	50	56250	16.28	298168	351563
②	2100	25	52500	8.72	159681	656250
Σ	3225		108750		457849	1007813

Problem 5.58

5.58 and 5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Reaction at A:

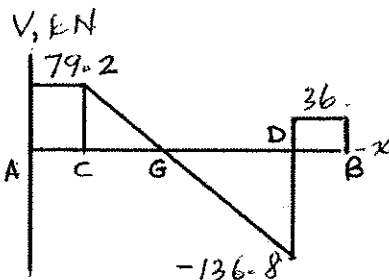
$$+\circlearrowleft \sum M_D = 0$$

$$-3.0A + (90)(2.4)(1.2) - (36)(0.6) = 0 \quad A = 79.2 \text{ kN} \uparrow$$

Reaction at D:

$$+\circlearrowright \sum M_A = 0$$

$$-(90)(2.4)(1.8) + 3.0D - (36)(3.6) = 0 \quad D = 172.8 \text{ kN} \uparrow$$



Areas of load diagram.

$$C \text{ to } D \quad (90)(2.4) = 216 \text{ kN}$$

Shear diagram.

A to C

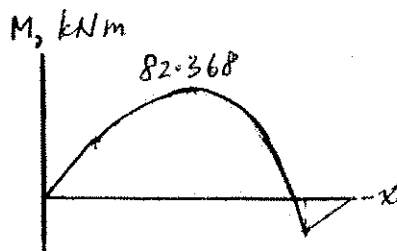
$$V = 79.2 \text{ kN}$$

$$V_D^- = 79.2 - 216 = -136.8 \text{ kN}$$

$$V_D^+ = -136.8 + 172.8 = 36 \text{ kN}$$

D to B

$$V = 36 \text{ kN}$$



$$\text{Over CD} \quad V = 79.2 - 90(x - 0.6)$$

$$\text{At G} \quad V = 79.2 - 90(x_G - 0.6) = 0 \quad x_G - 0.6 = 0.88 \text{ m}$$

Areas of shear diagram

$$A \text{ to } C \quad (0.6)(79.2) = 47.52 \text{ kNm}$$

$$C \text{ to } G \quad \frac{1}{2}(0.88)(79.2) = 34.848 \text{ kNm}$$

$$G \text{ to } D \quad \frac{1}{2}(1.52)(-136.8) = -103.968 \text{ kNm}$$

$$D \text{ to } B \quad (36)(0.6) = 21.6 \text{ kNm}$$

Bending moments. $M_A = 0$

$$M_C = 0 + 47.52 = 47.52 \text{ kNm}$$

$$M_G = 47.52 + 34.848 = 82.368 \text{ kNm}$$

$$M_D = 82.368 - 103.968 = -21.6 \text{ kNm}$$

$$M_B = -21.6 + 21.6 = 0$$

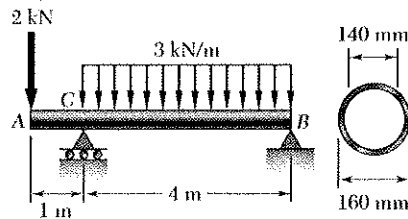
$$|M|_{\max} = M_G = 82.368 \text{ kNm}$$

$$\text{For W310x38.7 rolled steel section} \quad S = 549 \times 10^3 \text{ mm}^3$$

$$\text{Normal stress} \quad \sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{82.368 \times 10^3}{549 \times 10^{-6}} \quad \sigma_{\max} = 150 \text{ MPa} \leftarrow$$

Problem 5.59

5.58 and 5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

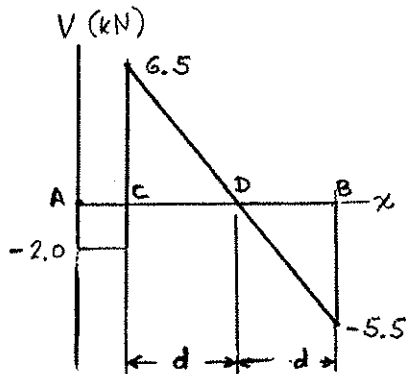


$$+\circlearrowleft \sum M_C = 0: (2)(1) - (3)(4)(2) + 4B = 0$$

$$B = 5.5 \text{ kN}$$

$$+\circlearrowleft \sum M_B = 0: (5)(2) + (3)(4)(2) - 4C = 0$$

$$C = 8.5 \text{ kN}$$



Shear: A to C: $V = -2 \text{ kN}$

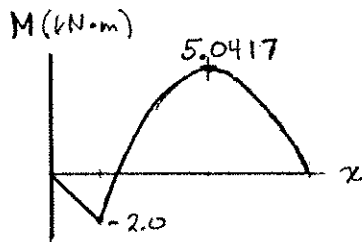
$$C^+: V = -2 + 8.5 = 6.5 \text{ kN}$$

$$D: V = 6.5 - (3)(4) = -5.5 \text{ kN}$$

Locate point D where $V = 0$.

$$\frac{d}{6.5} = \frac{4-d}{5.5} \quad 12d = 26$$

$$d = 2.1667 \text{ m} \quad 4-d = 3.8333 \text{ m}$$



Areas of the shear diagram:

$$A \text{ to } C: \int V dx = (-2.0)(1) = -2.0 \text{ kN}\cdot\text{m}$$

$$C \text{ to } D: \int V dx = \frac{1}{2}(2.1667)(6.5) = 7.0417 \text{ kN}\cdot\text{m}$$

$$D \text{ to } B: \int V dx = \frac{1}{2}(3.8333)(-5.5) = -5.0417 \text{ kN}\cdot\text{m}$$

Bending moments: $M_A = 0$

$$M_C = 0 - 2.0 = -2.0 \text{ kN}\cdot\text{m}$$

$$M_D = -2.0 + 7.0417 = 5.0417 \text{ kN}\cdot\text{m}$$

$$M_B = 5.0417 - 5.0417 = 0$$

$$\text{Maximum } |M| = 5.0417 \text{ kN}\cdot\text{m} = 5.0417 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{For pipe: } C_o = \frac{1}{2} d_o = \frac{1}{2}(160) = 80 \text{ mm}, \quad C_i = \frac{1}{2} d_i = \frac{1}{2}(140) = 70 \text{ mm}$$

$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = \frac{\pi}{4}[(80)^4 - (70)^4] = 13.3125 \times 10^6 \text{ mm}^4$$

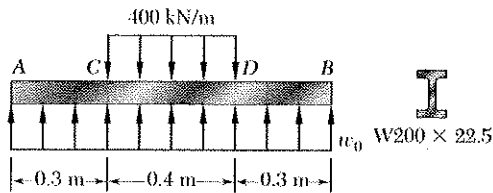
$$S = \frac{I}{C_o} = \frac{13.3125 \times 10^6}{80} = 166.406 \times 10^3 \text{ mm}^3 = 166.406 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress: } \sigma = \frac{M}{S} = \frac{5.0417 \times 10^3}{166.406 \times 10^{-6}} = 30.3 \times 10^6 \text{ Pa}$$

$$\sigma = 30.3 \text{ MPa}$$

Problem 5.60

5.60 and 5.61 Knowing that beam AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



$$+\uparrow \sum F_y = 0: \quad (1)(w_0) - (0.4)(400) = 0$$

$$w_0 = 160 \text{ kN/m}$$

Shear diagram. $V_A = 0$

$$V_C = 0 + (0.3)(160) = 48 \text{ kN}$$

$$V_D = 48 - (0.3)(400) + (0.3)(160) = -48 \text{ kN}$$

$$V_B = -48 + (0.3)(160) = 0$$

Locate point E where $V = 0$.

By symmetry E is the midpoint of CD.

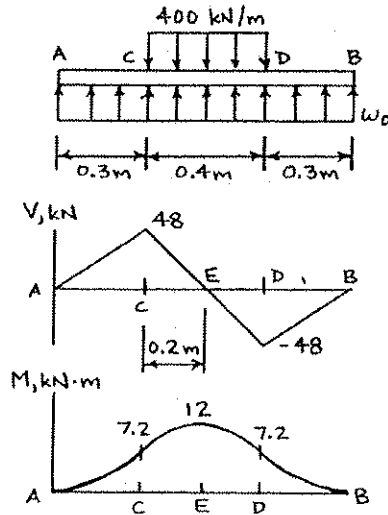
Areas of shear diagram.

$$A \text{ to } C: \quad \frac{1}{2}(0.3)(48) = 7.2 \text{ kN}\cdot\text{m}$$

$$C \text{ to } E: \quad \frac{1}{2}(0.2)(48) = 4.8 \text{ kN}\cdot\text{m}$$

$$E \text{ to } D: \quad \frac{1}{2}(0.2)(-48) = -4.8 \text{ kN}\cdot\text{m}$$

$$D \text{ to } B: \quad \frac{1}{2}(0.3)(-48) = -7.2 \text{ kN}\cdot\text{m}$$



Bending moments: $M_A = 0$

$$M_C = 0 + 7.2 = 7.2 \text{ kN}\cdot\text{m}$$

$$M_E = 7.2 + 4.8 = 12.0 \text{ kN}\cdot\text{m}$$

$$M_D = 12.0 - 4.8 = 7.2 \text{ kN}\cdot\text{m}$$

$$M_B = 7.2 - 7.2 = 0$$

$$|M|_{\text{max}} = 12.0 \text{ kN}\cdot\text{m} = 12.0 \times 10^3 \text{ N}\cdot\text{m}$$

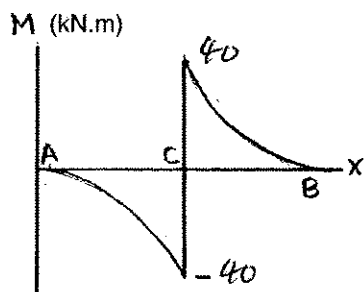
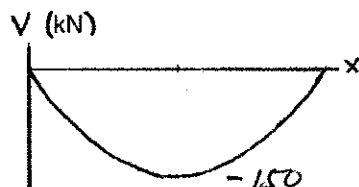
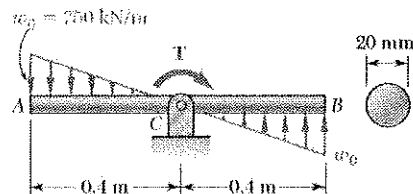
For W200x22.5 rolled steel shape, $S_x = 194 \times 10^3 \text{ mm}^3 = 194 \times 10^{-6} \text{ m}^3$

Normal stress;
$$\sigma = \frac{|M|}{S} = \frac{12.0 \times 10^3}{194 \times 10^{-6}} = 61.9 \times 10^6 \text{ Pa}$$

$$\sigma = 61.9 \text{ MPa}$$

Problem 5.61

5.61 Knowing that beam AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



A to C $0 < x < 0.4 \text{ m}$

$$w = 750 \left(1 - \frac{x}{0.4}\right) = 750 - 1875x$$

$$\frac{dV}{dx} = -w = 1875x - 750$$

$$V = V_A + \int_0^x (1875x - 750) dx$$

$$= 0 + 937.5x^2 - 750x = \frac{dM}{dx}$$

$$M = M_A + \int_0^x V dx$$

$$= 0 + \int_0^x (937.5x^2 - 750x) dx$$

$$= 312.5x^3 - 375x^2$$

At $x = 0.4 \text{ m}$, $V = -150 \text{ N}$
 $M = 40 \text{ N.m}$

C to B Use symmetry conditions.

Maximum $|M| = 40 \text{ kN.m}$.

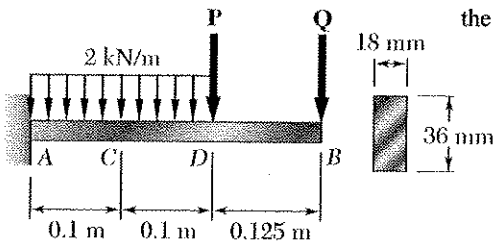
Cross section $c = \frac{d}{2} = \left(\frac{1}{2}\right)(20) = 10 \text{ mm}$

$$I = \frac{\pi}{4} c^4 = \left(\frac{\pi}{4}\right)(10)^4 = 7854 \text{ mm}^4$$

Normal stress $\sigma = \frac{|M|c}{I} = \frac{(40)(0.01)}{7854 \times 10^{-12}} = 50.9 \text{ MPa}$

Problem 5.62

*5.62 Beam AB supports a uniformly distributed load of 2 kN/m and two concentrated loads P and Q . It has been experimentally determined that the normal stress due to bending in the bottom edge of the beam is -56.9 MPa at A and -29.9 MPa at C . Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads P and Q .



$$I = \frac{1}{12}(18)(36)^3 = 69.984 \times 10^3 \text{ mm}^4$$

$$C = \frac{1}{2}d = 18 \text{ mm}$$

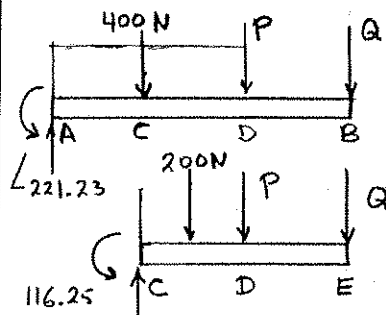
$$S = \frac{I}{C} = 3.888 \times 10^3 \text{ mm}^3 = 3.888 \times 10^{-6} \text{ m}^3$$

$$\text{At } A, M_A = S \sigma_A$$

$$M_A = (3.888 \times 10^{-6})(-56.9) = -221.25 \text{ N}\cdot\text{m}$$

$$\text{At } C, M_C = S \sigma_C$$

$$M_C = (3.888 \times 10^{-6})(-29.9) = -116.25 \text{ N}\cdot\text{m}$$



$$\sum M_A = 0:$$

$$221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \quad (1)$$

$$+\sum M_C = 0:$$

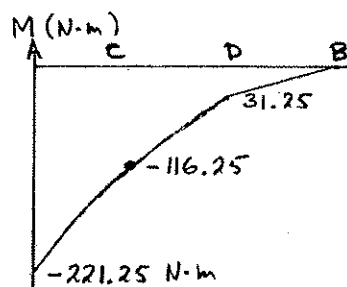
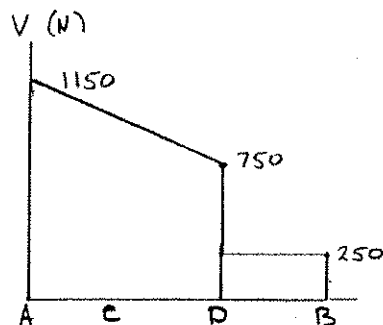
$$116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

$$0.1P + 0.225Q = 106.25 \quad (2)$$

Solving (1) and (2) simultaneously

$$P = 500 \text{ N} \quad \leftarrow$$

$$Q = 250 \text{ N} \quad \leftarrow$$



Reaction force at A

$$R_A - 400 - 500 - 250 = 0$$

$$R_A = 1150 \text{ N}\cdot\text{m}$$

$$V_A = 1150 \text{ N}$$

$$V_D = 250$$

$$M_A = -221.25 \text{ N}\cdot\text{m}$$

$$M_C = -116.25 \text{ N}\cdot\text{m}$$

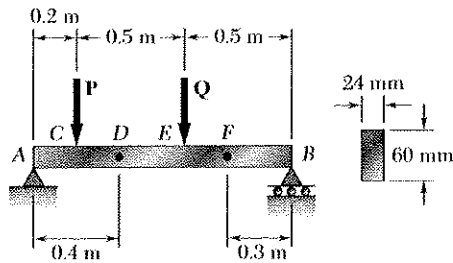
$$M_D = -31.25 \text{ N}\cdot\text{m}$$

$$|V|_{\max} = 1150 \text{ N} \quad \leftarrow$$

$$|M|_{\max} = 221 \text{ N}\cdot\text{m} \quad \leftarrow$$

Problem 5.63

*5.63 The beam AB supports two concentrated loads P and Q . The normal stress due to bending on the bottom edge of the beam is $+55 \text{ MPa}$ at D and $+37.5 \text{ MPa}$ at F . (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending that occurs in the beam.



$$I = \frac{1}{12}(24)(60)^3 = 432 \times 10^3 \text{ mm}^4$$

$$c = 30 \text{ mm}$$

$$S = \frac{I}{c} = 14.4 \times 10^3 \text{ mm}^3 = 14.4 \times 10^{-6} \text{ m}^3$$

$$M = S \sigma$$

$$\text{At } D, \quad M_D = (14.4 \times 10^{-6})(55 \times 10^6) = 792 \text{ N}\cdot\text{m}$$

$$\text{At } F, \quad M_F = (14.4 \times 10^{-6})(37.5 \times 10^6) = 540 \text{ N}\cdot\text{m}$$

Using free body FB , $\sum M_F = 0$:

$$-540 + 0.3B = 0$$

$$B = \frac{540}{0.3} = 1800 \text{ N}$$

Using free body $DEFB$, $\sum M_D = 0$:

$$-792 - 0.3Q + (0.8)(1800) = 0$$

$$Q = 2160 \text{ N}$$

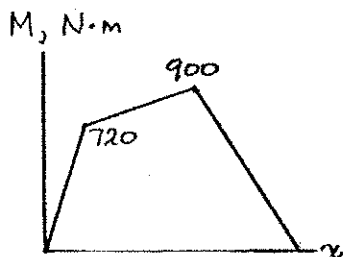
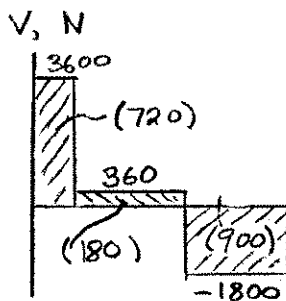
Using entire beam, $\sum M_A = 0$:

$$-0.2P - (0.7)(2160) + (1.2)(1800) = 0$$

$$P = 3240 \text{ N}$$

$$+\uparrow \sum F_y = 0: \quad A - 3240 - 2160 + 1800 = 0$$

$$A = 3600 \text{ N}$$



Shear diagram and its areas.

$$A \text{ to } C: \quad V = 3600 \text{ N}$$

$$C \text{ to } E: \quad V = 3600 - 3240 = 360 \text{ N}$$

$$E \text{ to } B: \quad V = 360 - 2160 = -1800 \text{ N}$$

$$A_{AC} = (0.2)(3600) = 720 \text{ N}\cdot\text{m}$$

$$A_{CE} = (0.5)(360) = 180 \text{ N}\cdot\text{m}$$

$$A_{EB} = (0.5)(-1800) = -900 \text{ N}\cdot\text{m}$$

Bending moments.

$$M_A = 0$$

$$M_C = 0 + 720 = 720 \text{ N}\cdot\text{m}$$

$$M_E = 720 + 180 = 900 \text{ N}\cdot\text{m}$$

$$M_B = 900 - 900 = 0$$

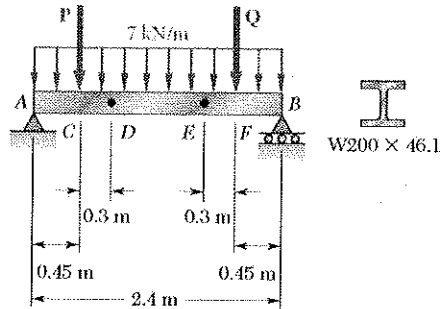
$$|M|_{\max} = 900 \text{ N}\cdot\text{m}$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{900}{14.4 \times 10^{-6}} = 62.5 \times 10^6 \text{ Pa}$$

$$(b) \quad \sigma_{\max} = 62.5 \text{ MPa}$$

Problem 5.64

*5.64 The beam AB supports a uniformly distributed load of 7 kN/m and two concentrated loads P and Q . The normal stress due to bending on the bottom edge of the lower flange is $+100 \text{ MPa}$ at D and $+73 \text{ MPa}$ at E . (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending that occurs in the beam.



For $W200 \times 46.1$ rolled steel section $S = 448 \times 10^3 \text{ mm}^3$

$$M = S \sigma$$

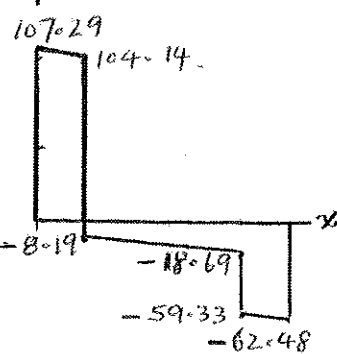
$$\text{At } D \quad M_D = (448 \times 10^3)(100 \times 10^6) = 44.8 \text{ kNm}$$

$$\text{At } E \quad M_E = (448 \times 10^3)(73 \times 10^6) = 32.7 \text{ kNm}$$

$$M_D = 44.8 \text{ kNm}$$

$$M_E = 32.7 \text{ kNm}$$

$V, \text{ kN}$



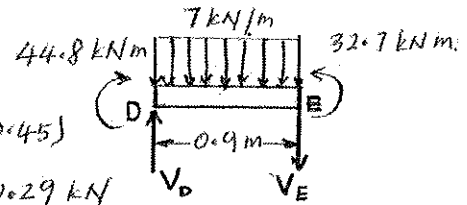
Use free body DE

$$+\circlearrowleft \sum M_E = 0$$

$$-44.8 + 32.7 + (7)(0.9)(0.45) - 0.9 V_D = 0$$

$$-0.9 V_D = 0$$

$$V_D = -10.29 \text{ kN}$$



$$+\circlearrowleft \sum M_D = 0$$

$$-44.8 + 32.7 - (7)(0.9)(0.45) - 0.9 V_E = 0$$

$$V_E = -16.59 \text{ kN}$$

Use free body ACD

$$+\circlearrowleft \sum M_A = 0$$

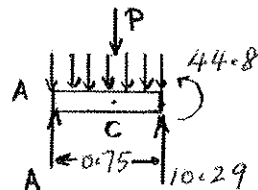
$$-0.45 P + (7)(0.75)(0.375) + (0.75)(10.29) + 44.8 = 0$$

$$P = 112.33 \text{ kN} \downarrow$$

$$+\uparrow \sum F_y = 0$$

$$A - (7)(0.75) + 10.29 - 112.33 = 0$$

$$A = 107.29 \text{ kN} \uparrow$$



Use free body EFB

$$+\circlearrowleft \sum M_B = 0$$

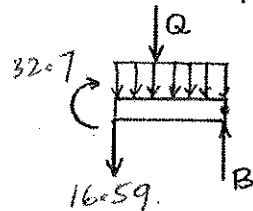
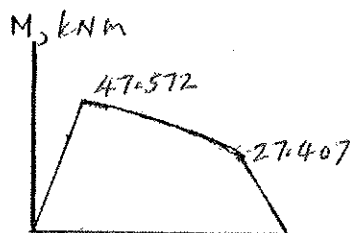
$$0.45 Q + (7)(0.75)(0.375) + (0.75)(16.59) - 32.7 = 0$$

$$Q = 40.64 \text{ kN}$$

$$+\uparrow \sum F_y = 0$$

$$B - 16.59 - (0.75)(7) - 40.64 = 0$$

$$B = 62.48 \text{ kN}$$



continued

Problem 5.64 continued

Areas of load diagram

$$\begin{aligned} A \text{ to } C & (0.45)(7) = 3.15 \text{ kN} \\ C \text{ to } F & (1.5)(7) = 10.5 \text{ kN} \\ F \text{ to } B & (0.45)(7) = 3.15 \text{ kN} \end{aligned}$$

Shear diagram

$$\begin{aligned} V_A &= 107.29 \text{ kN} \\ V_C^- &= 107.29 - 3.15 = 104.14 \text{ kN} \\ V_C^+ &= 104.14 - 112.33 = -8.19 \text{ kN} \\ V_F^- &= -8.19 - 10.5 = -18.69 \text{ kN} \\ V_F^+ &= -18.69 - 40.64 = -59.33 \text{ kN} \\ V_B &= -59.33 - 3.15 = -62.48 \text{ kN} \end{aligned}$$

Areas of shear diagram

$$\begin{aligned} A \text{ to } C & \frac{1}{2}(0.45)(107.29 + 104.14) = 47.572 \text{ kNm} \\ C \text{ to } F & \frac{1}{2}(1.5)(-8.19 - 18.69) = -20.16 \text{ kNm} \\ F \text{ to } B & \frac{1}{2}(0.45)(-59.33 - 62.48) = -27.407 \text{ kNm} \end{aligned}$$

Bending moments

$$\begin{aligned} M_A &= 0 \\ M_C &= 0 + 47.572 = 47.572 \text{ kNm} \\ M_F &= 47.572 - 20.16 = 27.412 \text{ kNm} \\ M_B &= 27.412 - 27.407 \approx 0 \end{aligned}$$

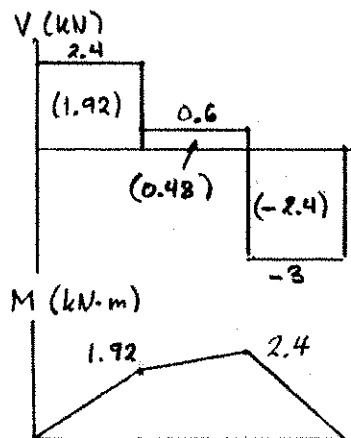
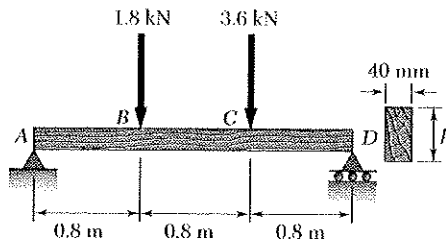
Maximum $|M|$ occurs at C $|M|_{\max} = 47.572 \text{ kNm}$

Maximum stress

$$\sigma = \frac{|M|_{\max}}{S} = \frac{47.572 \times 10^3}{.448 \times 10^{-6}} = 106.2 \text{ MPa}$$

Problem 5.65

5.65 and 5.66 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



$$+\circlearrowleft \sum M_D = 0:$$

$$-2.4A + (1.6)(1.8) + (0.8)(3.6) = 0$$

$$A = 2.4 \text{ kN}$$

$$+\circlearrowleft \sum M_A = 0:$$

$$-(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0$$

$$D = 3 \text{ kN}$$

Construct shear and bending moment diagrams.

$$|M|_{\max} = 2.4 \text{ kN}\cdot\text{m} = 2.4 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{2.4 \times 10^3}{12 \times 10^6} = 200 \times 10^{-6} \text{ m}^3$$

$$= 200 \times 10^3 \text{ mm}^3$$

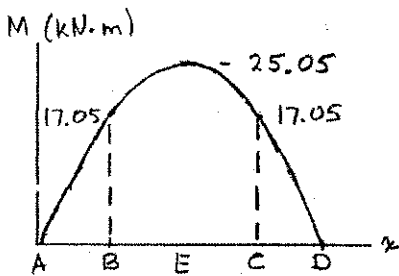
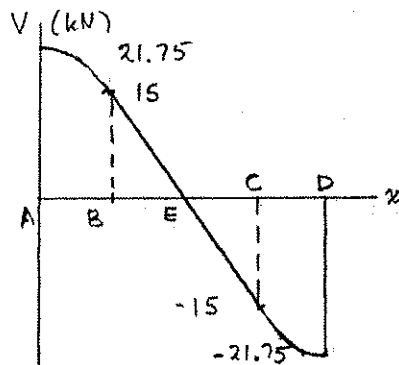
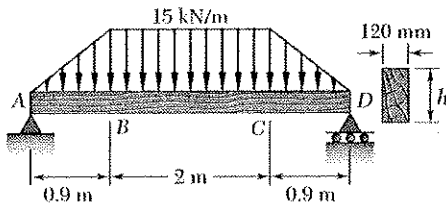
$$S = \frac{1}{6}bh^2 = \frac{1}{6}(40)h^2 = 200 \times 10^3$$

$$h^2 = \frac{(6)(200 \times 10^3)}{40} = 30 \times 10^3 \text{ mm}^2$$

$$h = 173.2 \text{ mm} \blacktriangleleft$$

Problem 5.66

5.65 and 5.66 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



By symmetry, $A = D$.

$$+\uparrow \sum F_y = 0: A + \frac{1}{2}(0.9)(15) + (2)(15) + \frac{1}{2}(0.9) + D = 0$$

$$A = D = 21.75 \text{ kN}$$

Shear diagram: $V_A = 21.75 \text{ kN}$

$$V_B = 21.75 - \frac{1}{2}(0.9)(15) = 15.0 \text{ kN}$$

$$V_C = 15.0 - (2)(15) = -15.0 \text{ kN}$$

$$V_D = -15.0 - \frac{1}{2}(0.9)(15) = -21.75 \text{ kN}$$

Locate point E where $V = 0$.

By symmetry, E is the midpoint of BC.

Areas of shear diagram.

$$A \text{ to } B: (0.9)(15) + \frac{2}{3}(0.9)(21.75 - 15) = 17.55 \text{ kN}\cdot\text{m}$$

$$B \text{ to } E: \frac{1}{2}(1)(15) = 7.5 \text{ kN}\cdot\text{m}$$

$$E \text{ to } C: \frac{1}{2}(1)(-15) = -7.5 \text{ kN}\cdot\text{m}$$

$$C \text{ to } D: \text{ By anti-symmetry } -17.55 \text{ kN}\cdot\text{m}$$

Bending moments: $M_A = 0$

$$M_B = 0 + 17.55 = 17.55 \text{ kN}\cdot\text{m}$$

$$M_E = 17.55 + 7.5 = 25.05 \text{ kN}\cdot\text{m}$$

$$M_C = 25.05 - 7.5 = 17.55 \text{ kN}\cdot\text{m}$$

$$M_D = 17.55 - 17.55 = 0$$

$$|M|_{\max} = 25.05 \text{ kN}\cdot\text{m} = 25.05 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{M}{S} \quad S = \frac{M}{\sigma} = \frac{25.05 \times 10^3}{12 \times 10^6} = 2.0875 \times 10^{-3} \text{ m}^3 = 2.0875 \times 10^6 \text{ mm}^3$$

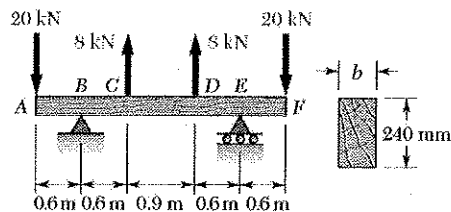
For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(2.0875 \times 10^6)}{120}} = 323 \text{ mm}$$

$$h = 323 \text{ mm} \quad \blacktriangleleft$$

Problem 5.67

5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

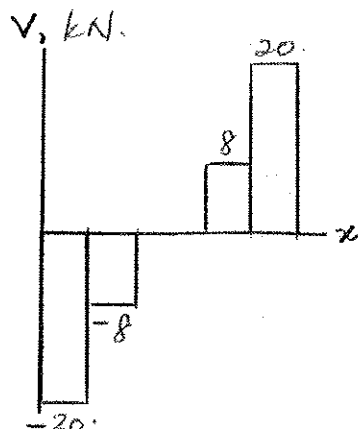


For equilibrium

$$B = E = 12 \text{ kN}$$

Shear diagram

$$\begin{aligned} A \text{ to } B^- & V = -20 \text{ kN} \\ B^+ \text{ to } C^- & V = -20 + 12 = -8 \\ C^+ \text{ to } D^- & V = -8 + 8 = 0 \\ D^+ \text{ to } E^- & V = 0 + 8 = 8 \text{ kN} \\ E^+ \text{ to } F & V = 8 + 12 = 20 \text{ kN} \end{aligned}$$

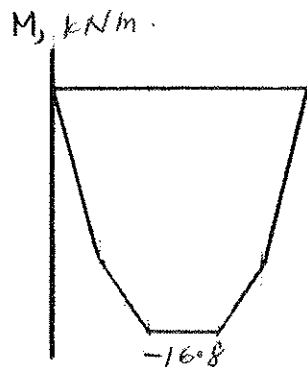


Areas of shear diagram

$$\begin{aligned} A \text{ to } B & (0.6)(-20) = -12 \text{ kNm} \\ B \text{ to } C & (0.6)(-8) = -4.8 \text{ kNm} \\ C \text{ to } D & (0.9)(0) = 0 \\ D \text{ to } E & (0.6)(8) = 4.8 \text{ kNm} \\ E \text{ to } F & (0.6)(20) = 12 \text{ kNm} \end{aligned}$$

Bending moments

$$\begin{aligned} M_A &= 0 \\ M_B &= 0 - 12 = -12 \text{ kNm} \\ M_C &= -12 - 4.8 = -16.8 \text{ kNm} \\ M_D &= -16.8 + 0 = -16.8 \text{ kNm} \\ M_E &= -16.8 + 4.8 = -12 \\ M_F &= -12 + 12 = 0 \end{aligned}$$



$$|M|_{\max} = 16.8 \text{ kNm}$$

Required value for S

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{16.8 \times 10^3}{12 \times 10^6} = 1400 \times 10^{-6} \text{ m}^3$$

For a rectangular section $I = \frac{1}{12}bh^3$, $c = \frac{1}{2}h$

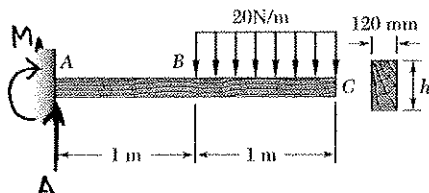
$$S = \frac{I}{c} = \frac{bh^2}{6} = \frac{(b)(0.24)^2}{6} = 0.0096 b$$

Equating the two expressions for S

$$\begin{aligned} 0.0096 b &= 1400 \times 10^{-6} \\ b &= 146 \text{ mm} \\ &= 0.146 \text{ m} \end{aligned}$$

Problem 5.68

5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

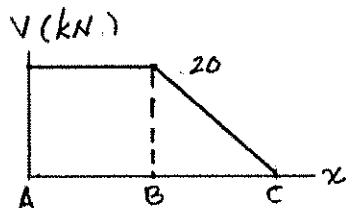


$$\text{Reactions: } +\uparrow \Sigma F_y = 0: A - (1)(20) = 0$$

$$A = 20 \text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0: -M_A - (20)(1)(1.5) = 0$$

$$M_A = -30 \text{ kNm}$$



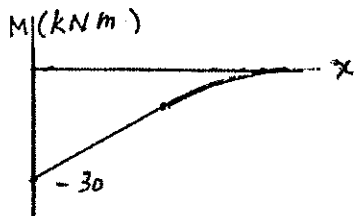
Shear diagram: $V_A = V_B = 20 \text{ kN}$

$$V_C = 20 - (1)(20) = 0$$

Areas of shear diagram.

A to B. $(1)(20) = 20 \text{ kNm}$

B to C. $\frac{1}{2}(1)(20) = 10 \text{ kNm}$



Bending moments: $M_A = -30 \text{ kNm}$

$$M_B = -30 + 20 = -10 \text{ kNm}$$

$$M_C = -10 + 10 = 0$$

$$|M|_{\max} = 30 \text{ kNm}$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S}$$

$$S = \frac{|M|_{\max}}{\sigma_{\max}} = \frac{30 \times 10^3}{12 \times 10^6} = 2.5 \times 10^{-3} \text{ m}^3$$

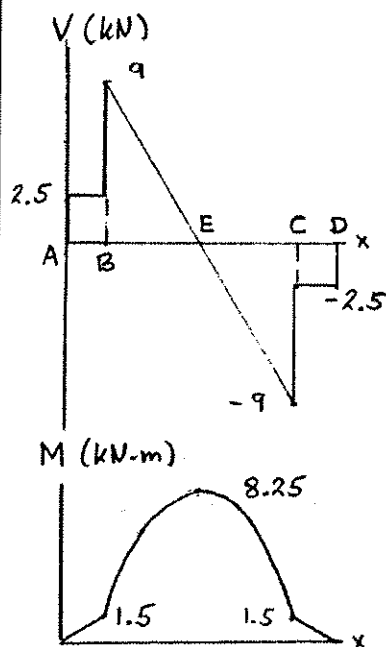
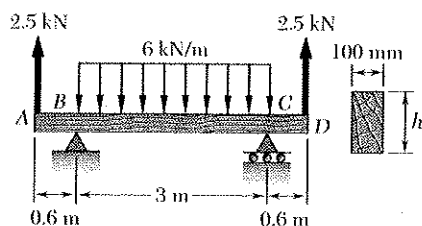
For a rectangular section, $S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(2.5 \times 10^{-3})}{120}} = 353.6 \text{ mm}$$

$$h = 354 \text{ mm} \quad \blacktriangleleft$$

Problem 5.69

5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



By symmetry, $B = C$

$$+\uparrow \sum F_y = 0: B + C + 2.5 + 2.5 - (3)(6) = 0$$

$$B = C = 6.5 \text{ kN}$$

Shear: A to B. $V = 2.5 \text{ kN}$

$$V_{B^+} = 2.5 + 6.5 = 9 \text{ kN}$$

$$V_{C^-} = 9 - (3)(6) = -9 \text{ kN}$$

$$C \text{ to } D. V = -9 + 6.5 = -2.5 \text{ kN}$$

Areas of the shear diagram.

$$A \text{ to } B. \int V dx = (0.6)(2.5) = 1.5 \text{ kN}\cdot\text{m}$$

$$B \text{ to } E. \int V dx = \left(\frac{1}{2}\right)(1.5)(9) = 6.75 \text{ kN}\cdot\text{m}$$

$$E \text{ to } C. \int V dx = -6.75 \text{ kN}\cdot\text{m}$$

$$C \text{ to } D. \int V dx = -1.5 \text{ kN}\cdot\text{m}$$

Bending moments. $M_A = 0$

$$M_B = 0 + 1.5 = 1.5 \text{ kN}\cdot\text{m}$$

$$M_E = 1.5 + 6.75 = 8.25 \text{ kN}\cdot\text{m}$$

$$M_C = 8.25 - 6.75 = 1.5 \text{ kN}\cdot\text{m}$$

$$M_D = 1.5 - 1.5 = 0$$

$$\text{Maximum } |M| = 8.25 \text{ kN}\cdot\text{m} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\text{min}} = \frac{|M|_{\text{max}}}{\sigma_{\text{all}}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$$

For a rectangular section, $S = \frac{1}{6} b h^2$

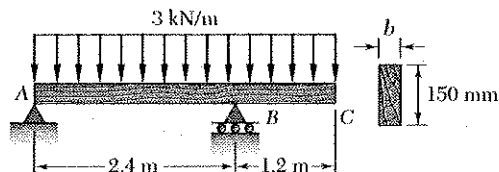
$$687.5 \times 10^3 = \left(\frac{1}{6}\right)(100) h^2$$

$$h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2$$

$$h = 203 \text{ mm}$$

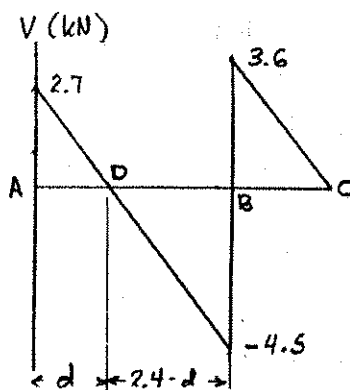
Problem 5.70

5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



$$+\circlearrowleft M_B = 0: -2.4 A + (0.6)(3.6)(3) = 0 \quad A = 2.7 \text{ kN}$$

$$+\circlearrowleft M_A = 0: -(1.8)(3.6)(3) + 2.4 B = 0 \quad B = 8.1 \text{ kN}$$

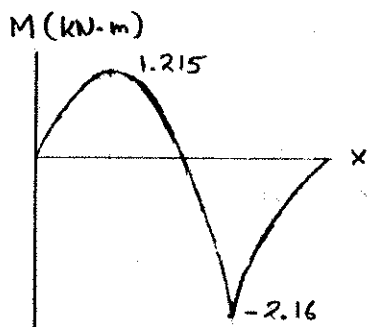


Shear: $V_A = 2.7 \text{ kN}$
 $V_B = 2.7 - (2.4)(3) = -4.5 \text{ kN}$
 $V_B^+ = -4.5 + 8.1 = 3.6 \text{ kN}$
 $V_C = 3.6 - (1.2)(3) = 0$

Locate point D where $V = 0$.

$$\frac{d}{2.7} = \frac{2.4-d}{4.5} \quad 7.2 d = 6.48$$

$$d = 0.9 \text{ m} \quad 2.4 - d = 1.5 \text{ m}$$



Areas of the shear diagram.

$$A \text{ to } D: \int V dx = \left(\frac{1}{2}\right)(0.9)(2.7) = 1.215 \text{ kN}\cdot\text{m}$$

$$D \text{ to } B: \int V dx = \left(\frac{1}{2}\right)(1.5)(-4.5) = -3.375 \text{ kN}\cdot\text{m}$$

$$B \text{ to } C: \int V dx = \left(\frac{1}{2}\right)(1.2)(3.6) = 2.16 \text{ kN}\cdot\text{m}$$

Bending moments: $M_A = 0$

$$M_D = 0 + 1.215 = 1.215 \text{ kN}\cdot\text{m}$$

$$M_B = 1.215 - 3.375 = -2.16 \text{ kN}\cdot\text{m}$$

$$M_C = -2.16 + 2.16 = 0$$

$$\text{Maximum } |M| = 2.16 \text{ kN}\cdot\text{m} = 2.16 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{2.16 \times 10^3}{12 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

For rectangular section,

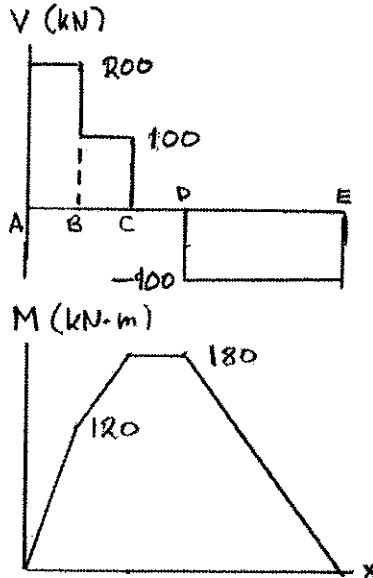
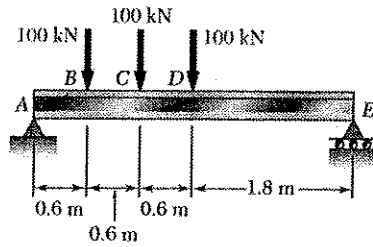
$$S = \frac{1}{6} b h^2 = \frac{1}{6} b (150)^2 = 180 \times 10^3$$

$$b = \frac{(6)(180 \times 10^3)}{150^2}$$

$$b = 48.0 \text{ mm} \quad \blacktriangleleft$$

Problem 5.71

5.71 and 5.72 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.



$$\sum M_E = 0 \quad -3.6 A + (3)(100) + (2.4)(100) + (1.8)(100) = 0$$

$$A = 200 \text{ kN}$$

$$\sum M_A = 0 \quad 3.6 E - (1.8)(100) - (1.2)(100) - (0.6)(100) = 0$$

$$E = 100 \text{ kN}$$

$$\text{Shear: } A \text{ to } B \quad V = 200 \text{ kN}$$

$$B \text{ to } C \quad V = 200 - 100 = 100 \text{ kN}$$

$$C \text{ to } D \quad V = 100 - 100 = 0$$

$$D \text{ to } E \quad V = 0 - 100 = -100 \text{ kN}$$

Areas under shear diagram

$$A \text{ to } B \quad \int V dx = (0.6)(200) = 120 \text{ kN}\cdot\text{m}$$

$$B \text{ to } C \quad \int V dx = (0.6)(100) = 60 \text{ kN}\cdot\text{m}$$

$$C \text{ to } D \quad \int V dx = 0$$

$$D \text{ to } E \quad \int V dx = (1.8)(-100) = -180 \text{ kN}\cdot\text{m}$$

Bending moments $M_A = 0$

$$M_B = 0 + 120 = 120 \text{ kN}\cdot\text{m}$$

$$M_C = 120 + 60 = 180 \text{ kN}\cdot\text{m}$$

$$M_D = 180 + 0 = 180 \text{ kN}\cdot\text{m}$$

$$M_E = 180 - 180 = 0$$

$$\text{Maximum } |M| = 180 \text{ kN}\cdot\text{m} = 180 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{180 \times 10^3}{160 \times 10^6} = 1.125 \times 10^{-3} \text{ m}^3 = 1125 \times 10^3 \text{ mm}^3$$

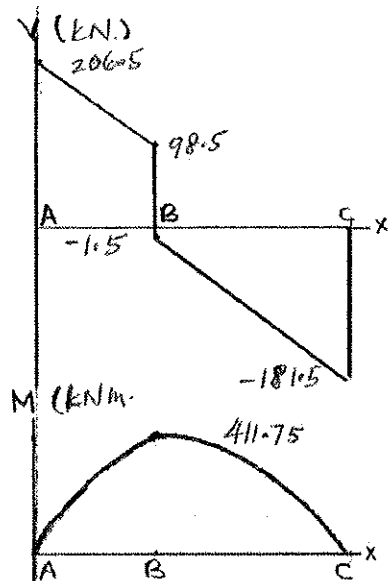
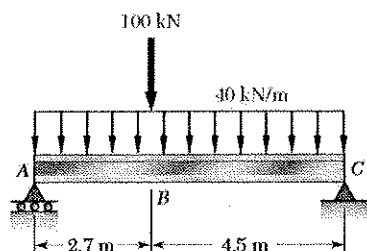
Shape	S (10^3 mm^3)
W 530 $\times$ 66	1340
W 460 $\times$ 74	1460
W 360 $\times$ 79	1280
W 250 $\times$ 101	1240

Lightest wide flange beam

W 530 $\times$ 66 @ 66 kg/m

Problem 5.72

5.71 and 5.72 Knowing that the allowable stress for the steel used is 165 MPa, select the most economical wide-flange beam to support the loading shown.



$$+\circlearrowleft \sum M_C = 0 \quad -7.2 A + (40)(7.2)(3.6) + (100)(4.5) = 0$$

$$A = 206.5 \text{ kN}$$

$$+\circlearrowleft \sum M_A = 0 \quad 7.2 C - (40)(7.2)(3.6) - (100)(2.7) = 0$$

$$C = 181.5 \text{ kN}$$

Shear: $V_A = 206.5$

$$V_B^- = 206.5 - (40)(2.7) = 98.5 \text{ kN}$$

$$V_B^+ = 98.5 - 100 = -1.5 \text{ kN}$$

$$V_C = -1.5 - (40)(4.5) = -181.5$$

Areas under shear diagram

$$A \text{ to } B \quad \int V dx = \left(\frac{1}{2}\right)(2.7)(206.5 + 98.5) = 411.75 \text{ kNm}$$

$$B \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(4.5)(-1.5 - 181.5) = -411.75 \text{ kNm}$$

Bending moments: $M_A = 0$

$$M_B = 0 + 411.75 = 411.75 \text{ kNm}$$

$$M_C = 411.75 - 411.75 = 0$$

$$\text{Maximum } |M| = 411.75 \text{ kNm}$$

$$\sigma_{\text{all}} = 165 \text{ MPa}$$

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{411.75 \times 10^3}{165 \times 10^6} = 2.495 \times 10^{-3} \text{ m}^3$$

$$= 2495 \times 10^{-3} \text{ mm}^3$$

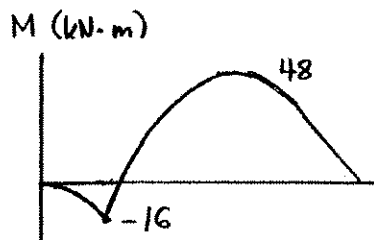
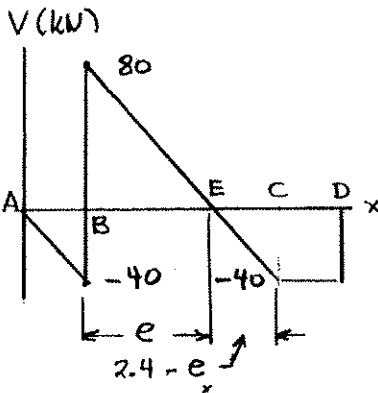
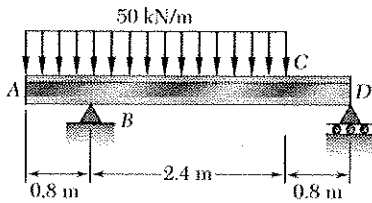
Shape	$S (\text{mm}^3 \times 10^3)$
W690 X 125	3510
W610 X 101	2530 ←
W530 X 150	3720
W460 X 113	2400
W360 X 216	3800

Lightest wide flange beam

$$W610 \times 101 @ 101 \text{ kg/m}$$

Problem 5.73

5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.



$$\sum M_D = 0: -3.2B + (2.4)(3.2)(50) = 0$$

$$B = 120 \text{ kN}$$

$$\sum M_B = 0: 3.2D - (0.8)(3.2)(50) = 0$$

$$D = 40 \text{ kN}$$

Shear: $V_A = 0$

$$V_B^- = 0 - (0.8)(50) = -40 \text{ kN}$$

$$V_B^+ = -40 + 120 = 80 \text{ kN}$$

$$V_C = 80 - (2.4)(50) = -40 \text{ kN}$$

$$V_D = -40 + 0 = -40 \text{ kN}$$

Locate point E where $V = 0$.

$$\frac{e}{80} = \frac{2.4 - e}{40} \quad 120e = 192$$

$$e = 1.6 \text{ m} \quad 2.4 - e = 0.8 \text{ m}$$

Areas: A to B, $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN}\cdot\text{m}$

B to E, $\int V dx = (\frac{1}{2})(1.6)(80) = 64 \text{ kN}\cdot\text{m}$

E to C, $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN}\cdot\text{m}$

C to D, $\int V dx = (0.8)(-40) = -32 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 - 16 = -16 \text{ kN}\cdot\text{m}$$

$$M_E = -16 + 64 = 48 \text{ kN}\cdot\text{m}$$

$$M_C = 48 - 16 = 32 \text{ kN}\cdot\text{m}$$

$$M_D = 32 - 32 = 0$$

$$\text{Maximum } |M| = 48 \text{ kN}\cdot\text{m} = 48 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

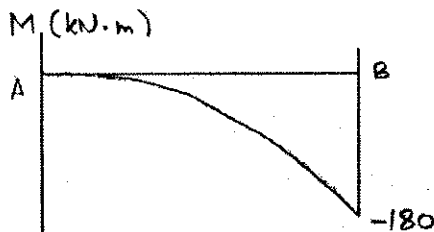
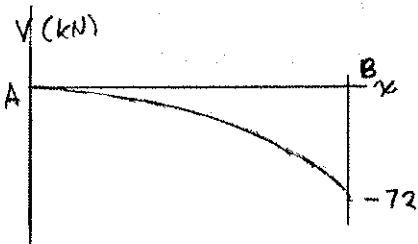
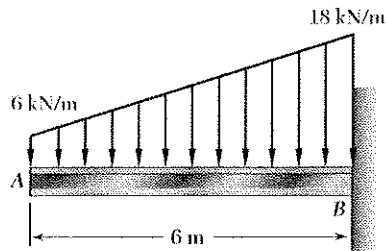
Shape	S (10^3 mm^3)
W 310 $\times$ 32.7	415
→ W 250 $\times$ 28.4	308
W 200 $\times$ 35.9	342

Lightest wide flange beam

W 250 $\times$ 28.4 @ 28.4 kg/m

Problem 5.74

5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.



$$w = (6 + \frac{18-6}{6}x) = (6 + 2x) \text{ kN/m}$$

$$\frac{dV}{dx} = -w = -6 - 2x$$

$$V = -6x - x^2 + C_1$$

$$V = 0 \text{ at } x = 0. \quad C_1 = 0$$

$$\frac{dM}{dx} = V = -6x - x^2$$

$$M = -3x^2 - \frac{1}{3}x^3 + C_2$$

$$M = 0 \text{ at } x = 0 \quad C_2 = 0$$

$$M = -3x^2 - \frac{1}{3}x^3$$

$$|M|_{\text{max}} \text{ occurs at } x = 6 \text{ m}$$

$$|M|_{\text{max}} = \left| -(3)(6)^2 - \left(\frac{1}{3}\right)(6)^3 \right| = 180 \text{ kN}\cdot\text{m} \\ = 180 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{180 \times 10^3}{160 \times 10^6} = 1.125 \times 10^{-3} \text{ m}^3 \\ = 1.125 \times 10^3 \text{ mm}^3$$

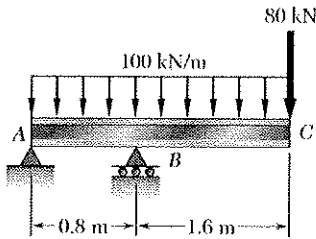
Lightest acceptable wide flange beam:

W 530 × 66

Shape	S (10^3 mm^3)
W 530 × 66	1346 ←
W 460 × 74	1460
W 410 × 85	1510
W 360 × 79	1280
W 310 × 107	1590
W 250 × 101	1240

Problem 5.75

5.75 and 5.76 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.



$$+\circlearrowleft \sum M_B = 0 \quad 0.8A - (0.4)(2.4)(100) - (1.6)(80) = 0$$

$$A = 280 \text{ kN} \downarrow$$

$$+\circlearrowleft \sum M_A = 0 \quad 0.8B - (1.2)(2.4)(100) - (2.4)(80) = 0$$

$$B = 600 \text{ kN} \uparrow$$

$$\text{Shear: } V_A = -280 \text{ kN}$$

$$V_B^- = -280 - (0.8)(100) = -360 \text{ kN}$$

$$V_B^+ = -360 + 600 = 240 \text{ kN}$$

$$V_C = 240 - (1.6)(100) = 80 \text{ kN}$$

Areas under shear diagram

$$A \text{ to } B \quad \left(\frac{1}{2}\right)(0.8)(-280 - 360) = -256 \text{ kN}\cdot\text{m}$$

$$B \text{ to } C \quad \left(\frac{1}{2}\right)(1.6)(240 + 80) = 256 \text{ kN}\cdot\text{m}$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 - 256 = -256 \text{ kN}\cdot\text{m}$$

$$M_C = -256 + 256 = 0$$

$$\text{Maximum } |M| = 256 \text{ kN}\cdot\text{m} = 256 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^3 \text{ mm}^3$$

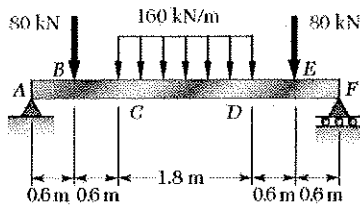
Shape	$S (10^3 \text{ mm}^3)$
S 510 × 98.3	1950
S 460 × 104	1685

Lightest S-section

S 510 × 98.3

Problem 5.76

5.75 and 5.76 Knowing that the allowable stress for the steel used is 165 MPa, select the most economical S-shape beam to support the loading shown.



From statics the reactions at A and F are

$$A = 224 \text{ kN} \uparrow \quad B = 224 \text{ kN} \uparrow$$

$$\text{Area of load diagram over CD} \quad (1.8)(160) = 288 \text{ kN}$$

Shear diagram.

$$\begin{aligned} A \text{ to } B & \quad V = 224 \text{ kN} \\ B \text{ to } C & \quad V = 224 - 80 = 144 \text{ kN} \\ \text{At } D & \quad V = 144 - 288 = -144 \text{ kN} \\ D \text{ to } E & \quad V = -144 \text{ kN} \\ E \text{ to } F & \quad V = -144 - 80 = -224 \text{ kN} \end{aligned}$$

$$\text{Over CD} \quad V = 144 - 160(x - 1.2) = 0$$

$$\text{At G} \quad V = 0 \quad x_G = 2.1 \text{ m}$$

Areas of shear diagram.

$$\begin{aligned} A \text{ to } B & \quad (0.6)(224) = 134.4 \text{ kNm} \\ B \text{ to } C & \quad (0.6)(144) = 86.4 \text{ kNm} \\ C \text{ to } G & \quad \frac{1}{2}(0.9)(144) = 129.6 \text{ kNm} \\ G \text{ to } D & \quad \frac{1}{2}(0.9)(-144) = -129.6 \text{ kNm} \\ D \text{ to } E & \quad (0.6)(-144) = -86.4 \text{ kNm} \\ E \text{ to } F & \quad (0.6)(-224) = -134.4 \text{ kNm} \end{aligned}$$

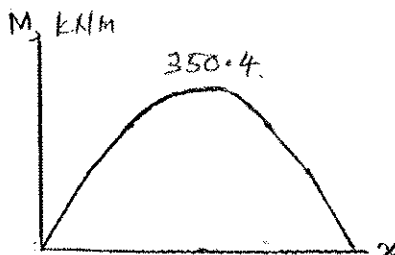
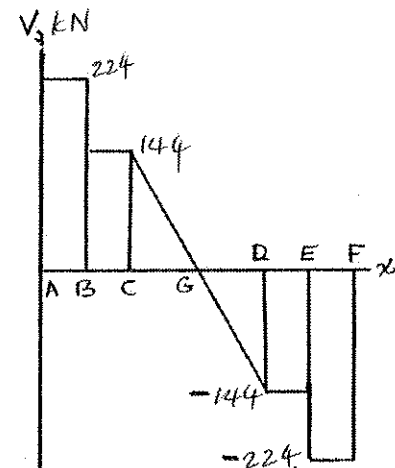
Bending moments. $M_A = 0$

$$\begin{aligned} M_B & = 0 + 134.4 = 134.4 \text{ kNm} \\ M_C & = 134.4 + 86.4 = 220.8 \text{ kNm} \\ M_G & = 220.8 + 129.6 = 350.4 \text{ kNm} \\ M_D & = 350.4 - 129.6 = 220.8 \text{ kNm} \\ M_E & = 220.8 - 86.4 = 134.4 \text{ kNm} \\ M_F & = 134.4 - 134.4 = 0 \end{aligned}$$

$$|M|_{\max} = 350.4 \text{ kNm}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{350.4 \times 10^3}{165 \times 10^6} = 2.121 \times 10^{-3} \text{ m}^3 = 2121 \times 10^3 \text{ mm}^3$$

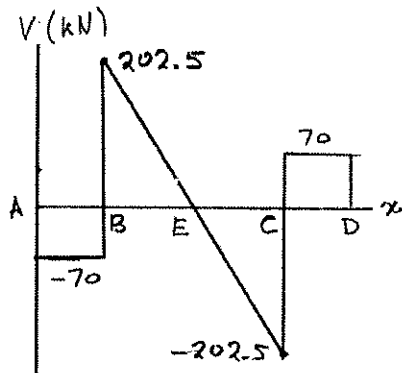
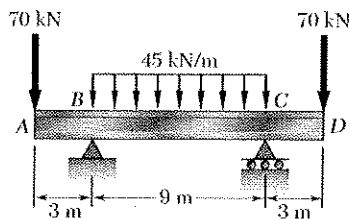
Use S 510 x 128



Shape	$S_x (\times 10^3 \text{ mm}^3)$
S 510 x 128	2550

Problem 5.77

5.77 and 5.78 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.



Reactions: By symmetry $B = C$

$$+\uparrow \sum F_y = 0: -70 + B - (9)(45) + C - 70 = 0$$

$$B = C = 272.5 \text{ kN} \uparrow$$

Shear: $V_A = -70 \text{ kN}$

$$V_B^- = -70 + 0 = -70 \text{ kN}$$

$$V_B^+ = -70 + 272.5 = 202.5 \text{ kN}$$

$$V_C^- = 202.5 - (9)(45) = -202.5 \text{ kN}$$

$$V_C^+ = -202.5 + 272.5 = 70 \text{ kN}$$

$$V_D = 70 \text{ kN}$$

Draw shear diagram. Locate point E where $V = 0$.

E is the midpoint of BC.

Areas of the shear diagram.

A to B. $\int V dx = (3)(-70) = -210 \text{ kN}\cdot\text{m}$

B to E. $\int V dx = \frac{1}{2}(4.5)(202.5) = 455.625 \text{ kN}\cdot\text{m}$

E to C. $\int V dx = \frac{1}{2}(4.5)(-202.5) = -455.625 \text{ kN}\cdot\text{m}$

C to D. $\int V dx = (3)(70) = 210 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 - 210 = -210 \text{ kN}\cdot\text{m}$$

$$M_E = -210 + 455.625 = 245.625 \text{ kN}\cdot\text{m}$$

$$M_C = 245.625 - 455.625 = -210 \text{ kN}\cdot\text{m}$$

$$M_D = -210 + 210 = 0$$

$$|M|_{\max} = 245.625 \text{ kN}\cdot\text{m} = 245.625 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\max} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$\sigma = \frac{|M|}{S}$$

$$S = \frac{|M|}{\sigma} = \frac{245.625 \times 10^3}{160 \times 10^6} = 1.5352 \times 10^{-3} \text{ m}^3$$

$$= 1535.2 \times 10^3 \text{ mm}^3$$

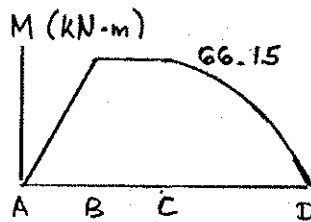
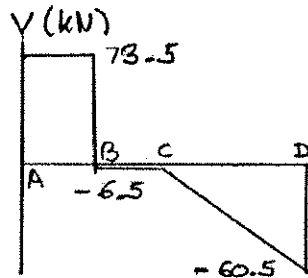
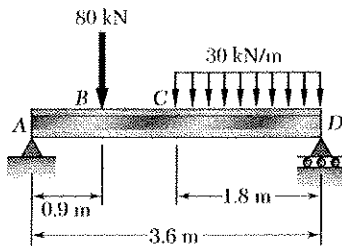
Shape	$S (10^3 \text{ mm}^3)$
S 610 x 119	2880
S 510 x 98.3	1950 ←
S 460 x 104	1685

Lightest S-shape

S 510 x 98.3

Problem 5.78

5.77 and 5.78 Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.



$$+\circlearrowleft \sum M_D = 0: -3.6 A + (2.7)(80) + (0.9)(1.8)(30) = 0$$

$$A = 73.5 \text{ kN} \uparrow$$

$$+\circlearrowleft \sum M_A = 0: 3.6 D - (0.9)(80) - (2.7)(1.8)(30) = 0$$

$$D = 60.5 \text{ kN} \uparrow$$

Shear: A to B. $V = 73.5 \text{ kN}$

B to C. $V = 73.5 - 80 = -6.5 \text{ kN}$

$$V_D = -6.5 - (1.8)(30) = -60.5 \text{ kN}$$

Areas of the shear diagram.

A to B. $\int V dx = (0.9)(73.5) = 66.15 \text{ kN-m}$

B to C. $\int V dx = (0.9)(-6.5) = -5.85 \text{ kN-m}$

C to D. $(\frac{1}{2})(1.8)(-6.5 - 60.5) = -60.30 \text{ kN-m}$

Bending moments: $M_A = 0$

$$M_B = 0 + 66.15 = 66.15 \text{ kN-m}$$

$$M_C = 66.15 - 5.85 = 60.30 \text{ kN-m}$$

$$M_D = 60.30 - 60.30 = 0$$

$$\text{Maximum } |M| = 66.15 \text{ kN-m} = 66.15 \times 10^3 \text{ N-m}$$

$$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

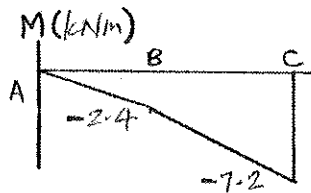
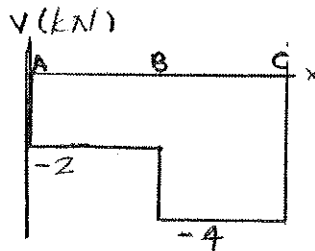
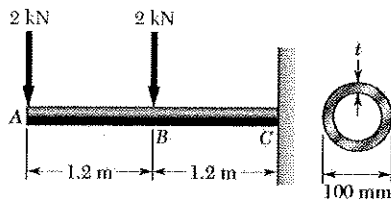
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.15 \times 10^3}{160 \times 10^6} = 413.4 \times 10^{-6} \text{ m}^3 = 413.4 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
S 380 x 64	971
S 310 x 47.3	593
S 250 x 52	482

Lightest S-section

$$S 310 \times 47.3 @ 47.3 \text{ kg/m}$$

Problem 5.79



5.79 A steel pipe of 10-mm diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from 6 mm to 24 mm in 3-mm increments, and that the allowable normal stress for the steel used is 165 MPa, determine the minimum wall thickness t that can be used.

Shear: A to B $V = -2 \text{ kN}$

B to C $V = -2 - 2 = -4 \text{ kN}$

Areas: A to B $-(1.2)(2) = -2.4 \text{ kN/m}$

B to C $(1.2)(-4) = -4.8 \text{ kN/m}$

Bending moments: $M_A = 0$

$M_B = 0 - 2.4 = -2.4 \text{ kN/m}$

$M_C = -2.4 - 4.8 = -7.2 \text{ kN/m}$

Maximum $|M| = 7.2 \text{ kN/m}$

$\sigma_{all} = 165 \text{ MPa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{7.2 \times 10^3}{165 \times 10^6} = 43.64 \times 10^{-6} \text{ m}^3 = 43.64 \times 10^{-3} \text{ mm}^3$$

$$I = \frac{\pi}{4} (C_2^4 - C_1^4)$$

$$C = C_2$$

$$C_2 = \frac{1}{2} d = 50 \text{ mm}$$

$$S = \frac{I}{C} = \frac{\pi}{4} \frac{C_2^4 - C_1^4}{C_2} = \frac{\pi}{4} \frac{50^4 - C_1^4}{50} = 43.64 \times 10^3 \text{ mm}^3$$

$$C_1^4 = 3471791$$

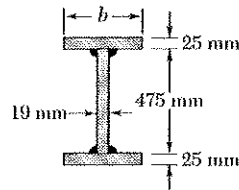
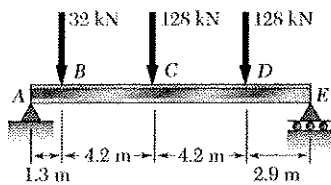
$$C_1 = 43.17 \text{ mm}$$

$$t_{min} = C_2 - C_1 = 50 - 43.17 = 6.83 \text{ mm}$$

Using 3 mm increments for design $t = 9 \text{ mm}$

Problem 5.80

5.80 Three steel plates are welded together to form the beam shown. Knowing that the allowable normal stress for the steel used is 154 MPa, determine the minimum flange width b that can be used.



Reactions. $+\circlearrowleft \sum M_E = 0:$

$$-12.6A + (11.3)(32) + (7.1)(128) + (2.9)(128) = 0$$

$$A = 130.3 \text{ kN} \quad \uparrow$$

$+\circlearrowright \sum M_A = 0:$

$$12.6E - (11.3)(32) - (5.5)(128) - (9.7)(128) = 0$$

$$E = 157.7 \text{ kN} \quad \uparrow$$

Shear: A to B. 130.3 kN

$$B \text{ to } C. \quad 130.3 - 32 = 98.3 \text{ kN}$$

$$C \text{ to } D. \quad 98.3 - 128 = -29.7 \text{ kN}$$

$$D \text{ to } E. \quad -29.7 - 128 = -157.7 \text{ kN}$$

Areas: A to B. $(1.3)(130.3) = 169.39 \text{ kNm}$

$$B \text{ to } C. \quad (4.2)(98.3) = 412.86 \text{ kNm}$$

$$C \text{ to } D. \quad (4.2)(-29.7) = -124.74 \text{ kNm}$$

$$D \text{ to } E. \quad (2.9)(-157.7) = -457.33 \text{ kNm}$$

Bending moments: $M_A = 0$

$$M_B = 0 + 169.39 = 169.39 \text{ kNm}$$

$$M_C = 169.39 + 412.86 = 582.25 \text{ kNm}$$

$$M_D = 582.25 - 124.74 = 457.51 \text{ kNm}$$

$$M_E = 457.51 - 457.33 = 0$$

$$\text{Maximum } |M| = 582.25 \text{ kNm}$$

$$\sigma_{all} = 154 \text{ MPa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{582.25 \times 10^3}{154 \times 10^6} = 3.7808 \times 10^{-3} \text{ m}^3 = 3.7808 \times 10^6 \text{ mm}^3$$

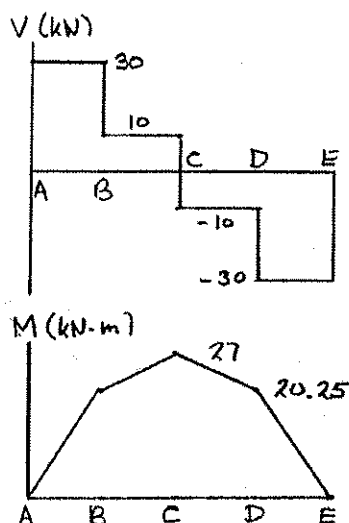
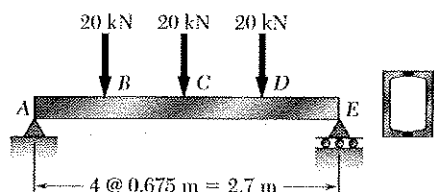
$$I = \frac{1}{12}(19)(475)^3 + 2\left[\frac{1}{12}(b)(25)^3 + (b)(25)(250)^2\right] = 169688802 + 3127604b$$

$$C = 237.5 + 25 = 262.5 \text{ mm}$$

$$S_{min} = \frac{I}{C} = 646433.5 + 11914.7b = 3.7808 \times 10^6 \quad b = 263 \text{ mm}$$

Problem 5.81

5.81 Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.



By symmetry, $A = E$

$$+\uparrow \sum F_y = 0: A + E - 20 - 20 - 20 = 0$$

$$A = E = 30 \text{ kN}$$

Shear: A to B. $V = 30 \text{ kN}$

B to C. $V = 30 - 20 = 10 \text{ kN}$

C to D. $V = 10 - 20 = -10 \text{ kN}$

D to E. $V = -10 - 20 = -30 \text{ kN}$

Areas: A to B. $(0.675)(30) = 20.25 \text{ kN}\cdot\text{m}$

B to C. $(0.675)(10) = 6.75 \text{ kN}\cdot\text{m}$

C to D. $(0.675)(-10) = -6.75 \text{ kN}\cdot\text{m}$

D to E. $(0.675)(-30) = -20.25 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$$M_B = 0 + 20.25 = 20.25 \text{ kN}\cdot\text{m}$$

$$M_C = 20.25 + 6.75 = 27 \text{ kN}\cdot\text{m}$$

$$M_D = 27 - 6.75 = 20.25 \text{ kN}\cdot\text{m}$$

$$M_E = 20.25 - 20.25 = 0$$

$$\text{Maximum } |M| = 27 \text{ kN}\cdot\text{m} = 27 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 150 \text{ MPa} = 150 \times 10^6 \text{ Pa}$$

For a section consisting of two channels,

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{27 \times 10^3}{150 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

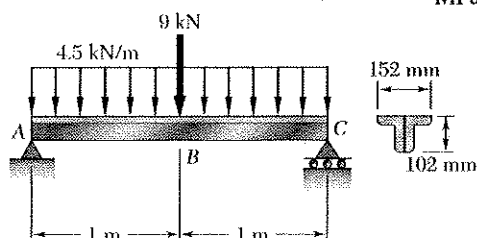
For each channel, $S_{\text{min}} = \left(\frac{1}{2}\right)(180 \times 10^3) = 90 \times 10^3 \text{ mm}^3$

Shape	$S (10^3 \text{ mm}^3)$
C 180 × 14.6	99.2
C 150 × 19.3	93.6

← Lightest channel section
C 180 × 14.6

Problem 5.82

5.82 Two L102 × 76 rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine the minimum angle thickness that can be used.



Reactions. By symmetry, $A = C$
 $\uparrow \sum F_y = 0: A - (2)(4.5) - 9 + C = 0$
 $A = C = 9 \text{ kN} \uparrow$

Shear: $V_A = 9 \text{ kN}$

$V_B^- = 9 - (1)(4.5) = 4.5 \text{ kN}$

$V_B^+ = 4.5 - 9 = -4.5 \text{ kN}$

$V_C = -4.5 - (1)(4.5) = -9 \text{ kN}$

Areas of shear diagram.

A to B. $\int V dx = \frac{1}{2}(1)(9 + 4.5) = 6.75 \text{ kN}\cdot\text{m}$

B to C. $\int V dx = \frac{1}{2}(1)(-9 - 4.5) = -6.75 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$M_B = 0 + 6.75 = 6.75 \text{ kN}\cdot\text{m}$

$M_C = 6.75 - 6.75 = 0$

Maximum $|M| = 6.75 \text{ kN}\cdot\text{m} = 6.75 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 140 \text{ MPa} = 140 \times 10^6 \text{ Pa}$

For the section of two angles, $S_{min} = \frac{|M|}{\sigma_{all}} = \frac{6.75 \times 10^3}{140 \times 10^6} = 48.21 \times 10^{-6} \text{ m}^3$
 $= 48.21 \times 10^3 \text{ mm}^3$

For each angle, $S_{min} = \frac{1}{2}(48.21) = 24.105 \times 10^3 \text{ mm}^3$

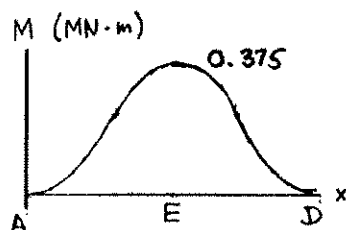
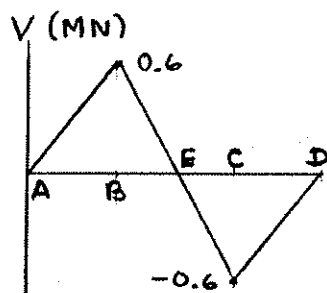
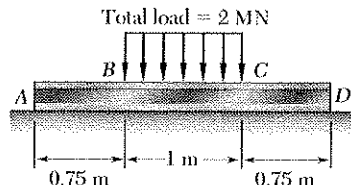
Shape	$S (10^3 \text{ mm}^3)$
L 102 × 76 × 12.7	31.1 ←
L 102 × 76 × 9.5	24.0
L 102 × 76 × 6.4	16.6

Lightest angle is

L 102 × 76 × 12.7

$t_{min} = 12.7 \text{ mm}$

Problem 5.83



5.83 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical wide-flange beam to support the loading shown.

Downward distributed load $w = \frac{2}{1.0} = 2 \text{ MN/m}$

Upward distributed reaction $q = \frac{2}{2.5} = 0.8 \text{ MN/m}$

Net distributed load over BC 1.2 MN/m

Shear: $V_A = 0$

$V_B = 0 + (0.75)(0.8) = 0.6 \text{ MN}$

$V_C = 0.6 - (1.0)(1.2) = -0.6 \text{ MN}$

$V_D = -0.6 + (0.75)(0.8) = 0$

Areas: A to B. $(\frac{1}{2})(0.75)(0.6) = 0.225 \text{ MN}\cdot\text{m}$

B to E. $(\frac{1}{2})(0.5)(0.6) = 0.150 \text{ MN}\cdot\text{m}$

E to C. $(\frac{1}{2})(0.5)(-0.6) = -0.150 \text{ MN}\cdot\text{m}$

C to D. $(\frac{1}{2})(0.75)(-0.6) = -0.225 \text{ MN}\cdot\text{m}$

Bending moments: $M_A = 0$

$M_B = 0 + 0.225 = 0.225 \text{ MN}\cdot\text{m}$

$M_E = 0.225 + 0.150 = 0.375 \text{ MN}\cdot\text{m}$

$M_C = 0.375 - 0.150 = 0.225 \text{ MN}\cdot\text{m}$

$M_D = 0.225 - 0.225 = 0$

Maximum $|M| = 0.375 \text{ MN}\cdot\text{m} = 375 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 170 \text{ MPa} = 170 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3} \text{ m}^3 = 2206 \times 10^3 \text{ mm}^3$

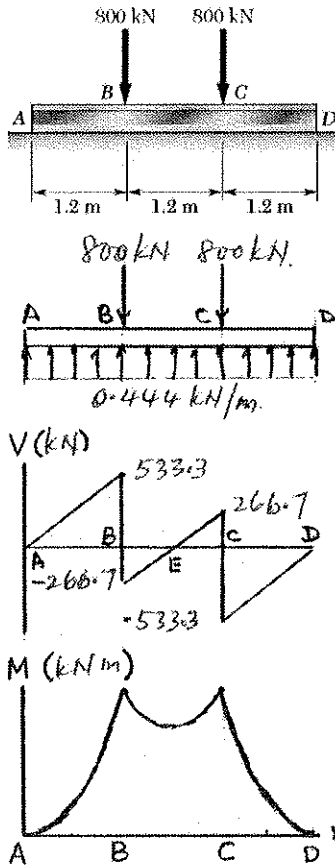
Shape	$S(10^3 \text{ mm}^3)$
W 690 × 125	3510
W 610 × 101	2530 ←
W 530 × 150	3720
W 460 × 113	2400

Lightest wide flange section

W 610 × 101

Problem 5.84

5.84 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 165 MPa, select the most economical wide-flange beam to support the loading shown.



$$\text{Distributed reaction } q = \frac{1600}{3.6} = 444.4 \text{ kN/m}$$

$$\text{Shear: } V_A = 0$$

$$V_B^- = 0 + (1.2)(444.4) = 533.3 \text{ kN}$$

$$V_B^+ = 533.3 - 0.8 = -266.7 \text{ kN}$$

$$V_C^- = -0.267 + 1.2(444.4) = 266.7 \text{ kN}$$

$$V_C^+ = 266.7 - 0.8 = -533.3 \text{ kN}$$

$$V_D = -533.3 + (1.2)(444.4) = 0 \text{ kN}$$

$$\text{Areas: } A \text{ to } B \left(\frac{1}{2}\right)(1.2)(533.3) = 320 \text{ kNm}$$

$$B \text{ to } E \left(\frac{1}{2}\right)(0.6)(-266.7) = -80 \text{ kNm}$$

$$E \text{ to } C \left(\frac{1}{2}\right)(0.6)(266.7) = 80 \text{ kNm}$$

$$C \text{ to } D \left(\frac{1}{2}\right)(1.2)(-533.3) = -320 \text{ kNm}$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 + 320 = 320 \text{ kNm}$$

$$M_E = 320 - 80 = 240 \text{ kNm}$$

$$M_C = 240 + 80 = 320 \text{ kNm}$$

$$M_D = 320 - 320 = 0$$

$$\text{Maximum } |M| = 320 \text{ kNm}$$

$$\sigma_{all} = 165 \text{ MPa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{320000}{165 \times 10^6} = 1.939 \times 10^{-3} \text{ m}^3 = 1.939 \times 10^3 \text{ mm}^3$$

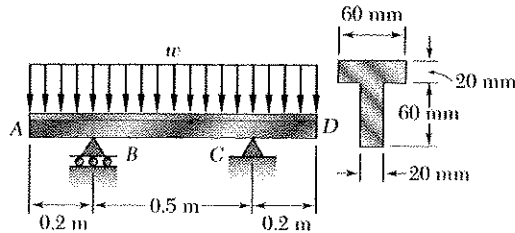
Shape	$S (\times 10^3 \text{ mm}^3)$
W530 x 92	2070
W460 x 113	2400
W410 x 114	2200
W360 x 122	2010
W310 x 143	2150
W250 x 167	2080

Lightest W-shaped section

W530 x 92

Problem 5.85

5.85 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is $+80$ MPa in tension and -130 MPa in compression.



Reactions. By symmetry, $B = C$

$$+\uparrow \Sigma F_y = 0: B + C - 0.9w = 0$$

$$B = C = 0.45w \uparrow$$

Shear: $V_A = 0$

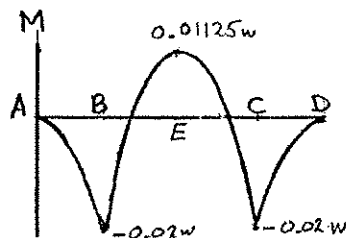
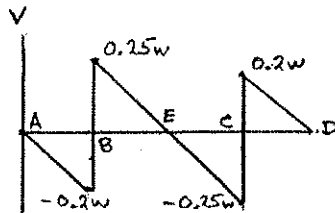
$$V_B^- = 0 - 0.2w = -0.2w$$

$$V_B^+ = -0.2w + 0.45w = 0.25w$$

$$V_C^- = 0.25w - 0.5w = -0.25w$$

$$V_C^+ = -0.25w + 0.45w = 0.2w$$

$$V_D = 0.2w - 0.2w = 0$$



$$\text{Areas: } A \text{ to } B, \frac{1}{2}(0.2)(-0.2w) = -0.02w$$

$$B \text{ to } E, \frac{1}{2}(0.25)(0.25w) = 0.03125w$$

Bending moments: $M_A = 0$

$$M_B = 0 - 0.02w = -0.02w$$

$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia.

Part	A, mm^2	$\bar{y}, \text{mm}$	$A\bar{y} (10^3 \text{mm}^3)$	d, mm	$Ad^2 (10^3 \text{mm}^4)$	$\bar{I} (10^3 \text{mm}^4)$
①	1200	70	84	20	480	40
②	1200	30	36	20	480	360
Σ	2400		120		960	400

$$\bar{y} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 1360 \times 10^3 \text{ mm}^4$$

$$\text{Top: } I/y = (1360 \times 10^3) / 30 = 45.333 \times 10^3 \text{ mm}^3 = 45.333 \times 10^{-6} \text{ m}^3$$

$$\text{Bottom: } I/y = (1360 \times 10^3) / (-50) = -27.2 \times 10^3 \text{ mm}^3 = -27.2 \times 10^{-6} \text{ m}^3$$

Bending moment limits ($M = -\sigma I/y$) and load limits w .

$$\text{Tension at B and C: } -0.02w = -(80 \times 10^6)(45.333 \times 10^{-6}) \quad w = 181.3 \times 10^3 \text{ N/m}$$

$$\text{Compression at B and C: } -0.02w = -(-130 \times 10^6)(27.2 \times 10^{-6}) \quad w = 176.8 \times 10^3 \text{ N/m}$$

$$\text{Tension at E: } 0.01125w = -(80 \times 10^6)(27.2 \times 10^{-6}) \quad w = 193.4 \times 10^3 \text{ N/m}$$

$$\text{Compression at E: } 0.01125w = -(-130 \times 10^6)(45.333 \times 10^{-6}) \quad w = 523.8 \times 10^3 \text{ N/m}$$

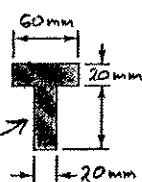
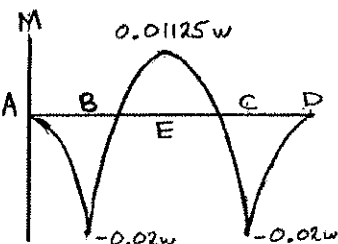
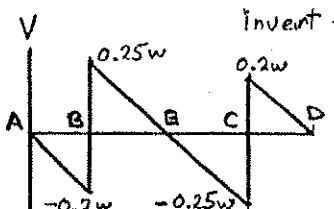
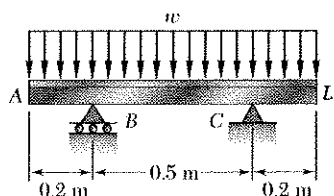
The smallest allowable load controls. $w = 176.8 \times 10^3 \text{ N/m}$

$$w = 176.8 \text{ kN/m}$$

Problem 5.86

5.86 Solve Prob. 5.85, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at B and C.

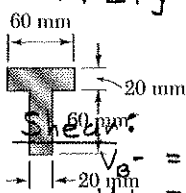
5.85 Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is $+80$ MPa in tension and -130 MPa in compression.



Reactions. By symmetry, $B = C$

$$+\uparrow \sum F_y = 0: B + C - 0.9w = 0$$

$$B = C = 0.45w \uparrow$$



$$V_A = 0$$

$$V_B^- = 0 - 0.2w = -0.2w$$

$$V_B^+ = -0.2w + 0.45w = 0.25w$$

$$V_C^- = 0.25w - 0.5w = -0.25w$$

$$V_C^+ = -0.25w + 0.45w = 0.2w$$

$$V_D = 0.2w - 0.2w = 0$$

$$\text{Areas: A to B. } \frac{1}{2}(0.2)(-0.2w) = -0.02w$$

$$\text{B to E. } \frac{1}{2}(0.25)(0.25w) = 0.03125w$$

$$\text{Bending moments: } M_A = 0$$

$$M_B = 0 - 0.02w = -0.02w$$

$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia.

Part	A, mm^2	$\bar{y}, \text{mm}$	$A\bar{y}, (10^3 \text{mm}^3)$	d, mm	$A d^2 (10^3 \text{mm}^4)$	$\bar{I}, (10^3 \text{mm}^4)$
①	1200	50	60	20	480	360
②	1200	10	12	20	480	40
Σ	2400		72		960	400

$$\bar{Y} = \frac{72 \times 10^3}{2400} = 30 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 1360 \times 10^3 \text{ mm}^4$$

$$\text{Top: } I/y = (1360 \times 10^3) / (50) = 27.2 \times 10^3 \text{ mm}^3 = 27.2 \times 10^{-6} \text{ m}^3$$

$$\text{Bottom: } I/y = (1360 \times 10^3) / (-30) = -45.333 \times 10^3 \text{ mm}^3 = -45.333 \times 10^{-6} \text{ m}^3$$

Bending moment limits ($M = -\sigma I/y$) and load limits w .

$$\text{Tension at B and C: } -0.02w = -(80 \times 10^6)(27.2 \times 10^{-6}) \quad w = 108.8 \times 10^3 \text{ N/m}$$

$$\text{Compression at B and C: } -0.02w = -(-130 \times 10^6)(45.333 \times 10^{-6}) \quad w = 294.7 \times 10^3 \text{ N/m}$$

$$\text{Tension at E: } 0.01125w = -(80 \times 10^6)(-45.333 \times 10^{-6}) \quad w = 322.4 \times 10^3 \text{ N/m}$$

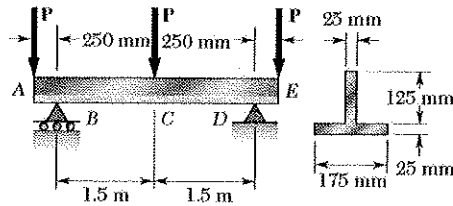
$$\text{Compression at E: } 0.01125w = -(-130 \times 10^6)(27.2 \times 10^{-6}) \quad w = 314.3 \times 10^3 \text{ N/m}$$

The smallest allowable load controls. $w = 108.8 \times 10^3 \text{ N/m}$

$$w = 108.8 \text{ kN/m}$$

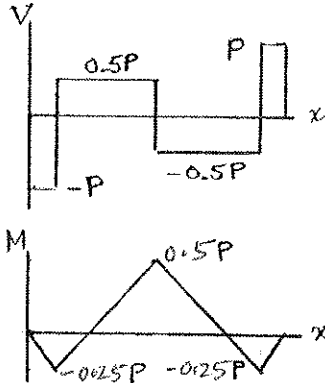
Problem 5.87

5.87 Determine the allowable value of P for the loading shown, knowing that the allowable normal stress is $+55 \text{ MPa}$ in tension and -125 MPa in compression.



Reactions. $B = D = 1.5P \uparrow$

Shear diagram. $A \text{ to } B^- \quad V = -P$
 $B^+ \text{ to } C^- \quad V = -P + 1.5P = 0.5P$
 $C^+ \text{ to } D^- \quad V = 0.5P - P = -0.5P$
 $D^+ \text{ to } E \quad V = -0.5P + 1.5P = P$

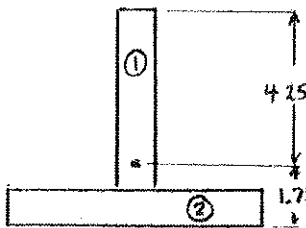


Areas. $A \text{ to } B \quad (0.25)(-P) = -0.25P$
 $B \text{ to } C \quad (1.5)(0.5P) = 0.75P$
 $C \text{ to } D \quad (1.5)(-0.5P) = -0.75P$
 $D \text{ to } E \quad (0.25)(P) = 0.25P$

Bending moments. $M_A = 0$
 $M_B = 0 - 0.25P = -0.25P$
 $M_C = -0.25P + 0.75P = 0.5P$
 $M_D = 0.5P - 0.75P = -0.25P$
 $M_E = -0.25P + 0.25P = 0$

Largest positive bending moment = $0.5P$
Largest negative bending moment = $-0.25P$

Centroid and moment of inertia.



Part	A, mm^2	$\bar{y}_0, \text{mm}$	$A\bar{y}_0, \text{mm}^3$	d, mm	Ad^2, mm^4	$\bar{I}_x, \text{mm}^4$
①	3125	87.5	273437	43.75	5.981×10^6	4.069×10^6
②	4375	12.5	54687	31.25	4.273×10^6	0.228×10^6
Σ	7500		328124		10.254×10^6	4.297×10^6

Top: $y = 106.25 \text{ mm}$
Bottom: $y = -43.75 \text{ mm}$

$$\bar{Y} = \frac{328124}{7500} = 43.75 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 14.554 \times 10^6 \text{ mm}^4$$

$$\sigma = -\frac{My}{I}$$

$$\text{Top, Tension } 55 \times 10^6 = -\frac{(-0.25P)(0.10625)}{14.544 \times 10^{-6}}$$

$$P = 30.1 \text{ kN}$$

$$\text{Top, Comp. } -125 \times 10^6 = -\frac{(0.5P)(0.10625)}{14.544 \times 10^{-6}}$$

$$P = 34.2 \text{ kN}$$

$$\text{Bot. Tension } 55 \times 10^6 = -\frac{(0.5P)(-0.04375)}{14.544 \times 10^{-6}}$$

$$P = 36.6 \text{ kN}$$

$$\text{Bot. Comp. } -125 \times 10^6 = -\frac{(-0.25P)(-0.04375)}{14.544 \times 10^{-6}}$$

$$P = 166.2 \text{ kN}$$

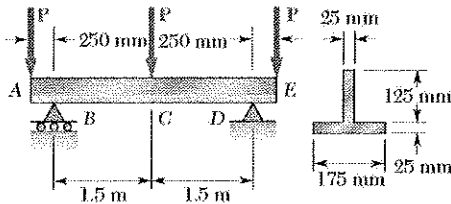
Smallest value of P is the allowable value

$$P = 30.1 \text{ kN}$$

Problem 5.88

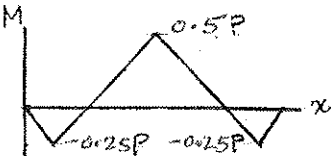
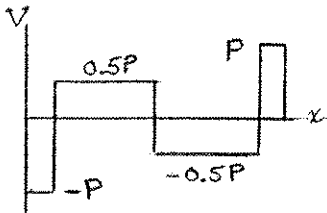
5.88 Solve Prob. 5.87, assuming that the T-shaped beam is inverted.

5.87 Determine the allowable value of P for the loading shown, knowing that the allowable normal stress is $+55 \text{ MPa}$ in tension and -125 MPa in compression.



Reactions. $B = D = 1.5 P \uparrow$

Shear diagram. $A \text{ to } B^- \quad V = -P$
 $B^+ \text{ to } C^- \quad V = -P + 1.5P = 0.5P$
 $C^+ \text{ to } D^- \quad V = 0.5P - P = -0.5P$
 $D^+ \text{ to } E \quad V = -0.5P + 1.5P = P$

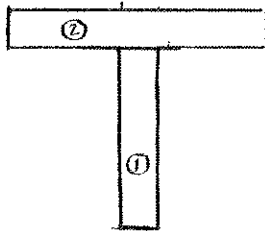


Areas. $A \text{ to } B \quad (0.25)(-P) = -0.25P$
 $B \text{ to } C \quad (1.5)(0.5P) = 0.75P$
 $C \text{ to } D \quad (1.5)(-0.5P) = -0.75P$
 $D \text{ to } E \quad (0.25)(P) = 0.25P$

Bending moments. $M_A = 0$
 $M_B = 0 - 0.25P = -0.25P$
 $M_C = -0.25P + 0.75P = 0.5P$
 $M_D = 0.5P - 0.75P = -0.25P$
 $M_E = -0.25P + 0.25P = 0$

Largest positive bending moment = $0.5P$
Largest negative bending moment = $-0.25P$

Centroid and moment of inertia.



Part	A, mm^2	$\bar{y}_0, \text{mm}$	$A\bar{y}_0, \text{mm}^3$	d, mm	Ad^2, mm^4	$\bar{I}, \text{mm}^4$
①	3125	62.5	195312	43.75	5.981×10^6	4.069×10^6
②	4375	137.5	601562	31.25	4.273×10^6	0.228×10^6
Σ	7500		796874		10.254×10^6	4.3×10^6

Top: $y = 43.75 \text{ mm}$
Bottom: $y = -106.25 \text{ mm}$

$$\bar{Y} = \frac{796874}{7500} = 106.25$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 14.544 \times 10^6 \text{ mm}^4$$

$$\sigma = -\frac{My}{I}$$

Top, Tension $55 \times 10^6 = -\frac{(-0.25P)(0.04375)}{4.544 \times 10^{-6}} \quad P = 73.1 \text{ kN}$

Top, Comp. $-125 \times 10^6 = -\frac{(0.5P)(0.04375)}{4.544 \times 10^{-6}} \quad P = 83.1 \text{ kN}$

Bot. Tension $55 \times 10^6 = -\frac{(0.5P)(-0.10625)}{4.544 \times 10^{-6}} \quad P = 15.1 \text{ kN}$

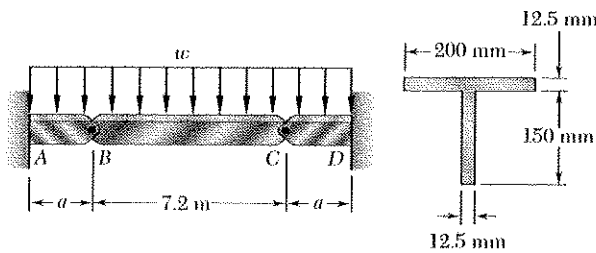
Bot. Comp. $-125 \times 10^6 = -\frac{(-0.25P)(-0.10625)}{4.544 \times 10^{-6}} \quad P = 68.4 \text{ kN}$

Smallest value of P is the allowable value

$$P = 15.1 \text{ kN}$$

Problem 5.89

5.89 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (a) the largest permissible value of w if beam BC is not to be overstressed, (b) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.



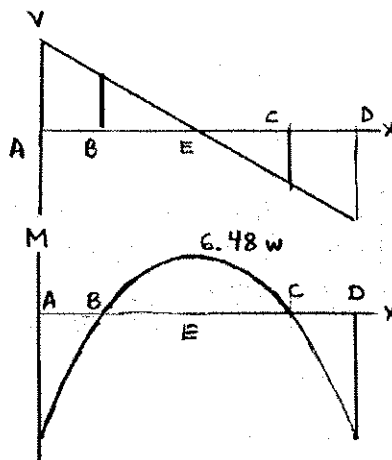
$$(a) \quad M_B = M_C = 0$$

$$V_B = -V_C = \left(\frac{1}{2}\right)(7.2)w = 3.6w$$

Area B to E of shear diagram,

$$\left(\frac{1}{2}\right)(3.6)(3.6w) = 6.48w$$

$$M_E = 0 + 6.48w = 6.48w$$

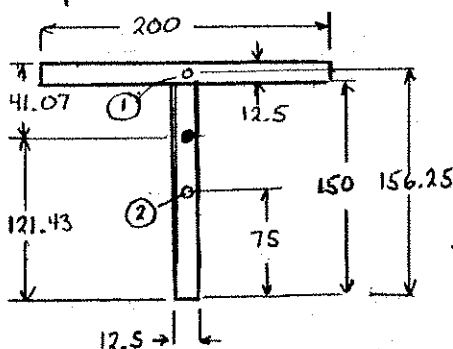


Centroid and moment of inertia

Part	A (mm ²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm ³)	d (mm)	Ad^2 (mm ⁴)	$\bar{I}$ (mm ⁴)
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{y} = \frac{531250}{4375} = 121.43 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 10.621 \times 10^6 \text{ mm}^4$$



Location	y (mm)	I/y (10 ³ mm ³)	also (10 ⁻⁶ m ³)
top	41.07	258.6	
bottom	-121.43	-87.47	

Bending moment limits: $M = -\sigma I/y$

$$\text{Tension at E: } -(110 \times 10^6)(-87.47 \times 10^{-6}) = 9.622 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Comp. at E: } -(-150 \times 10^6)(258.6 \times 10^{-6}) = 38.8 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Tension at A \& D: } -(110 \times 10^6)(258.6 \times 10^{-6}) = -28.45 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Comp. at A \& D: } -(-150 \times 10^6)(-87.47 \times 10^{-6}) = -13.121 \times 10^3 \text{ N}\cdot\text{m}$$

(a) Allowable load w .

$$6.48w = 9.622 \times 10^3$$

$$w = 1.485 \times 10^3 \text{ N/m}$$

$$w = 1.485 \text{ kN/m}$$

Shear at A. $V_A = (a + 3.6)w$

Area A to B of shear diagram: $\frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a + 7.2)w$

Bending moment at A (also D): $M_A = -\frac{1}{2}a(a + 7.2)w$

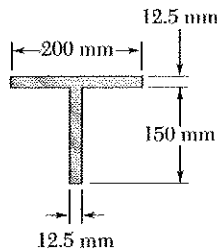
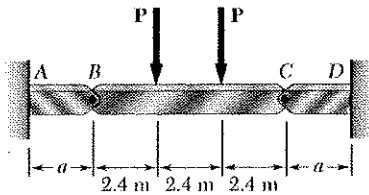
$$-\frac{1}{2}a(a + 7.2)(1.485 \times 10^3) = -13.121 \times 10^3$$

(b) Distance a . $\frac{1}{2}a^2 + 3.6a - 8.837 = 0$

$$a = 1.935 \text{ m}$$

Problem 5.90

5.90 Beams AB , BC , and CD have the cross section shown and are pin-connected at B and C . Knowing that the allowable normal stress is $+110$ MPa in tension and -150 MPa in compression, determine (a) the largest permissible value of P if beam BC is not to be overstressed, (b) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.



$$M_B = M_C = 0$$

$$V_B = -V_C = P$$

Area B to E of shear diagram.
 $2.4 P$

$$M_E = 0 + 2.4 P = 2.4 P = M_F$$

Centroid and moment of inertia.

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531250}{4375} = 121.43 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 10.621 \times 10^6 \text{ mm}^4$$

Location	$y(\text{mm})$	I/y (10^3 mm^3)	also (10^{-6} m^3)
top	41.07	258.6	
bottom	-121.43	-87.47	

Bending moment limits: $M = -\sigma I/y$

$$\text{Tension at E \& F: } -(110 \times 10^6)(-87.47 \times 10^{-6}) = 9.622 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Comp. at E \& F: } -(-150 \times 10^6)(258.6 \times 10^{-6}) = 38.8 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Tension at A \& D: } -(110 \times 10^6)(258.6 \times 10^{-6}) = -28.45 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Comp. at A \& D: } -(-150 \times 10^6)(-87.47 \times 10^{-6}) = -13.12 \times 10^3 \text{ N}\cdot\text{m}$$

(a) Allowable load P .

$$2.4 P = 9.622 \times 10^3$$

$$P = 4.01 \times 10^3 \text{ N}$$

$$P = 4.01 \text{ kN} \quad \blacktriangleleft$$

Shear at A $V_A = P$

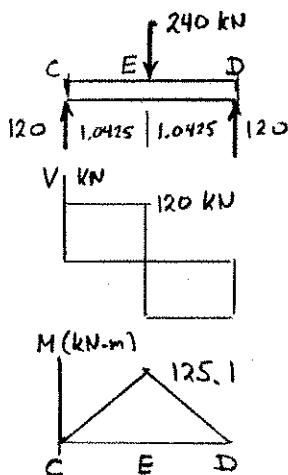
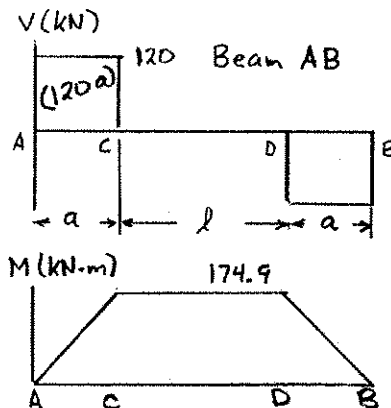
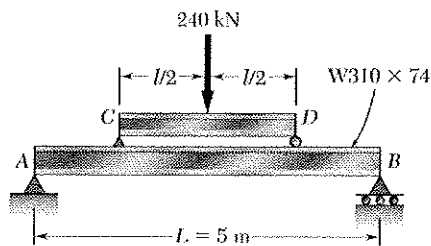
Area A to B of shear diagram: $aV_A = aP$

Bending moment at A. $M_A = -aP = -4.01 \times 10^3 a$

(b) Distance a . $-4.01 \times 10^3 a = -13.12 \times 10^3$

$$a = 3.27 \text{ m} \quad \blacktriangleleft$$

Problem 5.91



5.91 A 240-kN load is to be supported at the center of the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (a) the smallest allowable length l of beam CD if the $W310 \times 74$ beam AB is not to be overstressed, (b) the most economical W shape that can be used for beam CD . Neglect the weight of both beams.

For rolled steel section $W 310 \times 74$ of beam AB

$$S = 1060 \times 10^3 \text{ mm}^3 = 1060 \times 10^{-6} \text{ m}^3$$

$$\sigma_{all} = 165 \text{ MPa} = 165 \times 10^6 \text{ Pa}$$

Allowable bending moment.

$$M_{all} = S \sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3 \text{ N}\cdot\text{m} = 174.9 \text{ kN}\cdot\text{m}$$

(a) Beam AB .

Area A to C of shear diagram = $120 a$

Bending moment at C = $120 a$

$$120 a = 174.9 = 1.4575 \text{ m}$$

$$\text{Geometry: } 2a + l = 5 \quad l = 5 - 2a = 2.085 \text{ m}$$

(b) Beam CD (midpoint E)

Area C to E of shear diagram = $(1.0425)(120) = 125.1 \text{ kN}\cdot\text{m}$

Bending moment at E .

$$M = 125.1 \text{ kN}\cdot\text{m} = 125.1 \times 10^3 \text{ N}\cdot\text{m}$$

$$S_{min} = \frac{M}{\sigma_{all}} = \frac{125.1 \times 10^3}{165 \times 10^6} = 758.2 \times 10^{-6} \text{ m}^3 = 758.2 \times 10^3 \text{ mm}^3$$

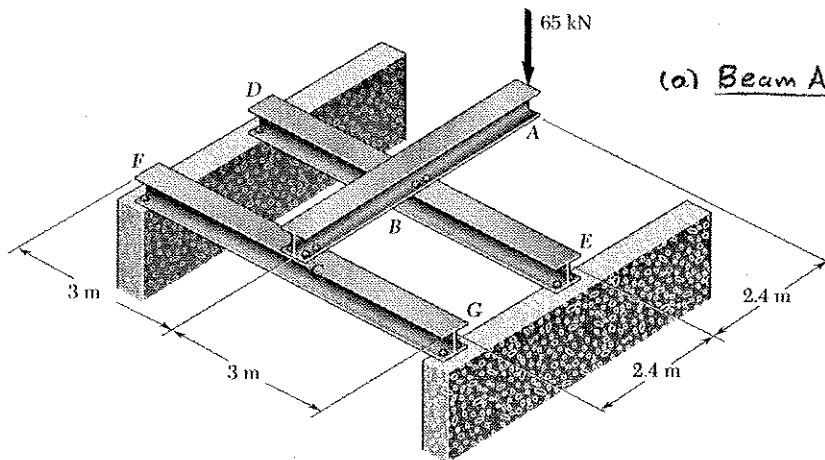
Shape	S (10^3 mm^3)
$W 460 \times 52$	942
$W 410 \times 46.1$	774 ←
$W 360 \times 52.8$	899
$W 310 \times 60$	851
$W 250 \times 67$	809
$W 200 \times 86$	853

Answer:

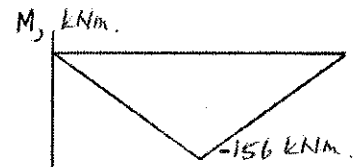
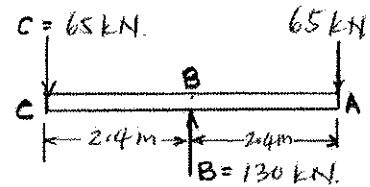
$W 410 \times 46.1$ ←

Problem 5.92

5.92 Beam ABC is bolted to beams DBE and FCG. Knowing that the allowable normal stress is 165 MPa, select the most economical wide-flange shape that can be used (a) for beam ABC, (b) for beam DBE, (c) for beam FCG.

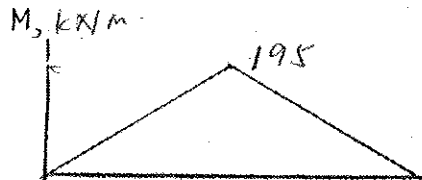
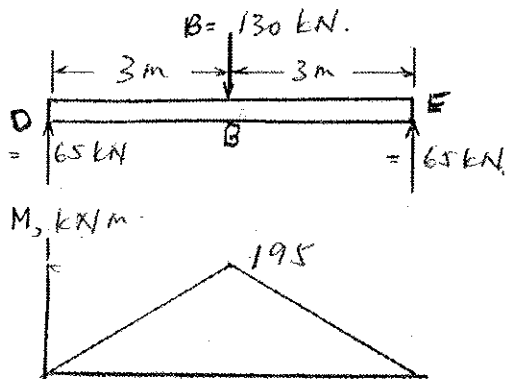


(a) Beam ABC



$$M_B = -(65)(2.4) = -156 \text{ kNm}$$

(b) Beam DBE



$$|M|_{\max} = 195 \text{ kNm}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{195 \times 10^3}{165 \times 10^6} = 1181 \times 10^3 \text{ mm}^3$$

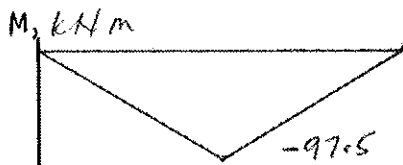
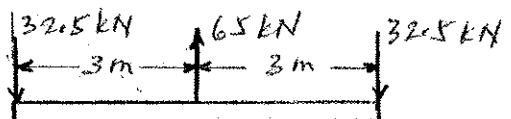
$$|M|_{\max} = 156 \text{ kNm}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{156 \times 10^3}{165 \times 10^6} = 945.5 \times 10^3 \text{ mm}^3$$

Shape	$S_x (\times 10^3 \text{ mm}^3)$
W530x66	1340
W460x74	1460
W410x60	1060 ← (a) W410x60
W360x64	1030
W310x74	1060
W250x101	1240

Shape	$S_x (\times 10^3 \text{ mm}^3)$
W530x66	1340 ← (b) W530x66
W460x74	1460
W410x85	1510
W360x74	1280
W310x107	1570
W250x101	1240

(c) Beam FCG

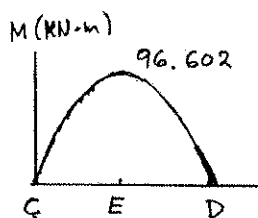
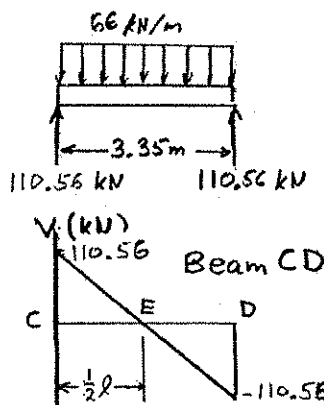
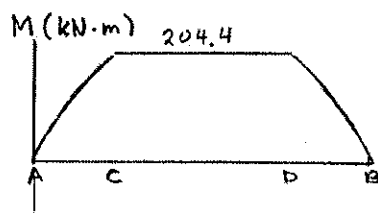
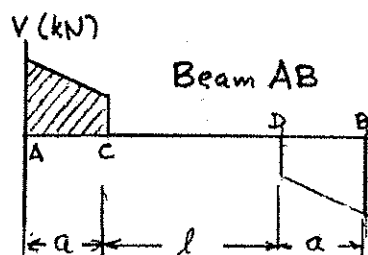
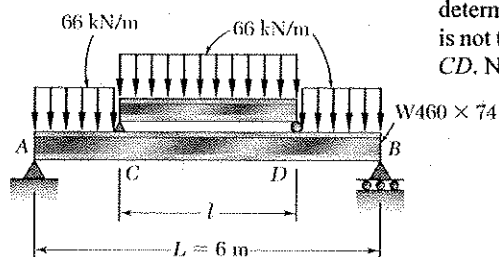


$$|M|_{\max} = 97.5 \text{ kNm}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{97500}{165 \times 10^6} = 590.1 \times 10^3 \text{ mm}^3$$

Shape	$S_x (\times 10^3 \text{ mm}^3)$
W460x52	942
W410x38.8	637 ← (c) W410x38.8
W360x44	693
W310x44.5	634
W250x58	693
W200x71	709

Problem 5.93



Shape	$S (10^3 \text{ mm}^3)$
W410 x 46.1	774
W360 x 44	693 ←
W310 x 52	748
W250 x 58	693
W200 x 71	709

5.93 A uniformly distributed load of 66 kN/m is to be supported over the 6-m span shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine (a) the smallest allowable length l of beam CD if the $W460 \times 74$ beam AB is not to be overstressed, (b) the most economical W shape that can be used for beam CD . Neglect the weight of both beams.

For $W460 \times 74$,

$$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$$

$$\sigma_{\text{all}} = 140 \text{ MPa} = 140 \times 10^6 \text{ Pa}$$

$$M_{\text{all}} = S \sigma_{\text{all}} = (1460 \times 10^{-6})(140 \times 10^6) = 204.4 \times 10^3 \text{ N}\cdot\text{m} = 204.4 \text{ kN}\cdot\text{m}$$

Reactions: By symmetry, $A = B$, $C = D$

$$+\uparrow \Sigma F_y = 0: A + B - (66)(6) = 0$$

$$A = B = 198 \text{ kN} = 198 \times 10^3 \text{ N}$$

$$+\Sigma F_y = 0: C + D - 66l = 0$$

$$C = D = (33l) \text{ kN} \quad (1)$$

Shear and bending moment in beam AB .

$$0 < x < a, \quad V = 198 - 66x \text{ kN}$$

$$M = 198x - 33x^2 \text{ kN}\cdot\text{m}$$

$$\text{At } C, \quad x = a \quad M = M_{\text{max}}$$

$$M = 198a - 33a^2 \text{ kN}\cdot\text{m}$$

$$\text{Set } M = M_{\text{all}} \quad 198a - 33a^2 = 204.4$$

$$33a^2 - 198a + 204.4 = 0$$

$$a = 4.6751 \text{ m}, \quad 1.32487 \text{ m}$$

$$\text{By geometry, } l = 6 - 2a = 3.35 \text{ m}$$

$$\text{From (1)} \quad C = D = 110.56 \text{ kN}$$

Draw shear and bending moment diagrams for beam CD . $V = 0$ at point E , the midpoint of CD .

$$\text{Area from A to E. } \int V dx = \frac{1}{2}(110.560)\left(\frac{1}{2}l\right) = 92.602 \text{ kN}\cdot\text{m}$$

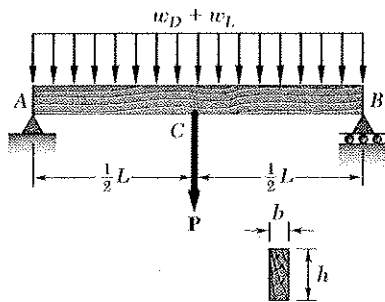
$$M_E = 92.602 \text{ kN}\cdot\text{m} = 92.602 \times 10^3 \text{ N}\cdot\text{m}$$

$$S_{\text{min}} = \frac{M_E}{\sigma_{\text{all}}} = \frac{92.602 \times 10^3}{140 \times 10^6} = 661.44 \times 10^{-6} \text{ m}^3$$

$$= 661.44 \times 10^3 \text{ mm}^3$$

Use $W360 \times 44$

Problem 5.94



*5.94 A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load P applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

$$L = 16 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m}$$

$$w_L = 600 \text{ N/m} = 0.6 \text{ kN/m}, \quad P = 6 \text{ kN}$$

Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8 \text{ kN}$

Area A to C of shear diagram.

$$\left(\frac{1}{2}\right)(8)(2.8) = 11.2 \text{ kN}\cdot\text{m}$$

Bending moment at C: $11.2 \text{ kN}\cdot\text{m} = 11.2 \times 10^3 \text{ N}\cdot\text{m}$

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8 \text{ kN}$

Shear at C⁻ $V = 7.8 - (8)(0.6) = 3 \text{ kV}$

Area A to C of shear diagram.

$$\left(\frac{1}{2}\right)(8)(7.8 + 3) = 43.2 \text{ kN}\cdot\text{m}$$

Bending moment at C: $43.2 \text{ kN}\cdot\text{m} = 43.2 \times 10^3 \text{ N}\cdot\text{m}$

Design: $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi S_{xx}$

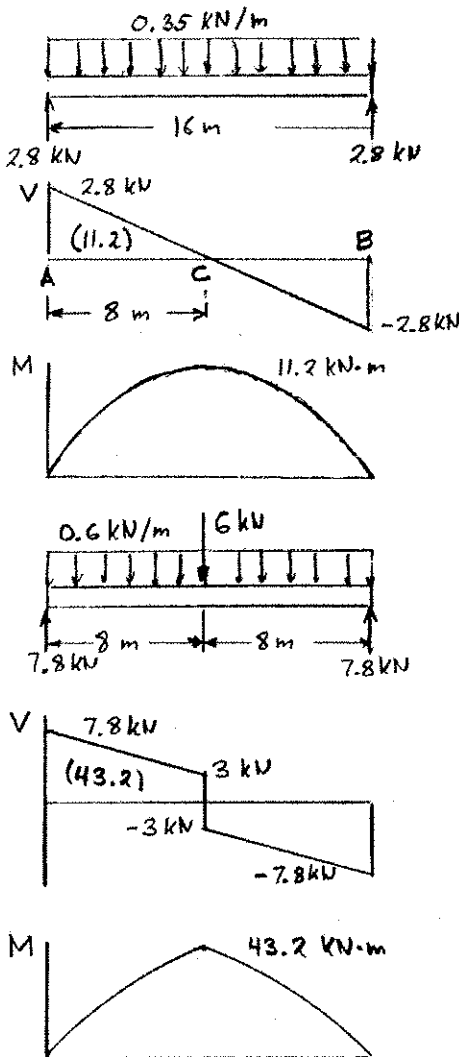
$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi S_{xx}} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

$$= 1.8347 \times 10^{-3} \text{ m}^3 = 1.8347 \times 10^6 \text{ mm}^3$$

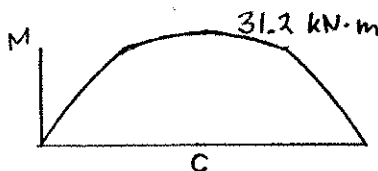
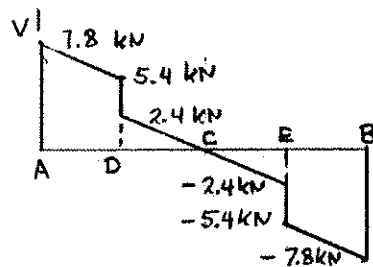
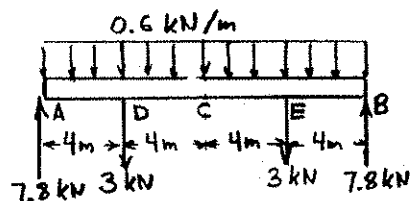
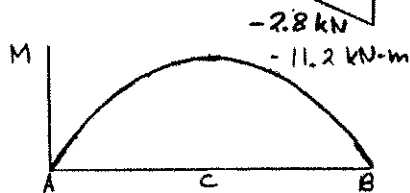
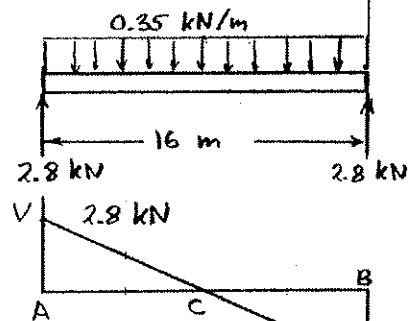
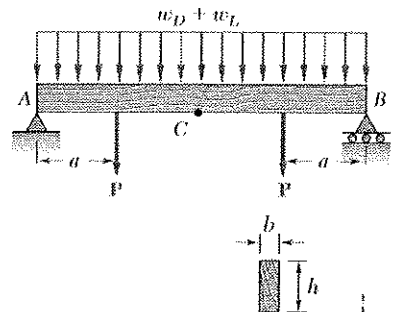
For a rectangular section, $S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}}$$

$$h = 383 \text{ mm} \blacktriangleleft$$



Problem 5.95



*5.95 Solve Prob. 5.94, assuming that the 6-kN concentrated load P applied to each beam is replaced by 3-kN concentrated loads P_1 and P_2 applied at a distance of 4 m from each end of the beams.

*5.94 A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load P applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

$$L = 16 \text{ m}, \quad a = 4 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m}$$

$$w_L = 600 \text{ N/m} = 0.6 \text{ kN/m} \quad P = 3 \text{ kN}$$

Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8 \text{ kN}$

Area A to C of shear diagram

$$\left(\frac{1}{2}\right)(8)(2.8) = 11.2 \text{ kN}\cdot\text{m}$$

Bending moment at C: $11.2 \text{ kN}\cdot\text{m} = 11.2 \times 10^3 \text{ N}\cdot\text{m}$

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8 \text{ kN}$

Shear at D^- $7.8 - (4)(0.6) = 5.4 \text{ kN}$

Shear at D^+ $5.4 - 3 = 2.4 \text{ kN}$

Area A to D $\left(\frac{1}{2}\right)(4)(7.8 + 5.4) = 26.4 \text{ kN}\cdot\text{m}$

Area D to C $\left(\frac{1}{2}\right)(4)(2.4) = 4.8 \text{ kN}\cdot\text{m}$

Bending moment at C $= 26.4 + 4.8 = 31.2 \text{ kN}\cdot\text{m}$
 $= 31.2 \times 10^3 \text{ N}\cdot\text{m}$

Design: $\gamma_D M_D + \gamma_L M_L = \phi M_u = \phi S_{xx} \sigma_U$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(31.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

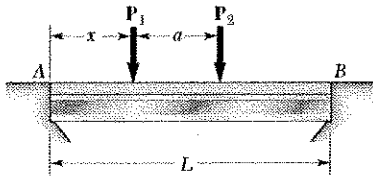
$$= 1.408 \times 10^{-3} \text{ m}^3 = 1.408 \times 10^6 \text{ mm}^3$$

For a rectangular section $S = \frac{1}{6} b h^2$

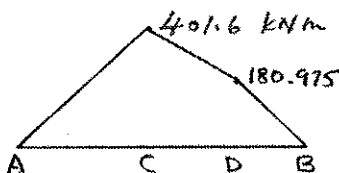
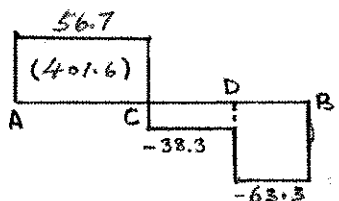
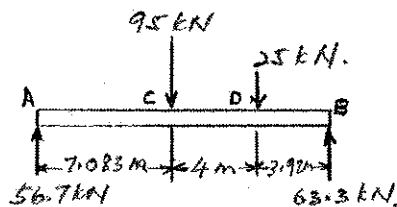
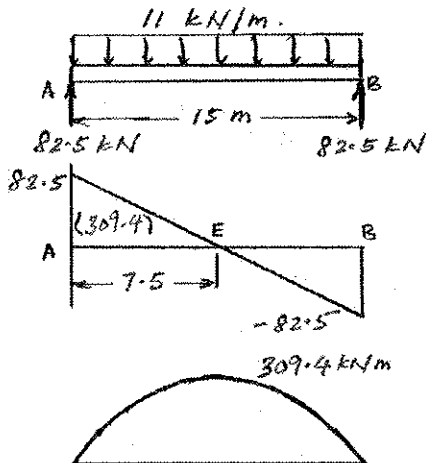
$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408 \times 10^6)}{75}}$$

$$h = 336 \text{ mm} \quad \blacktriangleleft$$

Problem 5.96



$$\begin{aligned} L &= 15 \text{ m} \\ a &= 4 \text{ m} \\ P_1 &= 95 \text{ kN} \\ P_2 &= 25 \text{ kN} \\ W &= 11 \text{ kN/m} \end{aligned}$$



*5.96 A bridge of length $L = 15 \text{ m}$ is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_u = 420 \text{ MPa}$. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 11 \text{ kN/m}$ on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 4 \text{ m}$ from each other will be driven across the bridge and that the resulting concentrated loads P_1 and P_2 exerted on each beam could be as large as 95 kN and 25 kN , respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [Hint: It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/(P_1 + P_2)$.]

Dead load: $R_A = R_B = \left(\frac{1}{2}\right)(15)(11) = 82.5 \text{ kN}$.

Area A to E of shear diagram $\left(\frac{1}{2}\right)(7.5)(82.5) = 309.4$

$M_{max} = 309.4 \text{ kNm}$ at point E

Live load: $U = \frac{aP_2}{2(P_1 + P_2)} = \frac{(4)(25)}{(2)(120)} = 0.417 \text{ m}$

$x = \frac{L}{2} - U = 7.5 - 0.417 = 7.083 \text{ m}$

$x + a = 7.083 + 4 = 11.083 \text{ m}$

$L - x - a = 15 - 11.083 = 3.92$

$\Sigma M_B = 0 \quad -15R_A + (7.92)(95) + (3.92)(25) = 0$
 $R_A = 56.7 \text{ kN}$.

Shear: A to C $V = 56.7 \text{ kN}$
 C to D $V = 56.7 - 95 = -38.3$
 D to B $V = -63.3$

Area A to C $(7.083)(56.7) = 401.6 \text{ kNm}$

Bending moment: $M_C = 401.6 \text{ kNm}$.

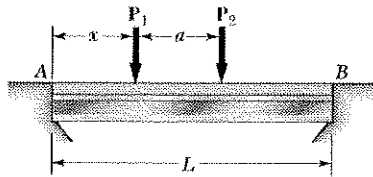
Design: $\gamma_D M_D + \gamma_L M_L = \phi M_u = \phi S_{all} S_{min}$

$S_{min} = \frac{\gamma_D M_D + \gamma_L M_L}{\phi S_{all}} = \frac{(1.25)(309.4 \times 10^3) + (1.75)(401.6 \times 10^3)}{(0.9)(420 \times 10^6)}$
 $= 2882.4 \times 10^{-6} \text{ m}^3 = 2882.4 \times 10^3 \text{ mm}^3$

Shape	$S (\times 10^3 \text{ mm}^3)$
W 690 X 125	3510
W 610 X 155	4220
W 530 X 150	3720
W 460 X 158	3340
W 360 X 216	3800

W 690 X 125

Problem 5.97



$$\begin{aligned} L &= 15 \text{ m} \\ a &= 4 \text{ m} \\ P_1 &= 95 \text{ kN} \\ P_2 &= 25 \text{ kN} \\ w &= 11 \text{ kN/m} \end{aligned}$$

***5.97** Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.96, determine how much heavier a truck could safely cross the bridge designed in that problem.

***5.96** A bridge of length $L = 15 \text{ m}$ is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 420 \text{ MPa}$. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 11 \text{ kN/m}$ on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 4 \text{ m}$ from each other will be driven across the bridge and that the resulting concentrated loads P_1 and P_2 exerted on each beam could be as large as 95 kN and 25 kN , respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [Hint: It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/(P_1 + P_2)$.]

See solution to Problem 5.94 for calculation of the following:

$$M_D = 309.4 \text{ kNm} \quad M_L = 401.6 \text{ kNm}$$

$$\text{For rolled steel section W690x125} \quad S = 3510 \times 10^3 \text{ mm}^3$$

Allowable live load moment M_L^*

$$\gamma_D M_D + \gamma_L M_L^* = \phi M_U = \phi \sigma_U S$$

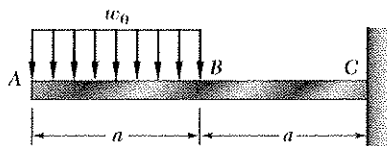
$$M_L^* = \frac{\phi \sigma_U S - \gamma_D M_D}{\gamma_L} = \frac{(0.9)(420 \text{ N/mm}^2)(3510 \times 10^6) - (1.25)(309.4 \times 10^3)}{1.75}$$

$$= 537.2 \text{ kNm}$$

$$\text{Ratio } \frac{M_L^*}{M_L} = \frac{537.2}{401.6} = 1.338 = 1 + 0.338$$

$$\text{Increase } 33.8\%$$

Problem 5.98



5.98 through 5.100 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

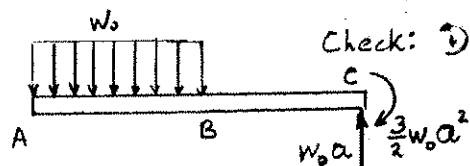
$$w = w_0 - w_0 \langle x-a \rangle^0 = -\frac{dV}{dx}$$

$$(a) \quad V = -w_0 x + w_0 \langle x-a \rangle^1 = \frac{dM}{dx}$$

$$M = -\frac{1}{2} w_0 x^2 + \frac{1}{2} w_0 \langle x-a \rangle^2$$

$$\text{At point C} \quad x = 2a$$

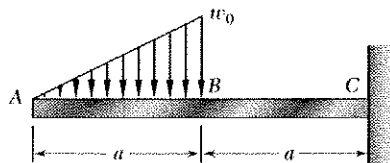
$$(b) \quad M_c = -\frac{1}{2} w_0 (2a)^2 + \frac{1}{2} w_0 a^2 = -\frac{3}{2} w_0 a^2$$



$$\text{Check: } \sum M_c = 0: \left(\frac{3a}{2}\right)(w_0 a) + M_c = 0$$

$$M_c = -\frac{3}{2} w_0 a^2$$

Problem 5.99



5.98 through 5.100 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

$$w = \frac{w_0 x}{a} - w_0 \langle x-a \rangle^0 - \frac{w_0}{a} \langle x-a \rangle^1 = -\frac{dV}{dx}$$

$$(a) \quad V = -\frac{w_0 x^2}{2a} + w_0 \langle x-a \rangle^1 + \frac{w_0}{2a} \langle x-a \rangle^2 = \frac{dM}{dx}$$

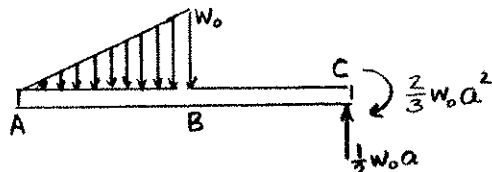
$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2} \langle x-a \rangle^2 + \frac{w_0}{6a} \langle x-a \rangle^3$$

$$\text{At point C} \quad x = 2a$$

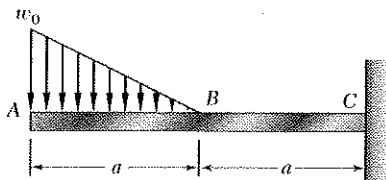
$$(b) \quad M_c = -\frac{w_0 (2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a} \quad M_c = -\frac{2}{3} w_0 a^2$$

$$\text{Check: } \sum M_c = 0: \left(\frac{4a}{3}\right)\left(\frac{1}{2} w_0 a\right) + M_c = 0$$

$$M_c = -\frac{2}{3} w_0 a^2$$



Problem 5.100



5.98 through 5.100 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a} \langle x-a \rangle' = -\frac{dw}{dx}$$

$$(a) V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a} \langle x-a \rangle^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a} \langle x-a \rangle^3$$

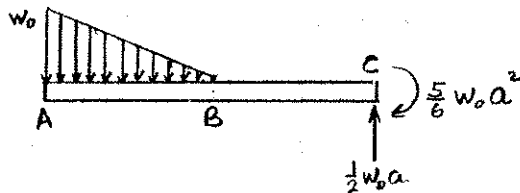
At point C $x = 2a$

$$(b) M_c = -\frac{w_0 (2a)^2}{2} + \frac{w_0 (2a)^3}{6a} - \frac{w_0 a^3}{6a} \quad M_c = -\frac{5}{6} w_0 a^2 \quad \blacktriangleleft$$

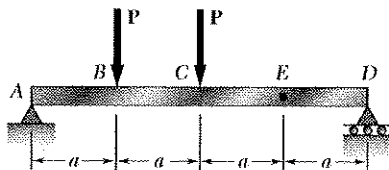
Check:

$$+\circlearrowleft \Sigma M_c = 0: \left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_c = 0$$

$$M_c = -\frac{5}{6} w_0 a^2$$



Problem 5.101



5.101 through 5.103 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of the beam to the right of E.

$$+\circlearrowleft \Sigma M_D = 0: -4aA + 3aP + 2aP = 0$$

$$A = 1.25 P$$

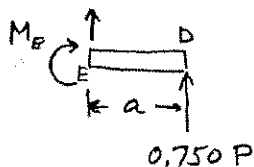
$$(a) V = 1.25 P - P \langle x-a \rangle^0 - P \langle x-2a \rangle^0 \quad \blacktriangleleft$$

$$M = 1.25 Px - P \langle x-a \rangle' - P \langle x-2a \rangle' \quad \blacktriangleleft$$

(b) At point E $x = 3a$

$$M_E = 1.25 P(3a) - P(2a) - P(a) = 0.750 Pa \quad \blacktriangleleft$$

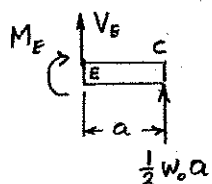
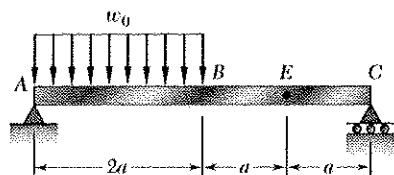
$$\text{Reaction: } +\uparrow \Sigma F_y = 0: A - P - P + D = 0 \quad D = 0.750 P \uparrow$$



$$+\circlearrowleft \Sigma M_E = 0 \quad -M_E + 0.750 Pa = 0$$

$$M_E = 0.750 Pa \quad \blacktriangleleft$$

Problem 5.102



5.101 through 5.103 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of the beam to the right of E .

$$\circlearrowleft \sum M_C = 0 \quad -4aA + (3a)(2aw_0) = 0 \quad A = \frac{3}{2}w_0a$$

$$w = w_0 - w_0 \langle x - 2a \rangle^0 = -\frac{dV}{dx}$$

$$(a) \quad V = -w_0x + w_0 \langle x - 2a \rangle^1 + \frac{3}{2}w_0a = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0x^2 + \frac{1}{2}w_0 \langle x - 2a \rangle^2 + \frac{3}{2}w_0ax + 0$$

$$\text{At point } E \quad x = 3a$$

$$(b) \quad M_E = -\frac{1}{2}w_0(3a)^2 + \frac{1}{2}w_0a^2 + \frac{3}{2}w_0a(3a)$$

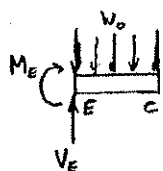
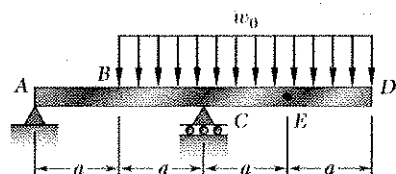
$$M_E = \frac{1}{2}w_0a^2 \quad \blacktriangleleft$$

$$\circlearrowleft \sum M_A = 0: \quad 4aC - (a)(2aw_0) = 0 \quad C = \frac{1}{2}w_0a$$

$$\circlearrowleft \sum M_E = 0: \quad -M_E + (a)(\frac{1}{2}w_0a) = 0$$

$$M_E = \frac{1}{2}w_0a^2 \quad \blacktriangleleft$$

Problem 5.103



$$\circlearrowleft \sum M_C = 0 \quad -2aA - (\frac{a}{2})(3aw_0) = 0 \quad A = -\frac{3}{4}w_0a$$

$$\circlearrowleft \sum M_A = 0 \quad 2aC + (\frac{5a}{2})(3aw_0) = 0 \quad C = \frac{15}{4}w_0a$$

$$w = w_0 \langle x - a \rangle^0 = -\frac{dV}{dx}$$

$$(a) \quad V = -w_0 \langle x - a \rangle^1 - \frac{3}{4}w_0a + \frac{15}{4}w_0a \langle x - 2a \rangle^0 = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0 \langle x - a \rangle^2 - \frac{3}{4}w_0ax + \frac{15}{4}w_0a \langle x - 2a \rangle^1 + 0$$

$$\text{At point } E \quad x = 3a$$

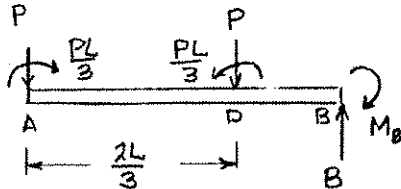
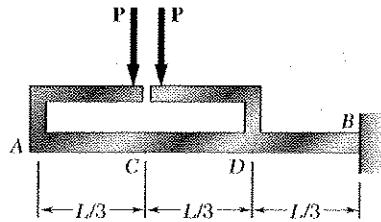
$$(b) \quad M_E = -\frac{1}{2}w_0(2a)^2 - \frac{3}{4}w_0a(3a) + \frac{15}{4}w_0a(a)$$

$$M_E = -\frac{1}{2}w_0a^2 \quad \blacktriangleleft$$

$$\text{Check: } + \sum M_E = 0: \quad -M_E - \frac{a}{2}(w_0a) = 0$$

$$M_E = -\frac{1}{2}w_0a^2$$

Problem 5.104



5.104 (a) Using singularity functions, write the equations for the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for M to determine the bending moment just to the right of point D .

$$(a) \quad V = -P - P\langle x - \frac{2L}{3} \rangle^0 = \frac{dM}{dx}$$

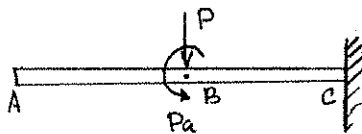
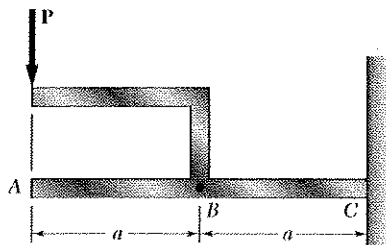
$$M = -Px + \frac{PL}{3} - P\langle x - \frac{2L}{3} \rangle' - \frac{PL}{3}\langle x - \frac{2L}{3} \rangle^0$$

Just to the right of $D \quad x = \frac{2L}{3}$

$$(b) \quad M_D^+ = -P(\frac{2L}{3}) + \frac{PL}{3} - P(0) - \frac{PL}{3}$$

$$M = -\frac{4PL}{3}$$

Problem 5.105



5.105 (a) Using singularity functions, write the equations for the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for M to determine the bending moment just to the right of point B .

$$(a) \quad V = -P\langle x - a \rangle^0$$

$$\frac{dM}{dx} = -P\langle x - a \rangle^0$$

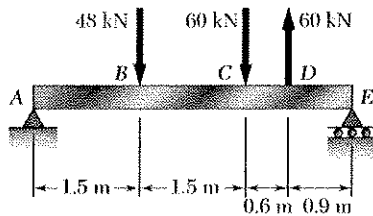
$$M = -P\langle x - a \rangle' - Pa\langle x - a \rangle^0$$

Just to the right of $B \quad x = a'$

$$(b) \quad M = -0 - Pa$$

$$M = -Pa$$

Problem 5.106



5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

$$+\circlearrowleft \sum M_E = 0: -4.5 R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

$$R_A = 40 \text{ kN}$$

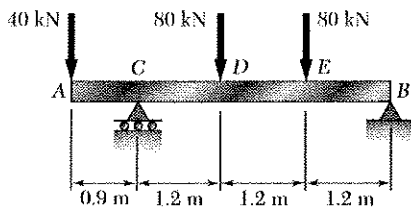
$$(a) V = 40 - 48 \langle x - 1.5 \rangle^0 - 60 \langle x - 3.0 \rangle^0 + 60 \langle x - 3.6 \rangle^0 \text{ kN}$$

$$M = 40x - 48 \langle x - 1.5 \rangle^1 - 60 \langle x - 3.0 \rangle^1 + 60 \langle x - 3.6 \rangle^1 \text{ kN}\cdot\text{m}$$

Pt.	x (m)	M (kN·m)
A	0	0
B	1.5	$(40)(1.5) = 60 \text{ kN}\cdot\text{m}$
C	3.0	$(40)(3.0) - (48)(1.5) = 48 \text{ kN}\cdot\text{m}$
D	3.6	$(40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2 \text{ kN}\cdot\text{m}$
E	4.5	$(40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$

$$(b) M_{\max} = 60 \text{ kN}\cdot\text{m}$$

Problem 5.107



5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

$$+\circlearrowleft \sum M_B = 0: (40)(4.5) - 3.6C + (80)(2.4) + (80)(1.2) = 0$$

$$C = 130 \text{ kN} \uparrow$$

$$(a) V = -40 + 130 \langle x - 0.9 \rangle^0 - 80 \langle x - 2.1 \rangle^0 - 80 \langle x - 3.3 \rangle^0 \text{ kN}$$

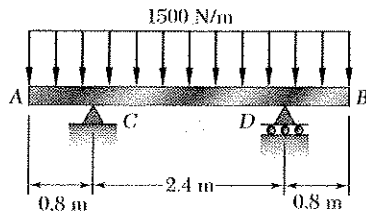
$$M = -40x + 130 \langle x - 0.9 \rangle^1 - 80 \langle x - 2.1 \rangle^1 - 80 \langle x - 3.3 \rangle^1 \text{ kN}\cdot\text{m}$$

Point	x (m)	M (kN·m)
A	0	0
C	0.9	$-(40)(0.9) = -36$
D	2.1	$-(40)(2.1) + (130)(1.2) = 72$
E	3.3	$-(40)(3.3) + (130)(2.4) - (80)(1.2) = 84 \leftarrow \text{maximum}$
B	4.5	$-(40)(4.5) + (130)(3.6) - (80)(2.4) - (80)(1.2) = 0$

(b)

$$|M|_{\max} = 84 \text{ kN}\cdot\text{m}$$

Problem 5.108



5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

$$w = 1.5 \text{ kN/m}$$

By statics, $C = D = 3 \text{ kN} \uparrow$

$$(a) \quad V = -1.5x + 3\langle x-0.8 \rangle^0 + 3\langle x-3.2 \rangle^0 \text{ kN} \quad \blacktriangleleft$$

$$M = -0.75x^2 + 3\langle x-0.8 \rangle^1 + 3\langle x-3.2 \rangle^1 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Locate point E where $V = 0$. Assume $x_C < x_E < x_D$

$$0 = -1.5x_E + 3(x_E - 0.8) + 0 \quad x_E = 2.0 \text{ m}$$

$$M_C = -(0.75)(0.8)^2 + 0 + 0 = -0.480 \text{ kN}\cdot\text{m}$$

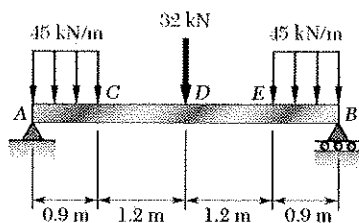
$$M_E = -(0.75)(2.0)^2 + (3)(1.2) + 0 = 0.600 \text{ kN}\cdot\text{m}$$

$$M_D = -(0.75)(3.2)^2 + (3)(2.4) + 0 = -0.480 \text{ kN}\cdot\text{m}$$

$$(b) \quad |M|_{\max} = 0.600 \text{ kN}\cdot\text{m}$$

$$600 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 5.109



5.106 through 5.109 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

$$\text{From symmetry, } A = \frac{1}{2}[(32) + 2(45)(0.9)] = 56.5 \text{ kN}$$

$$w = 45 - 45\langle x-0.9 \rangle^0 + 45\langle x-3.3 \rangle^0 = -\frac{dV}{dx}$$

$$(a) \quad V = 56.5 - 45x + 45\langle x-0.9 \rangle^1 - 32\langle x-2.1 \rangle^0 - 45\langle x-3.3 \rangle^1 \text{ kN} \quad \blacktriangleleft$$

$$M = 56.5x - 22.5x^2 + 22.5\langle x-0.9 \rangle^2 - 32\langle x-2.1 \rangle^1 - 22.5\langle x-3.3 \rangle^2 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$V_C = 56.5 - (45)(0.9) = 16 \text{ kN}$$

$$V_D^- = 56.5 - (45)(2.1) + (45)(1.2) = 16 \text{ kN}$$

$$V_D^+ = 56.5 - (45)(2.1) + (45)(1.2) - 32 = -16 \text{ kN}$$

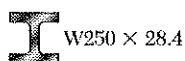
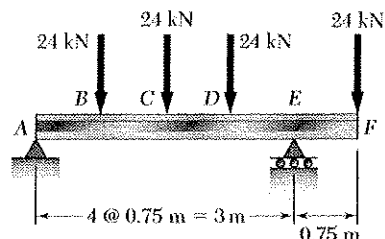
$$V_E = 56.5 - (45)(3.3) + (45)(2.4) - 32 = -16 \text{ kN}$$

$$V_B = 56.5 - (45)(4.2) + (45)(3.3) - 32 - (45)(0.9) = -56.5 \text{ kN}$$

← Changes sign

$$(b) \quad |M|_{\max} = M_D = (56.5)(2.1) - (22.5)(2.1)^2 + (22.5)(1.2)^2 - 0 - 0 = 51.825 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Problem 5.110



5.110 and 5.111 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.

$$+\circlearrowleft \sum M_E = 0;$$

$$-3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$$

$$R_A = 30 \text{ kN}$$

$$\circlearrowleft \sum M_A = 0; -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$$

$$R_E = 66 \text{ kN}$$

$$(a) V = 30 - 24\langle x - 0.75 \rangle^0 - 24\langle x - 1.5 \rangle^0 - 24\langle x - 2.25 \rangle^0 + 66\langle x - 3 \rangle^0 \text{ kN}$$

$$M = 30x - 24\langle x - 0.75 \rangle^1 - 24\langle x - 1.5 \rangle^1 - 24\langle x - 2.25 \rangle^1 + 66\langle x - 3 \rangle^1 \text{ kN}\cdot\text{m}$$

At x (m) M (kN·m)

B 0.75 $(30)(0.75) = 22.5 \text{ kN}\cdot\text{m}$

C 1.5 $(30)(1.5) - (24)(0.75) = 27 \text{ kN}\cdot\text{m}$

D 2.25 $(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5 \text{ kN}\cdot\text{m}$

E 3.0 $(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18 \text{ kN}\cdot\text{m}$

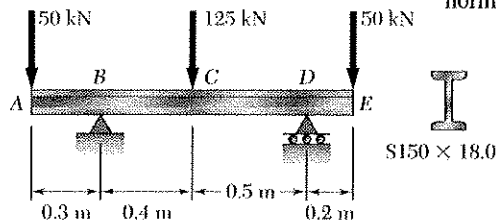
F 3.75 $(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$ ✓

$$\text{Maximum } |M| = 27 \text{ kN}\cdot\text{m} = 27 \times 10^3 \text{ N}\cdot\text{m}$$

For rolled steel section W250 x 28.4, $S = 308 \times 10^3 \text{ mm}^3$
 $= 308 \times 10^{-6} \text{ m}^3$

(b) Normal stress, $\sigma = \frac{|M|}{S} = \frac{27 \times 10^3}{308 \times 10^{-6}} = 87.7 \times 10^6 \text{ Pa} = 87.7 \text{ MPa}$ ✓

Problem 5.111



5.110 and 5.111 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.

$$+\circlearrowleft \sum M_D = 0;$$

$$(1.2)(50) - 0.9B + (0.5)(125) - (0.2)(50) = 0$$

$$B = 125 \text{ kN } \uparrow$$

$$+\circlearrowleft \sum M_B = 0;$$

$$(0.3)(50) - (0.4)(125) + 0.9D - (1.1)(50) = 0$$

$$D = 100 \text{ kN } \uparrow$$

$$(a) V = -50 + 125\langle x - 0.3 \rangle^0 - 125\langle x - 0.7 \rangle^0 + 100\langle x - 1.2 \rangle^0 \quad \text{kN}$$

$$M = -50x + 125\langle x - 0.3 \rangle^1 - 125\langle x - 0.7 \rangle^1 + 100\langle x - 1.2 \rangle^1 \quad \text{kN}\cdot\text{m}$$

Point	x (m)	M (kN·m)
B	0.3	$-(50)(0.3) + 0 - 0 + 0 = -15 \text{ kN}\cdot\text{m}$
C	0.7	$-(50)(0.7) + (125)(0.4) - 0 + 0 = 15 \text{ kN}\cdot\text{m}$
D	1.2	$-(50)(1.2) + (125)(0.9) - (125)(0.5) + 0 = -10 \text{ kN}\cdot\text{m}$
E	1.4	$-(50)(1.4) + (125)(1.1) - (125)(0.7) + (100)(0.2) = 0 \text{ (checks)}$

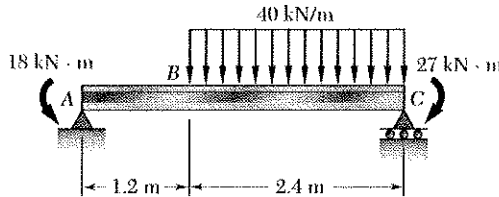
$$\text{Maximum } |M| = 15 \text{ kN}\cdot\text{m} = 15 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{For } S 150 \times 18.0 \text{ rolled steel section, } S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$$

$$(b) \text{ Normal stress, } \sigma = \frac{|M|}{S} = \frac{15 \times 10^3}{120 \times 10^{-6}} = 125 \times 10^6 \text{ Pa} \quad \sigma = 125.0 \text{ MPa}$$

Problem 5.112

5.112 and 5.113 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



S310 x 52

$$\sum M_C = 0:$$

$$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$$

$$R_A = 29.5 \text{ kN}$$

$$V = 29.5 - 40 \langle x - 1.2 \rangle^1 \text{ kN}$$

Point D. $V = 0$

$$29.5 - 40(x_D - 1.2) = 0$$

$$x_D = 1.9375 \text{ m}$$

$$M = -18 + 29.5x - 20 \langle x - 1.2 \rangle^2 \text{ kN}\cdot\text{m}$$

$$M_A = -18 \text{ kN}\cdot\text{m}$$

$$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN}\cdot\text{m}$$

$$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN}\cdot\text{m}$$

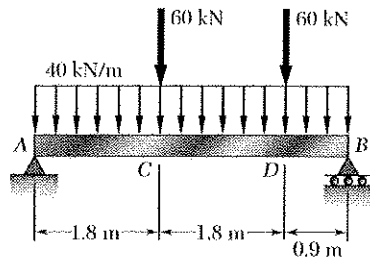
(a) Maximum $|M| = 28.278 \text{ kN}\cdot\text{m}$ at $x = 1.9375 \text{ m}$ ◀

For S310 x 52 rolled steel section, $S = 625 \times 10^3 \text{ mm}^3$
 $= 625 \times 10^{-6} \text{ m}^3$

(b) Normal stress: $\sigma = \frac{|M|}{S} = \frac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa}$

$$\sigma = 45.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 5.113



5.112 and 5.113 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.

$$\begin{aligned} +\circlearrowleft \sum M_B &= 0: \\ -4.5 A + (2.25)(4.5)(40) + (2.7)(60) + (0.9)(60) &= 0 \\ A &= 138 \text{ kN} \uparrow \end{aligned}$$

$$\begin{aligned} +\circlearrowleft \sum M_A &= 0: \\ -(2.25)(4.5)(40) - (1.8)(60) - (3.6)(60) + 4.5 B &= 0 \\ B &= 162 \text{ kN} \uparrow \end{aligned}$$

$$w = 40 \text{ kN} = \frac{dV}{dx}$$

$$V = -40x + 138 - 60\langle x-1.8 \rangle^0 - 60\langle x-3.6 \rangle^0 = \frac{dM}{dx}$$

$$M = -20x^2 - 138x - 60\langle x-1.8 \rangle^1 - 60\langle x-3.6 \rangle^1$$

$$V_C^+ = -(40)(1.8) + 138 - 60 = 6 \text{ kN}$$

$$V_D^- = -(40)(3.6) + 138 - 60 = -66 \text{ kN}$$

Locate point E where $V = 0$. It lies between C and D.

$$V_E = -40x_E + 138 - 60 + 0 = 0 \quad x_E = 1.95 \text{ m}$$

$$M_E = -(20)(1.95)^2 + (138)(1.95) - (60)(1.95 - 1.8) = 184 \text{ kN}\cdot\text{m}$$

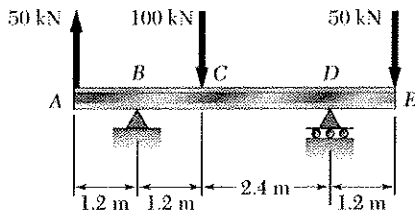
$$(a) |M|_{\max} = 184 \text{ kN}\cdot\text{m} = 184 \times 10^3 \text{ N}\cdot\text{m} \quad \text{at } x = 1.950 \text{ m}$$

$$\text{For } W 530 \times 66 \text{ rolled steel section } S = 1340 \times 10^3 \text{ mm}^3 = 1340 \times 10^{-6} \text{ m}^3$$

$$(b) \text{ Normal stress: } \sigma = \frac{|M|_{\max}}{S} = \frac{184 \times 10^3}{1340 \times 10^{-6}} = 137.3 \times 10^6 \text{ Pa}$$

$$\sigma = 137.3 \text{ MPa}$$

Problem 5.114



5.114 and 5.115 A beam is being designed to be supported and loaded as shown. (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the allowable normal stress for the steel to be used is 165 MPa, find the most economical wide-flange shape that can be used.

$$+\circlearrowleft \sum M_D = 0: -(4.8)(50) - 3.6B + (2.4)(100) - (1.2)(50) = 0$$

$$B = -16.67 \text{ kN} \quad B = 16.67 \text{ kN} \downarrow$$

$$+\circlearrowleft \sum M_B = 0: -(1.2)(50) - (1.2)(100) + 3.6D - (4.8)(50) = 0$$

$$D = 116.67 \text{ kN}$$

$$\text{Check: } +\uparrow \sum F_y = 50 - 16.67 - 100 + 116.67 - 50 = 0 \quad \checkmark$$

$$V = 50 - 16.67 \langle x - 1.2 \rangle^0 - 100 \langle x - 2.4 \rangle^0 + 116.67 \langle x - 4.8 \rangle^0$$

$$M = 50x - 16.67 \langle x - 1.2 \rangle^1 - 100 \langle x - 2.4 \rangle^0 + 116.67 \langle x - 4.8 \rangle^0$$

$$\text{At A. } x = 0, \quad M = 0$$

$$\text{At B. } x = 1.2 \text{ m}, \quad M = (50)(1.2) = 60 \text{ kNm}$$

$$\text{At C. } x = 2.4 \text{ m}, \quad M = (50)(2.4) - (16.67)(1.2) = 100 \text{ kNm}$$

$$\text{At D. } x = 4.8 \text{ m}, \quad M = (50)(4.8) - (16.67)(3.6) - (100)(2.4) = -60 \text{ kNm}$$

$$\text{At E. } x = 6.0 \text{ m}, \quad M = (50)(6) - (16.67)(4.8) - (100)(3.6) + (116.67)(1.2) = 0 \quad (\text{checks})$$

$$(a) \quad |M|_{\max} = 100 \text{ kNm at C} \quad \blacktriangleleft$$

$$(b) \quad |M|_{\max} = 100 \text{ kNm}$$

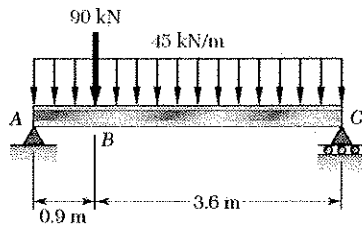
$$\sigma_{\text{all}} = 165 \text{ MPa}$$

$$\sigma = \frac{|M|}{S} \quad S_{\min} = \frac{|M|_{\max}}{\sigma} = \frac{100000}{165 \times 10^6} = 606.06 \times 10^{-6} \text{ m}^3 = 606.06 \times 10^3 \text{ mm}^3$$

From Appendix C, lightest wide flange shape is

$$W410 \times 38.8 \quad \blacktriangleleft$$

Problem 5.115



5.114 and 5.115 A beam is being designed to be supported and loaded as shown. (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the allowable normal stress for the steel to be used is 165 MPa, find the most economical wide-flange shape that can be used.

$$\sum M_C = 0 \quad -4.5 R_A + (45)(4.5)(2.25) + (90)(3.6) = 0$$

$$R_A = 173.25 \text{ kN}$$

$$W = 45 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 173.25 - 45x - 90\langle x - 0.9 \rangle^0 \text{ kN}$$

Location of point D where $V = 0$. Assume $0.9 < x_D < 4.5$

$$0 = 173.25 - 45x_D - 90 \quad x_D = 1.85 \text{ m}$$

$$M = 173.25x - 22.5x^2 - 90\langle x - 0.9 \rangle^1 \text{ kNm}$$

$$\text{At point D } (x = 1.85 \text{ m}) \quad M = (173.25)(1.85) - (22.5)(1.85)^2 - (90)(0.95) = 158 \text{ kNm}$$

(a) Maximum $|M| = 158 \text{ kNm}$ at $x = 1.85 \text{ m}$.

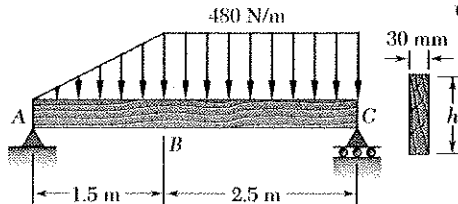
$$S_{\min} = \frac{M}{\sigma_{\text{all}}} = \frac{158 \times 10^3}{165 \times 10^6} = 957.6 \times 10^{-6} \text{ m}^3 = 957.6 \times 10^3 \text{ mm}^3$$

Shape	$S (\times 10^3 \text{ mm}^3)$
W 530 X 66	1340
W 460 X 74	1460
W 410 X 60	1060
W 360 X 64	1030
W 310 X 74	1060
W 250 X 101	1240

Answer W 410 X 60

Problem 5.116

5.116 and 5.117 A timber beam is being designed to be supported and loaded as shown. (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



$$480 \text{ N/m} = 0.48 \text{ kN/m}$$

$$\begin{aligned} \sum M_C = 0: \\ -4 R_A + (3')\left(\frac{1}{2}\right)(1.5)(0.48) + (1.25)(2.5)(0.48) = 0 \end{aligned}$$

$$R_A = 0.645 \text{ kN} \uparrow$$

$$W = \frac{0.48}{1.5} x - \frac{0.48}{1.5} \langle x - 1.5 \rangle' = 0.32 x - 0.32 \langle x - 1.5 \rangle' \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16 x^2 + 0.16 \langle x - 1.5 \rangle^2 \text{ kN}$$

Locate point D where $V = 0$. Assume $1.5 \text{ m} < x_D < 4 \text{ m}$.

$$\begin{aligned} 0 &= 0.645 - 0.16 x_D^2 + 0.16(x_D - 1.5)^2 \\ &= 0.645 - 0.16 x_D^2 + 0.16 x_D^2 - 0.48 x_D + 0.36 \end{aligned}$$

$$x_D = 2.09375 \text{ m}$$

$$M = 0.645 x - 0.05333 x^3 + 0.05333 \langle x - 1.5 \rangle^3 \text{ kN}\cdot\text{m}$$

$$\begin{aligned} \text{(a) At point D, } M_D &= (0.645)(2.09375) - (0.05333)(2.09375)^3 + (0.05333)(0.59375)^3 \\ &= 0.87211 \text{ kN}\cdot\text{m} \end{aligned}$$

$$S_{\min} = \frac{M_D}{\sigma_{\text{all}}} = \frac{0.87211 \times 10^3}{12 \times 10^6} = 72.6758 \times 10^{-6} \text{ m}^3 = 72.6758 \times 10^3 \text{ mm}^3$$

For a rectangular cross section $S = \frac{1}{6} b h^2$ $h = \sqrt{\frac{6S}{b}}$

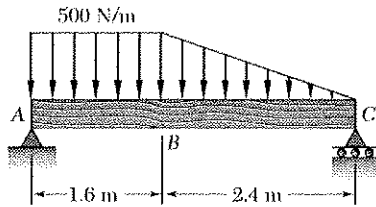
$$h_{\min} = \sqrt{\frac{(6)(72.6758 \times 10^3)}{30}} = 120.56 \text{ mm}$$

(b) At next larger 10-mm increment,

$$h = 130 \text{ mm}$$

Problem 5.117

5.116 and 5.117 A timber beam is being designed to be supported and loaded as shown. (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



$$500 \text{ N/m} = 0.5 \text{ kN/m}$$

$$+\circlearrowleft \sum M_c = 0:$$

$$-4R_A + (3.2)(1.6)(0.5) + (1.6)(\frac{1}{2})(2.4)(0.5) = 0$$

$$R_A = 0.880 \text{ kN} \uparrow$$

$$w = 0.5 - \frac{0.5}{2.4} \langle x - 1.6 \rangle^1 = 0.5 - 0.20833 \langle x - 1.6 \rangle^1 \text{ kN/m} = -\frac{dv}{dx}$$

$$V = 0.880 - 0.5x + 0.104167 \langle x - 1.6 \rangle^2 \text{ kN}$$

$$V_A = 0.880 \text{ kN}$$

$$V_B = 0.880 - (0.5)(1.6) = 0.080 \text{ kN}$$

$$V_C = 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520 \text{ kN} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Sign change}$$

locate point D (between B and C) where $V = 0$.

$$0 = 0.880 - 0.5x_D + 0.104167 (x_D - 1.6)^2$$

$$0.104167 x_D^2 - 0.83333 x_D + 1.14667 = 0$$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)}$$

$$= 4.0 \pm 2.2342 = \cancel{5.2342}, 1.7658 \text{ m}$$

$$M = 0.880x - 0.25x^2 + 0.347222 \langle x - 1.6 \rangle^3 \text{ kN-m}$$

$$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.347222)(0.1658)^3 = 0.776 \text{ kN-m}$$

(a) $M_{\max} = 0.776 \text{ kN-m}$ at $x = 1.7658 \text{ m}$

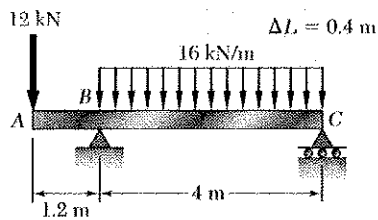
$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{0.776 \times 10^3}{12 \times 10^6} = 64.66 \times 10^{-6} \text{ m}^3 = 64.66 \times 10^3 \text{ mm}^3$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$ $h = \frac{6S}{b}$

$$h_{\min} = \sqrt{\frac{(6)(64.66 \times 10^3)}{30}} = 113.7 \text{ mm}$$

(b) At next higher 10-mm increment $h = 120 \text{ mm}$

Problem 5.118



5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL , starting at point A and ending at the right-hand support.

$$+\circlearrowleft \sum M_C = 0:$$

$$(5.2)(12) - 4B + (2)(4)(16) = 0$$

$$B = 47.6 \text{ kN } \uparrow$$

$$+\circlearrowleft \sum M_B = 0:$$

$$(1.2)(12) - (2)(4)(16) + 4C = 0$$

$$C = 28.4 \text{ kN } \uparrow$$

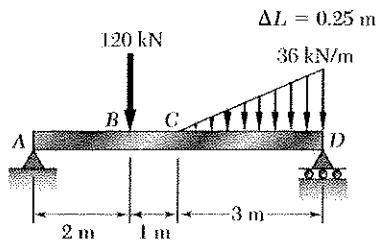
$$w = 16 \langle x - 1.2 \rangle^0 = -\frac{dV}{dx}$$

$$V = -16 \langle x - 1.2 \rangle^1 - 12 + 47.6 \langle x - 1.2 \rangle^0$$

$$M = -8 \langle x - 1.2 \rangle^2 - 12x + 47.6 \langle x - 1.2 \rangle^1$$

x m	V kN	M kN·m
0.0	-12.0	0.00
0.4	-12.0	-4.80
0.8	-12.0	-9.60
1.2	35.6	-14.40
1.6	29.2	-1.44
2.0	22.8	8.96
2.4	16.4	16.80
2.8	10.0	22.08
3.2	3.6	24.80
3.6	-2.8	24.96
4.0	-9.2	22.56
4.4	-15.6	17.60
4.8	-22.0	10.08
5.2	-28.4	-0.00

Problem 5.119



5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL , starting at point A and ending at the right-hand support.

$$\sum M_C = 0:$$

$$-6 R_A + (4)(120) + (1)(\frac{1}{2})(3)(36) = 0$$

$$R_A = 89 \text{ kN}$$

$$w = \frac{36}{3} \langle x-3 \rangle' = 12 \langle x-3 \rangle'$$

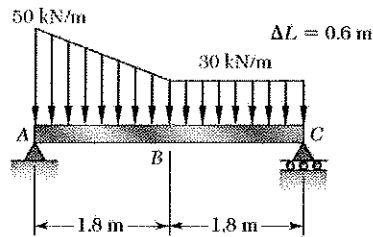
$$V = 89 - 120 \langle x-2 \rangle^0 - 6 \langle x-3 \rangle^2 \text{ kN}$$

$$M = 89x - 120 \langle x-2 \rangle' - 2 \langle x-3 \rangle^3 \text{ kN}\cdot\text{m}$$

x m	V kN	M kN·m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	-31.0	178.0
2.3	-31.0	170.3
2.5	-31.0	162.5
2.8	-31.0	154.8
3.0	-31.0	147.0
3.3	-31.4	139.2
3.5	-32.5	131.3
3.8	-34.4	122.9
4.0	-37.0	114.0
4.3	-40.4	104.3
4.5	-44.5	93.8
4.8	-49.4	82.0
5.0	-55.0	69.0
5.3	-61.4	54.5
5.5	-68.5	38.3
5.8	-76.4	20.2
6.0	-85.0	-0.0

Problem 5.120

5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL , starting at point A and ending at the right-hand support.



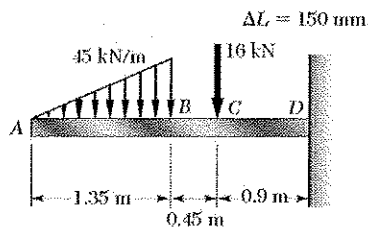
$$\begin{aligned} +\circlearrowleft \sum M_C &= 0 \\ -3.6 R_A + (30)(3.6)(1.8) + (20)(1.8)\left(\frac{1}{2}\right)(3) &= 0 \\ R_A &= 69 \text{ kN} \\ W &= 50 - \frac{20}{1.8}x + \frac{20}{1.8}\langle x-1.8 \rangle' \\ &= 50 - 11.11x + 11.11\langle x-1.8 \rangle' \end{aligned}$$

$$V = 69 - 50x + 5.56x^2 - 5.56\langle x-1.8 \rangle^2 \text{ kN} \rightarrow$$

$$M = 69x - 25x^2 + 1.852x^3 - 1.852\langle x-1.8 \rangle^3 \text{ kNm} \rightarrow$$

x m	V kN	M kNm.
0	69	0
0.6	41	32.8
1.2	17	50
1.8	-3	54
2.4	-20	46.8
3.0	-39	28.8
3.6	-57	0

Problem 5.121



5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL , starting at point A and ending at the right-hand support.

$$w = \frac{45}{1.35} x - 45 \langle x - 1.35 \rangle^0 - \frac{45}{1.35} \langle x - 1.35 \rangle^1$$

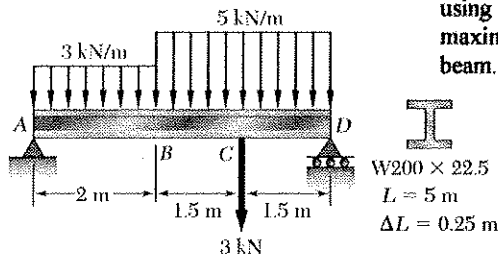
$$= 33.33x - 45 \langle x - 1.35 \rangle^0 - 33.33 \langle x - 1.35 \rangle^1 = -\frac{dV}{dx}$$

$$V = -16.67x^2 + 45 \langle x - 1.35 \rangle^1 + 16.67 \langle x - 1.35 \rangle^2 - 16 \langle x - 1.8 \rangle^0$$

$$M = -5.56x^3 + 22.5 \langle x - 1.35 \rangle^2 + 5.56 \langle x - 1.35 \rangle^3 - 16 \langle x - 1.8 \rangle^1$$

x m	V kN	M kNm
0	0	0
0.6	-5.32	-1.2
1.35	-27	-13.74
2.1	-43	-42.04
2.7	-43	-71.19

Problem 5.122



5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (b) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

$$\sum M_D = 0:$$

$$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$$

$$R_A = 10.2 \text{ kN}$$

$$W = 3 + 2\langle x-2 \rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 10.2 - 3x - 2\langle x-2 \rangle^1 - 3\langle x-3.5 \rangle^0 \text{ kN}$$

$$M = 10.2x - 1.5x^2 - \langle x-2 \rangle^2 - 3\langle x-3.5 \rangle^1 \text{ kN}\cdot\text{m}$$

x m	V kN	M kN·m	sigma MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.72	24.4
0.75	7.95	6.81	35.1
1.00	7.20	8.70	44.8
1.25	6.45	10.41	53.6
1.50	5.70	11.92	61.5
1.75	4.95	13.26	68.3
2.00	4.20	14.40	74.2
2.25	2.95	15.29	78.8
2.50	1.70	15.88	81.8
2.75	0.45	16.14	83.2
3.00	-0.80	16.10	83.0
3.25	-2.05	15.74	81.2
3.50	-6.30	15.07	77.7
3.75	-7.55	13.34	68.8
4.00	-8.80	11.30	58.2
4.25	-10.05	8.94	46.1
4.50	-11.30	6.27	32.3
4.75	-12.55	3.29	17.0
5.00	-13.80	-0.00	-0.0
2.83	0.05	16.164	83.3
2.84	0.00	16.164	83.3
2.85	-0.05	16.164	83.3

For rolled steel section
 $W200 \times 22.5$

$$S = 194 \times 10^3 \text{ mm}^3$$

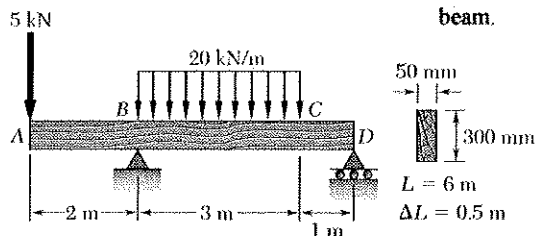
$$= 194 \times 10^{-6} \text{ m}^3$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{16.164 \times 10^3}{194 \times 10^{-6}}$$

$$= 83.3 \times 10^6 \text{ Pa}$$

$$= 83.3 \text{ MPa}$$

Problem 5.123



5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (b) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

$$\sum M_D = 0 :$$

$$-4R_B + (6)(5) + (2.5)(3)(20) = 0$$

$$R_B = 45 \text{ kN}$$

$$W = 20 \langle x-2 \rangle^0 - 20 \langle x-5 \rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = -5 + 45 \langle x-2 \rangle^0 - 20 \langle x-2 \rangle^1 + 20 \langle x-5 \rangle^1 \text{ kN}$$

$$M = -5x + 45 \langle x-2 \rangle^1 - 10 \langle x-2 \rangle^2 + 10 \langle x-5 \rangle^2 \text{ kN}\cdot\text{m}$$

x m	V kN	M kN·m	sigma MPa
0.00	-5	0.00	0.0
0.50	-5	-2.50	-3.3
1.00	-5	-5.00	-6.7
1.50	-5	-7.50	-10.0
2.00	40	-10.00	-13.3
2.50	30	7.50	10.0
3.00	20	20.00	26.7
3.50	10	27.50	36.7
4.00	0	30.00	40.0
4.50	-10	27.50	36.7
5.00	-20	20.00	26.7
5.50	-20	10.00	13.3
6.00	-20	0.00	0.0

$$\text{Maximum } |M| = 30 \text{ kN}\cdot\text{m} \text{ at } x = 4.0 \text{ m}$$

For rectangular cross section

$$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(50)(300)^2$$

$$= 750 \times 10^3 \text{ mm}^3$$

$$= 750 \times 10^{-6} \text{ m}^3$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{30 \times 10^3}{750 \times 10^{-6}}$$

$$= 40 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 40.0 \text{ MPa}$$

$$+\sum M_D = 0$$

$$-1.5R_A + (30)(0.45)(1.275) + (18)(0.6)(0.75) + (1.2)(0.45) = 0$$

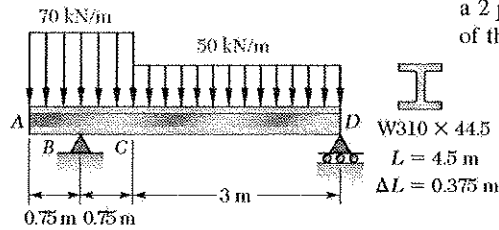
$$R_A = 17.235 \text{ kN}$$

$$M = 17.235x - 15x^2 + 6(x - 0.45)^2 + 9(x - 1.05)^2 - 1.2(x - 1.05) \text{ kNm}$$

[illegible]

$$\sigma = \frac{M}{S} = \frac{5.15 \times 10^3}{750000 \times 10^{-9}} = 6.87 \text{ MPa.}$$

Problem 5.125



5.124 and 5.125 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (b) using smaller increments if necessary, determine with a 2 percent accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

$$\sum M_B = 0$$

$$-3.75 R_B + (70)(1.5)(3.75) + (50)(3)(1.5) = 0$$

$$R_B = 165 \text{ kN}$$

$$w = 70 - 20 \langle x - 1.5 \rangle^0 \text{ kN/m}$$

$$V = -70x + 165 \langle x - 0.75 \rangle^0 + 20 \langle x - 1.5 \rangle^1 \text{ kN}$$

$$M = -35x^2 + 165 \langle x - 0.75 \rangle^1 + 10 \langle x - 1.5 \rangle^2 \text{ kNm}$$

x m	V kN	M kNm	sigma MPa
0	0	0	0
1.5	51.2	43.39	69.65
2.25	19.2	73.22	117.53
2.7	0	78.11	125.3
3.0	-12.8	75.94	121.87
4.5	-76.8	0	0

$$\text{Maximum } M = 78.11 \text{ kNm}$$

$$\text{at } x = 2.7 \text{ m}$$

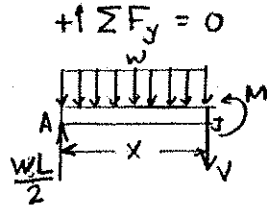
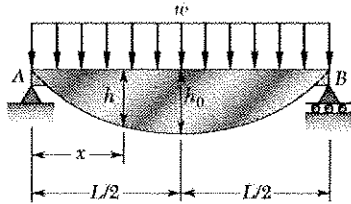
For rolled steel section W310x44.5

$$S = 634 \times 10^3 \text{ mm}^3$$

Maximum normal stress

$$\sigma = \frac{M}{S} = \frac{78.11 \times 10^6}{634 \times 10^3} = 123.2 \text{ MPa}$$

Problem 5.126



$$+\uparrow \sum F_y = 0$$

$$R_A + R_B - wL = 0 \quad R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \sum M_j = 0$$

$$\frac{wL}{2}x - wx \frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

For a rectangular cross section

$$S = \frac{1}{6}bh^2$$

Equating $\frac{1}{6}bh^2 = \frac{wx(L-x)}{2\sigma_{all}}$

$$h = \left\{ \frac{3wx(L-x)}{\sigma_{all}b} \right\}^{1/2}$$

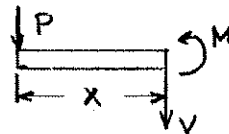
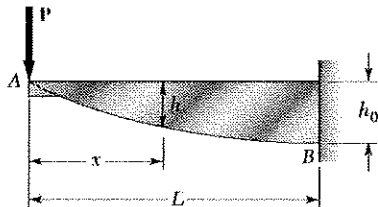
(a) At $x = \frac{L}{2}$ $h = h_0 = \left\{ \frac{3wL^2}{4\sigma_{all}b} \right\}^{1/2}$

$$h = h_0 \left[\frac{x}{L} \left(1 - \frac{x}{L} \right) \right]^{1/2}$$

(b) Solving for w $w = \frac{4\sigma_{all}bh_0^2}{3L^2} = \frac{(4)(165 \times 10^6)(0.03)(0.3)^2}{(3)(0.9)^2}$

$$= 733.3 \text{ kN/m}$$

Problem 5.127



$$V = -P$$

$$M = -Px \quad |M| = Px$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{Px}{\sigma_{all}}$$

For a rectangular cross section

$$S = \frac{1}{6}bh^2$$

Equating $\frac{1}{6}bh^2 = \frac{Px}{\sigma_{all}}$

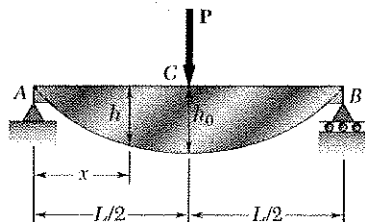
$$h = \left(\frac{6Px}{\sigma_{all}b} \right)^{1/2} \quad (1)$$

At $x = L$ $h = h_0 = \left\{ \frac{6PL}{\sigma_{all}b} \right\}^{1/2} \quad (2)$

(a) Divide Eq. (1) by Eq. (2) and solve for h $h = h_0(x/L)^{1/2}$

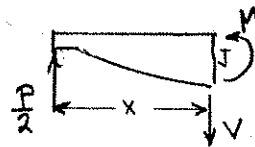
(b) Solving for P $P = \frac{\sigma_{all}bh_0^2}{6L} = \frac{(165 \times 10^6)(0.03)(0.3)^2}{(6)(0.9)} = 82.5 \text{ kN}$

Problem 5.128



5.128 and 5.129 The beam AB , consisting of an aluminum plate of uniform thickness b and length L , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

$$R_A = R_B = \frac{P}{2} \uparrow$$



$$\sum M_J = 0:$$

$$-\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \quad (0 < x < \frac{L}{2})$$

$$S = \frac{M}{\sigma_{all}} = \frac{Px}{2\sigma_{all}}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{Px}{2\sigma_{all}} \quad h = \sqrt{\frac{3Px}{\sigma_{all}b}}$

(a) At $x = \frac{L}{2}$, $h = h_0 = \sqrt{\frac{3PL}{2\sigma_{all}b}}$

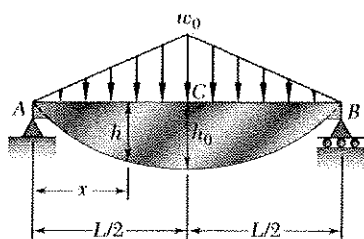
$$h = h_0 \sqrt{\frac{2x}{L}}, \quad 0 < x < \frac{L}{2}$$

For $x > \frac{L}{2}$, replace x by $L - x$.

(b) Solving for P , $P = \frac{2\sigma_{all}bh_0^2}{3L} = \frac{(2)(72 \times 10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60 \times 10^3 \text{ N}$

$$P = 60 \text{ kN}$$

Problem 5.129

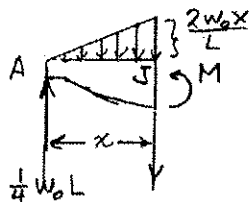


5.128 and 5.129 The beam AB , consisting of an aluminum plate of uniform thickness b and length L , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: A - \frac{1}{2} w_0 L + B = 0$$

$$A = B = \frac{1}{4} w_0 L \uparrow$$



For $0 \leq x \leq \frac{L}{2}$, $w = \frac{2w_0x}{L}$

$$\frac{dV}{dx} = -w = -\frac{2w_0x}{L}$$

$$V = C_1 - \frac{w_0x^2}{L}$$

At $x = 0$ $V = \frac{1}{4} w_0 L$

$$C_1 = \frac{1}{4} w_0 L$$

$$\frac{dM}{dx} = V = \frac{1}{4} w_0 L - \frac{w_0x^2}{L}$$

$$M = C_2 + \frac{1}{4} w_0 L x - \frac{1}{3} \frac{w_0x^3}{L}$$

At $x = 0$ $M = 0$

$$C_2 = 0$$

$$M = \frac{1}{12} \frac{w_0}{L} (3L^2x - 4x^3)$$

At $x = \frac{L}{2}$, $M = M_c = \frac{1}{12} \frac{w_0}{L} [3L^2(\frac{L}{2}) - 4(\frac{L}{2})^3] = \frac{1}{12} w_0 L^2$

For constant strength, $S = \frac{M}{\sigma_{all}}$ $S_o = \frac{M_o}{\sigma_{all}} = \frac{M_c}{\sigma_{all}}$

$$\frac{S}{S_o} = \frac{M}{M_o} = \frac{1}{L^3} (3L^2x - 4x^3)$$

For a rectangular section, $S = \frac{1}{6} b h^2$ $S_o = \frac{1}{6} b h_o^2$

(a) $\frac{S}{S_o} = \left(\frac{h}{h_o}\right)^2$ $h = h_o \sqrt{\frac{3L^2x - 4x^3}{L^3}}$

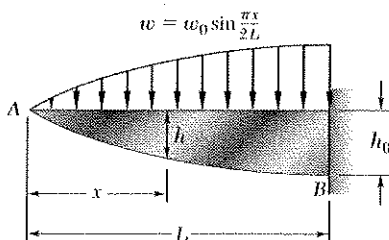
(b) Data: $L = 800$ mm $h_o = 200$ mm $b = 25$ mm $\sigma_{all} = 72$ MPa

$$S_o = \frac{1}{6} b h_o^2 = \frac{1}{6} (25)(200)^2 = 166.667 \times 10^3 \text{ mm}^3 = 166.667 \times 10^{-6} \text{ m}^3$$

$$M_c = \sigma_{all} S_o = (72 \times 10^6)(166.667 \times 10^{-6}) = 12 \times 10^3 \text{ N}\cdot\text{m}$$

$$w_0 = \frac{12 M_c}{L^2} = \frac{(12)(12 \times 10^3)}{(0.800)^2} = 225 \times 10^3 \text{ N/m} \quad w_0 = 225 \text{ kN/m}$$

Problem 5.130



5.130 and 5.131 The beam AB , consisting of a cast-iron plate of uniform thickness b and length L , is to support the distributed load $w(x)$ shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 . (b) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{2L}$$

$$V = -\frac{2w_0 L}{\pi} \cos \frac{\pi x}{2L} + C_1$$

$$V = 0 \text{ at } x = 0 \rightarrow C_1 = -\frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi} \left(1 - \cos \frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0 L}{\pi} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

$$|M| = \frac{2w_0 L}{\pi} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{2w_0 L}{\pi \sigma_{all}} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{2w_0 L}{\pi \sigma_{all}} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$

$$h = \left\{ \frac{12w_0 L}{\pi \sigma_{all} b} \left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right) \right\}^{1/2}$$

$$\text{At } x = L \quad h = h_0 = \left\{ \frac{12w_0 L^2}{\pi \sigma_{all} b} \left(1 - \frac{2}{\pi}\right) \right\}^{1/2} = 1.178 \sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$$

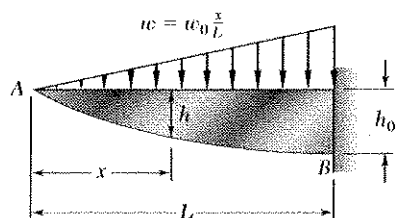
$$(a) \quad h = h_0 \left[\left(\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L} \right) / \left(1 - \frac{2}{\pi} \right) \right]^{1/2} = 1.659 h_0 \left[\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L} \right]^{1/2} \blacktriangleleft$$

Data: $L = 750$ mm = 0.75 m, $b = 30$ mm = 0.030 m

$w_0 = 300$ kN/m = 300×10^3 N/m, $\sigma_{all} = 200$ MPa = 200×10^6 Pa

$$(b) \quad h_0 = 1.178 \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 197.6 \times 10^{-3} \text{ m} \quad h_0 = 197.6 \text{ mm} \blacktriangleleft$$

Problem 5.131



5.130 and 5.131 The beam AB , consisting of a cast-iron plate of uniform thickness b and length L , is to support the distributed load $w(x)$ shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 . (b) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \quad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w_0 x^3}{6L\sigma_{all}}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{w_0 x^3}{6L\sigma_{all}} \quad h = \sqrt{\frac{w_0 x^3}{\sigma_{all} b L}}$

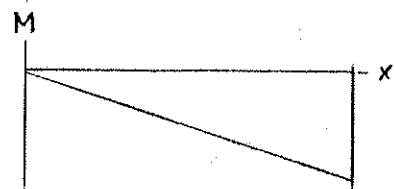
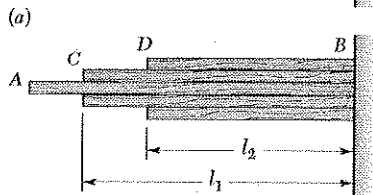
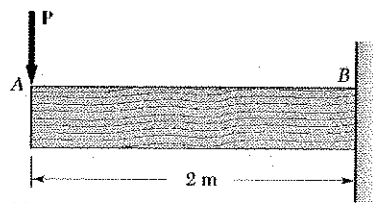
At $x = L$, $h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{all} b}} \quad (a) \quad h = h_0 \left(\frac{x}{L}\right)^{3/2}$

Data: $L = 750$ mm = 0.75 m, $b = 30$ mm = 0.030 m

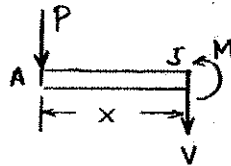
$w_0 = 300$ kN/m = 300×10^3 N/m, $\sigma_{all} = 200$ MPa = 200×10^6 Pa

(b) $h_0 = \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 167.7 \times 10^{-3}$ m $h_0 \approx 167.7$ mm

Problem 5.132



5.132 and 5.133 A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 250 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 50 × 250 mm cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the factor of safety as the original design.



$$+\circlearrowleft \sum M_J = 0$$

$$Px + M = 0 \quad M = -Px$$

$$|M| = Px$$

$$\text{At B} \quad |M|_B = M_{\max}$$

$$\text{At C} \quad |M|_C = M_{\max} x_C / 2$$

$$\text{At D} \quad |M|_D = M_{\max} x_D / 2$$

$$S_B = \frac{1}{6} b h^3 = \frac{1}{6} \cdot b (5b)^3 = \frac{25}{6} b^3$$

$$\text{A to C} \quad S_C = \frac{1}{6} \cdot b (b)^3 = \frac{1}{6} b^3$$

$$\text{C to D} \quad S_D = \frac{1}{6} b (3b)^3 = \frac{9}{6} b^3$$

$$\frac{|M|_C}{|M|_B} = \frac{x_C}{2} = \frac{S_C}{S_B} = \frac{1}{25}$$

$$x_C = \frac{(1)(2)}{25} = 0.08 \text{ m}$$

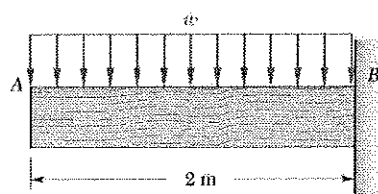
$$l_1 = 2 - 0.08 = 1.92 \text{ m} \quad \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \frac{x_D}{2} = \frac{S_D}{S_B} = \frac{9}{25}$$

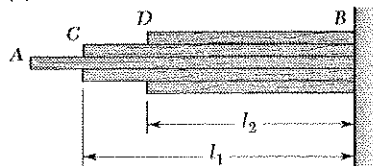
$$x_D = \frac{(9)(2)}{25} = 0.72 \text{ m}$$

$$l_2 = 2 - 0.72 = 1.28 \text{ m} \quad \blacktriangleleft$$

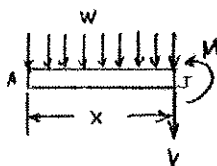
Problem 5.133



(a)



(b)



$$\sum M_J = 0 \quad w x \frac{x}{2} + M = 0$$

$$M = -\frac{wx^2}{2} \quad |M| = \frac{wx^2}{2}$$

$$\text{At B} \quad |M|_B = |M|_{\max}$$

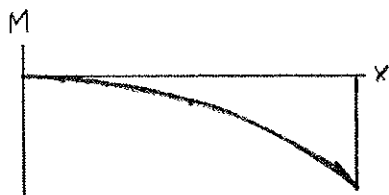
$$\text{At C} \quad |M|_C = |M|_{\max} (x_c/2)^2$$

$$\text{At D} \quad |M|_D = |M|_{\max} (x_D/2)^2$$

$$\text{At B} \quad S_B = \frac{1}{6} b h^2 = \frac{1}{6} b (5b)^2 = \frac{25}{6} b^3$$

$$\text{At C} \quad S_C = \frac{1}{6} b h^2 = \frac{1}{6} b (b)^2 = \frac{1}{6} b^3$$

$$\text{At D} \quad S_D = \frac{1}{6} b h^2 = \frac{1}{6} b (3b)^2 = \frac{9}{6} b^3$$



$$\frac{|M|_C}{|M|_B} = \left(\frac{x_c}{2} \right)^2 = \frac{S_C}{S_B} = \frac{1}{25}$$

$$x_c = \frac{2}{\sqrt{25}} = 0.4 \text{ m}$$

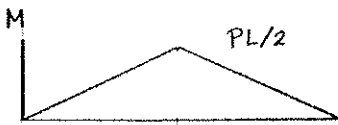
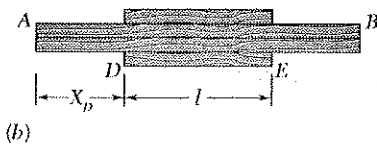
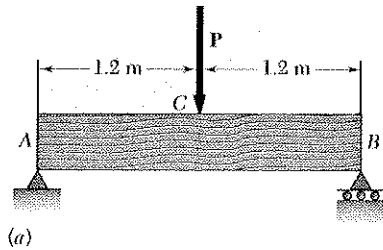
$$l_1 = 2 - 0.4 = 1.6 \text{ m}$$

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{2} \right)^2 = \frac{S_D}{S_B} = \frac{9}{25}$$

$$x_D = \frac{2\sqrt{9}}{\sqrt{25}} = 1.2 \text{ m}$$

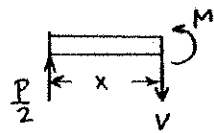
$$l_2 = 2 - 1.2 = 0.8 \text{ m}$$

Problem 5.134



5.134 and 5.135 A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

$$R_A = R_B = \frac{P}{2}$$



$$0 < x < \frac{l}{2}$$

$$\sum M_J = 0: -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \quad \text{or} \quad M = \frac{M_{max}x}{1.2}$$

Bending moment diagram is two straight lines.

$$\text{At } C \quad S_c = \frac{1}{6}bh_c^2$$

$$M_c = M_{max}$$

$$\text{At } D \quad S_D = \frac{1}{6}bh_D^2$$

$$M_D = \frac{M_{max}x_D}{1.2}$$

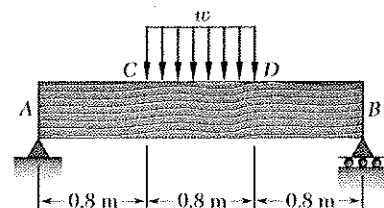
$$\frac{S_D}{S_c} = \frac{h_D^2}{h_c^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_c} = \frac{x_D}{1.2}$$

$$x_D = 0.3 \text{ m}$$

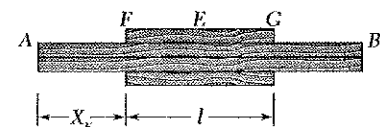
$$\frac{l}{2} = 1.2 - x_D = 0.9$$

$$l = 1.800 \text{ m} \quad \blacktriangleleft$$

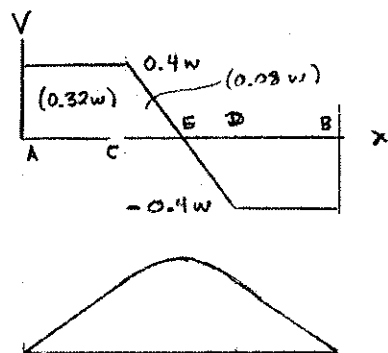
Problem 5.135



(a)



(b)



5.134 and 5.135 A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

$$R_A = R_B = \frac{0.8 w}{2} = 0.4 w$$

$$\text{Shear:} \quad \begin{array}{ll} \text{A to C} & V = 0.4 w \\ \text{D to B} & V = -0.4 w \end{array}$$

$$\text{Areas:} \quad \begin{array}{ll} \text{A to C} & (0.8)(0.4) w = 0.32 w \\ \text{C to E} & \left(\frac{1}{2}\right)(0.4)(0.4) w = 0.08 w \end{array}$$

Bending moments.

$$\text{At C} \quad M_C = 0.40 w$$

$$\text{A to C} \quad M = 0.40 w x$$

$$\text{At C} \quad S_c = \frac{1}{6} b h_c^2 \quad M_C = M_{\max} = 0.40 w$$

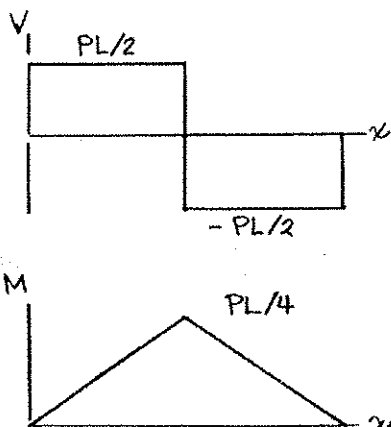
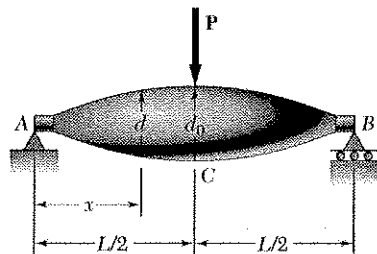
$$\text{At F} \quad S_F = \frac{1}{6} b h_F^2 \quad M_F = 0.40 w x_F$$

$$\frac{S_F}{S_c} = \frac{h_F^2}{h_c^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_C} = \frac{0.40 w x_F}{0.40 w}$$

$$x_F = 0.25 \text{ m} \quad \frac{l}{2} = 1.2 - x_F = 0.95 \text{ m}$$

$$l = 1.900 \text{ m}$$

Problem 5.136



5.136 and 5.137 A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x , L , and d_0 .

Draw shear and bending moment diagrams.

$$0 \leq x \leq \frac{L}{2}, \quad M = \frac{Px}{2}$$

$$\frac{L}{2} \leq x \leq L, \quad M = \frac{P(L-x)}{2}$$

For a solid circular section, $C = \frac{1}{2}d$

$$I = \frac{\pi}{4}C^4 = \frac{\pi}{64}d^4 \quad S = \frac{I}{C} = \frac{\pi}{32}d^3$$

For constant strength design, $\sigma = \text{constant}$.

$$S = \frac{M}{\sigma}$$

$$\text{For } 0 \leq x \leq \frac{L}{2}, \quad \frac{\pi}{32}d^3 = \frac{Px}{2} \quad (1a)$$

$$\text{For } \frac{L}{2} \leq x \leq L, \quad \frac{\pi}{32}d^3 = \frac{P(L-x)}{2} \quad (1b)$$

$$\text{At point C,} \quad \frac{\pi}{32}d_0^3 = \frac{PL}{4} \quad (2)$$

Dividing Eq.(1a) by Eq.(2),

$$0 \leq x \leq \frac{L}{2}, \quad \frac{d^3}{d_0^3} = \frac{2x}{L}$$

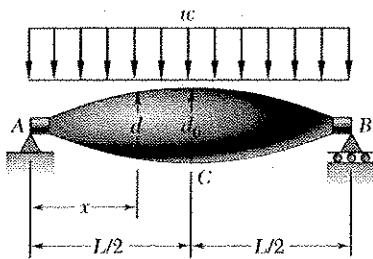
$$d = d_0 \left(\frac{2x}{L} \right)^{1/3} \quad \blacktriangleleft$$

Dividing Eq.(1b) by Eq.(2),

$$\frac{L}{2} \leq x \leq L, \quad \frac{d^3}{d_0^3} = \frac{2(L-x)}{L}$$

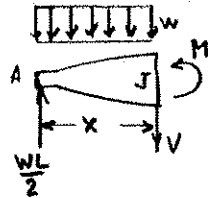
$$d = d_0 \left[\frac{2(L-x)}{L} \right]^{1/3} \quad \blacktriangleleft$$

Problem 5.137



5.136 and 5.137 A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x , L , and d_0 .

$$R_A = R_B = \frac{wL}{2}$$



$$+\circlearrowleft \sum M_J = 0:$$

$$-\frac{wL}{2}x + w x \frac{x}{2} + M = 0$$

$$M = \frac{w}{2} x (L - x)$$

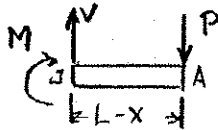
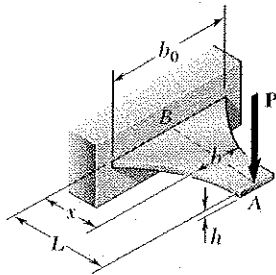
$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

For a solid circular cross section $c = \frac{d}{2}$ $I = \frac{\pi}{4} c^3$ $S = \frac{I}{c} = \frac{\pi d^3}{32}$

Equating, $\frac{\pi d^3}{32} = \frac{wx(L-x)}{2\sigma_{all}}$ $d = \left\{ \frac{16 wx(L-x)}{\pi \sigma_{all}} \right\}^{1/3}$

At $x = \frac{L}{2}$, $d = d_0 = \left\{ \frac{4 w L^2}{\pi \sigma_{all}} \right\}^{1/3}$ $d = d_0 \left\{ 4 \frac{x}{L} \left(1 - \frac{x}{L} \right) \right\}^{1/3}$

Problem 5.138



5.138 A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support the concentrated load P at point A . (a) Knowing that the beam is to be of constant strength, express b in terms of x , L , and b_0 . (b) Determine the smallest allowable value of h if $L = 300$ mm, $b_0 = 375$ mm, $P = 14.4$ kN, and $\sigma_{all} = 160$ MPa.

$$\circlearrowleft \sum M_J = 0: -M - P(L-x) = 0 \quad M = -P(L-x)$$

$$|M| = P(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P(L-x)}{\sigma_{all}}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

$$\text{Equating, } \frac{1}{6}bh^2 = \frac{P(L-x)}{\sigma_{all}} \quad b = \frac{6P(L-x)}{\sigma_{all}h^2}$$

$$\text{At } x=0, \quad b = b_0 = \frac{6PL}{\sigma_{all}h^2} \quad b = b_0\left(1 - \frac{x}{L}\right)$$

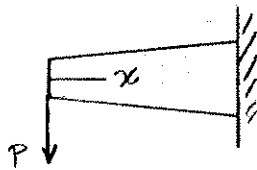
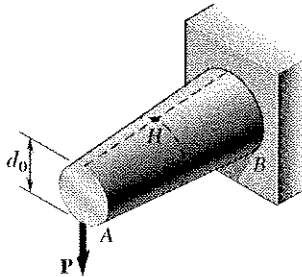
$$\text{Solving for } h, \quad h = \sqrt{\frac{6PL}{\sigma_{all}b_0}}$$

$$\text{Data: } L = 300 \text{ mm} = 0.300 \text{ m}, \quad b_0 = 375 \text{ mm} = 0.375 \text{ m}$$

$$P = 14.4 \text{ kN} = 14.4 \times 10^3 \text{ N} \cdot \text{m} \quad \sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$h = \sqrt{\frac{(6)(14.4 \times 10^3)(0.300)}{(160 \times 10^6)(0.375)}} = 20.8 \times 10^{-3} \text{ m} \quad h = 20.8 \text{ mm}$$

Problem 5.139



5.139 A transverse force P is applied as shown at end A of the conical taper AB . Denoting by d_0 the diameter of the taper at the A , show that the maximum normal stress occurs at point H , which is contained in a transverse section of diameter $d = 1.5 d_0$.

$$V = -P = \frac{dM}{dx} \quad M = -Px$$

$$\text{Let } d = d_0 + kx$$

$$\text{For a solid circular section, } I = \frac{\pi}{4} C^4 = \frac{\pi}{64} d^4$$

$$C = \frac{d}{2} \quad S = \frac{I}{C} = \frac{\pi}{32} d^3 = \frac{\pi}{32} (d_0 + kx)^3$$

$$\frac{dS}{dx} = \frac{3\pi}{32} (d_0 + kx)^2 k = \frac{3\pi}{32} d^2 k$$

$$\text{Stress } \sigma = \frac{|M|}{S} = \frac{Px}{S}$$

$$\text{At } H \quad \frac{d\sigma}{dx} = \frac{1}{S^2} (PS - Px_H \frac{dS}{dx}) = 0$$

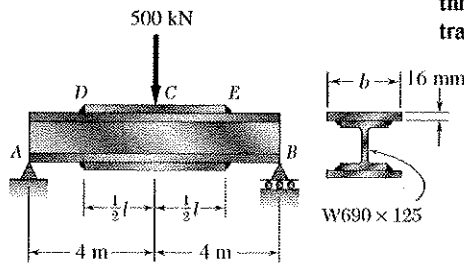
$$S - x_H \frac{dS}{dx} = \frac{\pi}{32} d^3 - x_H \frac{3\pi}{32} d^2 k$$

$$kx_H = \frac{1}{3} d = \frac{1}{3} (d_0 + kx_H) \quad kx_H = \frac{1}{2} d_0$$

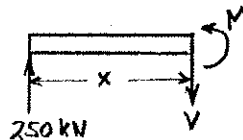
$$d = d_0 + \frac{1}{2} d_0 = \frac{3}{2} d_0 \quad d = 1.5 d_0 \quad \blacktriangle$$

Problem 5.140

5.140 Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4$ m and $b = 285$ mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



$$R_A = R_B = 250 \text{ kN} \uparrow$$



$$\sum M_J = 0:$$

$$-250x + M = 0$$

$$M = 250x \text{ kN}\cdot\text{m}$$

At center of beam, $x = 4$ m $M_c = (250)(4) = 1000 \text{ kN}\cdot\text{m}$

At D $x = \frac{1}{2}(8 - l) = \frac{1}{2}(8 - 4) = 2$ m $M_D = 500 \text{ kN}\cdot\text{m}$

At center of beam, $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 1190 \times 10^6 + 2 \left\{ (285)(16) \left(\frac{678}{2} + \frac{16}{2} \right)^2 + \frac{1}{12} (285)(16)^3 \right\}$$

$$= 2288 \times 10^6 \text{ mm}^4$$

$$c = \frac{678}{2} + 16 = 355 \text{ mm} \quad S = \frac{I}{c} = \frac{2288 \times 10^6}{355} = 6445 \times 10^3 \text{ mm}^3$$

$$= 6445 \times 10^{-6} \text{ m}^3$$

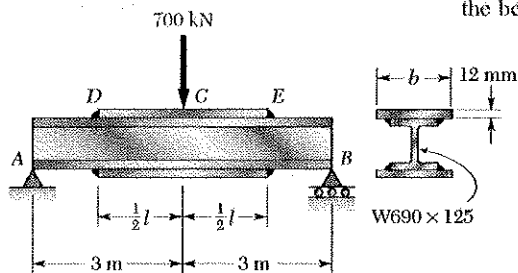
(a) Normal stress: $\sigma = \frac{M}{S} = \frac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6 \text{ Pa} = 155.2 \text{ MPa} \quad \blacktriangleleft$

At D, $S = 3510 \times 10^3 \text{ mm}^3 = 3510 \times 10^{-6} \text{ m}^3$

(b) Normal stress: $\sigma = \frac{M}{S} = \frac{500 \times 10^3}{3510 \times 10^{-6}} = 142.4 \times 10^6 \text{ Pa} = 142.4 \text{ MPa} \quad \blacktriangleleft$

Problem 5.141

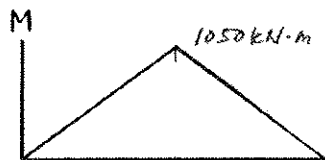
5.141 Two cover plates, each 12 mm thick, are welded to a W690 × 125 beam as shown. Knowing that $l = 3$ m and $b = 260$ mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



$$R_A = R_B = 350 \text{ kN}$$



$$\begin{aligned} \sum M_f &= 0 \\ -350x + M &= 0 \\ M &= 350x \text{ kNm} \end{aligned}$$



At C $x = 3$ m $M_C = 1050 \text{ kNm}$

At D $x = 1.5$ m $M_D = 525 \text{ kNm}$

At center of beam $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$\begin{aligned} I &= (1190 \times 10^6) + 2 \left\{ (12)(260) \left(6 + \frac{678}{2} \right)^2 + \frac{1}{12} (260)(12)^3 \right\} \\ &= 1932.8 \times 10^6 \text{ mm}^4 \end{aligned}$$

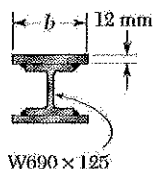
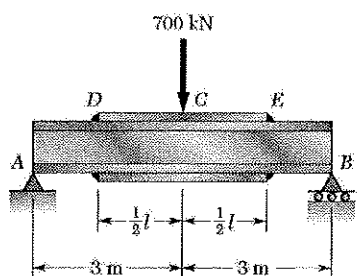
$$c = \left(\frac{678}{2} \right) + 12 = 351 \text{ mm}$$

(a) Normal stress $\sigma = \frac{Mc}{I} = \frac{(1050 \times 10^3)(0.351)}{1932.8 \times 10^6} = 190.7 \text{ MPa}$

At point D $S = 3510 \times 10^3 \text{ mm}^3 = 3510 \times 10^{-6} \text{ m}^3$

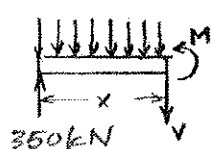
(b) Normal stress $\sigma = \frac{M}{S} = \frac{525 \times 10^3}{3510 \times 10^{-6}} = 149.6 \text{ MPa}$

Problem 5.142

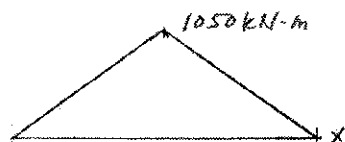


5.142 Two cover plates, each 12 mm thick, are welded to a W690 $\times$ 125 beam as shown. Knowing that $\sigma_{all} = 165 \text{ MPa}$ for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.

$$R_A = R_B = 350 \text{ kN.}$$



$$\begin{aligned} \sum M_J &= 0 \\ -350x + M &= 0 \\ M &= 350x \text{ kNm.} \end{aligned}$$



$$\text{At D} \quad S = 3510 \times 10^{-6} \text{ m}^3.$$

Allowable bending moment

$$\begin{aligned} M_{all} &= \sigma_{all} S = (165 \times 10^6) (3510 \times 10^{-6}) \\ &= 579.15 \text{ kNm.} \end{aligned}$$

$$\text{Set } M_D = M_{all}$$

$$350x_D = 579.15 \quad x_D = 1.66 \text{ m}$$

(a)

$$l = 6 - 2x_D = 2.68 \text{ m.}$$

At center of beam

$$M = (350)(3) = 1050 \text{ kNm}$$

$$S = \frac{M}{\sigma_{all}} = \frac{1050 \times 10^3}{165 \times 10^6} = 6.364 \times 10^{-3} \text{ m} = 6.364 \times 10^3 \text{ mm}^3$$

$$c = \left(\frac{678}{2} + 12 \right) = 351 \text{ mm.}$$

Required moment of inertia

$$I = Sc = 2233.8 \times 10^6 \text{ mm}^4.$$

$$\text{But } I = I_{beam} + 2I_{plate}$$

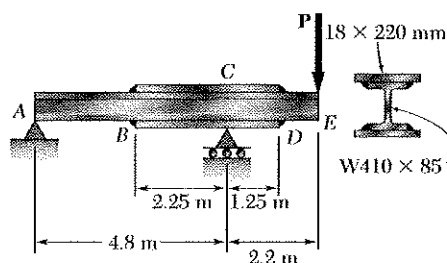
$$\begin{aligned} 2233.8 \times 10^6 &= (1190 \times 10^6) + 2 \left\{ (12)(b) \left(6 + \frac{678}{2} \right)^2 + \frac{1}{12} (b)(12)^3 \right\} \\ &= 1190 \times 10^6 + 2.857 \times 10^6 (b) \end{aligned}$$

(b)

$$b = 365.3 \text{ mm.}$$

Problem 5.143

5.143 Knowing that $\sigma_{all} = 150 \text{ MPa}$, determine the largest concentrated load P that can be applied at end E of the beam shown.



$$+\circlearrowleft \sum M_C = 0: -4.8A - 2.2P = 0$$

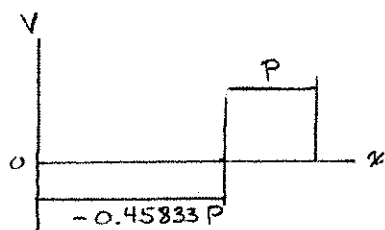
$$A = -0.45833P \quad A = 0.45833P \downarrow$$

$$+\circlearrowleft \sum M_A = 0: 4.8D - 7.0P = 0$$

$$D = 1.45833P \uparrow$$

$$\text{Shear: } A \text{ to } C, \quad V = -0.45833P$$

$$C \text{ to } E, \quad V = P$$



$$\text{Bending moments: } M_A = 0$$

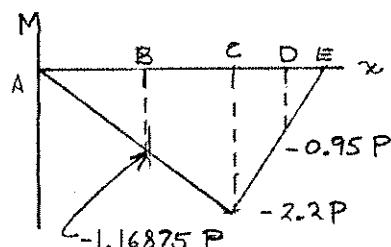
$$M_C = 0 + (4.8)(-0.45833P) = -2.2P$$

$$M_E = -2.2P + 2.2P = 0$$

$$M_B = \left(\frac{4.8 - 2.25}{4.8} \right)(-2.2P) = -1.16875P$$

$$M_D = \left(\frac{2.2 - 1.25}{2.2} \right)(-2.2P) = -0.95P$$

$$|M_D| < |M_B|$$



$$\text{For } W410 \times 85, \quad S = 1510 \times 10^3 \text{ mm}^3 = 1510 \times 10^{-6} \text{ m}^3$$

$$\text{Allowable value of } P \text{ based on strength at B. } \sigma = \frac{|M_B|}{S}$$

$$150 \times 10^6 = \frac{1.16875P}{1510 \times 10^{-6}} \quad P = 193.8 \times 10^3 \text{ N}$$

Section properties over portion BCD:

$$W410 \times 85: \quad d = 417 \text{ mm} \quad \frac{1}{2}d = 208.5 \text{ mm} \quad I_x = 315 \times 10^6 \text{ mm}^4$$

$$\text{Plate } A = (18)(220) = 3960 \text{ mm}^2 \quad d = 208.5 + \left(\frac{1}{2}\right)(18) = 217.5 \text{ mm}$$

$$= \frac{1}{12}(220)(18)^3 = 106.92 \times 10^3 \text{ mm}^4 \quad Ad^2 = 187.333 \times 10^6 \text{ mm}^4$$

$$= \bar{I} + Ad^2 = 187.440 \times 10^6 \text{ mm}^4$$

$$\text{For section: } I = 315 \times 10^6 + (2)(187.440 \times 10^6) = 689.88 \times 10^6 \text{ mm}^4$$

$$c = 208.5 + 18 = 226.5 \text{ mm}$$

$$S = \frac{I}{c} = \frac{689.88 \times 10^6}{226.5} = 3045.8 \times 10^3 \text{ mm}^3 = 3045.8 \times 10^{-6} \text{ m}^3$$

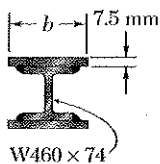
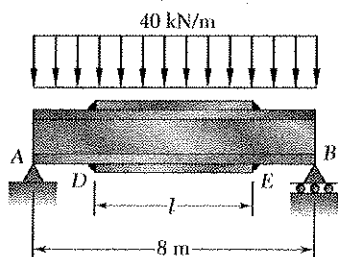
$$\text{Allowable load based on strength at C. } \sigma = \frac{|M_C|}{S}$$

$$150 \times 10^6 = \frac{2.2P}{3045.8 \times 10^{-6}} \quad P = 207.7 \times 10^3 \text{ N}$$

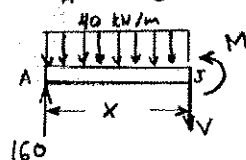
The smaller allowable load controls. $P = 193.8 \times 10^3 \text{ N} = 193.8 \text{ kN}$

Problem 5.144

5.144 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



$$R_A = R_B = 160 \text{ kN} \uparrow$$



$$+\circlearrowleft \sum M_J = 0:$$

$$-160x + (40x)\frac{x}{2} + M = 0$$

$$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$$

At center of beam $x = 4$ m $M_c = 320 \text{ kN}\cdot\text{m}$

At D $x = \frac{1}{2}(8-l) = 1.5$ m $M_D = 195 \text{ kN}\cdot\text{m}$

At center of beam

$$I = I_{\text{beam}} + 2I_{\text{plate}}$$

$$= 333 \times 10^6 + 2 \left\{ (200)(7.5) \left(\frac{457}{2} + \frac{7.5}{2} \right)^2 + \frac{1}{12} (200)(7.5)^3 \right\}$$

$$= 494.8 \times 10^6 \text{ mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$$

$$S = \frac{I}{c} = \frac{494.8 \times 10^6}{236} = 2097 \times 10^3 \text{ mm}^3$$

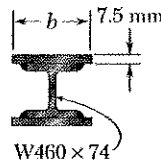
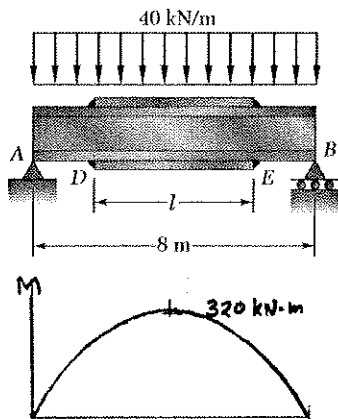
(a) Normal stress $\sigma = \frac{M}{S} = \frac{320 \times 10^3}{2097 \times 10^3} = 152.6 \times 10^6 \text{ Pa}$
 $\sigma = 152.6 \text{ MPa}$

At D $S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$

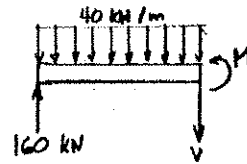
(b) Normal stress $\sigma = \frac{M}{S} = \frac{195 \times 10^3}{1460 \times 10^{-6}} = 133.6 \times 10^6 \text{ Pa}$
 $\sigma = 133.6 \text{ MPa}$

Problem 5.145

5.145 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $\sigma_{all} = 150 \text{ MPa}$ for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.



$$R_A = R_B = 160 \text{ kN} \uparrow$$



$$+\circlearrowleft \Sigma M_x = 0:$$

$$-160x + (40x)\left(\frac{x}{2}\right) + M = 0$$

$$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$$

For W 460 × 74 rolled steel beam

$$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$$

Allowable bending moment.

$$M_{all} = \sigma_{all} S = (150 \times 10^6)(1460 \times 10^{-6})$$

$$= 219 \times 10^3 \text{ N}\cdot\text{m} = 219 \text{ kN}\cdot\text{m}$$

To locate points D and E, set $M = M_{all}$.

$$160x - 20x^2 = 219$$

$$20x^2 - 160x + 219 = 0$$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} = \begin{matrix} 1.753 \text{ m} \\ 6.247 \text{ m} \end{matrix}$$

$$(a) \quad x_D = 1.753 \text{ m} \quad x_E = 6.247 \text{ m} \quad l = x_E - x_D = 4.49 \text{ m}$$

At center of beam, $M = 320 \text{ kN}\cdot\text{m} = 320 \times 10^3 \text{ N}\cdot\text{m}$

$$S = \frac{M}{\sigma_{all}} = \frac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6} \text{ m}^3 = 2133 \times 10^3 \text{ mm}^3$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$$

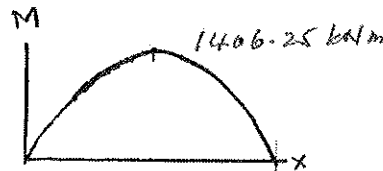
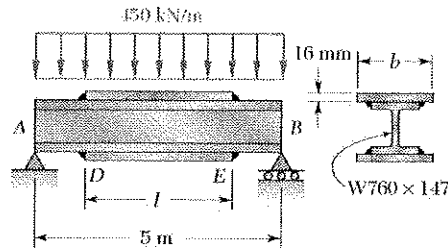
Required moment of inertia. $I = Sc = 503.4 \times 10^6 \text{ mm}^4$

But, $I = I_{beam} + 2I_{plate}$

$$\begin{aligned} 503.4 \times 10^6 &= 333 \times 10^6 + 2 \left\{ (b)(7.5) \left(\frac{457}{2} + \frac{7.5}{2} \right)^2 + \frac{1}{12} (b)(7.5)^3 \right\} \\ &= 333 \times 10^6 + 809.2 \times 10^3 b \end{aligned}$$

$$(b) \quad b = 211 \text{ mm}$$

Problem 5.146



5.146 Two cover plates, each 16 mm thick, are welded to a W760 $\times$ 147 beam as shown. Knowing that $l = 3$ m and $b = 300$ mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.

$$R_A = R_B = 1125 \text{ kN}$$

$$\sum M_J = 0$$

$$-1125x + 450x \frac{x}{2} + M = 0$$

$$M = 1125x - 225x^2 \text{ kNm}$$

$$\text{At center of beam } x = 2.5 \text{ m}$$

$$M_c = 1406.25 \text{ kNm}$$

$$\text{At point D, } x = \frac{1}{2}(5 - 3) = 1.0 \text{ m}$$

$$M_D = 900 \text{ kNm}$$

$$\text{At center of beam } I = I_{\text{beam}} + 2I_{\text{plate}}$$

$$I = (1660 \times 10^6) + 2 \left\{ (16)(300) \left(8 + \frac{755}{2} \right)^2 + \frac{1}{12} (300)(16)^3 \right\} = 3.07926 \times 10^9 \text{ mm}^4$$

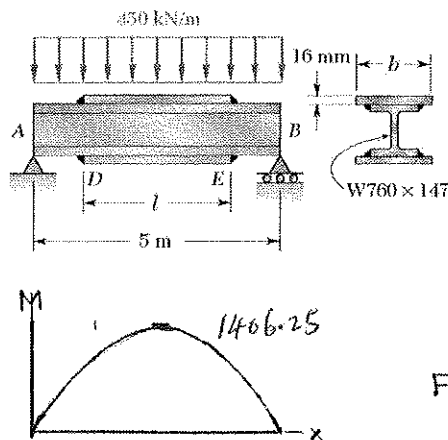
$$c = \left(\frac{755}{2} \right) + 16 = 392.5 \text{ mm}$$

$$(a) \text{ Normal stress } \sigma = \frac{Mc}{I} = \frac{(1406.25 \times 10^3)(0.3925)}{3.07926 \times 10^{-3}} = 179.2 \text{ MPa}$$

$$\text{At point D } S = 4410 \times 10^3 \text{ mm}^3$$

$$(b) \text{ Normal stress } \sigma = \frac{M}{S} = \frac{900 \times 10^3}{4410 \times 10^{-6}} = 204.1 \text{ MPa}$$

Problem 5.147



5.147 Two cover plates, each 16 mm thick, are welded to a W760 $\times$ 147 beam as shown. Knowing that $\sigma_{all} = 150$ MPa for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.

$$R_A = R_B = 1125 \text{ kN}$$

$$\begin{aligned} \sum M_A = 0 \\ -1125x + 450 \times \frac{x}{2} + M = 0 \\ M = 1125x - 225x^2 \text{ kNm} \end{aligned}$$

For W760 $\times$ 147 rolled steel section

$$S = 4410 \times 10^3 \text{ mm}^3$$

Allowable bending moment

$$\begin{aligned} M_{all} = \sigma_{all} S &= (150 \times 10^6)(4410 \times 10^{-6}) \\ &= 661.5 \text{ kNm} \end{aligned}$$

To locate points D and E, set $M = M_{all}$

$$1125x - 225x^2 = 661.5 \quad 225x^2 - 1125x + 661.5 = 0$$

$$x = \frac{1125 \pm [(1125)^2 - (4)(225)(661.5)]^{1/2}}{(2)(225)} = 4.32 \text{ m}, 0.68 \text{ m}$$

$$(a) \quad l = x_E - x_D = 4.32 - 0.68 = 3.64 \text{ m}$$

Center of beam $M = 1406.25 \text{ kNm}$

$$S = \frac{M}{\sigma_{all}} = \frac{1406.25 \times 10^3}{150 \times 10^6} = 9.375 \times 10^{-3} \text{ m}^3 = 9.375 \times 10^6 \text{ mm}^3$$

$$c = 16 + \frac{753}{2} = 392.5 \text{ mm}$$

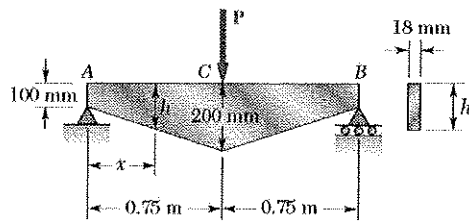
Required moment of inertia $I = Sc = 3679.7 \times 10^6 \text{ mm}^4$

But $I = I_{beam} + 2I_{plate}$

$$\begin{aligned} 3679.7 \times 10^6 &= (1660 \times 10^6) + 2 \left\{ (b)(16) \left(8 + \frac{753}{2} \right)^2 + \frac{1}{12} (b)(16)^3 \right\} \\ &= 1660 \times 10^6 + 4.7316 \times 10^6 b \end{aligned}$$

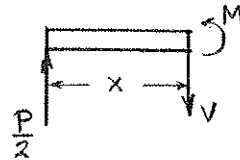
$$(b) \quad b = 426.9 \text{ mm}$$

Problem 5.148



5.148 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs, (b) the largest concentrated load P that can be applied, knowing that $\sigma_{all} = 165 \text{ MPa}$.

$$R_A = R_B = \frac{P}{2}$$



$$\sum M_f = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \quad (0 < x < \frac{L}{2})$$

For a tapered beam $h = a + kx$

For a rectangular cross section $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress $\sigma = \frac{M}{S} = \frac{3Px}{b(a + kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b} \frac{d}{dx} \left\{ \frac{x}{(a + kx)^2} \right\} = \frac{3P}{b} \frac{(a + kx)^2 - x \cdot 2(a + kx)k}{(a + kx)^4}$$

$$= \frac{3P}{b} \frac{a - kx}{(a + kx)^3} = 0 \quad x_m = \frac{a}{k}$$

Data: $a = 100 \text{ mm}$, $k = \frac{200 - 100}{750} = \frac{2}{15} \text{ mm/mm}$

$$(a) \quad x_m = \frac{100}{2/15} = 750 \text{ mm}$$

$$h_m = a + kx_m = 200 \text{ mm}$$

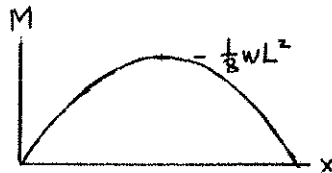
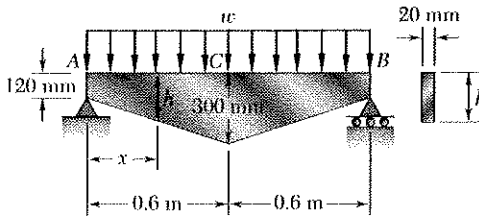
$$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)(18)(200)^2 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$$

$$M_m = \sigma_{all} S_m = (165 \times 10^6)(120 \times 10^{-6}) = 19.8 \text{ kNm}$$

$$(b) \quad P = \frac{2M_m}{x_m} = \frac{2(19.8)}{0.750} = 52.8 \text{ kN}$$

Problem 5.149

5.149 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs, (b) the largest distributed load w that can be applied, knowing that $\sigma_{all} = 140 \text{ MPa}$.



$$R_A = R_B = \frac{1}{2} wL \quad L = 1.2 \text{ m}$$

$$\begin{aligned} \sum M_J = 0: \\ -\frac{1}{2} wL + w \times \frac{x}{2} + M = 0 \\ M = \frac{w}{2} (Lx - x^2) \\ = \frac{w}{2} x (L - x) \end{aligned}$$

For the tapered beam, $h = a + kx$

$$a = 120 \text{ mm} \quad k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For rectangular cross section, $S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

$$\text{Bending stress: } \sigma = \frac{M}{S} = \frac{3w}{b} \frac{Lx - x^2}{(a + kx)^2}$$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$.

$$\begin{aligned} \frac{d\sigma}{dx} &= \frac{3w}{b} \frac{d}{dx} \left\{ \frac{Lx - x^2}{(a + kx)^2} \right\} = \frac{3w}{b} \left\{ \frac{(a + kx)^2 (L - 2x) - (Lx - x^2) 2(a + kx)k}{(a + kx)^3} \right\} \\ &= \frac{3w}{b} \left\{ \frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3} \right\} \\ &= \frac{3w}{b} \left\{ \frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a + kx)^3} \right\} \\ &= \frac{3w}{b} \left\{ \frac{aL - (2a + kL)x}{(a + kx)^3} \right\} = 0 \end{aligned}$$

$$(a) \quad x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24 \text{ m}$$

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

$$S_m = \frac{1}{6} b h_m^2 = \frac{1}{6} (20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$$

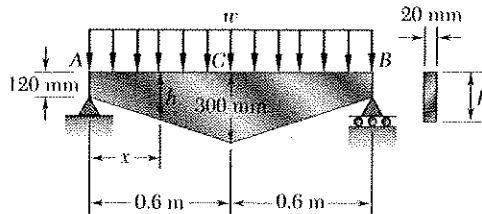
$$\text{Allowable value of } M_m: \quad M_m = S_m \sigma_{all} = (122.88 \times 10^{-6})(140 \times 10^6) \\ = 17.2032 \times 10^3 \text{ N}\cdot\text{m}$$

$$(b) \text{ Allowable value of } w: \quad w = \frac{2M_m}{x_m(L - x_m)} = \frac{(2)(17.2032 \times 10^3)}{(0.24)(0.96)} \\ = 149.3 \times 10^3 \text{ N/m}$$

$$w = 149.3 \text{ kN/m}$$

Problem 5.150

5.150 For the tapered beam shown, knowing that $w = 160 \text{ kN/m}$, determine (a) the transverse section in which the maximum normal stress occurs, (b) the corresponding value of the normal stress.



$$R_A = R_B = \frac{1}{2} wL \uparrow$$

$$\sum M_J = 0:$$

$$-\frac{1}{2} wLx + wx \frac{x}{2} + M = 0$$

$$M = \frac{w}{2} (Lx - x^2)$$

$$= \frac{w}{2} x (L - x)$$

where $w = 160 \text{ kN/m}$ and $L = 1.2 \text{ m}$.

For the tapered beam, $h = a + kx$

$$a = 120 \text{ mm} \quad k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For a rectangular cross section, $S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

$$\text{Bending stress: } \sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a + kx)^2}$$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b} \frac{d}{dx} \left\{ \frac{Lx - x^2}{(a + kx)^2} \right\} = \frac{3w}{b} \left\{ \frac{(a + kx)^2 (L - 2x) - (Lx - x^2) 2(a + kx)k}{(a + kx)^4} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a + kx)^3} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{aL - 2ax - kLx}{(a + kx)^3} \right\} = 0$$

$$(a) \quad x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)}$$

$$x_m = 0.240 \text{ m} \quad \blacktriangleleft$$

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

$$S_m = \frac{1}{6} b h_m^2 = \frac{1}{6} (20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$$

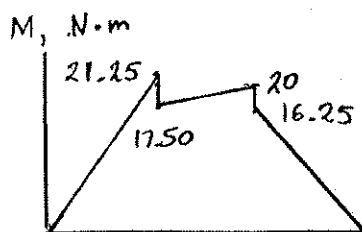
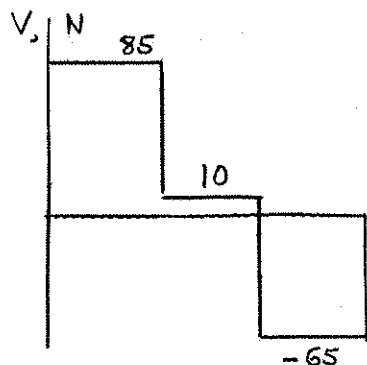
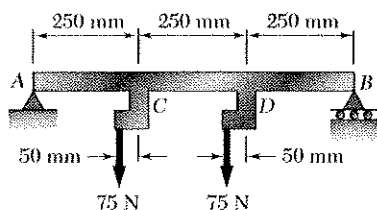
$$M_m = \frac{w}{2} x_m (L - x_m) = \frac{160 \times 10^3}{2} (0.24)(0.96) = 18.432 \times 10^3 \text{ N} \cdot \text{m}$$

$$(b) \text{ Maximum bending stress: } \sigma_m = \frac{M_m}{S_m} = \frac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6 \text{ Pa}$$

$$\sigma_m = 150.0 \text{ MPa} \quad \blacktriangleleft$$

Problem 5.151

5.151 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Reaction at A, $+\circlearrowleft M_B = 0$:

$$-0.750 R_A + (0.550)(75) + (0.300)(75) = 0$$

$$R_A = 85 \text{ N} \uparrow \quad \text{Also } R_B = 65 \text{ N} \uparrow$$

A to C, $V = 85 \text{ N}$

C to D, $V = 10 \text{ N}$

D to B, $V = -65 \text{ N}$

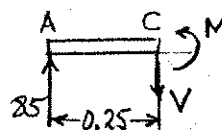
At A and B, $M = 0$

Just to the left of C,

$$+\circlearrowleft \Sigma M_C = 0$$

$$-(0.25)(85) + M = 0$$

$$M = 21.25 \text{ N}\cdot\text{m}$$

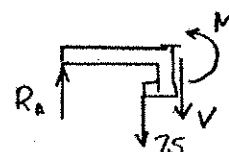


Just to the right of C,

$$+\circlearrowleft \Sigma M_C = 0$$

$$-(0.25)(85) + (0.050)(75) + M = 0$$

$$M = 17.50 \text{ N}\cdot\text{m}$$

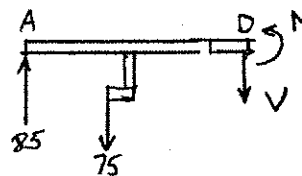


Just to the left of D,

$$+\circlearrowleft \Sigma M_D = 0$$

$$-(0.50)(85) + (0.300)(75) + M = 0$$

$$M = 20 \text{ N}\cdot\text{m}$$

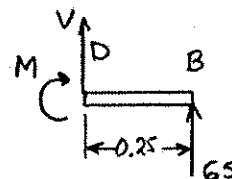


Just to the right of D,

$$+\circlearrowleft \Sigma M_D = 0$$

$$-M + (0.25)(65) = 0$$

$$M = 16.25 \text{ kN}$$

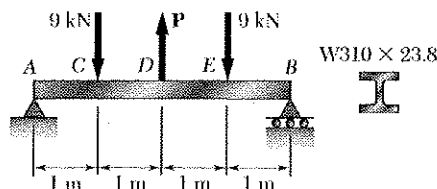


$$(a) \quad |V|_{\max} = 85.0 \text{ N}$$

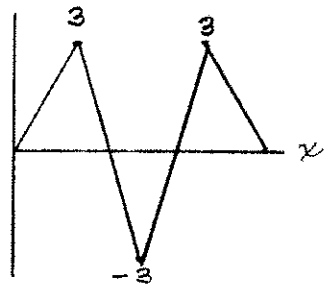
$$(b) \quad |M|_{\max} = 21.25 \text{ N}\cdot\text{m}$$

Problem 5.152

5.152 Determine (a) the magnitude of the upward force P for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (Hint: See hint of Prob. 5.27.)



$M, \text{ kN}\cdot\text{m}$



By symmetry the reactions at A and B are equal.

$$A = B$$

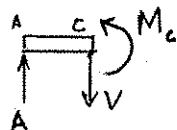
$$+\uparrow \sum F_y = 0: A - 9 + P - 9 + B = 0$$

$$A = B = 9 - \frac{1}{2}P$$

$$P = 18 - 2A$$

Also, by symmetry bending moments $M_C = M_E$.

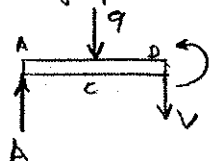
Using portion AC as a free body,



$$+\circlearrowleft \sum M_C = 0:$$

$$-1A + M_C = 0 \quad M_C = 1A$$

Using portion ACD as a free body,



$$+\circlearrowleft \sum M_D = 0:$$

$$-2A + (1)(9) + M_D = 0$$

$$M_D = 2A - 9$$

$$\text{Equate } M_C = -M_D$$

$$1A = 9 - 2A$$

$$A = 3 \text{ kN} \uparrow$$

$$\text{Then } P = 18 - (2)(3) = 12 \text{ kN}$$

$$(a) \quad P = 12.00 \text{ kN} \uparrow$$

$$M_C = 3 \text{ kN}\cdot\text{m}$$

$$M_D = (2)(3) - 9 = -3 \text{ kN}\cdot\text{m}$$

$$|M|_{\max} = 3 \text{ kN}\cdot\text{m} = 3 \times 10^3 \text{ N}\cdot\text{m}$$

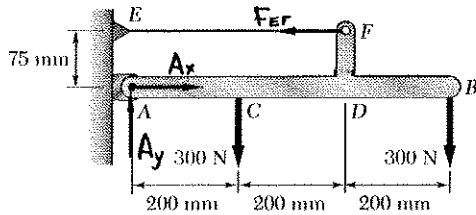
For $W310 \times 23.4$ rolled steel section, $S = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{ m}^3$

$$\text{Normal stress: } \sigma = \frac{M}{S} = \frac{3 \times 10^3}{280 \times 10^{-6}} = 10.71 \times 10^6 \text{ Pa}$$

$$(b) \quad \sigma = 10.71 \text{ MPa}$$

Problem 5.153

5.153 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



$$\uparrow \Sigma M_A = 0:$$

$$0.075 F_{EF} - (0.2)(300) - (0.6)(300) = 0$$

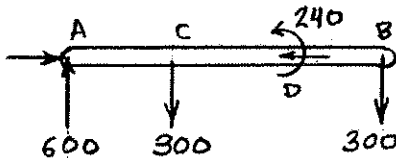
$$F_{EF} = 3.2 \times 10^3 \text{ N}$$

$$\rightarrow \Sigma F_x = 0: A_x - F_{EF} = 0 \quad A_x = 3.2 \times 10^3 \text{ N}$$

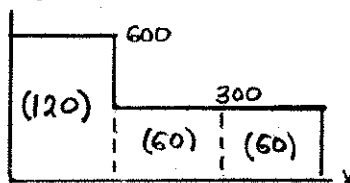
$$\uparrow \Sigma F_y = 0: A_y - 300 - 300 = 0$$

$$A_y = 600 \text{ N}$$

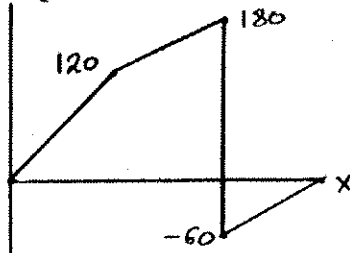
$$\text{Couple at D, } M_D = (0.075)(3.2 \times 10^3) = 240 \text{ N}\cdot\text{m}$$



$V \text{ (N)}$



$M \text{ (N}\cdot\text{m)}$



Shear diagram.

$$A \text{ to } C. \quad V = 600 \text{ N}$$

$$C \text{ to } B. \quad V = 600 - 300 = 300 \text{ N}$$

Areas of the shear diagram.

$$A \text{ to } C. \quad \int V dx = (0.2)(600) = 120 \text{ N}\cdot\text{m}$$

$$C \text{ to } D. \quad \int V dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$$

$$D \text{ to } B. \quad \int V dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$$

Bending moments.

$$M_A = 0$$

$$M_C = 0 + 120 = 120 \text{ N}\cdot\text{m}$$

$$M_D^- = 120 + 60 = 180 \text{ N}\cdot\text{m}$$

$$M_D^+ = 180 - 240 = -60 \text{ N}\cdot\text{m}$$

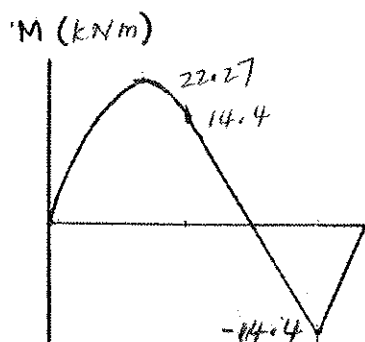
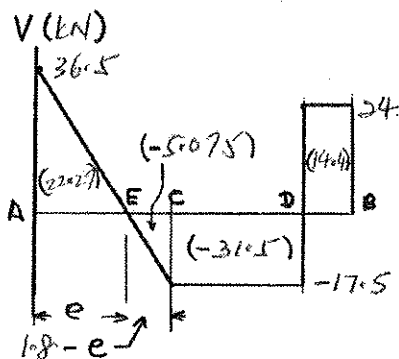
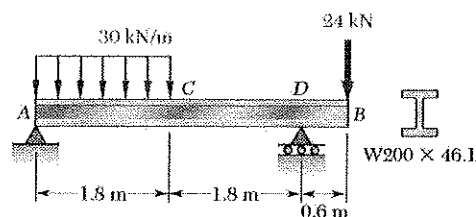
$$M_B = -60 + 60 = 0$$

$$(a) \quad \text{Maximum } |V| = 600 \text{ N}$$

$$(b) \quad \text{Maximum } |M| = 180 \text{ N}\cdot\text{m}$$

Problem 5.154

5.154 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



$$+\circlearrowleft M_D = 0$$

$$-3.6A + (30)(1.8)(2.7) - (24)(0.6) = 0$$

$$A = 36.5 \text{ kN.}$$

$$+\circlearrowleft M_A = 0$$

$$-(30)(1.8)(0.9) + 3.6D - (24)(4.2) = 0$$

$$D = 41.5.$$

Shear: $V_A = 36.5 \text{ kN.}$

$$V_C = 36.5 - (30)(1.8) = -17.5 \text{ kN}$$

$$C \text{ to } D \quad V = -17.5 \text{ kN.}$$

$$D \text{ to } B \quad V = -17.5 + 41.5 = 24 \text{ kN}$$

Locate point E where $V = 0$

$$\frac{e}{36.5} = \frac{1.8 - e}{17.5} \quad 54e = 65.7$$

$$e = 1.22 \quad 1.8 - e = 0.58 \text{ m.}$$

Areas under shear diagram

$$A \text{ to } E \quad \int V dx = \left(\frac{1}{2}\right)(1.22)(36.5) = 22.27 \text{ kN}\cdot\text{m.}$$

$$E \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(0.58)(17.5) = -5.075 \text{ kN}\cdot\text{m.}$$

$$C \text{ to } D \quad \int V dx = -(17.5)(1.8) = -31.5 \text{ kN}\cdot\text{m.}$$

$$D \text{ to } B \quad \int V dx = (24)(0.6) = 14.4 \text{ kN}\cdot\text{m.}$$

Bending moments: $M_A = 0$

$$M_E = 0 + 22.27 = 22.27 \text{ kN}\cdot\text{m.}$$

$$M_C = 22.27 - 5.075 = 17.195 \text{ kN}\cdot\text{m.}$$

$$M_D = 17.195 - 31.5 = -14.31 \text{ kN}\cdot\text{m.}$$

$$M_B = -14.31 + 14.4 = 0$$

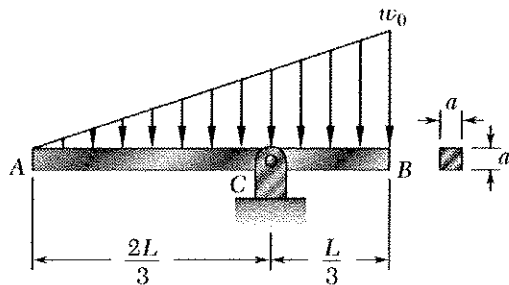
$$\text{Maximum } |M| = 22.27 \text{ kN}\cdot\text{m.}$$

$$\text{For } W200 \times 46.1 \text{ rolled steel section } S = 448 \times 10^3 \text{ mm}^3.$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{22.27 \times 10^3}{448 \times 10^3} = 49.7 \text{ MPa.}$$

Problem 5.155

5.155 Beam AB , of length L and square cross section of side a , is supported by a pivot at C and loaded as shown. (a) Check that the beam is in equilibrium. (b) Show that the maximum stress due to bending occurs at C and is equal to $w_0 L^2 / (1.5a)^3$.



Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

For the triangular distribution the centroid lies at $x = \frac{2L}{3}$.

$$W = \frac{1}{2} w_0 L$$

$$(a) +\uparrow \Sigma F_y = 0: R_0 - W = 0 \quad R_0 = \frac{1}{2} w_0 L$$

$$\circlearrowleft \Sigma M_c = 0: 0 = 0 \quad \text{equilibrium}$$

$$V = 0, M = 0, \text{ at } x = 0$$

$$0 < x < \frac{2L}{3},$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L} + C_1 = -\frac{w_0 x^2}{2L}$$

$$M = -\frac{w_0 x^3}{6L} + C_2 = -\frac{w_0 x^3}{6L}$$

Just to the left of C ,

$$V = -\frac{w_0 (2L/3)^2}{2L} = -\frac{2}{9} w_0 L$$

Just to the right of C ,

$$V = -\frac{2}{9} w_0 L + R_0 = \frac{5}{18} w_0 L$$

Note sign change. Maximum $|M|$ occurs at C .

$$M_c = -\frac{w_0 (2L/3)^3}{6L} = -\frac{4}{81} w_0 L^2$$

$$\text{Maximum } |M| = \frac{4}{81} w_0 L^2$$

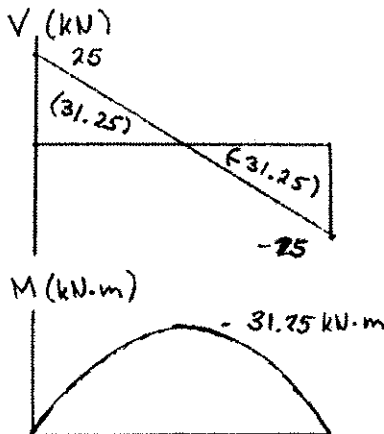
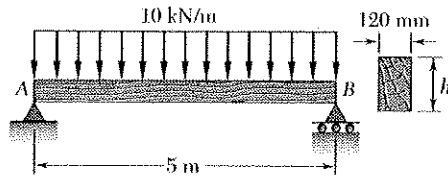
$$\text{For square cross section, } I = \frac{1}{12} a^4 \quad c = \frac{1}{2} a$$

$$(b) \sigma_m = \frac{|M|_{\max} c}{I} = \frac{\frac{4}{81} w_0 L^2 \cdot \frac{a}{2}}{\frac{1}{12} a^4} = \frac{8}{27} \frac{w_0 L^2}{a^3} = \left(\frac{2}{3}\right)^3 \frac{w_0 L^2}{a^3}$$

$$\sigma_m = \frac{w_0 L^2}{(1.5a)^3}$$

Problem 5.156

5.156 and 5.157 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



Reactions: $A = B$ by symmetry

$$+\uparrow \sum F_y = 0: A + B - (5)(10) = 0$$

$$A = B = 25 \text{ kN}$$

From bending moment diagram,

$$|M|_{\max} = 31.25 \text{ kN}\cdot\text{m} = 31.25 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{31.25 \times 10^3}{12 \times 10^6} = 2.604 \times 10^{-3} \text{ m}^3$$

$$= 2.604 \times 10^6 \text{ mm}^3$$

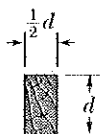
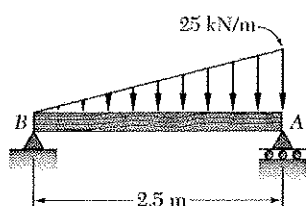
$$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(120) h^2 = 2.604 \times 10^6$$

$$h^2 = \frac{(6)(2.604 \times 10^6)}{120} = 130.21 \times 10^3 \text{ mm}^2$$

$$h = 361 \text{ mm} \quad \blacktriangleleft$$

Problem 5.157

5.156 and 5.157 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



$$\text{Distributed load } w = \frac{25}{2.5} x = 10x \text{ kN/m}$$

$$\frac{dV}{dx} = -w = -10x$$

$$V = -5x^2 + C_1 = \frac{dM}{dx}$$

$$M = -\frac{5}{3}x^3 + C_1x + C_2$$

$$M = 0 \text{ at } x = 0.$$

$$C_2 = 0$$

$$M = 0 \text{ at } x = 2.5 \text{ m.}$$

$$-\frac{5}{3}(2.5)^3 + C_1(2.5) = 0$$

$$C_1 = 10.417 \text{ kN}$$

$$V = -5x^2 + 10.417$$

$$V = 0 \quad -5x^2 + 10.417 = 0 \quad x = 1.4434 \text{ m}$$

$$M_{max} = -\frac{5}{3}(1.4434)^3 + (10.417)(1.4434)$$

$$= 10.023 \text{ kN}\cdot\text{m} = 10.023 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Required } S = \frac{M_{max}}{\sigma_{all}} = \frac{10.023 \times 10^3}{12 \times 10^6} = 835.29 \times 10^{-6} \text{ m}^3$$

$$= 835.29 \times 10^3 \text{ mm}^3$$

$$\text{For the rectangular section, } I = \frac{1}{12}(\frac{1}{2}d)(d)^3$$

$$c = \frac{1}{2}d \quad S = \frac{I}{c} = \frac{d^3}{12}$$

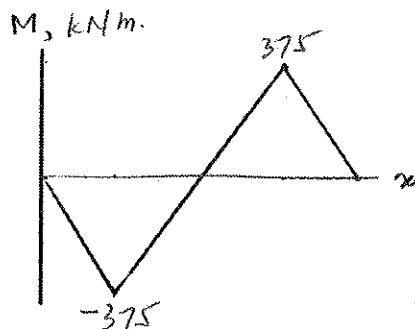
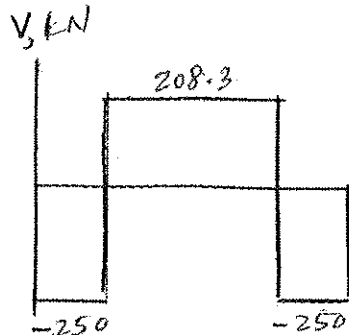
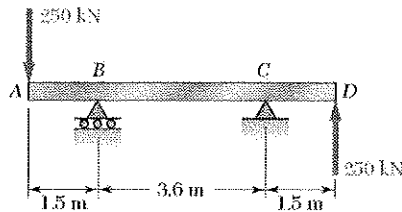
Equating expressions for S,

$$\frac{d^3}{12} = 835.29 \times 10^3$$

$$d = 216 \text{ mm} \quad \blacktriangleleft$$

Problem 5.158

5.158 Knowing that the allowable stress for the steel used is 165 MPa, select the most economical wide-flange beam to support the loading shown.



$$\sum M_C = 0$$

$$(250)(5.1) - 3.6B + (1.5)(250) = 0$$

$$B = 458.3 \text{ kN} \quad \uparrow$$

$$\sum M_B = 0$$

$$(250)(1.5) + 3.6C + (250)(5.1) = 0$$

$$C = -458.3 \text{ kN} \quad \text{or} \quad C = 458.3 \text{ kN} \quad \downarrow$$

Shear diagram.

$$A \text{ to } B^- \quad V = -250 \text{ kN}$$

$$B^+ \text{ to } C^- \quad V = -250 + 458.3 = 208.3 \text{ kN}$$

$$C^+ \text{ to } D \quad V = 208.3 - 458.3 = -250 \text{ kN}$$

Areas of shear diagram.

$$A \text{ to } B \quad (1.5)(-250) = -375 \text{ kNm}$$

$$B \text{ to } C \quad (3.6)(208.3) = 749.88 \text{ kNm}$$

$$C \text{ to } D \quad (1.5)(-250) = -375 \text{ kNm}$$

Bending moments.

$$M_A = 0$$

$$M_B = 0 - 375 = -375 \text{ kNm}$$

$$M_C = -375 + 749.88 = 374.88 \text{ kNm}$$

$$M_D = 374.88 - 375 \approx 0$$

$$|M|_{\max} = 375 \text{ kNm}$$

Required $S_{\min}$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{375 \times 10^3}{165 \times 10^6} = 2272.7 \times 10^{-6} \text{ m}^3 = 2273 \times 10^{-6} \text{ m}^3$$

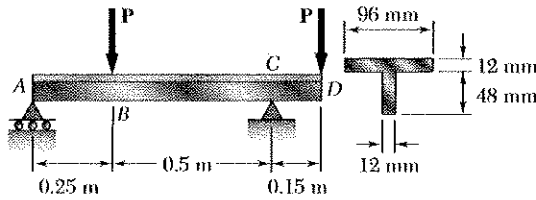
Shape	S (mm ³ × 10 ³)
W690 × 125	3510
W610 × 101	2530 ←
W530 × 150	3720
W460 × 113	2400
W360 × 216	3800

Lightest wide flange beam

$$W610 \times 101 @ 101 \text{ kg/m}$$

Problem 5.159

5.159 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is $+80$ MPa in tension and -140 MPa in compression.

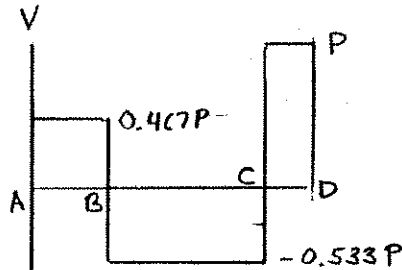


$$\sum M_C = 0: -0.75A + 0.5P - 0.15P = 0$$

$$A = 0.46667P$$

$$\sum M_A = 0: 0.75C - 0.25P - 0.9P = 0$$

$$C = 1.53333P$$



Shear: A to B. $V = 0.46667P$

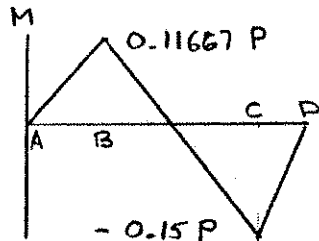
B to C. $V = 0.46667P - P = -0.53333P$

C to D. $V = -0.53333P + 1.53333P = P$

Areas: A to B. $(0.25)(0.46667P) = 0.11667P$

B to C. $(0.5)(-0.53333P) = -0.26667P$

C to D. $(0.15)P = 0.15P$



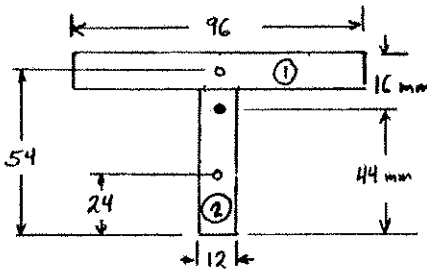
Bending moments: $M_A = 0$

$M_B = 0 + 0.11667P = 0.11667P$

$M_C = 0.11667P - 0.26667P = -0.15P$

$M_D = -0.15P + 0.15P = 0$

Centroid and moment of inertia.



Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	1152	54	62208	10	115200	13824
②	576	24	13824	20	230400	110592
Σ	1728		76032		345600	124416

$$\bar{Y} = \frac{76032}{1728} = 44 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 470016 \text{ mm}^4$$

Top: $y = 16 \text{ mm}$ $\frac{I}{y} = \frac{470016}{16} = 29.376 \times 10^3 \text{ mm}^3 = 29.376 \times 10^{-6} \text{ m}^3$

Bottom: $y = -44 \text{ mm}$ $\frac{I}{y} = \frac{470016}{44} = 10.682 \times 10^3 \text{ mm}^3 = -10.682 \times 10^{-6} \text{ m}^3$

Bending moment limits: $M = -\frac{I\sigma}{y}$

Tension at B. $= (-10.682 \times 10^{-6})(80 \times 10^6) = 854.56 \text{ N}\cdot\text{m}$

$\triangleleft B$

Comp. at B. $-(29.376 \times 10^{-6})(-140 \times 10^6) = 4.1126 \times 10^3 \text{ N}\cdot\text{m}$

Tension at C. $-(29.376 \times 10^{-6})(80 \times 10^6) = -2.35 \times 10^3 \text{ N}\cdot\text{m}$

Comp. at C. $-(-10.682 \times 10^{-6})(-140 \times 10^6) = -1.4955 \times 10^3 \text{ N}\cdot\text{m}$

$\triangleleft C$

Allowable load: $0.11667P = 854.56$

$P = 7.32 \times 10^3 \text{ N}$

$-0.15P = -1.4955 \times 10^3$

$P = 9.97 \times 10^3 \text{ N}$

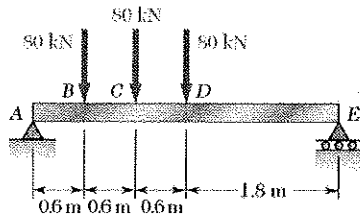
The smaller value is

$P = 7.32 \text{ kN}$

Problem 5.160

5.160

- (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown.
(b) Determine the maximum value of the bending moment in the beam.



$$+\sum M_E = 0 \quad -3.6A + (3)(80) + (2.4)(80) + (1.8)(80) = 0$$

$$A = 160 \text{ kN.}$$

$$(a) V = 160 - 80\langle x - 0.6 \rangle^0 - 80\langle x - 1.2 \rangle^0 - 80\langle x - 1.8 \rangle^0 \text{ kN}$$

$$M = 160x - 80\langle x - 0.6 \rangle^1 - 80\langle x - 1.2 \rangle^1 - 80\langle x - 1.8 \rangle^1 \text{ kNm.}$$

Values of V

A to B $V = 160 \text{ kN}$

B to C $V = 160 - 80 = 80 \text{ kN}$

C to D $V = 160 - 80 - 80 = 0$

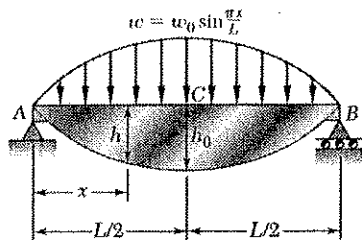
D to E $V = 160 - 80 - 80 - 80 = -80 \text{ kN}$

Bending moment is constant and maximum over C to D.

$$(b) \text{ At C } \quad x = 1.2 \text{ m} \quad M = (160)(1.2) - (80)(0.6) - 0 - 0 = 144 \text{ kNm}$$

Problem 5.161

5.161 The beam AB , consisting of an aluminum plate of uniform thickness b and length L , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x , L , and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.



$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$\text{At } A, \quad x = 0 \quad M = 0$$

$$0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$\text{At } B, \quad x = L \quad M = 0$$

$$0 = \frac{w_0 L^2}{\pi^2} \sin \pi + C_1 L \quad C_1 = 0$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

$$\text{For constant strength} \quad S = \frac{|M|}{\sigma_{all}} = \frac{w_0 L^2}{\pi^2 \sigma_{all}} \sin \frac{\pi x}{L}$$

$$\text{For a rectangular section} \quad I = \frac{1}{12} b h^3, \quad c = \frac{h}{2}, \quad S = \frac{I}{c} = \frac{1}{6} b h^2$$

$$\text{Equating the two expressions for } S \quad \frac{1}{6} b h^2 = \frac{w_0 L^2}{\pi^2 \sigma_{all}} \sin \frac{\pi x}{L} \quad (1)$$

$$\text{At } x = \frac{L}{2} \quad h = h_0 \quad \frac{1}{6} b h_0^2 = \frac{w_0 L^2}{\pi^2 \sigma_{all}} \quad (2)$$

$$(a) \text{ Dividing Eq. (1) by Eq. 2} \quad \frac{h^2}{h_0^2} = \sin \frac{\pi x}{L} \quad h = h_0 \left(\sin \frac{\pi x}{L} \right)^{1/2}$$

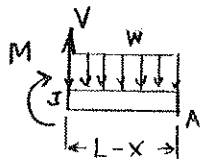
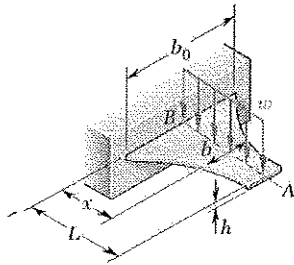
$$(b) \text{ Solving Eq. (1) for } w_0 \quad w_0 = \frac{\pi^2 \sigma_{all} b h_0^2}{6 L^2}$$

$$\text{Data: } \sigma_{all} = 72 \times 10^6 \text{ Pa}, \quad L = 800 \text{ mm} = 0.800 \text{ m}, \quad h_0 = 200 \text{ mm} = 0.200 \text{ m}, \\ b = 25 \text{ mm} = 0.025 \text{ m}$$

$$w_0 = \frac{\pi^2 (72 \times 10^6) (0.025) (0.200)^2}{(6) (0.800)^2} = 185.1 \times 10^3 \text{ N/m}$$

$$185.1 \text{ kN/m}$$

Problem 5.162



5.162 A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support the distributed load w along its center line AB . (a) Knowing that the beam is to be of constant strength, express b in terms of x , L , and b_0 . (b) Determine the maximum allowable value of w if $L = 0.4$ m, $b_0 = 200$ mm, $h = 20$ mm, and $\sigma_{all} = 165$ MPa.

$$+\circlearrowleft \sum M_J = 0 \quad -M - w(L-x) \frac{L-x}{2} = 0$$

$$M = -\frac{w(L-x)^2}{2} \quad |M| = \frac{w(L-x)^2}{2}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w(L-x)^2}{2\sigma_{all}}$$

For a rectangular cross section $S = \frac{1}{6}bh^2$

$$\frac{1}{6}bh^2 = \frac{w(L-x)^2}{2\sigma_{all}}$$

$$b = \frac{3w(L-x)^2}{\sigma_{all}h^2}$$

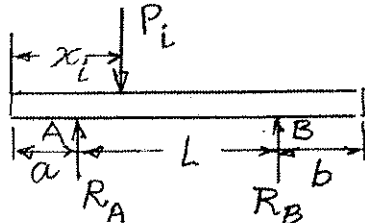
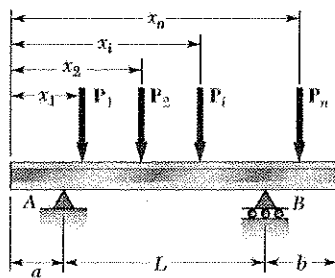
At $x = 0 \quad b = b_0 = \frac{3wL^2}{\sigma_{all}h^2}$

$$b = b_0 \left(1 - \frac{x}{L}\right)^2$$

Solving for $w \quad w = \frac{\sigma_{all}b_0h^2}{3L^2} = \frac{(165 \times 10^6)(0.2)(0.02)^2}{3(0.4)^2}$

$$= 27.5 \text{ kN/m}$$

PROBLEM 5.C1



5.C1 Several concentrated loads $P_i (i = 1, 2, \dots, n)$ can be applied to a beam as shown. Write a computer program that can be used to calculate the shear, bending moment, and normal stress at any point of the beam for a given loading of the beam and a given value of its section modulus. Use this program to solve Probs. 5.18, 5.21, and 5.25. (Hint: Maximum values will occur at a support or under a load.)

SOLUTION

REACTIONS AT A AND B

$$+\circlearrowleft \sum M_A = 0: R_B L - \sum_i P_i (x_i - a)$$

$$R_B = (1/L) \sum_i P_i (x_i - a)$$

$$R_A = \sum_i P_i - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text.)

WE DEFINE: IF $x \geq a$ THEN $STPA = 1$ ELSE $STPA = 0$

IF $x \geq a + L$ THEN $STPB = 1$ ELSE $STPB = 0$

IF $x \geq x_i$ THEN $STP(I) = 1$ ELSE $STP(I) = 0$

$$V = R_A STPA + R_B STPB - \sum_i P_i STP(I)$$

$$M = R_A (x - a) STPA + R_B (x - a - L) STPB - \sum_i P_i (x - x_i) STP(I)$$

$\sigma = M/S$, where S is obtained from Appendix C.

PROGRAM OUTPUTS

Problem 5.18

$R_A = 80.0 \text{ kN}$ $R_B = 80.0 \text{ kN}$

X m	V kN	M kN.m	Sigma MPa
2.00	0.00	104.00	139.0

Problem 5.25

$R_1 = 42.08$

$R_2 = 16.92 \text{ kN}$

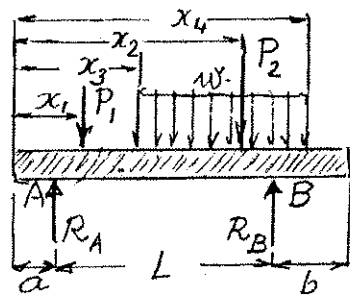
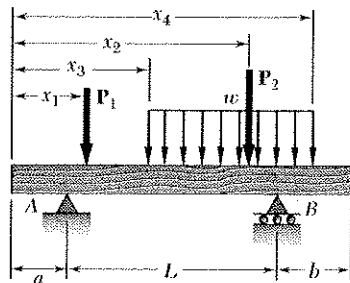
x m	V kN	M kN.m	Sigma MPa
0	-20.02	0	0
1.5	23.10	-30	-63.29
3.9	-16.94	25.38	53.56
5.4	-16.94	0	0

Problem 5.21

$R_1 = 210 \text{ kN}$

$R_2 = 90 \text{ kN}$

x m	V kN	M kN.m	Sigma MPa
0	-100	0	0
0.3	110	-38.31	-48
0.9	10	46	57.6
2.7	-90	54	86.4
3.3	0	0	0

PROBLEM 5.C2


5.C2 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value σ_{all} . Write a computer program that can be used to calculate at given intervals ΔL the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals ΔL indicated: (a) Prob. 5.65 ($\Delta L = 0.1$ m), (b) Prob. 5.69 ($\Delta L = 0.3$ m), (c) Prob. 5.70 ($\Delta L = 0.2$ m).

SOLUTION
REACTIONS AT A AND B

$$+\circlearrowleft \sum M_A = 0: R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3) \left(\frac{x_4 + x_3}{2} - a \right) = 0$$

$$R_B = \frac{1}{L} [P_1(x_1 - a) + P_2(x_2 - a) + \frac{1}{2} w(x_4 - x_3)(x_4 + x_3 - 2a)]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

$$\text{SET } n = (a + b + L) / \Delta L$$

$$\text{FOR } i = 0 \text{ TO } n: x = (i \Delta L)$$

$$\text{WE DEFINE: IF } x \geq a \text{ THEN STPA} = 1 \text{ ELSE STPA} = 0$$

$$\text{IF } x \geq a + L, \text{ THEN STPB} = 1 \text{ ELSE, STB} = 0$$

$$\text{IF } x \geq x_1, \text{ THEN STP1} = 1 \text{ ELSE STP1} = 0$$

$$\text{IF } x \geq x_2, \text{ THEN STP2} = 1 \text{ ELSE STP2} = 0$$

$$\text{IF } x \geq x_3, \text{ THEN STP3} = 1 \text{ ELSE STP3} = 0$$

$$\text{IF } x \geq x_4, \text{ THEN STP4} = 1 \text{ ELSE STP4} = 0$$

$$V = R_A \text{STPA} + R_B \text{STPB} - P_1 \text{STP1} - P_2 \text{STP2} - w(x - x_3) \text{STP3} + w(x - x_4) \text{STP4}$$

$$M = R_A(x - a) \text{STPA} + R_B(x - a - L) \text{STPB} - P_1(x - x_1) \text{STP1} - P_2(x - x_2) \text{STP2} - \frac{1}{2} w(x - x_3)^2 \text{STP3} + \frac{1}{2} w(x - x_4)^2 \text{STP4}$$

$$S_{min} = |M| / \sigma_{all}$$

IF UNKNOWN DIMENSION IS h:

$$\text{From } S = \frac{1}{6} t h^2, \text{ we have } h = \sqrt{6S/t}$$

IF UNKNOWN DIMENSION IS t:

$$\text{From } S = \frac{1}{6} t h^2, \text{ we have } t = 6S/h^2$$

(CONTINUED)

PROBLEM 5.C2 CONTINUED

PROGRAM OUTPUTS

Problem 5.65

RA =	2.40 kN	RB =	3.00 kN
X	V	M	H
m	kN	kN.m	mm
0.00	2.40	0.000	0.00
0.10	2.40	0.240	54.77
0.20	2.40	0.480	77.46
0.30	2.40	0.720	94.87
0.40	2.40	0.960	109.54
0.50	2.40	1.200	122.47
0.60	2.40	1.440	134.16
0.70	2.40	1.680	144.91
0.80	0.60	1.920	154.92
0.90	0.60	1.980	157.32
1.00	0.60	2.040	159.69
1.10	0.60	2.100	162.02
1.20	0.60	2.160	164.32
1.30	0.60	2.220	166.58
1.40	0.60	2.280	168.82
1.50	0.60	2.340	171.03
1.60	-3.00	2.400	173.21
1.70	-3.00	2.100	162.02
1.80	-3.00	1.800	150.00
1.90	-3.00	1.500	136.93
2.00	-3.00	1.200	122.47
2.10	-3.00	0.900	106.07
2.20	-3.00	0.600	86.60
2.30	-3.00	0.300	61.24
2.40	0.00	0.000	0.05

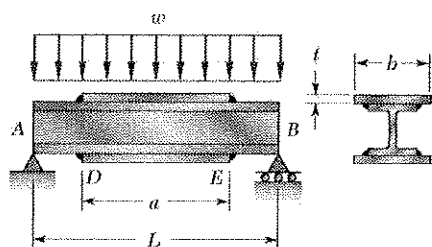
Problem 5.69

RA =	6.50 kN	RB =	6.50 kN
X	V	M	H
m	kN	kN.m	mm
0.00	2.50	0.000	0.00
0.30	2.50	0.750	61.24
0.60	9.00	1.500	86.60
0.90	7.20	3.930	140.18
1.20	5.40	5.820	170.59
1.50	3.60	7.170	189.34
1.80	1.80	7.980	199.75
2.10	-0.00	8.250	203.10
2.40	-1.80	7.980	199.75
2.70	-3.60	7.170	189.34
3.00	-5.40	5.820	170.59
3.30	-7.20	3.930	140.18
3.60	-2.50	1.500	86.60
3.90	-2.50	0.750	61.24
4.20	0.00	0.000	0.06

Problem 5.70

RA =	2.70 kN	RB =	8.10 kN
X	V	M	T
m	kN	kN.m	mm
0.00	2.70	0.000	0.00
0.20	2.10	0.480	10.67
0.40	1.50	0.840	18.67
0.60	0.90	1.080	24.00
0.80	0.30	1.200	26.67
1.00	-0.30	1.200	26.67
1.20	-0.90	1.080	24.00
1.40	-1.50	0.840	18.67
1.60	-2.10	0.480	10.67
1.80	-2.70	0.000	0.00
2.00	-3.30	-0.600	13.33
2.20	-3.90	-1.320	29.33
2.40	3.60	-2.160	48.00
2.60	3.00	-1.500	33.33
2.80	2.40	-0.960	21.33
3.00	1.80	-0.540	12.00
3.20	1.20	-0.240	5.33
3.40	0.60	-0.060	1.33
3.60	0.00	-0.000	0.00

PROBLEM 5.C3



5.C3 Two cover plates, each of thickness t , are to be welded to a wide-flange beam of length L , which is to support a uniformly distributed load w . Denoting by σ_{all} the allowable normal stress in the beam and in the plates, by d the depth of the beam, and by I_b and S_b , respectively, the moment of inertia and the section modulus of the cross section of the unreinforced beam about a horizontal centroidal axis, write a computer program that can be used to calculate the required value of (a) the length a of the plates, (b) the width b of the plates. Use this program to solve Prob. 5.145.

SOLUTION

(a) Required Length of Plates

$FB = AD: \sum M_D = 0: M_D + w x \left(\frac{x}{2}\right) - R_A x = 0$
 But: $R_A = \frac{1}{2} w L$ and $M_D = S \sigma_{all}$ Divide by $\frac{1}{2} w$:
 $x^2 - Lx + (2 S \sigma_{all} / w) = 0$ Set $k = \frac{2 S \sigma_{all}}{w}$
 $x^2 - Lx + k = 0$
 Solving the quadratic: $x = \frac{L - \sqrt{L^2 - 4k}}{2}$
 Compute x and $a = L - 2x$

(b) Required Width of Plates

At midpoint C of beam:
 $FB = AC: \sum M_C = 0: M_C + \frac{wL}{2} \left(\frac{L}{4}\right) - \frac{wL}{2} \left(\frac{L}{2}\right) = 0$
 Compute $M_C = \frac{1}{8} w L^2$
 Compute $C = t + \frac{1}{2} d$
 From $\sigma_{all} = \frac{M_C C}{I}$ compute $I = \frac{M_C C}{\sigma_{all}}$
 But $I = I_{beam} + I_{plates} = I_b + 2 \left[\frac{1}{12} b t^3 + b t \left(\frac{d+t}{2}\right)^2 \right]$
 Solving for b : $b = \frac{6(I - I_b)}{t [t^2 + 3(d+t)^2]}$

PROGRAM OUTPUTS

PROB. 5.145: $W 460 \times 74$, $\sigma_{all} = 150 \text{ MPa}$
 $w = 40 \text{ kN/m}$, $L = 8 \text{ m}$, $t = 7.5 \text{ mm}$
 $d = 457 \text{ mm}$, $I_b = 333 \times 10^6 \text{ mm}^4$, $S = 1460 \times 10^3 \text{ mm}^3$

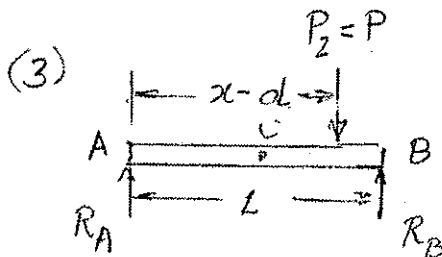
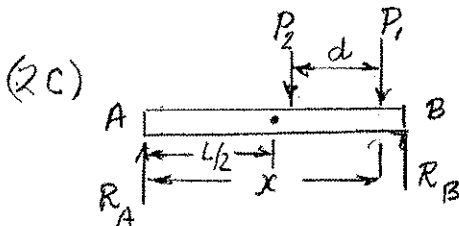
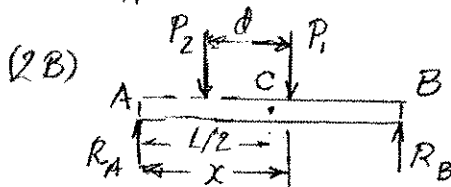
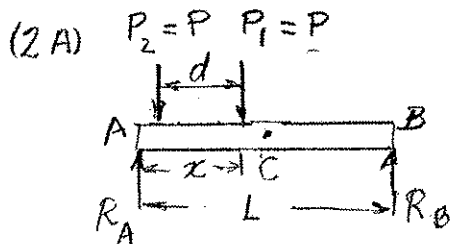
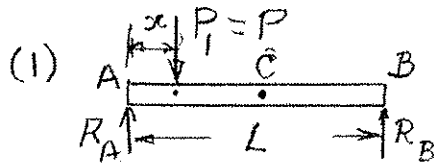
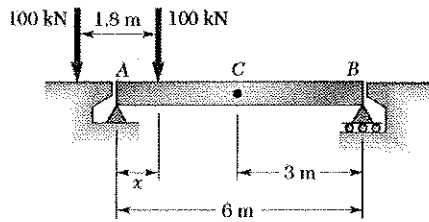
Problem 5.145

$a = 4.49 \text{ m}$
 $b = 211 \text{ mm}$

PROB. 5.143: $W 760 \times 147$, $\sigma_{all} = 150 \text{ MPa}$
 $w = 450 \text{ kN/m}$, $L = 4.8 \text{ m}$, $t = 16 \text{ mm}$
 $d = 376.5 \text{ mm}$, $I_b = 1660 \times 10^6 \text{ mm}^4$
 $S = 9.375 \times 10^6 \text{ mm}^3$

Problem 5.143

$a = 3.64 \text{ m}$
 $b = 426.9 \text{ mm}$

PROBLEM 5.C4


5.C4 Two 100-kN loads are maintained 1.8 m apart as they are moved slowly across the 6-m beam AB. Write a computer program and use it to calculate the bending moment under each load and at the midpoint C of the beam for values of x from 0 to 7 m at intervals $\Delta x = 0.45$ m.

SOLUTION

NOTATION: Length of beam = $L = 5.4$ m

Loads: $P_1 = P_2 = P = 100$ kN

Distance between loads = $d = 1.8$ m

We note that $d < L/2$

(1) FROM $x = 0$ TO $x = d$:

$$+\circlearrowleft \sum M_B = 0: P(L-x) - R_A L = 0$$

$$R_A = P(L-x)/L$$

Under P_1 : $M_1 = R_A x$

At C: $M_C = R_A(\frac{L}{2}) - P(\frac{L}{2} - x)$

(2) FROM $x = d$ TO $x = L$:

$$+\circlearrowleft \sum M_B = 0: P(L-x) + P(L-x+d) - R_A L = 0$$

$$R_A = P(2L - 2x + d)/L$$

Under P_1 : $M_1 = R_A x - Pd$

Under P_2 : $M_2 = R_A(x-d)$

(2A) FROM $x = d$ TO $x = L/2$:

$$M_C = R_A(\frac{L}{2}) - P(\frac{L}{2} - x) - P(\frac{L}{2} - x + d)$$

$$= R_A(L/2) - P(L - 2x + d)$$

(2B) FROM $x = L/2$ TO $x = L/2 + d$:

$$M_C = R_A(L/2) - P(\frac{L}{2} - x + d)$$

(2C) FROM $x = L/2 + d$ TO $x = L$:

$$M_C = R_A L/2$$

(3) FROM $x = L$ TO $x = L + d$:

$$+\circlearrowleft \sum M_B = 0: P(L-x+d) - R_A L = 0$$

$$R_A = P(L-x+d)/L$$

Under P_2 : $M_2 = R_A(x-d)$

At C: $M_C = R_A(L/2)$

(CONTINUED)

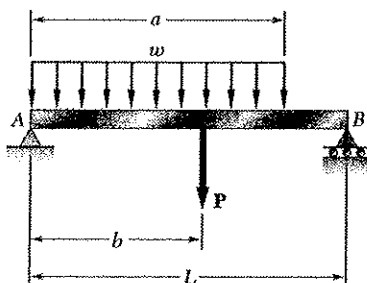
PROBLEM 5.C4 CONTINUED

PROGRAM OUTPUT

$$P = 100 \text{ kN}, \quad L = 5.4 \text{ m}, \quad D = 1.8 \text{ m}$$

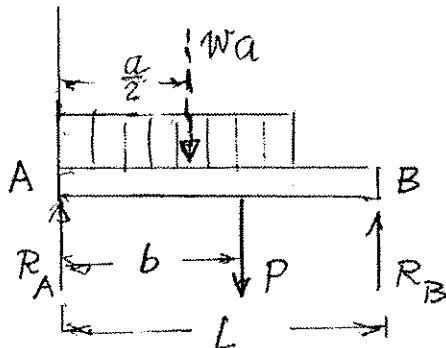
x m	MC kNm	M1 kNm	M2 kNm
0	0	0	0
0.9	50.85	84.75	0
1.8	101.7	135.6	0
2.7	203.4	203.4	135.6
3.6	203.4	203.4	203.4
4.5	203.4	135.6	203.4
5.4	101.7	0	135.6

PROBLEM 5.C5



5.C5 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.9$ m to the beam and loading of (a) Prob. 5.72, (b) Prob. 5.115.

SOLUTION



REACTIONS AT A AND B

USING FB DIAGRAM OF BEAM:

$$+\circlearrowleft \sum M_A = 0: R_B L + P b - w a (a/2) = 0$$

$$R_B = (1/L) (P b + \frac{1}{2} w a^2)$$

$$R_A = P + w a - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text).

SET $n = L/\Delta L$. FOR $i = 0$ TO n : $x = (\Delta L) i$

WE DEFINE: IF $x \geq a$ THEN $STPA = 1$ ELSE $STPA = 0$
IF $x \geq b$ THEN $STPB = 1$ ELSE $STPB = 0$

$$V = R_A - w x + w (x - a) STPA - P STPB$$

$$M = R_A x - \frac{1}{2} w x^2 + \frac{1}{2} w (x - a)^2 STPA - P (x - b) STPB$$

LOCATE AND PRINT (x, V) AND (x, M)

SEE NEXT PAGES FOR PROGRAM OUTPUTS

(CONTINUED)

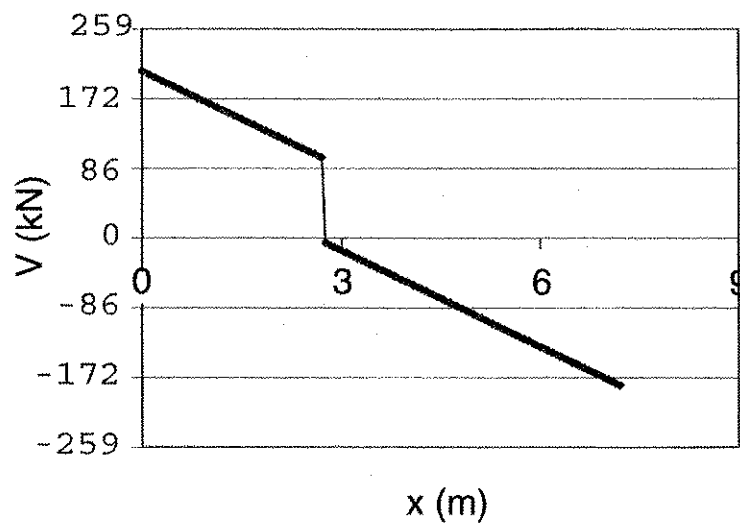
PROBLEM 5.C5 CONTINUED

PROGRAM OUTPUT

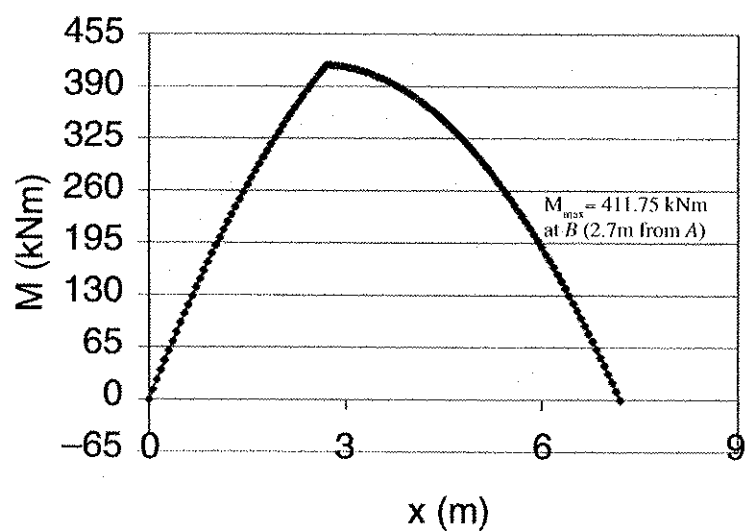
Problem 5.72

RA = 206.5 kN RB = 181.5 kN

Shear Diagram



Moment Diagram



(CONTINUED)

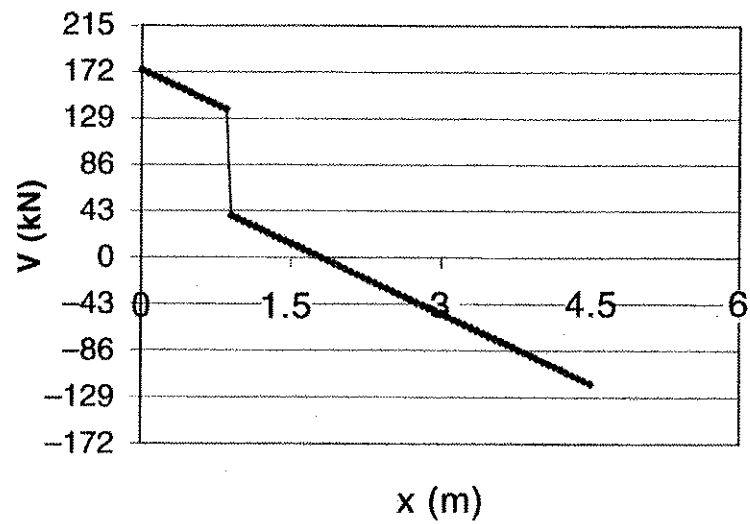
PROBLEM 5.C5 CONTINUED

PROGRAM OUTPUT FOR

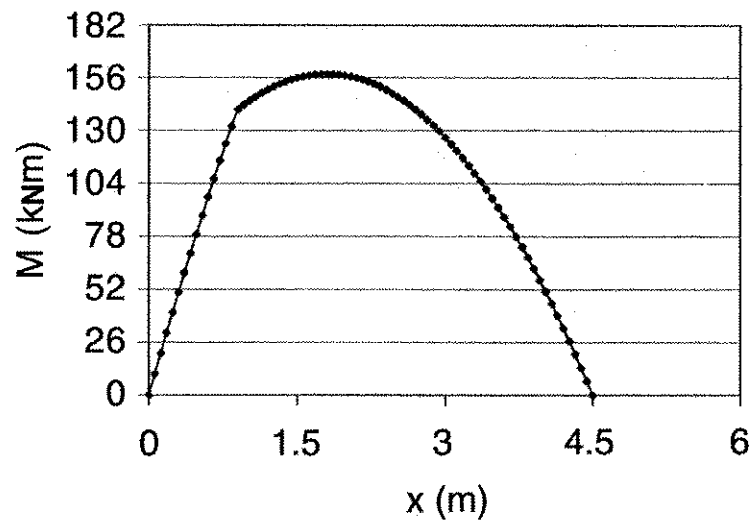
Problem 5.115

RA = 173.25 kN RB = 119.25 kN

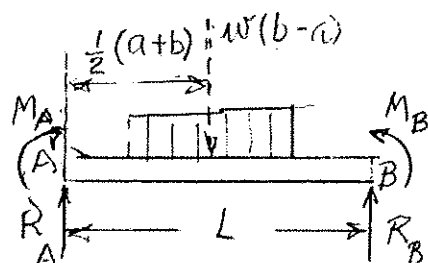
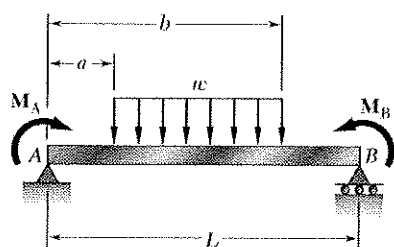
Shear Diagram



Moment Diagram



PROBLEM 5.C6



5.C6 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.025$ m to the beam and loading of Prob. 5.112

SOLUTION

REACTIONS AT A AND B

$$+\circlearrowleft \sum M_A = 0:$$

$$R_B L + M_B - M_A - w(b-a) \frac{1}{2}(a+b) = 0$$

$$R_B = (1/L) \left[M_A - M_B + \frac{1}{2} w(b^2 - a^2) \right]$$

$$R_A = w(b-a) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET $n = L/\Delta L$, FOR $i = 0$ TO n : $x = (\Delta L)i$

WE DEFINE: IF $x \geq a$ THEN $STPA = 1$ ELSE $STPA = 0$

IF $x \geq b$ THEN $STPB = 1$ ELSE $STPB = 0$

$$V = R_A - w(x-a)STPA + w(x-b)STPB$$

$$M = M_A + R_A x - \frac{1}{2} w(x-a)^2 STPA + \frac{1}{2} w(x-b)^2 STPB$$

LOCATE AND PRINT (x, V) AND (x, M)

PROGRAM OUTPUT ON NEXT PAGE

(CONTINUED)

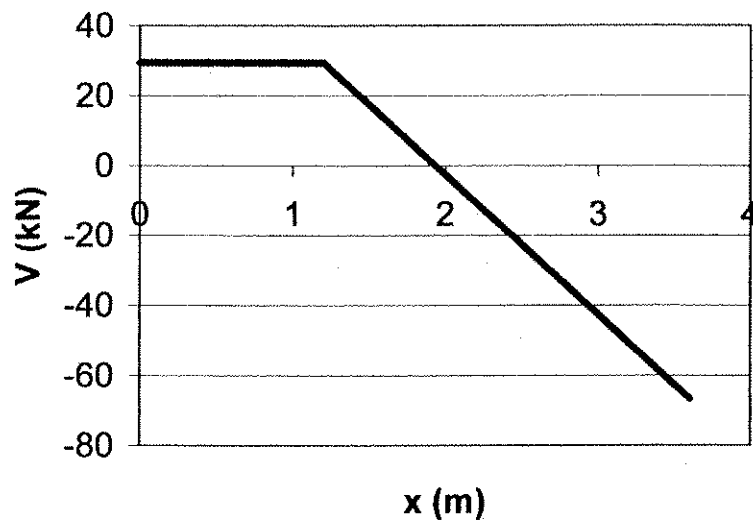
PROBLEM 5.C6 CONTINUED

PROGRAM OUTPUT

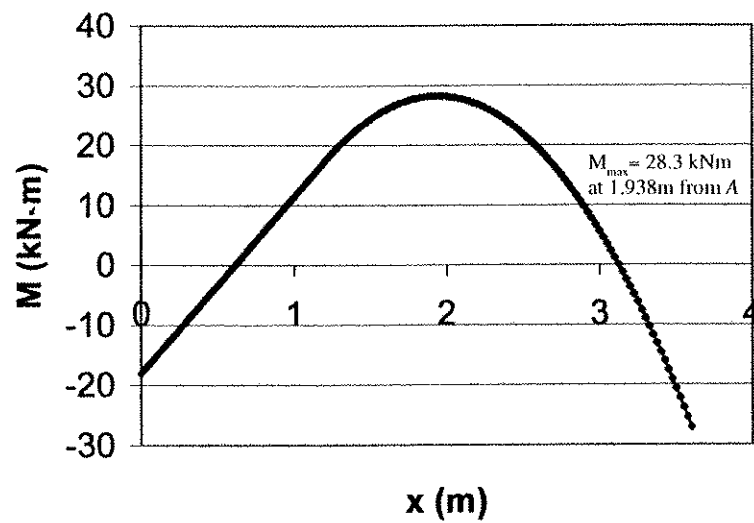
Problem 5.112

RA = 29.50 kN RB = 66.50 kN

Shear Diagram

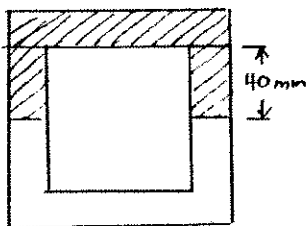
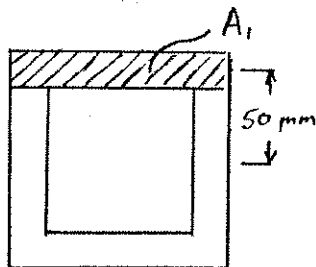
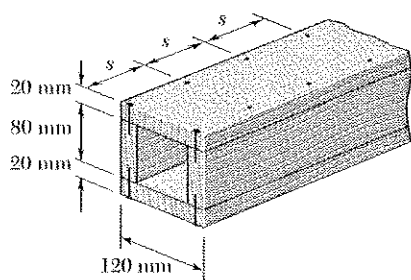


Moment Diagram



Chapter 6

Problem 6.1



6.1 A square box beam is made of two 20×80 -mm planks and two 20×120 -mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 50$ mm and that the allowable shearing force in each nail is 300 N, determine (a) the largest allowable vertical shear in the beam, (b) the corresponding maximum shearing stress in the beam.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12} (120)(120)^3 - \frac{1}{12} (80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4$$

$$= 13.8667 \times 10^{-6} \text{ m}^4$$

$$(a) A_1 = (120)(20) = 2400 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

$$Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$$

$$q_{\text{all}} = \frac{2 F_{\text{nail}}}{s} = \frac{(2)(300)}{50 \times 10^{-3}} = 12 \times 10^3 \text{ N}$$

$$q = \frac{VQ}{I}$$

$$V = \frac{qI}{Q} = \frac{(12 \times 10^3)(13.8667 \times 10^{-6})}{120 \times 10^{-6}}$$

$$= 1.38667 \times 10^3 \text{ N} = 1.387 \text{ kN}$$

$$(b) Q = Q_1 + (2)(20)(40)(20)$$

$$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$$

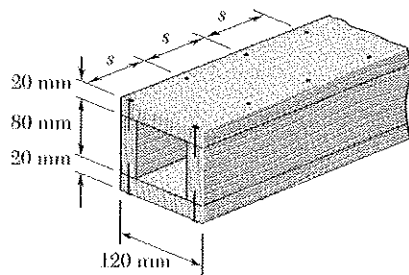
$$= 152 \times 10^{-6} \text{ m}^3$$

$$\tau_{\text{max}} = \frac{VQ}{It} = \frac{(1.38667 \times 10^3)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$$

$$= 380 \times 10^3 \text{ Pa}$$

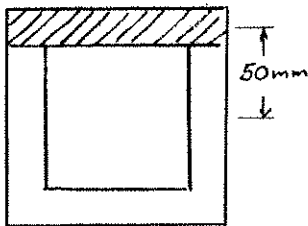
$$380 \text{ kPa}$$

Problem 6.2



6.2 A square box beam is made of two 20×80 -mm planks and two 20×120 -mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 30$ mm and that the vertical shear in the beam is $V = 1200$ N, determine (a) the shearing force in each nail, (b) the maximum shearing stress in the beam.

$$\begin{aligned} I &= \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3 \\ &= \frac{1}{12} (120)(120)^3 - \frac{1}{12} (80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4 \\ &= 13.8667 \times 10^{-6} \text{ m}^4 \end{aligned}$$



$$(a) \quad A_1 = (120)(20) = 2400 \text{ mm}^2$$

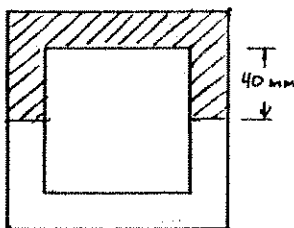
$$\bar{y}_1 = 50 \text{ mm}$$

$$Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(1200)(120 \times 10^{-6})}{13.8667 \times 10^{-6}} = 10.385 \times 10^3 \text{ N/m}$$

$$qs = 2F_{\text{nail}}$$

$$F_{\text{nail}} = \frac{qs}{2} = \frac{(10.385 \times 10^3)(30 \times 10^{-3})}{2} = 155.8 \text{ N}$$



$$(b) \quad Q = Q_1 + (2)(20)(40)(20)$$

$$= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3$$

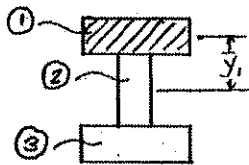
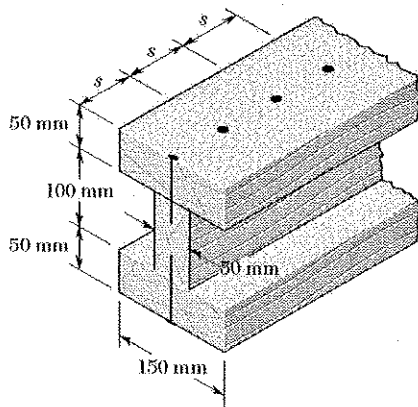
$$= 152 \times 10^{-6} \text{ m}^3$$

$$\tau_{\text{max}} = \frac{VQ}{It} = \frac{(1200)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^{-3})}$$

$$= 329 \times 10^3 \text{ Pa}$$

$$329 \text{ kPa}$$

Problem 6.3



6.3 Three boards, each 50 mm thick, are nailed together to form a beam that is subjected to a vertical shear. Knowing that the allowable shearing force in each nail is 600 N, determine the allowable shear if the spacing s between the nails is 75 mm.

$$I_1 = \frac{1}{12} b h^3 + A d^2$$

$$= \frac{1}{12} (150)(50)^3 + (150)(50)(75)^2 = 43.75 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12} (50)(100)^3 = 4.17 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 43.75 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 91.67 \times 10^6 \text{ mm}^4$$

$$Q = A_1 \bar{y}_1 = (150)(50)(75) = 562500 \text{ mm}^3$$

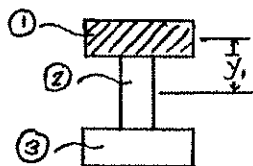
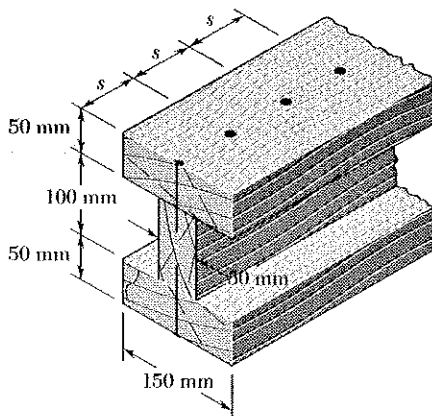
$$qs = F_{\text{nail}} \quad (1)$$

$$q = \frac{VQ}{I} \quad (2)$$

$$\text{Dividing Eq (2) by Eq (1)} \quad \frac{1}{s} = \frac{VQ}{F_{\text{nail}} I}$$

$$V = \frac{F_{\text{nail}} I}{Qs} = \frac{(600)(91.67 \times 10^6)}{(562500)(75)} = 1304 \text{ N}$$

Problem 6.4



6.4 Three boards, each 50 mm thick, are nailed together to form a beam that is subjected to a vertical shear of 1.2 kN. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s of the nails that can be used.

$$I_1 = \frac{1}{12} b h^3 + A d^2$$

$$= \frac{1}{12} (150)(50)^3 + (150)(50)(75)^2 = 43.75 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12} (50)(100)^3 = 4.17 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 43.75 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 91.67 \times 10^6 \text{ mm}^4$$

$$Q = A_1 \bar{y}_1 = (150)(50)(75) = 562.5 \times 10^3 \text{ mm}^3$$

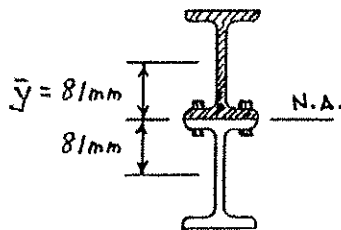
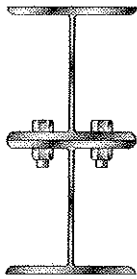
$$qs = F_{\text{nail}} \quad (1)$$

$$q = \frac{VQ}{I} \quad (2)$$

$$s = \frac{F_{\text{nail}} I}{VQ} = \frac{(400)(91.67 \times 10^6)}{(1200)(562.5 \times 10^3)}$$

$$s = 54.3 \text{ mm}$$

Problem 6.5



Shear: $q = \frac{VQ}{I}$

6.5 The composite beam shown is fabricated by connecting two W150 x 37.1 rolled-steel members, using bolts of 16-mm diameter spaced longitudinally every 150 mm. Knowing that the average allowable shearing stress in the bolts is 73 MPa, determine the largest allowable vertical shear in the beam.

W150 x 37.1: $A = 4370 \text{ mm}^2$, $d = 16 \text{ mm}$, $I_x = 22.2 \times 10^6 \text{ mm}^4$

$\bar{y} = \frac{1}{2}d = 81 \text{ mm}$

Composite: $I = 2 [22.2 \times 10^6 + (4370)(81)^2]$
 $= 101.743 \times 10^6 \text{ mm}^4$

$Q = A\bar{y} = (4370)(81) = 353970 \text{ mm}^3$

Bolts: $d = 16 \text{ mm}$, $\tau_{all} = 73 \text{ MPa}$, $S = 150 \text{ mm}$

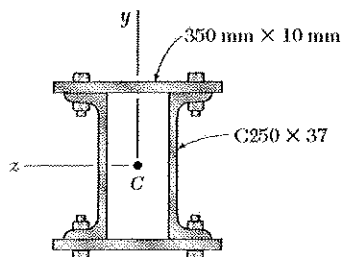
$A_{bolt} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2$

$F_{bolt} = \tau_{all} A_{bolt} = (73 \times 10^6)(201.06 \times 10^{-6}) = 14677 \text{ N}$

$q = \frac{2F_{bolt}}{S} = \frac{(2)(14677)}{150} = 195.69 \text{ N/mm}$

$V = \frac{Iq}{Q} = \frac{(101.743 \times 10^6)(195.69)}{353970}$ $V = 56.2 \text{ kN}$

Problem 6.6



6.6 A column is fabricated by connecting the rolled-steel members shown by bolts of 18-mm diameter spaced longitudinally every 125 mm. Determine the average shearing stress in the bolts caused by a shearing force of 120 kN parallel to the y axis.

Calculate moment of inertia.

Part	A (mm ²)	d (mm)	Ad ² (mm ⁴)	$\bar{I}$ (mm ⁴)
Top plate	3500	* 132	60.984 x 10 ⁶	29167
C250 x 37	4750	0		37.9 x 10 ⁶
C250 x 37	4750	0		37.9 x 10 ⁶
Bot. plate	3500	* 132	60.984 x 10 ⁶	29167
Σ			121.968 x 10 ⁶	75.858 x 10 ⁶

$$* d = \frac{254}{2} + \frac{10}{2} = 132 \text{ mm} = \bar{y}_1$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 121.968 \times 10^6 + 75.858 \times 10^6 = 197.83 \times 10^6 \text{ mm}^4$$

$$Q = A_{\text{plate}} \bar{y}_1 = (3500)(132) = 462000 \text{ mm}^3$$

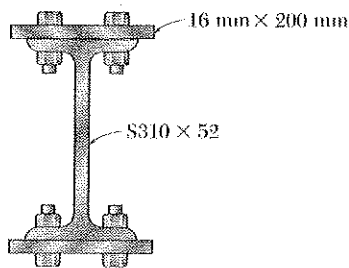
$$q = \frac{VQ}{I} = \frac{(120)(462000 \times 10^{-9})}{197.83 \times 10^{-6}} = 280.24 \text{ kN/m}$$

$$F_{\text{bolt}} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(280.24)(0.125) = 17.515 \text{ kN}$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} (18)^2 = 254.67 \text{ mm}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{17.515}{254.67 \times 10^{-6}} = 68.8 \text{ MPa}$$

Problem 6.7



6.7 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 × 200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

Calculate moment of inertia.

$$* d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

Part	A (mm ²)	d (mm)	Ad ² (10 ⁶ mm ⁴)	$\bar{I}$ (10 ⁶ mm ⁴)
Top plate	3200	*160.5	82.43	0.07
S310 x 52	6650	0		95.3
Bot. plate	3200	*160.5	82.43	0.07
Σ			164.86	95.44

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{\text{plate}} d_{\text{plate}} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} (18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

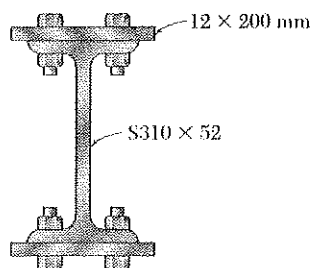
$$F_{\text{bolt}} = \tau_{\text{all}} A_{\text{bolt}} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

$$q_s = 2 F_{\text{bolt}} \quad q = \frac{2 F_{\text{bolt}}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \quad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N}$$

$$V = 193.5 \text{ kN} \quad \blacktriangleleft$$

Problem 6.8



$$* d = \frac{305}{2} + \frac{12}{2} = 158.5 \text{ mm}$$

6.8 Solve Prob. 6.7, assuming that the reinforcing plates are only 12 mm thick.

6.7 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 x 200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

Calculate moment of inertia.

Part	A (mm ²)	d (mm)	Ad ² (10 ⁶ mm ⁴)	$\bar{I}$ (10 ⁶ mm ⁴)
Top plate	2400	* 158.5	60.29	0.03
S 310 x 52	6650	0	0	95.3
Bot. plate	2400	* 158.5	60.29	0.03
Σ			120.58	95.36

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 215.94 \times 10^6 \text{ mm}^4 = 215.94 \times 10^{-6} \text{ m}^4$$

$$Q = A_{\text{plate}} d_{\text{plate}} = (200)(12)(158.5) = 380.4 \times 10^3 \text{ mm}^3 = 380.4 \times 10^{-6} \text{ m}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} (18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$F_{\text{bolt}} = \tau_{\text{all}} A_{\text{bolt}} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.902 \times 10^3 \text{ N}$$

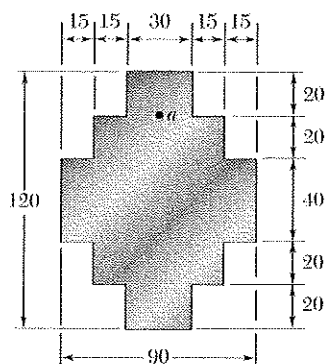
$$q_s = 2 F_{\text{bolt}} \quad q = \frac{2 F_{\text{bolt}}}{s} = \frac{(2)(22.903 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \quad V = \frac{Iq}{Q} = \frac{(215.94 \times 10^{-6})(381.7 \times 10^3)}{380.4 \times 10^{-6}} = 217 \times 10^3 \text{ N}$$

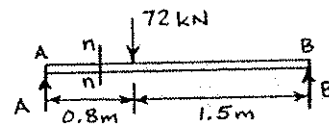
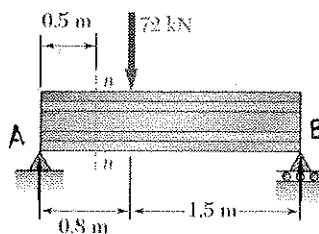
$$V = 217 \text{ kN}$$

Problem 6.9

6.9 through 6.12 For the beam and loading shown, consider section $n-n$ and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a .



Dimensions in mm



$$+\circlearrowleft \sum M_B = 0:$$

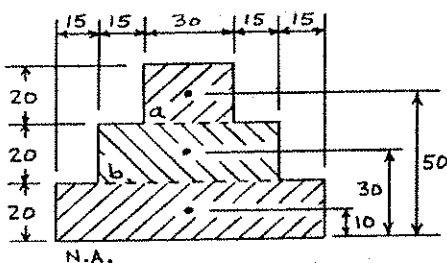
$$-2.3 A + (1.5)(72) = 0$$

$$A = 46.957 \text{ kN} \uparrow$$

At section $n-n$, $V = A = 46.957 \text{ kN}$

Calculate moment of inertia. $I = 2 \left[\frac{1}{12} (15)(40)^3 \right] + 2 \left[\frac{1}{12} (15)(80)^3 \right] + \frac{1}{12} (30)(120)^3$

$$= 5.76 \times 10^6 \text{ mm}^4 = 5.76 \times 10^{-6} \text{ m}^4$$



At a : $t_a = 30 \text{ mm} = 0.030 \text{ m}$

$$Q_a = (30 \times 20)(50) = 30 \times 10^3 \text{ mm}^3$$

$$= 30 \times 10^{-6} \text{ m}^3$$

$$\tau_a = \frac{V Q_a}{I t_a} = \frac{(46.957 \times 10^3)(30 \times 10^{-6})}{(5.76 \times 10^{-6})(0.030)}$$

$$= 8.15 \times 10^6 \text{ Pa} = 8.15 \text{ MPa}$$

At b : $t_b = 60 \text{ mm} = 0.060 \text{ m}$

$$Q_b = Q_a + (60 \times 20)(30) = 30 \times 10^3 + 36 \times 10^3 = 66 \times 10^3 \text{ mm}^3 = 66 \times 10^{-6} \text{ m}^3$$

$$\tau_b = \frac{V Q_b}{I t_b} = \frac{(46.957 \times 10^3)(66 \times 10^{-6})}{(5.76 \times 10^{-6})(0.060)} = 8.97 \times 10^6 \text{ Pa} = 8.97 \text{ MPa}$$

At $N.A$: $t_{NA} = 90 \text{ mm} = 0.090 \text{ m}$

$$Q_{NA} = Q_b + (90 \times 20)(10) = 66 \times 10^3 + 18 \times 10^3 = 84 \times 10^3 \text{ mm}^3 = 84 \times 10^{-6} \text{ m}^3$$

$$\tau_{NA} = \frac{V Q_{NA}}{I t_{NA}} = \frac{(46.957 \times 10^3)(84 \times 10^{-6})}{(5.76 \times 10^{-6})(0.090)} = 7.61 \times 10^6 \text{ Pa} = 7.61 \text{ MPa}$$

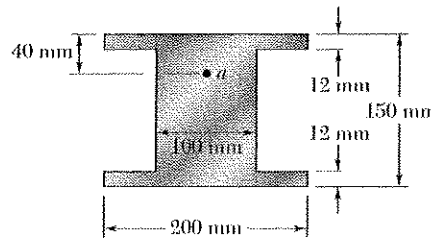
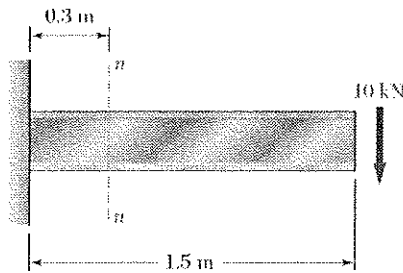
(a) $\tau_{\max}$ occurs at b .

$$\tau_{\max} = 8.97 \text{ MPa}$$

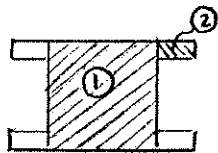
(b) $\tau_a = 8.15 \text{ MPa}$

Problem 6.10

6.9 through 6.12 For the beam and loading shown, consider section $n-n$ and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a .

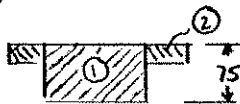


At section $n-n$
 $V = 10 \text{ kN}$



$$\begin{aligned} I &= I_1 + 4 I_2 \\ &= \frac{1}{12} b_1 h_1^3 + 4 \left(\frac{1}{12} b_2 h_2^3 + A_2 d_2^2 \right) \\ &= \frac{1}{12} (100)(150)^3 + 4 \left[\frac{1}{12} (50)(12)^3 + (50)(12)(69)^2 \right] \\ &= 28.125 \times 10^6 + 4 [0.0072 \times 10^6 + 2.8566 \times 10^6] \\ &= 39.58 \times 10^6 \text{ mm}^4 = 39.58 \times 10^{-6} \text{ m}^4 \end{aligned}$$

(a)



$$\begin{aligned} Q &= A_1 \bar{y}_1 + 2 A_2 \bar{y}_2 \\ &= (100)(75)(37.5) + (2)(50)(12)(69) \\ &= 364.05 \times 10^3 \text{ mm}^3 = 364.05 \times 10^{-6} \text{ m}^3 \\ t &= 100 \text{ mm} = 0.100 \text{ m} \end{aligned}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10 \times 10^3)(364.05 \times 10^{-6})}{(39.58 \times 10^{-6})(0.100)} = 920 \times 10^3 \text{ Pa} \quad \tau_{max} = 920 \text{ kPa} \rightarrow$$

(b)

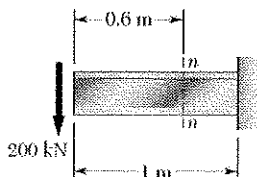
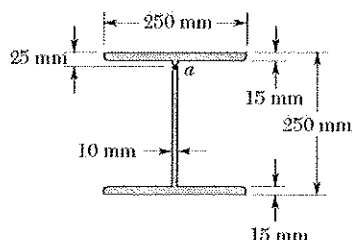


$$\begin{aligned} Q &= A_1 \bar{y}_1 + 2 A_2 \bar{y}_2 \\ &= (100)(40)(55) + (2)(50)(12)(69) \\ &= 302.8 \times 10^3 \text{ mm}^3 = 302.8 \times 10^{-6} \text{ m}^3 \\ t &= 100 \text{ mm} = 0.100 \text{ m} \end{aligned}$$

$$\tau_a = \frac{VQ}{It} = \frac{(10 \times 10^3)(302.8 \times 10^{-6})}{(39.58 \times 10^{-6})(0.100)} = 765 \times 10^3 \text{ Pa} \quad \tau_a = 765 \text{ kPa} \rightarrow$$

Problem 6.11

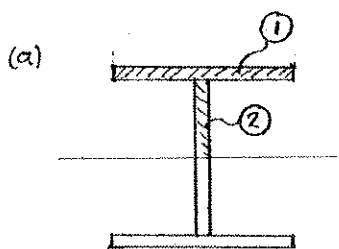
6.9 through 6.12 For the beam and loading shown, consider section $n-n$ and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a .



$$V = 200 \text{ kN}$$

Part	$A (\text{mm}^2)$	$d (\text{mm})$	$Ad^2 (\text{mm}^4)$	$\bar{I} (\text{mm}^4)$
Flange	3750	117.5	51773437.5	70312.5
Web	2200	0	0	8873333
Flange	3750	117.5	51773437.5	70312.5
Σ			103546875	9013958

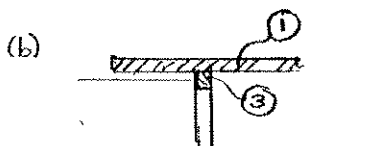
$$\begin{aligned}
 I &= \Sigma Ad^2 + \Sigma \bar{I} \\
 &= 103546875 + 9013958 \\
 &= 112.56 \times 10^6 \text{ mm}^4
 \end{aligned}$$



$$\begin{aligned}
 Q &= A_1 \bar{y}_1 + A_2 \bar{y}_2 \\
 &= (3750)(117.5) + (110)(10)(55) = 501125 \text{ mm}^3
 \end{aligned}$$

$$t = 10 \text{ mm}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(200 \times 10^3)(501125 \times 10^{-9})}{(112.56 \times 10^6)(0.01)} = 89 \text{ MPa}$$



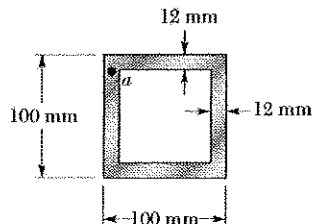
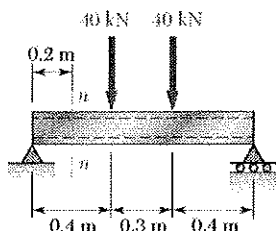
$$\begin{aligned}
 Q_a &= A_1 \bar{y}_1 + A_3 \bar{y}_3 \\
 &= (3750)(117.5) + (10)(10)(105) \\
 &= 451125 \text{ mm}^3
 \end{aligned}$$

$$t = 10 \text{ mm}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(200 \times 10^3)(451125 \times 10^{-9})}{(112.56 \times 10^6)(0.01)} = 80.1 \text{ MPa}$$

Problem 6.12

6.9 through 6.12 For the beam and loading shown, consider section $n-n$ and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a .

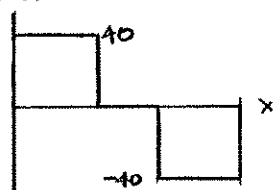


By symmetry $R_A = R_B$

$$+\uparrow \sum F_y = 0: R_A + R_B - 40 - 40 = 0$$

$$R_A = R_B = 40 \text{ kN}$$

$V \text{ (kN)}$

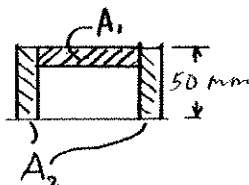


From the shear diagram, $V = 40 \text{ kN}$ at $n-n$.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12} (100)(100)^3 - \frac{1}{12} (76)(76)^3 = 5553152 \text{ mm}^4$$

(a)

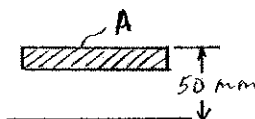


$$Q = A_1 \bar{y}_1 + A_2 \bar{y}_2 = (76)(12)(44) + 2(50)(12)(25) = 70128 \text{ mm}^3$$

$$t = 12 + 12 = 24 \text{ mm}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(40)(70128 \times 10^{-9})}{(5553152 \times 10^{-12})(24 \times 10^{-3})} = 21.05 \text{ MPa}$$

(b)

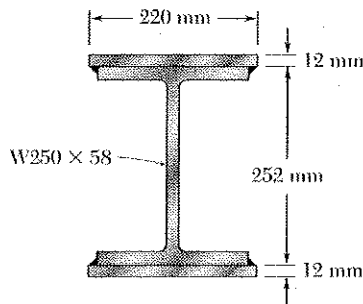


$$Q = A \bar{y} = (100)(12)(44) = 52800 \text{ mm}^3$$

$$t = 12 + 12 = 24 \text{ mm}$$

$$\tau = \frac{VQ}{It} = \frac{(40)(52800 \times 10^{-9})}{(5553152 \times 10^{-12})(0.024)} = 15.85 \text{ MPa}$$

Problem 6.13



$$* d = \frac{252}{2} + \frac{12}{2}$$

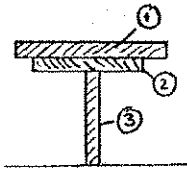
6.13 Two steel plates of 12×220 -mm rectangular cross section are welded to the W250 $\times$ 58 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

Calculate moment of inertia.

Part	A (mm ²)	d (mm)	Ad ² (10 ⁶ mm ⁴)	$\bar{I}$ (10 ⁶ mm ⁴)
Top plate	2640	* 132	45.999	0.032
W 250 $\times$ 58	7420	0	0	87.3
Bot. plate	2640	* 132	45.999	0.032
Σ			91.999	87.363

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 179.362 \times 10^6 \text{ mm}^4 = 179.362 \times 10^{-6} \text{ m}^4$$

$\tau_{\max}$ occurs at neutral axis. $t = 8.0 \text{ mm} = 8.0 \times 10^{-3} \text{ m}$



Part	A (mm ²)	$\bar{y}$ (mm)	$A\bar{y}$ (10 ³ mm ³)
① Top plate	2640	132	348.48
② Top flange	2740.5	119.25	326.805
③ Half web	900	56.25	50.625
Σ			725.91

Dimensions in mm: ① 12×220 ; ② 13.5×203 ; ③ 8.0×112.5

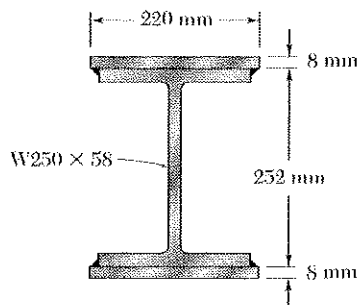
$$Q = \Sigma A\bar{y} = 725.91 \times 10^3 \text{ mm}^3 = 725.91 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} \quad V = \frac{It\tau}{Q} = \frac{(179.362 \times 10^{-6})(8.0 \times 10^{-3})(90 \times 10^6)}{725.91 \times 10^{-6}}$$

$$= 177.9 \times 10^3 \text{ N}$$

$$V = 177.9 \text{ kN}$$

Problem 6.14



$$* d = \frac{252}{2} + \frac{8}{2} =$$

6.14 Solve Prob. 6.13, assuming that the two steel plates are (a) replaced by 8×220 -mm plates, (b) removed.

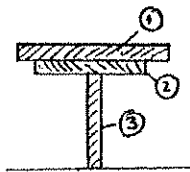
6.13 Two steel plates of 12×220 -mm rectangular cross section are welded to the W250 x 58 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

(a) Calculate moment of inertia.

Part	A (mm ²)	d (mm)	Ad ² (10 ⁶ mm ⁴)	$\bar{I}$ (10 ⁶ mm ⁴)
Top plate	1760	* 130	29.744	0.0094
W 250 x 58	7420	0	0	87.3
Bot. plate	1760	* 130	29.744	0.0094
Σ			59.488	87.312

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 146.807 \times 10^6 \text{ mm}^4 = 146.807 \times 10^{-6} \text{ m}^4$$

$\tau_{\max}$ occurs at neutral axis. $t = 8.0 \text{ mm} = 8.0 \times 10^{-3} \text{ m}$



Part	A (mm ²)	$\bar{y}$ (mm)	$A\bar{y}$ (10 ³ mm ³)
① Top plate	1760	130	228.8
② Top flange	2740.5	119.25	326.805
③ Half web	900	56.25	50.625
Σ			606.23

Dimensions in mm: ① 8×220 ; ② 13.5×203 ; ③ 8.0×112.5

$$Q = \Sigma A\bar{y} = 606.23 \times 10^3 \text{ mm}^3 = 606.23 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} \quad V = \frac{It\tau}{Q} = \frac{(146.807 \times 10^{-6})(8.0 \times 10^{-3})(90 \times 10^6)}{606.23 \times 10^{-6}}$$

$$= 174.4 \times 10^3 \text{ N} \quad V = 174.4 \text{ kN}$$

(b) With plates removed.

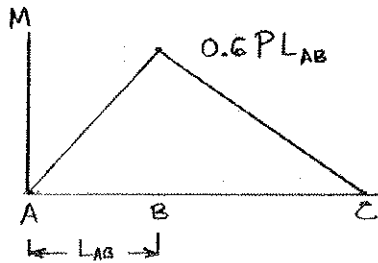
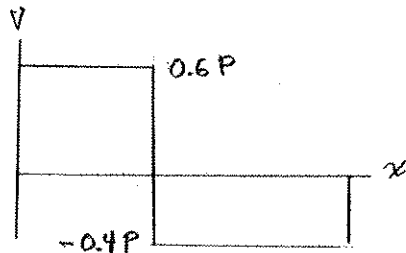
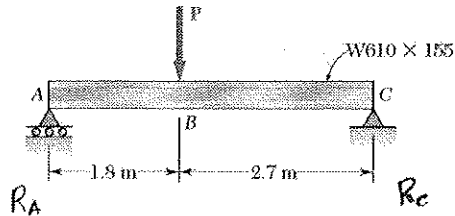
$$I = 87.3 \times 10^6 \text{ mm}^4 = 87.3 \times 10^{-6} \text{ m}^4 \quad t = 8.0 \times 10^{-3} \text{ m}$$

$$Q = (326.805 + 50.625) \times 10^3 = 377.43 \times 10^3 \text{ mm}^3 = 377.43 \times 10^{-6} \text{ m}^3$$

$$V = \frac{It\tau}{Q} = \frac{(87.3 \times 10^6)(8.0 \times 10^{-3})(90 \times 10^6)}{377.43 \times 10^{-6}}$$

$$= 166.5 \times 10^3 \text{ N} \quad V = 166.5 \text{ kN}$$

Problem 6.15



6.15 For the wide-flange beam with the loading shown, determine the largest load P that can be applied, knowing that the maximum normal stress is 165 MPa and the largest shearing stress using the approximation $\tau_m = V/A_{web}$ is 100 MPa.

$$+\circlearrowleft \sum M_C = 0 = -4.5R_A + 2.7P = 0$$

$$R_A = 0.6P$$

Draw shear and bending moment diagrams.

$$|V|_{max} = 0.6P \quad |M|_{max} = 0.6PL_{AB}$$

$$L_{AB} = 1.8 \text{ m}$$

Bending. For W610 x 155, $S = 4220 \times 10^3 \text{ mm}^3$

$$S = \frac{|M|_{max}}{\sigma_{all}} = \frac{0.6PL_{AB}}{\sigma_{all}}$$

$$P = \frac{\sigma_{all} S}{0.6L_{AB}} = \frac{(165)(4220 \times 10^3)}{(0.6)(1800)} = 644.7 \text{ kN} \approx 645 \text{ kN}$$

Shear. $A_{web} = d t_w$

$$= (611)(12.7)$$

$$= 7759.7 \text{ mm}^2$$

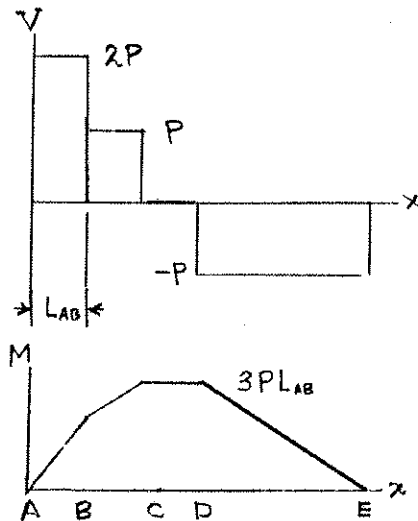
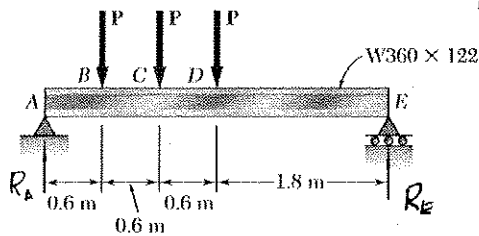
$$\tau = \frac{|V|_{max}}{A_{web}} = \frac{0.6P}{A_{web}}$$

$$P = \frac{\tau A_{web}}{0.6} = \frac{(100)(7759.7)}{0.6} = 1293 \text{ kN}$$

The smaller value of P is the allowable value. $P = 645 \text{ kN}$

Problem 6.16

6.16 For the wide-flange beam with the loading shown, determine the largest load P that can be applied, knowing that the maximum normal stress is 160 MPa and the largest shearing stress using the approximation $\tau_m = V/A_{web}$ is 100 MPa.



$$+\circlearrowleft \sum M_E = 0 = -3.6R_A + 3.0P + 2.4P + 1.8P = 0$$

$$R_A = 2P$$

Draw shear and bending moment diagrams.

$$M_B = 2PL_{AB}, \quad M_C = M_D = 3PL_{AB}$$

$$|V|_{max} = 2P \quad |M|_{max} = 3PL_{AB}$$

Bending. For W360 x 122 $S = 2010 \times 10^3 \text{ mm}^3$
 $= 2010 \times 10^{-6} \text{ m}^3$

$$\frac{|M|_{max}}{\sigma_{all}} = \frac{3PL_{AB}}{\sigma_{all}} = S$$

$$P = \frac{\sigma_{all} S}{3L_{AB}} = \frac{(160 \times 10^6)(2010 \times 10^3)}{(3)(0.6)} = 178.7 \times 10^3 \text{ N}$$

Shear $A_{web} = d t_w = (363)(13.0)$
 $= 4.719 \times 10^3 \text{ mm}^2 = 4.719 \times 10^{-3} \text{ m}^2$

$$\tau = \frac{|V|_{max}}{A_{web}} = \frac{2P}{A_{web}}$$

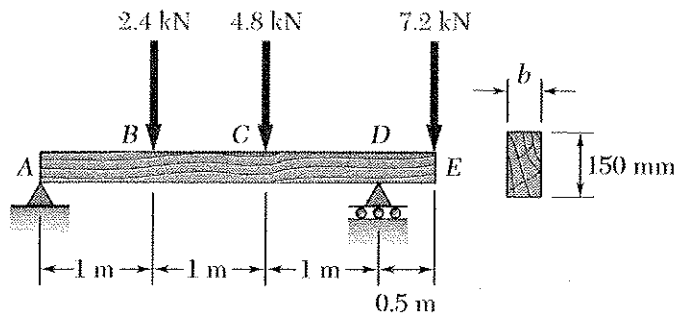
$$P = \frac{\tau A_{web}}{2} = \frac{(100 \times 10^6)(4.719 \times 10^{-3})}{2} = 236 \times 10^3 \text{ N}$$

The smaller value of P is the allowable one.

$$P = 178.7 \text{ kN} \quad \blacktriangleleft$$

Problem 6.17

6.17 For the beam and loading shown, determine the minimum required width b , knowing that for the grade of timber used, $\sigma_{all} = 12 \text{ MPa}$ and $\tau_{all} = 825 \text{ kPa}$.



$$+\circlearrowleft \sum M_D = 0:$$

$$-3A + (2)(2.4) + (1)(4.8) - (0.5)(7.2) = 0$$

$$A = 2 \text{ kN} \uparrow$$

Draw the shear and bending moment diagrams.

$$|V|_{max} = 7.2 \text{ kN} = 7.2 \times 10^3 \text{ N}$$

$$|M|_{max} = 3.6 \text{ kN}\cdot\text{m} = 3.6 \times 10^3 \text{ N}\cdot\text{m}$$

Bending: $\sigma = \frac{M}{S}$

$$S_{min} = \frac{|M|_{max}}{\sigma} = \frac{3.6 \times 10^3}{12 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

For a rectangular section, $S = \frac{1}{6} b h^2$

$$b = \frac{6S}{h^2} = \frac{(6)(300 \times 10^3)}{(150)^2} = 80 \text{ mm}$$

Shear: Maximum shearing stress occurs at the neutral axis of bending for a rectangular section.

$$A = \frac{1}{2} b h, \quad \bar{y} = \frac{1}{4} h, \quad Q = A\bar{y} = \frac{1}{8} b h^2$$

$$I = \frac{1}{12} b h^3 \quad t = b$$

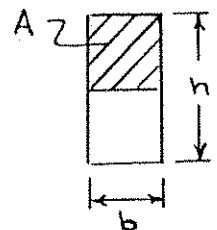
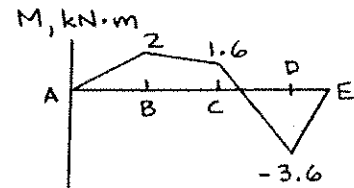
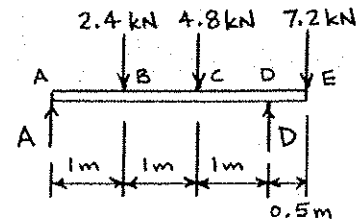
$$\tau = \frac{VQ}{It} = \frac{V(\frac{1}{8} b h^2)}{(\frac{1}{12} b h^3)(b)} = \frac{3}{2} \frac{V}{b h}$$

$$b = \frac{3V}{2h\tau} = \frac{(3)(7.2 \times 10^3)}{(2)(150 \times 10^{-3})(825 \times 10^3)} = 87.3 \times 10^3 \text{ m}$$

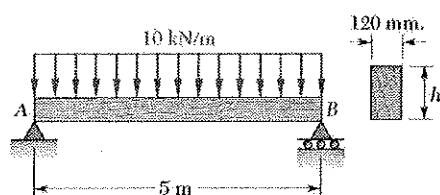
$$b = 87.3 \text{ mm}$$

The required value of b is the larger one.

$$b = 87.3 \text{ mm}$$



Problem 6.18



6.18 For the beam and loading shown, determine the minimum required depth h , knowing that for the grade of timber used, $\sigma_{all} = 12 \text{ MPa}$ and $\tau_{all} = 0.9 \text{ MPa}$.

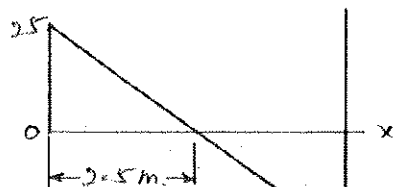
$$\text{Total load } (10)(5) = 50 \text{ kN}$$

$$\text{Reaction at A } R_A = 25 \text{ kN}$$

$$V_{max} = 25 \text{ kN}$$

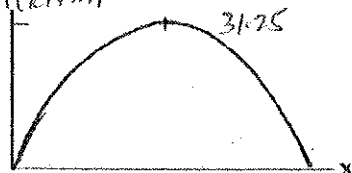
$$M_{max} = \frac{1}{2}(2.5)(25) = 31.25 \text{ kNm}$$

$V(\text{kN})$



-25

$M(\text{kNm})$



Bending. $S = \frac{1}{6}bh^2$ for rectangular section.

$$S = \frac{M_{max}}{\sigma_{all}} = \frac{31.25 \times 10^6}{12} = 2604167 \text{ mm}^3$$

$$h = \sqrt{\frac{6S}{b}} = \left[\frac{(6)(2604167)}{120} \right]^{1/2} = 360.8 \text{ mm}$$

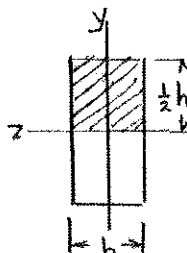
Shear. $I = \frac{1}{12}bh^3$ for rectangular section.

$$A = \frac{1}{2}bh$$

$$\bar{y} = \frac{1}{4}h$$

$$Q = A\bar{y} = \frac{1}{8}bh^2$$

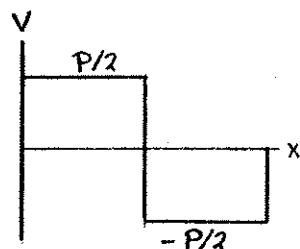
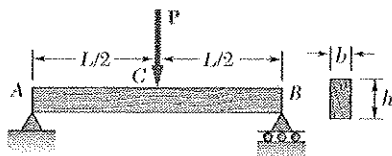
$$\tau_{max} = \frac{VQ}{Ib} = \frac{3V_{max}}{2bh}$$



$$h_v = \frac{3V_{max}}{2b\tau_{max}} = \frac{(3)(25 \times 10^3)}{(2)(120)(0.9)} = 347.2 \text{ mm}$$

The larger value of h is the minimum required depth. $h = 360.8 \text{ mm}$ ◀

Problem 6.19



6.19 A timber beam AB of length L and rectangular cross section carries a single concentrated load P at its midpoint C . (a) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (b) Determine the depth h and the width b of the beam, knowing that $L = 2$ m, $P = 40$ kN, $\tau_m = 960$ kPa, and $\sigma_m = 12$ MPa.

Reactions: $R_A = R_B = P/2 \uparrow$

(1) $V_{max} = R_A = \frac{P}{2}$

(2) $A = bh$ for rectangular section.

(3) $\tau_m = \frac{3}{2} \frac{V_{max}}{A} = \frac{3P}{4bh}$ for rectangular section.

(4) $M_{max} = \frac{PL}{4}$

(5) $S = \frac{1}{6} bh^2$ for rectangular section.

(6) $\sigma_m = \frac{M_{max}}{S} = \frac{3PL}{2bh^2}$

(a) $\frac{\tau_m}{\sigma_m} = \frac{h}{2L}$ ▶

(b) Solving for h , $h = \frac{2L\tau_m}{\sigma_m} = \frac{(2)(2)(960 \times 10^3)}{12 \times 10^6} = 320 \times 10^{-3}$ m

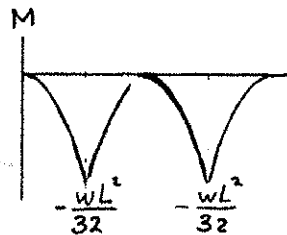
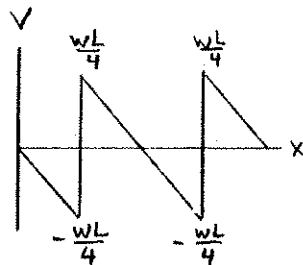
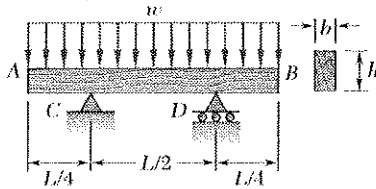
$h = 320$ mm ▶

Solving equation (3) for b ,

$b = \frac{3P}{4h\tau_m} = \frac{(3)(40 \times 10^3)}{(4)(320 \times 10^{-3})(960 \times 10^3)} = 97.7 \times 10^{-3}$ m

$b = 97.7$ mm ▶

Problem 6.20



6.20 A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown. (a) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (b) Determine the depth h and the width b of the beam, knowing that $L = 5$ m, $w = 8$ kN/m, $\tau_m = 1.08$ MPa, and $\sigma_m = 12$ MPa.

$$R_A = R_B = \frac{wL}{2}$$

From shear diagram, $|V|_m = \frac{wL}{4}$ (1)

For rectangular section, $A = bh$ (2)

$$\tau_m = \frac{3}{2} \frac{V_m}{A} = \frac{3wL}{8bh} \quad (3)$$

From bending moment diagram,

$$|M|_m = \frac{wL^2}{32} \quad (4)$$

For a rectangular cross section,

$$S = \frac{1}{6}bh^2 \quad (5)$$

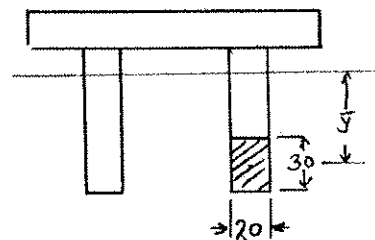
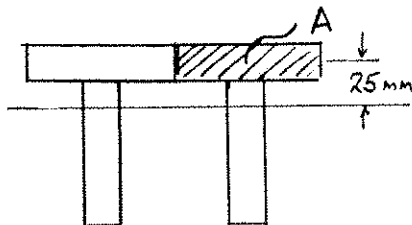
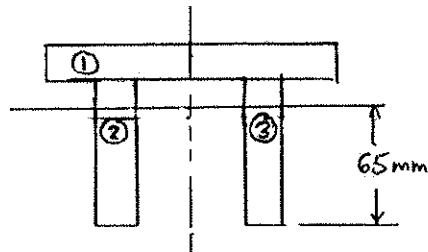
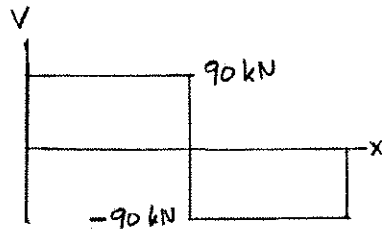
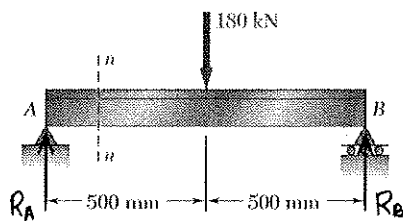
$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2} \quad (6)$$

(a) Dividing eq. (3) by eq. (6), $\frac{\tau_m}{\sigma_m} = \frac{2h}{L}$ $\blacktriangleleft$

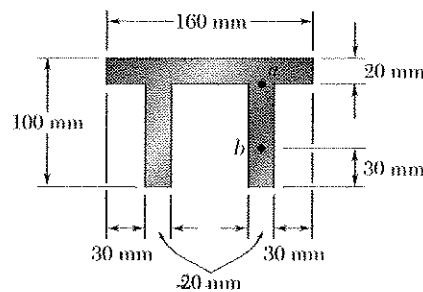
(b) Solving for h : $h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08 \times 10^6)}{(2)(12 \times 10^6)} = 225 \times 10^{-3} \text{ m}$
 $h = 225 \text{ mm}$ $\blacktriangleleft$

Solving eq. (3) for b : $b = \frac{3wL}{8h\tau_m} = \frac{(3)(8 \times 10^3)(5)}{(8)(225 \times 10^{-3})(1.08 \times 10^6)}$
 $= 61.7 \times 10^{-3} \text{ m}$ $b = 61.7 \text{ mm}$ $\blacktriangleleft$

Problem 6.21



6.21 and 6.22 For the beam and loading shown, consider section $n-n$ and determine the shearing stress at (a) point a, (b) point b.



Draw the shear diagram. $|V|_{max} = 90 \text{ kN}$

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	-25	1.000	0.8533
③	1600	40	64	-25	1.000	0.8533
Σ	6400		416		4.000	1.8133

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = (4.000 + 1.8133) \times 10^6 \text{ mm}^4 = 5.8133 \times 10^6 \text{ mm}^4 = 5.8133 \times 10^{-6} \text{ m}^4$$

$$(a) A = (80)(20) = 1600 \text{ mm}^2$$

$$\bar{y} = 25 \text{ mm}$$

$$Q_a = A\bar{y} = 40 \times 10^3 \text{ mm}^3 = 40 \times 10^{-6} \text{ m}^3$$

$$\tau_a = \frac{VQ_a}{I t} = \frac{(90 \times 10^3)(40 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^{-3})} = 31.0 \times 10^6 \text{ Pa}$$

$$\tau_a = 31.0 \text{ MPa}$$

$$(b) A = (30)(20) = 600 \text{ mm}^2$$

$$\bar{y} = 65 - 15 = 50 \text{ mm}$$

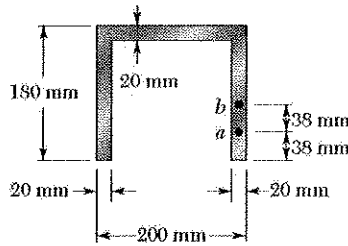
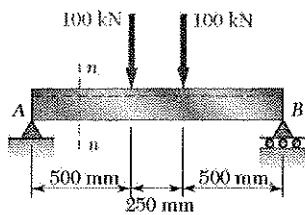
$$Q_b = A\bar{y} = 30 \times 10^3 \text{ mm}^3 = 30 \times 10^{-6} \text{ m}^3$$

$$\tau_b = \frac{VQ_b}{I t} = \frac{(90 \times 10^3)(30 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^{-3})} = 23.2 \times 10^6 \text{ Pa}$$

$$\tau_b = 23.2 \text{ MPa}$$

Problem 6.22

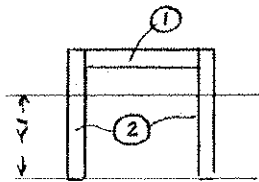
6.22 For the beam and loading shown, consider section $n-n$ and determine the shearing stress at (a) point a , (b) the shearing stress at point b .



$$R_A = R_B = 100 \text{ kN}$$

$$\text{At section } n-n \quad V = 100 \text{ kN}$$

Locate centroid and calculate moment of inertia.

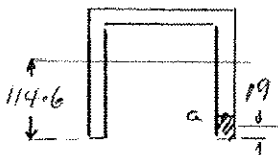


Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	3200	170	544000	55.4	9821312	106667
②	7200	90	648000	24.6	4357152	19440000
	10400		1192000		14178464	19546667

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{1192000}{10400} = 114.6 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 14178464 + 19546667 = 33.725 \times 10^6$$

(a)

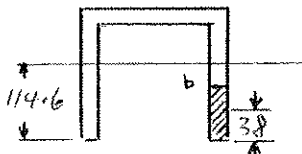


$$Q_a = A\bar{y} = (20)(38)(114.6 - 19) = 72656 \text{ mm}^3$$

$$t = 20 \text{ mm}$$

$$\tau_a = \frac{VQ}{It} = \frac{(100000)(72656)}{(33.725 \times 10^6)(20)} = 10.8 \text{ MPa}$$

(b)

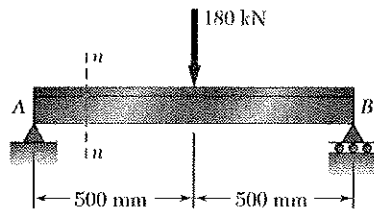


$$Q_b = A\bar{y} = (20)(76)(114.6 - 38) = 116432 \text{ mm}^3$$

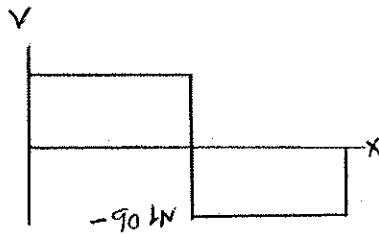
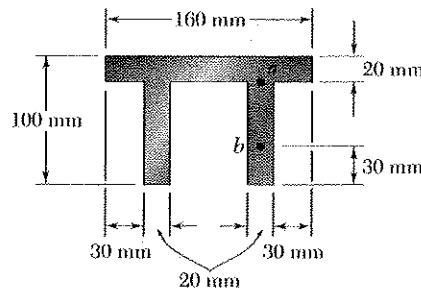
$$t = 20 \text{ mm}$$

$$\tau_b = \frac{VQ}{It} = \frac{(100000)(116432)}{(33.725 \times 10^6)(20)} = 17.3 \text{ MPa}$$

Problem 6.23



6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section $n-n$.

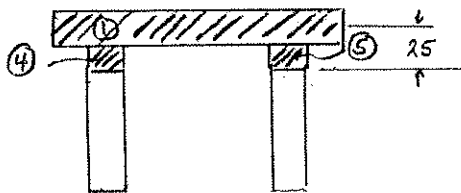
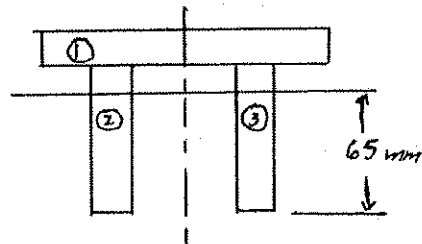


Draw the shear diagram. $|V|_{max} = 90 \text{ kN}$

Part	$A (\text{mm}^2)$	$\bar{y} (\text{mm})$	$A\bar{y} (10^3 \text{mm}^3)$	$d (\text{mm})$	$Ad^2 (10^6 \text{mm}^4)$	$\bar{I} (10^6 \text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	-25	1.000	0.8533
③	1600	40	64	-25	1.000	0.8533
Σ	6400		416		4.000	1.8133

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416 \times 10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = (4.000 + 1.8133) \times 10^6 \text{ mm}^4 = 5.8133 \times 10^6 \text{ mm}^4 = 5.8133 \times 10^{-6} \text{ m}^4$$



Part	$A (\text{mm}^2)$	$\bar{y} (\text{mm})$	$A\bar{y} (10^3 \text{mm}^3)$
①	3200	25	80
④	300	7.5	2.25
⑤	300	7.5	2.25
Σ			84.5

$$Q = \Sigma A\bar{y} = 84.5 \times 10^3 \text{ mm}^3 = 84.5 \times 10^{-6} \text{ m}^3$$

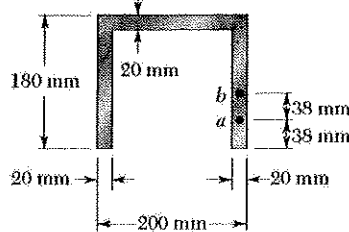
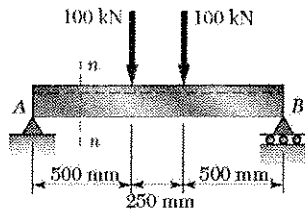
$$t = (2)(20) = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(90 \times 10^3)(84.5 \times 10^{-6})}{(5.8133 \times 10^{-6})(40 \times 10^{-3})} = 32.7 \times 10^6 \text{ Pa} \quad \tau_{max} = 32.7 \text{ MPa} \blacktriangleleft$$

Problem 6.24

6.24 For the beam and loading shown in Prob. 6.22, determine the largest shearing stress in section $n-n$.

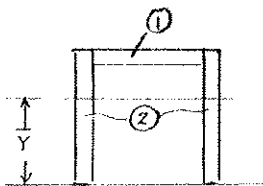
6.22 For the beam and loading shown, consider section $n-n$ and determine the shearing stress at (a) point a, (b) the shearing stress at point b.



$$R_A = R_B = 100 \text{ kN}$$

$$\text{At section } n-n \quad V = 100 \text{ kN}$$

Locate centroid and calculate moment of inertia.

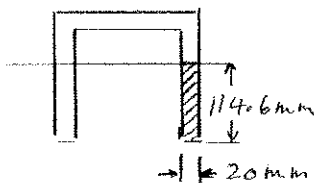


Part	A(mm ²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm ³)	d(mm)	Ad^2 (mm ⁴)	$\bar{I}$ (mm ⁴)
①	3200	170	544000	55.4	9821312	1066667
②	7200	90	648000	24.6	4357152	19440000
	10400		1192000		14178464	19546667

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{1192000}{10400} = 114.6 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 14178464 + 19546667 = 33.725 \times 10^6$$

Largest shearing stress occurs on section through centroid of entire cross section.

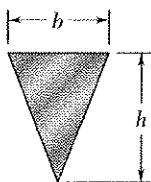


$$Q = A\bar{y} = (20)(114.6)\left(\frac{114.6}{2}\right) = 131331.6 \text{ mm}^3$$

$$t = 20 \text{ mm}$$

$$\tau = \frac{VQ}{It} = \frac{(100000)(131331.6)}{(33.725 \times 10^6)(20)} = 19.5 \text{ MPa}$$

Problem 6.25

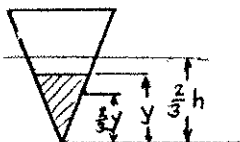


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear V . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k \frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$A = \frac{1}{2}bh \quad I = \frac{1}{36}bh^3$$



For a cut at location y ,

$$A(y) = \frac{1}{2} \left(\frac{b}{h}y \right) y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \frac{2}{3}h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h-y)$$

$$t(y) = \frac{by}{h}$$

$$\tau(y) = \frac{VQ}{It} = \frac{V \frac{by^2}{3}(h-y)}{\left(\frac{1}{36}bh^3 \right) \frac{by}{h}} = \frac{12Vy(h-y)}{bh^3} = \frac{12V}{bh^3}(hy - y^2)$$

(a) To find location of maximum of τ , set $\frac{d\tau}{dy} = 0$.

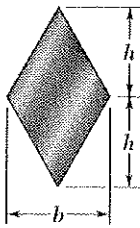
$$\frac{d\tau}{dy} = \frac{12V}{bh^3}(h - 2y_m) = 0$$

$$y_m = \frac{1}{2}h \quad \text{i.e. at mid-height} \quad \blacktriangleleft$$

$$(b) \quad \tau_m = \frac{12V}{bh^3}(hy_m - y_m^2) = \frac{12V}{bh^3} \left[\frac{1}{2}h^2 - \left(\frac{1}{2}h \right)^2 \right] = \frac{3V}{bh^2} = \frac{3}{2} \frac{V}{A}$$

$$k = \frac{3}{2} = 1.500 \quad \blacktriangleleft$$

Problem 6.26

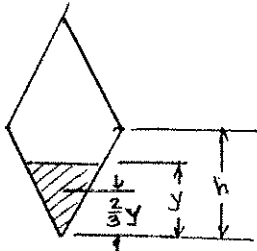


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear V . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k \frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$A = 2\left(\frac{1}{2}bh\right) = bh \quad I = 2\left(\frac{1}{12}bh^3\right) = \frac{1}{6}bh^3$$



For a cut at location y , where $y \leq h$

$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{2} - \frac{by^3}{3h}$$

$$t(y) = \frac{by}{h}$$

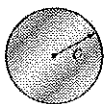
$$\tau(y) = \frac{VQ}{It} = V \left[\frac{6}{bh^2} \cdot \frac{h}{by} \cdot \frac{by^2}{2} - \frac{by^3}{3h} \right] = \frac{V}{bh} \left[3\left(\frac{y}{h}\right) - 2\left(\frac{y}{h}\right)^2 \right]$$

(a) To find the location of maximum of τ , set $\frac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{V}{bh^2} \left[3 - 4\frac{y}{h} \right] = 0 \quad \frac{y}{h} = \frac{3}{4} \quad \text{i.e. } \pm \frac{1}{4}h \text{ from neutral axis.} \quad \blacktriangleleft$$

$$(b) \quad \tau(y_m) = \frac{V}{bh} \left[3\left(\frac{3}{4}\right) - 2\left(\frac{3}{4}\right)^2 \right] = \frac{9}{8} \frac{V}{bh} = 1.125 \frac{V}{A} \quad k = 1.125 \quad \blacktriangleleft$$

Problem 6.27

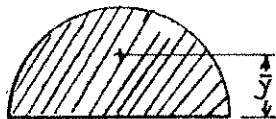


6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear V . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k \frac{V}{A}$$

where A is the cross-sectional area of the beam.

$$I = \frac{\pi}{4} c^4 \quad \text{and} \quad A = \pi c^2$$



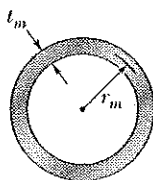
For semicircle, $A_s = \frac{\pi}{2} c^2$ $\bar{y} = \frac{4c}{3\pi}$

$$Q = A_s \bar{y} = \frac{\pi}{2} c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3} c^3$$

(a) $\tau_{\max}$ occurs at center where $t = 2c$. $\blacktriangleleft$

(b) $\tau_{\max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3} c^3}{\frac{\pi}{4} c^4 \cdot 2c} = \frac{4V}{3\pi c^2} = \frac{4}{3} \frac{V}{A} \quad k = \frac{4}{3} = 1.333 \quad \blacktriangleleft$

Problem 6.28



6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear V . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant k in the following expression for the maximum shearing stress

$$\tau_{\max} = k \frac{V}{A}$$

where A is the cross-sectional area of the beam.

For a thin walled circular section, $A = 2\pi r_m t_m$

$$J = A r_m^2 = 2\pi r_m^3 t_m, \quad I = \frac{1}{2} J = \pi r_m^3 t_m$$



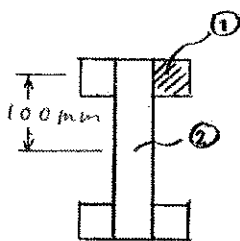
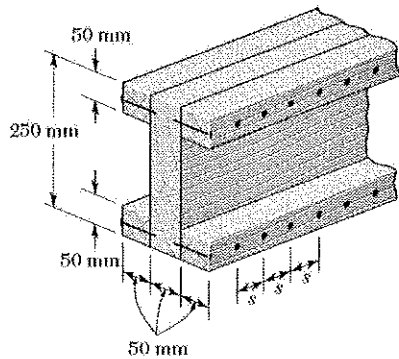
For a semicircular arc, $\bar{y} = \frac{2r_m}{\pi}$

$$A_s = \pi r_m t_m \quad Q = A_s \bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$$

(a) $t = 2t_m$ at neutral axis where maximum occurs. $\blacktriangleleft$

(b) $\tau_{\max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A} \quad k = 2.00 \quad \blacktriangleleft$

Problem 6.29



6.29 The built-up timber beam is subjected to a vertical shear of 5 kN. Knowing that the allowable shearing force in the nails is 300 N, determine the largest permissible spacing s of the nails.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12} (50)(50)^3 + (50)(50)(100)^2 = 25520833 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (50)(250)^3 = 65104167 \text{ mm}^4$$

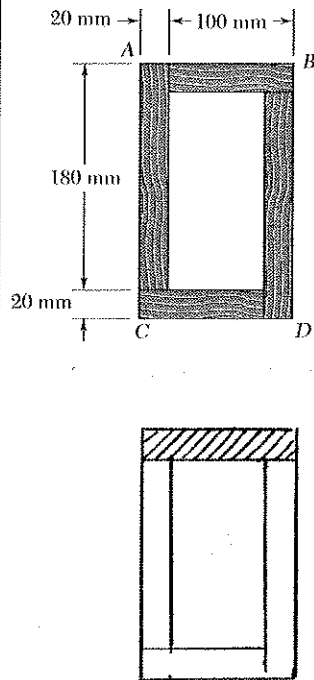
$$I = 4I_1 + I_2 = 167187499 \text{ mm}^4$$

$$Q = Q_1 = A_1 \bar{y}_1 = (50)(50)(100) = 250000 \text{ mm}^3$$

$$q = \frac{VQ}{I} = \frac{(5000)(250000)}{167187499} = 7.48 \text{ N/mm}$$

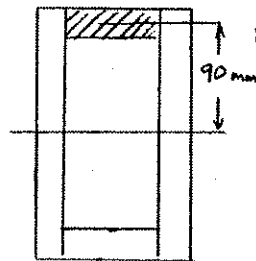
$$F_{\text{nail}} = qs \quad s = \frac{F_{\text{nail}}}{q} = \frac{300}{7.48} = 40.1 \text{ mm}$$

Problem 6.30



6.30 Two 20×100 -mm and two 20×180 -mm boards are glued together as shown to form a 120×200 -mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (a) at A, (b) at B.

$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \text{ mm}^4 = 52.693 \times 10^{-6} \text{ m}^4$$



$$(a) \quad Q_A = (80)(20)(90) = 144 \times 10^3 \text{ mm}^3 = 144 \times 10^{-6} \text{ m}^3$$

$$t_A = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$$

$$\tau_A = \frac{VQ_A}{It_A} = \frac{(3.5 \times 10^3)(144 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)}$$

$$= 239 \times 10^3 \text{ Pa} \quad \tau_A = 239 \text{ kPa} \quad \blacktriangleleft$$

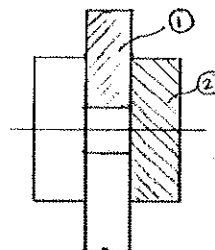
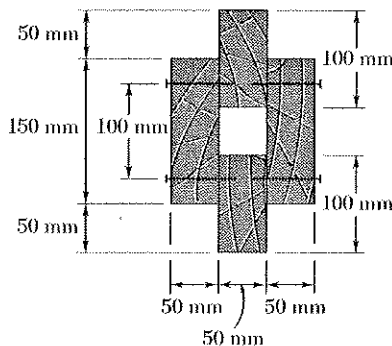
$$(b) \quad Q_B = (120)(20)(90) = 216 \times 10^3 \text{ mm}^3 = 216 \times 10^{-6} \text{ m}^3$$

$$t_B = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$$

$$\tau_B = \frac{VQ_B}{It_B} = \frac{(3.5 \times 10^3)(216 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} = 359 \times 10^3 \text{ Pa}$$

$$\tau_B = 359 \text{ kPa} \quad \blacktriangleleft$$

Problem 6.31



6.31 The built-up timber beam is subjected to a 6 -kN vertical shear. Knowing that the longitudinal spacing of the nails is $s = 60$ mm and that each nail is 90 mm long, determine the shearing force in each nail.

$$I_1 = \frac{1}{12}bh_1^3 + A_1d_1^2 = \frac{1}{12}(50)(100)^3 + (50)(100)(75)^2 = 32.292 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(150)^3 = 14.0625 \times 10^6 \text{ mm}^4$$

$$I = 2I_1 + 2I_2 = 92.71 \times 10^6 \text{ mm}^4 = 92.71 \times 10^{-6} \text{ m}^4$$

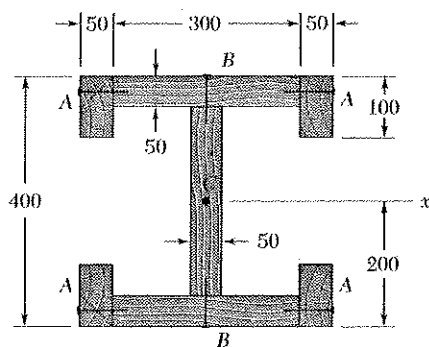
$$Q = Q_1 = A_1\bar{y}_1 = (50)(100)(75) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(6 \times 10^3)(375 \times 10^{-6})}{92.71 \times 10^{-6}} = 24.27 \times 10^3 \text{ N/m}$$

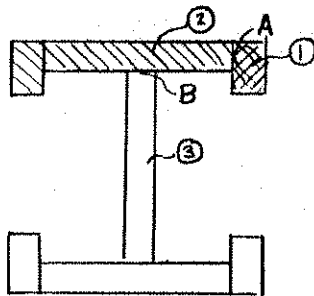
$$s = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

$$2F_{\text{nail}} = qs \quad F_{\text{nail}} = \frac{1}{2}qs = \frac{1}{2}(24.27 \times 10^3)(60 \times 10^{-3}) = 728 \text{ N} \quad \blacktriangleleft$$

Problem 6.32



Dimensions in mm



6.32 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (a) at A, (b) at B. (Given: $I_x = 1.504 \times 10^9 \text{ mm}^4$.)

$$I_x = 1.504 \times 10^9 \text{ mm}^4 = 1504 \times 10^{-6} \text{ m}^4$$

$$s_A = 60 \text{ mm} = 0.060 \text{ m}$$

$$s_B = 25 \text{ mm} = 0.025 \text{ m}$$

$$(a) \quad Q_A = Q_1 = A_1 \bar{y}_1 = (50)(100)(150) = 750 \times 10^3 \text{ mm}^3 = 750 \times 10^{-6} \text{ m}^3$$

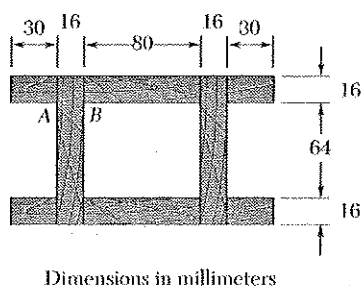
$$\begin{aligned} F_A &= q_A s_A \\ &= \frac{V Q_1 s_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}} \\ &= 239 \text{ N} \end{aligned}$$

$$(b) \quad Q_2 = A_2 \bar{y}_2 = (300)(50)(175) = 2625 \times 10^3 \text{ mm}^3$$

$$\begin{aligned} Q_B &= 2Q_1 + Q_2 = 4125 \times 10^3 \text{ mm}^3 \\ &= 4125 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$F_B = q_B s_B = \frac{V Q_B s_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}} \quad F_B = 549 \text{ N}$$

Problem 6.33



6.33 The built-up beam was made by gluing together several wooden planks. Knowing that the beam is subjected to a 5-kN shear, determine the average shearing stress in the glued joint (a) at A, (b) at B.

$$b_1 = 16 \text{ mm}, \quad h_1 = 16 + 64 + 16 = 96 \text{ mm}$$

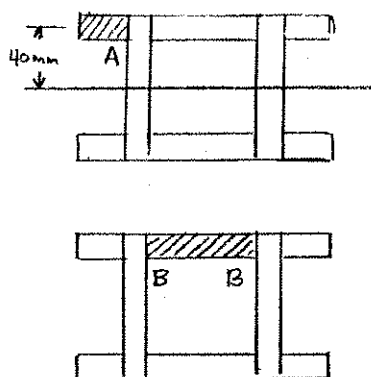
$$I_1 = \frac{1}{12} b_1 h_1^3 = 1.17965 \times 10^5 \text{ mm}^4$$

$$b_2 = 30 + 80 + 30 = 140 \text{ mm}, \quad t_2 = 16 \text{ mm}$$

$$A_2 = b_2 h_2 = 2240 \text{ mm}^2, \quad d_2 = \frac{64}{2} + \frac{16}{2} = 40 \text{ mm}$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = 3.63179 \times 10^6 \text{ mm}^4$$

$$I = 2I_1 + 2I_2 = 9.6229 \times 10^6 \text{ mm}^4 = 9.6229 \times 10^{-6} \text{ m}^4$$



$$(a) \quad A_A = (30)(16) = 480 \text{ mm}^2, \quad \bar{y}_A = 40 \text{ mm}$$

$$Q_A = A_A \bar{y}_A = 19.2 \times 10^3 \text{ mm}^3 = 19.2 \times 10^{-6} \text{ m}^3$$

$$t_A = 16 \text{ mm} = 0.016 \text{ m}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{(5 \times 10^3)(19.2 \times 10^{-6})}{(9.6229 \times 10^{-6})(0.016)}$$

$$= 624 \times 10^3 \text{ Pa}$$

$$\tau_A = 624 \text{ kPa} \quad \blacktriangleleft$$

$$(b) \quad A_B = (80)(16) = 1280 \text{ mm}^2, \quad \bar{y}_B = 40 \text{ mm}$$

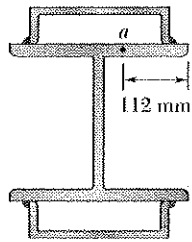
$$Q_B = A_B \bar{y}_B = 51.2 \times 10^3 \text{ mm}^3 = 51.2 \times 10^{-6} \text{ m}^3$$

$$t_B = (2)(16) = 32 \text{ mm} = 0.032 \text{ m}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{(5 \times 10^3)(51.2 \times 10^{-6})}{(9.6229 \times 10^{-6})(0.032)} = 831 \times 10^3 \text{ Pa}$$

$$\tau_B = 831 \text{ kPa} \quad \blacktriangleleft$$

Problem 6.34



6.34 The composite beam shown is made by welding C200 × 17.1 rolled-steel channels to the flanges of a W250 × 80 wide-flange rolled-steel shape. Knowing that the beam is subjected to a vertical shear of 200 kN, determine (a) the horizontal shearing force per meter at each weld, (b) the shearing stress at point *a* of the flange of the wide-flange shape.

For W250 × 80: $d = 256 \text{ mm}$, $t_f = 15.6 \text{ mm}$, $I_x = 126 \times 10^6 \text{ mm}^4$

For C200 × 17.1: $A = 2170 \text{ mm}^2$, $b_f = 57 \text{ mm}$, $t_f = 9.9 \text{ mm}$,
 $I_y = 0.538 \times 10^6 \text{ mm}^4$, $\bar{x} = 14.4 \text{ mm}$

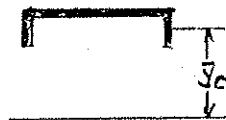
For the channel in the composite beam,

$$\bar{y}_c = \frac{256}{2} + 57 - 14.4 = 170.6 \text{ mm}$$

For the composite beam,

$$I = 126 \times 10^6 + 2[0.538 \times 10^6 + (2170)(170.6)^2]$$

$$= 253.39 \times 10^6 \text{ mm}^4 = 253.39 \times 10^{-6} \text{ m}^4$$



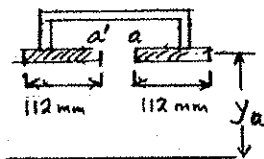
(a) For the two welds,

$$Q_w = A\bar{y}_c = (2170)(170.6)$$

$$= 370.20 \times 10^3 \text{ mm}^3 = 370.20 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(200 \times 10^3)(370.20 \times 10^{-6})}{253.39 \times 10^{-6}}$$

$$= 292 \times 10^3 \text{ N/m}$$



For one weld, $\frac{q}{2} = 146.1 \times 10^3 \text{ N/m}$

Shearing force per meter of weld: 146.1 kN/m

(b) For cuts at *a* and *a'* together,

$$A_a = 2(112)(15.6) = 3494.4 \text{ mm}^2 \quad \bar{y}_a = \frac{256}{2} - \frac{15.6}{2} = 120.2 \text{ mm}$$

$$Q_a = 370.20 \times 10^3 + (3494.4)(120.2) = 790.23 \times 10^3 \text{ mm}^3 = 790.23 \times 10^{-6} \text{ m}^3$$

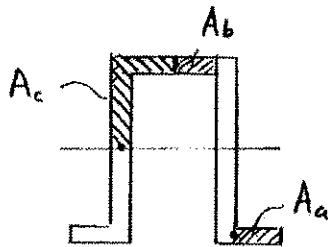
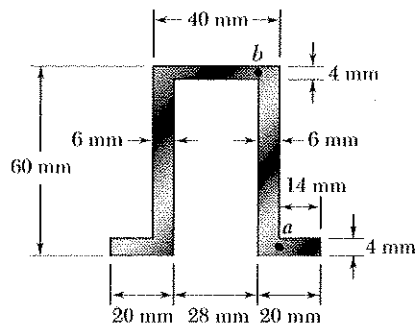
Since there are cuts at *a* and *a'*, $t = 2t_f = (2)(15.6) = 31.2 \text{ mm} = 0.0312 \text{ m}$.

$$\tau_a = \frac{VQ_a}{It} = \frac{(200 \times 10^3)(790.23 \times 10^{-6})}{(253.39 \times 10^6)(0.0312)} = 19.99 \times 10^6 \text{ Pa}$$

$$\tau_a = 19.99 \text{ MPa}$$

Problem 6.35

6.35 Knowing that a given vertical shear V causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress at (a) point a , (b) point b .



Neutral axis lies 30 mm above bottom.

$$\tau_c = \frac{VQ_c}{It}, \quad \tau_a = \frac{VQ_a}{It_a}, \quad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a}, \quad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$

$$Q_c = (6)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$$

$$t_c = 6 \text{ mm}$$

$$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$$

$$t_a = 4 \text{ mm}$$

$$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$$

$$t_b = 4 \text{ mm}$$

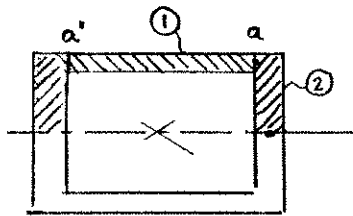
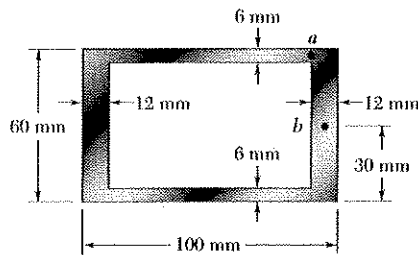
$$\tau_c = 75 \text{ MPa}$$

$$\tau_a = \frac{Q_a}{Q_c} \cdot \frac{t_c}{t_a} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

$$\tau_b = \frac{Q_b}{Q_c} \cdot \frac{t_c}{t_b} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

Problem 6.36

6.36 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 40 kN, determine the shearing stress at (a) point a, (b) point b.



$$I = \frac{1}{12}(100)(60)^3 - \frac{1}{12}(100-25)(60-12)^3$$

$$= 1108800 \text{ mm}^4$$

$$(a) \quad Q_a = A_1 \bar{y}_1 = (100-25)(6)\left(30 - \frac{6}{2}\right)$$

$$= 12150 \text{ mm}^3$$

$$t_a = (2)(6) = 12 \text{ mm}$$

$$\tau_a = \frac{V Q_a}{I t_a} = \frac{(40 \times 10^3)(12150 \times 10^{-9})}{(1108800 \times 10^{-12})(0.012)}$$

$$\tau_a = 36.5 \text{ MPa}$$

$$(b) \quad Q_b = 12150 + (2)(12)(30)\left(\frac{30}{2}\right)$$

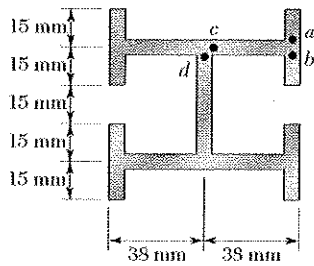
$$= 22950 \text{ mm}^3$$

$$t_b = (2)(12) = 24 \text{ mm}$$

$$\tau_b = \frac{V Q_b}{I t_b} = \frac{(40 \times 10^3)(22950 \times 10^{-9})}{(1108800 \times 10^{-12})(0.024)}$$

$$\tau_b = 34.5 \text{ MPa}$$

Problem 6.37



$$\tau = \frac{VQ}{It}$$

Since V , I , and t are constant, τ is proportional to Q .

$$\frac{\tau_a}{1953.125} = \frac{\tau_b}{2929.688} = \frac{\tau_c}{8876.563} = \frac{\tau_d}{18315.626} = \frac{\tau_m}{19315.626} = \frac{60}{19315.626}$$

$$\tau_a = 6.07 \text{ MPa}; \tau_b = 9.1 \text{ MPa}; \tau_c = 27.6 \text{ MPa}; \tau_d = 56.9 \text{ MPa}$$

6.37 An extruded beam has the cross section shown and a uniform wall thickness of 5 mm. Knowing that a given vertical shear V causes a maximum shearing stress $\tau = 60 \text{ MPa}$, determine the shearing stress at the four points indicated.

$$Q_a = (5)(12.5)(37.5 - 6.25) = 1953.125 \text{ mm}^3$$

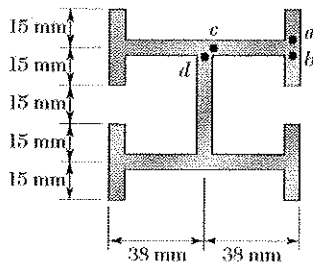
$$Q_b = (5)(12.5)(7.5 + 6.25) = 2929.688 \text{ mm}^3$$

$$Q_c = Q_a + Q_b + (35.5)(5)(22.5) = 8876.563 \text{ mm}^3$$

$$Q_d = 2Q_a + 2Q_b + (76)(5)(22.5) = 18315.626 \text{ mm}^3$$

$$Q_m = Q_d + (5)(20)(10) = 19315.626$$

Problem 6.38



6.38 Solve Prob. 6.37 assuming that the beam is subjected to a horizontal shear V .

6.37 An extruded beam has the cross section shown and a uniform wall thickness of 5 mm. Knowing that a given vertical shear V causes a maximum shearing stress $\tau = 60 \text{ MPa}$, determine the shearing stress at the four points indicated.

$$Q_a = (12.5)(5)(35.5) = 2218.75 \text{ mm}^3$$

$$Q_b = (12.5)(5)(35.5) = 2218.75 \text{ mm}^3$$

$$Q_c = Q_a + Q_b + (5)(35.5)(20) = 7987.5 \text{ mm}^3$$

$$Q_d = 0$$

$$Q_m = Q_c = 7987.5 \text{ mm}^3$$

$$\tau = \frac{VQ}{It}$$

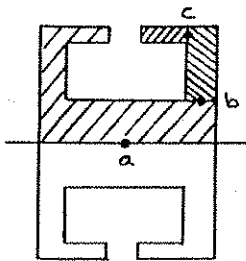
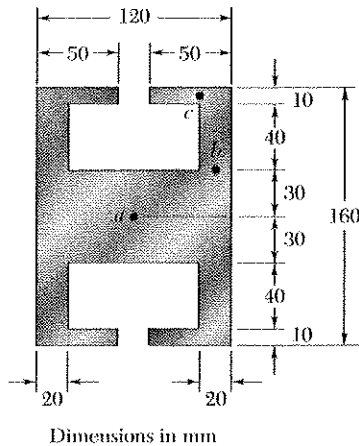
Since V , I , and t are constant, τ is proportional to Q .

$$\frac{\tau_a}{2218.75} = \frac{\tau_b}{2218.75} = \frac{\tau_c}{7987.5} = \frac{\tau_d}{0} = \frac{\tau_m}{7987.5} = \frac{60}{7987.5}$$

$$\tau_a = 16.7 \text{ MPa}; \tau_b = 16.7 \text{ MPa}; \tau_c = 60 \text{ MPa}; \tau_d = 0$$

Problem 6.39

6.39 Knowing that a given vertical shear V causes a maximum shearing stress of 75 MPa an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.



$$\tau = \frac{VQ}{It} \quad \tau \text{ is proportional to } Q/t.$$

Point c: $Q_c = (30)(10)(75) = 22.5 \times 10^3 \text{ mm}^3$

$$t_c = 10 \text{ mm}$$

$$Q_c/t_c = 2250 \text{ mm}^2$$

Point b: $Q_b = Q_c + (20)(50)(55) = 77.5 \times 10^3 \text{ mm}^3$

$$t_b = 20 \text{ mm}$$

$$Q_b/t_b = 3875 \text{ mm}^2$$

Point a: $Q_a = 2Q_b + (120)(30)(15) = 209 \times 10^3 \text{ mm}^3$

$$t_a = 120 \text{ mm}$$

$$Q_a/t_a = 1741.67 \text{ mm}^2$$

$(Q/t)_m$ occurs at b. $\tau_m = \tau_b = 75 \text{ MPa}$

$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

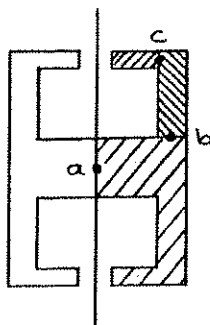
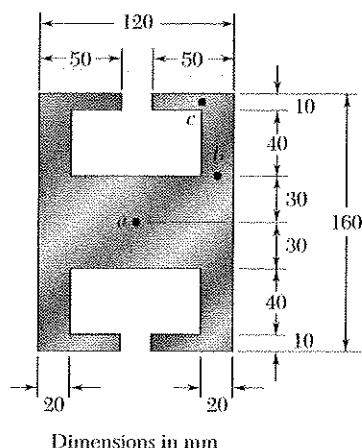
$$\frac{\tau_a}{1741.67 \text{ mm}^2} = \frac{75 \text{ MPa}}{3875 \text{ mm}^2} = \frac{\tau_c}{2250 \text{ mm}^2}$$

$$\tau_a = 33.7 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_b = 75.0 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_c = 43.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 6.40



6.40 Solve Prob. 6.39 assuming that the beam is subjected to a horizontal shear V .

6.39 Knowing that a given vertical shear V causes a maximum shearing stress of 75 MPa an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.

$$\tau = \frac{VQ}{It} \quad \tau \text{ is proportional to } Q/t$$

Point c: $Q_c = (10)(30)(25) = 7.5 \times 10^3 \text{ mm}^3$
 $t_c = 10 \text{ mm}$
 $Q_c/t_c = 750 \text{ mm}^2$

Point b: $Q_b = Q_c + (50)(20)(50) = 57.5 \times 10^3 \text{ mm}^3$
 $t_b = 20 \text{ mm}$
 $Q_b/t_b = 2875 \text{ mm}^2$

Point a: $Q_a = 2Q_b + (60)(60)(30) = 223 \times 10^3 \text{ mm}^3$
 $t_a = 60 \text{ mm}$
 $Q_a/t_a = 3716.7 \text{ mm}^2$

$(Q/t)_m$ occurs at a. $\tau_m = \tau_a = 75 \text{ MPa}$

$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

$$\frac{75 \text{ MPa}}{3716.7 \text{ mm}^2} = \frac{\tau_b}{2875 \text{ mm}^2} = \frac{\tau_c}{750 \text{ mm}^2}$$

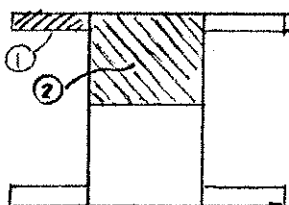
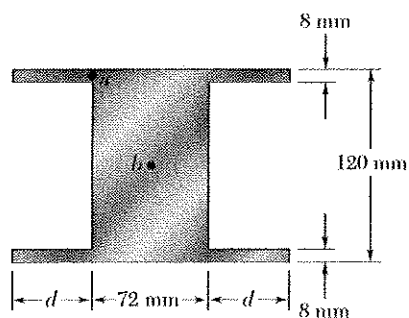
$$\tau_a = 75.0 \text{ MPa}$$

$$\tau_b = 58.0 \text{ MPa}$$

$$\tau_c = 15.13 \text{ MPa}$$

Problem 6.41

6.41 The vertical shear is 25 kN in a beam having the cross section shown. Knowing that $d = 50$ mm, determine the shearing stress at (a) point a, (b) point b.



$$I_1 = \frac{1}{12} (50)(8)^3 + (50)(8)(56)^2 = 1.25653 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} (72)(60)^3 = 5.184 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 = 15.3933 \times 10^6 \text{ mm}^4 = 15.3933 \times 10^{-6} \text{ m}^4$$

$$Q_1 = A_1 \bar{y}_1 = (50)(8)(56) = 22.4 \times 10^3 \text{ mm}^3 = 22.4 \times 10^{-6} \text{ m}^3$$

$$Q_2 = A_2 \bar{y}_2 = (72)(60)(30) = 129.6 \times 10^3 \text{ mm}^3 = 129.6 \times 10^{-6} \text{ m}^3$$

$$(a) \quad Q_a = Q_1 \quad t_a = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(25 \times 10^3)(22.4 \times 10^{-6})}{(15.3933 \times 10^{-6})(8 \times 10^{-3})} = 4.55 \times 10^6 \text{ Pa} \quad \tau_a = 4.55 \text{ MPa}$$

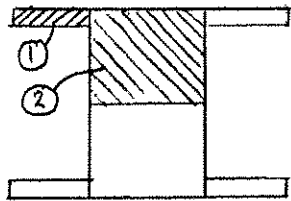
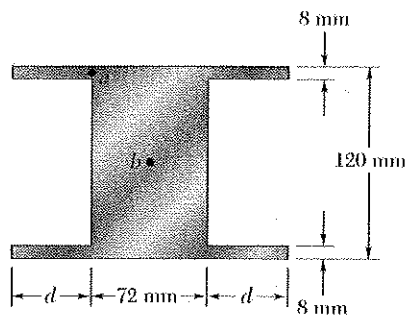
$$(b) \quad Q_b = 2Q_1 + Q_2 = 174.4 \times 10^{-6} \text{ m}^3$$

$$t_b = 72 \text{ mm} = 72 \times 10^{-3} \text{ m}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(25 \times 10^3)(174.4 \times 10^{-6})}{(15.3933 \times 10^{-6})(72 \times 10^{-3})} = 3.93 \times 10^6 \text{ Pa}$$

$$\tau_b = 3.93 \text{ MPa}$$

Problem 6.42



6.42 The vertical shear is 25 kN in a beam having the cross section shown. Determine (a) the distance d for which $\tau_a = \tau_b$, (b) the corresponding shearing stress at points a and b .

$$Q_1 = (d)(8)(56) = 448d \text{ mm}^3$$

$$Q_2 = (72)(60)(30) = 129.6 \times 10^3 \text{ mm}^3$$

$$Q_a = Q_1 = 448d \quad t_a = 8 \text{ mm}$$

$$Q_b = 2Q_1 + Q_2 = 896d + 129.6 \times 10^3 \text{ mm}^3$$

$$t_b = 72 \text{ mm}$$

$$\tau_a = \frac{VQ_a}{It_a}$$

$$\tau_b = \frac{VQ_b}{It_b}$$

$$\text{Since } \tau_a = \tau_b \quad \frac{Q_a}{t_a} = \frac{Q_b}{t_b}$$

$$(a) \quad \frac{448d}{8} = \frac{896d + 129.4 \times 10^3}{72} \quad d = 41.3 \text{ mm} \leftarrow$$

$$(b) \quad I_1 = \frac{1}{12}(d)(8)^3 + (d)(8)(56)^2 = 25.131d = 1.03696 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{3}(72)(60)^3 = 5.184 \times 10^6 \text{ mm}^4$$

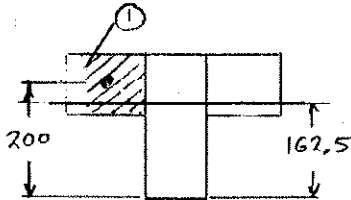
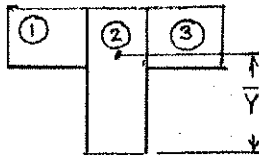
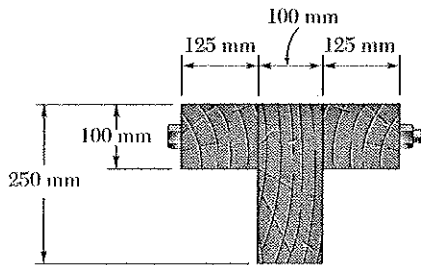
$$I = 4I_1 + 2I_2 = 14.5158 \times 10^6 \text{ mm}^4 = 14.5158 \times 10^{-6} \text{ m}^4$$

$$Q_a = 448d = (448)(41.263) = 18.4857 \times 10^3 \text{ mm}^3 = 18.4857 \times 10^{-6} \text{ m}^3$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(25 \times 10^3)(18.4857 \times 10^{-6})}{(14.5158 \times 10^{-6})(8 \times 10^{-3})} = 3.98 \times 10^6 \text{ Pa}$$

$$\tau_a = 3.98 \text{ MPa} \leftarrow$$

Problem 6.43



6.43 Three planks are connected as shown by bolts of 14-mm diameter spaced every 150 mm along the longitudinal axis of the beam. For a vertical shear of 10 kN, determine the average shearing stress in the bolts.

Locate neutral axis and compute moment of inertia.

PART	A (mm ²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm ³)	d (mm)	Ad^2 (mm ⁴)	$\bar{I}$ (mm ⁴)
①	12 500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
②	25 000	125	3.125×10^6	37.5	35.156×10^6	130.208×10^6
③	12 500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
Σ	50 000		8.125×10^6		70.312×10^6	151.041×10^6

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.125 \times 10^6}{50 \times 10^3} = 162.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 221.35 \times 10^6 \text{ mm}^4 = 221.35 \times 10^{-6} \text{ m}^4$$

$$Q = A_1 \bar{y}_1 = (12500)(37.5) = 468.75 \times 10^3 \text{ mm}^3 = 468.75 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(10 \times 10^3)(468.75 \times 10^{-6})}{221.35 \times 10^{-6}} = 21.177 \times 10^3 \text{ N}$$

$$F_{\text{bolt}} = q s = (21.177 \times 10^3)(150 \times 10^{-3}) = 3.1765 \times 10^3 \text{ N}$$

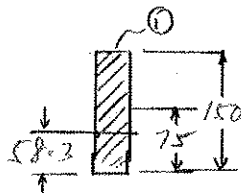
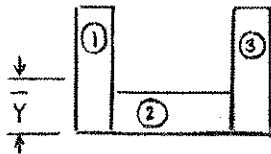
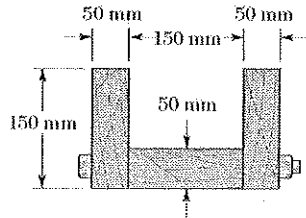
$$A_{\text{bolt}} = \frac{\pi}{4} (14)^2 = 153.938 \text{ mm}^2 = 153.938 \times 10^{-6} \text{ m}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{3.1765 \times 10^3}{153.938 \times 10^{-6}} = 20.6 \times 10^6 \text{ Pa}$$

$$\tau_{\text{bolt}} = 20.6 \text{ MPa}$$

Problem 6.44

6.44 A beam consists of three planks connected as shown by 10-mm-diameter bolts spaced every 300 mm along the longitudinal axis of the beam. Knowing that the beam is subjected to a 10-kN vertical shear, determine the average shearing stress in the bolts.



Locate neutral axis and compute moment of inertia.

Part	A (mm ²)	$\bar{y}$ (mm)	$A\bar{y}$ mm ³	d (mm)	Ad^2 (mm ⁴)	$\bar{I}$ (mm ⁴)
①	12	3	36	0.667	5.333	36
②	12	1	12	1.333	21.333	4
③	12	3	36	0.667	5.333	36
Σ	36		84		32	76

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{1312500}{22500} = 58.3 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 42.19 \times 10^6 \text{ mm}^4$$

$$Q = A_i \bar{y}_i = (7500)(75 - 58.3) = 125250 \text{ mm}^3$$

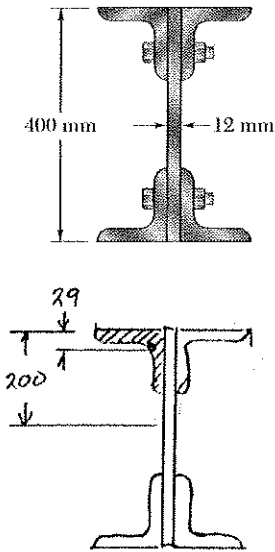
$$q = \frac{VQ}{I} = \frac{(10)(125250)}{42.19 \times 10^6} = 29.7 \text{ N/mm}$$

$$F_{\text{bolt}} = q s = (29.7)(300) = 8.91 \text{ kN}$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} (10)^2 = 78.54 \text{ mm}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{8910}{78.54} = 113.4 \text{ MPa}$$

Problem 6.45



6.45 Four L102 × 102 × 9.5 steel angle shapes and a 12 × 400-mm steel plate are bolted together to form a beam with the cross section shown. The bolts are of 22-mm diameter and are spaced longitudinally every 120 mm. Knowing that the beam is subjected to a vertical shear of 240 kN, determine the average shearing stress in each bolt.

$$\text{Angle: } A = 1850 \text{ mm}^2, \quad \bar{I} = 1.83 \times 10^6 \text{ mm}^4, \quad y = 29 \text{ mm}$$

$$d = 200 - 29 = 171 \text{ mm}$$

$$I_a = \bar{I} + A d^2 = 55.926 \times 10^6 \text{ mm}^4$$

$$\text{Plate: } I_p = \frac{1}{12} (12) (400)^3 = 64 \times 10^6 \text{ mm}^4$$

$$I = 4 I_a + I_p = 287.7 \times 10^6 \text{ mm}^4 = 287.7 \times 10^{-6} \text{ m}^4$$

$$Q = (1850)(171) = 316.35 \times 10^3 \text{ mm}^3 = 316.35 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(240 \times 10^3)(316.35 \times 10^{-6})}{287.7 \times 10^{-6}} = 263.9 \times 10^3 \text{ N/m}$$

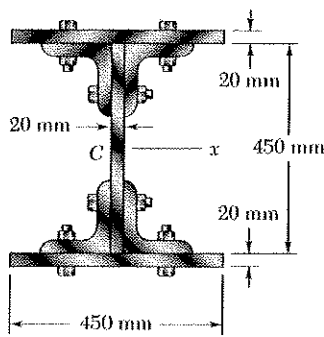
$$F_{\text{bolt}} = qs = (263.9 \times 10^3)(120 \times 10^{-3}) = 31.668 \times 10^3 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{31.668 \times 10^3}{380.13 \times 10^{-6}} = 83.3 \times 10^6 \text{ Pa}$$

$$\tau_{\text{bolt}} = 83.3 \text{ MPa} \quad \blacktriangleleft$$

Problem 6.46



6.46 Two 20 × 450-mm steel plates are bolted to four L 152 × 152 × 19.0 angles to form a beam with the cross section shown. The bolts have a 22-mm diameter and are spaced longitudinally every 125 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shear in the beam. (Given: $I_x = 1896 \times 10^6 \text{ mm}^4$).

$$\text{Flange: } I_f = \frac{1}{12} (450)(20)^3 + (450)(20)(235)^2 = 497.3 \times 10^6 \text{ mm}^4$$

$$\text{Web: } I_w = \frac{1}{12} (20)(450)^3 = 151.9 \times 10^6 \text{ mm}^4$$

$$\text{Angle: } \bar{I} = 11.6 \times 10^6 \text{ mm}^4, \quad A = 5420 \text{ mm}^2$$

$$y = 44.9 \text{ mm} \quad d = 225 - 44.9 = 180.1 \text{ mm}$$

$$I_a = \bar{I} + Ad^2 = 11.6 \times 10^6 + (5420)(180.1)^2 = 187.4 \times 10^6 \text{ mm}^4$$

$$I = 2I_f + I_w + 4I_a = 1896 \times 10^6 \text{ mm}^4 = 1896 \times 10^{-6} \text{ m}^4$$



$$Q_f = (450)(20)(235) = 2115 \times 10^3 \text{ mm}^3$$

$$Q_a = (5420)(180.1) = 976 \times 10^3 \text{ mm}^3$$

$$Q = Q_f + 2Q_a = 4067 \times 10^3 \text{ mm}^3 = 4067 \times 10^{-6} \text{ m}^3$$

$$A_{b,H} = \frac{\pi}{4} d_{b,H}^2 = \frac{\pi}{4} (22)^2 = 380.1 \text{ mm}^2 = 380.1 \times 10^{-6} \text{ m}^2$$

$$F_{b,H} = 2\tau_{b,H} A_b = (2)(90 \times 10^6)(380.1 \times 10^{-6}) = 68.42 \times 10^3 \text{ N}$$

$$q_{all} = \frac{F_{b,H}}{s} = \frac{68.42 \times 10^3}{0.125} = 547.36 \times 10^3 \text{ N/m}$$

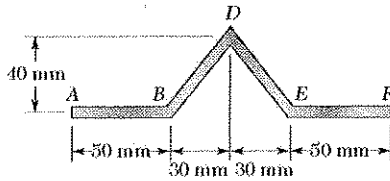
$$q = \frac{VQ}{I}$$

$$V_{all} = \frac{I q_{all}}{Q} = \frac{(1896 \times 10^{-6})(547.36 \times 10^3)}{4067 \times 10^{-6}} = 255 \times 10^3 \text{ N}$$

$$= 255 \text{ kN}$$

Problem 6.47

6.47 A plate of 6-mm thickness is corrugated as shown and then used as a beam. For a vertical shear of 5 kN, determine (a) the maximum shearing stress in the section, (b) the shearing stress at point B. Also sketch the shear flow in the cross section.



$$L_{BD} = \sqrt{(30)^2 + (40)^2} = 50 \text{ mm}$$

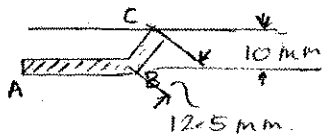
$$A_{BD} = (6)(50) = 300 \text{ mm}^2$$

Locate neutral axis and compute moment of inertia.

Part	A (mm ²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm ³)	d (mm)	Ad^2 (mm ⁴)	$\bar{I}$ (mm ⁴)	$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{12000}{1200} = 10 \text{ mm}$
AB	300	0	0	10	30000	neglect	
BD	300	20	6000	10	30000	* 40000	* $\frac{1}{12} A_{BD} h^2 = \frac{1}{12} (300)(40)^2 = 40000 \text{ mm}^4$
DE	300	20	6000	10	30000	* 40000	
EF	300	0	0	10	30000	neglect	
Σ	1200		12000		120000	80000	$I = \sum Ad^2 + \sum \bar{I} = 200000 \text{ mm}^4$

(a)

$$Q_m = Q_{AB} + Q_{BC}$$



$$Q_{AB} = (50)(6)(10) = 3000 \text{ mm}^3$$

$$Q_{BC} = (12.5)(6)(5) = 375 \text{ mm}^3$$

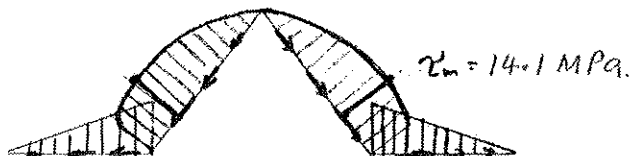
$$Q_m = 3375 \text{ mm}^3$$

$$\tau_m = \frac{V Q_m}{I t} = \frac{(5000)(3375)}{(200000)(6)} = 14.1 \text{ MPa}$$

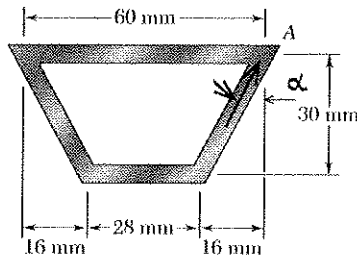
(b) $Q_B = Q_{AB} = 3000 \text{ mm}^3$

$$\tau_B = \frac{V Q_B}{I t} = \frac{(5000)(3000)}{(200000)(6)} = 12.5 \text{ MPa}$$

$$\tau_D = 0$$



Problem 6.48



6.48 An extruded beam has the cross section shown and a uniform wall thickness of 3 mm. For a vertical shear of 10 kN, determine (a) the shearing stress at point A, (b) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

$$\tan \alpha = \frac{16}{30} \quad \alpha = 28.07^\circ$$

$$\text{Side: } A = (3 \sec \alpha)(30) = 102 \text{ mm}^2$$

$$\bar{I} = \frac{1}{12} (3 \sec \alpha)(30)^3 = 7.6498 \times 10^3 \text{ mm}^4$$

Part	A (mm ²)	$\bar{y}_o$ (mm)	$A\bar{y}$ (10 ³ mm ³)	d (mm)	Ad^2 (10 ³ mm ⁴)	$\bar{I}$ (10 ³ mm ⁴)
Top	180	30	5.4	11.932	25.627	neglect
Side	102	15	1.53	3.077	0.966	7.6498
Side	102	15	1.53	3.077	0.966	7.6498
Bot	84	0	0	18.077	27.449	neglect
Σ	468		8.46		55.008	15.2996

$$\bar{y}_o = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.46 \times 10^3}{468} = 18.077 \text{ mm}$$

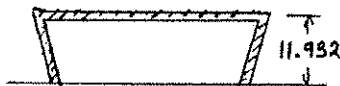
$$I = \Sigma Ad^2 + \Sigma \bar{I} = 70.31 \times 10^3 \text{ mm}^4 = 70.31 \times 10^{-9} \text{ m}^4$$

$$(a) \quad Q_A = (180)(11.932) = 2.14776 \times 10^3 \text{ mm}^3 = 2.14776 \times 10^{-6} \text{ m}^3$$

$$t = (2)(3 \times 10^{-3}) = 6 \times 10^{-3} \text{ m}$$

$$\tau_A = \frac{VQ}{It} = \frac{(10 \times 10^3)(2.14776 \times 10^{-6})}{(70.31 \times 10^{-9})(6 \times 10^{-3})} = 50.9 \times 10^6 \text{ Pa} \quad \tau_A = 50.9 \text{ MPa}$$

(b)



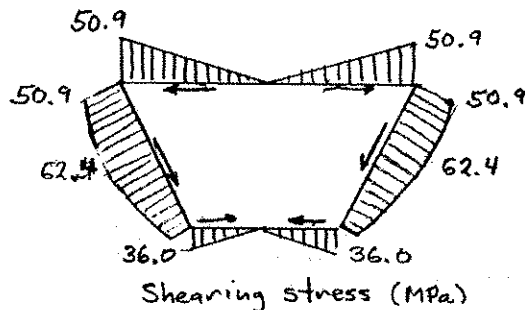
$$Q_m = Q_A + (2)(3 \sec \alpha)(11.932)(\frac{1}{2} \times 11.932)$$

$$= 2.14776 \times 10^3 + 484.06 = 2.6318 \times 10^3 \text{ mm}^3$$

$$= 2.6318 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau_m = \frac{VQ_m}{It} = \frac{(10 \times 10^3)(2.6318 \times 10^{-6})}{(70.31 \times 10^{-9})(6 \times 10^{-3})} = 62.4 \times 10^6 \text{ Pa} \quad \tau_m = 62.4 \text{ MPa}$$



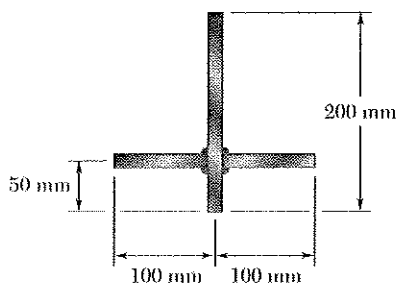
$$Q_B = (28)(3)(18.077) = 1.51847 \times 10^3 \text{ mm}^3$$

$$\tau_B = \frac{Q_B}{Q_A} \tau_A = \frac{1.51847 \times 10^3}{2.14776 \times 10^3} (50.9)$$

$$= 36.0 \text{ MPa}$$

Problem 6.49

6.49 Three plates, each 12 mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.

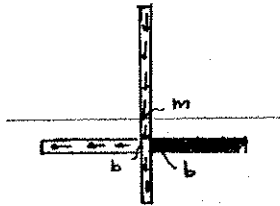


Locate neutral axis.

$$\Sigma A = (12)(200) + (2)(94)(12) = 4656 \text{ mm}^2$$

$$\Sigma A\bar{y} = (12)(200)(100) + (2)(94)(12)(50) = 352.8 \times 10^3 \text{ mm}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 75.77 \text{ mm}$$

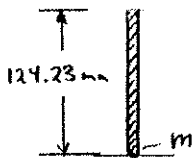


$$I = \frac{1}{12}(12)(200)^3 + (12)(200)(24.23)^2 + 2\left[\frac{1}{12}(94)(12)^3 + (94)(12)(25.77)^2\right] = 10.934 \times 10^6 \text{ mm}^4 = 10.934 \times 10^{-6} \text{ m}^4$$

$$Q = (94)(12)(25.77) = 29.07 \times 10^3 \text{ mm}^3 = 29.07 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(100 \times 10^3)(29.07 \times 10^{-6})}{10.934 \times 10^{-6}} = 266 \times 10^3 \text{ N/m} = 266 \text{ kN/m}$$

The maximum shear flow in the cross section occurs at the neutral axis.

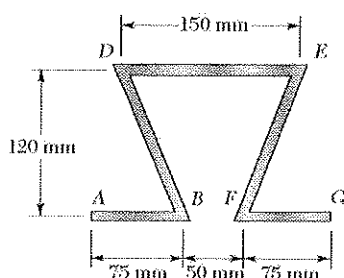


$$Q_m = (12)(124.23)\left(\frac{124.23}{2}\right) = 92.748 \times 10^3 \text{ mm}^3 = 92.748 \times 10^{-6} \text{ m}^3$$

$$q_m = \frac{VQ_m}{I} = \frac{(100 \times 10^3)(92.748 \times 10^{-6})}{(10.934 \times 10^{-6})} = 848 \times 10^3 \text{ N/m}$$

$$q_m = 848 \text{ kN/m}$$

Problem 6.50



6.50 A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 2.4 kN, determine (a) the thickness t for which the maximum shearing stress is 2 MPa, (b) the corresponding shearing stress at point E. Also sketch the shear flow in the cross section.

$$L_{BD} = L_{EF} = \sqrt{120^2 + 50^2} = 130 \text{ mm}$$

Neutral axis lies at 60 mm above AB.

Calculate I .

$$I_{AB} = (75t)(60)^2 = 270000t$$

$$I_{BD} = \frac{1}{12}(130t)(120)^2 = 156000t$$

$$I_{DE} = (150t)(60)^2 = 540000t$$

$$I_{EF} = I_{DB} = 156000t$$

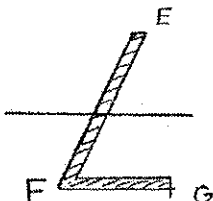
$$I_{FG} = I_{AB} = 270000t$$

$$I = \sum I = 1392000t$$

(a) At point C: $Q_c = Q_{AB} + Q_{BC} = (75t)(60) + (65t)(30) = 6450t$

$$\tau = \frac{VQ_c}{It} \quad \therefore \quad t = \frac{VQ}{\tau I} = \frac{(2400)(6450t)}{(2)(1392000t)} = 5.56 \text{ mm}$$

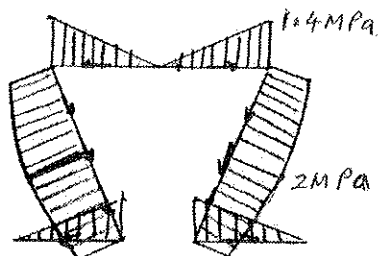
(b) $I = (1392000)(5.56) = 7739520 \text{ mm}^4$



$$Q_E = Q_{EF} + Q_{FG}$$

$$= 0 + (75)(5.56)(60) = 25020 \text{ mm}^3$$

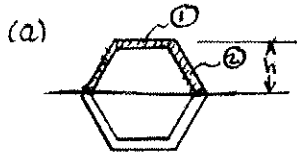
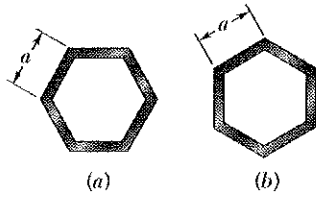
$$\tau_E = \frac{VQ_E}{It} = \frac{(2400)(25020)}{(7739520)(5.56)} = 1.4 \text{ MPa}$$



Shearing stress (MPa)

Problem 6.51

6.51 and 6.52 An extruded beam has a uniform wall thickness t . Denoting by V the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{\max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



$$h = \frac{\sqrt{3}}{2} a$$

$$A_1 = A_2 = at$$

$$I_1 = A_1 h^2 = at h^2 = \frac{3}{4} a^3 t$$

$$I_2 = \frac{1}{3} A_2 h^2 = \frac{1}{3} at \frac{3}{4} a^2 = \frac{1}{4} a^3 t$$

$$I = 2I_1 + 4I_2 = \frac{5}{2} a^3 t$$

$$Q_1 = A_1 h = \frac{\sqrt{3}}{2} a^2 t$$

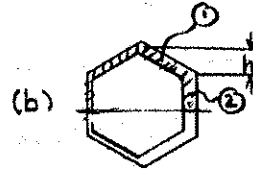
$$Q_2 = A_2 \frac{h}{2} = \frac{\sqrt{3}}{4} a^2 t$$

$$Q_m = Q_1 + 2Q_2 = \sqrt{3} a^2 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \sqrt{3} a^2 t}{(\frac{5}{2} a^3 t)(2t)} = \frac{\sqrt{3}}{5} \frac{V}{at}$$

$$= \frac{6\sqrt{3}}{5} \frac{V}{6at} = \frac{6\sqrt{3}}{5} \frac{V}{A} = k \frac{V}{A}$$

$$k = \frac{6\sqrt{3}}{5} = 2.08 \quad \blacktriangleleft$$



$$h = \frac{a}{2}$$

$$A_1 = at \quad A_2 = \frac{1}{2} at$$

$$\begin{aligned} I_1 &= \bar{I}_1 + A_1 d^2 \\ &= \frac{1}{12} at h^2 + at \left(\frac{a}{2} + \frac{h}{2} \right)^2 \\ &= \frac{1}{48} a^3 t + \frac{9}{16} a^3 t = \frac{7}{12} a^3 t \end{aligned}$$

$$I_2 = \frac{1}{3} t \left(\frac{a}{2} \right)^3 = \frac{1}{24} a^3 t$$

$$I = 4I_1 + 4I_2 = \frac{5}{2} a^3 t$$

$$Q_1 = at \left(\frac{a}{2} + \frac{h}{2} \right) = \frac{3}{4} a^2 t$$

$$Q_2 = \left(\frac{1}{2} at \right) \left(\frac{a}{4} \right) = \frac{1}{8} a^2 t$$

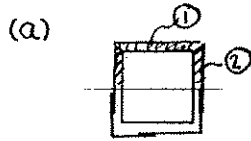
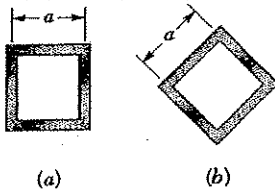
$$Q = 2Q_1 + 2Q_2 = \frac{7}{4} a^2 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \cdot \frac{7}{4} a^2 t}{(\frac{5}{2} a^3 t)(2t)}$$

$$= \frac{7}{20} \frac{V}{at} = \frac{42}{20} \frac{V}{6at} = \frac{21}{10} \frac{V}{A}$$

$$= k \frac{V}{A} \quad k = \frac{21}{10} = 2.10 \quad \blacktriangleleft$$

Problem 6.52



$$I_1 = (at)\left(\frac{a}{2}\right)^2 = \frac{1}{4}a^3t$$

$$I_2 = \frac{1}{3}t\left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3t$$

$$I = 2I_1 + 4I_2 = \frac{2}{3}a^3t$$

$$Q_1 = (at)\left(\frac{a}{2}\right) = \frac{1}{2}a^2t$$

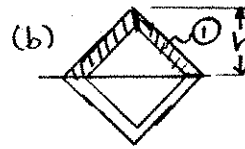
$$Q_2 = \left(\frac{1}{2}at\right)\left(\frac{a}{4}\right) = \frac{1}{8}a^2t$$

$$Q = Q_1 + 2Q_2 = \frac{3}{4}a^2t$$

$$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{3}{4}a^2t\right)}{\left(\frac{2}{3}a^3t\right)(2t)} =$$

$$= \frac{9}{16} \frac{V}{at} = \frac{9}{4} \frac{V}{4at} = \frac{9}{4} \frac{V}{A}$$

$$= k \frac{V}{A} \therefore k = \frac{9}{4} = 2.25 \quad \blacktriangleleft$$



$$h = \frac{1}{2}\sqrt{2}a$$

$$I_1 = \frac{1}{3}A, h^2 = \left(\frac{1}{2}at\right)\left(\frac{\sqrt{2}}{2}a\right)^2 = \frac{1}{6}a^3t$$

$$I = 4I_1 = \frac{2}{3}a^3t$$

$$Q_1 = at\left(\frac{h}{2}\right) = \frac{1}{4}\sqrt{2}a^2t$$

$$Q = 2Q_1 = \frac{1}{2}\sqrt{2}a^2t$$

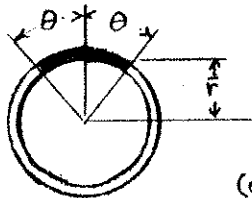
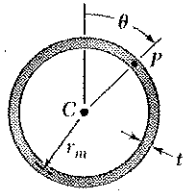
$$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{1}{2}\sqrt{2}a^2t\right)}{\left(\frac{2}{3}a^3t\right)(2t)}$$

$$= \frac{3\sqrt{2}}{8} \frac{V}{at} = \frac{3\sqrt{2}}{2} \frac{V}{4at}$$

$$= \frac{3\sqrt{2}}{2} \frac{V}{A} = k \frac{V}{A}$$

$$k = \frac{3\sqrt{2}}{2} = 2.12 \quad \blacktriangleleft$$

Problem 6.53



6.53 (a) Determine the shearing stress at point P of a thin-walled pipe of the cross section shown caused by a vertical shear V . (b) Show that the maximum shearing stress occurs for $\theta = 90^\circ$ and is equal to $2V/A$, where A is the cross-sectional area of the pipe.

$$A = 2\pi r_m t \quad J = A r_m^2 = 2\pi r_m^3 t \quad I = \frac{1}{2}J = \pi r_m^3 t$$

$$\bar{r} = \frac{\sin \theta}{\theta} \quad \text{for a circular arc}$$

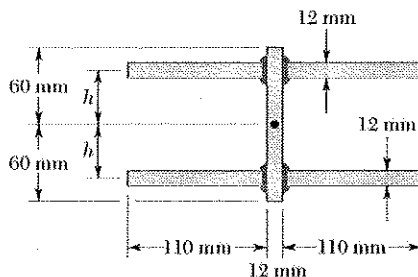
$$A_p = 2r\theta t$$

$$Q_p = A_p \bar{r} = 2rt \sin \theta$$

$$(a) \quad \tau_p = \frac{VQ_p}{I(2t)} = \frac{(V)(2rt \sin \theta)}{(\pi r_m^3 t)(2t)} = \frac{V \sin \theta}{\pi r_m t}$$

$$(b) \quad \tau_m = \frac{2V \sin \frac{\pi}{2}}{2\pi r_m t} = \frac{2V}{A}$$

Problem 6.54



6.54 The design of a beam requires welding four horizontal plates to a vertical 12×120 -mm plate as shown. For a vertical shear V , determine the dimension h for which the shear flow through the welded surface is maximum.

Horizontal plate:

$$I_h = \frac{1}{12}(110)(12)^3 + (110)(12)h^2$$

$$= 15840 + 1728 h^2$$

$$\text{Vertical plate: } I_v = \frac{1}{12}(12)(120)^3 = 1728000 \text{ mm}^3$$

$$\text{Whole section: } I = 4I_h + I_v = 6912 h^2 + 1791360 \text{ mm}^4$$

$$\text{For one horizontal plate: } Q = (110)(12)h = 1320 h \text{ mm}^3$$

$$q = \frac{VQ}{I} = \frac{V1320h}{6912h^2 + 1791360}$$

To maximize q set $\frac{dq}{dh} = 0$

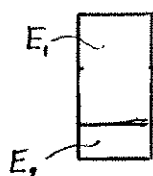
$$1320V \frac{(6912h^2 + 1791360) - 13824h^2}{(6912h^2 + 1791360)^2} = 0 \quad h = 19.4 \text{ mm}$$

Problem 6.55

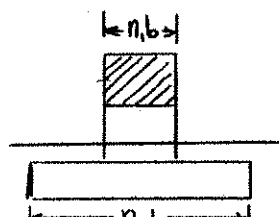
6.55 For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

$$\tau_{ave} = \frac{VQ}{It}$$

remains valid provided that both Q and I are computed by using the transformed section of the beam (see Sect 4.6) and provided further that t is the actual width of the beam where τ_{ave} is computed.



Actual Section



Transformed Section

Let E_{ref} be a reference modulus of elasticity.

$$n_1 = \frac{E_1}{E_{ref}}, \quad n_2 = \frac{E_2}{E_{ref}}, \quad \text{etc.}$$

Widths b of actual section are multiplied by n 's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{nMy}{I}$$

where I is the moment of inertia of the transformed cross section and y is measured from the centroid of the transformed section.

The horizontal shearing force over length Δx is

$$\Delta H = -\int (\Delta \sigma_x) dA = -\int \frac{n(\Delta M)y}{I} dA = -\frac{(\Delta M)}{I} \int ny dA = -\frac{Q(\Delta M)}{I}$$

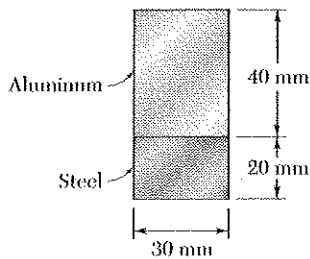
$$Q = \int ny dA = \text{first moment of transformed section.}$$

$$\text{Shear flow: } q = \frac{\Delta H}{\Delta x} = \frac{\Delta M}{\Delta x} \frac{Q}{I} = \frac{VQ}{I}$$

q is distributed over actual width t , thus $\tau = \frac{q}{t}$.

$$\tau = \frac{VQ}{It}$$

Problem 6.56

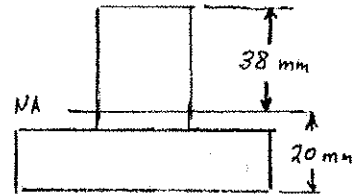


6.56 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 20 kN and that the modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum, determine (a) the average stress at the bonded surface, (b) the maximum shearing stress in the beam. (Hint: Use the method indicated in Prob. 6.55.)

$n = 1$ in aluminum

$$n = \frac{210}{70} = 3.0 \text{ in steel}$$

Locate centroid and compute moment of inertia of transformed section.



Part	A, mm^2	nA, mm^2	$\bar{y}, \text{mm}$	$nA\bar{y}, \text{mm}^3$	d, mm	nAd^2, mm^4	$n\bar{I}, \text{mm}^4$
Alum.	1200	1200	40	48000	18	388.8×10^3	160×10^3
Steel.	600	1800	10	18000	12	259.2×10^3	60×10^3
		3000		66000		648×10^3	220×10^3

$$\bar{Y} = \frac{\sum nA\bar{y}}{\sum nA} = \frac{66000}{3000} = 22 \text{ mm}$$

$$I = \sum n\bar{I} + \sum nAd^2 = 868 \times 10^3 \text{ mm}^4 = 868 \times 10^{-9} \text{ m}^4$$

(a) At the bonded surface, $Q = (30)(40)(18) = 21.6 \times 10^3 \text{ mm}^3 = 21.6 \times 10^{-6} \text{ m}^3$

$$\tau = \frac{VQ}{It} = \frac{(20 \times 10^3)(21.6 \times 10^{-6})}{(868 \times 10^{-9})(0.030)} = 16.59 \times 10^6 \text{ Pa}$$

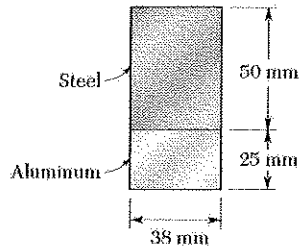
$$\tau = 16.59 \text{ MPa}$$

(b) At the neutral axis, $Q = (30)(38)\left(\frac{38}{2}\right) = 21.66 \times 10^3 \text{ mm}^3 = 21.66 \times 10^{-6} \text{ m}^3$

$$\tau = \frac{VQ}{It} = \frac{(20 \times 10^3)(21.66 \times 10^{-6})}{(868 \times 10^{-9})(0.030)} = 16.64 \times 10^6 \text{ Pa}$$

$$\tau = 16.64 \text{ MPa}$$

Problem 6.57



6.57 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 16 kN and that the modulus of elasticity is 200 GPa for the steel and 73 GPa for the aluminum, determine (a) the average stress at the bonded surface, (b) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

$$n = 1 \text{ in aluminum}$$

$$n = \frac{200}{73} = 2.74 \text{ in steel}$$

Part	$nA \text{ (mm}^2\text{)}$	$\bar{y} \text{ (mm)}$	$nA\bar{y} \text{ (mm}^3\text{)}$	$d \text{ (mm)}$	$nAd^2 \text{ (mm}^4\text{)}$	$n\bar{I} \text{ (mm}^4\text{)}$
Steel	5206	50	260300	5.8	175130	1084583
Alum.	950	12.5	11875	31.7	954646	49479
Σ	6156		272175		1129776	1134062

$$\bar{y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{272175}{6156} = 44.2 \text{ mm}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 2263838 \text{ mm}^4$$

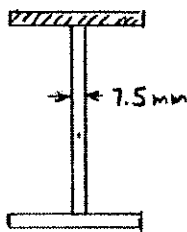
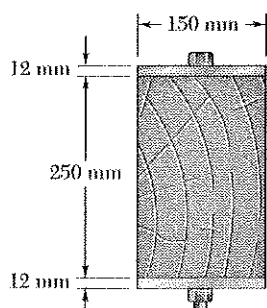
(a) At the bonded surface: $Q = (950)(31.7) = 30115 \text{ mm}^3$

$$\tau = \frac{VQ}{It} = \frac{(16000)(30115)}{(2263838)(38)} = 5.6 \text{ MPa}$$

(b) At the neutral axis: $Q = (2.74)(38)\left(\frac{30.8}{2}\right) = 49386 \text{ mm}^3$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(16000)(49386)}{(2263838)(38)} = 9.2 \text{ MPa}$$

Problem 6.58



6.58 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (a) the average shearing stress in the bolts, (b) the shearing stress at the center of the cross section. (Hint: Use the method indicated in Prob. 6.55.)

$$\text{Let } E_{\text{ref}} = E_s = 200 \text{ GPa}$$

$$n_s = 1 \quad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = \frac{1}{20}$$

Widths of transformed section:

$$b_s = 150 \text{ mm} \quad b_w = \left(\frac{1}{20}\right)(150) = 7.5 \text{ mm}$$

$$\begin{aligned} I &= 2 \left[\frac{1}{12} (150)(12)^3 + (150)(12)(125 + 6)^2 \right] \\ &\quad + \frac{1}{12} (7.5)(250)^3 \\ &= 2 \left[0.0216 \times 10^6 + 30.890 \times 10^6 \right] + 9.766 \times 10^6 \\ &= 71.589 \times 10^6 \text{ mm}^4 = 71.589 \times 10^{-6} \text{ m}^4 \end{aligned}$$

$$\begin{aligned} (a) \quad Q_1 &= (150)(12)(125 + 6) = 235.8 \times 10^3 \text{ mm}^3 \\ &= 235.8 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$q = \frac{VQ_1}{I} = \frac{(4 \times 10^3)(235.8 \times 10^{-6})}{71.589 \times 10^{-6}} = 13.175 \times 10^3 \text{ N/m}$$

$$F_{\text{bolt}} = q s = (23.187 \times 10^3)(200 \times 10^{-3}) = 2.635 \times 10^3 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \left(\frac{\pi}{4}\right)(12)^2 = 113.1 \text{ mm}^2 = 113.1 \times 10^{-6} \text{ m}^2$$

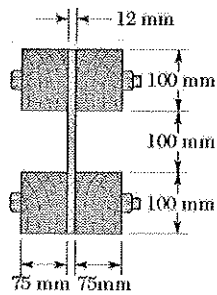
$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{2.635 \times 10^3}{113.1 \times 10^{-6}} = 23.3 \times 10^6 \text{ Pa} = 23.3 \text{ MPa}$$

$$\begin{aligned} (b) \quad Q_2 &= Q_1 + (7.5)(125)(62.5) = 235.8 \times 10^3 + 58.594 \times 10^3 = 294.4 \times 10^3 \text{ mm}^3 \\ &= 294.4 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$t = 150 \text{ mm} = 150 \times 10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_2}{I t} = \frac{(4 \times 10^3)(294.4 \times 10^{-6})}{(71.589 \times 10^{-6})(150 \times 10^{-3})} = 109.7 \times 10^3 \text{ Pa} = 109.7 \text{ kPa}$$

Problem 6.59



6.59 A composite beam is made by attaching the timber and steel portions shown with bolts of 16-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 13 GPa for the wood and 200 GPa for the steel. For a vertical shear of 16 kN, determine (a) the average shearing stress in the bolts, (b) the shearing stress at the center of the cross section. (Hint: Use the method indicated in Prob. 6.55.)

$$\text{Let } E_{\text{ref}} = E_w = 13 \text{ GPa.}$$

$$n_s = \frac{E_s}{E_w} = \frac{200}{13} = 15.385$$

$$\text{For one timber, } I_w = \frac{1}{12} (75)(100)^3 + (75)(100)(100)^2 \\ = 81.25 \times 10^6 \text{ mm}^4$$

$$\text{For steel plate, } I_s = \frac{1}{12} (12)(300)^3 = 27 \times 10^6 \text{ mm}^4$$

$$\text{For transformed section, } I = n_s I_s + 4 I_w = 740.4 \times 10^6 \text{ mm}^4$$

$$(a) \quad Q_1 = (75)(100)(100) = 750000 \text{ mm}^3$$

$$q = \frac{V Q_1}{I} = \frac{(16)(750000)}{740.4 \times 10^6} = 16.2 \text{ N/m}$$

$$F_{\text{bolt}} = q s = (16.2)(200) = 3240 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (16)^2 = 201 \text{ mm}^2$$

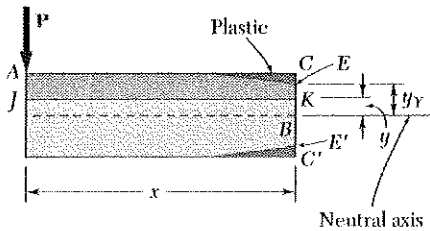
$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{3240}{201} = 16.1 \text{ MPa}$$

$$(b) \quad Q_2 = 2Q_1 + (15.385)(12)(150)(75) = 357697.5 \text{ mm}^3$$

$$t = 12 \text{ mm}$$

$$\tau_c = \frac{V Q_2}{I t} = \frac{(16000)(357697.5)}{(740.4 \times 10^6)(12)} = 6.4 \text{ MPa}$$

Problem 6.60



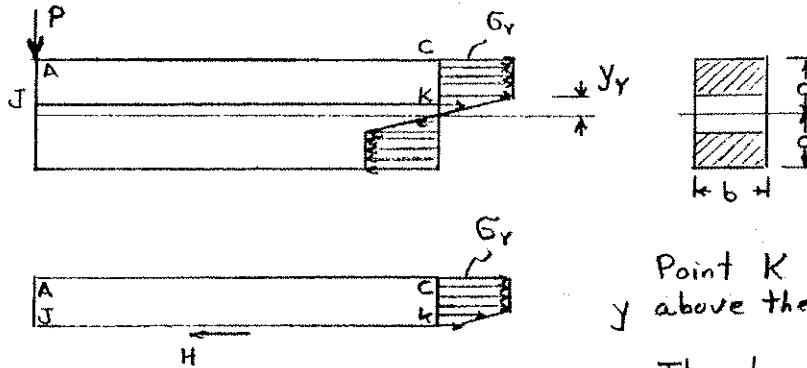
6.60 Consider the cantilever beam AB discussed in Sec. 6.8 and the portion $ACKJ$ of the beam that is located to the left of the transverse section CC' and above the horizontal plane JK , where K is a point at a distance $y < y_r$ above the neutral axis (Fig. 6.60). (a) Recalling that $\sigma_x = \sigma_y$ between C and E and $\sigma_x = (\sigma_y/y_r)y$ between E and K , show that the magnitude of the horizontal shearing force H exerted on the lower face of the portion of beam $ACKJ$ is

$$H = \frac{1}{2} b \sigma_y \left(2c - y_r - \frac{y^2}{y_r} \right)$$

(b) Observing that the shearing stress at K is

$$\tau_{xy} = \lim_{\Delta A \rightarrow 0} \frac{\Delta H}{\Delta A} = \lim_{\Delta x \rightarrow 0} \frac{1}{b} \frac{\Delta H}{\Delta x} = \frac{1}{b} \frac{\partial H}{\partial x}$$

and recalling that y_r is a function of x defined by Eq. (6.14), derive Eq. (6.15).



Point K is located a distance y above the neutral axis.

The stress distribution is given by

$$\sigma = \sigma_r \frac{y}{y_r} \quad \text{for } 0 \leq y < y_r \quad \text{and} \quad \sigma = \sigma_r \quad \text{for } y_r \leq y \leq c.$$

For equilibrium of horizontal forces acting on $ACKJ$,

$$\begin{aligned} H &= \int \sigma dA = \int_y^{y_r} \frac{\sigma_r y b}{y_r} dy + \int_{y_r}^c \sigma_r b dy = \frac{\sigma_r b}{y_r} \left(\frac{y_r^2 - y^2}{2} \right) + \sigma_r b (c - y_r) \\ &= \frac{1}{2} b \sigma_r \left(2c - y_r - \frac{y^2}{y_r} \right) \end{aligned} \quad \blacktriangleleft (a)$$

Note that y_r is a function of x .

$$\tau_{xy} = \frac{1}{b} \frac{\partial H}{\partial x} = \frac{1}{2} \sigma_r \left(-\frac{\partial y_r}{\partial x} + \frac{y^2}{y_r^2} \frac{dy_r}{dx} \right) = -\frac{1}{2} \sigma_r \left(1 - \frac{y^2}{y_r^2} \right) \frac{dy_r}{dx}$$

$$\text{But } M = Px = \frac{3}{2} M_y \left(1 - \frac{1}{3} \frac{y_r^2}{c^2} \right)$$

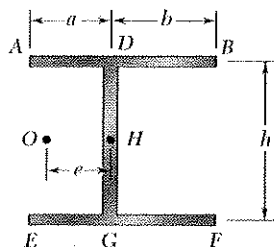
$$\text{Differentiating, } \frac{dM}{dx} = P = \frac{3}{2} M_y \left(-\frac{2}{3} \frac{y_r}{c^2} \frac{dy_r}{dx} \right)$$

$$\frac{dy_r}{dx} = -\frac{P c^2}{y_r M_y} = -\frac{P c^2}{y_r \frac{3}{2} \sigma_r b c^2} = -\frac{3 P}{2 \sigma_r b y_r}$$

$$\text{Then, } \tau_{xy} = \frac{1}{2} \sigma_r \left(1 - \frac{y^2}{y_r^2} \right) \frac{3 P}{2 \sigma_r b y_r} = \frac{3 P}{4 b y_r} \left(1 - \frac{y^2}{y_r^2} \right) \quad \blacktriangleleft (b)$$

Problem 6.61

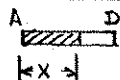
6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{OG} = \frac{1}{12}th^3 \quad I = \sum I = \frac{1}{12}t(6a+6b+h)h^2$$

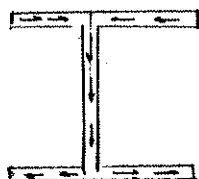
Part AD: $Q = tx \frac{h}{2} = \frac{1}{2}thx$



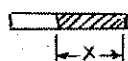
$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_1 = \int \tau dA = \int_0^a \frac{Vhx}{2I} t dx = \frac{Vht}{2I} \int_0^a x dx$$

$$= \frac{Vht}{2I} \left. \frac{x^2}{2} \right|_0^a = \frac{Vhta^2}{4I}$$



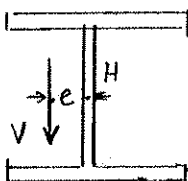
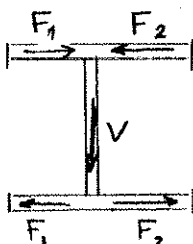
Part BD: $Q = tx \frac{h}{2} = \frac{1}{2}thx$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau dA = \int_0^b \frac{Vhx}{2I} t dx = \frac{Vht}{2I} \int_0^b x^2 dx$$

$$= \frac{Vht}{2I} \left. \frac{x^2}{2} \right|_0^b = \frac{Vhtb^2}{4I}$$



$$\sum \mathcal{M}_H = \sum \mathcal{M}_H:$$

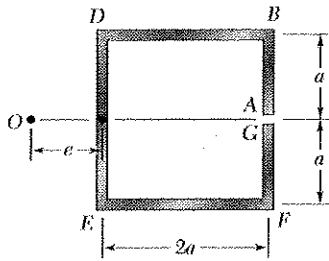
$$Ve = F_2 n - F_1 h = \frac{Vht^2(b^2 - a^2)}{4I}$$

$$= \frac{Vh^2t(b^2 - a^2)}{4 \cdot \frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2 - a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2 - a^2)}{6(a+b) + h}$$

Problem 6.62

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = I_{FG} = \frac{1}{3} t a^3 \quad I_{DB} = I_{EF} = 2 a t a^3 + \frac{1}{12} 2 a t t^3 \approx 2 t a^3$$

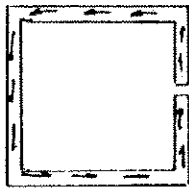
$$I_{DE} = \frac{1}{12} t (2a)^3 = \frac{2}{3} t a^3 \quad I = \sum I = \frac{16}{3} t a^3$$

Part AB: $A = t y \quad \bar{y} = \frac{y}{2} \quad Q = \frac{1}{2} t y^2$

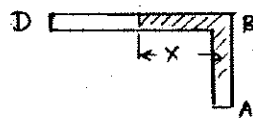
$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2} t y^2}{\frac{16}{3} t a^3 t} = \frac{3V y^2}{32 a^3 t}$$



$$F_1 = \int \tau dA = \int_0^a \tau t dy = \frac{3V}{32 a^3} \int_0^a y^2 dt = \frac{1}{32} V$$



Part BD:



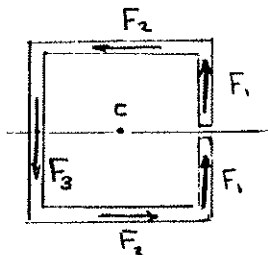
$$Q = Q_B + t x a = \frac{1}{2} t a^2 + t a x$$

$$\tau = \frac{VQ}{It} = \frac{Vt}{\frac{16}{3} a^3 t} \left(\frac{1}{2} a^2 + a x \right)$$

$$\frac{3V}{32 a^2} (a + 2x)$$

$$F_2 = \int \tau dA = \int_0^{2a} \frac{3V}{32 a^2} (a + 2x) dx$$

$$= \frac{3V}{32 a^2} (a x + x^2) \Big|_0^{2a} = \frac{3V}{32 a^2} (2a^2 + 4a^2) = \frac{9}{16} V$$



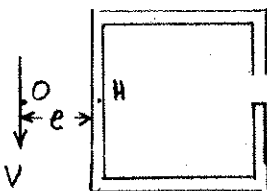
$$\sum M_H = \sum M_H:$$

$$V e = (2a)(2F_1) + (2a)(F_2)$$

$$= \frac{1}{8} V a + \frac{9}{8} V a = \frac{5}{4} V a$$

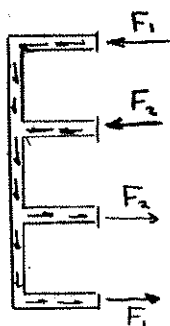
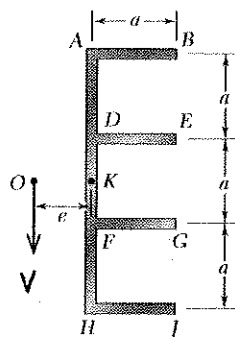
$$e = \frac{5}{4} a$$

$$= 1.250 a$$



Problem 6.63

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

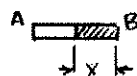


$$I_{AB} = I_{HJ} = at \left(\frac{3a}{2} \right)^2 + \frac{1}{12} at^3 \approx \frac{9}{4} ta^3$$

$$I_{DE} = I_{FG} = at \left(\frac{a}{2} \right)^2 + \frac{1}{12} at^3 \approx \frac{1}{4} ta^3$$

$$I_{AH} = \frac{1}{12} t(3a)^3 = \frac{9}{4} ta^3 \quad I = \sum I = \frac{29}{4} ta^3$$

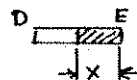
Part AB: $A = tx \quad \bar{y} = \frac{3a}{2} \quad Q = \frac{3}{2} atx$



$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{3}{2} atx}{\frac{29}{4} ta^3 t} = \frac{6Vx}{29a^2 t}$$

$$F_1 = \int \tau dA = \int_0^a \frac{6Vx}{29a^2 t} t dx = \frac{6V}{29a^2} \int_0^a x dx = \frac{3}{29} V$$

Part DE: $A = tx \quad \bar{y} = \frac{a}{2} \quad Q = \frac{1}{2} atx$



$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2} atx}{\frac{29}{4} ta^3 t} = \frac{2Vx}{29a^2 t}$$

$$F_2 = \int \tau dA = \int_0^a \frac{2Vx}{29a^2 t} t dx = \frac{2V}{29a^2} \int_0^a x dx = \frac{1}{29} V$$

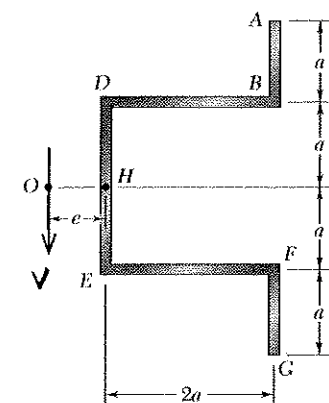
$$\sum M_K = \sum M_K:$$

$$Ve = F_1(3a) + F_2(a) = \frac{9}{29} Va + \frac{1}{29} Va = \frac{10}{29} Va$$

$$e = \frac{10}{29} a = 0.345 a$$

Problem 6.64

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

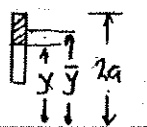


$$I_{AB} = I_{FG} = \frac{1}{12} t a^3 + (t a) \left(\frac{3a}{2} \right)^2 = \frac{7}{3} t a^3$$

$$I_{DB} = I_{EF} = (2at) a^2 + \frac{1}{12} (2a) t^3 \approx 2a^3 t$$

$$I_{DE} = \frac{1}{12} t (2a)^3 = \frac{2}{3} t a^3 \quad I = \sum I = \frac{28}{3} t a^3$$

Part AB: $A = t(2a - y) \quad \bar{y} = \frac{2a + y}{2}$



$$Q = A \bar{y} = \frac{1}{2} t (2a - y) (2a + y) = \frac{1}{2} t (4a^2 - y^2)$$

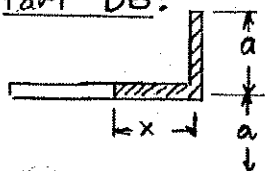
$$\tau = \frac{VQ}{It} = \frac{V}{2I} (4a^2 - y^2)$$

$$F_1 = \int \tau dA = \int_a^{2a} \frac{V}{2I} (4a^2 - y^2) t dy$$

$$= \frac{Vt}{2I} \left(4a^2 y - \frac{y^3}{3} \right) \Big|_a^{2a} = \frac{Vt a^3}{2I} \left[(4)(2) - \frac{(2)^3}{3} - (4)(1) + \frac{(1)^3}{3} \right]$$

$$= \frac{5}{6} \frac{V t a^3}{I} = \frac{5}{56} V$$

Part DB:



$$Q = (t a) \frac{3a}{2} + t x a = t a \left(\frac{3a}{2} + x \right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I} \left(\frac{3a}{2} + x \right)$$

$$F_2 = \int \tau dA = \int_0^{2a} \frac{Va}{I} \left(\frac{3a}{2} + x \right) t dx = \frac{Vta}{I} \int_0^{2a} \left(\frac{3a}{2} + x \right) dx$$

$$= \frac{Vta}{I} \left(\frac{3ax}{2} + \frac{x^2}{2} \right) \Big|_0^{2a} = \frac{Vta^3}{I} \left[\frac{(3)(2)}{2} + \frac{(2)^2}{2} \right]$$

$$= 5 \frac{V t a^3}{I} = \frac{15}{28} V$$

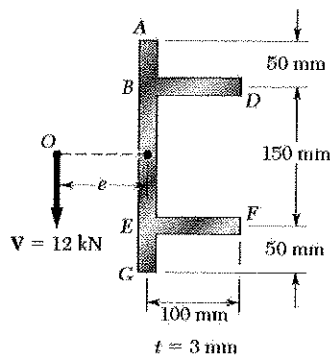
$$\sum M_H = 0$$

$$Ve = F_2(2a) - 2F_1(2a) = \frac{30}{28} Va - \frac{20}{56} Va = \frac{5}{7} Va$$

$$e = \frac{5}{7} a = 0.714 a$$

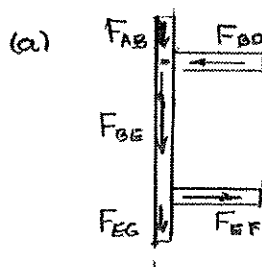
Problem 6.65

6.65 and 6.66 An extruded beam has the cross section as shown. Determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by the 12 kN vertical shearing force applied at O .

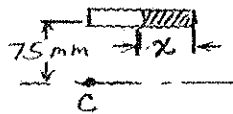


Part	$A (\text{mm}^2)$	$d (\text{mm})$	$Ad^2 (\text{mm}^4)$	$\bar{I} (\text{mm}^4)$
BD	300	75	1687500	≈ 0
ABEG	750	0	0	3906250
EF	300	75	1687500	≈ 0
Σ	2250		3375000	3906250

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 7281250 \text{ mm}^4$$



Part BD



$$Q(x) = 75 t x$$

$$q(x) = \frac{V Q(x)}{I} = \frac{V}{I} (75 t x)$$

$$F_{BD} = \frac{75 V t}{I} \int_0^{100} x dx = \frac{75 V t}{I} (5000) = \frac{375000 V t}{I}$$

$$\text{Its moment about H: } (M_{BD})_H = 75 F_{BD} = \frac{28.125 \times 10^6 V t}{I}$$

$$\text{Part EF: By same method, } F_{EF} = \frac{9.375 \times 10^6 V t}{I} \quad (M_{EF})_H = \frac{28.125 \times 10^6 V t}{I}$$

Moments of F_{AB} , F_{BE} , and F_{EG} about H are zero.

$$V e = \Sigma M_H = \frac{28.125 \times 10^6 V t}{I} + \frac{28.125 \times 10^6 V t}{I} = \frac{56.25 \times 10^6 V t}{I}$$

$$e = \frac{56.25 \times 10^6 t}{I} = \frac{(56.25 \times 10^6)(3)}{7281250} = 23.2 \text{ mm}$$

(b) At A, D, F, and G: $Q = 0$ $\tau_A = \tau_D = \tau_F = \tau_G = 0$

Just above B: $Q_1 = Q_{AB} = (50t)(100) = 5000t$

$$\tau_1 = \frac{V Q_1}{I t} = \frac{(12000)(5000t)}{(7281250)t} = 8.2 \text{ MPa}$$

Just to the right of B: $Q_2 = Q_{BD} = (75t)(100) = 7500t$

$$\tau_2 = \frac{V Q_2}{I t} = \frac{(12000)(7500t)}{(7281250)t} = 12.4 \text{ MPa}$$

Just below B: $Q_3 = Q_1 + Q_2 = 12500t$

$$\tau_3 = \frac{V Q_3}{I t} = \frac{(12000)(12500t)}{(7281250)t} = 20.6 \text{ MPa}$$

Continued on next page.

Problem 6.65 continued.

At H (neutral axis): $Q_H = Q_3 + Q_{BH} = 12500t + t(75)(37.5) = 15312.5t$

$$\tau_H = \frac{VQ_H}{It} = \frac{(12000)(15312.5t)}{(7281250)t} = 25.2 \text{ MPa}$$

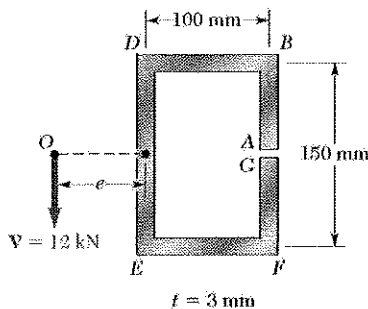
By symmetry: $\tau_4 = \tau_3 = 20.6 \text{ MPa}$

$$\tau_5 = \tau_2 = 12.4 \text{ MPa}$$

$$\tau_6 = \tau_1 = 8.2 \text{ MPa}$$

Problem 6.66

6.65 and 6.66 An extruded beam has the cross section shown. Determine (a) the location of the shear center O, (b) the distribution of the shearing stresses caused by the 12 kN vertical shearing force applied at O.



$$I_{AB} = \frac{1}{3}(3)(75)^3 = 42187.5 \text{ mm}^4$$

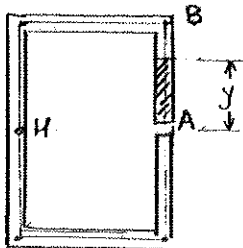
$$I_{BD} = \frac{1}{12}(100)(3)^3 + (100)(3)(75)^2 = 1687725 \text{ mm}^4$$

$$I_{DE} = \frac{1}{12}(3)(150)^3 = 843750 \text{ mm}^4$$

$$I_{EF} = I_{BD} = 1687725 \text{ mm}^4$$

$$I_{FG} = I_{AB} = 42187.5 \text{ mm}^4$$

$$I = \sum I = 5062950 \text{ mm}^4$$



(a) Part AB: $Q(y) = ty \frac{y}{2} = 0.5 ty^2$

$$q(y) = \frac{VQ(y)}{I} = \frac{0.5 Vt}{I} y^2$$

$$F_{AB} = \int_0^{75} q(y) dy = \frac{0.5 Vt}{I} \int_0^{75} y^2 dy = 70312.5 \frac{Vt}{I} \uparrow$$

Its moment about H is $100 F_{AB} = 7031250 \frac{Vt}{I} \curvearrowright$

$$Q_B = (0.5)(t)(75)^2 = 2812.5t$$

Part BD: $Q(x) = Q_B + xt(75) = (2812.5 + 75x)t$

$$q(x) = \frac{VQ(x)}{I} = \frac{Vt}{I} (2812.5 + 75x)$$

$$F_{BD} = \int_0^{100} q(x) dx = \frac{Vt}{I} \int_0^{100} (2812.5 + 75x) dx = 656250 \frac{Vt}{I} \leftarrow$$

Its moment about H: $75 F_{BD} = 49218750 \frac{Vt}{I} \curvearrowright$

$$Q_D = [2812.5 + (75)(100)]t = 10312.5t$$

Continued on next page.

Problem 6.66 continued.

Part EF: By symmetry with part BD, $F_{EF} = 656250 \frac{Vt}{I} \rightarrow$

Its moment about H is $75 F_{EF} = 49218750 \frac{Vt}{I} \rightarrow$

Part FG: By symmetry with part AB, $F_{FG} = 703125 \frac{Vt}{I} \uparrow$

Its moment about H is $100 F_{FG} = 7031250 \frac{Vt}{I} \rightarrow$

Moment about H of force in part DE is zero.

$$V e = \sum M_H = \frac{Vt}{I} (2[49218750 + 7031250]) = \frac{112.5 \times 10^6 Vt}{I}$$

$$e = \frac{112.5 \times 10^6 t}{I} = \frac{(112.5 \times 10^6)(3)}{5062950} = 66.7 \text{ mm}$$

(b) $Q_A = Q_G = 0$

$$\tau_A = \tau_G = 0$$

$$Q_B = Q_F = 2812.5$$

$$\tau_B = \tau_F = \frac{V Q_B}{I t} = \frac{(12000)(2812.5 t)}{5062950 t} = 6.7 \text{ MPa}$$

$$Q_D = Q_E = 10312.5 t$$

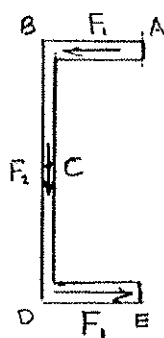
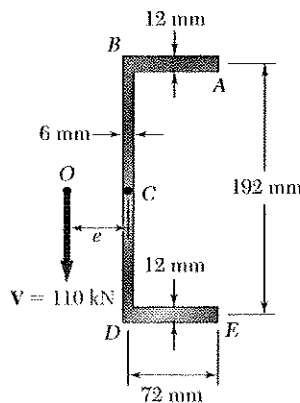
$$\tau_D = \tau_E = \frac{V Q_D}{I t} = \frac{(12000)(10312.5 t)}{5062950 t} = 24.4 \text{ MPa}$$

At H (neutral axis): $Q_H = Q_D + t(75)\left(\frac{75}{2}\right) = 13125 t$

$$\tau_H = \frac{V Q_H}{I t} = \frac{(12000)(13125 t)}{5062950 t} = 31.1 \text{ MPa}$$

Problem 6.67

6.67 and 6.68 For an extruded beam having the cross section shown, determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by the vertical shearing force V shown applied at O .



$$I = 2 \left[\left(\frac{1}{12} \right) (72) (12)^3 + (72) (12) \left(\frac{192}{2} \right)^2 \right] + \frac{1}{12} (6) (192)^3$$

$$= 19.4849 \times 10^6 \text{ mm}^4 = 19.4849 \times 10^{-6} \text{ m}^4$$

Part AB: $A = 12x$ $Q = A\bar{y} = (12x) \left(\frac{192}{2} \right) = 1152x$

$$q = \frac{VQ}{I} = \frac{1152 Vx}{I}$$

$x = 0$ at point A. $x = l_{AB} = 72 \text{ mm}$ at point B.

$$F_1 = \int_{x_A}^{x_B} q dx = \int_0^{72} \frac{1152 Vx}{I} dx = \frac{1152 V}{I} \frac{(72)^2}{2}$$

$$= \frac{(576)(72)^2}{19.4849 \times 10^6} V = 0.153246 V$$

$$+\circlearrowleft \sum M_C = +\circlearrowleft M_C: Ve = (0.153246 V)(192)$$

(a) $e = 29.423 \text{ mm}$ $e = 29.4 \text{ mm}$ $\blacktriangleleft$

(b) Point A: $x = 0$ $Q = 0$, $q = 0$ $\tau_A = 0$ $\blacktriangleleft$

Point B in Part AB. $x = 72 \text{ mm}$

$$Q = (1152)(72) = 82.944 \times 10^3 \text{ mm}^3 = 82.944 \times 10^{-6} \text{ m}^3$$

$$t = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_B = \frac{VQ}{It} = \frac{(110 \times 10^3)(82.944 \times 10^{-6})}{(19.4849 \times 10^6)(0.012)}$$

$$= 39.0 \times 10^6 \text{ Pa} \quad \tau_B = 39.0 \text{ MPa in AB} \quad \blacktriangleleft$$

Part BD:

Point B: $y = 96 \text{ mm}$ $Q = 82.944 \times 10^3 \text{ mm}^3 = 82.944 \times 10^{-6} \text{ m}^3$

$$t = 6 \text{ mm} = 0.006 \text{ m}$$

$$\tau_B = \frac{VQ}{It} = \frac{(110 \times 10^3)(82.944 \times 10^{-6})}{(19.4849 \times 10^6)(0.006)}$$

$$= 78.0 \times 10^6 \text{ Pa} \quad \tau_B = 78.0 \text{ MPa in BD} \quad \blacktriangleleft$$

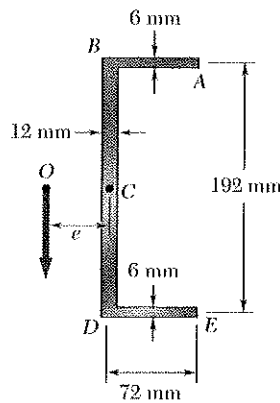
Point C: $y = 0$, $t = 6 \text{ mm} = 0.006 \text{ m}$

$$Q = 82.944 \times 10^3 + (6)(96) \left(\frac{96}{2} \right) = 110.592 \times 10^3 \text{ mm}^3 = 110.592 \times 10^{-6} \text{ m}^3$$

$$\tau_C = \frac{VQ}{It} = \frac{(110 \times 10^3)(110.592 \times 10^{-6})}{(19.4849 \times 10^6)(0.006)} = 104.1 \times 10^6 \text{ Pa} \quad \tau_C = 104.1 \text{ MPa} \quad \blacktriangleleft$$

Problem 6.68

6.67 and 6.68 For an extruded beam having the cross section shown, determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by the vertical shearing force V shown applied at O .



$$I = 2 \left[\left(\frac{1}{12} (72) (6)^3 + (72) (6) \left(\frac{192}{2} \right)^2 \right) \right] + \frac{1}{12} (12) (192)^3$$

$$= 15.0431 \times 10^6 \text{ mm}^4 = 15.0431 \times 10^{-6} \text{ m}^4$$

Part AB: $A = 6x$ $Q = A\bar{y} = (6x) \left(\frac{192}{2} \right) = 576x$

$$q = \frac{VQ}{I} = \frac{576 Vx}{I}$$

$x = 0$ at point A. $x = l_{AB} = 72 \text{ mm}$ at point B

$$F_1 = \int_{x_A}^{x_B} q dx = \int_0^{72} \frac{576 Vx}{I} dx = \frac{576 V}{I} \frac{(72)^2}{2}$$

$$= \frac{(288)(72)^2}{15.0431 \times 10^6} V = 0.099247 V$$

$+5M_c = +5M_c$ $Ve = (0.099247) V (192)$

(a) $e = 19.0555 \text{ mm}$

$e = 19.06 \text{ mm}$

(b) Point A: $x = 0$ $Q = 0$, $q = 0$ $\tau_A = 0$

Point B in Part AB: $x = 72 \text{ mm}$

$$Q = (576)(72) = 41.472 \times 10^3 \text{ mm}^3 = 41.472 \times 10^{-6} \text{ m}^3$$

$$t = 6 \text{ mm} = 0.006 \text{ m}$$

$$\tau_B = \frac{VQ}{It} = \frac{(110 \times 10^3)(41.472 \times 10^{-6})}{(15.0431 \times 10^{-6})(0.006)}$$

$$= 50.5 \times 10^6 \text{ Pa}$$

$\tau_B = 50.5 \text{ MPa}$

Part BD:

Point B: $y = 96 \text{ mm}$ $Q = 41.472 \times 10^3 \text{ mm}^3 = 41.472 \times 10^{-6} \text{ m}^3$

$$t = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_B = \frac{VQ}{It} = \frac{(110 \times 10^3)(41.472 \times 10^{-6})}{(15.0431 \times 10^{-6})(0.012)}$$

$$= 25.271 \times 10^6 \text{ Pa}$$

$\tau_B = 25.3 \text{ MPa}$

Point C: $y = 0$ $t = 0.012 \text{ m}$

$$Q = 41.472 \times 10^3 + (12)(96) \left(\frac{96}{2} \right) = 96.768 \times 10^3 \text{ mm}^3 = 96.768 \times 10^{-6} \text{ m}^3$$

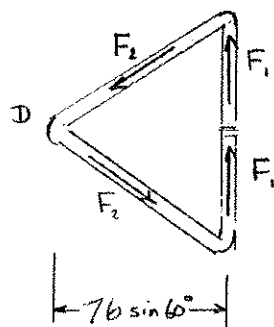
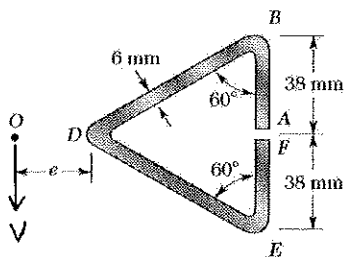
$$\tau_c = \frac{VQ}{It} = \frac{(110 \times 10^3)(96.768 \times 10^{-6})}{(15.0431 \times 10^{-6})(0.012)}$$

$$= 58.987 \times 10^6 \text{ Pa}$$

$\tau_c = 59.0 \text{ MPa}$

Problem 6.69

6.69 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{AB} = \frac{1}{3} (6)(38)^3 = 109744 \text{ mm}^4$$

$$L_{BD} = 76 \text{ mm} \quad A_{BD} = (76)(6) = 456 \text{ mm}^2$$

$$I_{BD} = \frac{1}{3} A_{BD} h^2 = \frac{1}{3} (456)(38)^2 = 219488 \text{ mm}^4$$

$$I = (2)(109744) + (2)(219488) = 658464 \text{ mm}^4$$

Part AB: $A = 6y$ $\bar{y} = \frac{1}{2}y$ $Q = A\bar{y} = 3y^2$

$$\tau = \frac{VQ}{It} = \frac{V(3y^2)}{(658464)(6)} = 759 \times 10^{-9} Vy^2$$

$$F_1 = \int \tau dA = \int_0^{38} 759 \times 10^{-9} Vy^2 (6 dy)$$

$$= 1.518 \times 10^{-6} V y^3 \Big|_0^{38}$$

$$= 0.0833 V$$

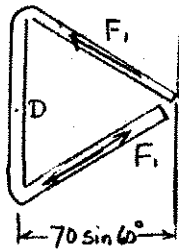
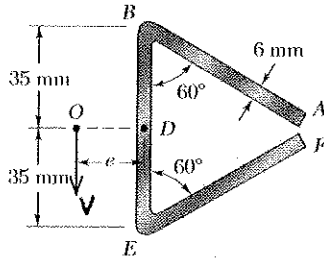
$$\sum M_D = \sum M_D: Ve = 2 F_1 (76 \sin 60^\circ)$$

$$Ve = (2)(0.08333)V(76 \sin 60^\circ)$$

$$e = (2)(0.08333)(76 \sin 60^\circ) = 10.97 \text{ mm}$$

Problem 6.70

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$I_{DB} = \frac{1}{3}(6)(35)^3 = 85.75 \times 10^3 \text{ mm}^4$$

$$L_{AB} = 70 \text{ mm} \quad A_{AB} = (70)(6) = 420 \text{ mm}^2$$

$$I_{AB} = \frac{1}{3} A_{AB} h^2 = \left(\frac{1}{3}\right)(420)(35)^2 = 171.5 \times 10^3 \text{ mm}^4$$

$$I = (2)(85.75 \times 10^3) + (2)(171.5 \times 10^3) = 514.5 \times 10^3 \text{ mm}^4$$

Part AB: $A = ts = 6s$

$$\bar{y} = \frac{1}{2} s \sin 30^\circ = \frac{1}{4} s$$

$$Q = A\bar{y} = \frac{3}{2} s^2$$

$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$$

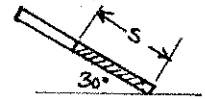
$$F_1 = \int \tau dA = \int_0^{70} \frac{3Vs^2}{2It} t ds = \frac{3V}{I} \int_0^{70} s^2 ds$$

$$= \frac{(3)(70)^3}{(2)(3)I} V = \frac{1}{3} V$$

$$\begin{aligned} \sum M_O = 0: \quad V e &= 2(F_1 \cos 60^\circ)(70 \sin 60^\circ) \\ &= 20.2 V \end{aligned}$$

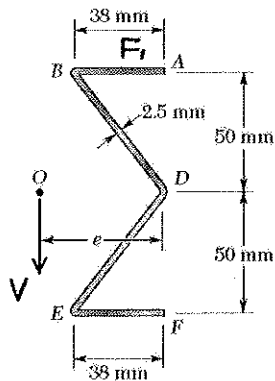
Dividing by V ,

$$e = 20.2 \text{ mm}$$



Problem 6.71

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



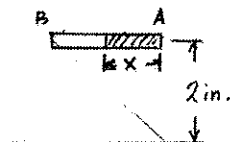
$$I_{AB} = \frac{1}{12} (38)(2.5)^3 + (38)(2.5)(50)^2 = 237549 \text{ mm}^4$$

$$L_{BD} = \sqrt{38^2 + 50^2} = 62.8 \text{ mm} \quad A_{BD} = (62.8)(2.5) = 157 \text{ mm}^2$$

$$I_{BD} = \frac{1}{3} A_{BD} h^2 = \frac{1}{3} (157)(50)^2 = 130833 \text{ mm}^4$$

$$I = 2 I_{AB} + 2 I_{BD} = 736766 \text{ mm}^4$$

Part AB:



$$A(x) = tx = 2.5x, \quad \bar{y} = 50 \text{ mm}$$

$$Q(x) = A(x)\bar{y} = 125x \text{ mm}^3$$

$$q(x) = \frac{VQ(x)}{I} = \frac{125Vx}{I}$$

$$F_1 = \int_0^{38} q(x) dx = \frac{125V}{I} \int_0^{38} x dx$$

$$= \frac{(125)(38)^2}{2} \frac{V}{I} = 90250 \frac{V}{I}$$

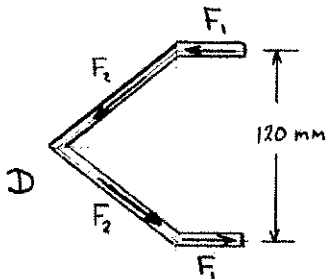
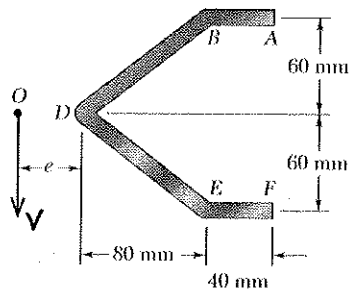
Like wise, by symmetry, in part EF: $F_1 = 90250 \frac{V}{I}$

$$+\circlearrowleft \sum M_O = +\circlearrowleft \sum M_O: \quad V e = 100 F_1 = 9025000 \frac{V}{I} = 12.25 V$$

Dividing by V ,

$$e = 12.25 \text{ mm}$$

Problem 6.72



6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

$$I_{AB} = (40t)(60)^2 = 144 \times 10^3 t$$

$$l_{DB} = \sqrt{80^2 + 60^2} = 100 \text{ mm} \quad A_{DB} = 100t$$

$$I_{DB} = \frac{1}{3} A_{DB} h^2 = \frac{1}{3} (100t)(60)^2 = 120 \times 10^3 t$$

$$I = 2I_{AB} + 2I_{DB} = 528 \times 10^3 t$$

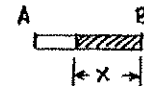
Part AB: $A = tx \quad \bar{y} = 60 \text{ mm}$

$$Q = A\bar{y} = 60tx \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60Vx}{I}$$

$$F_1 = \int \tau dA = \int_0^{40} \frac{60Vx}{I} t dx = \frac{60Vt}{I} \int_0^{40} x dx$$

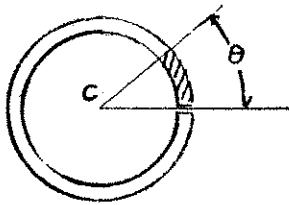
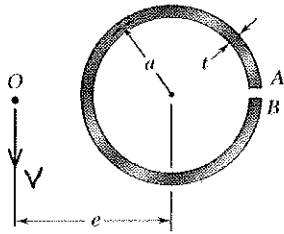
$$= \frac{60Vt}{I} \frac{x^2}{2} \Big|_0^{40} = \frac{(60)(30)^2 Vt}{(2)(528 \times 10^3)t} = 0.051136 V$$



$$\sum M_O = \sum M_O: Ve = (0.051136 V)(120) \quad e = (0.051136)(120) = 6.14 \text{ mm}$$

Problem 6.73

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



For whole cross section, $A = 2\pi at$

$$J = Aa^2 = 2\pi a^3 t \quad I = \frac{1}{2}J = \pi a^3 t$$

Use polar coordinate θ for partial cross section.

$$A = st = a\theta t \quad s = \text{arc length}$$

$$\bar{r} = a \frac{\sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{1}{2}\theta$$

$$\bar{y} = \bar{r} \sin \alpha = a \frac{\sin^2 \alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t a \frac{\sin^2 \alpha}{\alpha} = a^2 t \cdot 2 \sin^2 \alpha \\ = a^2 t \cdot 2 \sin^2 \frac{\theta}{2} = a^2 t (1 - \cos \theta)$$

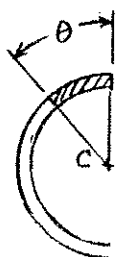
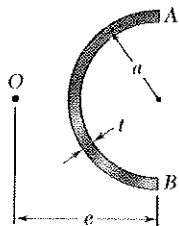
$$\tau = \frac{VQ}{It} = \frac{Va^2}{I} (1 - \cos \theta)$$

$$M_c = \int a \tau dA = \int_0^{2\pi} \frac{Va^3}{I} (1 - \cos \theta) t a d\theta = \frac{Va^4 t}{I} (\theta - \sin \theta) \Big|_0^{2\pi} \\ = \frac{2\pi Va^4 t}{\pi a^3 t} = 2aV$$

$$\text{But } M_c = Ve, \quad \text{hence } e = 2a$$

Problem 6.74

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



For a thin-walled hollow circular cross section $A = 2\pi at$

$$J = a^2 A = 2\pi a^3 t \quad I = \frac{1}{2} J = \pi a^3 t$$

For the half-pipe section $I = \frac{\pi}{2} a^3 t$

Use polar coordinate θ for partial cross section,

$$A = st = a\theta t \quad s = \text{arc length}$$

$$\bar{r} = a \frac{\sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{\theta}{2}$$

$$\bar{y} = \bar{r} \cos \alpha = a \frac{\sin \alpha \cos \alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t a \frac{\sin \alpha \cos \alpha}{\alpha} = a^2 t (2 \sin \alpha \cos \alpha) \\ = a^2 t \sin 2\alpha = a^2 t \sin \theta$$

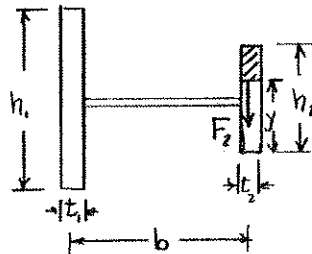
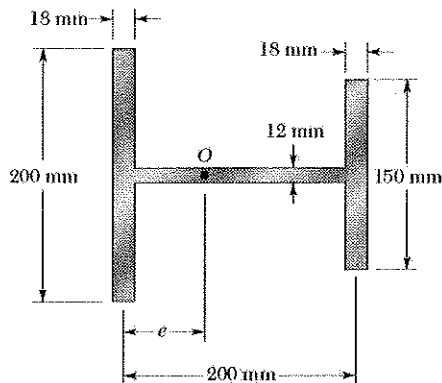
$$\tau = \frac{VQ}{It} = \frac{Va^2}{I} \sin \theta$$

$$M_H = \int a \tau dA = \int_0^\pi a \frac{Va^2}{I} \sin \theta t a d\theta = \frac{Va^4 t}{I} [-\cos \theta]_0^\pi \\ = 2 \frac{Va^4 t}{I} = \frac{4}{\pi} Va$$

$$\text{But } M_H = Ve, \quad \text{hence } e = \frac{4}{\pi} a = 1.273 a$$

Problem 6.75

6.75 and 6.76 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.



$$I = \frac{1}{12} t_1 h_1^3 + \frac{1}{12} t_2 h_2^3$$

Right flange:

$$A = (\frac{1}{2} h_2 - y) t_2$$

$$\bar{y} = \frac{1}{2} (\frac{1}{2} h_2 + y) t_2$$

$$Q = A \bar{y}$$

$$= \frac{1}{2} (\frac{1}{2} h_2 - y) (\frac{1}{2} h_2 + y) t_2$$

$$= \frac{1}{2} (\frac{1}{4} h_2^2 - y^2) t_2$$

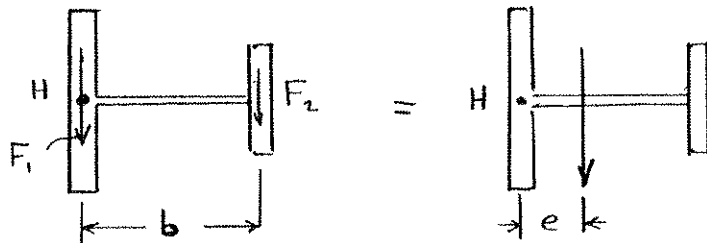
$$\tau = \frac{VQ}{It_2} = \frac{V}{2It_2} (\frac{1}{4} h_2^2 - y^2) t_2$$

$$F_2 = \int \tau dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2It_2} (\frac{1}{4} h_2^2 - y^2) t_2 dy = \frac{Vt_2}{2I} (\frac{1}{4} h_2^2 y - \frac{y^3}{3}) \Big|_{-h_2/2}^{h_2/2}$$

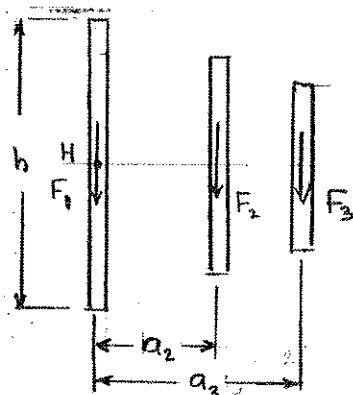
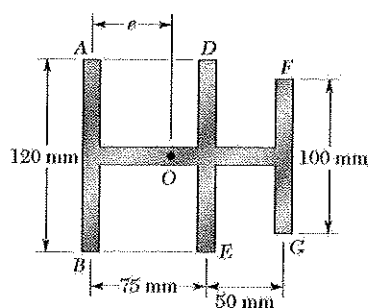
$$= \frac{Vt_2}{2I} \left\{ \frac{1}{4} h_2^2 \frac{h_2}{2} - \frac{1}{3} \left(\frac{h_2}{2} \right)^3 + \frac{1}{4} h_2^2 \frac{h_2}{2} - \frac{1}{3} \left(\frac{h_2}{2} \right)^3 \right\} = \frac{Vt_2 h_2^3}{12I} = \frac{V t_2 h_2^3}{t_1 h_1^3 + t_2 h_2^3}$$

$$\sum M_H = \sum M_H: -Ve = -F_2 b = -V \frac{t_2 h_2^3 b}{t_1 h_1^3 + t_2 h_2^3}$$

$$e = \frac{t_2 h_2^3 b}{t_1 h_1^3 + t_2 h_2^3} = \frac{(18)(150)^3(200)}{(18)(200)^3 + (18)(150)^3} = 59.3 \text{ mm}$$



Problem 6.76



6.75 and 6.76 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.

$$\text{Let } h_1 = \overline{AB}, \quad h_2 = \overline{DE}, \quad \text{and } h_3 = \overline{FG}.$$

$$I = \frac{1}{12} t (h_1^3 + h_2^3 + h_3^3)$$

$$\text{Part AB: } A = \left(\frac{1}{2} h_1 - y\right) t$$

$$\bar{y} = \frac{1}{2} \left(\frac{1}{2} h_1 + y\right)$$

$$Q = A \bar{y} = \frac{1}{2} t \left(\frac{1}{2} h_1 - y\right) \left(\frac{1}{2} h_1 + y\right) \\ = \frac{1}{2} t \left(\frac{1}{4} h_1^2 - y^2\right)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I} \left(\frac{1}{4} h_1^2 - y^2\right)$$

$$F_1 = \int \tau dA = \int_{-\frac{1}{2} h_1}^{\frac{1}{2} h_1} \frac{V}{2I} \left(\frac{1}{4} h_1^2 - y^2\right) t dy \\ = \frac{Vt}{2I} \left(\frac{1}{4} h_1^2 y - \frac{y^3}{3}\right) \Big|_{-\frac{1}{2} h_1}^{\frac{1}{2} h_1} \\ = \frac{Vt}{I} \left(\frac{1}{4} h_1^2 \frac{1}{2} h_1 - \frac{1}{3} \left(\frac{h_1}{2}\right)^3\right) = \frac{Vt h_1^3}{12 I} \\ = \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$

Likewise, for Part DE:

$$F_2 = \frac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$$

and for Part FG:

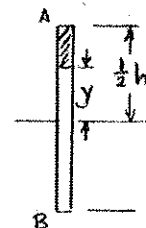
$$F_3 = \frac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$$

$$+\circlearrowleft \sum M_H = +\circlearrowleft \sum M_H:$$

$$Ve = a_2 F_2 + a_3 F_3 = \frac{a_2 h_2^3 + a_3 h_3^3}{h_1^3 + h_2^3 + h_3^3} V$$

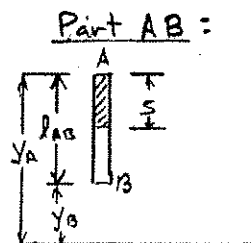
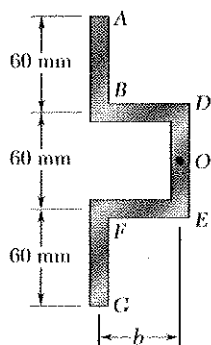
Dividing by V ,

$$e = \frac{a_2 h_2^3 + a_3 h_3^3}{h_1^3 + h_2^3 + h_3^3} = \frac{(75)(120)^3 + (125)(100)^3}{120^3 + 120^3 + 100^3} \\ = 57.1 \text{ mm}$$



Problem 6.77

6.77 and 6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



Part AB :

$$A(s) = ts$$

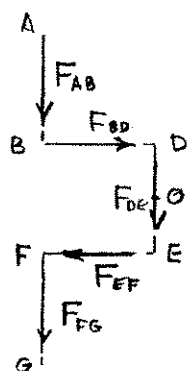
$$\bar{y}(s) = y_A - \frac{1}{2}s$$

$$Q(s) = A(s)\bar{y}(s) = ty_A s - \frac{1}{2}ts^2$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vt}{I}(y_A s - \frac{1}{2}s^2)$$

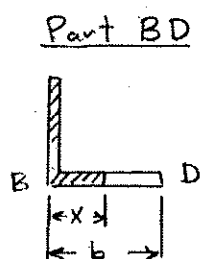
$$F_{AB} = \int_0^{l_{AB}} q(s) ds$$

$$= \frac{Vt}{I} \left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6} \right) \downarrow$$



At B: $Q_B = ty_A l_{AB} - \frac{1}{2}t l_{AB}^2$

By symmetry, $F_{FG} = F_{AB}$



Part BD :

$$A(x) = tx$$

$$Q(x) = Q_B + y_B A(x) = Q_B + ty_B x$$

$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(Q_B + ty_B x)$$

$$F_{BD} = \int_0^b q(x) dx = \frac{V}{I} (Q_B b + \frac{1}{2}ty_B b^2) \rightarrow$$

By symmetry, $F_{EF} = F_{BD}$

F_{DE} is not required, since its moment about O is zero.

$$\sum M_O = 0: \quad b(F_{AB} + F_{FG}) - y_B F_{BD} + y_F F_{EF} = 0$$

$$2b F_{AB} - 2y_B F_{BD} = 0$$

$$2b \cdot \frac{Vt}{I} \left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6} \right) - 2y_B \frac{V}{I} (Q_B b + \frac{1}{2}ty_B b^2) = 0$$

$$\frac{2Vt}{I} \left\{ \frac{1}{2} y_A l_{AB}^2 - \frac{1}{6} l_{AB}^3 \right\} b - \frac{2Vt}{I} \left\{ (y_A l_{AB} - \frac{1}{2} l_{AB}^2) y_B b + \frac{1}{2} y_B^2 b^2 \right\} = 0$$

Dividing by $\frac{2Vt}{I}$ and substituting numerical data,

$$\left\{ \frac{1}{2} (90)(60)^2 - \frac{1}{6} (60)^3 \right\} b - \left[(90)(60) - \frac{1}{2} (60)^2 \right] (30)b + \frac{1}{2} (30)^2 b^2 = 0$$

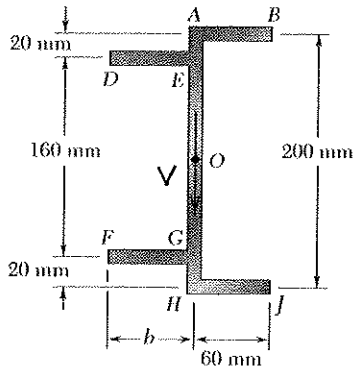
$$126 \times 10^3 b - 108 \times 10^3 b + 450 b^2 = 0$$

$$18 \times 10^3 b - 450 b^2 = 0$$

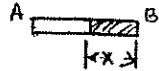
$$b = 0 \text{ and } b = 40.0 \text{ mm}$$

Problem 6.78

6.77 and 6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



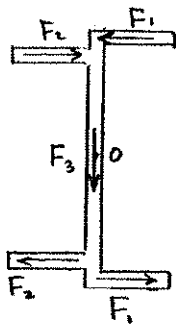
Part AB:



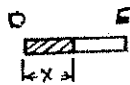
$$A = tx, \quad \bar{y} = 100 \text{ mm}, \quad Q = 100 tx$$

$$q = \frac{VQ}{I} = \frac{100 V tx}{I}$$

$$F_1 = \int_{x_A}^{x_B} q dx = 100 \frac{Vt}{I} \int_0^{60} x dx = \frac{(100)(60)^2}{2} \frac{Vt}{I} = 180 \times 10^3 \frac{Vt}{I}$$



Part DE:



$$A = tx, \quad \bar{y} = 80 \text{ mm}, \quad Q = 80 tx$$

$$q = \frac{VQ}{I} = \frac{80 V tx}{I}$$

$$F_2 = \int_{x_0}^x q dx = 80 \frac{Vt}{I} \int_0^a x dx = (40 a^2) \frac{Vt}{I}$$

$$+\circlearrowleft \sum M_O = 0: (200)(180 \times 10^3) \frac{Vt}{I} - (160)(40 a^2) \frac{Vt}{I} = 0$$

$$a^2 = \frac{(200)(180 \times 10^3)}{(160)(40)} = 5625 \text{ mm}^2$$

$$a = 75.0 \text{ mm}$$

Problem 6.79

6.79 For the angle shape and loading of Sample Prob. 6.6, check that $\int q dz = 0$ along the horizontal leg of the angle and $\int q dy = P$ along its vertical leg.

Refer to Sample Prob. 6.6.

Along horizontal leg: $\tau_t = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3} (a^2 - 4az + 3z^2)$

$$\int q dz = \int_0^a \tau_t t dz = \frac{3P}{4a^3} \int_0^a (a^2 - 4az + 3z^2) dz = \frac{3P}{4a^3} \left(a^2 z - 4a \frac{z^2}{2} + 3 \frac{z^3}{3} \right) \Big|_0^a = \frac{3P}{4a^3} (a^3 - 2a^3 + a^3) = 0$$

Along vertical leg: $\tau_c = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3} (a^2 + 4ay - 5y^2)$

$$\int q dy = \int_0^a \tau_c t dy = \frac{3P}{4a^3} \int_0^a (a^2 + 4ay - 5y^2) dy = \frac{3P}{4a^3} \left(a^2 y + 4a \frac{y^2}{2} - 5 \frac{y^3}{3} \right) \Big|_0^a = \frac{3P}{4a^3} (a^3 + 2a^3 - \frac{5}{3}a^3) = \frac{3P}{4a^3} \cdot \frac{4}{3}a^3 = P$$

Problem 6.80

6.80 For the angle shape and loading of Sample Prob. 6.6, (a) determine the points where the shearing stress is maximum and the corresponding values of the stress, (b) verify that the points obtained are located on the neutral axis corresponding to the given loading.

Refer to Sample Prob. 6.6.

(a) Along vertical leg: $\tau_c = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3} (a^2 + 4ay - 5y^2)$

$$\frac{d\tau_c}{dy} = \frac{3P}{4ta^3} (4a - 10y) = 0 \quad y = \frac{2}{5}a$$

$$\tau_m = \frac{3P}{4ta^3} \left[a^2 + (4a)\left(\frac{2}{5}a\right) - (5)\left(\frac{2}{5}a\right)^2 \right] = \frac{3P}{4ta^3} \left(\frac{9}{5}a^2 \right) = \frac{27}{20} \frac{P}{ta}$$

Along horizontal leg: $\tau_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3} (a^2 - 4az + 3z^2)$

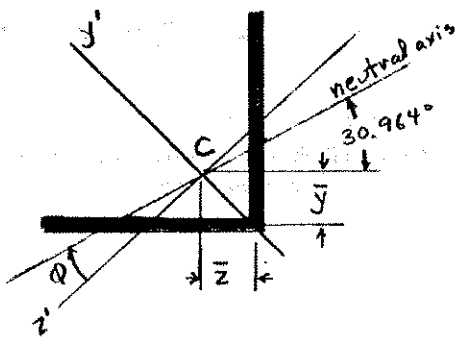
$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3} (-4a + 6z) = 0 \quad z = \frac{2}{3}a$$

$$\tau_m = \frac{3P}{4ta^3} \left[a^2 - (4a)\left(\frac{2}{3}a\right) + (3)\left(\frac{2}{3}a\right)^2 \right] = \frac{3P}{4ta^3} \left(-\frac{5}{3}a^2 \right) = -\frac{1}{4} \frac{P}{ta}$$

At the corner: $y=0, z=0 \quad \tau = \frac{3}{4} \frac{P}{ta}$

(b) $I_{y'} = \frac{1}{3} ta^3 \quad I_{z'} = \frac{1}{12} ta^3 \quad \theta = 45^\circ$

$$\tan \varphi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{1}{4} \quad \varphi = 14.036^\circ$$



$$\theta - \varphi = 45 - 14.036 = 30.964^\circ$$

$$\bar{y} = \frac{\sum A \bar{y}}{\sum A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

$$\bar{z} = \frac{\sum A \bar{z}}{\sum A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

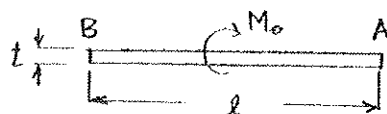
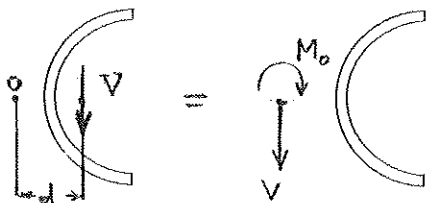
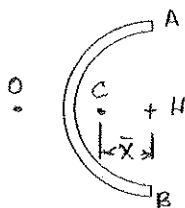
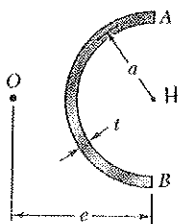
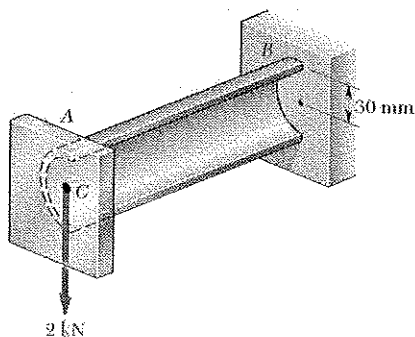
Neutral axis intersects vertical leg at

$$y = \bar{y} + \bar{z} \tan 30.964^\circ = \left(\frac{1}{4} + \frac{1}{4} \tan 30.964^\circ \right) a = 0.400a = \frac{2}{5}a$$

Neutral axis intersects horizontal leg at

$$z = \bar{z} + \bar{y} \tan (45^\circ + \varphi) = \left(\frac{1}{4} + \frac{1}{4} \tan 59.036^\circ \right) a = 0.6667a = \frac{2}{3}a$$

Problem 6.81



***6.81** The cantilever beam AB , consisting of half of a thin-walled pipe of 30-mm mean radius and 10-mm wall thickness, is subjected to a 2-kN vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum shearing stress in the beam. (Hint: The shear center O of this cross section was shown in Prob. 6.73 to be located twice as far from its vertical diameter as its centroid C .)

From the solution to Problem 6.73,

$$I = \frac{\pi}{2} a^3 t$$

$$Q = a^2 t \sin \theta$$

$$e = \frac{4}{\pi} a$$

$$Q_{\max} = a^2 t$$

For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$

At each section of the beam, the shearing force V is equal to P . Its line of action passes through the centroid C . The moment arm of its moment about the shear center O is

$$d = e - \bar{x} = \frac{4}{\pi} a - \frac{2}{\pi} a = \frac{2}{\pi} a$$

(a) Equivalent force-couple system at O .

$$V = P$$

$$M_o = Vd = \frac{2}{\pi} Pa$$

$$\text{Data: } P = 2 \text{ kN}$$

$$a = 30 \text{ mm}$$

$$V = 2 \text{ kN}$$

$$M_o = 38.2 \text{ Nm}$$

$$M_o = \left(\frac{2}{\pi}\right)(2000)(30)$$

(b) Shearing stresses

$$(1) \text{ Due to } V, \quad \tau_v = \frac{V Q_{\max}}{I t}$$

$$\tau_v = \frac{(P)(a^2 t)}{\left(\frac{\pi}{2} a^3 t\right)(t)} = \frac{2P}{\pi a t} = \frac{(2)(2000)}{\pi(0.03)(0.01)} = 4.24 \text{ MPa}$$

(2) Due to M_o , the torque.

For a long rectangular section of length l and width t the shearing stress due to torque M_o is

$$\tau_m = \frac{M_o}{C_1 l t^2} \quad \text{where} \quad C_1 = \frac{1}{3} \left(1 - 0.630 \frac{t}{l}\right)$$

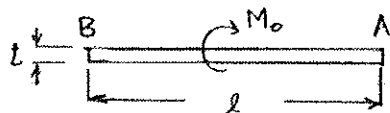
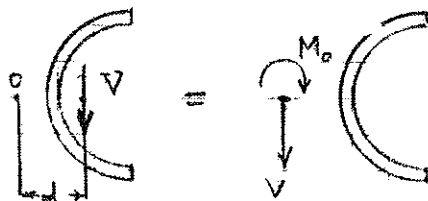
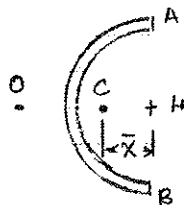
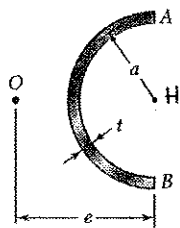
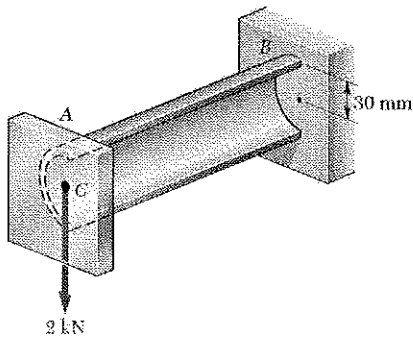
$$\text{Data: } l = \pi a = \pi(30) = 94.25 \text{ mm} \quad t = 10 \text{ mm}$$

$$C_1 = 0.311$$

$$\tau_m = \frac{38200}{(0.311)(94.25)(10)^2} = 13.03 \text{ MPa}$$

$$\text{By superposition} \quad \tau = \tau_v + \tau_m = 4.24 + 13.03 = 17.3 \text{ MPa}$$

Problem 6.82



***6.82** Solve Prob. 6.81, assuming that the thickness of the beam is reduced to 6 mm.

***6.81** The cantilever beam AB, consisting of half of a thin-walled pipe of 30-mm mean radius and 10-mm wall thickness, is subjected to a 2-kN vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum shearing stress in the beam. (Hint: The shear center O of this cross section was shown in Prob. 6.74 to be located twice as far from its vertical diameter as its centroid C.)

From the solution to Problem 6.73

$$I = \frac{\pi}{2} a^3 t \quad Q = a^2 t \sin \theta$$

$$e = \frac{4}{\pi} a \quad Q_{max} = a^2 t$$

For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$

At each section of the beam, the shearing force V is equal to P. Its line of action passes through the centroid C. The moment arm of its moment about the shear center O is

$$d = e - \bar{x} = \frac{4}{\pi} a - \frac{2}{\pi} a = \frac{2}{\pi} a$$

(a) Equivalent force-couple system at O.

$$V = P \quad M_o = Vd = \frac{2}{\pi} Pa$$

$$\text{Data: } P = 2 \text{ kN} \quad a = 30 \text{ mm}$$

$$V = 2 \text{ kN} \quad \blacktriangleleft$$

$$M_o = 38.2 \text{ Nm} \quad \blacktriangleleft$$

(b) Shearing stresses

$$(1) \text{ Due to } V, \quad \tau_v = \frac{V Q_{max}}{I t}$$

$$\tau_v = \frac{(P)(a^2 t)}{(\frac{\pi}{2} a^3 t)(t)} = \frac{2P}{\pi a t} = \frac{(2)(2000)}{\pi(30)(6)} = 7.07 \text{ MPa}$$

(2) Due to M_o , the torque.

For a long rectangular section of length l and width t the shearing stress due to torque M_o is

$$\tau_m = \frac{M_o}{C_1 l t^2} \quad \text{where } C_1 = \frac{1}{3} \left(1 - 0.630 \frac{t}{l} \right)$$

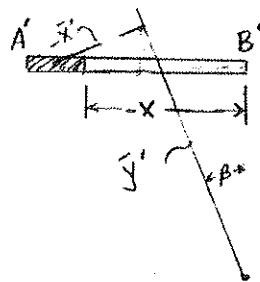
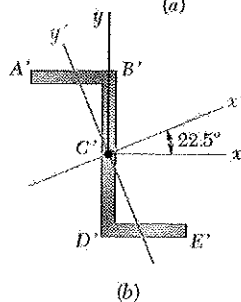
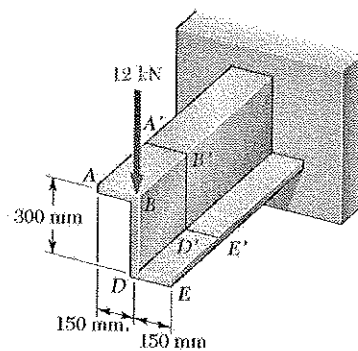
$$\text{Data: } l = \pi a = \pi(30) = 94.25 \text{ mm} \quad t = 6 \text{ mm}$$

$$C_1 = 0.31996$$

$$\tau_m = \frac{38200}{(0.31996)(94.25)(36)} = 35.19 \text{ MPa}$$

$$\text{By superposition } \tau = \tau_v + \tau_m = 7.07 + 35.19 = 42.3 \text{ MPa}$$

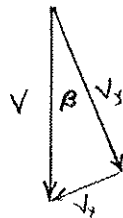
Problem 6.83



*6.83 The cantilever beam shown consists of a Z shape of 6-mm thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_x = 69.2 \times 10^6 \text{ mm}^4$ and are $I_y = 5.7 \times 10^6 \text{ mm}^4$.

$$V = 12 \text{ kN}$$

$$\beta = 22.5^\circ$$



$$V_{x'} = V \sin \beta \quad V_{y'} = V \cos \beta$$

In upper horizontal leg use coordinate x ($-150 \text{ mm} \leq x \leq 0$)

$$A = 6(150 + x) \text{ mm.}$$

$$\bar{x} = \frac{1}{2}(-150 + x) \text{ mm.}$$

$$\bar{y} = 150 \text{ mm.}$$

$$\bar{x}' = \bar{x} \cos \beta + \bar{y} \sin \beta$$

$$\bar{y}' = \bar{y} \cos \beta - \bar{x} \sin \beta$$

$$\text{Due to } V_{x'}: \tau_1 = \frac{V_{x'} A \bar{x}'}{I_y t}$$

$$\tau_1 = \frac{(V \sin \beta)(6)(150 + x) \left[\frac{1}{2}(-150 + x) \cos \beta + 150 \sin \beta \right]}{(5.7 \times 10^6)(6)}$$

$$= 8.056 \times 10^{-4} (150 + x) (-11.9 + 0.46194x)$$

$$\text{Due to } V_{y'}: \tau_2 = \frac{V_{y'} A \bar{y}'}{I_x t} = \frac{(V \cos \beta)(6)(150 + x) \left[150 \cos \beta - \frac{1}{2}(-150 + x) \sin \beta \right]}{(69.2 \times 10^6)(6)}$$

$$= 1.602 \times 10^{-4} (150 + x) [167.3 - 0.19134x]$$

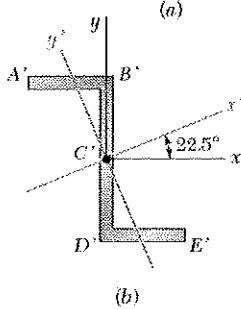
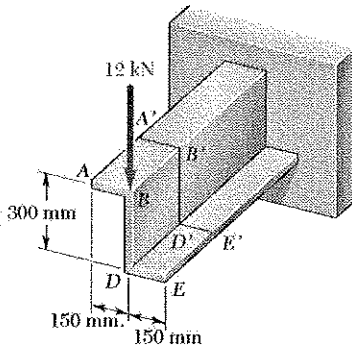
$$\text{Total: } \tau_1 + \tau_2 = (150 + x) [0.0172 + 0.0003415x]$$

$x \text{ (mm)}$	-150	-125	-100	-75	-50	-25	0
$\tau \text{ (MPa)}$	0	-0.643	-0.848	-0.631	0.0125	1.083	2.58

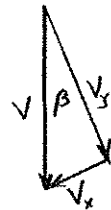
Problem 6.84

***6.84** For the cantilever beam and loading of Prob. 6.83, determine the distribution of the shearing stress along line $B'D'$ in the vertical web of the Z shape.

***6.83** The cantilever beam shown consists of a Z shape of 6-mm thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 69.2 \times 10^6 \text{ mm}^4$ and are $I_{y'} = 5.7 \times 10^6 \text{ mm}^4$.



$$V = 12 \text{ kN} \quad \beta = 22.5^\circ$$



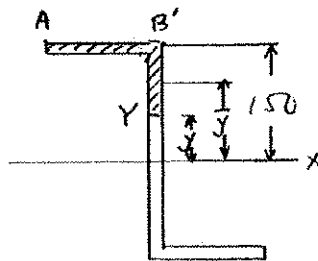
$$V_{x'} = V \sin \beta \quad V_{y'} = V \cos \beta$$

For part AB' $A = (6)(150) = 900 \text{ mm}^2$
 $\bar{x} = -75 \text{ mm}, \bar{y} = 150 \text{ mm}$

For part $B'Y$

$$A = 6(150 - y)$$

$$\bar{x} = 0 \quad \bar{y} = \frac{1}{2}(150 + y)$$



$$x' = x \cos \beta + y \sin \beta$$

$$y' = y \cos \beta - x \sin \beta$$

Due to $V_{x'}$: $\tau_1 = \frac{V_{x'}(A_{AB} \bar{x}'_{AB} + A_{BY} \bar{x}'_{BY})}{I_{y'} t}$

$$\tau_1 = \frac{(V \sin \beta) [(900)(-75 \cos \beta + 150 \sin \beta) + 6(150 - y) \frac{1}{2}(150 + y) \sin \beta]}{(5.7 \times 10^6)(6)}$$

$$= \frac{(V \sin \beta) [-10700 + 25831 - 1.148 y^2]}{34.2 \times 10^6} = 2.0317 - 0.0001541 y^2$$

Due to $V_{y'}$: $\tau_2 = \frac{V_{y'}(A_{AB} \bar{y}'_{AB} + A_{BY} \bar{y}'_{BY})}{I_{x'} t}$

$$\tau_2 = \frac{(V \cos \beta) [(900)(150 \cos \beta + 75 \sin \beta) + 6(150 - y) \frac{1}{2}(150 + y) \cos \beta]}{(69.2 \times 10^6)(6)}$$

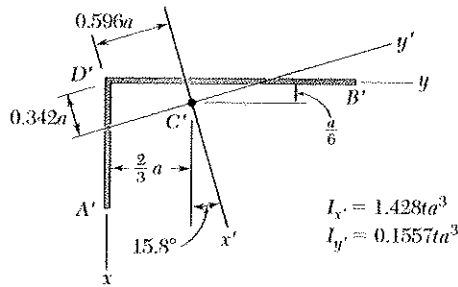
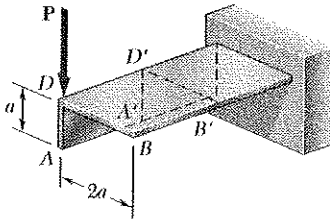
$$= \frac{(V \cos \beta) [187086 + 62361 - 2.772 y^2]}{(69.2 \times 10^6)(6)} = 6.66 - 7.402 \times 10^{-5} y^2$$

Total: $\tau_1 + \tau_2 = 8.6917 - 2.2812 \times 10^{-4} y^2$

$y \text{ (mm)}$	0	± 50	± 100	± 150
$\tau \text{ (MPa)}$	8.692	8.121	6.41	3.56

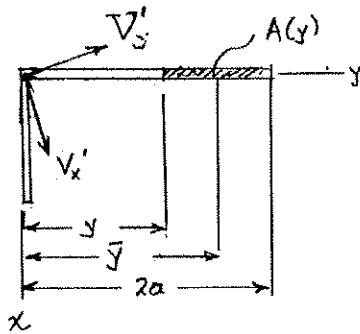
Problem 6.85

*6.85 Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.



$$I_{x'} = 1.428ta^3$$

$$I_{y'} = 0.1557ta^3$$



$$\beta = 15.8^\circ \quad V_{x'}' = P \cos \beta \quad V_{y'}' = -P \sin \beta$$

$$A(y) = (2a - y)t \quad \bar{y} = \frac{1}{2}(2a + y), \quad \bar{x} = 0$$

Coordinate transformation.

$$y' = (y - \frac{2}{3}a) \cos \beta - (x - \frac{1}{6}a) \sin \beta$$

$$x' = (x - \frac{1}{6}a) \cos \beta + (y - \frac{2}{3}a) \sin \beta$$

In particular,

$$\bar{y}' = (\bar{y} - \frac{2}{3}a) \cos \beta - (\bar{x} - \frac{1}{6}a) \sin \beta$$

$$= (\frac{1}{2}y + \frac{1}{3}a) \cos \beta - (-\frac{1}{6}a) \sin \beta$$

$$= 0.48111 y + 0.36612 a$$

$$\bar{x}' = (\bar{x} - \frac{1}{6}a) \cos \beta + (\bar{y} - \frac{2}{3}a) \sin \beta$$

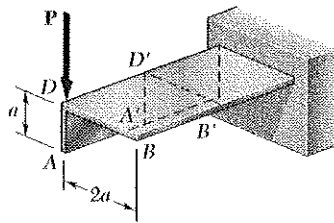
$$= (-\frac{1}{6}a) \cos \beta + (\frac{1}{2}y + \frac{1}{3}a) \sin \beta$$

$$= 0.13614 y - 0.06961 a$$

$$\begin{aligned} \tau &= \frac{V_{x'}' A \bar{x}'}{I_{y'}' t} + \frac{V_{y'}' A \bar{y}'}{I_{x'}' t} \\ &= \frac{(P \cos \beta)(2a - y)(t)(0.13614 y - 0.06961 a)}{(0.1557 t a^3)(t)} \\ &\quad + \frac{(-P \sin \beta)(2a - y)(0.48111 y + 0.36612 a)}{(1.428 a^3 t)(t)} \\ &= \frac{P(2a - y)(0.750 y - 0.500 a)}{t a^3} \end{aligned}$$

$y(a)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$-\tau \left(\frac{P}{at} \right)$	-1.000	-0.417	0	0.250	0.333	0.250	0

Problem 6.86



*6.86 For the angle shape and loading of Prob. 6.85, determine the distribution of the shearing stresses along line $D'A'$ in the vertical leg.

*6.85 Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.

$$\beta = 15.8^\circ \quad V_x' = P \cos \beta \quad V_y' = -P \sin \beta$$

$$A(x) = (a-x)t \quad \bar{x} = \frac{1}{2}(a+x), \quad \bar{y} = 0$$

Coordinate transformation.

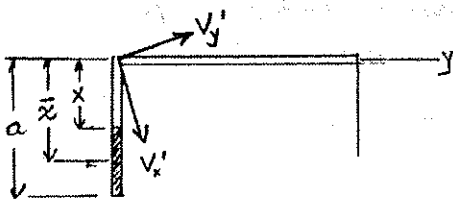
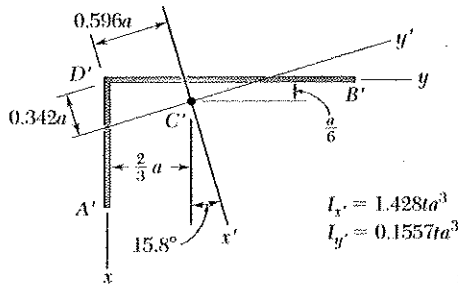
$$y' = (y - \frac{2}{3}a) \cos \beta - (x - \frac{1}{6}a) \sin \beta$$

$$x' = (x - \frac{1}{6}a) \cos \beta + (y - \frac{2}{3}a) \sin \beta$$

In particular,

$$\begin{aligned} \bar{y}' &= (\bar{y} - \frac{2}{3}a) \cos \beta - (\bar{x} - \frac{1}{6}a) \sin \beta \\ &= (-\frac{2}{3}a) \cos \beta - (\frac{1}{2}x + \frac{1}{3}a) \sin \beta \\ &= -0.13614x - 0.73224a \end{aligned}$$

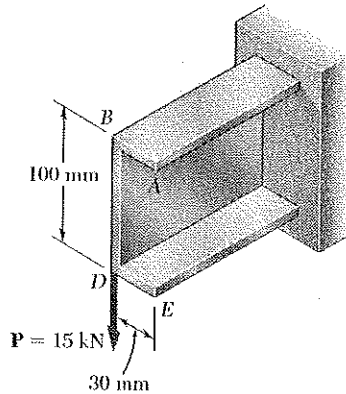
$$\begin{aligned} \bar{x}' &= (\bar{x} - \frac{1}{6}a) \cos \beta + (\bar{y} - \frac{2}{3}a) \sin \beta \\ &= (\frac{1}{2}x + \frac{1}{3}a) \cos \beta + (-\frac{2}{3}a) \sin \beta \\ &= 0.48111x + 0.13922a \end{aligned}$$



$$\begin{aligned} + \downarrow \tau &= \frac{V_x' A(x) \bar{x}'}{I_y' t} - \frac{V_y' A(x) \bar{y}'}{I_x' t} \\ &= \frac{(P \cos \beta)(a-x)(t)(0.48111x + 0.13922a)}{(0.1557 ta^3)(t)} \\ &\quad + \frac{(-P \sin \beta)(a-x)(t)(-0.13614x - 0.73224a)}{(1.428 ta^3)(t)} \\ &= \frac{P(a-x)(3.00x + 1.000a)}{ta^3} \end{aligned}$$

$x(a)$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$\tau \downarrow (\frac{P}{at})$	1.000	1.250	1.333	1.250	1.000	0.583	0

Problem 6.87



*6.87 A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load P acts at a point in the midplane of the web of the channel, determine (a) the torque T that would cause the channel to twist in the same way that it does under the load P , (b) the maximum shearing stress in the channel caused by the load P .

Use results of Example 6.06 with $b = 30 \text{ mm}$,

$h = 100 \text{ mm}$, and $t = 8 \text{ mm}$.

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} t h^3 (6b + h) = \frac{1}{12} (8)(100)^3 [(6)(30) + 100] \\ = 1.86667 \times 10^6 \text{ mm}^4 = 1.86667 \times 10^{-6} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$

$$(a) T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3}) = 144.64 \text{ N}\cdot\text{m}$$

Stress at neutral axis due to V :

$$Q = b t \frac{h}{2} + t \left(\frac{h}{2} \right) \left(\frac{h}{4} \right) \\ = \frac{1}{8} t h (h + 4b) \\ = \frac{1}{8} (8)(100) [100 + (4)(30)] \\ = 22 \times 10^3 \text{ mm}^3 = 22 \times 10^{-6} \text{ m}^3$$

$$t = 8 \times 10^{-3} \text{ m}$$

$$\tau_v = \frac{VQ}{It} = \frac{(15 \times 10^3)(22 \times 10^{-6})}{(1.86667 \times 10^{-6})(8 \times 10^{-3})} = 22.10 \times 10^6 \text{ Pa} = 22.10 \text{ MPa}$$

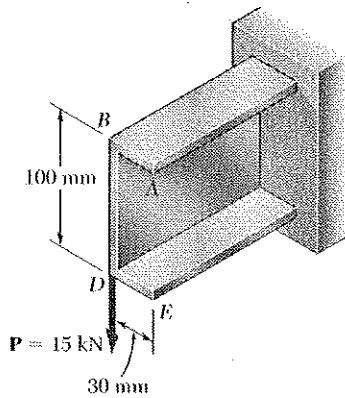
Stress due to T : $a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_1 = \frac{1}{3} \left(1 - 0.630 \frac{t}{a} \right) = \frac{1}{3} \left[1 - (0.630) \frac{8}{160} \right] = 0.3228$$

$$\tau_v = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.3228)(0.160)(8 \times 10^{-3})^2} = 43.76 \times 10^6 \text{ Pa} \\ = 43.76 \text{ MPa}$$

$$(b) \text{ By superposition, } \tau_{\max} = \tau_v + \tau_T = \tau_{\max} = 65.9 \text{ MPa}$$

Problem 6.88



*6.88 Solve Prob. 6.87, assuming that a 6-mm-thick plate is bent to form the channel shown.

*6.87 A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load P acts at a point in the midplane of the web of the channel, determine (a) the torque T that would cause the channel to twist in the same way that it does under the load P , (b) the maximum shearing stress in the channel caused by the load P .

Use results of Example 6.06 with $b = 30 \text{ mm}$

$h = 100 \text{ mm}$, and $t = 6 \text{ mm}$

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{3 \times 30}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} t h^2 (6b + h) = \frac{1}{12} (6)(100)^2 [(6)(30) + 100] \\ = 1.400 \times 10^6 \text{ mm}^4 = 1.400 \times 10^{-6} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$

$$(a) T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3}) = 144.64 \text{ N}\cdot\text{m}$$

Stress at neutral axis due to V

$$Q = bt \frac{h}{2} + t \left(\frac{h}{2} \times \frac{h}{4} \right) \\ = \frac{1}{8} t h (h + 4b) \\ = \frac{1}{8} (6)(100) [100 + (4)(30)] \\ = 16.5 \times 10^3 \text{ mm}^3 = 16.5 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau_v = \frac{VQ}{It} = \frac{(15 \times 10^3)(16.5 \times 10^{-6})}{(1.400 \times 10^{-6})(6 \times 10^{-3})} = 29.46 \times 10^6 \text{ Pa} = 29.46 \text{ MPa}$$

Stress due to T

$$a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$$

$$c_1 = \frac{1}{3} \left(1 - 0.630 \frac{t}{a} \right) = \frac{1}{3} \left[1 - (0.630) \frac{6}{160} \right] = 0.32546$$

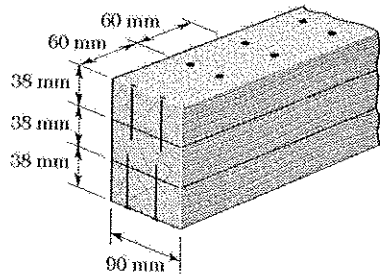
$$\tau_t = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.32546)(0.160)(6 \times 10^{-3})^2} = 77.16 \times 10^6 \text{ Pa} = 77.16 \text{ MPa}$$

(b) By superposition

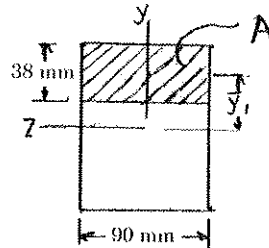
$$\tau_{\max} = \tau_v + \tau_t$$

$$\tau_{\max} = 106.6 \text{ MPa}$$

Problem 6.89



6.89 Three boards, each of 38×90 -mm rectangular cross section, are nailed together to form a beam that is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.



$$I = \frac{1}{12} b h^3 = \frac{1}{12} (90) (114)^3 = 110112 \times 10^6 \text{ mm}^4$$

$$A = (90)(38) = 3420 \text{ mm}^2$$

$$\bar{y}_1 = 38 \text{ mm}$$

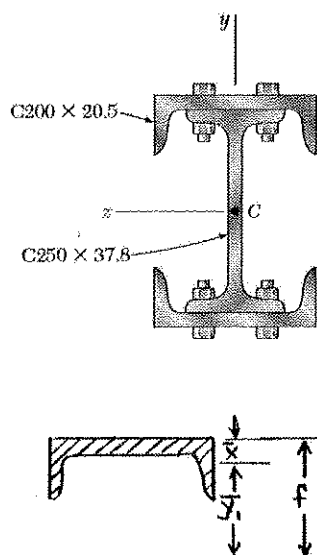
$$Q = A \bar{y}_1 = 129960 \text{ mm}^3$$

$$q = \frac{VQ}{I} = \frac{(1)(129960 \times 10^{-9})}{(110112) \times 10^{-6}} = 11.7 \text{ kN/m}$$

$$q_s = 2 F_{\text{nail}}$$

$$F_{\text{nail}} = \frac{qs}{2} = \frac{(11.7)(0.06)}{2} = 351 \text{ N}$$

Problem 6.90



6.90 A column is fabricated by connecting the rolled-steel members shown by bolts of 18-mm diameter spaced longitudinally every 125 mm. Determine the average shearing stress in the bolts caused by a shearing force of 120 kN parallel to the y axis.

Geometry

$$f = \left(\frac{d}{2}\right)_s + (t_w)_c$$

$$= \frac{254}{2} + 7.7 = 134.7 \text{ mm}$$

$$\bar{x} = 13.9 \text{ mm}$$

$$\bar{y}_1 = f - \bar{x} = 134.7 - 13.9 = 120.8 \text{ mm}$$

Determine moment of inertia.

Part	A (mm ²)	d (mm)	A d ² (mm ⁴)	$\bar{I}$ (mm ⁴)
C200x20.5	2660	120.8	38.816x10 ⁶	0.625x10 ⁶
S250x37.8	7.46	0	0	57.1x10 ⁶
C200x20.5	2660	120.8	38.816x10 ⁶	0.625x10 ⁶
Σ			77.63x10 ⁶	52.35x10 ⁶

$$I = \Sigma A d^2 + \Sigma \bar{I} = (77.63 + 52.35) \times 10^6 = 130 \times 10^6$$

$$Q = A \bar{y}_1 = (2660)(120.8) = 321328 \text{ mm}^3$$

$$q = \frac{VQ}{I} = \frac{(120)(321328 \times 10^{-9})}{130 \times 10^6} = 296.6 \text{ kN/m}$$

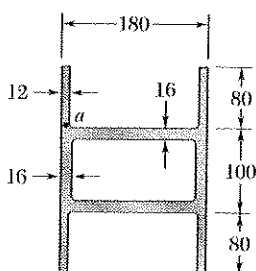
$$F_{\text{bolt}} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(296.6)(0.125) = 18.54 \text{ kN}$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} (18)^2 = 254.47 \text{ mm}^2$$

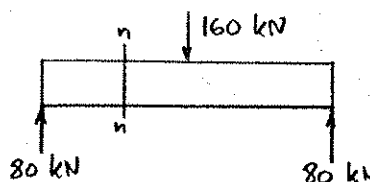
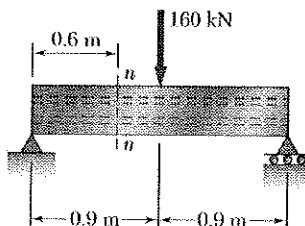
$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{18540}{254.47 \times 10^{-6}} = 72.9 \text{ MPa}$$

Problem 6.91

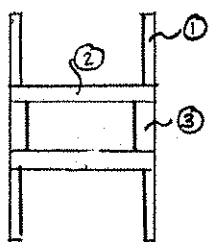
6.91 For the beam and loading shown, consider section $n-n$ and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a .



Dimensions in mm



At section $n-n$ $V = 80 \text{ kN}$



Consider cross section as composed of rectangles of types ①, ②, and ③.

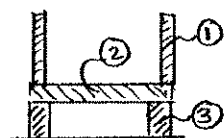
$$I_1 = \frac{1}{12} (12)(80)^3 + (12)(80)(90)^2 = 8.288 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} (180)(16)^3 + (180)(16)(42)^2 = 5.14176 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12} (16)(68)^3 = 419.24 \times 10^3 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + 2I_3 = 44.274 \times 10^6 \text{ mm}^4 \\ = 44.274 \times 10^{-6} \text{ m}^4$$

(a) Calculate Q at neutral axis.



$$Q_1 = (12)(80)(90) = 86.4 \times 10^3 \text{ mm}^4$$

$$Q_2 = (180)(16)(42) = 120.96 \times 10^3 \text{ mm}^4$$

$$Q_3 = (16)(34)(17) = 9.248 \times 10^3 \text{ mm}^4$$

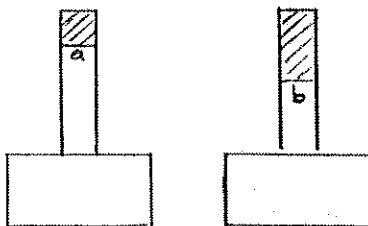
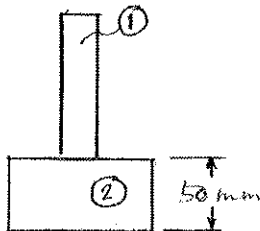
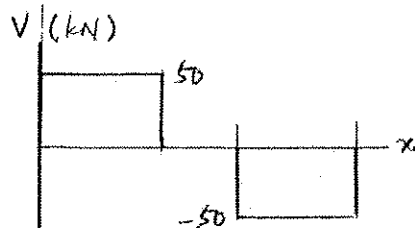
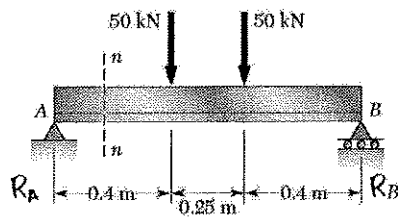
$$Q = 2Q_1 + Q_2 + 2Q_3 = 312.256 \times 10^3 \text{ mm}^3 = 312.256 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(312.256 \times 10^{-6})}{(44.274 \times 10^{-6})(2 \times 16 \times 10^{-3})} = 17.63 \times 10^6 \text{ Pa} \quad 17.63 \text{ MPa} \blacktriangleleft$$

(b) At point a , $Q = Q_1 = 86.4 \times 10^3 \text{ mm}^4 = 86.4 \times 10^{-6} \text{ m}^4$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(86.4 \times 10^{-6})}{(44.274 \times 10^{-6})(12 \times 10^{-3})} = 13.01 \times 10^6 \text{ Pa} \quad 13.01 \text{ MPa} \blacktriangleleft$$

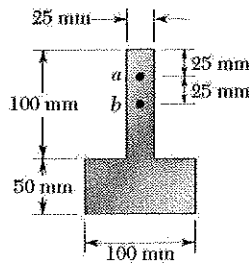
Problem 6.92



(a)

(b)

6.92 For the beam and loading shown, consider section $n-n$ and determine the shearing stress at (a) point a , (b) point b .



$$R_A = R_B = 50 \text{ kN}$$

Draw shear diagram.

$$V = 50 \text{ kN}$$

Determine section properties.

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	2500	100	250000	50	6250000	2083333
②	5000	25	125000	-25	3125000	1041667
Σ	7500		375000		9375000	3125000

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{375000}{7500} = 50 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 12.5 \times 10^6 \text{ mm}^4$$

$$(a) \quad A = 625 \text{ mm}^2 \quad \bar{y} = 87.5 \text{ mm} \quad Q_a = A\bar{y} = 54687.5 \text{ mm}^3$$

$$t = 25 \text{ mm}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(50000)(54687.5)}{(12.5 \times 10^6)(25)} = 8.75 \text{ MPa} \quad \blacktriangleleft$$

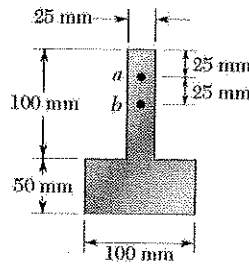
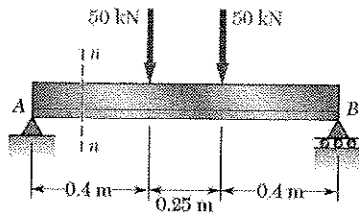
$$(b) \quad A = 1250 \text{ mm}^2 \quad \bar{y} = 75 \text{ mm} \quad Q_b = A\bar{y} = 93750 \text{ mm}^3$$

$$t = 25 \text{ mm}$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(50000)(93750)}{(12.5 \times 10^6)(25)} = 15 \text{ MPa} \quad \blacktriangleleft$$

Problem 6.93

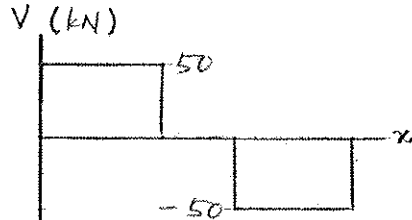
6.93 For the beam and loading shown, determine the largest shearing stress in section $n-n$.



$$R_A = R_B = 50 \text{ kN}$$

Draw shear diagram.

$$V = 50 \text{ kN}$$

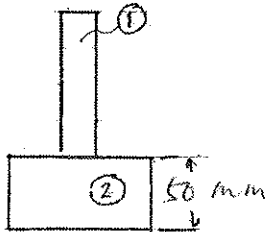


Determine section properties.

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	2500	100	250000	50	6250000	208333.3
②	5000	25	125000	-25	3125000	104166.7
Σ	7500		375000		9375000	312500.0

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{375000}{7500} = 50 \text{ mm.}$$

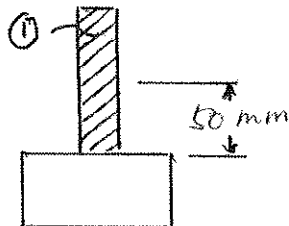
$$I = \Sigma Ad^2 + \Sigma \bar{I} = 12.5 \times 10^6 \text{ mm}^4$$



$$Q = A_1 \bar{y}_1 = (2500)(50) = 125000$$

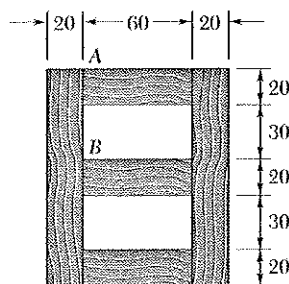
$$t = 25 \text{ mm}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(50000)(125000)}{(12.5 \times 10^6)(25)} = 2.0 \text{ MPa.}$$

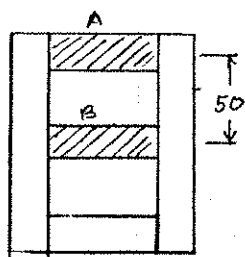


Problem 6.94

6.94 Several planks are glued together to form the box beam shown. Knowing that the beam is subjected to a vertical shear of 3 kN, determine the average shearing stress in the glued joint (a) at A, (b) at B.



Dimensions in mm



$$I_A = \frac{1}{12} b h^3 + A d^2 = \frac{1}{12} (60)(20)^3 + (60)(20)(50)^2$$

$$= 3.04 \times 10^6 \text{ mm}^4$$

$$I_B = \frac{1}{12} b h^3 = \frac{1}{12} (60)(20)^3 = 0.04 \times 10^6 \text{ mm}^4$$

$$I_C = \frac{1}{12} b h^3 = \frac{1}{12} (20)(120)^3 = 2.88 \times 10^6 \text{ mm}^4$$

$$I = 2I_A + I_B + 2I_C = 11.88 \times 10^6 \text{ mm}^4$$

$$= 11.88 \times 10^{-6} \text{ m}^4$$

$$Q_A = A \bar{y} = (60)(20)(50) = 60 \times 10^3 \text{ mm}^3 = 60 \times 10^{-6} \text{ m}^3$$

$$t = 20 \text{ mm} + 20 \text{ mm} = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$(a) \quad \tau_A = \frac{VQ}{It} = \frac{(3 \times 10^3)(60 \times 10^{-6})}{(11.88 \times 10^{-6})(40 \times 10^{-3})} = 379 \times 10^3 \text{ Pa}$$

$$\tau_A = 379 \text{ kPa}$$

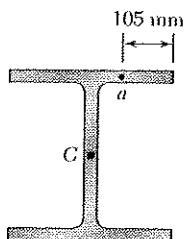
$$Q_B = 0$$

$$(b) \quad \tau_B = \frac{VQ_B}{It} = 0$$

$$\tau_B = 0$$

Problem 6.95

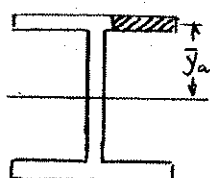
6.95 Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (a) at point A, (b) at the centroid C of the section.



For W360 × 122, $d = 363 \text{ mm}$, $b_f = 257 \text{ mm}$, $t_f = 21.70 \text{ mm}$, $t_w = 13.0 \text{ mm}$

$$I = 365 \times 10^6 \text{ mm}^4 = 365 \times 10^{-6} \text{ m}^4$$

(a)



$$A_a = (105)(21.70) = 2278.5 \text{ mm}^2$$

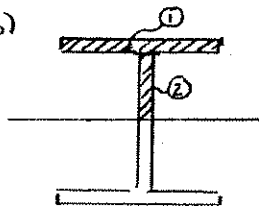
$$\bar{y}_a = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$$

$$Q_a = A_a \bar{y}_a = 388.8 \times 10^3 \text{ mm}^3 = 388.8 \times 10^{-6} \text{ m}^3$$

$$t_a = t_f = 21.70 \text{ mm} = 21.7 \times 10^{-3} \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(388.8 \times 10^{-6})}{(365 \times 10^{-6})(21.7 \times 10^{-3})} = 12.27 \times 10^6 \text{ Pa} = 12.27 \text{ MPa}$$

(b)



$$A_1 = b_f t_f = (257)(21.70) = 5577 \text{ mm}^2$$

$$\bar{y}_1 = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$$

$$A_2 = t_w \left(\frac{d}{2} - t_f \right) = (13.0)(159.8) = 2077 \text{ mm}^2$$

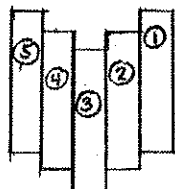
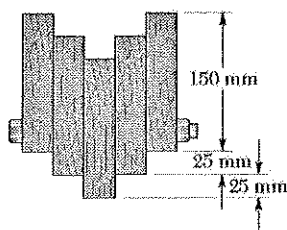
$$\bar{y}_2 = \frac{1}{2} \left(\frac{d}{2} - t_f \right) = 79.9 \text{ mm}$$

$$Q_c = \sum A \bar{y} = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3 \text{ mm}^3 = 1117.7 \times 10^{-6} \text{ m}^3$$

$$t_c = t_w = 13.0 \text{ mm} = 13 \times 10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(365 \times 10^{-6})(13 \times 10^{-3})} = 58.9 \times 10^6 \text{ Pa} = 58.9 \text{ MPa}$$

Problem 6.96



6.96 A beam consists of five planks of 38×150 -mm cross section connected by steel bolts with a longitudinal spacing of 220 mm. Knowing that the shear in the beam is vertical and equal to 8 kN and that the allowable average shearing stress in each bolt is 50 MPa, determine the smallest permissible bolt diameter that may be used.

Part	$A(\text{mm}^2)$	$\bar{y}_0(\text{m})$	$A\bar{y}_0(\text{mm}^3)$	$\bar{y}(\text{mm})$	$A\bar{y}^2(\text{mm}^4)$	$\bar{I} \text{ mm}^4$
①	5700	125	712500	20	2280000	10687500
②	5700	100	570000	-5	142500	10687500
③	5700	75	427500	-30	5130000	10687500
④	5700	100	570000	-5	142500	10687500
⑤	5700	125	712500	20	2280000	10687500
Σ	28500		2992500		9.975×10^6	53.4375×10^6

$$\bar{y}_0 = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2992500}{28500} = 105 \text{ mm}$$

$$I = \Sigma A\bar{y}^2 + \Sigma \bar{I} = 63.4125 \times 10^6$$

Between ① and ②: $Q_{12} = Q_1 = A\bar{y}_1 = (5700)(20) = 114000 \text{ mm}^3$

Between ② and ③: $Q_{23} = Q_1 + A\bar{y}_2 = 114000 + (5700)(-5) = 85500 \text{ mm}^3$

$q = \frac{VQ}{I}$ Maximum q is based on $Q_{12} = 114000 \text{ mm}^3$

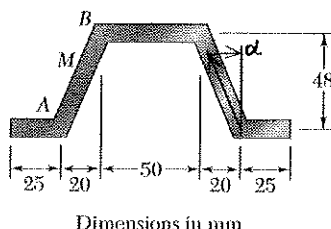
$$q = \frac{(8)(114000)}{63.4125 \times 10^6} = 0.01438 \text{ kN/mm}$$

$$F_{\text{bolt}} = qs = (0.01438)(220) = 3.164 \text{ kN}$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} \quad A_{\text{bolt}} = \frac{F_{\text{bolt}}}{\tau_{\text{bolt}}} = \frac{3164}{50} = 63.28 \text{ mm}^2$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 \quad d_{\text{bolt}} = \sqrt{\frac{4A_{\text{bolt}}}{\pi}} = \sqrt{\frac{(4)(63.28)}{\pi}} = 9 \text{ mm}$$

Problem 6.97



6.97 A plate of 4-mm thickness is bent as shown and then used as a beam. For a vertical shear of 12 kN, determine (a) the shearing stress at point A, (b) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

$$\tan \alpha = \frac{20}{48} \quad \alpha = 22.62^\circ$$

$$\text{Slanted side: } A_s = (4 \sec \alpha)(48) = 208 \text{ mm}^2$$

$$\bar{I}_s = \frac{1}{12}(4 \sec \alpha)(48)^3 = 39.936 \times 10^3 \text{ mm}^4$$

$$\text{Top: } I_T = \frac{1}{12}(50)(4)^3 + (50)(4)(24)^2 = 115.46 \times 10^3 \text{ mm}^4$$

$$\text{Bottom: } I_B = I_T = 115.46 \times 10^3 \text{ mm}^4$$

$$I = 2\bar{I}_s + I_T + I_B = 310.8 \times 10^3 \text{ mm}^4 = 310.8 \times 10^{-9} \text{ m}^4$$

$$(a) \quad Q_A = (25)(4)(24) = 2.4 \times 10^3 \text{ mm}^3 = 2.4 \times 10^{-6} \text{ m}^3$$

$$t = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$$

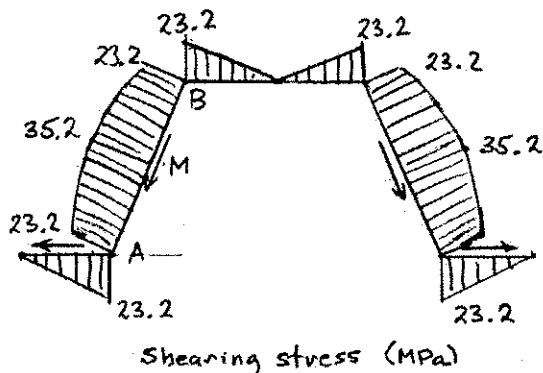
$$\tau_A = \frac{VQ_A}{It} = \frac{(12 \times 10^3)(2.4 \times 10^{-6})}{(310.8 \times 10^{-9})(4 \times 10^{-3})} = 23.2 \times 10^6 \text{ Pa} = 23.2 \text{ MPa}$$

(b) Maximum shearing occurs at point M, 24 mm above the bottom

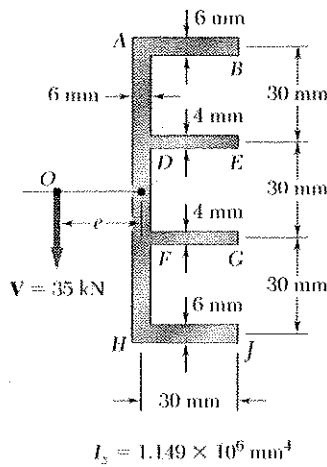
$$Q_M = Q_A + (4 \sec \alpha)(24)(12) = 2.4 \times 10^3 + 1.248 \times 10^3 = 3.648 \times 10^3 \text{ mm}^3 = 3.648 \times 10^{-6} \text{ m}^3$$

$$\tau_M = \frac{VQ_M}{It} = \frac{(12 \times 10^3)(3.648 \times 10^{-6})}{(310.8 \times 10^{-9})(4 \times 10^{-3})} = 35.2 \times 10^6 \text{ Pa} = 35.2 \text{ MPa}$$

$$Q_B = Q_A \quad \tau_B = \tau_A = 23.2 \text{ MPa}$$



Problem 6.98



6.98 and 6.99 For an extruded beam having the cross section shown, determine (a) the location of the shear center O, (b) the distribution of the shearing stresses caused by the vertical shearing force V shown applied at O.

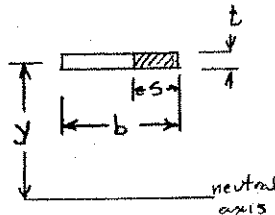
$$I_{AB} = I_{HJ} = \frac{1}{12}(30)(6)^3 + (30)(6)(45)^2 = 0.365 \times 10^6 \text{ mm}^4$$

$$I_{DE} = I_{FG} = \frac{1}{12}(30)(4)^3 + (30)(4)(15)^2 = 0.02716 \times 10^6 \text{ mm}^4$$

$$I_{AK} = \frac{1}{12}(6)(90)^3 = 0.3645 \times 10^6 \text{ mm}^4$$

$$I = \sum I = 1.14882 \times 10^6 \text{ mm}^4$$

(a) For a typical flange,



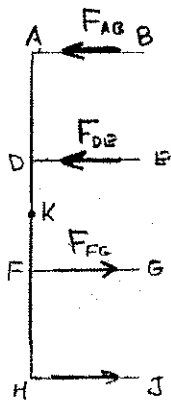
$$A(s) = ts$$

$$Q(s) = yts$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vyt s}{I}$$

$$F = \int_0^b q(s) ds$$

$$= \frac{Vyt b^2}{2I}$$



Flange AB: $F_{AB} = \frac{V(45)(6)(30^2)}{(2)(1.14882 \times 10^6)} = 0.10576 V \leftarrow$

Flange DE: $F_{DE} = \frac{V(15)(4)(30^2)}{(2)(1.14882 \times 10^6)} = 0.023502 V \leftarrow$

Flange FG: $F_{FG} = 0.023502 V \rightarrow$

Flange HJ: $F_{HJ} = 0.10576 V \rightarrow$

$$\sum \bar{M}_K = 0$$

$$Ve = 45 F_{AB} + 15 F_{DE} + 15 F_{FG} + 45 F_{HJ} = 10.223 V$$

Dividing by V,

$$e = 10.22 \text{ mm}$$

(b) Calculation of shearing stresses.

$$V = 35 \times 10^3 \text{ N} \quad I = 1.14882 \times 10^6 \text{ mm}^4$$

At B, E, G, and J: $\tau = 0$

At A and H: $Q = (30)(6)(45) = 8.1 \times 10^3 \text{ mm}^3 = 8.1 \times 10^{-6} \text{ m}^3$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(8.1 \times 10^{-6})}{(1.14882 \times 10^6)(6 \times 10^{-3})} = 41.1 \times 10^6 \text{ Pa} = 41.1 \text{ MPa}$$

Continued on next page.

Problem 6.98 continued

Just above D and just below F:

$$Q = 8.1 \times 10^3 + (6)(30)(30) = 13.5 \times 10^3 \text{ mm}^3 = 13.5 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(13.5 \times 10^{-6})}{(1.14882 \times 10^{-6})(6 \times 10^{-3})} = 68.5 \times 10^6 \text{ Pa} = 68.5 \text{ MPa} \quad \blacktriangleleft$$

Just to right of D and just to the right of F:

$$Q = (30)(4)(15) = 1.8 \times 10^3 \text{ mm}^3 = 1.8 \times 10^{-6} \text{ m}^3 \quad t = 4 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(1.8 \times 10^{-6})}{(1.14882 \times 10^{-6})(4 \times 10^{-3})} = 13.71 \times 10^6 \text{ Pa} = 13.71 \text{ MPa} \quad \blacktriangleleft$$

Just below D and just above F:

$$Q = 13.5 \times 10^3 + 1.8 \times 10^3 = 15.3 \times 10^3 \text{ mm}^3 = 15.3 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

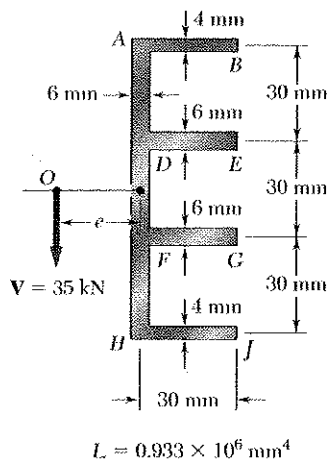
$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(15.3 \times 10^{-6})}{(1.14882 \times 10^{-6})(6 \times 10^{-3})} = 77.7 \times 10^6 \text{ Pa} = 77.7 \text{ MPa} \quad \blacktriangleleft$$

$$\text{At K: } Q = 15.3 \times 10^3 + (6)(15)(7.5) = 15.975 \times 10^3 \text{ mm}^3 = 15.975 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(15.975 \times 10^{-6})}{(1.14882 \times 10^{-6})(6 \times 10^{-3})} = 81.1 \times 10^6 \text{ Pa} = 81.1 \text{ MPa} \quad \blacktriangleleft$$

Problem 6.99

6.98 and 6.99 For an extruded beam having the cross section shown, determine (a) the location of the shear center O , (b) the distribution of the shearing stresses caused by the vertical shearing force V shown applied at O .



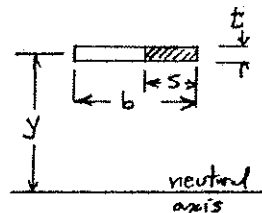
$$I_{AB} = I_{HJ} = \frac{1}{12}(30)(4)^3 + (30)(4)(45)^2 = 0.24316 \times 10^6 \text{ mm}^4$$

$$I_{DE} = I_{FG} = \frac{1}{12}(30)(6)^3 + (30)(6)(15)^2 = 0.04104 \times 10^6 \text{ mm}^4$$

$$I_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645 \times 10^6 \text{ mm}^4$$

$$I = \Sigma I = 0.9329 \times 10^6 \text{ mm}^4$$

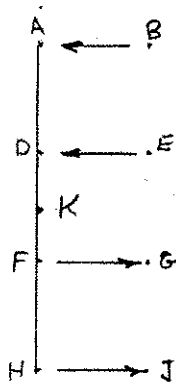
(a) For a typical flange, $A(s) = ts$



$$Q(s) = yts$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vyts}{I}$$

$$F = \int_0^b q(s) ds = \frac{Vy t b^2}{2I}$$



$$\text{Flange AB: } F_{AB} = \frac{V(45)(4)(30)^2}{(2)(0.9329 \times 10^6)} = 0.086826 V \leftarrow$$

$$\text{Flange DE: } F_{DE} = \frac{V(15)(6)(30)^2}{(2)(0.9329 \times 10^6)} = 0.043413 V \leftarrow$$

$$\text{Flange FG: } F_{FG} = 0.043413 V \rightarrow$$

$$\text{Flange HJ: } F_{HJ} = 0.086826 V \rightarrow$$

$$+\circlearrowleft \Sigma M_K = +\circlearrowleft \Sigma M_K:$$

$$Ve = 45 F_{AB} + 15 F_{DE} + 15 F_{FG} + 45 F_{HJ} = 9.1167 V$$

Dividing by V ,

$$e = 9.12 \text{ mm}$$

(b) Calculation of shearing stresses.

$$V = 35 \times 10^3 \text{ N} \quad I = 0.9329 \times 10^6 \text{ mm}^4$$

At B, E, G, and J: $\tau = 0$

$$\text{At A and H: } Q = (30)(4)(45) = 5.4 \times 10^3 \text{ mm}^3 = 5.4 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(35 \times 10^3)(5.4 \times 10^{-6})}{0.9329 \times 10^{-6}} = 202.59 \times 10^3 \text{ N/m}$$

Just to the right of A and H: $t = 4 \times 10^{-3} \text{ m}$

$$\tau = \frac{q}{t} = \frac{202.59 \times 10^3}{4 \times 10^{-3}} = 50.6 \times 10^6 \text{ Pa} = 50.6 \text{ MPa}$$

Continued on next page.

Problem 6.99 continued

Just below A and just above H: $t = 6 \times 10^{-3} \text{ m}$

$$\tau = \frac{Q}{It} = \frac{202.59 \times 10^3}{6 \times 10^{-3}} = 33.8 \times 10^6 \text{ Pa} = 33.8 \text{ MPa} \quad \blacktriangleleft$$

Just above D and just below F: $t = 6 \times 10^{-3} \text{ m}$

$$Q = 5.4 \times 10^3 + (6)(30)(30) = 10.8 \times 10^3 \text{ mm}^3 = 10.8 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(10.8 \times 10^{-6})}{(0.9329 \times 10^{-6})(6 \times 10^{-3})} = 67.5 \times 10^6 \text{ Pa} = 67.5 \text{ MPa} \quad \blacktriangleleft$$

Just to the right of D and E: $t = 6 \times 10^{-3} \text{ m}$

$$Q = (30)(6)(15) = 2.7 \times 10^3 \text{ mm}^3 = 2.7 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(2.7 \times 10^{-6})}{(0.9329 \times 10^{-6})(6 \times 10^{-3})} = 16.88 \times 10^6 \text{ Pa} = 16.88 \text{ MPa} \quad \blacktriangleleft$$

Just below D and just above F: $t = 6 \times 10^{-3} \text{ m}$

$$Q = 10.8 \times 10^3 + 2.7 \times 10^3 = 13.5 \times 10^3 \text{ mm}^3 = 13.5 \times 10^{-6} \text{ m}^3$$

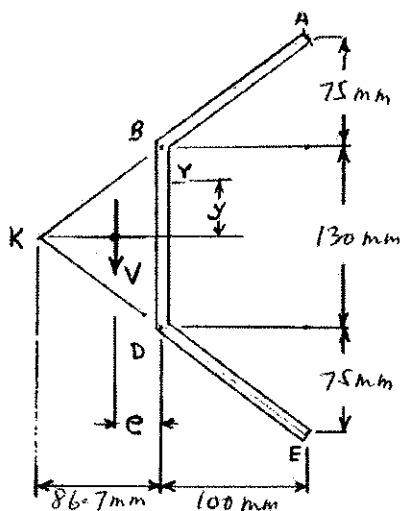
$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(13.5 \times 10^{-6})}{(0.9329 \times 10^{-6})(6 \times 10^{-3})} = 84.4 \times 10^6 \text{ Pa} = 84.4 \text{ MPa} \quad \blacktriangleleft$$

At K: $Q = 13.5 \times 10^3 + (6)(15)(7.5) = 14.175 \times 10^3 \text{ mm}^3 = 14.175 \times 10^{-6} \text{ m}^3$

$$\tau = \frac{VQ}{It} = \frac{(35 \times 10^3)(14.175 \times 10^{-6})}{(0.9329 \times 10^{-6})(6 \times 10^{-3})} = 88.6 \times 10^6 \text{ Pa} = 88.6 \text{ MPa} \quad \blacktriangleleft$$

Problem 6.100

6.100 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



$$L_{AB} = \sqrt{100^2 + 75^2} = 125 \text{ mm} \quad A_{AB} = 125t$$

$$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12} (125t)(75)^2 + (125t)(100)^2 = 1308594 t \text{ mm}^4$$

$$I_{BD} = \frac{1}{12} (t)(130)^3 = 183083 t \text{ mm}^4$$

$$I = 2I_{AB} + I_{BD} = 2.8 \times 10^6 t \text{ mm}^4$$

$$\text{In part BD: } Q = Q_{AB} + Q_{BD}$$

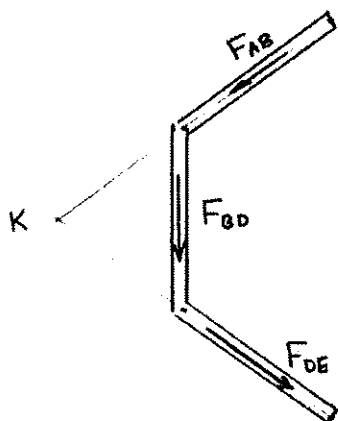
$$Q = (125t)(100) + (65-y)t\left(\frac{1}{2}\right)(65+y) = 12500t + 2112.5t - \frac{1}{2}ty^2 = (14612.5 - \frac{1}{2}y^2)t$$

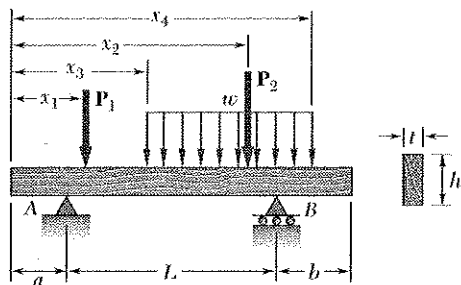
$$\tau = \frac{VQ}{It} \quad F_{BD} = \int \tau dA = \int_{-65}^{65} \frac{V(14612.5 - \frac{1}{2}y^2)t}{It} \cdot t dy = \frac{Vt}{I} \int_{-65}^{65} (14612.5 - \frac{1}{2}y^2) dy = \frac{Vt}{I} [14612.5y - \frac{1}{6}y^3]_{-65}^{65} = \frac{Vt}{I} \cdot 2 \left[(14612.5)(65) - \frac{(65)^3}{6} \right] = \frac{Vt(1899623)}{2.8 \times 10^6 t} = 0.678 V$$

$$\sum M_K = \sum M_K: -V(86.7 - e) = -86.7(0.678 V)$$

$$e = 86.7 [1 - 0.678] = 27.9 \text{ mm}$$

Note that the lines of action of F_{AB} and F_{BD} pass through point K . Thus, these forces have zero moment about point K .



PROBLEM 6.C1


6.C1 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values σ_{all} and τ_{all} . Measuring x from end A, write a computer program to calculate for successive cross sections, from $x = 0$ to $x = L$ and using given increments Δx , the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement, (2) the allowable shearing stress requirement. Use this program to design the beams of uniform cross section of the following problems, assuming $\sigma_{all} = 12 \text{ MPa}$ and $\tau_{all} = 825 \text{ kPa}$, and using the increments indicated: (a) Prob. 5.65 ($\Delta x = 0.1 \text{ m}$), (b) Prob. 5.157 ($\Delta x = 0.2 \text{ m}$).

SOLUTION

See solution of P 5.C2 for the determination of R_A , R_B , $V(x)$, and $M(x)$.
We recall that

$$V(x) = R_A \text{STPA} + R_B \text{STPB} - P_1 \text{STP1} - P_2 \text{STP2} - w(x-x_3) \text{STP3} + w(x-x_4) \text{STP4}$$

$$M(x) = R_A(x-a) \text{STPA} + R_B(x-a-L) \text{STPB} - P_1(x-x_1) \text{STP1} - P_2(x-x_2) \text{STP2} - \frac{1}{2} w(x-x_3)^2 \text{STP3} + \frac{1}{2} w(x-x_4)^2 \text{STP4}$$

where STPA , STPB , STP1 , STP2 , STP3 , and STP4 are step functions defined in P 5.C2.

(1) TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

If unknown dimension is h :

$$S_{min} = |M| / \sigma_{all} \cdot \text{From } S = \frac{1}{6} t h^2, \text{ we have } h_0 = h = \sqrt{6S/t}$$

If unknown dimension is t :

$$S_{min} = |M| / \sigma_{all} \cdot \text{From } S = \frac{1}{6} t h^2, \text{ we have } t_0 = t = 6S/h^2$$

(2) TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

$$\text{We use Eq. (6.10), page 378: } \tau_{max} = \frac{3M}{2A} = \frac{3|M|}{2th}$$

$$\text{If unknown dimension is } h: h_0 = h = \frac{3M}{2t\tau_{all}}$$

$$\text{If unknown dimension is } t: t_0 = t = \frac{3M}{2h\tau_{all}}$$

(CONTINUED)

PROBLEM 6.C1 CONTINUED
PROGRAM OUTPUTS
Problem 5.65

RA = 2.40 kN RB = 3.00 kN

X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	2.40	0.000	0.00	109.09
0.10	2.40	0.240	54.77	109.09
0.20	2.40	0.480	77.46	109.09
0.30	2.40	0.720	94.87	109.09
0.40	2.40	0.960	109.54	109.09
0.50	2.40	1.200	122.47	109.09
0.60	2.40	1.440	134.16	109.09
0.70	2.40	1.680	144.91	109.09
0.80	0.60	1.920	154.92	27.27
0.90	0.60	1.980	157.32	27.27
1.00	0.60	2.040	159.69	27.27
1.10	0.60	2.100	162.02	27.27
1.20	0.60	2.160	164.32	27.27
1.30	0.60	2.220	166.58	27.27
1.40	0.60	2.280	168.82	27.27
1.50	0.60	2.340	171.03	27.27
1.60	-3.00	2.400	<u>173.21</u>	136.36
1.70	-3.00	2.100	162.02	136.36
1.80	-3.00	1.800	150.00	136.36
1.90	-3.00	1.500	136.93	136.36
2.00	-3.00	1.200	122.47	136.36
2.10	-3.00	0.900	106.07	136.36
2.20	-3.00	0.600	86.60	136.36
2.30	-3.00	0.300	61.24	136.36
2.40	0.00	0.000	0.05	0.00

Problem 5.157

RA = 25.00 kN RB = 25.00 kN

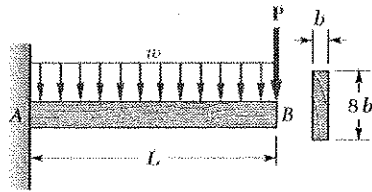
X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	25.00	0.000	0.00	378.79
0.20	23.00	4.800	141.42	348.48
0.40	21.00	9.200	195.79	318.18
0.60	19.00	13.200	234.52	287.88
0.80	17.00	16.800	264.58	257.58
1.00	15.00	20.000	288.68	227.27
1.20	13.00	22.800	308.22	196.97
1.40	11.00	25.200	324.04	166.67
1.60	9.00	27.200	336.65	136.36
1.80	7.00	28.800	346.41	106.06
2.00	5.00	30.000	353.55	75.76
2.20	3.00	30.800	358.24	45.45
2.40	1.00	31.200	360.56	15.15
2.60	-1.00	31.200	360.56	15.15
2.80	-3.00	30.800	358.24	45.45
3.00	-5.00	30.000	353.55	75.76
3.20	-7.00	28.800	346.41	106.06
3.40	-9.00	27.200	336.65	136.36
3.60	-11.00	25.200	324.04	166.67
3.80	-13.00	22.800	308.22	196.97
4.00	-15.00	20.000	288.68	227.27
4.20	-17.00	16.800	264.58	257.58
4.40	-19.00	13.200	234.52	287.88
4.60	-21.00	9.200	195.79	318.18
4.80	-23.00	4.800	141.42	348.48
5.00	0.00	0.000	0.00	0.00

The smallest allowable value of h is the largest of the values shown in the last two columns.

For Prob. 5.65, $h = h_G = 173.2 \text{ mm}$.

For Prob. 5.157, $h = h_2 = 379 \text{ mm}$.

PROBLEM 6.C2



6.C2 A cantilever timber beam AB of length L and of uniform rectangular section shown supports a concentrated load P at its free end and a uniformly distributed load w along its entire length. Write a computer program to determine the length L and the width b of the beam for which both the maximum normal stress and the maximum shearing stress in the beam reach their largest allowable values. Assuming $\sigma_{all} = 12.4 \text{ MPa}$ and $\tau_{all} = 827 \text{ kPa}$, use this program to determine the dimensions L and b when (a) $P = 4.448 \text{ kN}$ and $w = 0$, (b) $P = 0$ and $w = 2.2 \text{ kN/m}$, (c) $P = 2.224 \text{ kN}$ and $w = 2.2 \text{ kN/m}$.

SOLUTION

Both the maximum shear and the maximum bending moment occur at A. We have

$$V_A = P + wL$$

$$M_A = PL + \frac{1}{2} wL^2$$

TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

$$\sigma_{all} = \frac{M_A}{S} = \frac{M_A}{\frac{1}{6} b(8b)^2} = \frac{3M_A}{32b^3}$$

$$b_o = b = \left[\frac{3}{32} \frac{M_A}{\sigma_{all}} \right]^{1/3}$$

TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378:

$$\tau_{all} = \frac{3V}{2A} = \frac{3}{2} \frac{V_A}{b(8b)} = \frac{3V_A}{16b^2}$$

$$b_s = b = \left[\frac{3}{16} \frac{V_A}{\tau_{all}} \right]^{1/2}$$

PROGRAM

For $L=0$, $V_A = P$ and $b_s > 0$, while $M_A = 0$ and $b_o = 0$.

Starting with $L=0$ and using increments $\Delta L = 0.001 \text{ in.}$, we increase L until b_o and b_s become equal. We then print L and b .

PROGRAM OUTPUTS

For $P = 4.448 \text{ kN}$, $w = 0.0 \text{ kN/m}$

Increment = 2mm

$L = 952 \text{ mm}$, $b = 31.75 \text{ mm}$

For $P = 0 \text{ kN}$, $w = 2.2 \text{ kN/m}$

Increment = 2mm

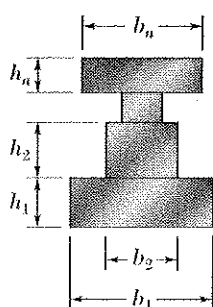
$L = 1786 \text{ mm}$, $b = 29.77 \text{ mm}$

For $P = 2.224 \text{ kN}$, $w = 2.2 \text{ kN/m}$

Increment = 2mm

$L = 1518 \text{ mm}$, $b = 35.46 \text{ mm}$

PROBLEM 6.C3



6.C3 A beam having the cross section shown is subjected to a vertical shear V . Write a computer program that can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (a) Prob. 6.10, (b) Prob. 6.12, (c) Prob. 6.21.

SOLUTION

1. Enter V and the number n of rectangles.

2. For $i = 1$ to n , enter the dimensions b_i and h_i .

3. Determine the area $A_i = b_i h_i$ of each rectangle.

4. Determine the elevation of the centroid of each rectangle:

$$\bar{y}_i = \sum_{k=1}^i h_k - 0.5 h_i$$

and the elevation $\bar{y}$ of the centroid of the entire section:

$$\bar{y} = (\sum_i A_i \bar{y}_i) / (\sum_i A_i)$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[\frac{1}{12} b_i h_i^3 + A_i (\bar{y}_i - \bar{y})^2 \right]$$

6. For each surface separating two rectangles i and $i+1$, determine Q_i of the area below that surface:

$$Q_i = \sum_{k=1}^i A_k (\bar{y}_k - \bar{y})$$

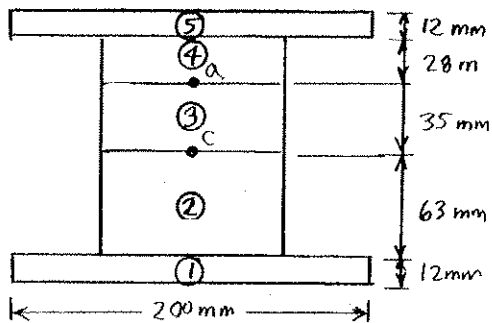
7. Select for t_i the smaller of b_i and b_{i+1} .

The shearing stress on the surface between the rectangles i and $i+1$ is

$$\tau_i = \frac{V Q_i}{I t_i} \quad \blacktriangleleft$$

(CONTINUED)

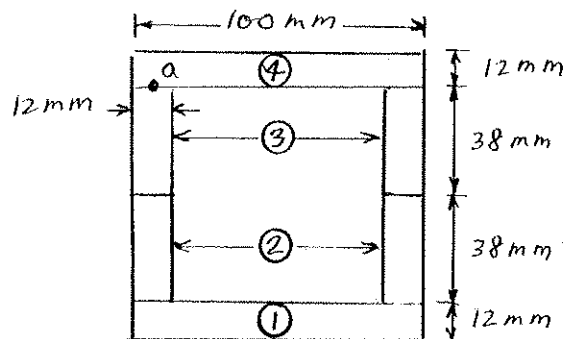
PROBLEM 6.C3 CONTINUED



PROGRAM OUTPUTS

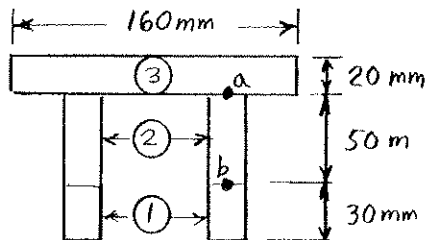
Problem 6.10

Shearing force = 10 kN
 YBAR = 75.000 mm above base
 $I = 39.580 \times 10^{-6} \text{ mm}^4$
 Between elements 1 and 2:
 TAU = 418.39 kPa
 Between elements 2 and 3:
 TAU = 919.78 kPa ◀ (a)
 Between elements 3 and 4:
 TAU = 765.03 kPa ◀ (b)
 Between elements 4 and 5:
 TAU = 418.39 kPa



Problem 6.12

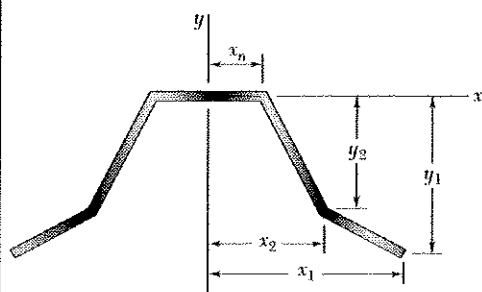
Shearing force = 40 kN
 YBAR = 50 mm
 $I = 5.553 \times 10^{-6} \text{ mm}^4$
 Between elements 1 and 2:
 TAU = 15.85 MPa
 Between elements 2 and 3:
 TAU = 21.05 MPa ◀ (a)
 Between elements 3 and 4:
 TAU = 15.85 MPa ◀ (b)



Problem 6.21

Shearing force = 90 kN
 YBAR = 65.000 mm
 $I = 58.133 \times 10^{-6} \text{ mm}^4$
 Between elements 1 and 2:
 TAU = 23.222 MPa ◀ (b)
 Between elements 2 and 3:
 TAU = 30.963 MPa ◀ (a)

PROBLEM 6.C4



6.C4 A plate of uniform thickness t is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that can be used to determine the distribution of shearing stresses caused by a vertical shear V . Use this program (a) to solve Prob. 6.47, (b) to find the shearing stress at a point E for the shape and load of Prob. 6.50, assuming a thickness $t = 6$ mm.

SOLUTION

For each element on the right-hand side, we compute (for $i = 1$ to n):

$$\text{Length of element} = L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$$

$$\text{Area of element} = A_i = t L_i \quad \text{where } t = 6 \text{ mm}$$

$$\text{Distance from } x \text{ axis to centroid of element} = \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$$

Distance from x axis to centroid of section:

$$\bar{y} = (\sum A_i \bar{y}_i) / \sum A_i$$

Note that $y_n = 0$ and that $x_{n+1} = y_{n+1} = 0$

Moment of inertia of section about centroidal axis:

$$\bar{I} = 2 \sum A_i \left[\frac{1}{12} (y_i - y_{i+1})^2 + (\bar{y}_i - \bar{y})^2 \right]$$

Computation of Q at point P where stress is desired

$Q = \sum A_i (\bar{y}_i - \bar{y})$ where sum extends to the areas located between one end of section and point P .

Shearing stress at P :

$$\tau = \frac{VQ}{I t}$$

NOTE: $\tau_{\max}$ occurs on neutral axis, i.e., for $y_P = \bar{y}$.

PROGRAM OUTPUTS

Part (a):

$$I = 0.2 \times 10^6 \text{ mm}^4$$

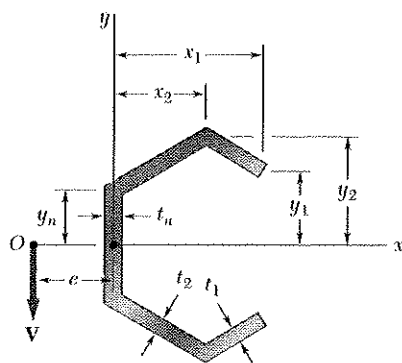
$$\tau_{\max} = 14.06 \text{ MPa}$$

$$\tau_B = 12.5 \text{ MPa}$$

Part (b):

$$I = 8.352 \times 10^6 \text{ mm}^4$$

$$\tau_E = 1.293 \text{ MPa}$$

PROBLEM 6.C5


6.C5 The cross section of an extruded beam is symmetric with respect to the x axis and consists of several straight segments as shown. Write a computer program that can be used to determine (a) the location of the shear center O , (b) the distribution of shearing stresses caused by a vertical force applied at O . Use this program to solve Probs. 6.66 and 6.70.

SOLUTION

SINCE SECTION IS SYMMETRIC WITH x AXIS, COMPUTATIONS WILL BE DONE FOR TOP HALF.

FOR $L = 1$ TO $n+1$ (NOTE: $n+1$ IS THE ORIGIN)

ENTER x_L, y_L

COMPUTE LENGTH OF EACH SEGMENT

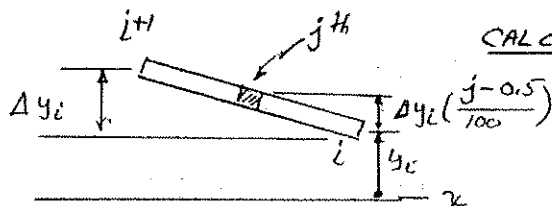
FOR $L = 1$ TO n

$$\Delta x_L = x_{L+1} - x_L$$

$$\Delta y_L = y_{L+1} - y_L$$

$$L = (\Delta x_L^2 + \Delta y_L^2)^{1/2}$$

CALCULATE MOMENT OF INERTIA I_x



CONSIDER EACH SEGMENT AS MADE OF 100 EQUAL PARTS

FOR $L = 1$ TO n

$$\Delta \text{AREA} = L_L t_L / 100$$

FOR $j = 1$ TO 100

$$y = y_L + \Delta y_L (j - 0.5) / 100$$

$$\Delta I = (\Delta \text{AREA}) y^2$$

$$I_x = I_x + \Delta I$$

SINCE ONLY TOP HALF WAS USED

$$I_x = 2 I_x$$

CALCULATE SHEARING STRESS AT ENDS OF SEGMENTS AND SHEAR FORCES IN SEGMENTS

FOR $L = 1$ TO n

$$\Delta \text{AREA} = L_L t_L / 100, \tau_{\text{new}} = \tau_{\text{next}}$$

FOR $j = 1$ TO 100

$$y = y_L + \Delta y_L (j - 0.5) / 100$$

$$\Delta Q = (\Delta \text{AREA}) y$$

$$\tau_{\text{old}} = \tau_{\text{new}}, Q = Q + \Delta Q$$

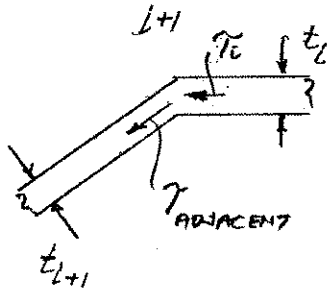
$$\tau_{\text{new}} = V Q / I_x t_L$$

$$\tau_{\text{ave}} = 0.5 (\tau_{\text{old}} + \tau_{\text{new}})$$

$$\tau = \tau + \tau_{\text{ave}}$$

CONTINUED

PROBLEM 6.C5 - CONTINUED



$$\begin{aligned} \text{FORCE}_L &= \tau_L (\text{AREA}) \\ \tau_L &= VQ / I_x t_L \\ (\tau_{\text{ADJACENT}})_L &= VQ / I_x t_{L+1} \\ Q_L &= Q \\ \tau_{\text{max}} &= (\tau_{\text{ADJACENT}})_L \end{aligned}$$

LOCATION OF SHEAR CENTER
 CALCULATE MOMENT OF SHEAR
 FORCES ABOUT ORIGIN

For $L = 1$ TO n

$$\begin{aligned} (F_x)_L &= \text{FORCE}_L (\Delta x_L) / L_L \\ (F_y)_L &= \text{FORCE}_L (\Delta y_L) / L_L \\ \text{MOMENT}_L &= -(F_x)_L y_L + (F_y)_L x_L \\ \text{MOMENT} &= \text{MOMENT} + \text{MOMENT}_L \end{aligned}$$

FOR WHOLE SECTION $\text{MOMENT} = 2(\text{MOMENT})$
 SHEAR CENTER IS AT
 $e = \text{MOMENT} / V$

PROGRAM OUTPUT

Prob. 6.66

	T (K) mm	X (K) mm	Y (K) mm	L (K) mm
1	3	100	.00	75
2	3	100	75	100
3	3	.00	75	75
4	3	.00	.00	

Moment of inertia: $I_x = 5.063 \times 10^6 \text{ mm}^4$ Shear = 2 kN

Junction of segments	Q mm**3	Tau Before MPa	Tau After MPa	Force in segment kN
1 and 2	8435.7	6.66	6.66	0.499
2 and 3	30937.5	24.44	24.44	4.640
3 and 4	39375	31.11	31.11	6.461

Moment of shear forces about origin: $M = 800 \text{ kNm}$
 + counterclockwise

Distance from origin to shear center: $e = 66.667 \text{ mm}$

CONTINUED

PROBLEM 6.C5 - PROGRAM PRINTOUTS CONTINUED

Prob. 6.70

	T (K) mm	X (K) mm	Y (K) mm	L (K) mm
1	6.00	60.62	.00	70.00
2	6.00	.00	35.00	35.00
3	6.00	.00	.00	

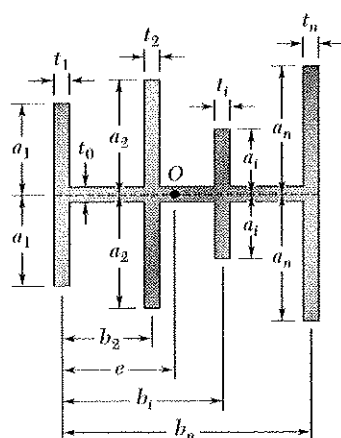
Moment of inertia: $I_x = 514487. \text{ mm}^4$ Shear = 1000.000 N

Junction of segments	Q mm^3	Tau Before MPa	Tau After MPa	Force in segment kN
1 and 2	7350.00	2.38	2.38	335.01
2 and 3	11025.00	3.57	3.57	666.27

Moment of shear forces about origin: $M = 20.309 \text{ N}\cdot\text{m}$
+ counterclockwise

Distance from origin to shear center: $e = 20.309 \text{ mm}$

PROBLEM 6.C6



6.C6 A thin-walled beam has the cross section shown. Write a computer program that can be used to determine the location of the shear center O of the cross section. Use the program to solve Prob. 6.75.

SOLUTION

Distribution of shearing stresses in element i

Let V = shear in cross section

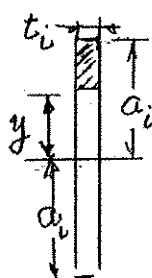
$\bar{I}$ = Centroidal moment of inertia of section

We have for shaded area

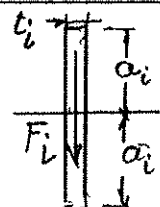
$$Q = A \bar{y} = t_i (a_i - y) \frac{a_i + y}{2}$$

$$= \frac{1}{2} t_i (a_i^2 - y^2)$$

$$\tau = \frac{QV}{It_i} = \frac{V}{2I} (a_i^2 - y^2)$$



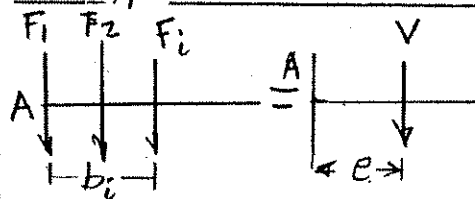
Force exerted on element i



$$F_i = \int_{-a_i}^{a_i} \tau (t_i dy) = \frac{V t_i}{2I} \int_{-a_i}^{a_i} (a_i^2 - y^2) dy$$

$$= \frac{V t_i}{I} \int_0^{a_i} (a_i^2 - y^2) dy = \frac{V t_i}{I} (a_i^3 - \frac{1}{3} a_i^3) = \frac{2}{3} \frac{V}{I} t_i a_i^3$$

The system of the forces F_i must be equivalent to V at shear center.



$$\sum F_i = \sum F: \frac{2}{3} \frac{V}{I} \sum t_i a_i^3 = V \quad (1)$$

$$\sum M_A = \sum M_A: \frac{2}{3} \frac{V}{I} \sum t_i a_i^3 b_i = e V \quad (2)$$

$$\text{Divide (2) by (1): } e = \frac{\sum t_i a_i^3 b_i}{\sum t_i a_i^3}$$

PROGRAM OUTPUT:

Problem 6.75

For element 1:

$$t = 18 \text{ mm}, a = 100 \text{ mm}, b = 0$$

For element 2:

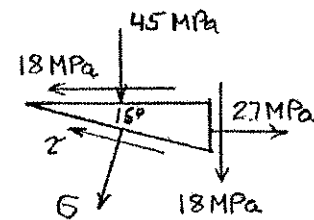
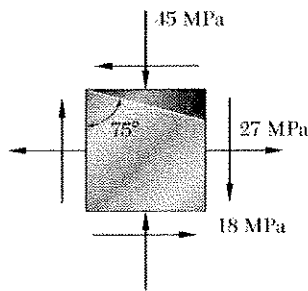
$$t = 18 \text{ mm}, a = 75 \text{ mm}, b = 200 \text{ mm}$$

$$\text{Answer: } e = 59.34 \text{ mm}$$

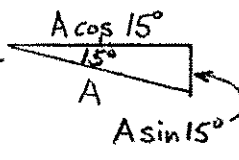
Chapter 7

Problem 7.1

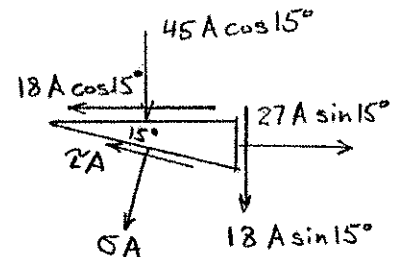
7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



Stresses



Areas



Forces

$$+\nearrow \sum F = 0 :$$

$$\sigma A + 18 A \cos 15^\circ \sin 15^\circ + 45 A \cos 15^\circ \cos 15^\circ + 27 A \sin 15^\circ \sin 15^\circ + 18 A \sin 15^\circ \cos 15^\circ = 0$$

$$\sigma = -18 \cos 15^\circ \sin 15^\circ - 45 \cos^2 15^\circ + 27 \sin^2 15^\circ - 18 \sin 15^\circ \cos 15^\circ$$

$$\sigma = -49.2 \text{ MPa} \quad \blacktriangleleft$$

$$+\searrow \sum F = 0 :$$

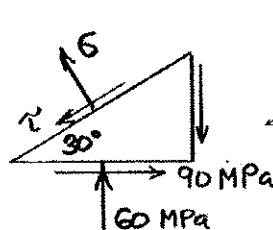
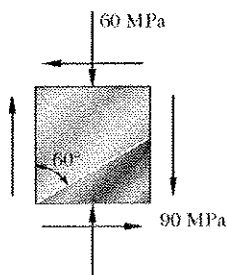
$$\tau A + 18 A \cos 15^\circ \cos 15^\circ - 45 A \cos 15^\circ \sin 15^\circ - 27 A \sin 15^\circ \cos 15^\circ - 18 A \sin 15^\circ \sin 15^\circ = 0$$

$$\tau = -18 (\cos^2 15^\circ - \sin^2 15^\circ) + (45 + 27) \cos 15^\circ \sin 15^\circ$$

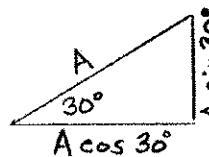
$$\tau = 2.41 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.2

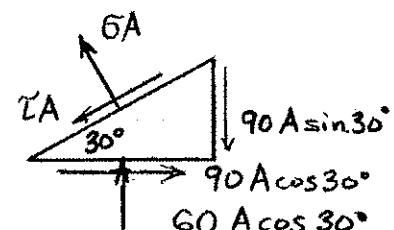
7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



Stresses



Areas



Forces

$$+\nearrow \sum F = 0 :$$

$$60 A - 90 A \sin 30^\circ \cos 30^\circ - 90 A \cos 30^\circ \sin 30^\circ + 60 A \cos 30^\circ \cos 30^\circ = 0$$

$$\sigma = 180 \sin 30^\circ \cos 30^\circ - 60 \cos^2 30^\circ = 32.9 \text{ MPa} \quad \blacktriangleleft$$

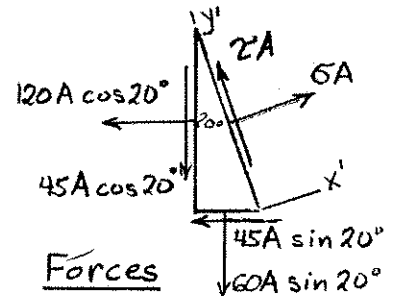
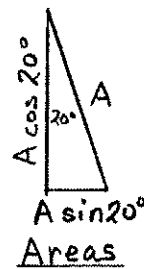
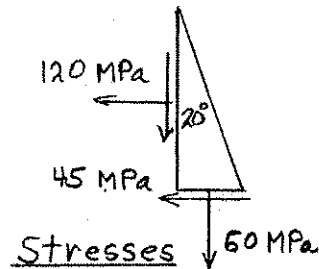
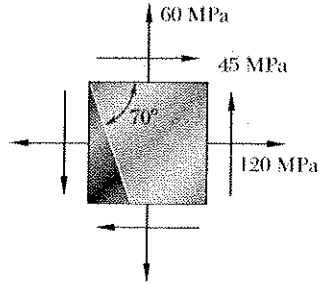
$$+\searrow \sum F = 0 :$$

$$\tau A + 90 A \sin 30^\circ \sin 30^\circ - 90 A \cos 30^\circ \cos 30^\circ - 60 A \cos 30^\circ \sin 30^\circ = 0$$

$$\tau = 90 (\cos^2 30^\circ - \sin^2 30^\circ) + 60 \cos 30^\circ \sin 30^\circ = 71.0 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.3

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



$$\rightarrow \Sigma F = 0:$$

$$\sigma A - 120 A \cos 20^\circ \cos 20^\circ - 45 A \cos 20^\circ \sin 20^\circ - 45 A \sin 20^\circ \cos 20^\circ - 60 A \sin 20^\circ \sin 20^\circ = 0$$

$$\sigma = 120 \cos^2 20^\circ + 45 \cos 20^\circ \sin 20^\circ + 45 \sin 20^\circ \cos 20^\circ + 60 \sin^2 20^\circ$$

$$\sigma = 14.19 \text{ MPa} \quad \blacktriangleleft$$

$$\uparrow \Sigma F = 0:$$

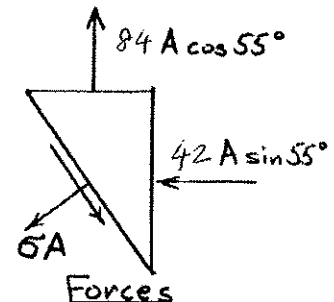
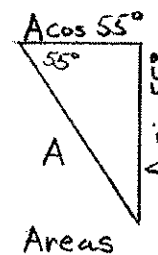
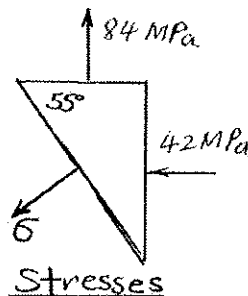
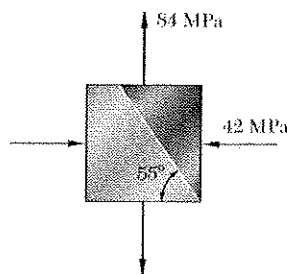
$$\tau A + 120 A \cos 20^\circ \sin 20^\circ - 45 A \cos 20^\circ \cos 20^\circ + 45 A \sin 20^\circ \sin 20^\circ - 60 A \sin 20^\circ \cos 20^\circ = 0$$

$$\tau = -120 \cos 20^\circ \sin 20^\circ + 45 (\cos^2 20^\circ - \sin^2 20^\circ) + 60 \sin 20^\circ \cos 20^\circ$$

$$\tau = 15.19 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.4

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



$$\rightarrow \Sigma F = 0$$

$$\sigma A - 84 A \cos 55^\circ \cos 55^\circ + 42 A \sin 55^\circ \sin 55^\circ = 0$$

$$\sigma = 84 \cos^2 55^\circ - 42 \sin^2 55^\circ = -0.546 \text{ MPa} \quad \blacktriangleleft$$

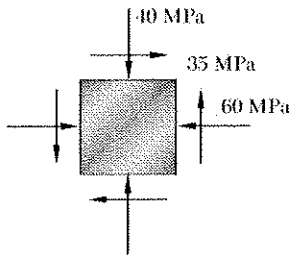
$$\uparrow \Sigma F = 0$$

$$\tau A - 84 A \cos 55^\circ \sin 55^\circ - 42 A \sin 55^\circ \cos 55^\circ = 0$$

$$\tau = 126 \cos 55^\circ \sin 55^\circ = 59.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.5

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35)}{-60 - (-40)} = -3.50$$

$$2\theta_p = -74.05^\circ \quad \theta_p = -37.0^\circ, 53.0^\circ \quad \blacktriangleleft$$

$$(b) \sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-60 - 40}{2} \pm \sqrt{\left(\frac{-60 + 40}{2}\right)^2 + (35)^2}$$

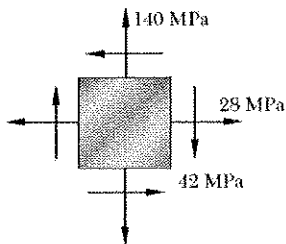
$$= -50 \pm 36.4 \text{ MPa}$$

$$\sigma_{max} = -13.60 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = -86.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.6

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



$$\sigma_x = 28 \text{ MPa} \quad \sigma_y = 140 \text{ MPa} \quad \tau_{xy} = -42 \text{ MPa}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-42)}{28 - 140} = 0.750$$

$$2\theta_p = 36.87^\circ \quad \theta_p = 18.43^\circ, 108.43^\circ \quad \blacktriangleleft$$

$$(b) \sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{28 + 140}{2} \pm \sqrt{\left(\frac{28 - 140}{2}\right)^2 + (-42)^2}$$

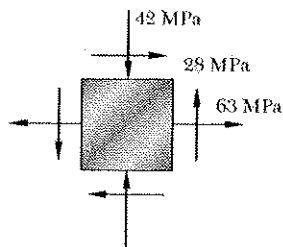
$$= 84 \pm 70$$

$$\sigma_{max} = 154 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = 14 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.7

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



$$\sigma_x = 63 \text{ MPa} \quad \sigma_y = -42 \text{ MPa} \quad \tau_{xy} = 28 \text{ MPa}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(28)}{63 - (-42)} = 0.5333$$

$$2\theta_p = 28.07^\circ \quad \theta_p = 14.04^\circ, 104.04^\circ$$

$$(b) \sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{63 - 42}{2} \pm \sqrt{\left(\frac{63 + 42}{2}\right)^2 + 28^2}$$

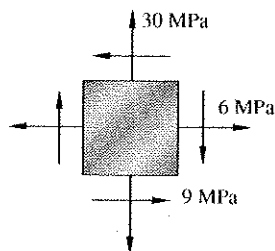
$$= 10.5 \pm 59.5$$

$$\sigma_{max} = 70 \text{ MPa}$$

$$\sigma_{min} = -49 \text{ MPa}$$

Problem 7.8

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



$$\sigma_x = 6 \text{ MPa} \quad \sigma_y = 30 \text{ MPa} \quad \tau_{xy} = -9 \text{ MPa}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-9)}{6 - 30} = -0.750$$

$$2\theta_p = 36.87^\circ \quad \theta_p = 18.4^\circ, 108.4^\circ$$

$$(b) \sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{6 + 30}{2} \pm \sqrt{\left(\frac{6 - 30}{2}\right)^2 + (-9)^2}$$

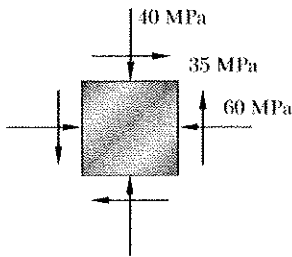
$$= 18 \pm 15$$

$$\sigma_{max} = 33.0 \text{ MPa}$$

$$\sigma_{min} = 3.00 \text{ MPa}$$

Problem 7.9

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the corresponding normal stress.



$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{-60 + 40}{(2)(35)} = 0.2857$$

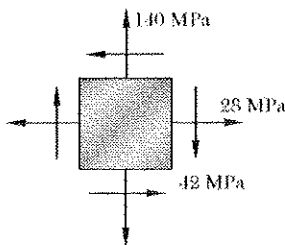
$$2\theta_s = 15.95^\circ \quad \theta_s = 8.0^\circ, 98.0^\circ \quad \blacktriangleleft$$

$$\begin{aligned} \tau_{max} &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{\left(\frac{-60 + 40}{2}\right)^2 + (35)^2} \quad \tau_{max} = 36.4 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$

$$(b) \sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{-60 - 40}{2} \quad \sigma' = -50.0 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.10

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the corresponding normal stress.



$$\sigma_x = 28 \text{ MPa} \quad \sigma_y = 140 \text{ MPa} \quad \tau_{xy} = -42 \text{ MPa}$$

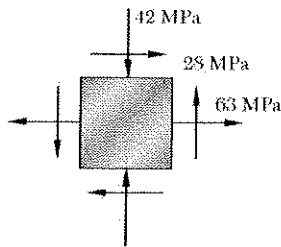
$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{28 - 140}{(2)(-42)} = -1.3333$$

$$2\theta_s = -53.13^\circ \quad \theta_s = -26.57^\circ, 63.43^\circ \quad \blacktriangleleft$$

$$\begin{aligned} (b) \tau_{max} &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{\left(\frac{28 - 140}{2}\right)^2 + (-42)^2} = 70 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$

$$(c) \sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{28 + 140}{2} = 84 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.11



7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the corresponding normal stress.

$$\sigma_x = 63 \text{ MPa} \quad \sigma_y = -42 \text{ MPa} \quad \tau_{xy} = 28 \text{ MPa}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{63 + 42}{(2)(28)} = -1.875$$

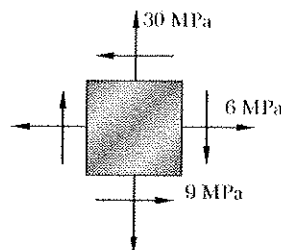
$$2\theta_s = -61.93^\circ \quad \theta_s = -30.96^\circ, 59.04^\circ \quad \blacktriangleleft$$

$$(b) \tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{63 + 42}{2}\right)^2 + (28)^2} = 59.5 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{63 - 42}{2} = 10.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.12



$$\sigma_x = 6 \text{ MPa} \quad \sigma_y = 30 \text{ MPa} \quad \tau_{xy} = -9 \text{ MPa}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{6 - 30}{(2)(-9)} = -1.33333$$

$$2\theta_s = -53.13^\circ \quad \theta_s = -26.6^\circ, 63.4^\circ \quad \blacktriangleleft$$

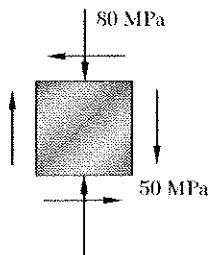
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{6 - 30}{2}\right)^2 + (-9)^2} \quad \tau_{\max} = 15.00 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{6 + 30}{2} \quad \sigma' = 18.00 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.13



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

$$\sigma_x = 0 \quad \sigma_y = -80 \text{ MPa} \quad \tau_{xy} = -50 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(a) $\theta = -25^\circ \quad 2\theta = -50^\circ$

$$\sigma_{x'} = -40 + 40 \cos(-50^\circ) - 50 \sin(-50^\circ)$$

$$\sigma_{x'} = 24.0 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -40 \sin(-50^\circ) - 50 \cos(-50^\circ)$$

$$\tau_{x'y'} = -1.5 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40 \cos(-50^\circ) + 50 \sin(-50^\circ)$$

$$\sigma_{y'} = -104.0 \text{ MPa} \quad \blacktriangleleft$$

(b) $\theta = 10^\circ \quad 2\theta = 20^\circ$

$$\sigma_{x'} = -40 + 40 \cos(20^\circ) - 50 \sin(20^\circ)$$

$$\sigma_{x'} = -19.5 \text{ MPa} \quad \blacktriangleleft$$

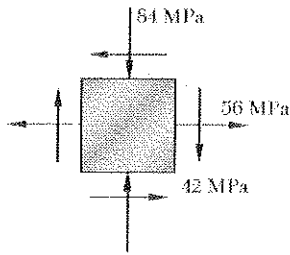
$$\tau_{x'y'} = -40 \sin(20^\circ) - 50 \cos(20^\circ)$$

$$\tau_{x'y'} = -60.7 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40 \cos(20^\circ) + 50 \sin(20^\circ)$$

$$\sigma_{y'} = -60.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.14



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated (a) 25° clockwise, (b) 10° counterclockwise.

$$\sigma_x = 56 \text{ MPa} \quad \sigma_y = -84 \text{ MPa} \quad \tau_{xy} = -42 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -14 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = 70 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(a) $\theta = -25^\circ \quad 2\theta = -50^\circ$

$$\sigma_{x'} = -14 + 70 \cos(-50^\circ) - 42 \sin(-50^\circ) = 63.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -70 \sin(-50^\circ) - 42 \cos(-50^\circ) = 26.6 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = -14 - 70 \cos(-50^\circ) + 42 \sin(-50^\circ) = -91.1 \text{ MPa} \quad \blacktriangleleft$$

(b) $\theta = 10^\circ \quad 2\theta = 20^\circ$

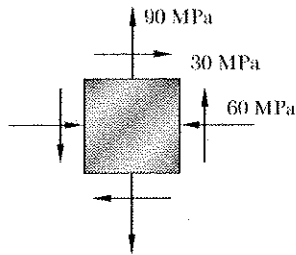
$$\sigma_{x'} = -14 + 70 \cos(20^\circ) - 42 \sin(20^\circ) = 37.4 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -70 \sin(20^\circ) - 42 \cos(20^\circ) = -63.4 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = -14 - 70 \cos(20^\circ) + 42 \sin(20^\circ) = -65.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.15

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = 90 \text{ MPa} \quad \tau_{xy} = 30 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = -75 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{xy'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(a) $\theta = -25^\circ \quad 2\theta = -50^\circ$

$$\sigma_{x'} = 15 - 75 \cos(-50^\circ) + 30 \sin(-50^\circ)$$

$$\sigma_{x'} = -56.2 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{xy'} = +75 \sin(-50^\circ) + 30 \cos(-50^\circ)$$

$$\tau_{xy'} = -38.2 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = 15 + 75 \cos(-50^\circ) - 30 \sin(-50^\circ)$$

$$\sigma_{y'} = 86.2 \text{ MPa} \quad \blacktriangleleft$$

(b) $\theta = 10^\circ \quad 2\theta = 20^\circ$

$$\sigma_{x'} = 15 - 75 \cos(20^\circ) + 30 \sin(20^\circ)$$

$$\sigma_{x'} = -45.2 \text{ MPa} \quad \blacktriangleleft$$

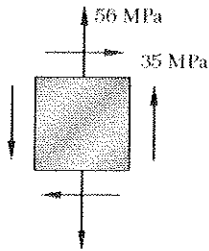
$$\tau_{xy'} = +75 \sin(20^\circ) + 30 \cos(20^\circ)$$

$$\tau_{xy'} = 53.8 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = 15 + 75 \cos(20^\circ) - 30 \sin(20^\circ)$$

$$\sigma_{y'} = 75.2 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.16



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated (a) 25° clockwise, (b) 10° counterclockwise.

$$\sigma_x = 0 \quad \sigma_y = 56 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 28 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = -28 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(a) $\theta = -25^\circ \quad 2\theta = -50^\circ$

$$\sigma_{x'} = 28 - 28 \cos(-50^\circ) + 35 \sin(-50^\circ) = -16.8 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = 28 \sin(-50^\circ) + 35 \cos(-50^\circ) = 1.05 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = 28 + 28 \cos(-50^\circ) - 35 \sin(-50^\circ) = 72.8 \text{ MPa} \quad \blacktriangleleft$$

(b) $\theta = 10^\circ \quad 2\theta = 20^\circ$

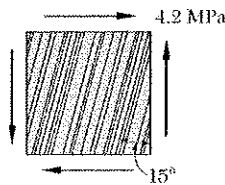
$$\sigma_{x'} = 28 - 28 \cos(20^\circ) + 35 \sin(20^\circ) = 13.7 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = 28 \sin(20^\circ) + 35 \cos(20^\circ) = 42.5 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = 28 + 28 \cos(20^\circ) - 35 \cos(20^\circ) = 42.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.17

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.



$$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = 4.2 \text{ MPa}$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$(a) \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -0 + 4.2 \cos(-30^\circ)$$

$$= 3.64 \text{ MPa} \quad \blacktriangleleft$$

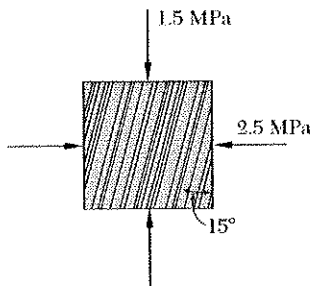
$$(b) \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= 0 + 0 + 4.2 \sin(-30^\circ)$$

$$= -2.1 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.18

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.



$$\sigma_x = -2.5 \text{ MPa} \quad \sigma_y = -1.5 \text{ MPa} \quad \tau_{xy} = 0$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$(a) \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{-2.5 - (-1.5)}{2} \sin(-30^\circ) + 0$$

$$\tau_{x'y'} = -0.250 \text{ MPa} \quad \blacktriangleleft$$

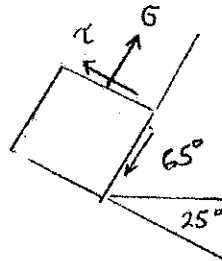
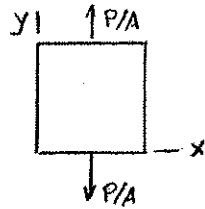
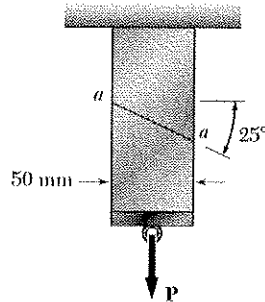
$$(b) \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-2.5 + (-1.5)}{2} + \frac{-2.5 - (-1.5)}{2} \cos(-30^\circ) + 0$$

$$\sigma_{x'} = -2.43 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.19

7.19 Two members of uniform cross section 50×80 mm are glued together along plane $a-a$ that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load P that can be applied.



For plane $a-a$ $\theta = 65^\circ$.

$$\sigma_x = 0, \quad \tau_{xy} = 0, \quad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta = 0 + \frac{P}{A} \sin^2 65^\circ + 0$$

$$P = \frac{A \sigma}{\sin^2 65^\circ} = \frac{(50 \times 10^{-3})(80 \times 10^{-3})(800 \times 10^3)}{\sin^2 65^\circ} = 3.90 \times 10^3 \text{ N}$$

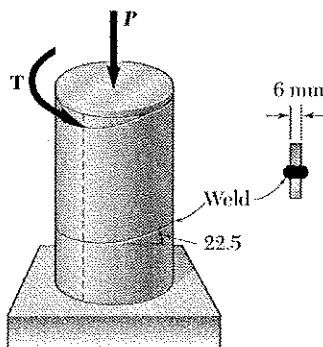
$$\tau = -(\sigma_x - \sigma_y) \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta) = \frac{P}{A} \sin 65^\circ \cos 65^\circ + 0$$

$$P = \frac{A \tau}{\sin 65^\circ \cos 65^\circ} = \frac{(50 \times 10^{-3})(80 \times 10^{-3})(600 \times 10^3)}{\sin 65^\circ \cos 65^\circ} = 6.27 \times 10^3 \text{ N}$$

Allowable value of P is the smaller one.

$$P = 3.70 \text{ kN} \quad \blacktriangleleft$$

Problem 7.20



7.20 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 160-kN axial force P and an 800 N · m torque T , each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

$$d_2 = 0.3 \text{ m}, \quad C_2 = \frac{1}{2} d_2 = 0.15 \text{ m}, \quad t = 0.006 \text{ m}$$

$$C_1 = C_2 - t = 0.144$$

$$A = \pi (C_2^2 - C_1^2) = \pi (0.15^2 - 0.144^2) = 5541.8 \times 10^{-6} \text{ m}^2$$

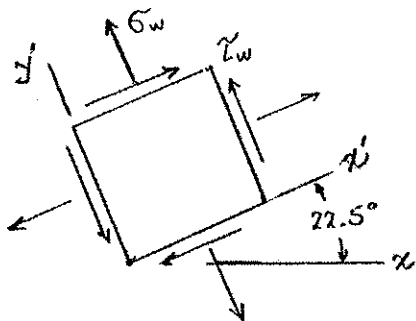
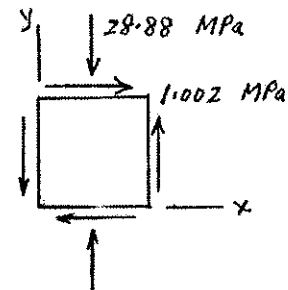
$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.15^4 - 0.144^4) = 119.8 \times 10^{-6} \text{ m}^4$$

Stresses

$$\sigma = -\frac{P}{A} = -\frac{160 \times 10^3}{5541 \times 10^{-6}} = -28.88 \text{ MPa}$$

$$\tau = \frac{TC_2}{J} = \frac{(800)(0.15)}{119.8 \times 10^{-6}} = 1002 \text{ MPa}$$

$$\sigma_x = 0, \quad \sigma_y = -28.88 \text{ MPa}, \quad \tau_{xy} = 1002 \text{ MPa}$$



Choose the x' and y' axes respectively tangential and normal to the weld.

Then, $\sigma_w = \sigma_{y'}$ and $\tau_w = \tau_{x'y'}$

$$\theta = 22.5^\circ$$

$$\begin{aligned} \sigma_{y'} &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \\ &= \frac{(-28.88)}{2} - \frac{[-(-28.88)]}{2} \cos 45^\circ - 1002 \sin 45^\circ \\ &= -25.36 \text{ MPa} \end{aligned}$$

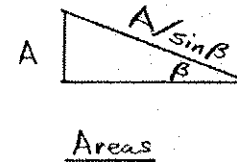
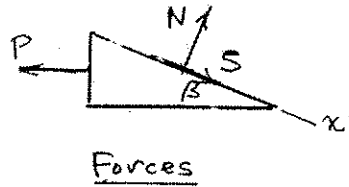
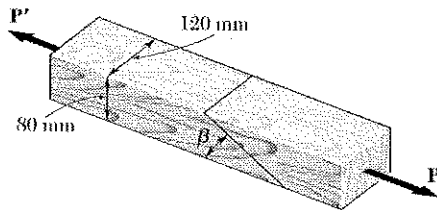
$$\sigma_w = -25.4 \text{ MPa} \quad \blacktriangleleft$$

$$\begin{aligned} \tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{[-(-28.88)]}{2} \sin 45^\circ + 1002 \cos 45^\circ \\ &= -9.5 \text{ MPa} \end{aligned}$$

$$\tau_w = -9.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.21

7.21 Two wooden members of 80×120 -mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25^\circ$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (a) the in-plane shearing stress parallel to the splice, (b) the normal stress perpendicular to the splice.



$$A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$(b) \quad +\uparrow \Sigma F_{y'} = 0: \quad N - P \sin \beta = 0 \quad N = P \sin \beta = (10 \times 10^3) \sin 25^\circ = 4.226 \times 10^3 \text{ N}$$

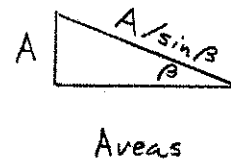
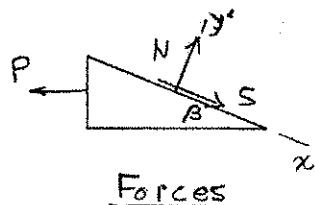
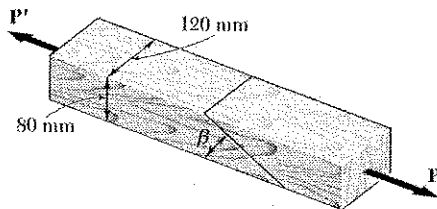
$$\sigma = \frac{N}{A/\sin \beta} = \frac{(4.226 \times 10^3) \sin 25^\circ}{9.6 \times 10^3} = 186.0 \times 10^3 \text{ Pa} = 186.0 \text{ kPa} \quad \blacktriangleleft$$

$$(a) \quad +\rightarrow \Sigma F_{x'} = 0: \quad S - P \cos \beta = 0 \quad S = P \cos \beta = (10 \times 10^3) \cos 25^\circ = 9.063 \times 10^3 \text{ N}$$

$$\tau = \frac{S}{A/\sin \beta} = \frac{(9.063 \times 10^3) \sin 25^\circ}{9.6 \times 10^3} = 399 \times 10^3 \text{ Pa} = 399 \text{ kPa} \quad \blacktriangleleft$$

Problem 7.22

7.22 Two wooden members of 80×120 -mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22^\circ$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load P that can be applied.



$$A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$N_{all} = \sigma_{all} A/\sin \beta = \frac{(400 \times 10^3)(9.6 \times 10^{-3})}{\sin 22^\circ} = 10.251 \times 10^3 \text{ N}$$

$$+\uparrow \Sigma F_{y'} = 0: \quad N - P \sin \beta = 0 \quad P = \frac{N}{\sin \beta} = \frac{10.251 \times 10^3}{\sin 22^\circ} = 27.4 \times 10^3 \text{ N}$$

$$S_{all} = \tau_{all} A/\sin \beta = \frac{(600 \times 10^3)(9.6 \times 10^{-3})}{\sin 22^\circ} = 15.376 \times 10^3 \text{ N}$$

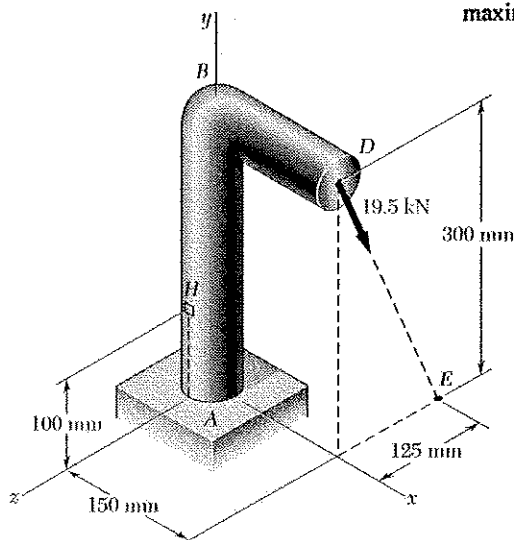
$$+\rightarrow \Sigma F_{x'} = 0: \quad S - P \cos \beta = 0 \quad P = \frac{S}{\cos \beta} = \frac{15.376 \times 10^3}{\cos 22^\circ} = 16.58 \times 10^3 \text{ N}$$

The smaller value for P governs.

$$P = 16.58 \text{ kN} \quad \blacktriangleleft$$

Problem 7.23

7.23 A 19.5-kN force is applied at point *D* of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point *H*.



$$l_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

Resolve the 19.5 kN force *F* at point *D* into *x*, *y*, and *z* components.

$$F_x = 0$$

$$F_y = -\frac{300}{325} (19.5) = -18 \text{ kN} = -18 \times 10^3 \text{ N}$$

$$F_z = -\frac{125}{325} (19.5) = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

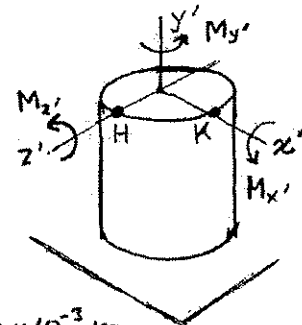
Determine the force-couple system at the point on the *y*-axis where it intersects the plane containing elements *H* and *K*.

$$F_x' = 0, \quad F_y' = -18 \times 10^3 \text{ N}, \quad F_z' = -7.5 \times 10^3 \text{ N}$$

$$M_x' = -(7.5 \text{ kN})(300 \text{ mm} - 100 \text{ mm}) = -1.5 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y' = (7.5 \text{ kN})(150 \text{ mm}) = 1.125 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_z' = -(18 \text{ kN})(150 \text{ mm}) = -2.7 \times 10^3 \text{ N}\cdot\text{m}$$



Properties of section. (Circle) $c = \frac{1}{2}d = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$

$$A = \pi c^2 = \pi (30)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

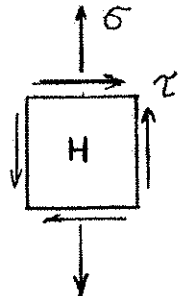
$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (30)^4 = 636.17 \times 10^3 \text{ mm}^4 = 636.17 \times 10^{-9} \text{ m}^4$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (30)^4 = 1.27235 \times 10^6 \text{ mm}^4 = 1.27235 \times 10^{-6} \text{ m}^4$$

$$\text{(Semicircle)} \quad Q = \frac{2}{3} c^3 = \frac{2}{3} (30)^3 = 18 \times 10^3 \text{ mm}^3 = 18 \times 10^{-6} \text{ m}^3$$

$$t = d = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

Stresses at H.



$$\sigma = \frac{F_y}{A} - \frac{M_x c}{I} = \frac{-18 \times 10^3}{2.8274 \times 10^{-3}} - \frac{(-1.5 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^{-9}}$$

$$= 64.370 \times 10^6 \text{ Pa} = 64.37 \text{ MPa}$$

$$\tau = \frac{F_z Q}{I t} + \frac{M_y c}{J} = 0 + \frac{(1.125 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}}$$

$$= 26.526 \times 10^6 \text{ Pa} = 26.526 \text{ MPa}$$

Continued

Problem 7.23 continued

Principal stresses

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max,min} = \frac{0 + 64.37}{2} \pm \sqrt{\left(\frac{0 - 64.37}{2}\right)^2 + (26.526)^2}$$

$$= -32.185 \pm 41.707 \text{ MPa}$$

$$\sigma_{max} = 73.9 \text{ MPa}$$

$$\sigma_{min} = -9.52 \text{ MPa} \blacktriangleleft$$

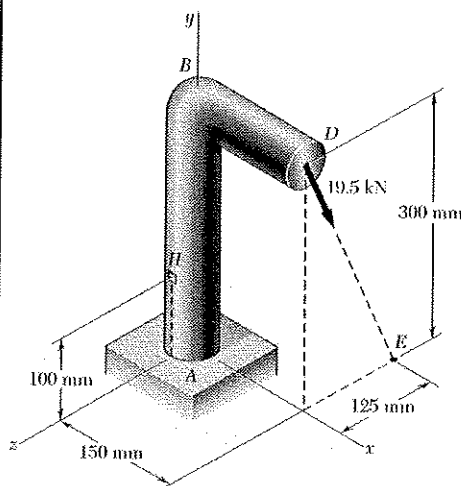
Maximum shearing stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = 41.7 \text{ MPa} \blacktriangleleft$$

Problem 7.24

7.24 A 19.5-kN force is applied at point D of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point K.



$$l_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

Resolve the 19.5 kN force F at point D into x, y, and z components.

$$F_x = 0$$

$$F_y = -\frac{300}{325} (19.5) = -18 \text{ kN} = -18 \times 10^3 \text{ N}$$

$$F_z = -\frac{125}{325} (19.5) = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

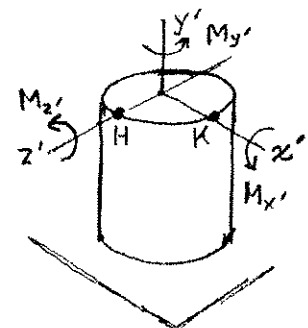
Determine the force-couple system at the point on the y-axis where it intersects the plane containing elements H and K.

$$F_x' = 0, \quad F_y' = -18 \times 10^3 \text{ N}, \quad F_z' = -7.5 \times 10^3 \text{ N}$$

$$M_x' = -(7.5 \text{ kN})(300 \text{ mm} - 100 \text{ mm}) = -1.5 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y' = (7.5 \text{ kN})(150 \text{ mm}) = 1.125 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_z' = -(18 \text{ kN})(150 \text{ mm}) = -2.7 \times 10^3 \text{ N}\cdot\text{m}$$



continued

Problem 7.24 continued

Properties of section. (Circle) $c = \frac{1}{2}d = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$

$$A = \pi c^2 = \pi (30)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-6} \text{ m}^2$$

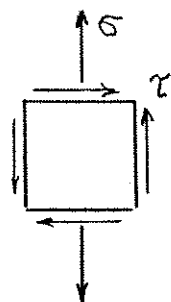
$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (30)^4 = 636.17 \times 10^3 \text{ mm}^4 = 636.17 \times 10^{-9} \text{ m}^4$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (30)^4 = 1.27235 \times 10^6 \text{ mm}^4 = 1.27235 \times 10^{-6} \text{ m}^4$$

(Semicircle) $Q = \frac{2}{3} c^3 = \frac{2}{3} (30)^3 = 18 \times 10^3 \text{ mm}^3 = 18 \times 10^{-6} \text{ m}^3$

$$t = d = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

Stresses at K.



$$\sigma = \frac{F_y}{A} + \frac{M_z c}{I} = \frac{-18 \times 10^3}{2.8274 \times 10^{-6}} + \frac{(-2.7 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^{-9}}$$

$$= -133.69 \times 10^6 \text{ Pa} = -133.69 \text{ MPa}$$

$$\tau = -\frac{F_z Q}{I t} + \frac{M_y c}{J} = -\frac{(-7.5 \times 10^3)(18 \times 10^{-6})}{(636.17 \times 10^{-9})(60 \times 10^{-3})} + \frac{(1.125 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}}$$

$$= 3.5368 \times 10^6 + 26.526 \times 10^6$$

$$= 30.06 \times 10^6 \text{ Pa} = 30.06 \text{ MPa}$$

Principal stresses

$$\sigma_{\max, \min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{\max, \min} = \frac{0 - 133.69}{2} \pm \sqrt{\left(\frac{0 + 133.69}{2}\right)^2 + (30.06)^2}$$

$$= -66.845 \pm 73.293 \text{ MPa}$$

$$\sigma_{\max} = 6.45 \text{ MPa} \quad \blacktriangleleft$$

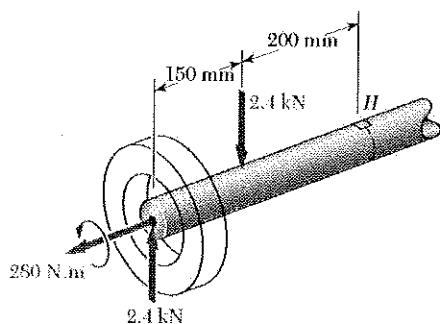
$$\sigma_{\min} = -140.2 \text{ MPa} \quad \blacktriangleleft$$

Maximum shearing stress

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = 73.3 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.25



7.25 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 30 mm, determine (a) the principal planes and principal stresses at point *H* located on top of the axle, (b) the maximum shearing stress at the same point.

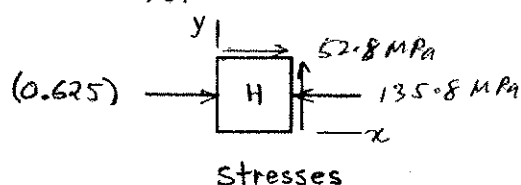
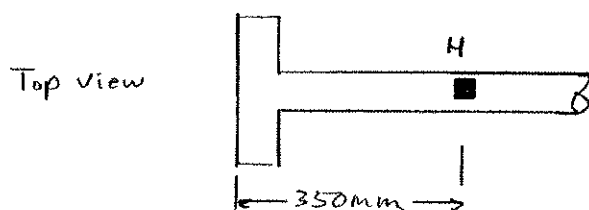
$$c = \frac{1}{2}d = \frac{1}{2}(30) = 15 \text{ mm}$$

$$\text{Torsion: } \gamma = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\gamma = \frac{(2)(280)}{\pi(15)^3} = 52.8 \text{ kPa}$$

$$\text{Bending: } I = \frac{\pi}{4}c^4 = 39761 \text{ mm}^4$$

$$M = (150)(2400) = 360000 \text{ N·mm} \quad \sigma = -\frac{My}{I} = -\frac{(36 \times 10^4)(15)}{39761} = -135.8 \text{ MPa}$$



$$\sigma_x = -135.8 \text{ MPa} \quad \sigma_y = 0 \quad \tau_{xy} = 52.8 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -67.9 \text{ MPa}$$

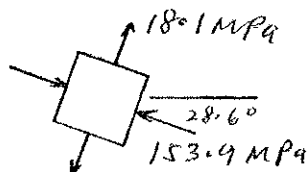
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-67.9)^2 + (52.8)^2} = 86 \text{ MPa}$$

$$(a) \quad \sigma_1 = \sigma_{ave} + R = -67.9 + 86 = 18.1$$

$$\sigma_2 = \sigma_{ave} - R = -67.9 - 86 = -153.9 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(52.8)}{-67.9} = -1.5552$$

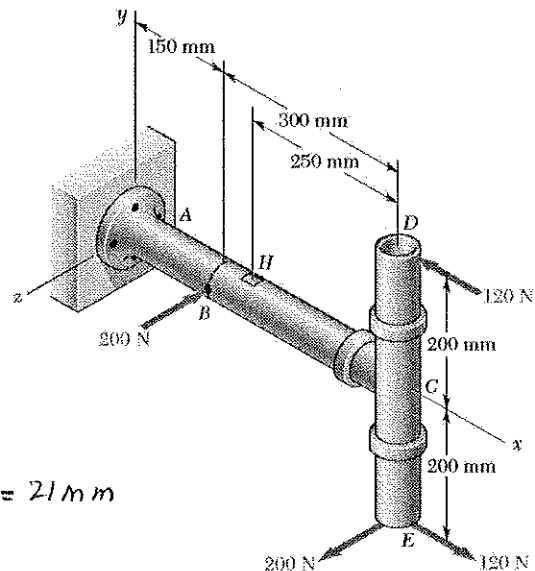
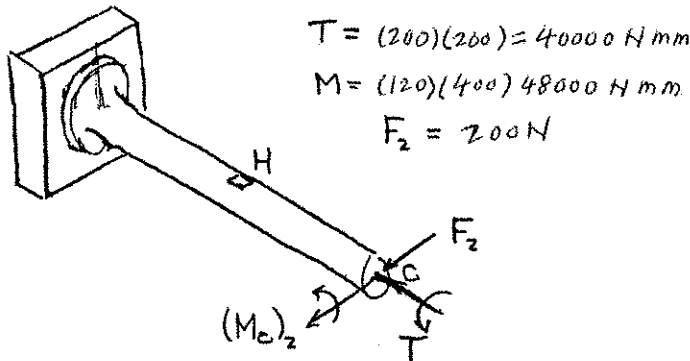
$$\theta_p = -28.6^\circ \text{ and } 61.4^\circ$$



$$(b) \quad \tau_{max} = R = 86 \text{ MPa}$$

Problem 7.26

Replace forces on pipe DCE by an equivalent force-couple system at C.



Cross section. $C_1 = \frac{d_i}{2} = 19 \text{ mm}$ $C_2 = \frac{d_o}{2} = 21 \text{ mm}$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 100782 \text{ mm}^4$$

$$I = \frac{1}{2}J = 50391 \text{ mm}^4$$

$$Q_y = \frac{2}{3}(C_2^3 - C_1^3) = \frac{2}{3}(21^3 - 19^3) = 1601 \text{ mm}^3$$

$$t = C_2 - C_1 = 2 \text{ mm}$$

At the section containing element H

$$T = 40000 \text{ Nmm}, \quad M_2 = 48000 \text{ Nmm}, \quad V_2 = 200 \text{ N}$$

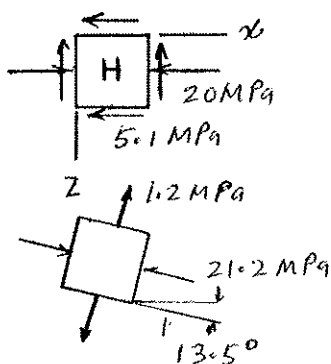
Stresses.

Torsion: $\tau_{zx} = -\frac{Tc}{J} = -\frac{(40000)(21)}{100782} = -8.3 \text{ MPa}$

Bending: $\sigma_x = -\frac{My}{I} = -\frac{(48000)(21)}{50391} = -20 \text{ MPa}$

Transverse Shear: $\tau_{zx} = \frac{VQ}{I(2t)} = \frac{(200)(1601)}{(50391)(2)} = 3.2 \text{ MPa}$

Total: $\sigma_z = 0$, $\sigma_x = -20 \text{ MPa}$, $\tau_{zx} = -8.3 + 3.2 = -5.1 \text{ MPa}$



(a) $\tan 2\theta_p = \frac{2\tau_{zx}}{\sigma_z - \sigma_x} = \frac{(2)(-5.1)}{0 + 20} = -0.51$

$\theta_p = -13.5^\circ$ and 76.5° from z-axis.

$$\sigma_{ave} = \frac{1}{2}(\sigma_z + \sigma_x) = \frac{1}{2}(0 - 20) = -10 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} = \sqrt{10^2 + 5.1^2} = 11.2 \text{ MPa}$$

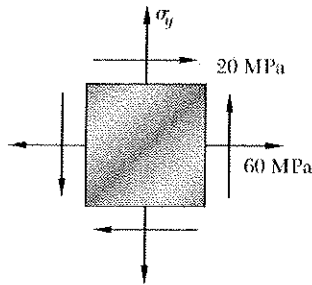
$\sigma_1 = \sigma_{ave} + R = 1.2 \text{ MPa}$ at 13.5° from z-axis.

$\sigma_2 = \sigma_{ave} - R = -21.2 \text{ MPa}$ at 76.5° from z-axis.

(b) $\tau_{max} = R$

$\tau_{max} = 11.2 \text{ MPa}$

Problem 7.27



7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$

$$\text{Let } u = \frac{\sigma_x - \sigma_y}{2}. \quad \text{Then } \sigma_y = \sigma_x - 2u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = 75 \text{ MPa}$$

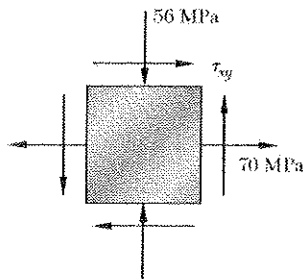
$$u = \pm \sqrt{R^2 - \tau_{xy}^2} = \pm \sqrt{75^2 - 20^2} = 72.284 \text{ MPa}$$

$$\sigma_y = \sigma_x - 2u = 60 \mp (2)(72.284) = -84.6 \text{ MPa or } 205 \text{ MPa}$$

Largest value of σ_y is required.

$$\sigma_y = 205 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.28



7.28 For the state of plane stress shown, determine (a) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 84 MPa, (b) the corresponding principal stresses.

$$\sigma_x = 70 \text{ MPa}, \quad \sigma_y = -56 \text{ MPa}, \quad \tau_{xy} = ?$$

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{70 - (-56)}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{63^2 + \tau_{xy}^2} = 84 \text{ MPa}$$

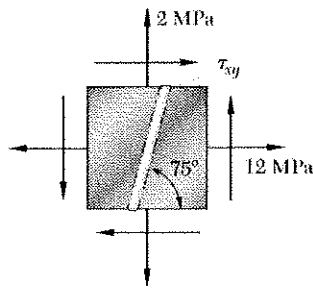
$$(a) \quad \tau_{xy} = \sqrt{84^2 - 63^2} = 55.6 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 7 + 84 = 91 \text{ MPa} \quad \blacktriangleleft$$

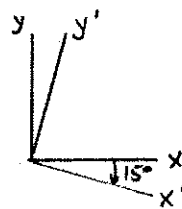
$$\sigma_b = \sigma_{ave} - R = 7 - 84 = -77 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.29



7.29 For the state of plane stress shown, determine (a) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (b) the corresponding principal stresses.

$$\sigma_x = 12 \text{ MPa}, \quad \sigma_y = 2 \text{ MPa}, \quad \tau_{xy} = ?$$



Since $\tau_{xy'} = 0$, x' -direction is a principal direction.

$$\theta_p = -15^\circ$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$(a) \quad \tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y) \tan 2\theta_p = \frac{1}{2}(12 - 2) \tan(-30^\circ) \quad \tau_{xy} = -2.89 \text{ MPa} \quad \leftarrow$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7 \text{ MPa}$$

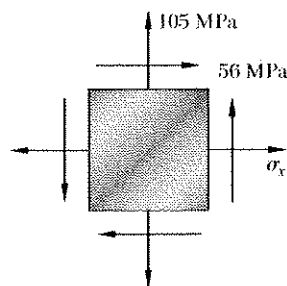
$$(b) \quad \sigma_a = \sigma_{ave} + R = 7 + 5.7735$$

$$\sigma_a = 12.77 \text{ MPa} \quad \leftarrow$$

$$\sigma_b = \sigma_{ave} - R = 7 - 5.7735$$

$$\sigma_b = 1.226 \text{ MPa} \quad \leftarrow$$

Problem 7.30



7.30 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 70 MPa.

$$\sigma_x = ? \quad , \quad \sigma_y = 105 \text{ MPa} \quad , \quad \tau_{xy} = 56 \text{ MPa}$$

$$\text{Let } u = \frac{\sigma_x - \sigma_y}{2} \quad \sigma_x = \sigma_y + 2u$$

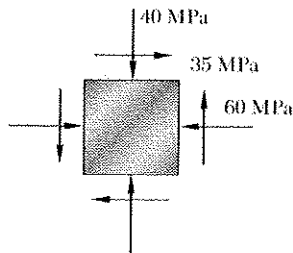
$$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 70 \text{ MPa}$$

$$u = \pm \sqrt{R^2 - \tau_{xy}^2} = \pm \sqrt{70^2 - 56^2} = \pm 42 \text{ MPa}$$

$$\sigma_x = \sigma_y + 2u = 105 \pm (2)(42) = 189 \text{ MPa or } 21 \text{ MPa}$$

$$\text{Allowable range} \quad 21 \text{ MPa} \leq \sigma_x \leq 189 \text{ MPa} \quad \leftarrow$$

Problem 7.31



7.31 Solve Probs. 7.5 and 7.9, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the corresponding normal stress.

$$\sigma_x = -60 \text{ MPa}, \quad \sigma_y = -40 \text{ MPa}, \quad \tau_{xy} = 35 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$$

Plotted points for Mohr's circle.

$$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (-50 \text{ MPa}, 0)$$

$$(a) \tan \beta = \frac{GX}{CG} = \frac{35}{10} = 3.500$$

$$\beta = 74.05^\circ$$

$$\theta_B = -\frac{1}{2}\beta = -37.03^\circ$$

$$\alpha = 180^\circ - \beta = 105.95^\circ$$

$$\theta_A = \frac{1}{2}\alpha = 52.97^\circ$$

$$R = \sqrt{CG^2 + GX^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa}$$

$$(b) \sigma_{min} = \sigma_{ave} - R = -50 - 36.4$$

$$\sigma_{min} = -86.4 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4$$

$$\sigma_{max} = -13.6 \text{ MPa}$$

$$(a') \theta_D = \theta_B + 45^\circ = 7.97^\circ$$

$$\theta_D = 8.0^\circ$$

$$\theta_E = \theta_A + 45^\circ = 97.97^\circ$$

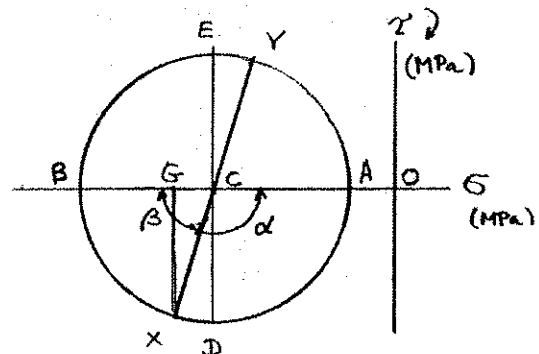
$$\theta_E = 98.0^\circ$$

$$\tau_{max} = R = 36.4 \text{ MPa}$$

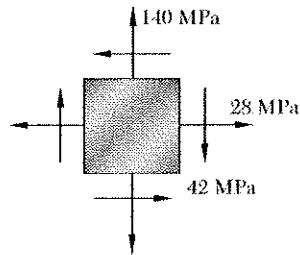
$$\tau_{max} = 36.4 \text{ MPa}$$

$$(b') \sigma' = \sigma_{ave} = -50 \text{ MPa}$$

$$\sigma' = -50.0 \text{ MPa}$$



Problem 7.32



7.32 Solve Probs. 7.6 and 7.10, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the corresponding normal stress.

$$\sigma_x = 28 \text{ MPa}, \quad \sigma_y = -140 \text{ MPa}, \quad \tau_{xy} = -42 \text{ MPa}$$

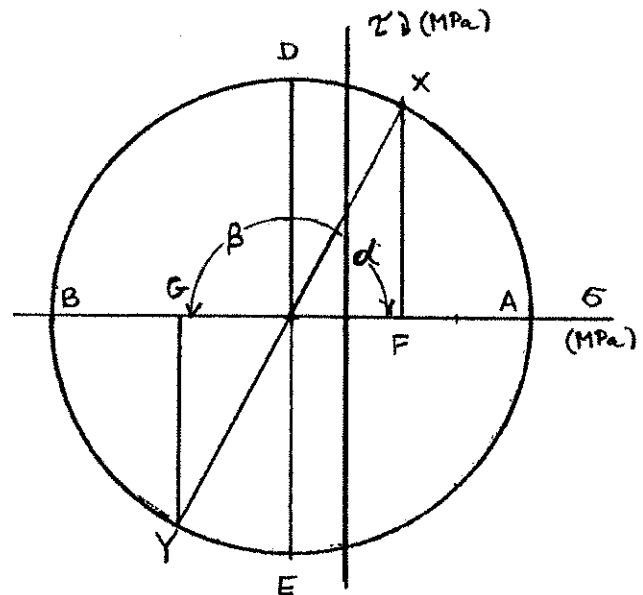
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -56 \text{ MPa}$$

Plotted points for Mohr's circle.

$$X: (\sigma_x, -\tau_{xy}) = (28 \text{ MPa}, 42 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (-140 \text{ MPa}, -42 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (-56 \text{ MPa}, 0)$$



$$(a) \tan \alpha = \frac{FX}{CF} = \frac{42}{84} = 0.5$$

$$\alpha = 26.57^\circ$$

$$\theta_a = -\frac{1}{2}\alpha = -13.29^\circ$$

$$\beta = 180^\circ - \alpha = 153.43^\circ$$

$$\theta_b = \frac{1}{2}\beta = 76.72^\circ$$

$$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(84)^2 + (42)^2} = 93.9 \text{ MPa}$$

$$(b) \sigma_a = \sigma_{max} = \sigma_{ave} + R = -56 + 93.9$$

$$\sigma_{max} = 37.9 \text{ MPa}$$

$$\sigma_{min} = \sigma_{min} = \sigma_{ave} - R = -56 - 93.9$$

$$\sigma_{min} = -149.9 \text{ MPa}$$

$$(a') \theta_d = \theta_a + 45^\circ = 31.71^\circ$$

$$\theta_d = 31.7^\circ$$

$$\theta_e = \theta_b + 45^\circ = 121.72^\circ$$

$$\theta_e = 121.7^\circ$$

$$\tau_{max} = R$$

$$\tau_{max} = 93.9 \text{ MPa}$$

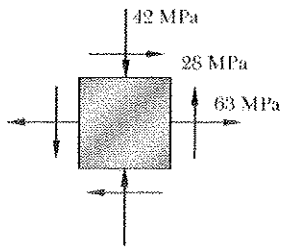
$$(b') \sigma' = \sigma_{ave}$$

$$\sigma' = -56 \text{ MPa}$$

Problem 7.33

7.33 Solve Prob. 7.11, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the corresponding normal stress.



$$\sigma_x = 63 \text{ MPa} \quad \sigma_y = -42 \text{ MPa} \quad \tau_{xy} = 28 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 10.5 \text{ MPa}$$

Points

$$X: (\sigma_x, -\tau_{xy}) = (63 \text{ MPa}, 28 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (-42 \text{ MPa}, 28 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (10.5 \text{ MPa}, 0)$$

$$\tan \alpha = \frac{FX}{CF} = \frac{28}{52.5} = 0.5333$$

$$\alpha = 28.07^\circ$$

$$\theta_a = \frac{1}{2} \alpha = 14.04^\circ$$

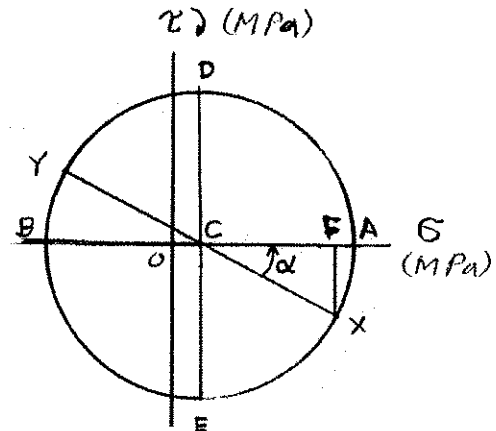
$$\theta_d = \theta_a + 45^\circ = 59.04^\circ \quad \blacktriangleleft$$

$$\theta_e = \theta_a - 45^\circ = -30.96^\circ \quad \blacktriangleleft$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{52.5^2 + 28^2} = 59.5 \text{ MPa}$$

$$\tau_{max} = R = 59.5 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma' = \sigma_{ave} = 10.5 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.34

7.34 Solve Prob. 7.12, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the corresponding normal stress.

$$\sigma_x = 6 \text{ MPa} \quad \sigma_y = 30 \text{ MPa} \quad \tau_{xy} = -9 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(6 + 30) = 18 \text{ MPa}$$

Plotted points for Mohr's circle.

$$X: (\sigma_x, -\tau_{xy}) = (6 \text{ MPa}, 9 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (30 \text{ MPa}, -9 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (18 \text{ MPa}, 0)$$

$$\tan \alpha = \frac{FX}{FE} = \frac{9}{12} = 0.75$$

$$\alpha = 36.87^\circ$$

$$\theta_b = \frac{1}{2}\alpha = 18.43^\circ$$

$$(a) \quad \theta_d = \theta_b - 45^\circ$$

$$\theta_d = -26.6^\circ$$

$$\theta_e = \theta_b + 45^\circ$$

$$\theta_e = 63.4^\circ$$

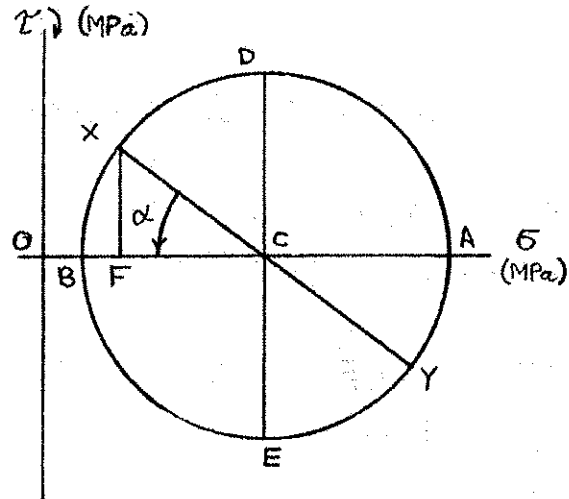
$$R = \sqrt{CF^2 + FX^2} = \sqrt{12^2 + 9^2} = 15 \text{ MPa}$$

$$\tau_{max}(\text{in-plane}) = R$$

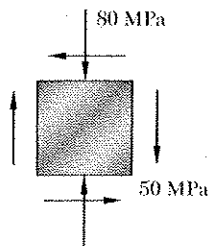
$$\tau_{max}(\text{in-plane}) = 15.00 \text{ MPa}$$

$$(b) \quad \sigma' = \sigma_{ave}$$

$$\sigma' = 18.00 \text{ MPa}$$



Problem 7.35



7.35 Solve Prob. 7.13, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

$$\sigma_x = 0, \quad \sigma_y = -80 \text{ MPa}, \quad \tau_{xy} = -50 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$$

Plotted points for Mohr's circle.

$$X: (0, 50 \text{ MPa})$$

$$Y: (-80 \text{ MPa}, -50 \text{ MPa})$$

$$C: (-40 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{50}{40} = 1.25$$

$$2\theta_p = 51.34^\circ$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{40^2 + 50^2}$$

$$= 64.03 \text{ MPa}$$

(a) $\theta = 25^\circ \downarrow$, $2\theta = 50^\circ \downarrow$

$$\phi = 51.34^\circ - 50^\circ = 1.34^\circ \downarrow$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = 24.0 \text{ MPa} \leftarrow$$

$$\tau_{x'y'} = -R \sin \phi = -1.5 \text{ MPa} \leftarrow$$

$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -104.0 \text{ MPa} \leftarrow$$

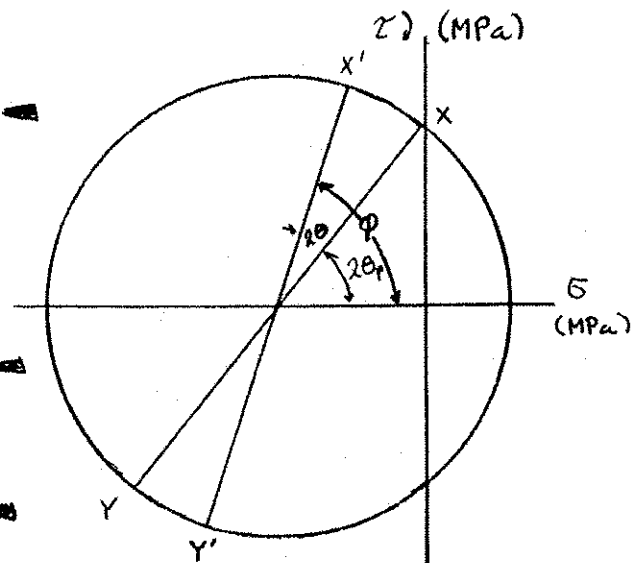
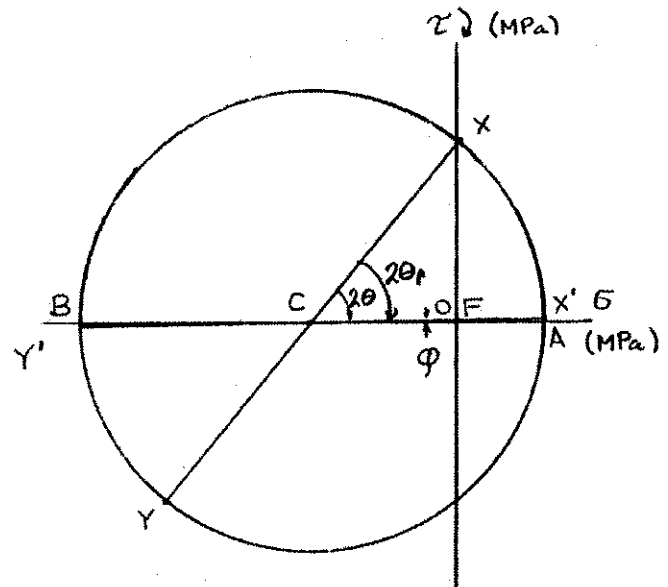
(b) $\theta = 10^\circ \uparrow$, $2\theta = 20^\circ \uparrow$

$$\phi = 51.34^\circ + 20^\circ = 71.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = -19.5 \text{ MPa} \leftarrow$$

$$\tau_{x'y'} = -R \sin \phi = -60.7 \text{ MPa} \leftarrow$$

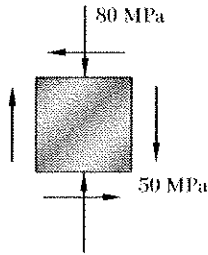
$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -60.5 \text{ MPa} \leftarrow$$



Problem 7.36

7.36 Solve Prob. 7.14, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$\begin{aligned}\sigma_x &= 0 & \sigma_y &= -80 \text{ MPa} & \tau_{xy} &= -50 \text{ MPa} \\ \sigma_{ave} &= \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}\end{aligned}$$

Points

$$X: (0, 50 \text{ MPa})$$

$$Y: (-80 \text{ MPa}, -50 \text{ MPa})$$

$$C: (-40 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{50}{40} = 1.25$$

$$2\theta_p = 51.34^\circ$$

$$\begin{aligned}R &= \sqrt{CF^2 + FX^2} = \sqrt{40^2 + 50^2} \\ &= 64.03 \text{ MPa}\end{aligned}$$

$$(a) \theta = 25^\circ \quad 2\theta = 50^\circ$$

$$\phi = 51.34^\circ - 50^\circ = 1.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = 24.0 \text{ MPa}$$

$$\tau_{xy'} = -R \sin \phi = -1.5 \text{ MPa}$$

$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -104.0 \text{ MPa}$$

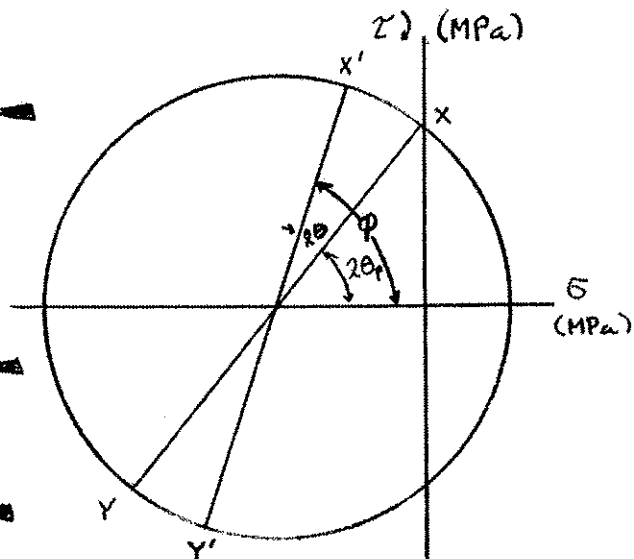
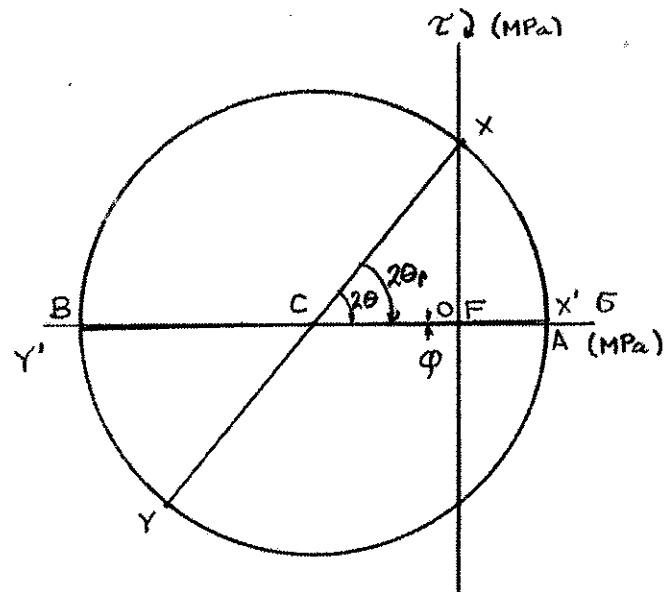
$$(b) \theta = 10^\circ \quad 2\theta = 20^\circ$$

$$\phi = 51.34^\circ + 20^\circ = 71.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = -19.5 \text{ MPa}$$

$$\tau_{xy'} = +R \sin \phi = -60.7 \text{ MPa}$$

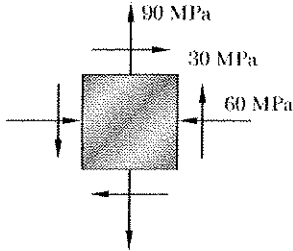
$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -60.5 \text{ MPa}$$



Problem 7.37

7.37 Solve Prob. 7.15, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$\sigma_x = -60 \text{ MPa}, \quad \sigma_y = 90 \text{ MPa}, \quad \tau_{xy} = 30 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa}$$

Plotted points for Mohr's circle.

$$X: (-60 \text{ MPa}, -30 \text{ MPa})$$

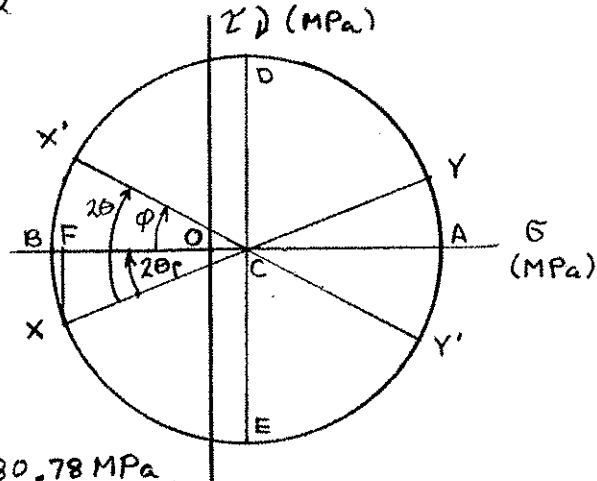
$$Y: (90 \text{ MPa}, 30 \text{ MPa})$$

$$C: (15 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{30}{75} = 0.4$$

$$2\theta_p = 21.80^\circ \quad \theta_p = 10.90^\circ$$

$$R = \sqrt{FC^2 + FX^2} = \sqrt{75^2 + 30^2} = 80.78 \text{ MPa}$$



(a) $\theta = 25^\circ$, $2\theta = 50^\circ$

$$\phi = 2\theta - 2\theta_p = 50^\circ - 21.80^\circ = 28.20^\circ$$

$$\sigma_{x'} = \sigma_{ave} - R \cos \phi = -56.2 \text{ MPa}$$

$$\tau_{x'y'} = -R \sin \phi = -38.2 \text{ MPa}$$

$$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 86.2 \text{ MPa}$$

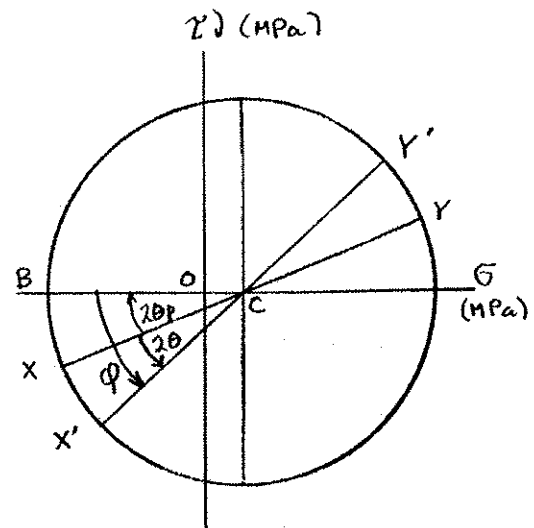
(b) $\theta = 10^\circ$, $2\theta = 20^\circ$

$$\phi = 2\theta_p + 2\theta = 21.80^\circ + 20^\circ = 41.80^\circ$$

$$\sigma_{x'} = \sigma_{ave} - R \cos \phi = -45.2 \text{ MPa}$$

$$\tau_{x'y'} = R \sin \phi = 53.8 \text{ MPa}$$

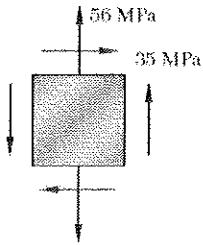
$$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 75.2 \text{ MPa}$$



Problem 7.38

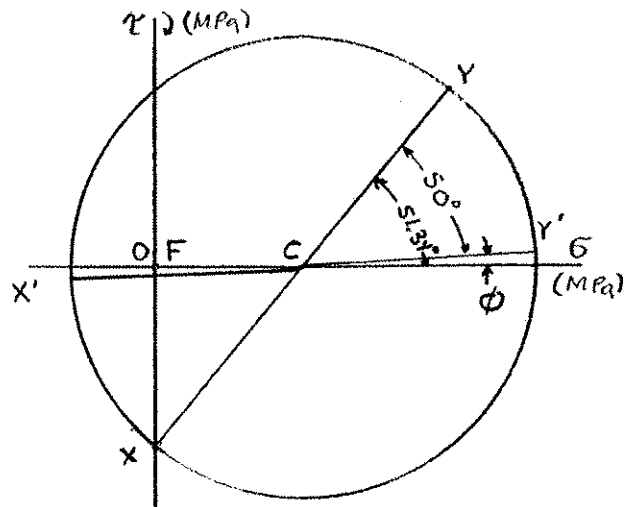
7.38 Solve Prob. 7.16, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated (a) 25° clockwise, (b) 10° counterclockwise.



$$\sigma_x = 0 \quad \sigma_y = 56 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 28 \text{ MPa}$$



Points:

$$X: (0, -35 \text{ MPa})$$

$$Y: (56 \text{ MPa}, 35 \text{ MPa})$$

$$C: (28 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{35}{28} = 1.25$$

$$2\theta_p = 51.34^\circ$$

$$R = \sqrt{FC^2 + FX^2} = \sqrt{28^2 + 35^2}$$

$$= 44.8 \text{ MPa}$$

(a) $\theta = 25^\circ \rightarrow 2\theta = 50^\circ \rightarrow$

$$\phi = 51.34^\circ - 50^\circ = 1.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} - R \cos \phi = -16.8 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = R \sin \phi = 1.05 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 72.8 \text{ MPa} \quad \blacktriangleleft$$

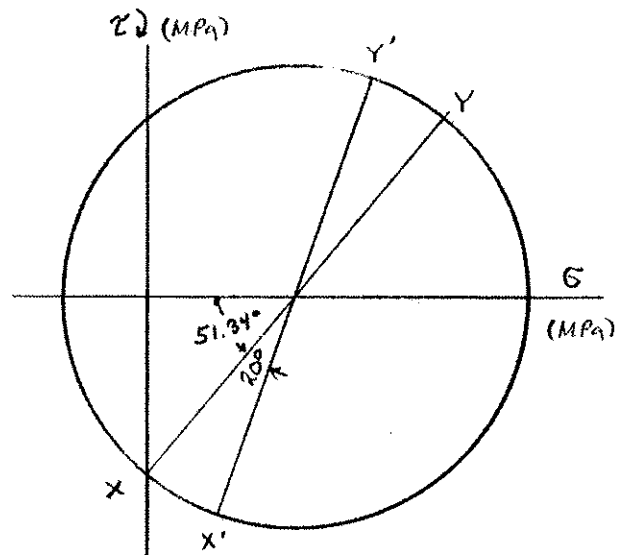
(b) $\theta = 10^\circ \rightarrow 2\theta = 20^\circ \rightarrow$

$$\phi = 51.34^\circ + 20^\circ = 71.34^\circ$$

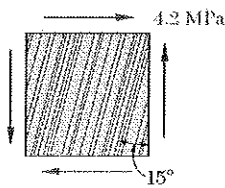
$$\sigma_{x'} = \sigma_{ave} - R \cos \phi = 13.7 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = R \sin \phi = 42.3 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 42.4 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.39



7.39 Solve Prob. 7.17, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

$$\sigma_x = \sigma_y = 0 \quad \tau_{xy} = 2.8 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Points

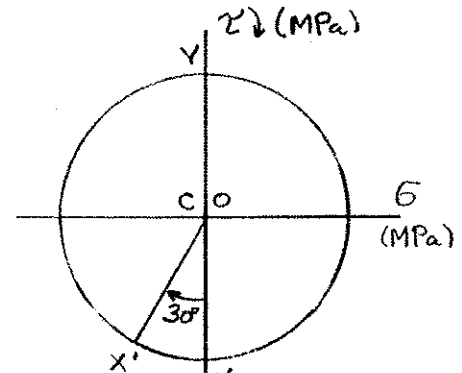
$$X: (\sigma_x, -\tau_{xy}) = (0, -4.2 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (0, 4.2 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (0, 0)$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

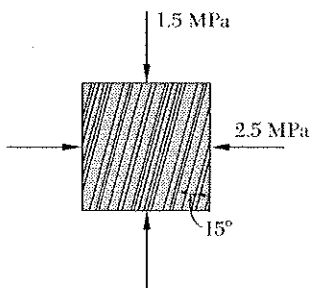
$$\overline{CX} = R = 4.2 \text{ MPa}$$



$$(a) \tau_{x'y'} = R \cos 30^\circ = 6.00 \cos 30^\circ = 3.16 \text{ MPa}$$

$$(b) \sigma_{x'} = \sigma_{ave} - R \sin 30^\circ = -6.00 \sin 30^\circ = -2.1 \text{ MPa}$$

Problem 7.40



7.40 Solve Prob. 7.18, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

$$\sigma_x = -2.5 \text{ MPa} \quad \sigma_y = -1.5 \text{ MPa} \quad \tau_{xy} = 0$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2.0 \text{ MPa}$$

Plotted points for Mohr's circle.

$$X: (\sigma_x, -\tau_{xy}) = (-2.5 \text{ MPa}, 0)$$

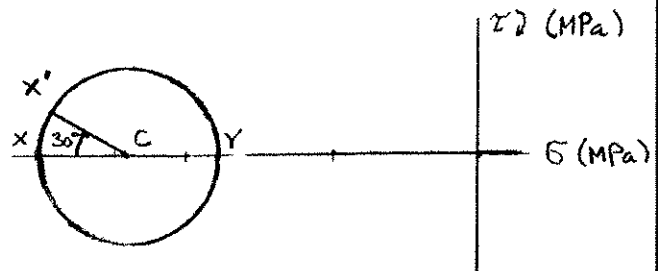
$$Y: (\sigma_y, \tau_{xy}) = (-1.5 \text{ MPa}, 0)$$

$$C: (\sigma_{ave}, 0) = (-2.0 \text{ MPa}, 0)$$

$$\theta = -15^\circ, \quad 2\theta = -30^\circ$$

$$\overline{CX} = 0.5 \text{ MPa}$$

$$R = 0.5 \text{ MPa}$$



$$(a) \tau_{x'y'} = -\overline{CX'} \sin 30^\circ = -R \sin 30^\circ = -0.5 \sin 30^\circ$$

$$\tau_{x'y'} = -0.25 \text{ MPa}$$

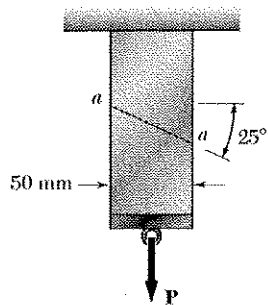
$$(b) \sigma_{x'} = \sigma_{ave} - \overline{CX'} \cos 30^\circ = -2.0 - 0.5 \cos 30^\circ =$$

$$\sigma_{x'} = -2.43 \text{ MPa}$$

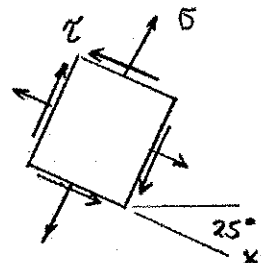
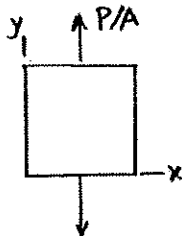
Problem 7.41

7.41 Solve Prob. 7.19, using Mohr's circle.

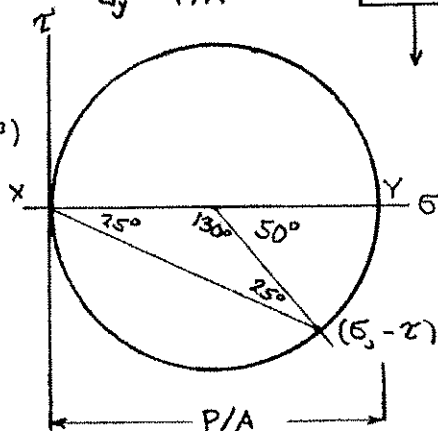
7.19 Two members of uniform cross section 50×80 mm are glued together along plane $a-a$, that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load P that can be applied.



$$\begin{aligned}\sigma_x &= 0 \\ \tau_{xy} &= 0 \\ \sigma_y &= P/A\end{aligned}$$



$$\begin{aligned}A &= (50 \times 10^{-3})(80 \times 10^{-3}) \\ &= 4 \times 10^{-3} \text{ m}^2\end{aligned}$$



$$\sigma = \frac{P}{2A} (1 + \cos 50^\circ)$$

$$P = \frac{2A\sigma}{1 + \cos 50^\circ}$$

$$P \leq \frac{(2)(4 \times 10^{-3})(800 \times 10^3)}{1 + \cos 50^\circ}$$

$$P \leq 3.90 \times 10^3 \text{ N}$$

$$\tau = \frac{P}{2A} \sin 50^\circ$$

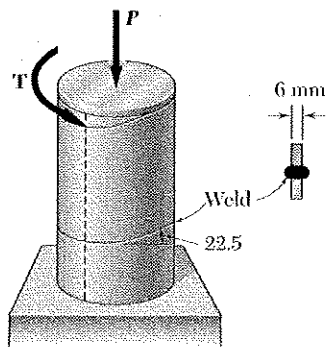
$$P = \frac{2A\tau}{\sin 50^\circ} \leq \frac{(2)(4 \times 10^{-3})(600 \times 10^3)}{\sin 50^\circ} = 6.27 \times 10^3 \text{ N}$$

Choosing the smaller value,

$$P = 3.90 \text{ kN} \quad \blacktriangle$$

Problem 7.42

7.42 Solve Prob. 7.20, using Mohr's circle.



7.20 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 160-kN axial force P and an 800 N · m torque T , each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

$$d_2 = 0.3 \text{ m} \quad C_2 = \frac{1}{2} d_2 = 0.15 \text{ m}, \quad t = 0.006 \text{ m}$$

$$C_1 = C_2 - t = 0.144 \text{ m}$$

$$A = \pi (C_2^2 - C_1^2) = \pi (0.15^2 - 0.144^2) = 5541 \times 10^{-6} \text{ m}^2$$

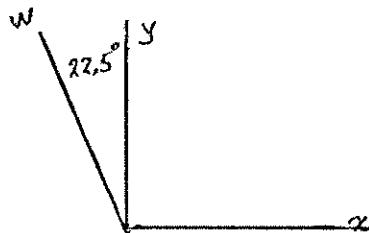
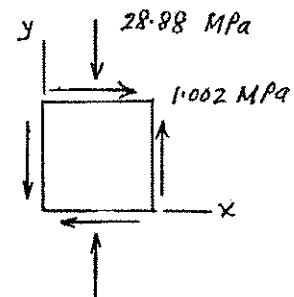
$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (0.15^4 - 0.144^4) = 119.8 \times 10^{-6} \text{ m}^4$$

Stresses

$$\sigma = -\frac{P}{A} = -\frac{160 \times 10^3}{5541 \times 10^{-6}} = -28.88 \text{ MPa}$$

$$\tau = \frac{T C_2}{J} = \frac{(800)(0.15)}{119.8 \times 10^{-6}} = 1002 \text{ MPa}$$

$$\sigma_x = 0, \quad \sigma_y = -28.88 \text{ MPa}, \quad \tau_{xy} = 1002 \text{ MPa}$$

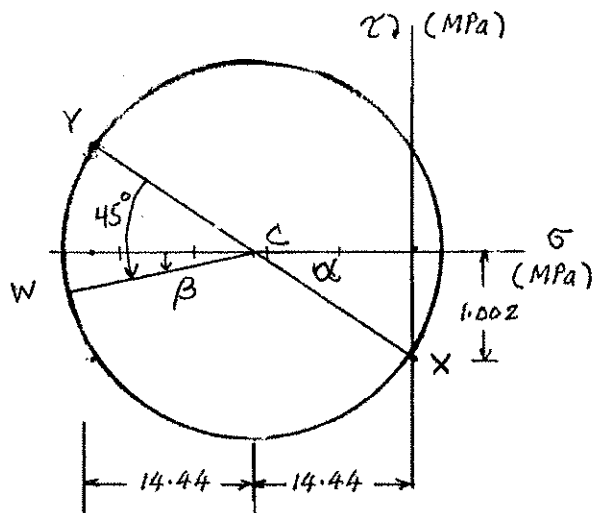


Draw the Mohr's circle.

$$X: (0, -1002 \text{ MPa})$$

$$Y: (-28.88 \text{ MPa}, 1002 \text{ MPa})$$

$$C: (-14.44, 0)$$



$$\tan \alpha = \frac{1002}{14.44} = 0.06939$$

$$\alpha = 4.0^\circ$$

$$\beta = (2)(22.5^\circ) - \alpha = 41^\circ$$

$$R = \sqrt{(14.44)^2 + (1002)^2} = 14.47 \text{ MPa}$$

$$\sigma_w = -14.44 - 14.47 \cos 41^\circ$$

$$\sigma_w = -25.4 \text{ MPa}$$

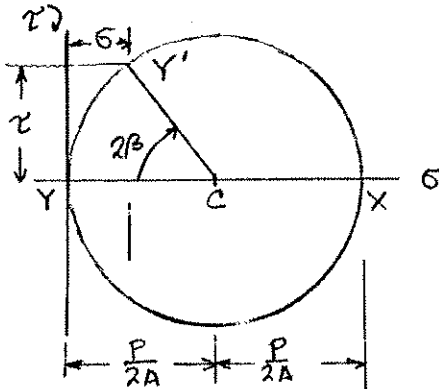
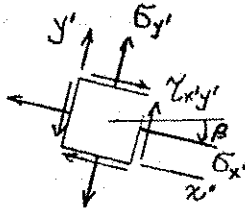
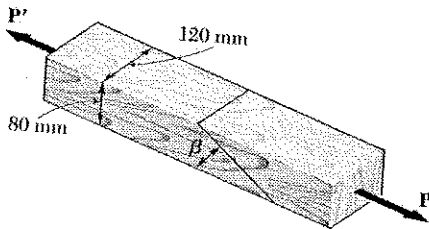
$$\tau_w = -14.47 \sin 41^\circ$$

$$\tau_w = -9.5 \text{ MPa}$$

Problem 7.43

7.43 Solve Prob. 7.21, using Mohr's circle.

7.21 Two wooden members of 80×120 -mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25^\circ$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (a) the in-plane shearing stress parallel to the splice, (b) the normal stress perpendicular to the splice.



$$\sigma_x = \frac{P}{A}, \quad \sigma_y = 0, \quad \tau_{xy} = 0$$

Plotted points for Mohr's circle.

$$X: (P/A, 0), \quad Y: (0, 0)$$

$$C: (P/2A, 0)$$

$$R = CX = \frac{P}{2A}$$

Coordinates of point Y' .

$$\sigma = \frac{P}{2A} (1 - \cos 2\beta), \quad \tau = \frac{P}{2A} \sin 2\beta$$

$$\text{Data: } A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

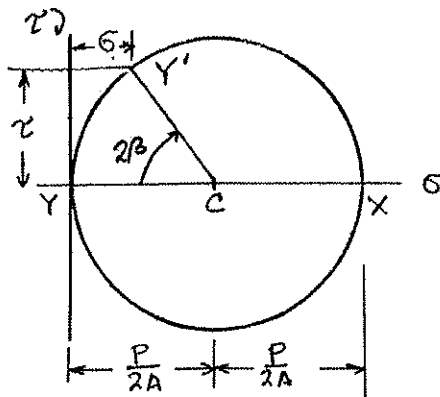
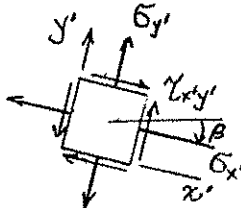
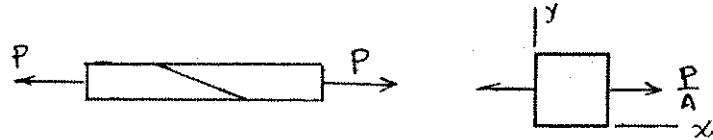
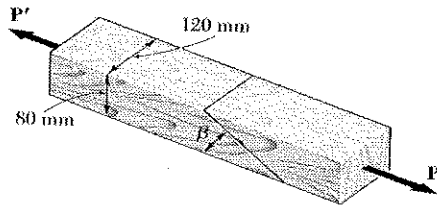
$$(a) \quad \tau = \frac{(10 \times 10^3) \sin 50^\circ}{(2)(9.6 \times 10^{-3})} = 399 \times 10^3 \text{ Pa} = 399 \text{ kPa} \quad \leftarrow$$

$$(b) \quad \sigma = \frac{(10 \times 10^3)(1 - \cos 50^\circ)}{(2)(9.6 \times 10^{-3})} = 186.0 \times 10^3 \text{ Pa} = 186.0 \text{ kPa} \quad \leftarrow$$

Problem 7.44

7.44 Solve Prob. 7.22, using Mohr's circle.

7.22 Two wooden members of 80×120 -mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22^\circ$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice) and 600 kPa in shear (parallel to the splice), determine the largest centric load P that can be applied.



$$\sigma_x = \frac{P}{A}, \quad \sigma_y = 0, \quad \tau_{xy} = 0$$

Plotted points for Mohr's circle.

$$X: \left(\frac{P}{A}, 0\right), \quad Y: (0, 0)$$

$$C: \left(\frac{P}{2A}, 0\right)$$

$$R = CX = \frac{P}{2A}$$

Coordinates of point Y'

$$\sigma = \frac{P}{2A}(1 - \cos 2\beta), \quad \tau = \frac{P}{2A} \sin 2\beta$$

$$\text{Data: } A = (80)(120) = 9.6 \times 10^3 \text{ mm}^2 = 9.6 \times 10^{-3} \text{ m}^2$$

$$\text{If } \sigma = 400 \text{ kPa} = 400 \times 10^3 \text{ Pa},$$

$$P = \frac{2A\sigma}{1 - \cos 2\beta} = \frac{(2)(9.6 \times 10^{-3})(400 \times 10^3)}{(1 - \cos 44^\circ)}$$

$$= 27.4 \times 10^3 \text{ N} = 27.4 \text{ kN}$$

$$\text{If } \tau = 600 \text{ kPa} = 600 \times 10^3 \text{ Pa},$$

$$P = \frac{2A\tau}{\sin 2\beta} = \frac{(2)(9.6 \times 10^{-3})(600 \times 10^3)}{\sin 44^\circ}$$

$$= 16.58 \times 10^3 \text{ N} = 16.58 \text{ kN}$$

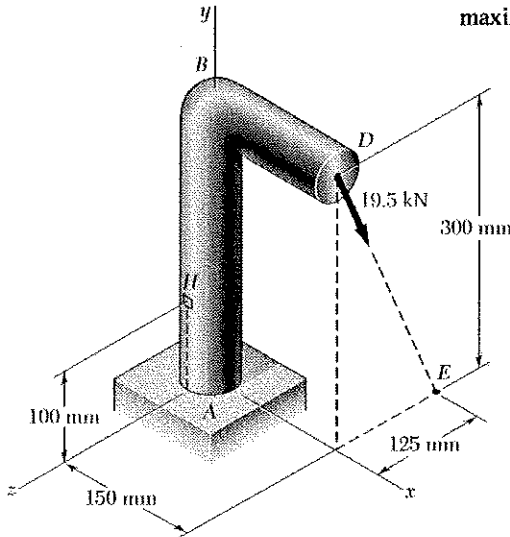
The smaller value of P governs.

$$P = 16.58 \text{ kN}$$

Problem 7.45

7.45 Solve Prob. 7.23, using Mohr's circle.

7.23 A 19.5-kN force is applied at point D of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point H.



$$l_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

Resolve the 19.5 kN force F at point D into x, y, and z components.

$$F_x = 0$$

$$F_y = -\frac{300}{325} (19.5) = -18 \text{ kN} = -18 \times 10^3 \text{ N}$$

$$F_z = -\frac{125}{325} (19.5) = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

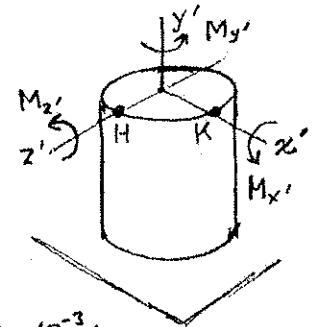
Determine the force-couple system at the point on the y-axis where it intersects the plane containing elements H and K.

$$F_x' = 0, \quad F_y' = -18 \times 10^3 \text{ N}, \quad F_z' = -7.5 \times 10^3 \text{ N}$$

$$M_x' = -(7.5 \text{ kN})(300 \text{ mm} - 100 \text{ mm}) = -1.5 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y' = (7.5 \text{ kN})(150 \text{ mm}) = 1.125 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_z' = -(18 \text{ kN})(150 \text{ mm}) = -2.7 \times 10^3 \text{ N}\cdot\text{m}$$



Properties of section. (Circle) $c = \frac{1}{2}d = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$

$$A = \pi c^2 = \pi (30)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-6} \text{ m}^2$$

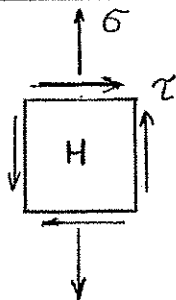
$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (30)^4 = 636.17 \times 10^3 \text{ mm}^4 = 636.17 \times 10^{-9} \text{ m}^4$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (30)^4 = 1.27235 \times 10^6 \text{ mm}^4 = 1.27235 \times 10^{-6} \text{ m}^4$$

$$\text{(Semicircle)} \quad Q = \frac{2}{3} c^3 = \frac{2}{3} (30)^3 = 18 \times 10^3 \text{ mm}^3 = 18 \times 10^{-6} \text{ m}^3$$

$$t = d = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

Stresses at H.



$$\sigma = \frac{F_y}{A} - \frac{M_x c}{I} = \frac{-18 \times 10^3}{2.8274 \times 10^{-3}} - \frac{(-1.5 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^{-9}}$$

$$= 64.370 \times 10^6 \text{ Pa} = 64.37 \text{ MPa}$$

$$\tau = \frac{F_z Q}{I t} + \frac{M_y c}{J} = 0 + \frac{(1.125 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}}$$

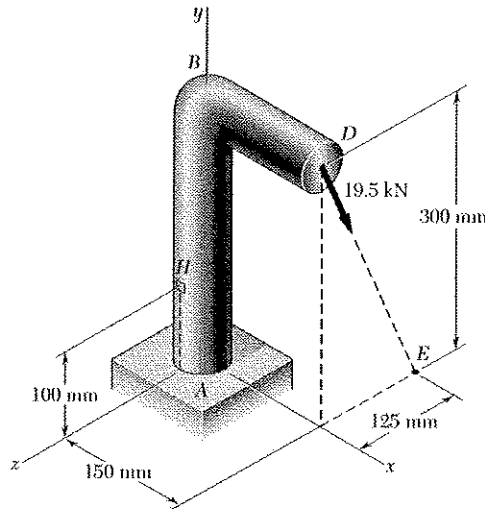
$$= 26.526 \times 10^6 \text{ Pa} = 26.526 \text{ MPa}$$

continued after Prob. 7.46

Problem 7.46

7.46 Solve Prob. 7.24, using Mohr's circle.

7.24 A 19.5-kN force is applied at point D of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point K.



$$l_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

Resolve the 19.5 kN force F at point D into x, y, and z components.

$$F_x = 0$$

$$F_y = -\frac{300}{325} (19.5) = -18 \text{ kN} = -18 \times 10^3 \text{ N}$$

$$F_z = -\frac{125}{325} (19.5) = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

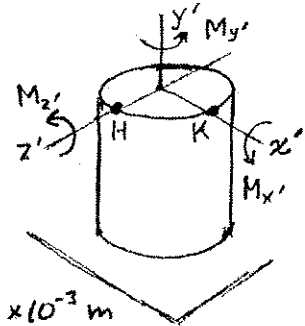
Determine the force-couple system at the point on the y-axis where it intersects the plane containing elements H and K.

$$F_x' = 0, \quad F_y' = -18 \times 10^3 \text{ N}, \quad F_z' = -7.5 \times 10^3 \text{ N}$$

$$M_x' = -(7.5 \text{ kN})(300 \text{ mm} - 100 \text{ mm}) = +1.5 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y' = (7.5 \text{ kN})(150 \text{ mm}) = 1.125 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_z' = -(18 \text{ kN})(150 \text{ mm}) = -2.7 \times 10^3 \text{ N}\cdot\text{m}$$



Properties of section. (Circle) $c = \frac{1}{2}d = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$

$$A = \pi c^2 = \pi (30)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

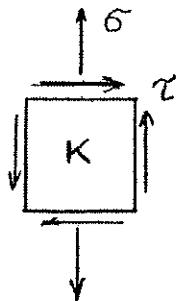
$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (30)^4 = 636.17 \times 10^3 \text{ mm}^4 = 636.17 \times 10^{-9} \text{ m}^4$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (30)^4 = 1.27235 \times 10^6 \text{ mm}^4 = 1.27235 \times 10^{-6} \text{ m}^4$$

$$\text{(Semicircle)} \quad Q = \frac{2}{3} c^3 = \frac{2}{3} (30)^3 = 18 \times 10^3 \text{ mm}^3 = 18 \times 10^{-6} \text{ m}^3$$

$$t = d = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

Stresses at K.



$$\sigma = \frac{F_y}{A} + \frac{M_y c}{I} = \frac{-18 \times 10^3}{2.8274 \times 10^{-3}} + \frac{(-2.7 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^{-9}}$$

$$= -133.69 \times 10^6 \text{ Pa} = -133.69 \text{ MPa}$$

$$\tau = -\frac{F_z Q}{I t} + \frac{M_z c}{J} = -\frac{(-7.5 \times 10^3)(18 \times 10^{-6})}{(636.17 \times 10^{-9})(60 \times 10^{-3})} + \frac{(1.125 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}}$$

$$= 3.5368 \times 10^6 + 26.526 \times 10^6 \text{ Pa}$$

$$= 30.06 \times 10^6 \text{ Pa} = 30.06 \text{ MPa}$$

continued

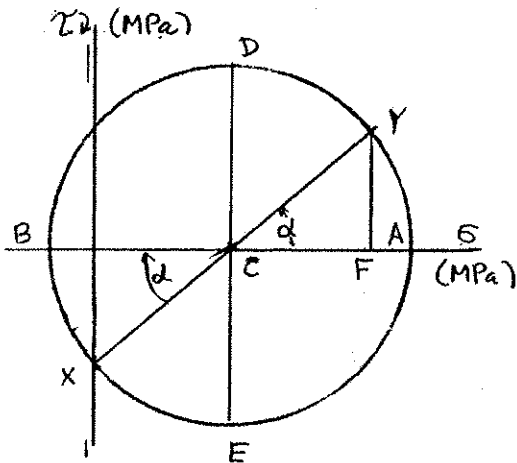
Problem 7.45 continued

Plotted points for Mohr's circle.

$$X: (\sigma_x, -\tau_{xy}) = (0, -26.53 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (64.37 \text{ MPa}, 26.53 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (32.19 \text{ MPa}, 0)$$



$$R = \sqrt{CF^2 + FY^2} = \sqrt{32.19^2 + 26.53^2} = 41.71 \text{ MPa}$$

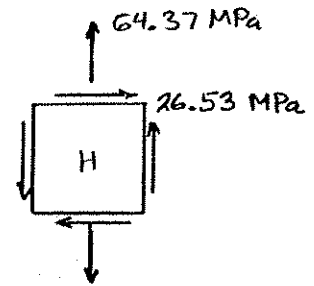
$$\tan \alpha = \frac{FY}{CF} = \frac{26.53}{32.19} = 0.8242$$

$$\alpha = 39.49^\circ \quad \theta_b = 19.7^\circ \quad \theta_a = 70.3^\circ$$

$$\sigma_a = \sigma_{ave} + R = 73.9 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -9.5 \text{ MPa}$$

$$\tau_{max} = 7 \quad \tau_{max} = 41.7 \text{ MPa}$$



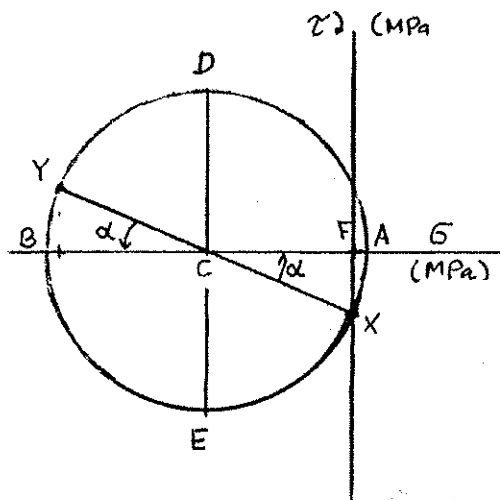
Problem 7.46 continued

Plotted points for Mohr's circle.

$$X: (\sigma_x, -\tau_{xy}) = (0, -30.06 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (-133.69 \text{ MPa}, 30.06 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (-66.85 \text{ MPa}, 0)$$



$$R = \sqrt{CF^2 + FY^2} = \sqrt{66.85^2 + 30.06^2} = 73.3 \text{ MPa}$$

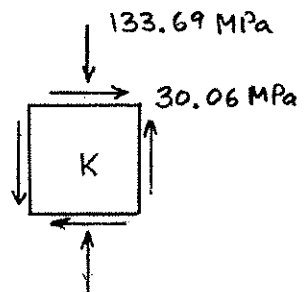
$$\tan \alpha = \frac{FY}{CF} = \frac{30.06}{66.85} = 0.4497$$

$$\alpha = 24.2^\circ \quad \theta_a = 12.1^\circ \quad \theta_b = 177.9^\circ$$

$$\sigma_a = \sigma_{ave} + R = 6.45 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -140.2 \text{ MPa}$$

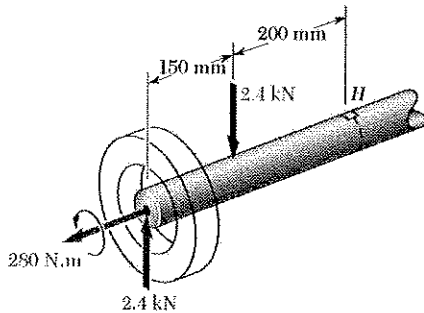
$$\tau_{max} = R = 73.3 \text{ MPa}$$



Problem 7.47

7.47 Solve Prob. 7.25, using Mohr's circle.

7.25 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 30 mm, determine (a) the principal planes and principal stresses at point *H* located on top of the axle, (b) the maximum shearing stress at the same point.

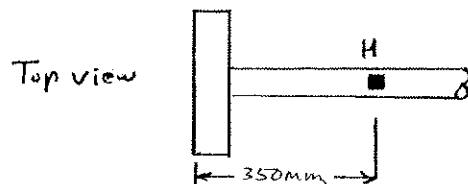


$$c = \frac{1}{2}d = \frac{1}{2}(30) = 15 \text{ mm}$$

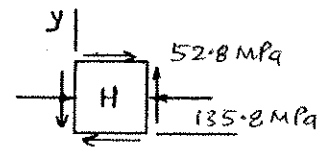
$$\text{Torsion: } \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(280)}{\pi(15)^3} = 52.8 \text{ MPa}$$

$$I = \frac{\pi c^4}{4} = 39761 \text{ mm}^4$$

$$M = (150)(2400) = 360000 \text{ Nmm} \quad \sigma = -\frac{My}{I} = -\frac{(360000)(15)}{39761} = -135.8 \text{ MPa}$$

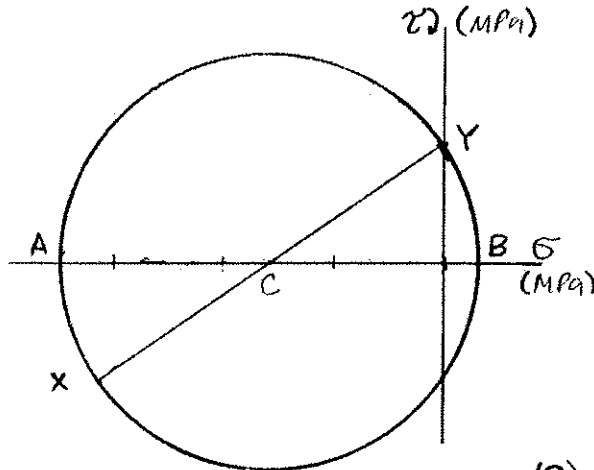


Stresses



$$\sigma_x = -135.8 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 52.8 \text{ MPa}$$

Plotted points. $X: (-135.8, -52.8); Y: (0, 52.8); C(-67.9, 0)$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -67.9 \text{ MPa}$$

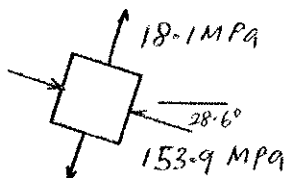
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-135.8}{2}\right)^2 + (52.8)^2} = 86 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(52.8)}{-135.8} = -1.5552$$

$$(a) \quad \theta_a = 28.6^\circ, \quad \theta_b = 61.4^\circ$$

$$\sigma_a = \sigma_{ave} - R = -67.9 - 86 = -153.9 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} + R = -67.9 + 86 = 18.1 \text{ MPa}$$

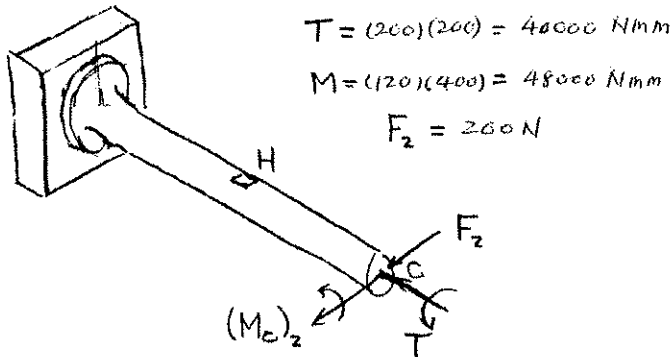


$$(b) \quad \tau_{max} = R = 86 \text{ MPa}$$

Problem 7.48

7.48 Solve Prob. 7.26, using Mohr's circle.

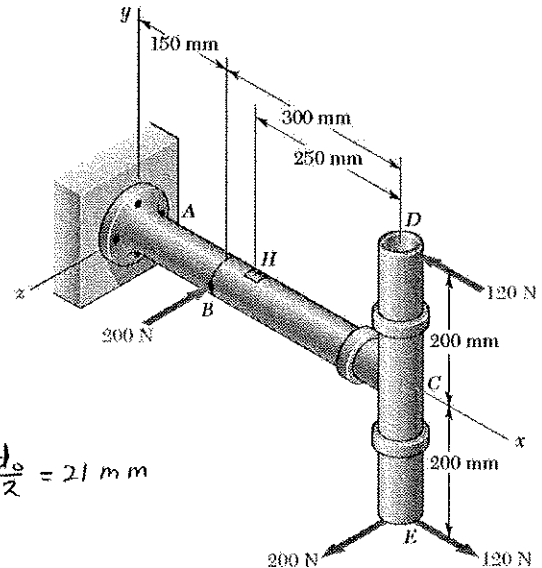
Replace forces on pipe DCE by an equivalent force-couple system at C.



$$T = (200)(200) = 40000 \text{ Nmm}$$

$$M = (120)(400) = 48000 \text{ Nmm}$$

$$F_2 = 200 \text{ N}$$



Cross section. $C_1 = \frac{d_i}{2} = 19 \text{ mm}$ $C_2 = \frac{d_o}{2} = 21 \text{ mm}$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 100782 \text{ mm}^4$$

$$I = \frac{1}{2}J = 50391 \text{ mm}^4$$

$$Q_y = \frac{2}{3}(C_2^3 - C_1^3) = \frac{2}{3}(21^3 - 19^3) = 1601 \text{ mm}^3$$

$$t = C_2 - C_1 = 2 \text{ mm}$$



At the section containing element H

$$T = 40000 \text{ Nmm}, \quad M_z = 48000 \text{ Nmm}, \quad V_z = 200 \text{ N}$$

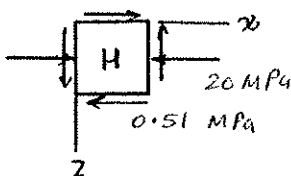
Stresses.

Torsion: $\tau_{zx} = -\frac{T C}{J} = -\frac{(40000)(21)}{100782} = -8.3 \text{ MPa}$

Bending: $\sigma_x = -\frac{M_z y}{I} = -\frac{(48000)(21)}{50391} = -20 \text{ MPa}$

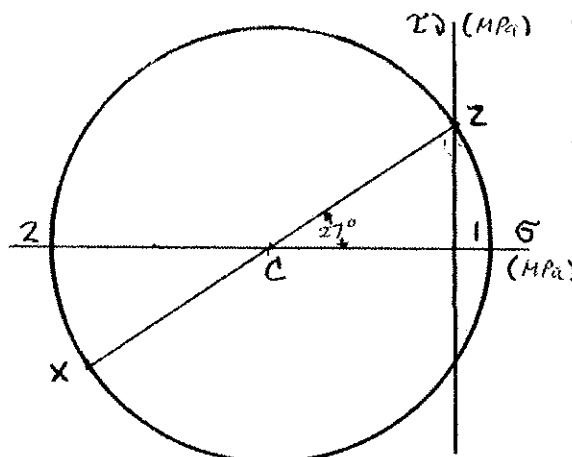
Transverse Shear: $\tau_{zx} = \frac{V_z Q_y}{I(2t)} = \frac{(200)(1601)}{(50391)(2)} = 3.2 \text{ MPa}$

Total: $\sigma_z = 0, \quad \sigma_x = -20, \quad \tau_{zx} = -8.3 + 3.2 = -5.1 \text{ MPa}$



$$\theta_1 = 13.5^\circ$$

$$\theta_2 = 76.5^\circ$$



Plotted points
Z: $(0, 5.1)$
X: $(-20, -5.1)$
C: $(-10, 0)$

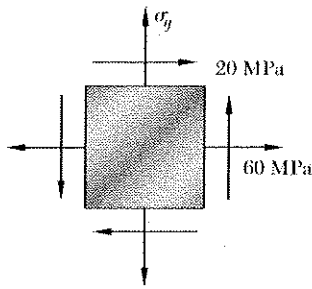
$$\sigma_1 = 1.2 \text{ MPa}$$

$$\sigma_2 = -21.2 \text{ MPa}$$

$$\tau_{max} = R = 11.2 \text{ MPa}$$

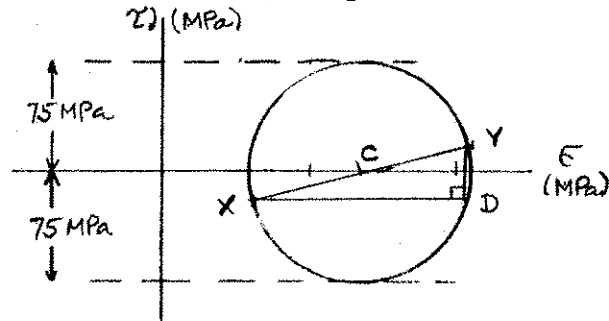
Problem 7.49

7.49 Solve Prob. 7.27, using Mohr's circle.



7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$



$$\text{Given: } \tau_{\max} = R = 75 \text{ MPa}$$

$$\overline{XY} = 2R = 150 \text{ MPa}$$

$$\overline{DY} = (2)(\tau_{xy}) = 40 \text{ MPa}$$

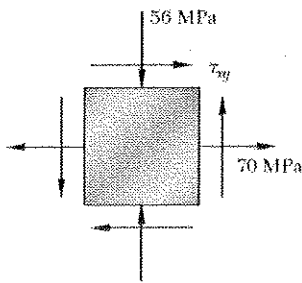
$$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{150^2 - 40^2} = 144.6 \text{ MPa}$$

$$\sigma_y = \sigma_x + \overline{XD} = 60 + 144.6$$

$$\sigma_y = 205 \text{ MPa} \quad \blacktriangleleft$$

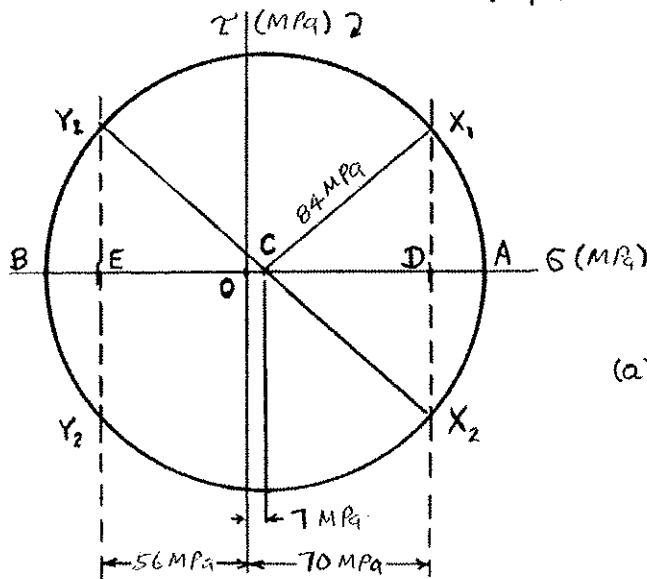
Problem 7.50

7.50 Solve Prob. 7.28, using Mohr's circle.



7.28 For the state of plane stress shown, determine (a) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 84 MPa, (b) the corresponding principal stresses.

The center of the Mohr's circle lies at point C with coordinates $(\frac{\sigma_x + \sigma_y}{2}, 0) = (\frac{10 - 56}{2}, 0) = (-7, 0)$. The radius of the circle is $\tau_{\max(\text{in-plane})} = 84 \text{ MPa}$



The stress point $(\sigma_x, -\tau_{xy})$ lie along the line X_1X_2 of the Mohr circle diagram. The extreme points with $R \leq 84 \text{ MPa}$ are X_1 and X_2 .

(a) The largest allowable value of τ_{xy} is obtained from triangle CDX_1 ,

$$\overline{DX_1}^2 = \overline{DX_2}^2 = \overline{CX_1}^2 - \overline{CD}^2$$

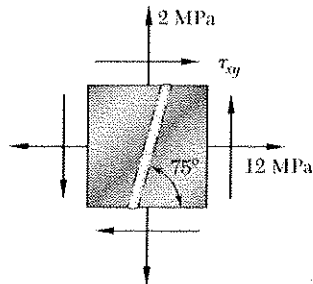
$$\tau_{xy} = \sqrt{84^2 - 63^2} = 55.6 \text{ MPa} \quad \blacktriangleleft$$

(b) The principal stresses are $\sigma_a = 7 + 84 = 91 \text{ MPa} \quad \blacktriangleleft$

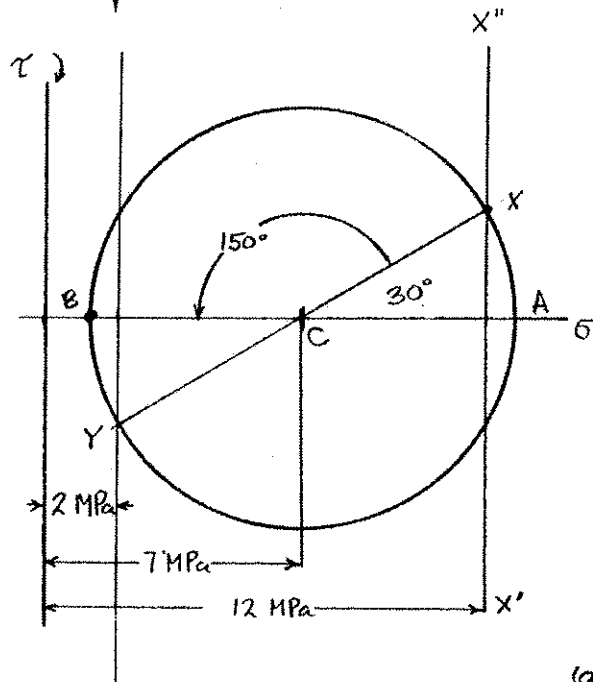
$$\sigma_b = 7 - 84 = -77 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.51

7.51 Solve Prob. 7.29, using Mohr's circle.



7.29 For the state of plane stress shown, determine (a) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (b) the corresponding principal stresses.



Point X of Mohr's circle must lie on $X'X''$ so that $\sigma_x = 12 \text{ MPa}$. Likewise, point Y lies on line $Y'Y''$ so that $\sigma_y = 2 \text{ MPa}$. The coordinates of C are

$$\frac{2+12}{2}, 0 = (7 \text{ MPa}, 0).$$

Counterclockwise rotation through 150° brings line CX to CB, where $\tau = 0$.

$$\begin{aligned} R &= \frac{\sigma_x - \sigma_y}{2} \sec 30^\circ \\ &= \frac{12 - 2}{2} \sec 30^\circ \\ &= 5.77 \text{ MPa} \end{aligned}$$

$$\begin{aligned} (a) \quad \tau_{xy} &= -\frac{\sigma_x - \sigma_y}{2} \tan 30^\circ \\ &= -\frac{12 - 2}{2} \tan 30^\circ \\ \tau_{xy} &= -2.89 \text{ MPa} \end{aligned}$$

$$(b) \quad \sigma_a = \sigma_{ave} + R = 7 + 5.77$$

$$\sigma_a = 12.77 \text{ MPa}$$

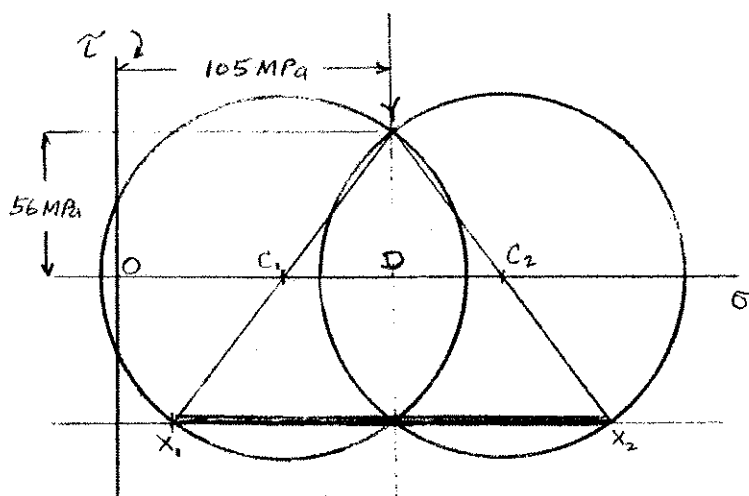
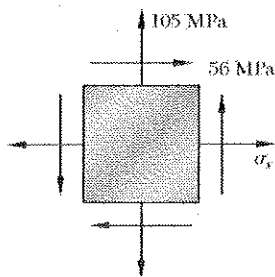
$$\sigma_b = \sigma_{ave} - R = 7 - 5.77$$

$$\sigma_b = 1.23 \text{ MPa}$$

Problem 7.52

7.52 Solve Prob. 7.30, using Mohr's circle.

7.30 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 70 MPa.



For the Mohr's circle, point Y lies at (105 MPa, 56 MPa).

The radius of limiting circles is $R = 70$ MPa

Let C_1 be the location of the left most limiting circle and C_2 be that of the right most one.

$$\overline{C_1 Y} = 70 \text{ MPa}$$

$$\overline{C_2 Y} = 70 \text{ MPa}$$

Noting right triangles $C_1 D Y$ and $C_2 D Y$

$$\overline{C_1 D}^2 + \overline{D Y}^2 = \overline{C_1 Y}^2 \quad \overline{C_2 D}^2 + 56^2 = 70^2 \quad \overline{C_1 D} = 42 \text{ MPa}$$

Coordinates of point C_1 are $(0, 105 - 42) = (0, 63 \text{ MPa})$

Likewise, coordinates of point C_2 are $= (0, 105 + 42) = (0, 147 \text{ MPa})$

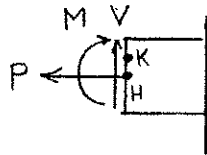
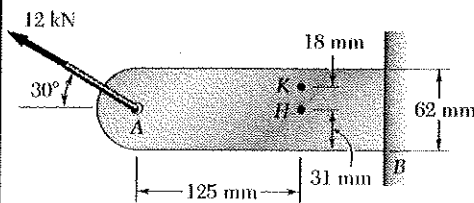
Coordinates of point X_1 $(133 - 42, -56) = (21 \text{ MPa}, -56 \text{ MPa})$

Coordinates of point X_2 $(144 + 42, -56) = (186 \text{ MPa}, -56 \text{ MPa})$

The point $(\sigma_x, -\tau_{xy})$ must lie on the line $X_1 X_2$

$$\text{Thus} \quad 21 \text{ MPa} \leq \sigma_x \leq 186 \text{ MPa}$$

Problem 7.53



7.53 Knowing that the bracket AB has a uniform thickness of 15 mm, determine (a) the principal planes and principal stresses at point H, (b) the maximum shearing stress at point H.

Resolve the 12 kN force F at point A into x and y components.

$$F_x = -F \cos 30^\circ = -(12) \cos 30^\circ = -10.39 \text{ kN}$$

$$F_y = F \sin 30^\circ = (12) \sin 30^\circ = 6 \text{ kN}$$

At the section containing points H and K,

$$P = -F_x = 10.39 \text{ kN}, \quad V = F_y = 6 \text{ kN}$$

$$M = (0.125)(6000) = 750 \text{ N m}$$

Section properties, $t = 0.015 \text{ m}$,

$$A = (0.015)(0.062) = 93 \times 10^{-5} \text{ m}^2$$

$$I = \frac{1}{12}(0.015)(0.062)^3 = 297 \times 10^{-9} \text{ m}^4, \quad c = 0.031 \text{ m}$$

At point H, $y = 0$,

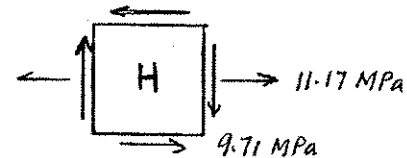
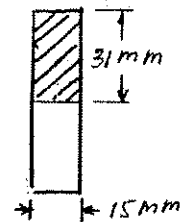
$$Q = (0.015)(0.031)\left(\frac{0.031}{2}\right) = 7.207 \times 10^{-6} \text{ m}^3$$

Stresses at point H.

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{10.39 \times 10^3}{93 \times 10^{-5}} + 0 = 11.17 \text{ MPa}$$

$$\tau = \frac{VQ}{IE} = \frac{(6000)(7.207 \times 10^{-6})}{(297 \times 10^{-9})(0.015)} = 9.71 \text{ MPa}$$

$$\sigma_x = 11.17 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = -9.71 \text{ MPa}$$



Mohr's circle

$$X: (\sigma_x, -\tau_{xy}) = (11.17 \text{ MPa}, 9.71 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (0, -9.71 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (5.585 \text{ MPa}, 0)$$

$$\frac{\sigma_x - \sigma_y}{2} = 5.585$$

$$R = \sqrt{5.585^2 + 9.71^2} = 11.2 \text{ MPa}$$

$$\tan 2\theta_p = \frac{9.71}{5.585} = 1.7386 \quad 2\theta_p = 60.0^\circ \Rightarrow \theta_p = 30.0^\circ$$

$$(a) \quad \sigma_{max} = \sigma_{ave} + R = 5.585 + 11.2$$

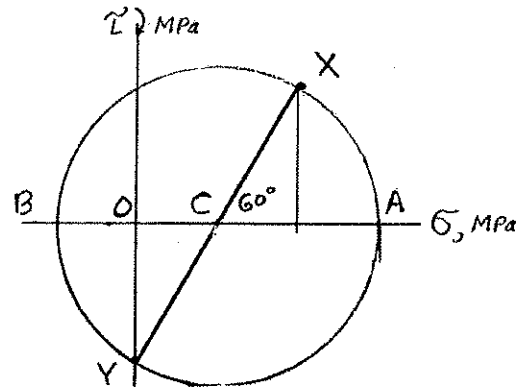
$$\sigma_{min} = \sigma_{ave} - R = 5.585 - 11.2$$

$$(b) \quad \tau_{max} = R = 11.2 \text{ MPa}$$

$$\sigma_{max} = 16.8 \text{ MPa at } 30.0^\circ$$

$$\sigma_{min} = -5.6 \text{ MPa at } 60.0^\circ$$

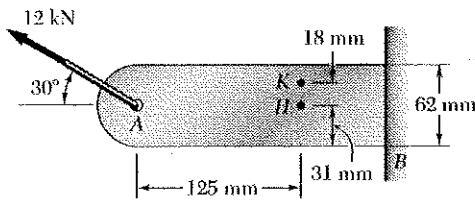
$$\tau_{max} = 11.2 \text{ MPa}$$



Problem 7.54

7.54 Solve Prob. 7.53, considering point K.

7.53 Knowing that the bracket AB has a uniform thickness of 15 mm, determine (a) the principal planes and principal stresses at point H, (b) the maximum shearing stress at point H.



Resolve the 12 kN force F at point A into x and y components.

$$F_x = -F \cos 30^\circ = -(12) \cos 30^\circ = -10.39 \text{ kN}$$

$$F_y = F \sin 30^\circ = (12) \sin 30^\circ = 6 \text{ kN}$$

At the section containing points H and K,

$$P = -F_x = 10.39 \text{ kN}, \quad V = F_y = 6 \text{ kN}$$

$$M = (0.125)(600) = 750 \text{ Nm}$$

Section properties. $t = 0.015 \text{ m}$, $A = (0.015)(0.062) = 93 \times 10^{-5} \text{ m}^2$

$$I = \frac{1}{12}(0.015)(0.062)^3 = 297 \times 10^{-9} \text{ m}^4, \quad C = 0.031 \text{ m}$$

At point K, $y = 0.018 \text{ m}$

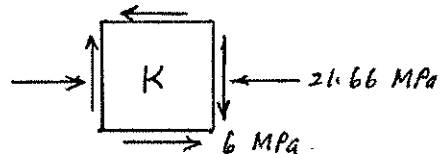
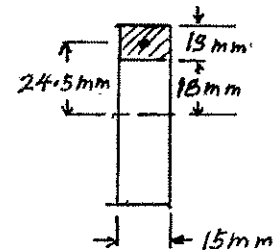
$$Q = (0.015)(0.013)(0.0245) = 4.777 \times 10^{-6} \text{ m}^3$$

Stresses at point K.

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{10390}{93 \times 10^{-5}} - \frac{(750)(0.018)}{297 \times 10^{-9}} = -34.28 \text{ MPa}$$

$$\tau = \frac{VQ}{It} = \frac{(6000)(4.777 \times 10^{-6})}{(297 \times 10^{-9})(0.015)} = 6.43 \text{ MPa}$$

$$\sigma_x = -21.66 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = -6.43 \text{ MPa}$$



Mohr's circle

$$X: (\sigma_x, -\tau_{xy}) = (-21.66 \text{ MPa}, 6.43 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (0, -6.43 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (-10.83 \text{ MPa}, 0)$$

$$\frac{\sigma_x - \sigma_y}{2} = -10.83 \text{ MPa}$$

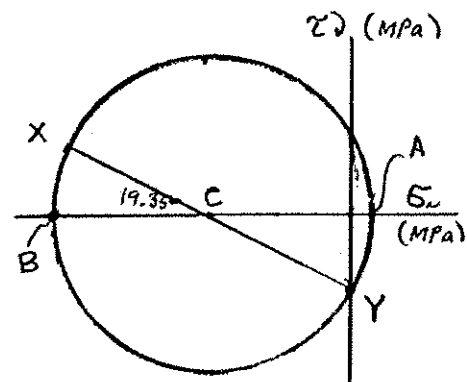
$$R = \sqrt{(10.83)^2 + (6.43)^2} = 12.81 \text{ MPa}$$

$$\tan 2\theta_p = \frac{6.43}{10.83} = 0.5937 \quad 2\theta_p = 30.56^\circ$$

$$(a) \quad \sigma_{max} = \sigma_{ave} + R = -10.83 + 12.81$$

$$(b) \quad \sigma_{min} = \sigma_{ave} - R = -10.83 - 12.81$$

$$\tau_{max} = R = 12.81 \text{ MPa}$$



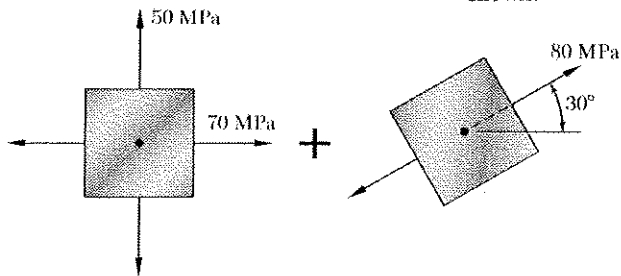
$$\sigma_{max} = 1.97 \text{ MPa at } 74.7^\circ$$

$$\sigma_{min} = -23.81 \text{ MPa at } 10.3^\circ$$

$$\tau_{max} = 12.8 \text{ MPa}$$

Problem 7.55

7.55 through 7.58 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

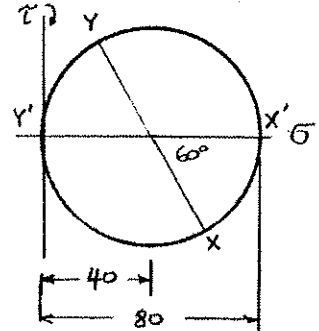


Mohr's circle for 2nd stress state

$$\sigma_x = 40 + 40 \cos 60^\circ = 60 \text{ MPa}$$

$$\sigma_y = 40 - 40 \cos 60^\circ = 20 \text{ MPa}$$

$$\tau_{xy} = 40 \sin 60^\circ = 34.64 \text{ MPa}$$

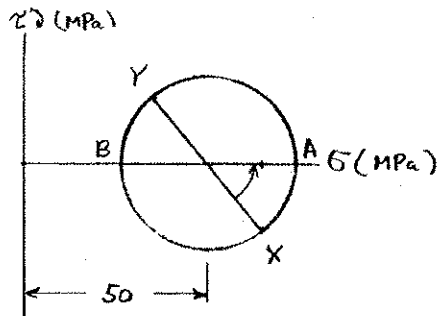


Resultant stresses

$$\sigma_x = 70 + 60 = 130 \text{ MPa}$$

$$\sigma_y = 50 + 20 = 70 \text{ MPa}$$

$$\tau_{xy} = 0 + 34.64 = 34.64 \text{ MPa}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(130 + 70) = 100 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(34.64)}{130 - 70} = 1.1547$$

$$2\theta_p = 49.11^\circ \quad \theta_a = 24.6^\circ \quad \theta_b = 114.6^\circ$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 45.8 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R$$

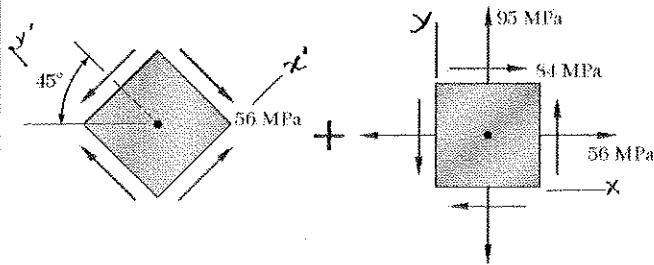
$$\sigma_a = 145.8 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R$$

$$\sigma_b = 54.2 \text{ MPa}$$

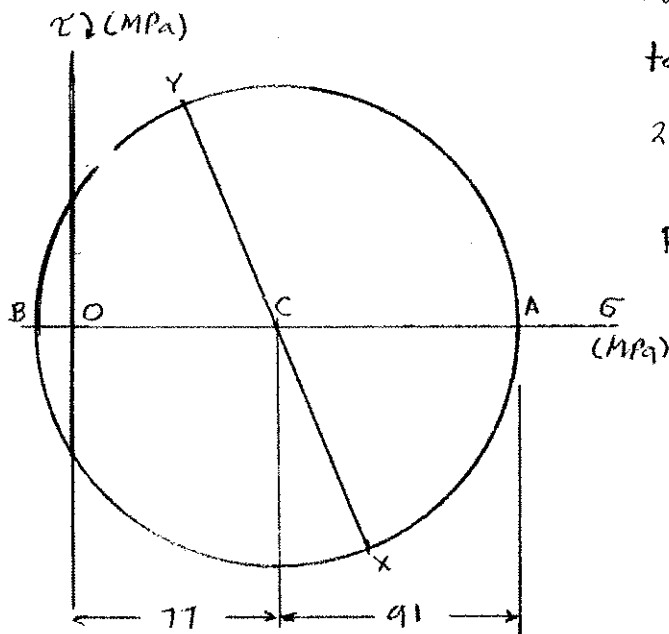
Problem 7.56

7.55 through 7.58 Determine the principal planes and the principal stress for the state of plane stress resulting from the superposition of the two states of stress shown.

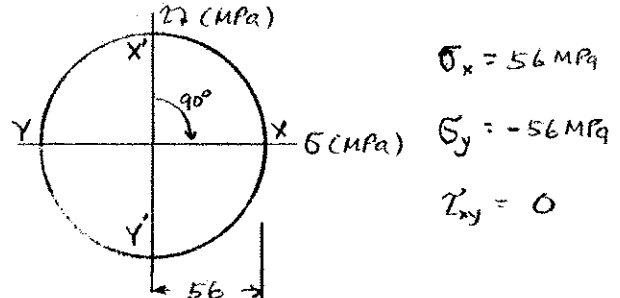


Resultant stresses

$$\begin{aligned}\sigma_x &= 56 + 56 = 112 \text{ MPa} \\ \sigma_y &= -56 + 98 = 42 \text{ MPa} \\ \tau_{xy} &= 84 + 0 = 84 \text{ MPa}\end{aligned}$$



Mohr's circle for 1st stress state.



$$\sigma_x = 56 \text{ MPa}$$

$$\sigma_y = -56 \text{ MPa}$$

$$\tau_{xy} = 0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 71 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(84)}{170} = 2.4$$

$$2\theta_p = 67.38^\circ \quad \theta_a = 33.69^\circ \quad \theta_b = 123.69^\circ$$

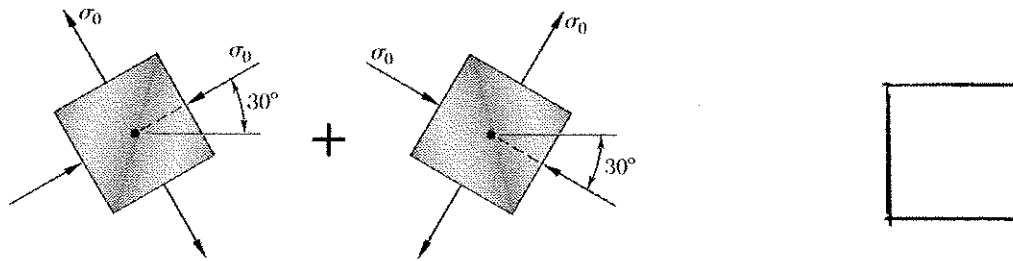
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{35^2 + 84^2} = 91 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 168 \text{ MPa}$$

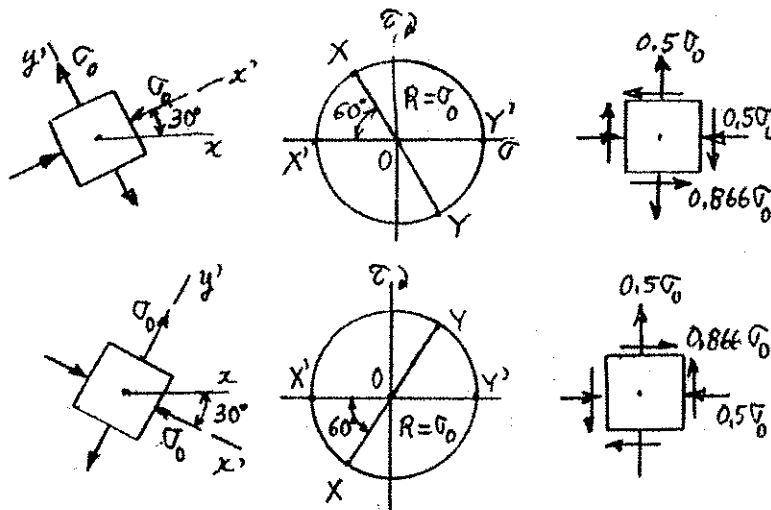
$$\sigma_b = \sigma_{ave} - R = -14 \text{ MPa}$$

Problem 7.57

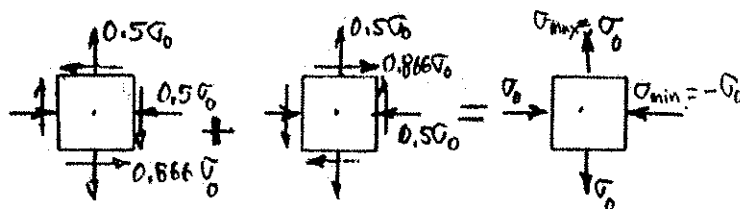
7.55 through 7.58 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



Express each state of stress in terms of components acting on the element shown at the right.



Add like components of the two states of stress.



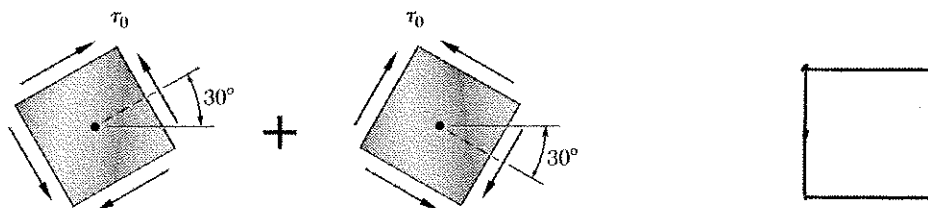
$$\theta_p = 0 \text{ and } 90^\circ$$

$$\sigma_{\max} = \sigma_0$$

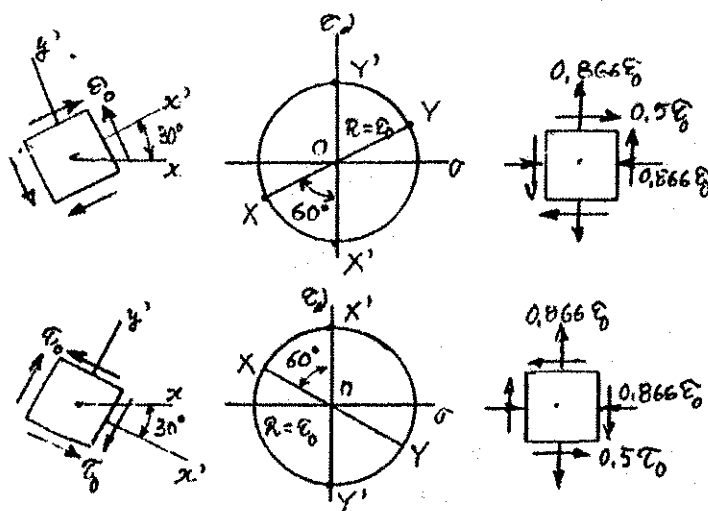
$$\sigma_{\min} = -\sigma_0$$

Problem 7.58

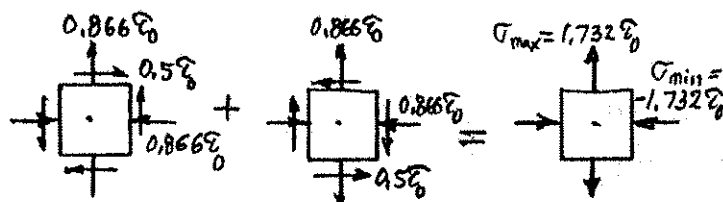
7.55 through 7.58 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



Express each state of stress in terms of components acting on the element shown at the right.



Add like components of the two states of stress.



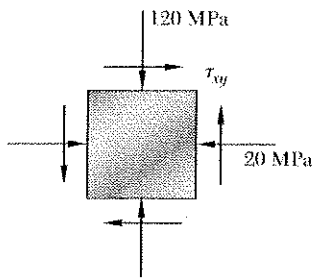
$$\theta_p = 0 \text{ and } 90^\circ$$

$$\sigma_{max} = 1.732 \sigma_0$$

$$\sigma_{min} = -1.732 \sigma_0$$

Problem 7.59

7.59 For the element shown, determine the range of values of τ_{xy} for which the maximum tensile stress is equal to or less than 60 MPa.



$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -70 \text{ MPa}$$

$$\text{Set } \sigma_{max} = 60 \text{ MPa} = \sigma_{ave} + R$$

$$R = \sigma_{max} - \sigma_{ave} = 130 \text{ MPa}$$

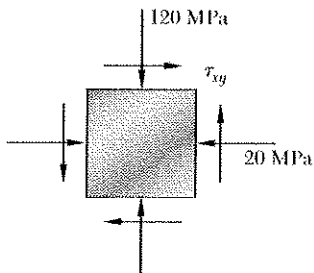
$$\text{But, } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{130^2 - 50^2} = 120 \text{ MPa}$$

$$\text{Range of } \tau_{xy}: \quad -120 \text{ MPa} \leq \tau_{xy} \leq 120 \text{ MPa}$$

Problem 7.60

7.60 For the element shown, determine the range of values of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 150 MPa.



$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = 50 \text{ MPa}$$

$$\text{Set } \tau_{max(in-plane)} = R = 150 \text{ MPa}$$

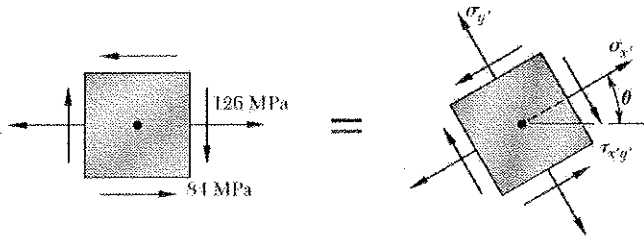
$$\text{But } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{150^2 - 50^2} = 141.4 \text{ MPa}$$

$$\text{Range of } \tau_{xy}: \quad -141.4 \text{ MPa} \leq \tau_{xy} \leq 141.4 \text{ MPa}$$

Problem 7.61

7.61 For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 140 MPa.



$$\sigma_x = 126 \text{ MPa}, \sigma_y = 0$$

$$\tau_{xy} = -84 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 63 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{63^2 + 84^2} = 105 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-84)}{126} = -\frac{4}{3}$$

$$2\theta_p = -53.13^\circ$$

$$\theta_a = -26.565^\circ$$

$\sigma_{x'} \leq 140 \text{ MPa}$ for states of stress corresponding to arc HBK of Mohr's circle. From the circle

$$R \cos 2\phi = 140 - 63 = 77 \text{ MPa}$$

$$\cos 2\phi = \frac{77}{105} = 0.73333$$

$$2\phi = 42.833^\circ \quad \phi = 21.417^\circ$$

$$\theta_H = \theta_a + \phi = -26.565^\circ + 21.417^\circ = -5.15^\circ$$

$$2\theta_K = 2\theta_H + 360^\circ - 4\phi = -10.297^\circ + 360^\circ - 85.666^\circ = 264.037^\circ$$

$$\theta_K = 132.02^\circ$$

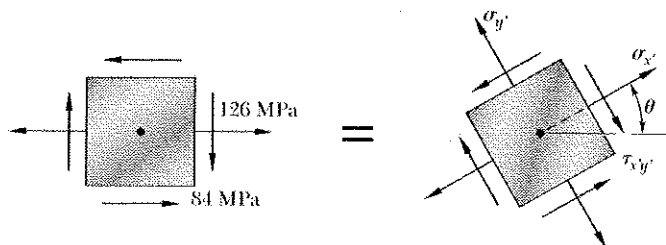
Permissible range of θ is $\theta_H \leq \theta \leq \theta_K$

$$-5.15^\circ \leq \theta \leq 132.02^\circ$$

$$\text{Also } 174.85^\circ \leq \theta \leq 312.02^\circ$$

Problem 7.62

7.62 For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 140 MPa.



$$\sigma_x = 126 \text{ MPa}, \sigma_y = 0$$

$$\tau_{xy} = -84 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 63 \text{ MPa}$$

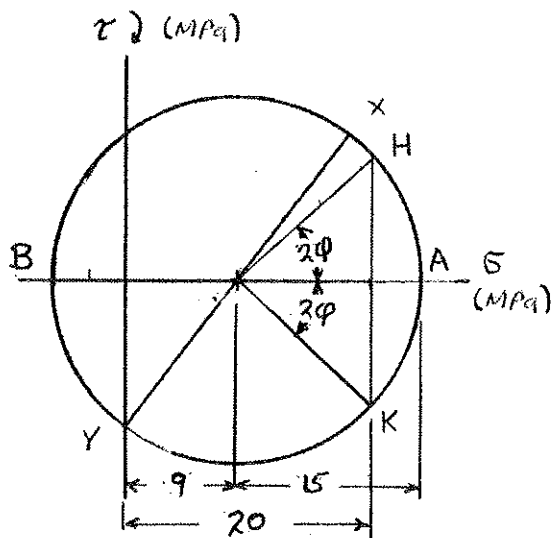
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{63^2 + 84^2} = 105 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-84)}{126} = -\frac{4}{3}$$

$$2\theta_p = -53.13^\circ$$

$$\theta_a = -26.565^\circ$$



$\sigma_{x'} \leq 140 \text{ MPa}$ for states of stress corresponding to arc HBK of Mohr's circle. From the circle

$$R \cos 2\phi = 140 - 63 = 77 \text{ MPa}$$

$$\cos 2\phi = \frac{77}{105} = 0.73333$$

$$2\phi = 42.833^\circ \quad \phi = 21.417^\circ$$

$$\theta_H = \theta_a + \phi = -26.565^\circ + 21.417^\circ = -5.15^\circ$$

$$2\theta_K = 2\theta_H + 360^\circ - 4\phi = -10.297^\circ + 360^\circ - 85.666^\circ = 264.037^\circ$$

$$\theta_K = 132.02^\circ$$

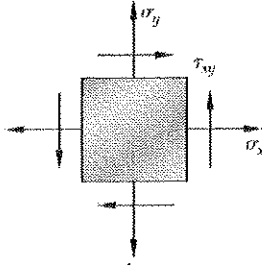
Permissible range of θ is $\theta_H \leq \theta \leq \theta_K$

$$-5.15^\circ \leq \theta \leq 132.02^\circ$$

$$\text{Also } 174.85^\circ \leq \theta \leq 312.02^\circ$$

Problem 7.63

7.63 For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 98 \text{ MPa}$, $\sigma_y = 63 \text{ MPa}$, and $\sigma_{\min} = 35 \text{ MPa}$. Determine (a) the orientation of the principal planes, (b) the principal stress $\sigma_{\max}$, (c) the maximum in-plane shearing stress.



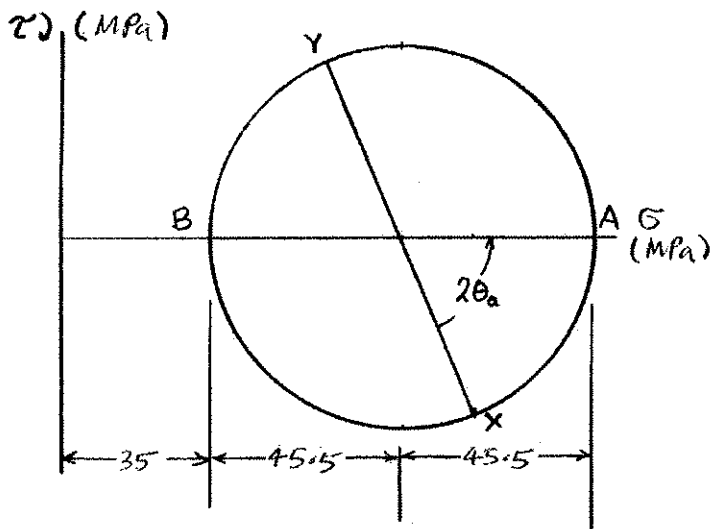
$$\sigma_x = 98 \text{ MPa}, \sigma_y = 63 \text{ MPa}, \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 80.5 \text{ MPa}$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \quad \therefore \quad R = \sigma_{\text{ave}} - \sigma_{\min} \\ = 80.5 - 35 = 45.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm \sqrt{45.5^2 - 17.5^2} = \pm 42 \text{ MPa}$$

But it is given that τ_{xy} is positive, thus $\tau_{xy} = +42 \text{ MPa}$



$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \\ = \frac{(2)(42)}{35} = 2.4$$

$$2\theta_p = 67.38^\circ$$

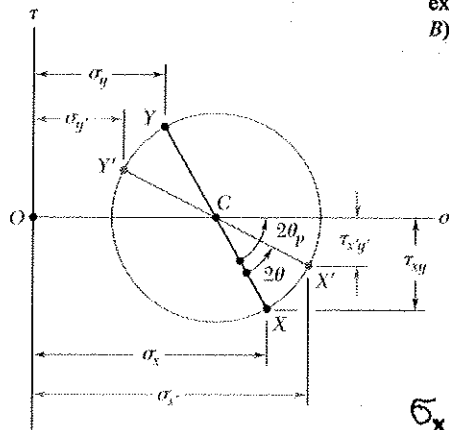
$$\theta_a = 33.69^\circ$$

$$\theta_b = 123.69^\circ$$

$$(b) \sigma_{\max} = \sigma_{\text{ave}} + R \\ = 126 \text{ MPa}$$

$$(c) \tau_{\max(\text{in-plane})} = R \\ = 45.5 \text{ MPa}$$

Problem 7.64



7.64 The Mohr's circle shown corresponds to the state of stress given in Fig. 7.5a and b. Noting that $\sigma_{x'} = OC + (CX) \cos(2\theta_p - 2\theta)$ and that $\tau_{x'y'} = (CX) \sin(2\theta_p - 2\theta)$, derive the expressions for $\sigma_{x'}$ and $\tau_{x'y'}$ given in Eqs. (7.5) and (7.6), respectively. [Hint: Use $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and $\cos(A + B) = \cos A \cos B - \sin A \sin B$.]

$$\overline{OC} = \frac{1}{2}(\sigma_x + \sigma_y) \quad \overline{CX'} = \overline{CX}$$

$$\overline{CX'} \cos 2\theta_p = \overline{CX} \cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2}$$

$$\overline{CX'} \sin 2\theta_p = \overline{CX} \sin 2\theta_p = \tau_{xy}$$

$$\sigma_{x'} = \overline{OC} + \overline{CX'} \cos(2\theta_p - 2\theta)$$

$$= \overline{OC} + \overline{CX'} (\cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta)$$

$$= \overline{OC} + \overline{CX'} \cos 2\theta_p \cos 2\theta + \overline{CX'} \sin 2\theta_p \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

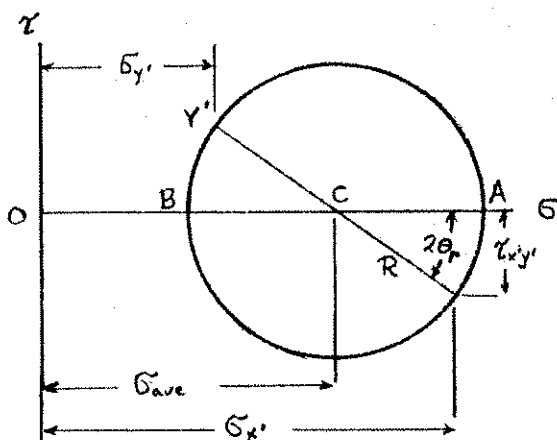
$$\tau_{x'y'} = \overline{CX'} \sin(2\theta_p - 2\theta) = \overline{CX'} (\sin 2\theta_p \cos 2\theta - \cos 2\theta_p \sin 2\theta)$$

$$= \overline{CX'} \sin 2\theta_p \cos 2\theta - \overline{CX'} \cos 2\theta_p \sin 2\theta$$

$$= \tau_{xy} \cos 2\theta - \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

Problem 7.65

7.65 (a) Prove that the expression $\sigma_x \sigma_y - \tau_{xy}^2$, where σ_x , σ_y , and τ_{xy} are components of the stress along the rectangular axes x' and y' , is independent of the orientation of these axes. Also, show that the given expression represents the square of the tangent drawn from the origin of the coordinates to Mohr's circle. (b) Using the invariance property established in part a, express the shearing stress τ_{xy} in terms of σ_x , σ_y , and the principal stresses σ_{max} and σ_{min} .



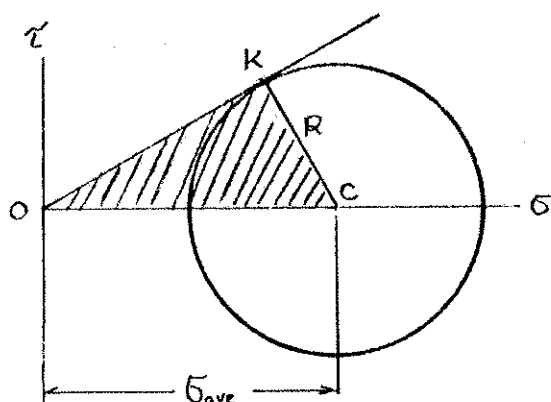
(a) From Mohr's circle,

$$\tau_{xy'} = R \sin 2\theta_p$$

$$\sigma_{x'} = \sigma_{ave} + R \cos 2\theta_p$$

$$\sigma_{y'} = \sigma_{ave} - R \cos 2\theta_p$$

$$\begin{aligned} \sigma_{x'} \sigma_{y'} - \tau_{xy'}^2 &= \sigma_{ave}^2 - R^2 \cos^2 2\theta_p - R^2 \sin^2 2\theta_p \\ &= \sigma_{ave}^2 - R^2; \text{ independent of } \theta_p. \end{aligned}$$



Draw line $\overline{OK}$ from origin tangent to the circle at K. Triangle OCK is a right triangle.

$$\overline{OC}^2 = \overline{OK}^2 + \overline{CK}^2$$

$$\begin{aligned} \overline{OK}^2 &= \overline{OC}^2 - \overline{CK}^2 \\ &= \sigma_{ave}^2 - R^2 \\ &= \sigma_{x'} \sigma_{y'} - \tau_{xy'}^2 \end{aligned}$$

(b) Applying above to σ_x, σ_y , and τ_{xy} and to σ_a, σ_b ,

$$\sigma_x \sigma_y - \tau_{xy}^2 = \sigma_a \sigma_b - \tau_{ab}^2 = \sigma_{ave}^2 - R^2$$

$$\text{But } \tau_{ab} = 0, \quad \sigma_a = \sigma_{max}, \quad \sigma_b = \sigma_{min}$$

$$\sigma_x \sigma_y - \tau_{xy}^2 = \sigma_{max} \sigma_{min}$$

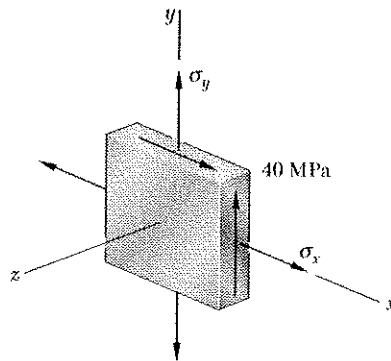
$$\tau_{xy}^2 = \sigma_x \sigma_y - \sigma_{max} \sigma_{min}$$

$$\tau_{xy} = \pm \sqrt{\sigma_x \sigma_y - \sigma_{max} \sigma_{min}}$$

The sign cannot be determined from above equations.

Problem 7.66

7.66 For the state of plane stress shown, determine the maximum shearing stress when (a) $\sigma_x = 0$ and $\sigma_y = 60$ MPa, (b) $\sigma_x = 105$ MPa and $\sigma_y = 45$ MPa. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



$$(a) \quad \sigma_x = 0, \quad \sigma_y = 60 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-30)^2 + 40^2} = 50 \text{ MPa}$$

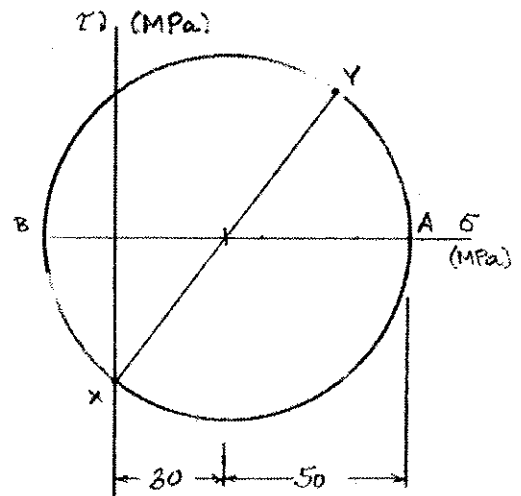
$$\sigma_a = \sigma_{ave} + R = 80 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = -20 \text{ MPa} \quad (\text{min})$$

$$\sigma_c = 0$$

$$\sigma_{max} = 80 \text{ MPa}, \quad \sigma_{min} = -20 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 50 \text{ MPa} \quad \blacktriangleleft$$



$$(b) \quad \sigma_x = 105 \text{ MPa}, \quad \sigma_y = 45 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa}$$

$$\sigma_{ave} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-30)^2 + 40^2} = 50 \text{ MPa}$$

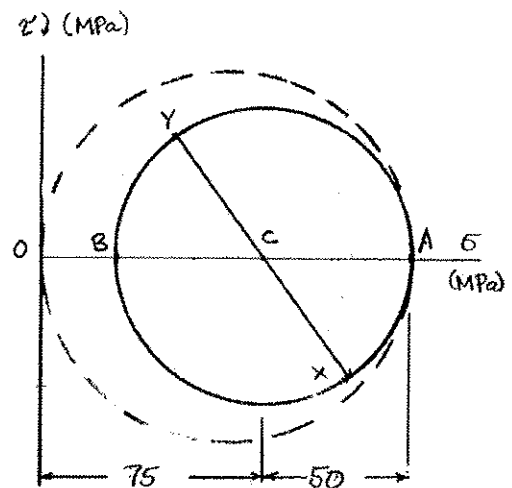
$$\sigma_a = \sigma_{ave} + R = 125 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = 25 \text{ MPa}$$

$$\sigma_c = 0 \quad (\text{min})$$

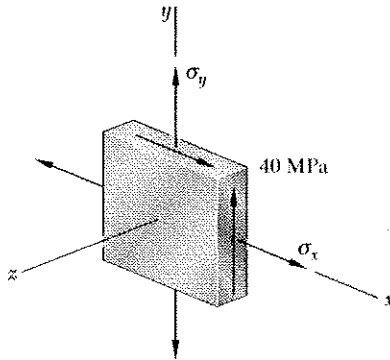
$$\sigma_{max} = 125 \text{ MPa}, \quad \sigma_{min} = 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 62.5 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.67

7.67 For the state of plane stress shown, determine the maximum shearing stress when (a) $\sigma_x = 30$ MPa and $\sigma_y = 90$ MPa, (b) $\sigma_x = 70$ MPa and $\sigma_y = 10$ MPa. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 30$ MPa $\sigma_y = 90$ MPa $\tau_{xy} = 40$ MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 60 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + 40^2} = 50 \text{ MPa}$$

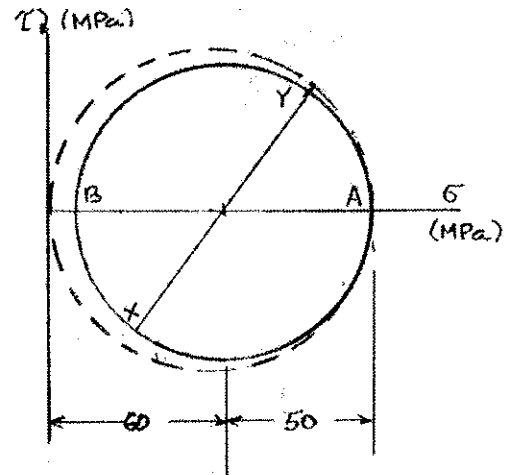
$$\sigma_a = \sigma_{ave} + R = 60 + 50 = 110 \text{ MPa (max)}$$

$$\sigma_b = \sigma_{ave} - R = 60 - 50 = 10 \text{ MPa}$$

$$\sigma_c = 0$$

$$\tau_{max(in-plane)} = R = 50 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 55 \text{ MPa} \blacktriangleleft$$



(b) $\sigma_x = 70$ MPa $\sigma_y = 10$ MPa $\tau_{xy} = 40$ MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + 40^2} = 50 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 90 \text{ MPa (max)}$$

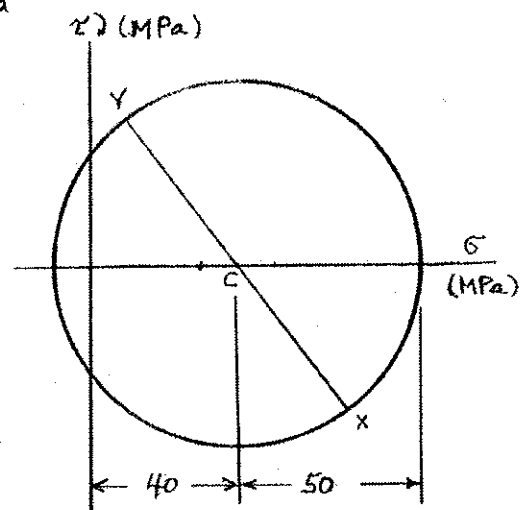
$$\sigma_b = \sigma_{ave} - R = -10 \text{ MPa (min)}$$

$$\sigma_c = 0$$

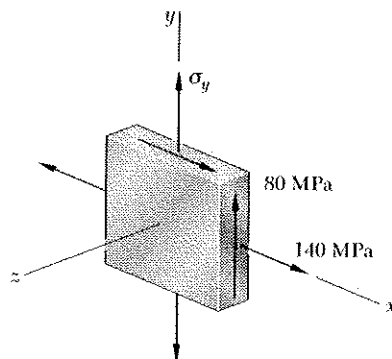
$$\sigma_{max} = 90 \text{ MPa}$$

$$\sigma_{min} = -10 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 50 \text{ MPa} \blacktriangleleft$$



Problem 7.68

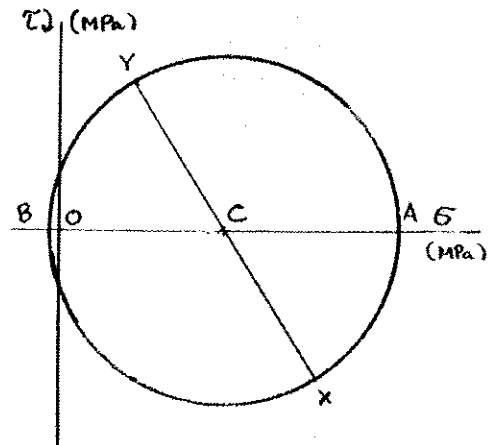


7.68 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_y = 40 \text{ MPa}$, (b) $\sigma_y = 120 \text{ MPa}$. (Hint: Consider both in-plane and out-of-plane shearing stresses.)

(a) $\sigma_x = 140 \text{ MPa}$, $\sigma_y = 40 \text{ MPa}$, $\tau_{xy} = 80 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{50^2 + 80^2} = 94.34 \text{ MPa}$$



$$\sigma_a = \sigma_{ave} + R = 184.34 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = -4.34 \text{ MPa} \quad (\text{min})$$

$$\sigma_c = 0$$

$$\tau_{\text{max (in-plane)}} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

$$\tau_{\text{max}} = \frac{1}{2}(\sigma_{\text{max}} - \sigma_{\text{min}}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.3 \text{ MPa}$$

$$\tau_{\text{max}} = 94.3 \text{ MPa} \quad \blacktriangleleft$$

(b) $\sigma_x = 140 \text{ MPa}$, $\sigma_y = 120 \text{ MPa}$, $\tau_{xy} = 80 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 210.62 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = 49.38 \text{ MPa}$$

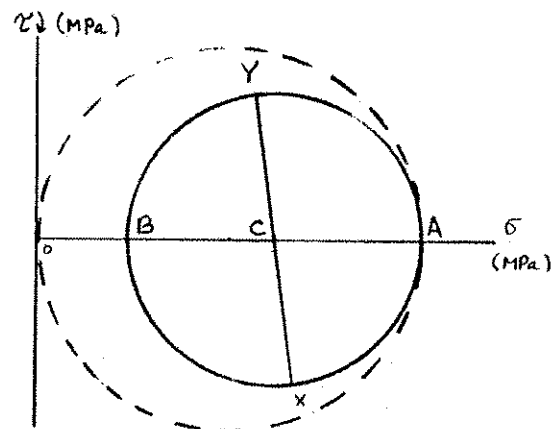
$$\sigma_c = 0 \quad (\text{min})$$

$$\sigma_{\text{max}} = \sigma_a = 210.62 \text{ MPa}$$

$$\sigma_{\text{min}} = \sigma_c = 0$$

$$\tau_{\text{max (in-plane)}} = R = 80.62 \text{ MPa}$$

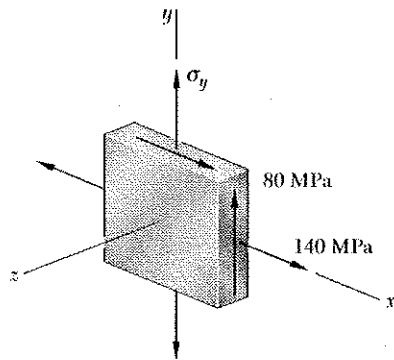
$$\tau_{\text{max}} = \frac{1}{2}(\sigma_{\text{max}} - \sigma_{\text{min}}) = 105.3 \text{ MPa}$$



$$\tau_{\text{max}} = 105.3 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.69

7.69 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_y = 20 \text{ MPa}$, (b) $\sigma_y = 140 \text{ MPa}$. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 140 \text{ MPa}$, $\sigma_y = 20 \text{ MPa}$
 $\tau_{xy} = 80 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$

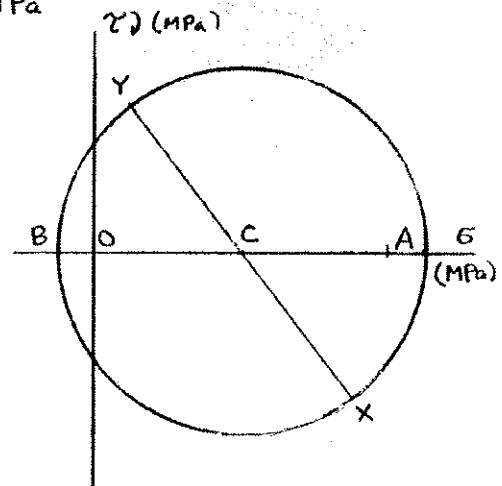
$$\sigma_a = \sigma_{ave} + R = 80 + 100 = 180 \text{ MPa (max)}$$

$$\sigma_b = \sigma_{ave} - R = 80 - 100 = -20 \text{ MPa (min)}$$

$$\sigma_c = 0$$

$$\tau_{max(inplane)} = \frac{1}{2}(\sigma_a - \sigma_b) = 100 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 100 \text{ MPa}$$



$$\tau_{max} = 100 \text{ MPa} \leftarrow$$

(b) $\sigma_x = 140 \text{ MPa}$, $\sigma_y = 140 \text{ MPa}$

$$\tau_{xy} = 80 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 140 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 80^2} = 80 \text{ MPa}$$

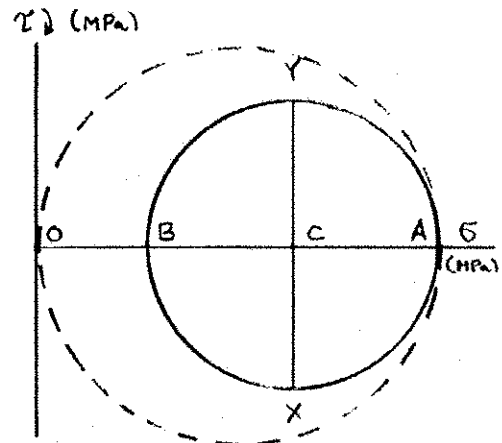
$$\sigma_a = \sigma_{ave} + R = 220 \text{ MPa (max)}$$

$$\sigma_b = \sigma_{ave} - R = 60 \text{ MPa}$$

$$\sigma_c = 0 \quad (\text{min})$$

$$\tau_{max(inplane)} = \frac{1}{2}(\sigma_a - \sigma_b) = 80 \text{ MPa}$$

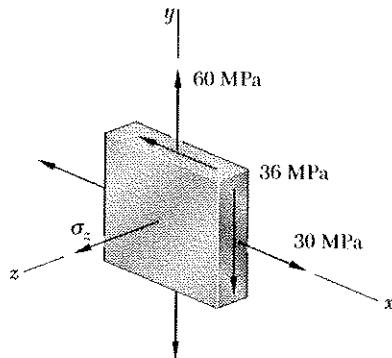
$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 110 \text{ MPa}$$



$$\tau_{max} = 110 \text{ MPa} \leftarrow$$

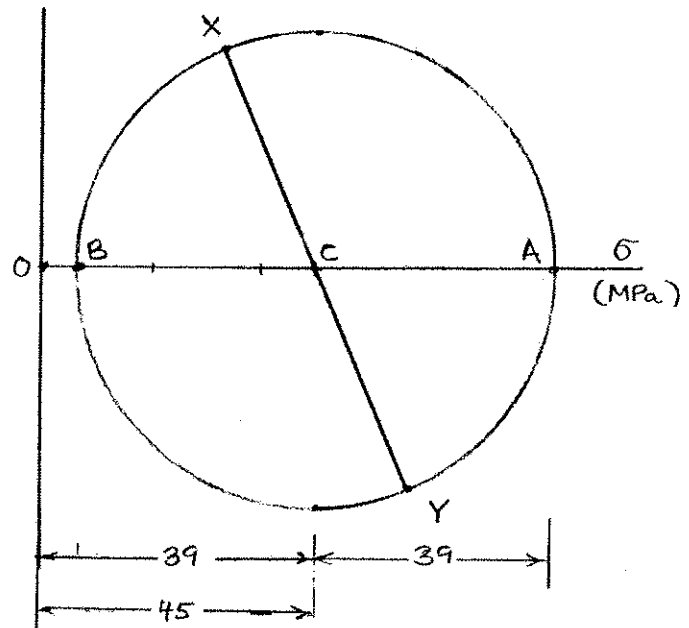
Problem 7.70

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = +24$ MPa, (b) $\sigma_z = -24$ MPa, (c) $\sigma_z = 0$.



$$\sigma_x = 30 \text{ MPa}, \quad \sigma_y = 60 \text{ MPa}, \quad \tau_{xy} = -36 \text{ MPa}$$

τ_{xy} (MPa)



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(15)^2 + (-36)^2} = 39 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 84 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 6 \text{ MPa}$$

(a) $\sigma_z = +24 \text{ MPa}$

$\sigma_a = 84 \text{ MPa}$

$\sigma_b = 6 \text{ MPa}$

$\sigma_{max} = 84 \text{ MPa}$

$\sigma_{min} = 6 \text{ MPa}$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 39 \text{ MPa}$

(b) $\sigma_z = -24 \text{ MPa}$

$\sigma_a = 84 \text{ MPa}$

$\sigma_b = 6 \text{ MPa}$

$\sigma_{max} = 84 \text{ MPa}$

$\sigma_{min} = -24 \text{ MPa}$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 54 \text{ MPa}$

(c) $\sigma_z = 0$

$\sigma_a = 84 \text{ MPa}$

$\sigma_b = 6 \text{ MPa}$

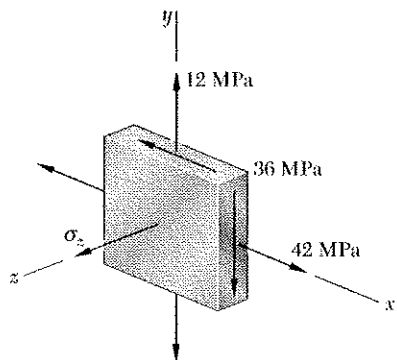
$\sigma_{max} = 84 \text{ MPa}$

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 42 \text{ MPa}$

Problem 7.71

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = +24$ MPa, (b) $\sigma_z = -24$ MPa, (c) $\sigma_z = 0$.



$$\sigma_x = 42 \text{ MPa}, \quad \sigma_y = 12 \text{ MPa}, \quad \tau_{xy} = -36 \text{ MPa}$$

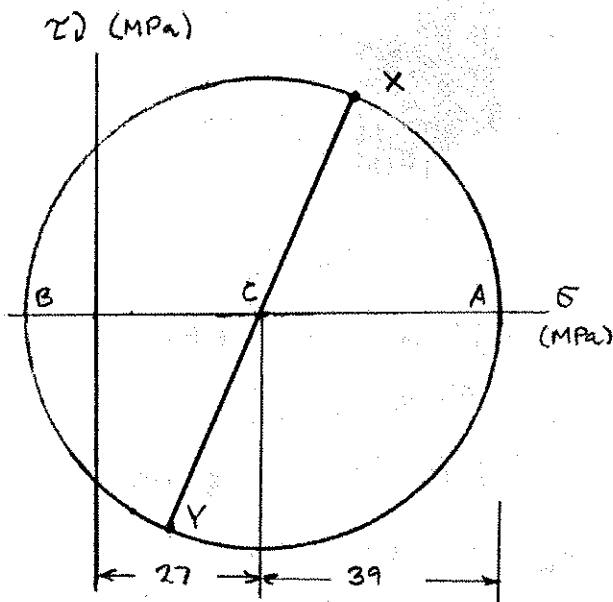
$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 27 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(15)^2 + (-36)^2} = 39 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 66 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -12 \text{ MPa}$$



(a) $\sigma_z = +24 \text{ MPa}$

$$\sigma_a = 66 \text{ MPa}$$

$$\sigma_b = -12 \text{ MPa}$$

$$\sigma_{max} = 66 \text{ MPa}$$

$$\sigma_{min} = -12 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 39 \text{ MPa}$$

(b) $\sigma_z = -24 \text{ MPa}$

$$\sigma_a = 66 \text{ MPa}$$

$$\sigma_b = -12 \text{ MPa}$$

$$\sigma_{max} = 66 \text{ MPa}$$

$$\sigma_{min} = -24 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 45 \text{ MPa}$$

(c) $\sigma_z = 0$

$$\sigma_a = 66 \text{ MPa}$$

$$\sigma_b = -12 \text{ MPa}$$

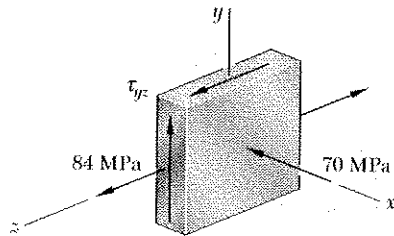
$$\sigma_{max} = 66 \text{ MPa}$$

$$\sigma_{min} = -12 \text{ MPa}$$

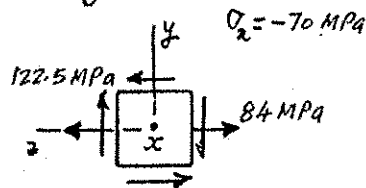
$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 39 \text{ MPa}$$

Problem 7.72

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a) $\tau_{yz} = 122.5 \text{ MPa}$, (b) $\tau_{yz} = 56 \text{ MPa}$, (c) $\tau_{yz} = 0$.



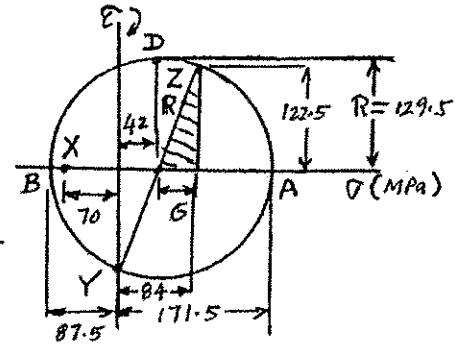
(a) $\tau_{yz} = 122.5 \text{ MPa}$



$$R = \sqrt{(42)^2 + (122.5)^2} = 129.5$$

$$\sigma_A = 42 + 129.5 = 171.5$$

$$\sigma_B = 42 - 129.5 = -87.5$$



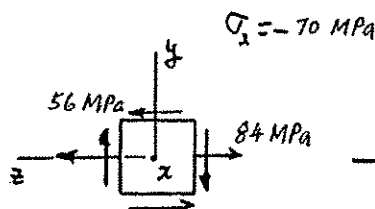
$$\sigma_{\max} = \sigma_A = 171.5 \text{ MPa}$$

$$\sigma_{\min} = \sigma_B = -87.5 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 129.5 \text{ MPa}$$

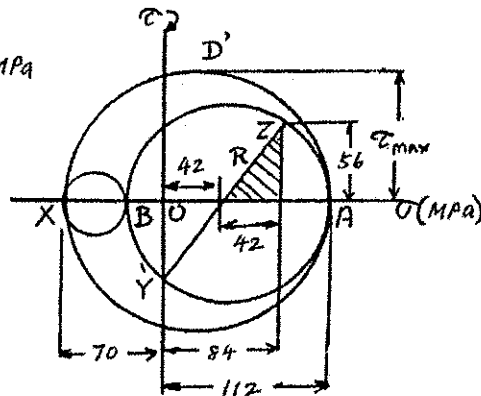
(b) $\tau_{yz} = 56 \text{ MPa}$



$$R = \sqrt{(42)^2 + (56)^2} = 70$$

$$\sigma_A = 42 + 70 = 112$$

$$\sigma_B = 42 - 70 = -28$$



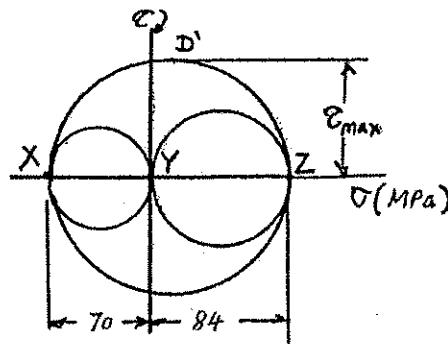
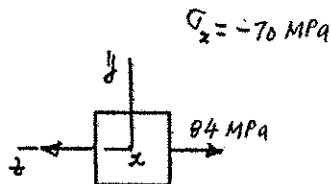
$$\sigma_{\max} = \sigma_A = 112 \text{ MPa}$$

$$\sigma_{\min} = \sigma_B = -28 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 91 \text{ MPa}$$

(c) $\tau_{yz} = 0$



$$\sigma_{\max} = \sigma_z = 84 \text{ MPa}$$

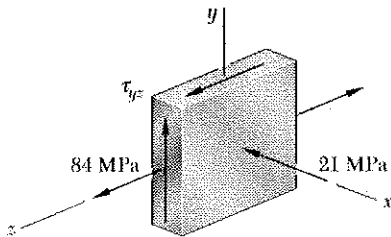
$$\sigma_{\min} = \sigma_x = -70 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

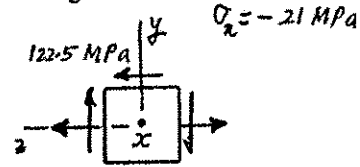
$$\tau_{\max} = 77 \text{ MPa}$$

Problem 7.73

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a) $\tau_{yz} = 122.5$ MPa, (b) $\tau_{yz} = 56$ MPa, (c) $\tau_{yz} = 0$.



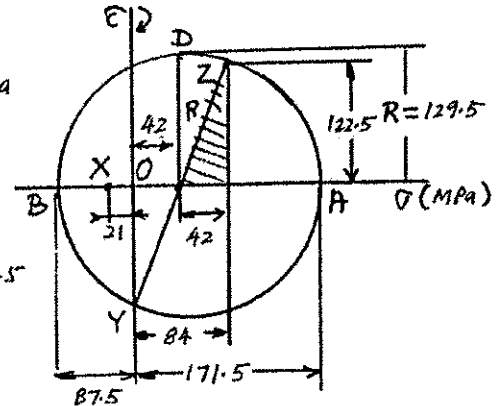
(a) $\tau_{yz} = 122.5$ MPa



$$R = \sqrt{(42)^2 + (122.5)^2} = 129.5$$

$$\sigma_A = 42 + 129.5 = 171.5$$

$$\sigma_B = 42 - 129.5 = -87.5$$



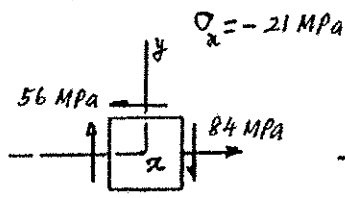
$$\sigma_{\max} = \sigma_A = 171.5 \text{ MPa}$$

$$\sigma_{\min} = \sigma_B = -87.5 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 129.5 \text{ MPa}$$

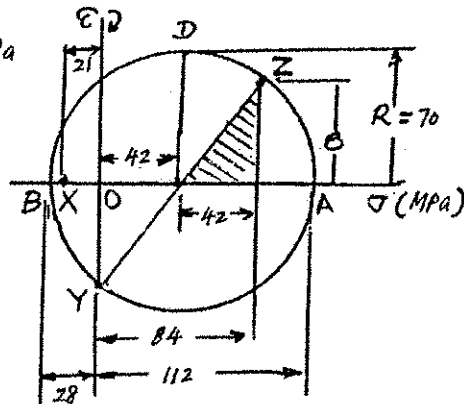
(b) $\tau_{yz} = 56$ MPa



$$R = \sqrt{(42)^2 + (56)^2} = 70$$

$$\sigma_A = 42 + 70 = 112$$

$$\sigma_B = 42 - 70 = -28$$



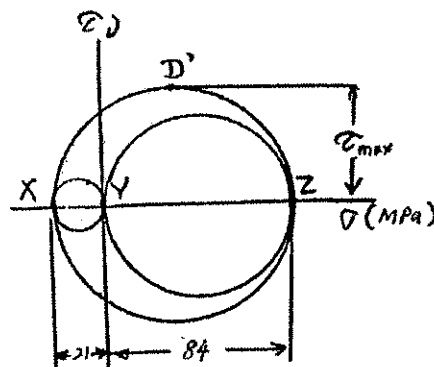
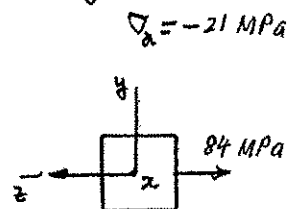
$$\sigma_{\max} = \sigma_A = 112 \text{ MPa}$$

$$\sigma_{\min} = \sigma_B = -28 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 70 \text{ MPa}$$

(c) $\tau_{yz} = 0$



$$\sigma_{\max} = \sigma_z = 84 \text{ MPa}$$

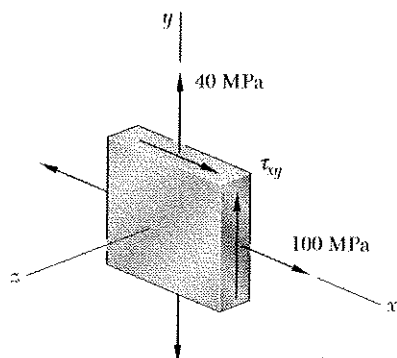
$$\sigma_{\min} = \sigma_x = -21 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 52.5 \text{ MPa}$$

Problem 7.74

7.74 For the state of plane stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (a) 60 MPa, (b) 78 MPa.



$$\sigma_x = 100 \text{ MPa}, \quad \sigma_y = 40 \text{ MPa}, \quad \sigma_z = 0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 70 \text{ MPa}$$

(a) $\tau_{max} = 60 \text{ MPa}$

If σ_z is σ_{min} , then $\sigma_{max} = \sigma_{min} + 2\tau_{max}$.

$$\sigma_{max} = 0 + 2(60) = 120 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R$$

$$R = \sigma_{max} - \sigma_{ave} = 120 - 70 = 50 \text{ MPa}$$

$$\sigma_b = \sigma_{max} - 2R = 120 - 100 = 20 \text{ MPa} > 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\tau_{xy} = \sqrt{50^2 - 30^2}$$

$$\tau_{xy} = 40 \text{ MPa} \quad \blacktriangleleft$$

(b) $\tau_{max} = 78 \text{ MPa}$

If σ_z is σ_{min} , then $\sigma_{max} = \sigma_{min} + 2\tau_{max} = 0 + 2(78) = 156 \text{ MPa}$.

$$\sigma_{max} = \sigma_{ave} + R$$

$$R = \sigma_{max} - \sigma_{ave} = 156 - 70 = 86 \text{ MPa} > \tau_{max} = 78 \text{ MPa}$$

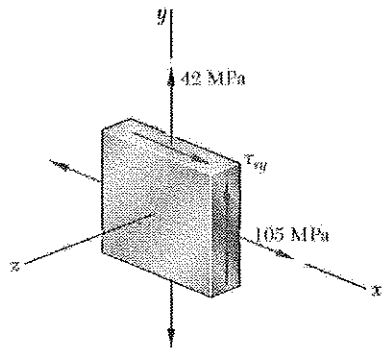
Set $R = \tau_{max} = 78 \text{ MPa}$. $\sigma_{min} = \sigma_{ave} - R = -8 \text{ MPa} < 0$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{78^2 - 30^2}$$

$$\tau_{xy} = 72 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.75



7.75 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (a) 63 MPa, (b) 84 MPa.

$$\sigma_x = 105 \text{ MPa} \quad \sigma_y = 42 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 73.5 \text{ MPa}$$

$$U = \frac{\sigma_x - \sigma_y}{2} = 31.5 \text{ MPa}$$

$$\tau_{xy} \text{ (MPa)}$$

(a) For $\tau_{max} = 63 \text{ MPa}$

Center of Mohr's circle lies at point C. Lines marked (a) show the limits on τ_{max} . Limit on σ_{max} is $\sigma_{max} = 2\tau_{max} = 126 \text{ MPa}$. For the Mohr's circle $\sigma_a = \sigma_{max}$ corresponds to point A_a .

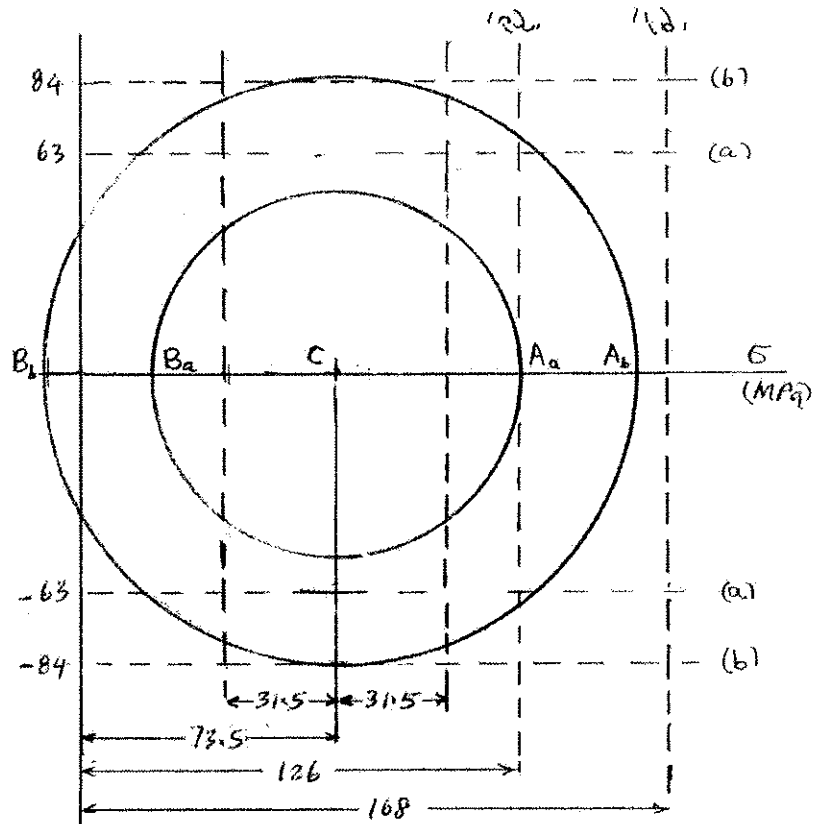
$$R = \sigma_a - \sigma_{ave} = 126 - 73.5 = 52.5 \text{ MPa}$$

$$R = \sqrt{U^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm \sqrt{R^2 - U^2}$$

$$= \pm \sqrt{52.5^2 - 31.5^2}$$

$$= \pm 42 \text{ MPa}$$



(b) For $\tau_{max} = 84 \text{ MPa}$

Center of Mohr's circle lies at point C.

$$R = 84 \text{ MPa}$$

$$\tau_{xy} = \pm \sqrt{R^2 - U^2} = \pm 78.7 \text{ MPa}$$

Checking

$$\sigma_a = 73.5 + 84 = 157.5 \text{ MPa}$$

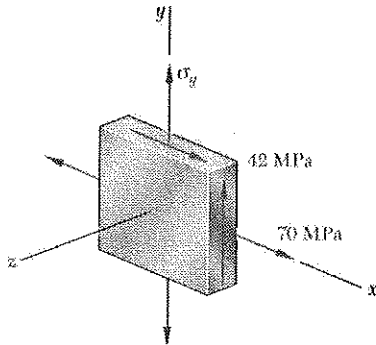
$$\sigma_b = 73.5 - 84 = -10.5 \text{ MPa}$$

$$\sigma_c = 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 84 \text{ MPa} \quad \text{O.K.}$$

Problem 7.76

7.76 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 52.5 MPa.



$$\sigma_x = 70 \text{ MPa}, \quad \tau_{xy} = 42 \text{ MPa}, \quad \tau_{max} = 52.5 \text{ MPa}$$

$$\text{Let } u = \frac{\sigma_y - \sigma_x}{2} \quad \sigma_y = 2u + \sigma_x$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} \quad u = \pm \sqrt{R^2 - \tau_{xy}^2}$$

$$\text{Case 1} \quad \tau_{max} = R = 52.5 \text{ MPa}, \quad u = \pm 31.5 \text{ MPa}$$

$$(1a) \quad u = +31.5 \text{ MPa} \quad \sigma_y = 2u + \sigma_x = 133 \text{ MPa} \quad \text{rejected}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 101.5 \text{ MPa}, \quad \sigma_a = \sigma_{ave} + R = 154 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 49 \text{ MPa}$$

$$\sigma_{max} = 154 \text{ MPa}, \quad \sigma_{min} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 77 \text{ MPa} \neq 52.5 \text{ MPa}$$

$$(1b) \quad u = -31.5 \text{ MPa} \quad \sigma_y = 2u + \sigma_x = 7 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 38.5 \text{ MPa}, \quad \sigma_a = \sigma_{ave} + R = 91 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -14 \text{ MPa}$$

$$\sigma_{max} = 91 \text{ MPa}, \quad \sigma_{min} = -14 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 52.5 \text{ MPa} \quad \text{OK.}$$

$$\text{Case 2} \quad \text{Assume } \sigma_{min} = 0 \quad \sigma_{max} = 2\tau_{max} = 105 \text{ MPa} = \sigma_a$$

$$\sigma_a = \sigma_{ave} + R = \sigma_x + u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2$$

$$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + u^2 = u^2 + \tau_{xy}^2$$

$$2u = \frac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \frac{(105 - 70)^2 - 42^2}{105 - 70} = -15.4 \text{ MPa}$$

$$u = -7.7 \text{ MPa}$$

$$\sigma_y = 2u + \sigma_x = 54.6 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 62.3 \text{ MPa}$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = 42.7 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 105 \text{ MPa} \quad \checkmark$$

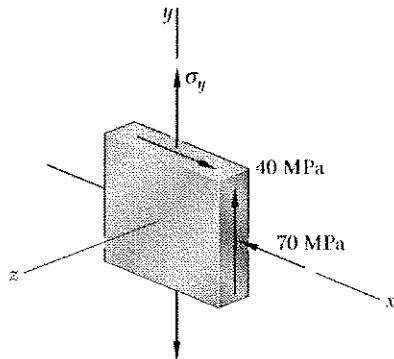
$$\sigma_b = \sigma_{ave} - R = 19.6 \text{ MPa}$$

$$\sigma_{max} = 105 \text{ MPa}, \quad \sigma_{min} = 0$$

$$\tau_{max} = 52.5 \text{ MPa} \quad \checkmark$$

Problem 7.77

7.77 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 75 MPa.



$$\sigma_x = -70 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa}$$

$$\text{Let } U = \frac{\sigma_y - \sigma_x}{2} \quad \sigma_y = 2U + \sigma_x$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + U$$

$$R = \sqrt{U^2 + \tau_{xy}^2} \quad U = \pm \sqrt{R^2 - \tau_{xy}^2}$$

Case (1) $\tau_{max} = R = 75 \text{ MPa}, \quad U = \pm \sqrt{75^2 - 40^2} = \pm 63.44 \text{ MPa}$

(1a) $U = +63.44 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = 56.9 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.56 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 68.44 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -81.56 \text{ MPa}$$

$$\sigma_c = 0 \quad \sigma_{max} = 68.44 \text{ MPa}, \quad \sigma_{min} = -81.56 \text{ MPa} \quad \tau_{max} = 75 \text{ MPa}$$

(1b) $U = -63.44 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = -196.88 \text{ MPa} \text{ (reject)}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -133.44 \text{ MPa} \quad \sigma_a = \sigma_{ave} + R = -58.44 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -208.44 \text{ MPa}, \quad \sigma_c = 0, \quad \sigma_{max} = 0$$

$$\sigma_{min} = -208.44 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 104.22 \text{ MPa} \neq 75 \text{ MPa}$$

$$\sigma_y = 56.9 \text{ MPa} \quad \blacktriangleleft$$

Case (2) Assume $\sigma_{max} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 75 \text{ MPa}$

$$\sigma_{min} = -150 \text{ MPa} = \sigma_b$$

$$\sigma_b = \sigma_{ave} - R = \sigma_x + U - \sqrt{U^2 + \tau_{xy}^2}$$

$$\sqrt{U^2 + \tau_{xy}^2} = -\sigma_x + U - \sigma_b$$

$$U^2 + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)U + U^2$$

$$2U = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} \quad \frac{(40)^2 - (-70 + 150)^2}{-70 + 150} = -160 \text{ MPa}$$

$$U = -30 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = -130 \text{ MPa}$$

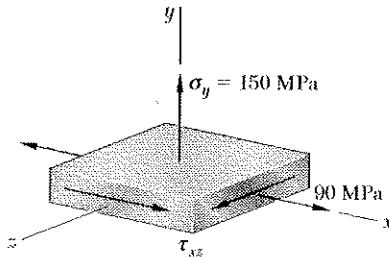
$$R = \sqrt{U^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\sigma_a = \sigma_b + 2R = -150 + 100 = -50 \text{ MPa} \quad \text{O.K.}$$

$$\sigma_y = -130.0 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.78

7.78 For the state of stress shown, determine the range of values of τ_{xz} for which the maximum shearing stress is equal to or less than 90 MPa.



$$\sigma_x = 90 \text{ MPa}, \quad \sigma_z = 0, \quad \sigma_y = 150 \text{ MPa}$$

For Mohr's circle of stresses in zx -plane,

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 45 \text{ MPa}$$

$$U = \frac{\sigma_x - \sigma_z}{2} = 45$$

Assume $\sigma_{max} = \sigma_y = 150 \text{ MPa}$.

$$\begin{aligned} \sigma_{min} &= \sigma_b = \sigma_{max} - 2\tau_{max} \\ &= 150 - (2)(90) = -30 \text{ MPa} \end{aligned}$$

$$\begin{aligned} R &= \sigma_{ave} - \sigma_b \\ &= 45 - (-30) = 75 \text{ MPa} \end{aligned}$$

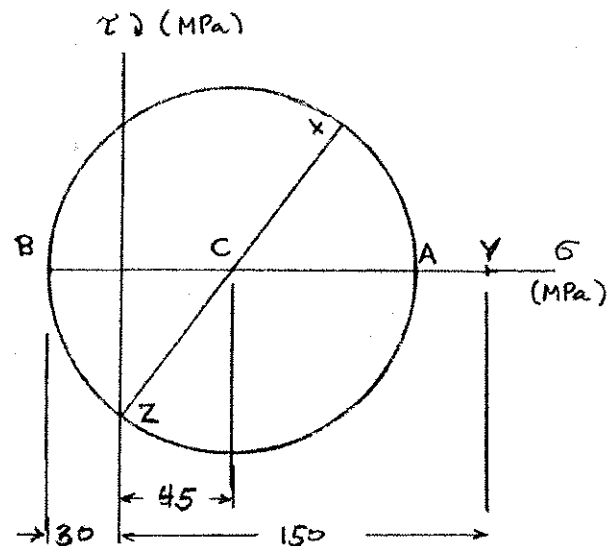
$$\begin{aligned} \sigma_a &= \sigma_{ave} + R \\ &= 45 + 75 = 120 \text{ MPa} < \sigma_y \end{aligned}$$

O.K

$$R = \sqrt{U^2 + \tau_{xz}^2}$$

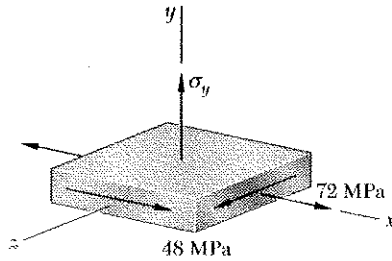
$$\begin{aligned} \tau_{xz} &= \pm \sqrt{R^2 - U^2} \\ &= \pm \sqrt{75^2 - 45^2} = \pm 60 \text{ MPa} \end{aligned}$$

$$-60 \text{ MPa} \leq \tau_{xz} \leq 60 \text{ MPa}$$



Problem 7.79

7.79 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 64 MPa.



$$\sigma_x = 72 \text{ MPa} \quad \sigma_z = 0 \quad \tau_{xz} = 48 \text{ MPa}$$

Mohr's circle for stresses in xz -plane

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 36 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{36^2 + 48^2} = 60 \text{ MPa}$$

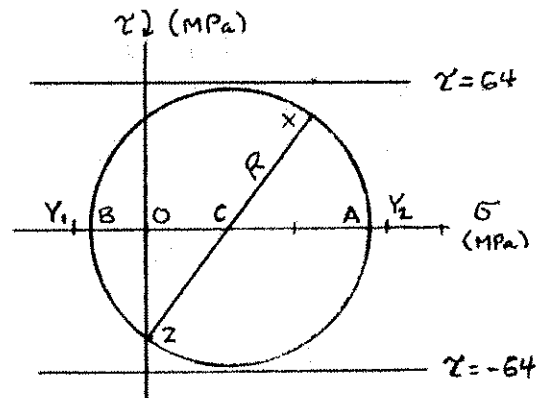
$$\sigma_a = \sigma_{ave} + R = 96 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -24 \text{ MPa}$$

Assume $\sigma_{max} = \sigma_a = 96 \text{ MPa}$.

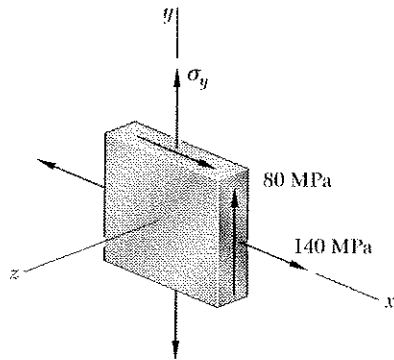
$$\begin{aligned} \sigma_y = \sigma_{min} &= \sigma_{max} - 2\tau_{max} \\ &= 96 - (2)(64) = -32 \text{ MPa} \end{aligned} \quad \blacktriangleleft$$

Assume $\sigma_{min} = \sigma_b = -24 \text{ MPa}$.

$$\begin{aligned} \sigma_y = \sigma_{max} &= \sigma_{min} + 2\tau_{max} \\ &= -24 + (2)(64) = 104 \text{ MPa} \end{aligned} \quad \blacktriangleleft$$



Problem 7.80



*7.80 For the state of stress of Prob. 7.69, determine (a) the value of σ_y for which the maximum shearing stress is as small as possible, (b) the corresponding value of the shearing stress.

$$\text{Let } u = \frac{\sigma_x - \sigma_y}{2} \quad \sigma_y = \sigma_x - 2u$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$$

Assume τ_{max} is the in-plane shearing stress, $\tau_{max} = R$

Then, $\tau_{max}(\text{in-plane})$ is minimum if $u = 0$.

$$\sigma_y = \sigma_x - 2u = \sigma_x = 140 \text{ MPa}, \quad \sigma_{ave} = \sigma_x - u = 140 \text{ MPa}$$

$$R = |\tau_{xy}| = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 140 + 80 = 220 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 140 - 80 = 60 \text{ MPa}$$

$$\sigma_{max} = 220 \text{ MPa}, \quad \sigma_{min} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 110 \text{ MPa}$$

Assumption is incorrect.

$$\text{Assume } \sigma_{max} = \sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_{min} = 0 \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}\sigma_a$$

$$\frac{d\sigma_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0 \quad (\text{no minimum})$$

Optimum value for u occurs when $\tau_{max}(\text{out-of-plane}) = \tau_{max}(\text{in-plane})$

$$\frac{1}{2}(\sigma_a + R) = R \quad \text{or} \quad \sigma_a = R \quad \text{or} \quad \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + u^2 = u^2 + \tau_{xy}^2$$

$$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3 \text{ MPa} \quad u = 47.14 \text{ MPa}$$

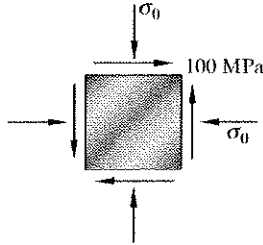
$$(a) \quad \sigma_y = \sigma_x - 2u = 140 - 94.3$$

$$\sigma_y = 45.7 \text{ MPa} \quad \leftarrow$$

$$(b) \quad R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 92.9 \text{ MPa}$$

$$\tau_{max} = 92.9 \text{ MPa} \quad \leftarrow$$

Problem 7.81



7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (a) $\sigma_0 = 200$ MPa, (b) $\sigma_0 = 240$ MPa, (c) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.

$$\sigma_{ave} = -\sigma_0 \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a) $\sigma_0 = 200 \text{ MPa} \quad \sigma_{ave} = -200 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -100 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -300 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 264.56 \text{ MPa} < 325 \text{ MPa} \text{ (No yielding)}$$

$$F.S. = \frac{325}{264.56}$$

$$F.S. = 1.228 \blacktriangleleft$$

(b) $\sigma_0 = 240 \text{ MPa} \quad \sigma_{ave} = -240 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -140 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -340 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 295.97 \text{ MPa} < 325 \text{ MPa} \text{ (No yielding)}$$

$$F.S. = \frac{325}{295.97}$$

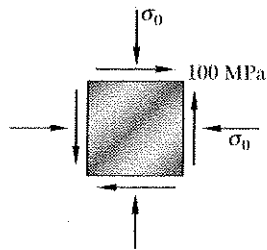
$$F.S. = 1.098 \blacktriangleleft$$

(c) $\sigma_0 = 280 \text{ MPa} \quad \sigma_{ave} = -280 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -180 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -380 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 329.24 \text{ MPa} > 325 \text{ MPa} \text{ (Yielding occurs)}$$

Problem 7.82



7.82 Solve Prob. 7.81, using the maximum-shearing-stress criterion.

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (a) $\sigma_0 = 200$ MPa, (b) $\sigma_0 = 240$ MPa, (c) $\sigma_0 = 280$ MPa. If yield does occur, determine the corresponding factor of safety.

$$\sigma_{ave} = -\sigma_0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a) $\sigma_0 = 200 \text{ MPa}$, $\sigma_{ave} = -200 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -100 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -300 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -300 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 300 \text{ MPa} < 325 \text{ MPa} \quad (\text{No yielding})$$

$$F.S. = \frac{325}{300}$$

$$F.S. = 1.083$$

(b) $\sigma_0 = 240 \text{ MPa}$, $\sigma_{ave} = -240 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -140 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -340 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -340 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 340 \text{ MPa} > 325 \text{ MPa} \quad (\text{Yielding occurs})$$

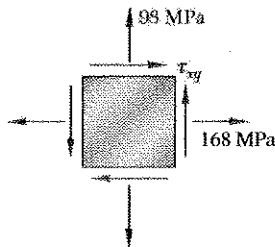
(c) $\sigma_0 = 280 \text{ MPa}$, $\sigma_{ave} = -280 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = -180 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -380 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -380 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 380 \text{ MPa} > 325 \text{ MPa} \quad (\text{Yielding occurs})$$

Problem 7.83



7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 210$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a) $\tau_{xy} = 42$ MPa, (b) $\tau_{xy} = 84$ MPa, (c) $\tau_{xy} = 98$ MPa. If yield does not occur, determine the corresponding factor of safety.

$$\sigma_x = 168 \text{ MPa} \quad \sigma_y = 98 \text{ MPa} \quad \sigma_z = 0$$

$$\text{For stresses in } xy\text{-plane: } \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 133 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = 35 \text{ MPa}$$

$$(a) \quad \tau_{xy} = 42 \text{ MPa} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (42)^2} = 54.7 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 187.7 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 78.3 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 163.3 \text{ MPa} < 210 \text{ MPa} \quad (\text{No yielding})$$

$$F.S. = \frac{210}{163.3} = 1.286$$

$$(b) \quad \tau_{xy} = 84 \text{ MPa} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (84)^2} = 91 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 224 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 42 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 206.2 \text{ MPa} < 210 \text{ MPa} \quad (\text{No yielding})$$

$$F.S. = \frac{210}{206.2} = 1.018$$

$$(c) \quad \tau_{xy} = 98 \text{ MPa} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(35)^2 + (98)^2} = 104.1 \text{ MPa}$$

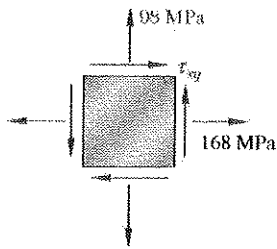
$$\sigma_a = \sigma_{ave} + R = 237.1 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 28.9 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 224 \text{ MPa} > 210 \text{ MPa} \quad (\text{Yielding occurs})$$

Problem 7.84

7.84 Solve Prob. 7.83, using the maximum-shearing-stress criterion.

7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 210$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a) $\tau_{xy} = 42$ MPa, (b) $\tau_{xy} = 84$ MPa, (c) $\tau_{xy} = 98$ MPa. If yield does not occur, determine the corresponding factor of safety.



$$\sigma_x = 168 \text{ MPa} \quad \sigma_y = 98 \text{ MPa} \quad \sigma_z = 0$$

For stresses in xy-plane $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 133 \text{ MPa}$

$$\frac{\sigma_x - \sigma_y}{2} = 35 \text{ MPa}$$

(a) $\tau_{xy} = 42 \text{ MPa}$ $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 54.7 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = 187.7 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 78.3 \text{ MPa}$$

$$\sigma_{max} = 187.7 \text{ MPa}, \quad \sigma_{min} = 0$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 187.7 \text{ MPa} < 210 \text{ MPa} \quad (\text{No yielding})$$

$$F.S. = \frac{210}{187.7} = 1.119$$

(b) $\tau_{xy} = 84 \text{ MPa}$ $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 91 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = 224 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 42 \text{ MPa}$$

$$\sigma_{max} = 224 \text{ MPa} \quad \sigma_{min} = 0$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 224 \text{ MPa} > 210 \text{ MPa} \quad (\text{Yielding occurs})$$

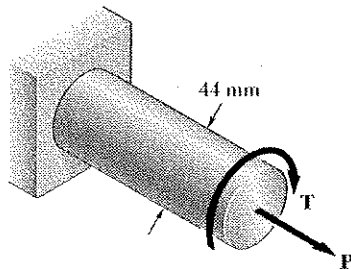
(c) $\tau_{xy} = 98 \text{ MPa}$ $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 104.1 \text{ MPa}$

$$\sigma_a = \sigma_{ave} + R = 237.1 \text{ MPa} \quad \sigma_b = \sigma_{ave} - R = 28.9 \text{ MPa}$$

$$\sigma_{max} = 237.1 \text{ MPa} \quad \sigma_{min} = 0$$

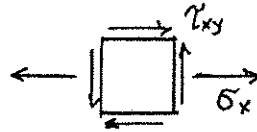
$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 237.1 > 210 \text{ MPa} \quad (\text{Yielding occurs})$$

Problem 7.85



7.85 The 44-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the force P for which yield occurs when $T = 1500$ Nm.

Let the x-axis lie along the shaft axis.



$$\sigma_y = 0, \quad \sigma_z = 0$$

$$\sigma_x = \frac{P}{A}, \quad \tau_{xy} = \frac{Tc}{J}$$

Section properties: $c = \frac{1}{2}d = 0.022$ m

$$A = \pi c^2 = \pi (0.022)^2 = 152.05 \times 10^{-6} \text{ m}^2, \quad J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.022)^4 = 367.97 \times 10^{-9} \text{ m}^4$$

From torsion, $\tau_{xy} = \frac{Tc}{J} = \frac{(1500)(0.022)}{367.97 \times 10^{-9}} = 89.68 \text{ MPa}$.

From Mohr's circle, $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} =$$

$$\sigma_a = \sigma_{ave} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Maximum shearing stress criterion.

$$\sigma_{max} = \sigma_a \quad \sigma_{min} = \sigma_b$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = \sigma_a - \sigma_b = 2\sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} = \sigma_Y$$

$$\sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y \quad \sigma_x = \sqrt{\sigma_Y^2 - 4\tau_{xy}^2}$$

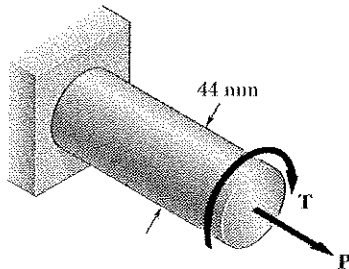
$$\sigma_x = \sqrt{(250)^2 - 4(89.68)^2} = 174.16 \text{ MPa}$$

$$P = \sigma_x A = (174.16 \times 10^6)(152.05 \times 10^{-6})$$

$$P = 264.8 \text{ kN}$$

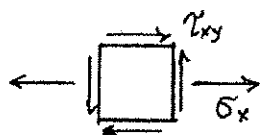
Problem 7.86

7.86 Solve Prob. 7.87, using the maximum-distortion-energy criterion.



7.85 The 44-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the force P for which yield occurs when $T = 1500$ Nm.

Let the x-axis lie along the shaft axis.



$$\begin{aligned}\sigma_y &= 0, & \sigma_z &= 0 \\ \sigma_x &= \frac{P}{A}, & \tau_{xy} &= \frac{Tc}{J}\end{aligned}$$

Section properties: $c = \frac{1}{2}d = 0.022$ m

$$A = \pi c^2 = \pi (0.022)^2 = 152.05 \times 10^{-6} \text{ m}^2, \quad J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.022)^4 = 367.97 \times 10^{-9} \text{ m}^4$$

$$\text{From torsion, } \tau_{xy} = \frac{Tc}{J} = \frac{(1500)(0.022)}{367.97 \times 10^{-9}} = 89.68 \text{ MPa.}$$

$$\text{From Mohr's circle, } \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R =$$

Distortion energy criterion

$$\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b = \sigma_Y^2$$

$$(\sigma_{ave} + R)^2 + (\sigma_{ave} - R)^2 - (\sigma_{ave} + R)(\sigma_{ave} - R) = \sigma_Y^2$$

$$\sigma_{ave}^2 + 3R^2 = \sigma_Y^2$$

$$\left(\frac{\sigma_x}{2}\right)^2 + 3\left[\frac{\sigma_x^2}{4} + \tau_{xy}^2\right] = \sigma_Y^2 + 3\tau_{xy}^2 = \sigma_Y^2$$

$$\sigma_x = \sqrt{\sigma_Y^2 - 3\tau_{xy}^2}$$

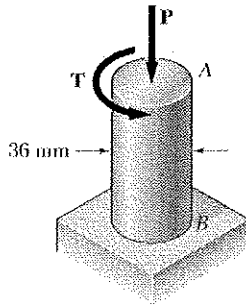
$$\sigma_x = \sqrt{(250)^2 - (3)(89.68)^2} = 195.89 \text{ MPa}$$

$$P = \sigma_x A = (195.89 \times 10^6)(152.05 \times 10^{-6})$$

$$P = 297.9 \text{ kN}$$

Problem 7.87

7.87 The 36-mm-diameter shaft is made of a grade of steel with a 250-MPa tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque T for which yield occurs when $P = 200$ kN.



$$P = 200 \text{ kN} = 200 \times 10^3 \text{ N} \quad C = \frac{1}{2}d = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$A = \pi C^2 = \pi (18 \times 10^{-3})^2 = 1.01788 \times 10^{-3} \text{ m}^2$$

$$\sigma_y = -\frac{P}{A} = -\frac{200 \times 10^3}{1.01788 \times 10^{-3}} = 196.488 \times 10^6 \text{ Pa} \\ = 196.488 \text{ MPa}$$

$$\sigma_x = 0 \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_y = 98.244 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R \text{ (positive)} \quad \sigma_b = \sigma_{ave} - R \text{ (negative)}$$

$$|\sigma_a - \sigma_b| = 2R \quad |\sigma_a - \sigma_b| > |\sigma_a| \quad |\sigma_a - \sigma_b| > |\sigma_b|$$

Maximum shear stress criterion under the above conditions:

$$|\sigma_a - \sigma_b| = 2R = \sigma_Y = 250 \text{ MPa} \quad R = 125 \text{ MPa}$$

Equating expressions for R ,

$$125 = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

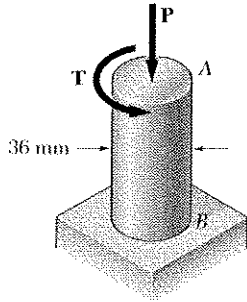
$$\tau_{xy} = \sqrt{(125)^2 - (98.244)^2} = 77.286 \text{ MPa} = 77.286 \times 10^6 \text{ Pa}$$

Torsion $J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (18 \times 10^{-3})^4 = 164.896 \times 10^{-9} \text{ m}^4$

$$\tau_{xy} = \frac{Tc}{J} \quad T = \frac{J \tau_{xy}}{c} = \frac{(164.896 \times 10^{-9})(77.286 \times 10^6)}{18 \times 10^{-3}} \\ = 708 \text{ N}\cdot\text{m}$$

$$T = 708 \text{ N}\cdot\text{m}$$

Problem 7.88



7.88 Solve Prob. 7.87 using the maximum-distortion-energy criterion.

7.87 The 36-mm-diameter shaft is made of a grade of steel with a 250-MPa tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque T for which yield occurs when $P = 200$ kN.

$$P = 200 \text{ kN} = 200 \times 10^3 \text{ N} \quad c = \frac{1}{2}d = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$A = \pi c^2 = \pi (18 \times 10^{-3})^2 = 1.01788 \times 10^{-3} \text{ m}^2$$

$$\begin{aligned} \sigma_y &= -\frac{P}{A} = -\frac{200 \times 10^3}{1.01788 \times 10^{-3}} = 196.488 \times 10^6 \text{ Pa} \\ &= 196.488 \text{ MPa} \end{aligned}$$

$$\sigma_x = 0 \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_y = 98.244 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R \quad \sigma_b = \sigma_{ave} - R$$

Distortion energy criterion.

$$\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b = \sigma_y^2$$

$$(\sigma_{ave} + R)^2 + (\sigma_{ave} - R)^2 - (\sigma_{ave} + R)(\sigma_{ave} - R) = \sigma_y^2$$

$$\sigma_{ave}^2 + 3R^2 = \sigma_y^2$$

$$(98.244)^2 + 3[(98.244)^2 + \tau_{xy}^2] = (250)^2$$

$$\tau_{xy} = 89.242 \text{ MPa}$$

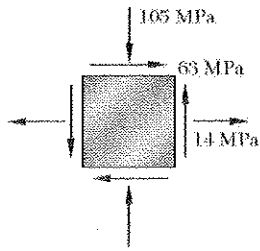
Torsion $J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (18 \times 10^{-3})^4 = 164.846 \times 10^{-9} \text{ m}^4$

$$\tau_{xy} = \frac{Tc}{J} \quad T = \frac{J \tau_{xy}}{c} = \frac{(164.846 \times 10^{-9})(89.242 \times 10^6)}{18 \times 10^{-3}}$$

$$= 818 \text{ N}\cdot\text{m}$$

$$T = 818 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

Problem 7.89



7.89 and 7.90 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 210$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.

$$\sigma_x = 14 \text{ MPa} \quad \sigma_y = -105 \text{ MPa} \quad \tau_{xy} = 63 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -45.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{59.5^2 + 63^2} = 86.7 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 41.2 \text{ MPa}$$

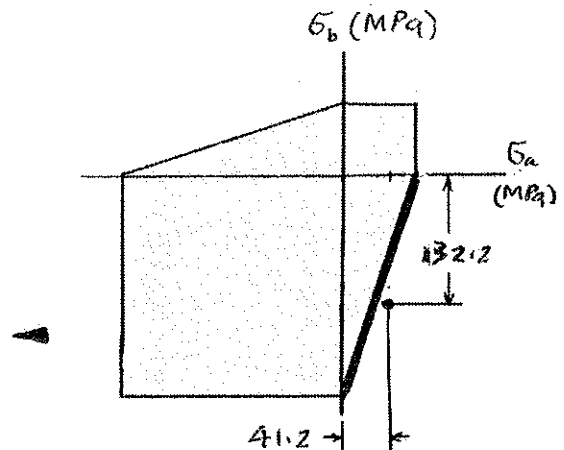
$$\sigma_b = \sigma_{ave} - R = -132.2 \text{ MPa}$$

Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

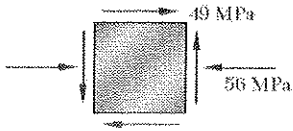
$$\frac{41.2}{70} - \frac{(-132.2)}{210} = 1.218 > 1$$

Rupture will occur.



Problem 7.90

7.89 and 7.90 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 210$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.



$$\sigma_x = -56 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 49 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -28 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{28^2 + 49^2} = 56.4 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = -28 + 56.4 = 28.4 \text{ MPa}$$

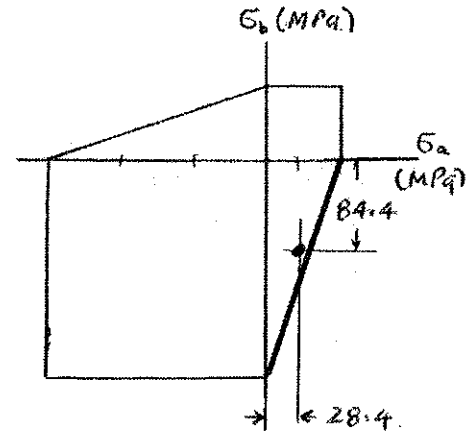
$$\sigma_b = \sigma_{ave} - R = -28 - 56.4 = -84.4 \text{ MPa}$$

Equation of 4th quadrant of boundary

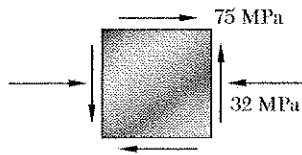
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{28.4}{70} - \frac{(-84.4)}{210} = 0.808 < 1$$

(No rupture)



Problem 7.91



7.91 and 7.92 The state of plane stress shown is expected in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

$$\sigma_x = -32 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 75 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -16 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(16)^2 + (75)^2} = 76.69 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = -16 + 76.69 = 60.69 \text{ MPa}$$

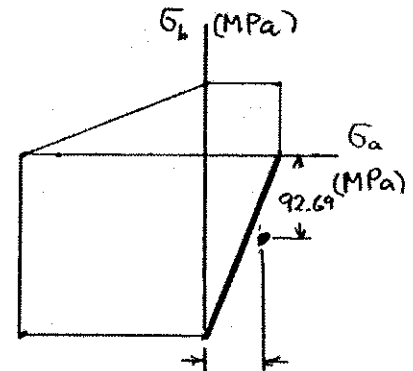
$$\sigma_b = \sigma_{ave} - R = -16 - 76.69 = -92.69 \text{ MPa}$$

Equation of 4th quadrant of boundary.

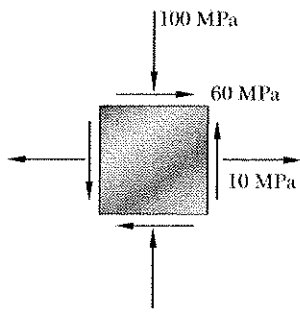
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{60.69}{80} - \frac{(-92.69)}{200} = 1.222 > 1$$

Rupture will occur. ◀



Problem 7.92



7.91 and 7.92 The state of plane stress shown is expected in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

$$\sigma_x = 10 \text{ MPa}, \quad \sigma_y = -100 \text{ MPa}, \quad \tau_{xy} = 60 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{10 - 100}{2} = -45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(55)^2 + (60)^2} = 81.39 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = -45 + 81.39 = 36.39 \text{ MPa}$$

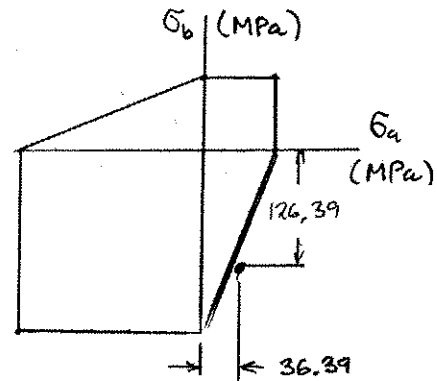
$$\sigma_b = \sigma_{ave} - R = -45 - 81.39 = -126.39 \text{ MPa}$$

Equation of 4th quadrant of boundary.

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{36.39}{80} - \frac{(-126.39)}{200} = 1.087 > 1$$

Rupture will occur. ◀



Problem 7.93

7.93 The state of plane stress shown will occur at a critical point in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 170$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



$$\sigma_x = 56 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = \tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{28^2 + \tau_0^2}, \quad \tau_0 = \pm \sqrt{R^2 - 28^2}$$

$$\sigma_a = \sigma_{ave} + R = (28 + R) \text{ MPa} \quad \sigma_b = \sigma_{ave} - R = (28 - R) \text{ MPa}$$

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

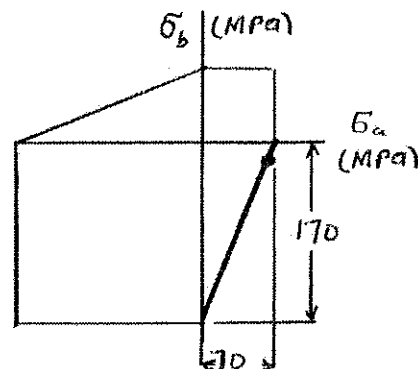
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{28+R}{70} - \frac{28-R}{170} = 1$$

$$\left(\frac{1}{70} + \frac{1}{170}\right)R = 1 - \frac{28}{70} + \frac{28}{170}$$

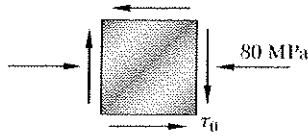
$$R = 37.9 \text{ MPa}$$

$$\tau_0 = \pm \sqrt{37.9^2 - 28^2} = \pm 25.5 \text{ MPa}$$



Problem 7.94

7.94 The state of plane stress shown will occur at a critical point in a pipe made of an aluminum alloy for which $\sigma_{UT} = 75 \text{ MPa}$ and $\sigma_{UC} = 150 \text{ MPa}$. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



$$\sigma_x = -80 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = -\tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2} \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R, \quad \sigma_b = \sigma_{ave} - R, \quad \tau_0 = \pm \sqrt{R^2 - 40^2}$$

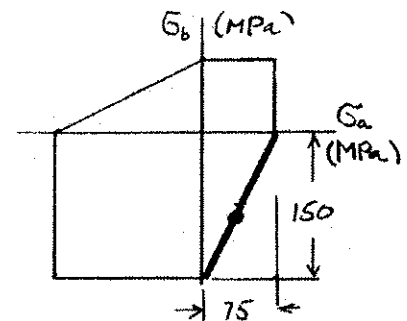
Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$

$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$

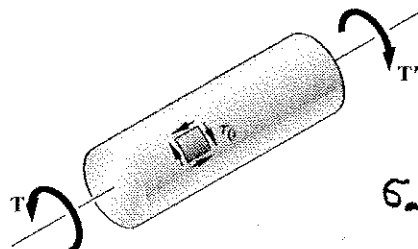
$$R = 63.33 \text{ MPa}, \quad \tau_0 = \pm \sqrt{63.33^2 - 40^2}$$



$$\tau_0 = \pm 49.1 \text{ MPa} \blacktriangleleft$$

Problem 7.95

7.95 The case-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 175$ MPa. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_0 that should be expected at rupture.



$$\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = -\tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$$

$$\sigma_a = \sigma_{ave} + R = R$$

$$\sigma_b = \sigma_{ave} - R = -R$$

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

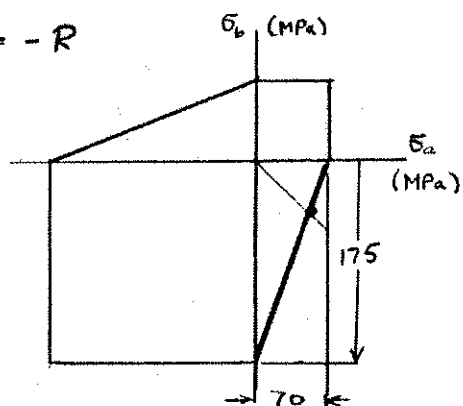
$$\frac{R}{70} - \frac{-R}{175} = 1$$

$$\left(\frac{1}{70} + \frac{1}{175}\right) R = 1$$

$$R = 50 \text{ MPa}$$

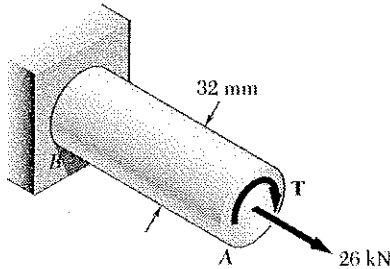
$$\tau_0 = R$$

$$\tau_0 = 50.0 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.96

7.96 The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 60 \text{ MPa}$ and $\sigma_{UC} = 120 \text{ MPa}$. Using Mohr's criterion, determine the magnitude of the torque T for which failure should be expected.



$$P = 26 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} (32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{26 \times 10^3}{804.25 \times 10^{-6}} = 32.328 \times 10^6 \text{ Pa} = 32.328 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(32.328 + 0) = 16.164 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(32.328 - 0) = 16.164 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 16.164 + R \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = 16.164 - R \text{ MPa}$$

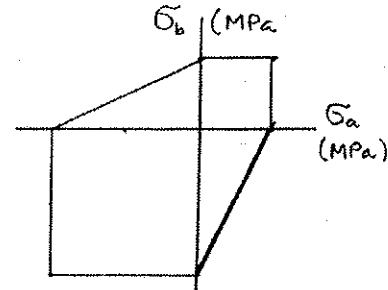
Since $|\sigma_{ave}| < R$, stress point lies in the 4th quadrant. Equation of the 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{16.164 + R}{60} - \frac{16.164 - R}{120} = 1$$

$$\left(\frac{1}{60} + \frac{1}{120}\right)R = 1 - \frac{16.164}{60} + \frac{16.164}{120}$$

$$R = 34.612 \text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{34.612^2 - 16.164^2} = 30.606 \text{ MPa}$$

$$= 30.606 \times 10^6 \text{ Pa}$$

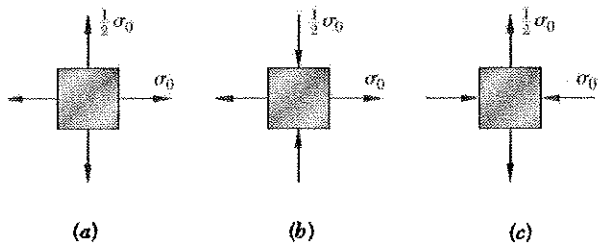
For torsion $\tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$ where $c = \frac{1}{2}d = 16 \text{ mm} = 16 \times 10^{-3} \text{ m}$

$$T = \frac{\pi}{2} c^3 \tau_{xy} = \frac{\pi}{2} (16 \times 10^{-3})^3 (30.606 \times 10^6)$$

$$T = 196.9 \text{ N}\cdot\text{m} \blacktriangleleft$$

Problem 7.97

7.97 A machine component is made of a grade of cast iron for which $\sigma_{UT} = 56 \text{ MPa}$ and $\sigma_{UC} = 140 \text{ MPa}$. For each of the states of stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which rupture of the component should be expected.



$$(a) \quad \sigma_a = \sigma_0, \quad \sigma_b = \frac{1}{2} \sigma_0$$

Stress point lies in 1st quadrant.

$$\sigma_a = \sigma_0 = \sigma_{UT} = 56 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \sigma_a = \sigma_0, \quad \sigma_b = -\frac{1}{2} \sigma_0$$

Stress point lies in 4th quadrant.

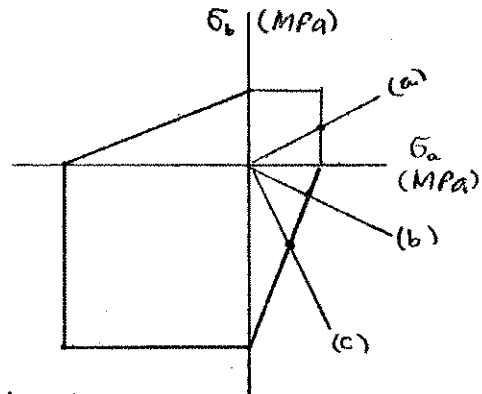
Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{56} - \frac{-\frac{1}{2}\sigma_0}{140} = 1 \quad \sigma_0 = 46.7 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \sigma_a = \frac{1}{2} \sigma_0, \quad \sigma_b = -\sigma_0, \quad 4\text{th quadrant}$$

$$\frac{\frac{1}{2}\sigma_0}{56} - \frac{-\sigma_0}{140} = 1 \quad \sigma_0 = 62.2 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.98

7.98 A basketball has a 300-mm outer diameter and a 3-mm wall thickness. Determine the normal stress in the wall when the basketball is inflated to a 120-kPa gage pressure.

$$r = \frac{1}{2}d - t = 147 \text{ mm} = 147 \times 10^{-3} \text{ m} \quad p = 120 \times 10^3 \text{ Pa}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(120 \times 10^3)(147 \times 10^{-3})}{(2)(3 \times 10^{-3})} = 2.94 \times 10^6 \text{ Pa} \quad \sigma = 2.94 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.99

7.99 A spherical pressure vessel of 1.2-m outer diameter is to be fabricated from a steel having an ultimate stress $\sigma_U = 450 \text{ MPa}$. Knowing that a factor of safety of 4.0 is desired and that the gage pressure can reach 3 MPa, determine the smallest wall thickness that should be used.

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450}{4} = 112.5 \text{ MPa} \quad r = \frac{1}{2}d - t = (0.6 - t) \text{ m}$$

$$\sigma_{all} = \frac{pr}{2t} \quad 2\sigma_{all}t = pr$$

$$(2)(112.5)t = 3(0.6 - t) \quad 228t = 1.8$$

$$t = 7.89 \times 10^{-3} \text{ m} \quad t_{min} = 7.89 \text{ mm} \quad \blacktriangleleft$$

Problem 7.100

7.100 A spherical gas container made of steel has a 6-m outer diameter and a wall thickness of 9 mm. Knowing that the internal pressure is 500 kPa, determine the maximum normal stress and the maximum shearing stress in the container.

$$r = \frac{1}{2}d - t = \frac{6}{2} - 9 \times 10^{-3} = 2.991 \text{ m} \quad t = 9 \times 10^{-3} \text{ m}$$

$$p = 500 \times 10^3 \text{ Pa}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(500 \times 10^3)(2.991)}{(2)(9 \times 10^{-3})} = 83.1 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = \sigma_1 \quad \sigma_{max} = 83.1 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} \approx -p = 0.5 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 41.8 \times 10^6 \text{ Pa} \quad \tau_{max} = 41.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.101

7.101 A spherical pressure vessel has an outer diameter of 3 m and a wall thickness of 12 mm. Knowing that for the steel used $\sigma_{all} = 80 \text{ MPa}$, $E = 200 \text{ GPa}$, and $\nu = 0.29$, determine (a) the allowable gage pressure, (b) the corresponding increase in the diameter of the vessel.

$$r = \frac{1}{2}d - t = \frac{1}{2}(3) - 12 \times 10^{-3} = 1.488 \text{ m} \quad \sigma_1 = \sigma_2 = \sigma_{all} = 80 \times 10^6 \text{ Pa}$$

$$(a) \quad \sigma_1 = \sigma_2 = \frac{pr}{2t} \quad p = \frac{2t\sigma_1}{r} = \frac{(2)(12 \times 10^{-3})(80 \times 10^6)}{1.488}$$

$$p = 1.290 \times 10^6 \text{ Pa} \quad p = 1.290 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1 = \frac{1-0.29}{200 \times 10^9}(80 \times 10^6) = 284 \times 10^{-6}$$

$$\Delta d = d\epsilon_1 = (3)(284 \times 10^{-6}) = 852 \times 10^{-6} \text{ m} \quad \Delta d = 0.852 \text{ mm} \quad \blacktriangleleft$$

Problem 7.102

7.102 A spherical gas container having an outer diameter of 4.5 m and a wall thickness of 22 mm is made of a steel for which $E = 200 \text{ GPa}$ and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 1.7 MPa, determine (a) the maximum normal stress in the container, (b) the increase in the diameter of the container.

$$r = \frac{1}{2}d - t = \frac{1}{2}(4500) - 22 = 2228 \text{ mm} \quad t = 22 \text{ mm}$$

$$(a) \quad \sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.7)(2228)}{(2)(22)} = 86.1 \text{ MPa} \quad 86.1 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \epsilon_1 = \epsilon_2 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1$$

$$= \frac{1-0.29}{200 \times 10^9}(86.1) = 305.66 \times 10^{-6}$$

$$\Delta d = (d)(\epsilon_1) = (4500)(305.66 \times 10^{-6}) = 1.38 \text{ mm} \quad \blacktriangleleft$$

Problem 7.103

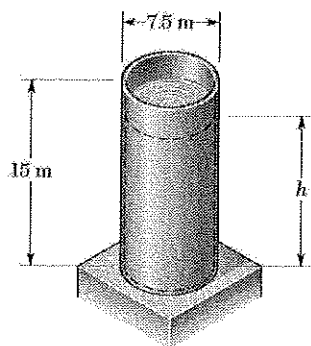
7.103 The maximum gage pressure is known to be 10 MPa in a spherical steel pressure vessel having a 200-mm outer diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_U = 400 \text{ MPa}$, determine the factor of safety with respect to tensile failure.

$$r = \frac{1}{2}d - t = \frac{1}{2}(200) - 6 = 94 \text{ mm} = 94 \times 10^{-3} \text{ m} \quad t = 6 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(10 \times 10^6)(94 \times 10^{-3})}{(2)(6 \times 10^{-3})} = 78.333 \times 10^6 \text{ Pa}$$

$$F.S. = \frac{\sigma_U}{\sigma_{max}} = \frac{400 \times 10^6}{78.333 \times 10^6} \quad F.S. = 5.11 \quad \blacktriangleleft$$

Problem 7.104



But $p = \gamma h$

7.104 The unpressurized cylindrical storage tank shown has a 5-mm wall thickness and is made of a steel having a 420-MPa ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Density of water = 9810 N/m³.)

$$d_o = 7500 \text{ mm}$$

$$r = \frac{1}{2}d - t = 3750 - 5 = 3745 \text{ mm}$$

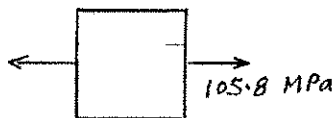
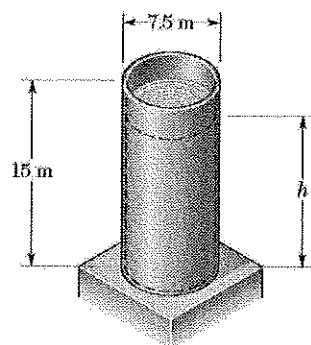
$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{420}{4.0} = 105 \text{ MPa}$$

$$\sigma_{all} = \frac{pr}{t}$$

$$PF \frac{t\sigma_{all}}{r} = \frac{(5)(105)}{3745} = 140.2 \text{ kPa}$$

$$h = \frac{p}{\gamma_g} = \frac{140200}{(1000)(9.81)} = 14.3 \text{ m}$$

Problem 7.105



7.105 For the storage tank of Prob. 7.104, determine the maximum normal stress and the maximum shearing stress in the cylindrical wall when the tank is filled to capacity ($h = 14.4 \text{ m}$).

7.104 The unpressurized cylindrical storage tank shown has a 5-mm wall thickness and is made of steel having a 420-MPa ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Specific weight of water = 9810 N/m³.)

$$d_o = 7.5 \text{ m}$$

$$t = 0.005 \text{ m}$$

$$r = \frac{1}{2}d - t = 3.745 \text{ m}$$

$$p = \gamma h = (9810)(14.4) = 141264 \text{ Pa}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(141264)(3.745)}{0.005} = 105.81 \text{ MPa}$$

$$\sigma_{max} = \sigma_1$$

$$\sigma_{max} = 105.8 \text{ MPa}$$

$$\sigma_{min} \approx 0 \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) \quad \tau_{max} = 52.9 \text{ MPa}$$

Problem 7.106

7.106 A standard-weight steel pipe of 300-mm nominal diameter carries water under a pressure of 2.8 MPa. (a) Knowing that the outside diameter is 320 mm and the wall thickness is 10 mm, determine the maximum tensile stress in the pipe. (b) Solve part a, assuming an extra-strong pipe is used, of 320 mm outside diameter and 12-mm wall thickness.

$$(a) \quad d_o = 0.32 \text{ m} \quad t = 0.01 \text{ m} \quad r = \frac{1}{2} d_o - t = 0.15 \text{ m}$$

$$\sigma = \frac{pr}{t} = \frac{(2.8)(0.15)}{0.01} = 42 \text{ MPa}$$

$$\sigma = 42 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad d_o = 0.32 \text{ m} \quad t = 0.012 \text{ m} \quad r = \frac{1}{2} d_o - t = 0.148 \text{ m}$$

$$\sigma = \frac{pr}{t} = \frac{(2.8)(0.148)}{0.012} = 34.5 \text{ MPa}$$

$$\sigma = 34.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.107

7.107 A storage tank contains liquified propane under a pressure of 1.5 MPa at a temperature of 38°C. Knowing that the tank has an outer diameter of 320 mm and a wall thickness of 3 mm, determine the maximum normal stress and the maximum shearing stress in the tank.

$$r = \frac{1}{2} d - t = \frac{1}{2}(320) - 3 = 157 \text{ mm} \quad t = 3 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5)(157)}{3} = 78.5 \text{ MPa}$$

$$\sigma_{\max} = \sigma_1 = 78.5 \text{ MPa}$$

$$\sigma_{\max} = 78.5 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{\min} \approx -p = -1.5 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 40 \text{ MPa}$$

$$\tau_{\max} = 40 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.108

7.108 The bulk storage tank shown in Fig. 7.49 has an outer diameter of 3.5 m and a wall thickness of 20 mm. At a time when the internal pressure of the tank is 1.2 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

$$r = \frac{1}{2} d - t = \frac{1}{2}(3.5) - 20 \times 10^{-3} = 1.73 \text{ m}$$

$$t = 20 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2 \times 10^6)(1.73)}{20 \times 10^{-3}} = 103.8 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = \sigma_1 = 103.8 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = 103.8 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{\min} = -p \approx 1.2 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 52.5 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 52.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.109

7.109 Determine the largest internal pressure that can be applied to a cylindrical tank of 1.75-m outer diameter and 16-mm wall thickness if the ultimate normal stress of the steel used is 450 MPa and a factor of safety of 5.0 is desired.

$$\sigma_1 = \frac{\sigma_u}{F.S.} = \frac{450 \text{ MPa}}{5} = 90 \text{ MPa} = 90 \times 10^6 \text{ Pa}$$

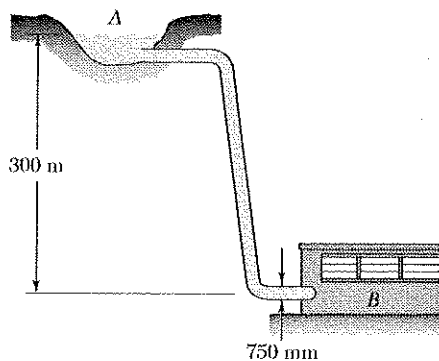
$$r = \frac{1}{2}d - t = \frac{1.75}{2} - 16 \times 10^{-3} = 0.859 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} \quad p = \frac{t\sigma_1}{r} = \frac{(16 \times 10^{-3})(90 \times 10^6)}{0.859} = 1.676 \times 10^6 \text{ Pa}$$

$$\sigma_1 = 1.676 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.110

7.110 A steel penstock has a 750-mm outer diameter, a 12-mm wall thickness, and connects a reservoir at A with a generating station at B. Knowing that the density of water is 1000 kg/m³, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.



$$r = \frac{1}{2}d - t = \frac{1}{2}(750) - 12 = 363 \text{ mm} = 363 \times 10^{-3} \text{ m}$$

$$t = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$p = \rho g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(300 \text{ m})$$

$$= 2.943 \times 10^6 \text{ Pa}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(2.943 \times 10^6)(363 \times 10^{-3})}{12 \times 10^{-3}} = 89.0 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = \sigma_1$$

$$\sigma_{\max} = 89.0 \text{ MPa} \quad \blacktriangleleft$$

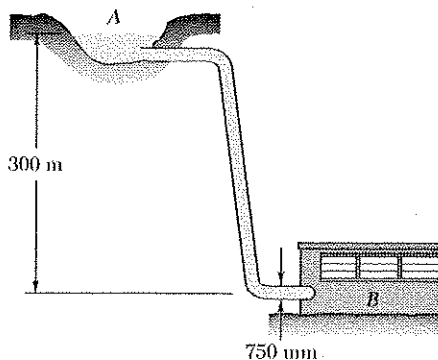
$$\sigma_{\min} = -p \times 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 44.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.111

7.111 A steel penstock has a 750-mm outer diameter and connects a reservoir at A with a generating station at B. Knowing that the density of water is 1000 kg/m³ and that the allowable normal stress in the steel is 85 MPa, determine the smallest thickness that can be used for the penstock.



$$p = \rho g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(300 \text{ m})$$

$$= 2.943 \times 10^6 \text{ Pa}$$

$$\sigma_1 = 85 \text{ MPa} = 85 \times 10^6 \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(750 \times 10^{-3}) - t = 0.375 - t$$

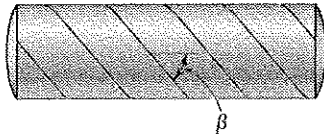
$$\sigma_1 = \frac{pr}{t}$$

$$85 \times 10^6 = \frac{(2.943 \times 10^6)(0.375 - t)}{t}$$

$$(87.943 \times 10^6)t = 1.103625 \times 10^6 \quad t = 12.547 \times 10^{-3} \text{ m}$$

$$t = 12.55 \text{ mm} \quad \blacktriangleleft$$

Problem 7.112



7.112 The steel pressure tank shown has a 750-mm inner diameter and a 9-mm wall thickness. Knowing that the butt welded seams form an angle $\beta = 50^\circ$ with the longitudinal axis of the tank and that the gage pressure in the tank is 1.4 MPa, determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

$$r = \frac{1}{2}d = 375 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.4)(375)}{9} = 58.3 \text{ MPa}$$

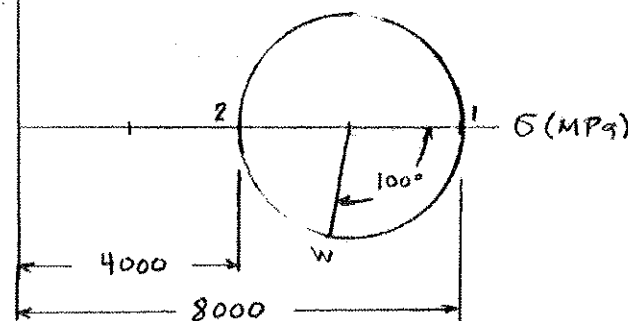
$$\sigma_2 = \frac{1}{2}\sigma_1 = 29.15 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 43.725 \text{ MPa}$$

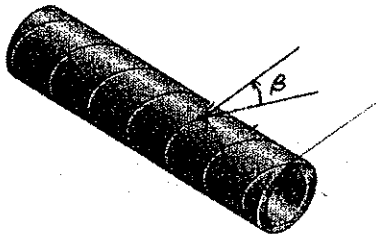
$$R = \frac{\sigma_1 - \sigma_2}{2} = 14.575 \text{ MPa}$$

$$(a) \quad \sigma_w = \sigma_{ave} + R \cos 100^\circ = 41.2 \text{ MPa}$$

$$(b) \quad \tau_w = R \sin 100^\circ = 14.4 \text{ MPa}$$



Problem 7.113



7.113 The pressurized tank shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not be larger than 85 percent of the maximum stress in the tank.

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

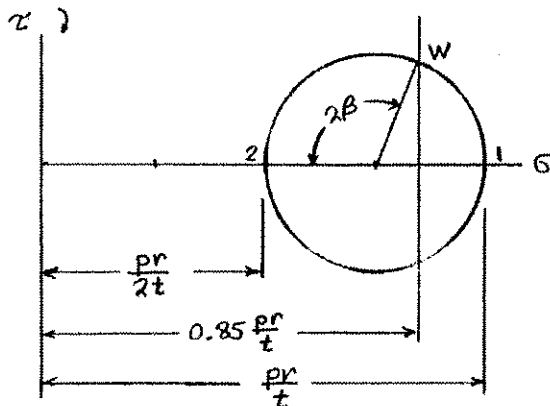
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} - R \cos 2\beta$$

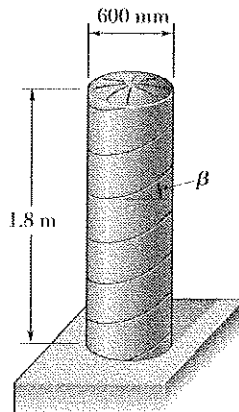
$$0.85 \frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4} \cos 2\beta \right) \frac{pr}{t}$$

$$\cos 2\beta = -4 \left(0.85 - \frac{3}{4} \right) = -0.4$$

$$2\beta = 113.6^\circ \quad \beta = 56.8^\circ$$



Problem 7.114

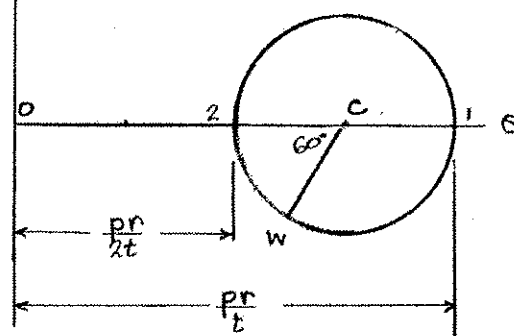


7.114 The cylindrical portion of the compressed air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30^\circ$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.

$$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 6 = 292 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{1}{2} \frac{pr}{t}$$

τ



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

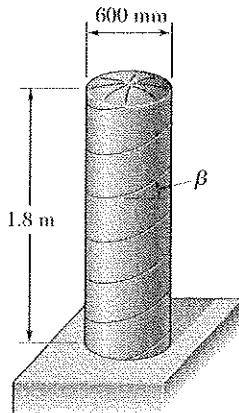
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

$$\begin{aligned} \sigma_w &= \sigma_{ave} + R \cos 60^\circ \\ &= \frac{5}{8} \frac{pr}{t} \end{aligned}$$

$$p = \frac{8}{5} \frac{\sigma_w t}{r}$$

$$p = \frac{8}{5} \frac{(75)(8)}{292} = 3.29 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.115

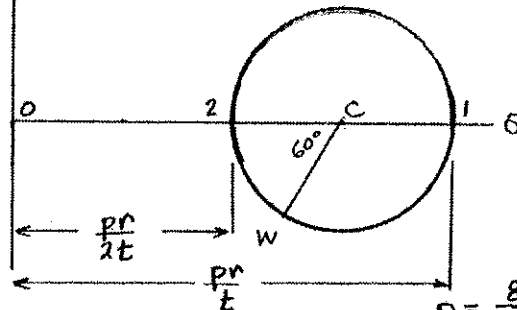


7.115 The cylindrical portion of the compressed air tank shown is fabricated of 8-mm-thick plate welded along a helix forming an angle $\beta = 30^\circ$ with the horizontal. Determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.

$$r = \frac{1}{2}d - t = \frac{1}{2}(600) - 8 = 292 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t}, \quad \sigma_2 = \frac{1}{2} \frac{pr}{t}$$

τ



$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

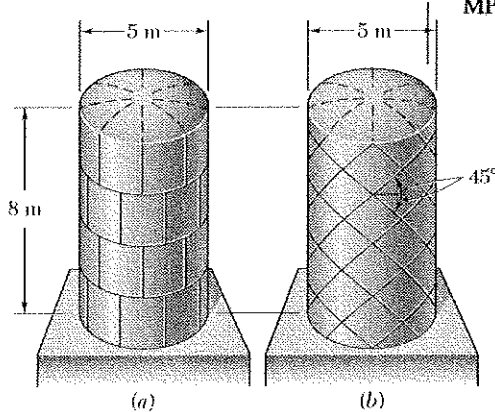
$$\begin{aligned} \tau_w &= R \sin 60^\circ \\ &= \frac{\sqrt{3}}{8} \frac{pr}{t} \end{aligned}$$

$$p = \frac{8}{\sqrt{3}} \frac{\tau_w t}{r}$$

$$p = \frac{8}{\sqrt{3}} \frac{(30)(8)}{292} = 3.80 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.116

7.116 Square plates, each of 16-mm thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed air tank. Knowing that the allowable normal stress perpendicular to the weld is 65 MPa, determine the largest allowable gage pressure in each case.



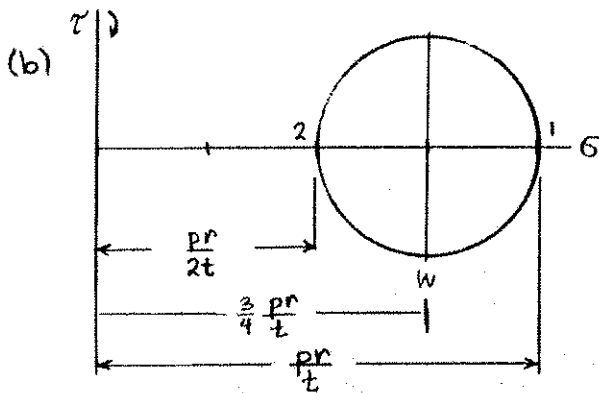
$$r = \frac{1}{2}d - t = \frac{1}{2}(5) - 16 \times 10^{-3} = 2.484 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$(a) \quad \sigma_1 = 65 \text{ MPa} = 65 \times 10^6 \text{ Pa}$$

$$p = \frac{\sigma_1 t}{r} = \frac{(65 \times 10^6)(16 \times 10^{-3})}{2.484} = 419 \times 10^3 \text{ Pa}$$

$$p = 419 \text{ kPa}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

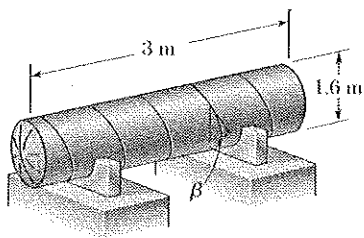
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

$$\beta = \pm 45^\circ$$

$$\begin{aligned} \sigma_w &= \sigma_{ave} + R \cos \beta \\ &= \frac{3}{4} \frac{pr}{t} \end{aligned}$$

$$p = \frac{4 \sigma_w t}{3r} = \frac{(4)(65 \times 10^6)(16 \times 10^{-3})}{(3)(2.484)} = 558 \times 10^3 \text{ Pa} \quad p = 558 \text{ kPa}$$

Problem 7.117



7.117 The pressure tank shown has an 8-mm wall thickness and butt welded seams forming an angle $\beta = 20^\circ$ with a transverse plane. For a gage pressure of 600 kPa, determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

$$d = 1.6 \text{ m} \quad t = 8 \times 10^{-3} \text{ m} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(600 \times 10^3)(0.792)}{8 \times 10^{-3}} = 59.4 \times 10^6 \text{ Pa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(600 \times 10^3)(0.792)}{(2)(8 \times 10^{-3})} = 29.7 \times 10^6 \text{ Pa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 44.55 \times 10^6 \text{ Pa}$$

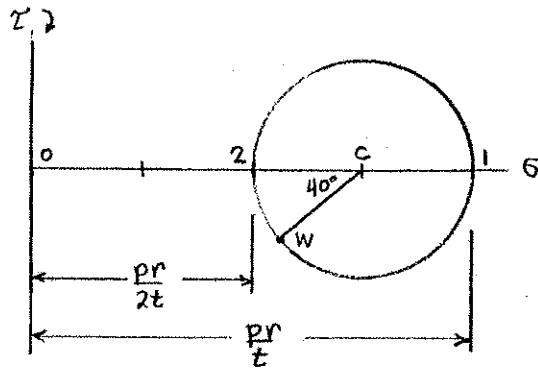
$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = 14.85 \times 10^6 \text{ Pa}$$

$$(a) \sigma_w = \sigma_{ave} - R \cos 40^\circ = 33.17 \times 10^6 \text{ Pa}$$

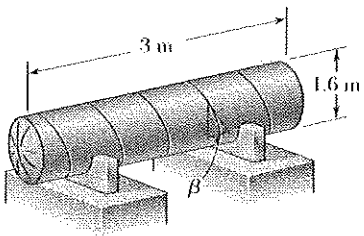
$$\sigma_w = 33.2 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \tau_w = R \sin 40^\circ = 9.55 \times 10^6 \text{ Pa}$$

$$\tau_w = 9.55 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.118



7.118 For the tank of Prob. 7.117, determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 120 MPa and the allowable shearing stress parallel to the weld is 80 MPa.

7.117 The pressure tank shown has a 8-mm wall thickness and butt welded seams forming an angle $\beta = 20^\circ$ with a transverse plane. For a gage pressure of 600 kPa, determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

$$d = 1.6 \text{ m}, \quad t = 8 \times 10^{-3} \text{ m}, \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{4} \frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} - R \cos 40^\circ$$

$$= \left(\frac{3}{4} - \frac{1}{4} \cos 40^\circ \right) \frac{pr}{t} = 0.5585 \frac{pr}{t}$$

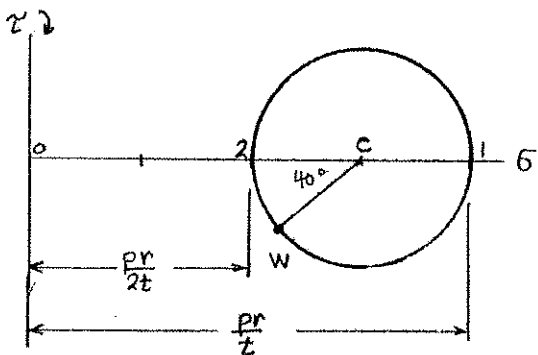
$$p = \frac{\sigma_w t}{0.5585 r} = \frac{(120 \times 10^6)(8 \times 10^{-3})}{(0.5585)(0.792)} = 2.17 \times 10^6 \text{ Pa} = 2.17 \text{ MPa}$$

$$\tau_w = R \sin 40^\circ = \left(\frac{1}{4} \sin 40^\circ \right) \frac{pr}{t} = 0.1607 \frac{pr}{t}$$

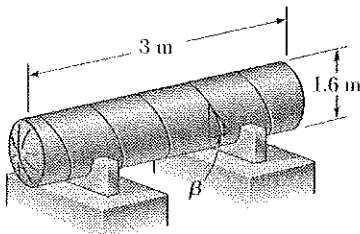
$$p = \frac{\tau_w t}{0.1607 r} = \frac{(80 \times 10^6)(8 \times 10^{-3})}{(0.1607)(0.792)} = 5.03 \times 10^6 \text{ Pa} = 5.06 \text{ MPa}$$

The largest allowable pressure is the smaller value.

$$p = 2.17 \text{ MPa} \quad \blacktriangleleft$$

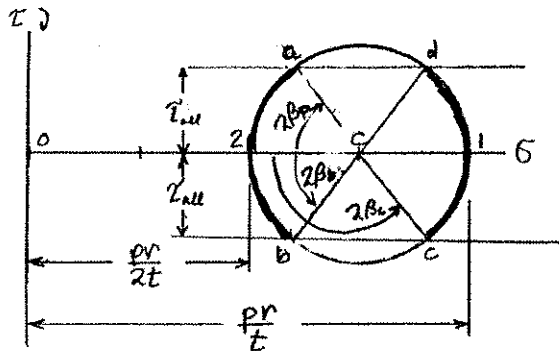


Problem 7.119



7.119 For the tank of Prob. 7.117, determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 12 MPa when the gage pressure is 600 kPa.

7.117 The pressure tank shown has a 8-mm wall thickness and butt welded seams forming an angle $\beta = 20^\circ$ with a transverse plane. For a gage pressure of 600 kPa, determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.



$$d = 1.6 \text{ m} \quad t = 8 \times 10^{-3} \text{ mm}$$

$$r = \frac{1}{2}d - t = 0.792 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(600 \times 10^3)(0.792)}{8 \times 10^{-3}} = 59.4 \times 10^6 \text{ Pa} = 59.4 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 29.7 \text{ MPa}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 14.85 \text{ MPa}$$

$$\tau_w = R |\sin 2\beta|$$

$$|\sin 2\beta_a| \frac{\tau_w}{R} = \frac{12}{14.85} = 0.80808$$

$$2\beta_a = -53.91^\circ$$

$$\beta_a = +27.0^\circ$$

$$2\beta_b = +53.91^\circ$$

$$\beta_b = 27.0^\circ$$

$$2\beta_c = 180^\circ - 53.91^\circ = +126.09^\circ$$

$$\beta_c = 63.0^\circ$$

$$2\beta_d = 180^\circ + 53.91^\circ = +233.91^\circ$$

$$\beta_d = -117.0^\circ$$

Let the total range of values for β be $-180^\circ < \beta \leq 180^\circ$

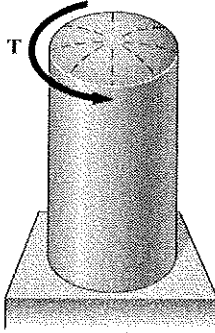
Safe ranges for β :

$$-27.0^\circ \leq \beta \leq 27.0^\circ$$

$$\text{and } 63.0^\circ \leq \beta \leq 117.0^\circ$$

Problem 7.120

7.120 A torque of magnitude $T = 12 \text{ kN} \cdot \text{m}$ is applied to the end of a tank containing compressed air under a pressure of 8 MPa. Knowing that the tank has a 180-mm inner diameter and a 12-mm wall thickness, determine the maximum normal stress and the maximum shearing stress in the tank.



$$d = 180 \text{ mm} \quad r = \frac{1}{2}d = 90 \text{ mm} \quad t = 12 \text{ mm}$$

Torsion: $C_1 = 90 \text{ mm} \quad C_2 = 90 + 12 = 102 \text{ mm}$

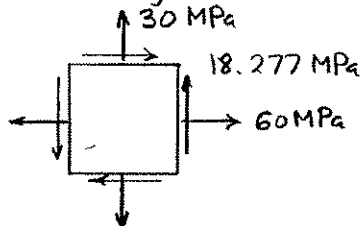
$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12 \times 10^3)(102 \times 10^{-3})}{66.968 \times 10^{-6}} = 18.277 \text{ MPa}$$

Pressure: $\sigma_x = \frac{pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa} \quad \sigma_y = \frac{pr}{2t} = 30 \text{ MPa}$

Summary of stresses:

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = 30 \text{ MPa}, \quad \tau_{xy} = 18.277 \text{ MPa}$$



$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 68.64 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 21.36 \text{ MPa}$$

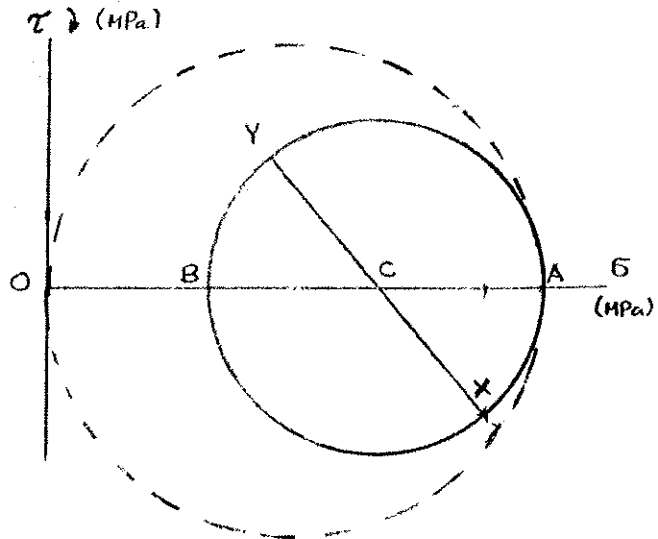
$$\sigma_c \approx 0$$

$$\sigma_{max} = 68.6 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = 0$$

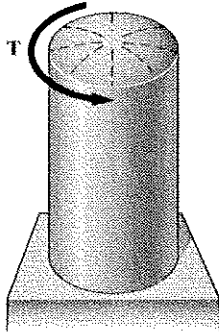
$$\tau_{max} = \frac{1}{2} (\sigma_{max} - \sigma_{min})$$

$$\tau_{max} = 34.3 \text{ MPa} \quad \blacktriangleleft$$



Problem 7.121

7.121 The tank shown has a 180-mm inner diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress is 75 MPa.



$$r = \frac{1}{2} d = \left(\frac{1}{2}\right)(180) = 90 \text{ mm}$$

$$t = 12 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 30 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_1 + \sigma_2) = 45 \text{ MPa}$$

$$\sigma_{max} = 75 \text{ MPa}$$

$$R = \sigma_{max} - \sigma_{ave} = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98 \text{ MPa}$$

$$= 25.98 \times 10^6 \text{ Pa}$$

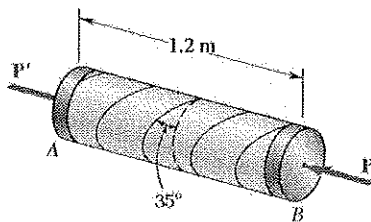
Torsion: $c_1 = 90 \text{ mm}$ $c_2 = 90 + 12 = 102 \text{ mm}$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau_{xy} = \frac{Tc}{J} \quad T = \frac{J \tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3 \text{ N}\cdot\text{m}$$

$$T = 17.06 \text{ kN}\cdot\text{m}$$

Problem 7.122



7.122 A pressure vessel of 250-mm inner diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 40-kN centric axial forces P and P' are applied to the end plates. Determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

$$r = \frac{1}{2}d = \frac{1}{2}(250) = 125 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$$

$$\sigma_2 = \frac{Pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$$

$$r_o = r + t = 125 + 6 = 131 \text{ mm}$$

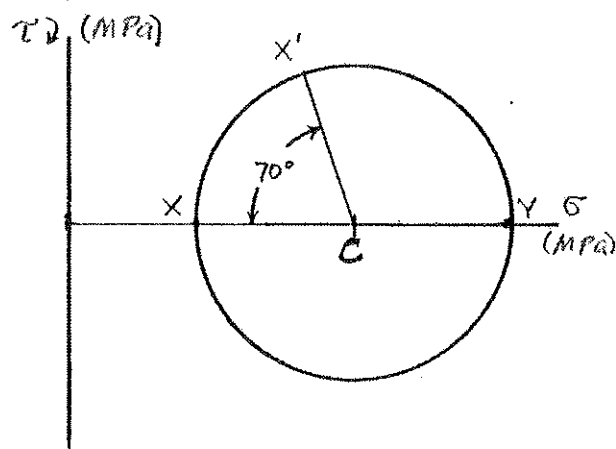
$$A = \pi(r_o^2 - r^2) = \pi(131^2 - 125^2) = 4825 \text{ mm}^2$$

$$\sigma = -\frac{P}{A} = -\frac{40000}{4825} = -8.29 \text{ MPa}$$

Total stresses. Longitudinal $\sigma_x = 20.83 - 8.29 = 15.54 \text{ MPa}$.

Circumferential $\sigma_y = 41.67 \text{ MPa}$.

Shear $\tau_{xy} = 0$



Plotted points for Mohr's circle.

$$X: (15.54, 0)$$

$$Y: (41.67, 0)$$

$$C: (28.605, 0)$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.605 \text{ MPa}$$

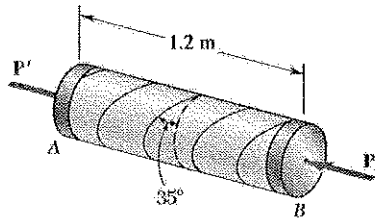
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left[\frac{(15.54 - 41.67)}{2}\right]^2 + 0} = 13.065 \text{ MPa}$$

$$(a) \sigma_{x'} = \sigma_{ave} + R \cos 70^\circ = 28.605 - 13.065 \cos 70^\circ = 24.1 \text{ MPa} \quad \blacktriangleleft$$

$$(b) |\tau_{xy}| = R \sin 70^\circ = 13.065 \sin 70^\circ = 12.3 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.123



7.123 Solve Prob. 7.122, assuming that the magnitude P of the two forces is increased to 120 kN.

7.122 A pressure vessel of 250-mm inner diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 40-kN centric axial forces P and P' are applied to the end plates. Determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

$$r = \frac{1}{2}d = \frac{1}{2}(250) = 125 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$$

$$\sigma_2 = \frac{Pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$$

$$r_o = r + t = 125 + 6 = 131 \text{ mm}$$

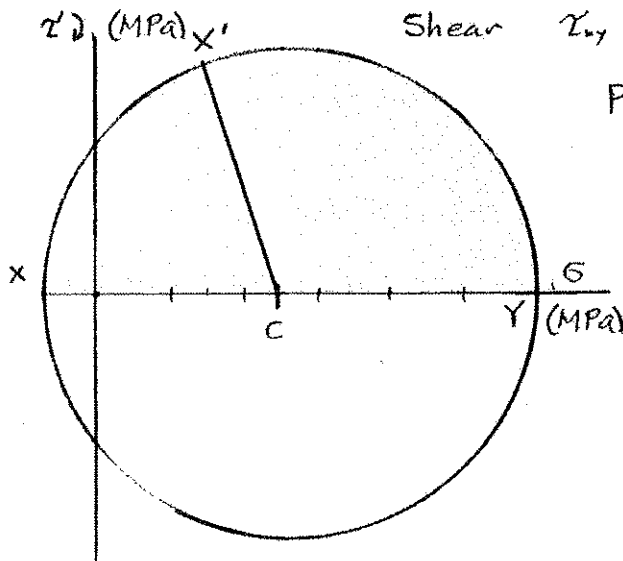
$$A = \pi(r_o^2 - r^2) = \pi(131^2 - 125^2) = 4825 \text{ mm}^2$$

$$\sigma = -\frac{P}{A} = -\frac{120 \times 10^3}{4825} = -24.87$$

Total stresses. Longitudinal $\sigma_x = 20.83 - 24.87 = -4.04 \text{ MPa}$

Circumferential $\sigma_y = 41.67 \text{ MPa}$

Shear $\tau_{xy} = 0$



Plotted points for Mohr's circle.

$$X: (-4.04, 0)$$

$$Y: (41.67, 0)$$

$$C: (18.815, 0)$$

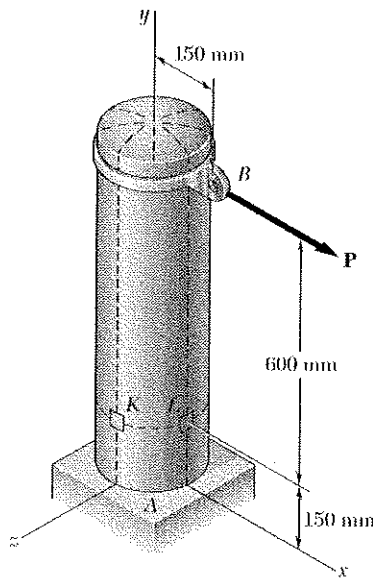
$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 18.815 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-4.04 - 41.67}{2}\right)^2 + 0} = 22.855 \text{ MPa}$$

$$(a) \quad \sigma_{x'} = \sigma_{ave} - R \cos 70^\circ = 18.815 - 22.855 \cos 70^\circ = 11 \text{ MPa}$$

$$(b) \quad |\tau_{x'y'}| = R \sin 70^\circ = 22.855 \sin 70^\circ = 21.5 \text{ MPa}$$

Problem 7.124



7.124 The compressed-air tank AB has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force P is applied at B in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point K .

Consider element at point K .

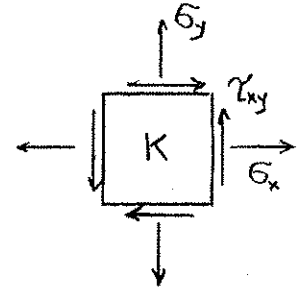
Stresses due to internal pressure.

$$p = 5 \text{ MPa} = 5 \times 10^6 \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117 \text{ mm}$$

$$\sigma_x = \frac{pr}{t} = \frac{(5 \times 10^6)(117 \times 10^{-3})}{(8 \times 10^{-3})} = 73.125 \text{ MPa}$$

$$\sigma_y = \frac{pr}{2t} = \frac{(5 \times 10^6)(117 \times 10^{-3})}{(2)(8 \times 10^{-3})} = 36.563 \text{ MPa}$$



Stress due to bending moment. Point K is on the neutral axis.

$$\sigma_y = 0$$

Stress due to transverse shear. $V = P = 40 \times 10^3 \text{ N}$

$$c_2 = \frac{1}{2}d = 125 \text{ mm} \quad c_1 = c_2 - t = 117 \text{ mm}$$

$$Q = \frac{2}{3}(c_2^3 - c_1^3) = \frac{2}{3}(125^3 - 117^3) = 234.34 \times 10^3 \text{ mm}^3 = 234.34 \times 10^{-6} \text{ m}^3$$

$$I_y = \frac{\pi}{4}(c_2^4 - c_1^4) = \frac{\pi}{4}(125^4 - 117^4) = 44.573 \times 10^6 \text{ mm}^4 = 44.573 \times 10^{-6} \text{ m}^4$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{PQ}{I(2t)} = \frac{(40 \times 10^3)(234.34 \times 10^{-6})}{(44.573 \times 10^{-6})(16 \times 10^{-3})} = 13.144 \times 10^6 \text{ Pa} = 13.144 \text{ MPa}$$

Total stresses $\sigma_x = 73.125 \text{ MPa}$, $\sigma_y = 36.563 \text{ MPa}$, $\tau_{xy} = 13.144 \text{ MPa}$

Mohr's circle

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 54.844 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(18.281)^2 + (13.144)^2} = 22.516 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 77.360 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 32.328 \text{ MPa}$$

continued

Problem 7.124 continued

Principal stresses.

$$\sigma_a = 77.4 \text{ MPa}, \sigma_b = 32.3 \text{ MPa}$$

The 3rd principal stress is the radial stress.

$$\sigma_2 \approx 0$$

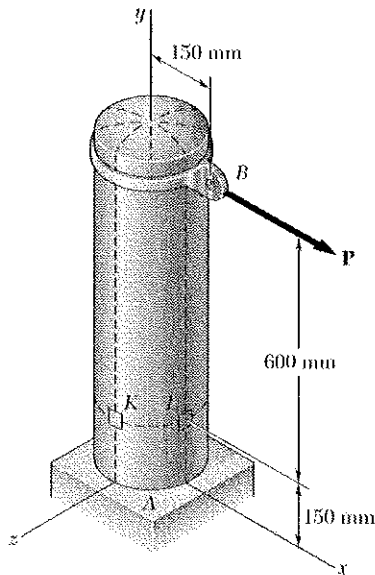
$$\sigma_{\max} = 77.4 \text{ MPa}, \sigma_{\min} = 0$$

Maximum shearing stress.

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 38.7 \text{ MPa}$$

Problem 7.125

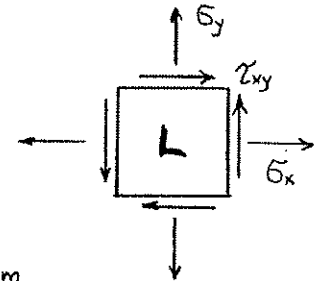


7.125 In Prob. 7.124, determine the maximum normal stress and the maximum shearing stress at point L.

7.124 The compressed-air tank AB has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force P is applied at B in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point K.

Consider element at point L.

Stresses due to internal pressure.



$$p = 5 \text{ MPa} = 5 \times 10^6 \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117 \text{ mm}$$

$$\sigma_x = \frac{pr}{t} = \frac{(5 \times 10^6)(117 \times 10^{-3})}{8 \times 10^{-3}} = 73.125 \text{ MPa}$$

$$\sigma_y = \frac{pr}{2t} = \frac{(5 \times 10^6)(117 \times 10^{-3})}{(2)(8 \times 10^{-3})} = 36.563 \text{ MPa}$$

Stress due to bending moment. $M = (40 \text{ kN})(600 \text{ mm}) = 24000 \text{ N} \cdot \text{m}$

$$c_2 = \frac{1}{2}d = 125 \text{ mm}, c_1 = c_2 - t = 125 - 8 = 117 \text{ mm}$$

$$I = \frac{\pi}{4}(c_2^4 - c_1^4) = \frac{\pi}{4}(125^4 - 117^4) = 44.573 \times 10^6 \text{ mm}^4 = 44.573 \times 10^{-6} \text{ m}^4$$

$$\sigma_y = -\frac{Mc_1}{I} = -\frac{(24000)(125 \times 10^{-3})}{44.573 \times 10^{-6}} = -67.305 \text{ MPa}$$

Stress due to transverse shear. Point L lies in a plane of symmetry.

$$\tau_{xy} = 0$$

Total stresses. $\sigma_x = 73.125 \text{ MPa}, \sigma_y = -30.742 \text{ MPa}, \tau_{xy} = 0$

Principal stresses. Since $\tau_{xy} = 0$, σ_x and σ_y are principal stresses.

The 3rd principal stress is in the radial direction. $\sigma_2 \approx 0$.

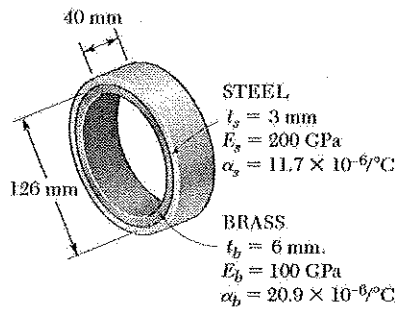
$$\sigma_{\max} = 73.125 \text{ MPa}, \sigma_{\min} = 0 \quad / \quad \sigma_a = 73.1 \text{ MPa}, \sigma_b = -30.7 \text{ MPa}, \sigma_2 = 0$$

Maximum shearing stress

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$$\tau_{\max} = 51.9 \text{ MPa}$$

Problem 7.126



7.126 A brass ring of 126-mm outer diameter and 6-mm thickness fits exactly inside a steel ring of 126-mm inner diameter and 3-mm thickness when the temperature of both rings is 10°C . Knowing that the temperature of both rings is then raised to 52°C , determine (a) the tensile stress in the steel ring, (b) the corresponding pressure exerted by the brass ring on the steel ring.

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p , $\sigma_s = \frac{pr}{t_s}$ (1)

Corresponding strain $\epsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$

Strain due to temperature change $\epsilon_{st} = \alpha_s \Delta T$

Total strain $\epsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \epsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p , $\sigma_b = -\frac{pr}{t_b}$

Corresponding strains $\epsilon_{bp} = -\frac{pr}{E_b t_b}$, $\epsilon_{bt} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \epsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b

$$\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \quad (2)$$

Data: $\Delta T = 52^\circ\text{C} - 10^\circ\text{C} = 42^\circ\text{C}$

$$r = \frac{1}{2}d = \frac{1}{2}(126) = 63 \text{ mm}$$

From Eq. (2) $\left[\frac{63 \times 10^{-3}}{(200 \times 10^9)(0.003)} + \frac{63 \times 10^{-3}}{(100 \times 10^9)(0.006)} \right] p = (20.9 - 11.7)(10^{-6})(38)$

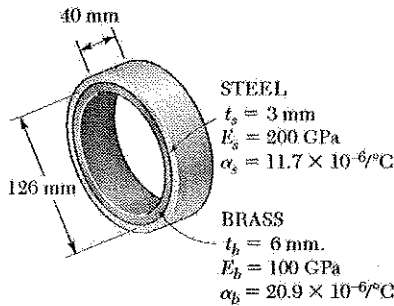
$$2.1 \times 10^{-10} p = 3.496 \times 10^{-4} \quad p = 1.67 \text{ MPa}$$

From Eq. (1) $\sigma_s = \frac{pr}{t_s} = \frac{(1.67)(63)}{3} = 35.1 \text{ MPa}$

(a) $\sigma_s = 35.1 \text{ MPa}$

(b) $p = 1.67 \text{ MPa}$

Problem 7.127



7.127 Solve Prob. 7.126, assuming that the brass ring is 3 mm thick and the steel ring is 6 mm thick.

7.126 A brass ring of 126-mm outer diameter and 6-mm thickness fits exactly inside a steel ring of 126-mm inner diameter and 3-mm thickness when the temperature of both rings is 10°C. Knowing that the temperature of both rings is then raised to 52°C, determine (a) the tensile stress in the steel ring, (b) the corresponding pressure exerted by the brass ring on the steel ring.

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p , $\sigma_s = \frac{pr}{t_s} \quad (1)$

Corresponding strain $\epsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$

Strain due to temperature change $\epsilon_{sT} = \alpha_s \Delta T$

Total strain $\epsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \epsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p , $\sigma_b = -\frac{pr}{t_b}$

Corresponding strains $\epsilon_{bp} = -\frac{pr}{E_b t_b}$, $\epsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \epsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b

$$\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \quad (2)$$

Data: $\Delta T = 52^\circ\text{C} - 10^\circ\text{C} = 42^\circ\text{C}$

$$r = \frac{1}{2}d = \frac{1}{2}(126) = 63 \text{ mm}$$

$$\text{From Eq. (2)} \left[\frac{63 \times 10^{-3}}{(200 \times 10^9)(0.003)} + \frac{63 \times 10^{-3}}{(100 \times 10^9)(0.006)} \right] p = (20.9 - 11.7)(10^{-6})38.$$

$$2.625 \times 10^{-10} p = 3.496 \times 10^{-4} \quad p = 1.33 \text{ MPa}$$

$$\text{From Eq. (1)} \quad \sigma_s = \frac{pr}{t_s} = \frac{(1.33)(63)}{3} = 13.3 \text{ MPa}$$

(a) $\sigma_s = 14 \text{ MPa}$

(b) $p = 1.33 \text{ MPa}$

Problem 7.128

7.128 through 7.131 For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$\underline{\epsilon_x = -240 \mu, \epsilon_y = +320 \mu, \gamma_{xy} = -330 \mu, \theta = 65^\circ} = +65^\circ$$

$$\frac{\epsilon_x + \epsilon_y}{2} = 40 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -280 \mu \quad \frac{\gamma_{xy}}{2} = -165 \mu$$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$= 40 + (-280) \cos 130^\circ + (-165) \sin 130^\circ$$

$$\epsilon_{x'} = 93.6 \mu \quad \blacktriangleleft$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$= 40 - (-280) \cos 130^\circ - (-165) \sin 130^\circ$$

$$\epsilon_{y'} = -13.58 \mu \quad \blacktriangleleft$$

$$\gamma_{x'y'} = -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta$$

$$= -(-240 - 320) \sin 130^\circ + (-330) \cos 130^\circ$$

$$\gamma_{x'y'} = 641 \mu \quad \blacktriangleleft$$

Problem 7.129

7.128 through 7.131 For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$\underline{\epsilon_x = +240 \mu, \epsilon_y = +160 \mu, \gamma_{xy} = +150 \mu, \theta = 60^\circ} = -60^\circ$$

$$\frac{\epsilon_x + \epsilon_y}{2} = +200 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = 40 \mu \quad \frac{\gamma_{xy}}{2} = 75 \mu$$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$= 200 + 40 \cos(-120^\circ) + 75 \sin(-120^\circ)$$

$$\epsilon_{x'} = 115.0 \mu \quad \blacktriangleleft$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$= 200 - 40 \cos(-120^\circ) - 75 \sin(-120^\circ)$$

$$\epsilon_{y'} = 285 \mu \quad \blacktriangleleft$$

$$\gamma_{x'y'} = -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta$$

$$= -(240 - 160) \sin(-120^\circ) + 150 \cos(-120^\circ)$$

$$\gamma_{x'y'} = -5.72 \mu \quad \blacktriangleleft$$

Problem 7.130

7.128 through 7.131 For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$\underline{\epsilon_x = 350 \mu, \epsilon_y = 0, \gamma_{xy} = +120 \mu, \theta = 15^\circ} \Rightarrow -15^\circ$$

$$\frac{\epsilon_x + \epsilon_y}{2} = 175 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = 175 \mu \quad \frac{\gamma_{xy}}{2} = 60 \mu$$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$= 175 + 175 \cos(-30^\circ) + 60 \sin(-30^\circ)$$

$$\epsilon_{x'} = 297 \mu$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin(-30^\circ)$$

$$= 175 - 175 \cos(-30^\circ) - 60 \sin(-30^\circ)$$

$$\epsilon_{y'} = 53.4 \mu$$

$$\gamma_{x'y'} = -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta$$

$$= -(350 - 0) \sin(-30^\circ) + 120 \cos(-30^\circ)$$

$$\gamma_{x'y'} = 279 \mu$$

Problem 7.131

7.128 through 7.131 For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$\underline{\epsilon_x = 0, \epsilon_y = +320 \mu, \gamma_{xy} = -100 \mu, \theta = 30^\circ} \Rightarrow +30^\circ$$

$$\frac{\epsilon_x + \epsilon_y}{2} = 160 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -160 \mu$$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$= 160 - 160 \cos 60^\circ - \frac{100}{2} \sin 60^\circ$$

$$\epsilon_{x'} = +36.7 \mu$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$= 160 + 160 \cos 60^\circ + \frac{100}{2} \sin 60^\circ$$

$$\epsilon_{y'} = +283 \mu$$

$$\gamma_{x'y'} = -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta$$

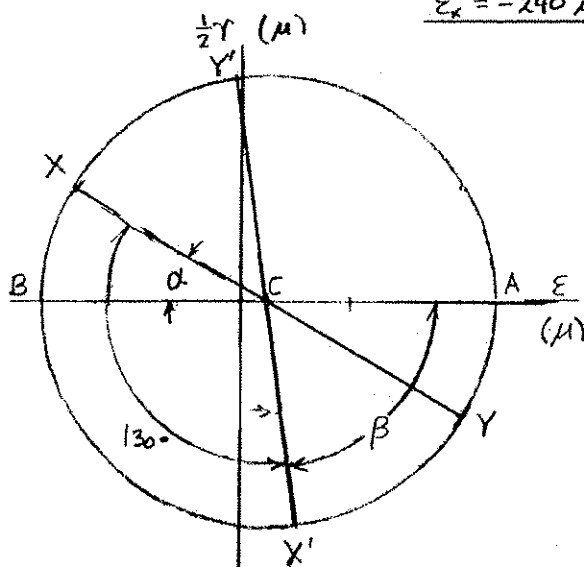
$$= -(0 - 320) \sin 60^\circ - 100 \cos 60^\circ$$

$$\gamma_{x'y'} = +227 \mu$$

Problem 7.132

7.132 through 7.135 For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$\epsilon_x = -240 \mu, \epsilon_y = 320 \mu, \gamma_{xy} = -330 \mu, \theta = 65^\circ$$



$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 40 \mu$$

Plotted points for Mohr's circle:

$$X: (-240 \mu, 165 \mu)$$

$$Y: (320 \mu, -165 \mu)$$

$$C: (40 \mu, 0)$$

$$\tan \alpha = \frac{165}{280} = 0.58929 \quad \alpha = 30.5^\circ$$

$$\beta = 180^\circ - 130^\circ + \alpha = 80.5^\circ$$

$$R = \sqrt{(280)^2 + (165)^2} = 325 \mu$$

$$\epsilon_{x'} = \epsilon_{ave} + R \cos \beta$$

$$\epsilon_{x'} = 93.6 \mu$$

$$\epsilon_{y'} = \epsilon_{ave} - R \cos \beta$$

$$\epsilon_{y'} = -13.6 \mu$$

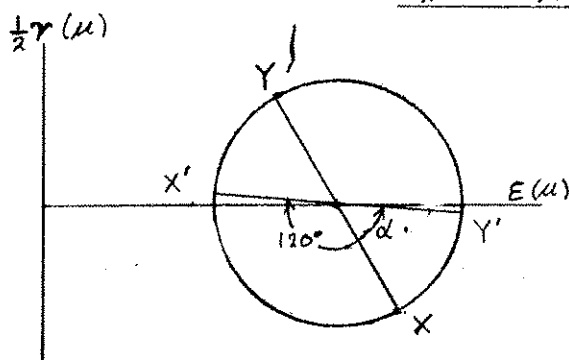
$$\frac{1}{2} \gamma_{x'y'} = R \sin \beta = 320.5 \mu$$

$$\gamma_{x'y'} = 641 \mu$$

Problem 7.133

7.132 through 7.135 For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$\epsilon_x = +240 \mu, \epsilon_y = +160 \mu, \gamma_{xy} = +150 \mu, \theta = 60^\circ$$



Plotted points for Mohr's circle:

$$X: (+240 \mu, 75 \mu)$$

$$Y: (+160 \mu, -75 \mu)$$

$$C: (+200 \mu, 0)$$

$$\tan \alpha = \frac{75}{40} = 1.875 \quad \alpha = 61.93^\circ$$

$$R = \sqrt{(40 \mu)^2 + (75 \mu)^2} = 85 \mu$$

$$\beta = 2\theta - \alpha = -120^\circ - 61.93^\circ = -181.93^\circ$$

$$\epsilon_{x'} = \epsilon_{ave} + R \cos \beta = 200 \mu + (85 \mu) \cos(-181.93^\circ)$$

$$\epsilon_{x'} = 115.0 \mu$$

$$\epsilon_{y'} = \epsilon_{ave} - R \cos \beta = 200 \mu - (85 \mu) \cos(-181.93^\circ)$$

$$\epsilon_{y'} = 285 \mu$$

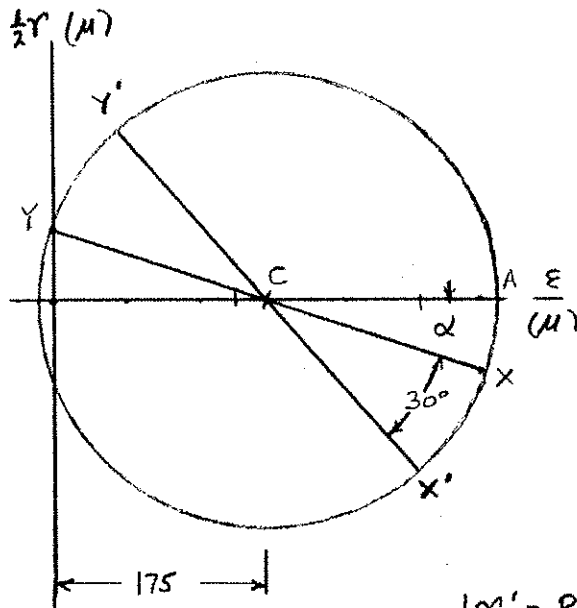
$$\frac{1}{2} \gamma_{x'y'} = -R \sin \beta = -85 \sin(-181.93^\circ) = -2.86 \mu$$

$$\gamma_{x'y'} = -5.72 \mu$$

Problem 7.134

7.132 through 7.135 For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$E_x = 350 \mu, E_y = 0, \gamma_{xy} = +120 \mu, \theta = 15^\circ$$



$$E_{ave} = \frac{1}{2}(E_x + E_y) = 175 \mu$$

Plotted points for Mohr's circle:

$$X: (350 \mu, -60 \mu)$$

$$Y: (0, +60 \mu)$$

$$C: (175, 0)$$

$$\tan \alpha = \frac{60}{175} = 0.34286 \quad \alpha = 18.92^\circ$$

$$\beta = \alpha + 30^\circ = 48.92^\circ$$

$$R = \sqrt{(175)^2 + (60)^2} = 185 \mu$$

$$E_{x'} = E_{ave} + R \cos \beta \quad E_{x'} = 297 \mu$$

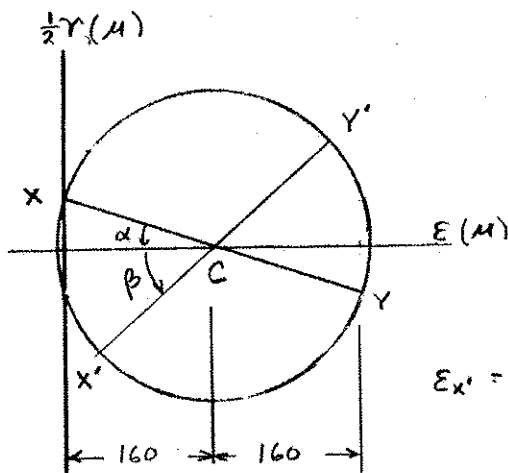
$$E_{y'} = E_{ave} - R \cos \beta \quad E_{y'} = 53.4 \mu$$

$$\frac{1}{2} \gamma_{x'y'} = R \sin \beta = 139.5 \mu \quad \gamma_{x'y'} = 279 \mu$$

Problem 7.135

7.132 through 7.135 For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ .

$$E_x = 0, E_y = +320 \mu, \gamma_{xy} = -100 \mu, \theta = 30^\circ$$



Plotted points for Mohr's circle:

$$X: (0, 50 \mu)$$

$$Y: (320 \mu, -50 \mu)$$

$$C: (160 \mu, 0)$$

$$\tan \alpha = \frac{50}{160} \quad \alpha = 17.35^\circ$$

$$R = \sqrt{(160 \mu)^2 + (50 \mu)^2} = 167.63 \mu$$

$$\beta = 2\theta - \alpha = 60^\circ - 17.35^\circ = 42.65^\circ$$

$$E_{x'} = E_{ave} - R \cos \beta = 160 \mu - (167.63 \mu) \cos 42.65^\circ \quad E_{x'} = 36.7 \mu$$

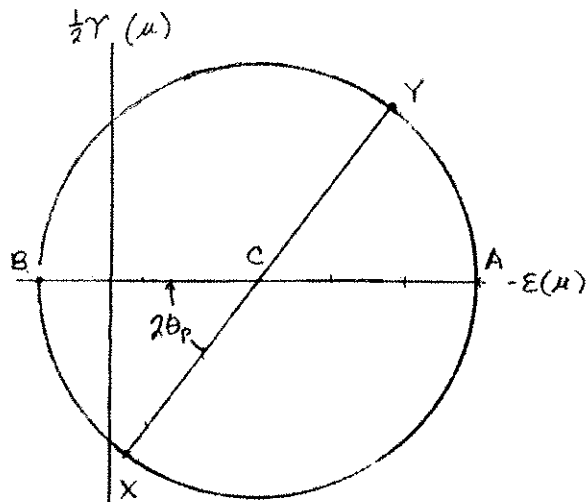
$$E_{y'} = E_{ave} + R \cos \beta = 160 \mu + (167.63 \mu) \cos 42.65^\circ \quad E_{y'} = 283 \mu$$

$$\frac{1}{2} \gamma_{x'y'} = R \sin \beta = (167.63 \mu) \sin 42.65^\circ \quad \gamma_{x'y'} = 227 \mu$$

Problem 7.136

7.136 through 7.139 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$$\epsilon_x = +30 \mu, \quad \epsilon_y = +570 \mu, \quad \gamma_{xy} = +720 \mu$$



Plotted points for Mohr's circle:

$$X: (30 \mu, -360 \mu)$$

$$Y: (570 \mu, +360 \mu)$$

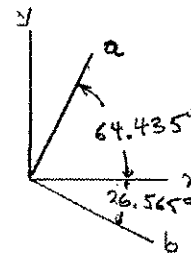
$$C: (300 \mu, 0)$$

$$\tan 2\theta_p = \frac{-360}{270} = -1.3333$$

$$2\theta_p = -53.13^\circ$$

$$(a) \quad \theta_b = -26.565^\circ \quad \blacktriangleleft$$

$$\theta_a = 64.435^\circ \quad \blacktriangleleft$$



$$R = \sqrt{(270 \mu)^2 + (360 \mu)^2} = 450 \mu$$

$$\epsilon_a = \epsilon_{ave} + R = 300 \mu + 450 \mu$$

$$\epsilon_a = 750 \mu \quad \blacktriangleleft$$

$$\epsilon_b = \epsilon_{ave} - R = 300 \mu - 450 \mu$$

$$\epsilon_b = -150 \mu \quad \blacktriangleleft$$

$$(b) \quad \gamma_{max (in-plane)} = 2R = 900 \mu \quad \blacktriangleleft$$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{1/3}{2/3} (750 \mu - 150 \mu)$$

$$\epsilon_c = -300 \mu \quad \blacktriangleleft$$

$$\epsilon_{max} = \epsilon_a = 750 \mu, \quad \epsilon_{min} = \epsilon_c = -300 \mu$$

$$(c) \quad \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 750 \mu - (-300 \mu)$$

$$\gamma_{max} = 1050 \mu \quad \blacktriangleleft$$

Problem 7.137

7.136 through 7.139 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

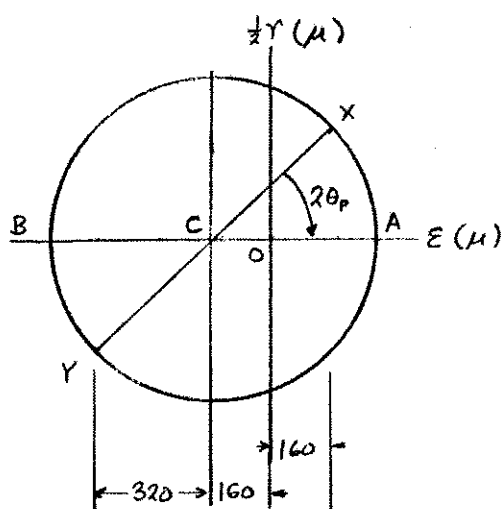
$$\epsilon_x = +160 \mu, \quad \epsilon_y = -480 \mu, \quad \gamma_{xy} = -600 \mu$$

(a) For Mohr's circle of strain, plot points

$$X: (160 \mu, 300 \mu)$$

$$Y: (-480 \mu, -300 \mu)$$

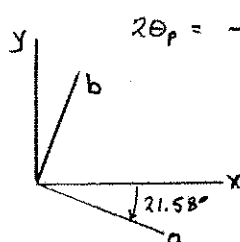
$$C: (-160 \mu, 0)$$



$$(a) \tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-300}{320} = -0.9375$$

$$2\theta_p = -43.15^\circ \quad \theta_p = -21.58^\circ$$

$$\text{and } -21.58 + 90 = 68.42^\circ$$



$$\theta_a = -21.58^\circ$$

$$\theta_b = 68.42^\circ$$

$$R = \sqrt{(320 \mu)^2 + (300 \mu)^2} = 438.6 \mu$$

$$\epsilon_a = \epsilon_{ave} + R = -160 \mu + 438.6 \mu \quad \epsilon_a = +278.6 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -160 \mu - 438.6 \mu \quad \epsilon_b = -598.6 \mu$$

$$(b) \frac{1}{2} \gamma_{(\max, \text{ in-plane })} = R \quad \gamma_{(\max, \text{ in-plane })} = 2R = 877 \mu$$

$$(c) \epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3} (160 \mu - 480 \mu)$$

$$\epsilon_c = 160.0 \mu$$

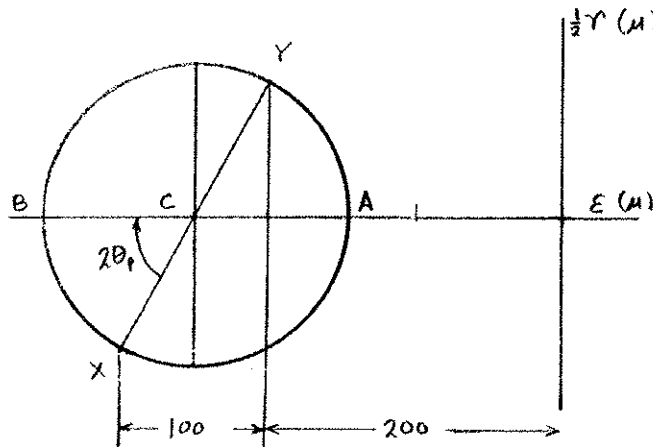
$$\epsilon_{\max} = 278.6 \mu \quad \epsilon_{\min} = -598.6 \mu$$

$$\gamma_{\max} = \epsilon_{\max} - \epsilon_{\min} = 278.6 \mu + 598.6 \mu \quad \gamma_{\max} = 877 \mu$$

Problem 7.138

7.136 through 7.139 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$$\epsilon_x = -600 \mu, \quad \epsilon_y = -400 \mu, \quad \gamma_{xy} = +350 \mu$$



Plotted points for Mohr's circle:

$$X: (-600 \mu, -175 \mu)$$

$$Y: (-400 \mu, +175 \mu)$$

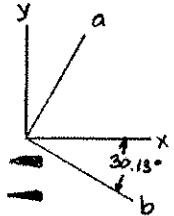
$$C: (-500 \mu, 0)$$

$$\tan 2\theta_p = -\frac{175}{100}$$

$$2\theta_p = -60.26^\circ$$

$$\theta_b = -30.13^\circ$$

$$\theta_a = 59.87^\circ$$



$$R = \sqrt{(100 \mu)^2 + (175 \mu)^2} = 201.6 \mu$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = -500 \mu + 201.6 \mu$$

$$\epsilon_a = -298 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -500 \mu - 201.6 \mu$$

$$\epsilon_b = -702 \mu$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 403 \mu$$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3} (-600 \mu - 400 \mu)$$

$$\epsilon_c = 500 \mu$$

$$\epsilon_{max} = 500 \mu \quad \epsilon_{min} = -702 \mu$$

$$(c) \quad \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 500 \mu + 702 \mu$$

$$\gamma_{max} = 1202 \mu$$

Problem 7.139

7.136 through 7.139 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

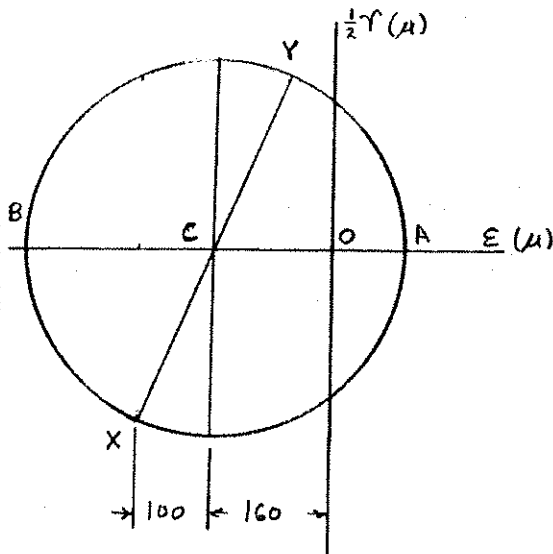
$$\epsilon_x = -260 \mu, \quad \epsilon_y = -60 \mu, \quad \gamma_{xy} = +480 \mu$$

For Mohr's circle of strain plot points

$$X: (-260 \mu, -240 \mu)$$

$$Y: (-60 \mu, 240 \mu)$$

$$C: (-160 \mu, 0)$$

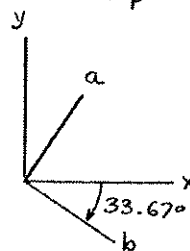


$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{480}{-260 - 60} = -2.4$$

$$2\theta_p = -67.38^\circ$$

$$\theta_b = -33.67^\circ$$

$$\theta_a = 56.31^\circ$$



$$R = \sqrt{(100 \mu)^2 + (240 \mu)^2}$$

$$R = 260 \mu$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = -160 \mu + 260 \mu$$

$$\epsilon_a = 100 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -160 \mu - 260 \mu$$

$$\epsilon_b = -420 \mu$$

$$(b) \quad \frac{1}{2} \gamma_{max, (in-plane)} = R \quad \gamma_{max, (in-plane)} = 2R \quad \gamma_{max, (in-plane)} = 520 \mu$$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3} (-260 - 60) = 160 \mu$$

$$\epsilon_{max} = 160 \mu$$

$$\epsilon_{min} = -420 \mu$$

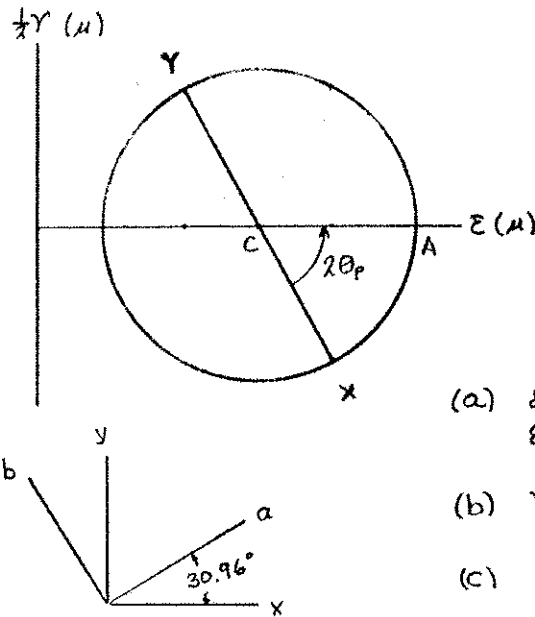
$$(c) \quad \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 160 \mu + 420 \mu$$

$$\gamma_{max} = 580 \mu$$

Problem 7.140

7.140 through 7.143 For the given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$\epsilon_x = +400 \mu, \quad \epsilon_y = +200 \mu, \quad \gamma_{xy} = 375 \mu$$



Plotted points for Mohr's circle:

$$X: (+400 \mu, -187.5 \mu)$$

$$Y: (+200 \mu, +187.5 \mu)$$

$$C: (+300 \mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{375}{400 - 200} = 1.875$$

$$2\theta_p = 61.93^\circ \quad \theta_a = 30.96^\circ, \quad \theta_b = 120.96^\circ$$

$$R = \sqrt{(100 \mu)^2 + (187.5 \mu)^2} = 212.5 \mu$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = 300 \mu + 212.5 \mu = 512.5 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = 300 \mu - 212.5 \mu = 87.5 \mu$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 425 \mu$$

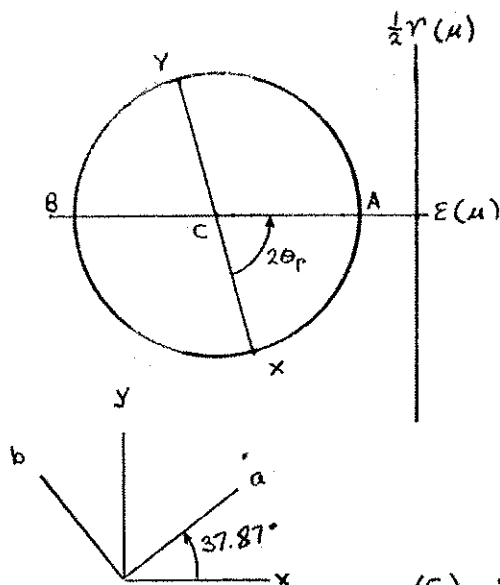
$$(c) \quad \epsilon_c = 0 \quad \epsilon_{max} = 512.5 \mu \quad \epsilon_{min} = 0$$

$$\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 512.5 \mu$$

Problem 7.141

7.140 through 7.143 For the given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$\epsilon_x = -180 \mu, \quad \epsilon_y = -260 \mu, \quad \gamma_{xy} = +315 \mu$$



Plotted points for Mohr's circle:

$$X: (-180 \mu, -157.5 \mu) \quad Y: (-260 \mu, +157.5 \mu)$$

$$C: (-220 \mu, 0)$$

$$(a) \quad \tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{315}{-180 - (-260)} = \frac{315}{80} = 3.9375$$

$$2\theta_p = 75.75^\circ \quad \theta_a = 37.87^\circ$$

$$\theta_b = 127.87^\circ$$

$$R = \sqrt{(40 \mu)^2 + (157.5 \mu)^2} = 162.5 \mu$$

$$\epsilon_a = \epsilon_{ave} + R = -220 \mu + 162.5 \mu = -57.5 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -220 \mu - 162.5 \mu = -382.5 \mu$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 325 \mu$$

$$(c) \quad \epsilon_c = 0 \quad \epsilon_{max} = 0, \quad \epsilon_{min} = -382.5 \mu$$

$$\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 0 + 382.5 \mu = 382.5 \mu$$

Problem 7.142

7.140 through 7.143 For the given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$\epsilon_x = +60 \mu, \quad \epsilon_y = +240 \mu, \quad \gamma_{xy} = -50 \mu$$

Plotted points: $X: (60 \mu, 25 \mu)$
 $Y: (240 \mu, -25 \mu)$ $C: (150 \mu, 0)$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-50}{60 - 240} = 0.277778$$

$$2\theta_p = 15.52^\circ \quad \theta_b = 7.76^\circ \quad \theta_a = 97.76^\circ$$

$$R = \sqrt{(90 \mu)^2 + (25 \mu)^2} = 93.4 \mu$$

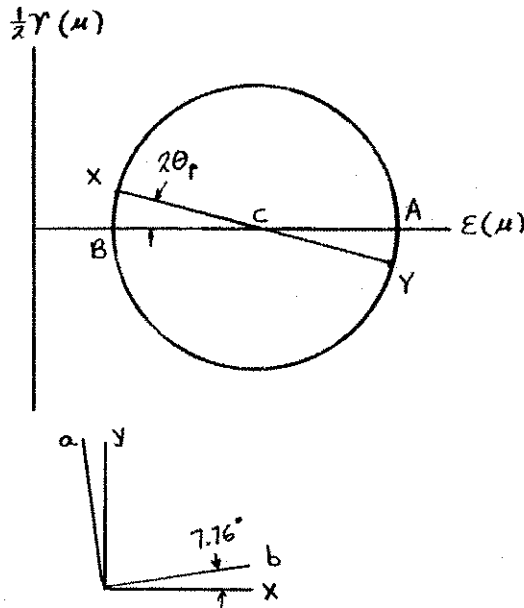
$$(a) \quad \epsilon_a = \epsilon_{ave} + R = 150 \mu + 93.4 \mu = 243.4 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = 150 \mu - 93.4 \mu = 56.6 \mu$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 186.8 \mu$$

$$(c) \quad \epsilon_c = 0 \quad \epsilon_{max} = 243.4 \mu \quad \epsilon_{min} = 0$$

$$\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 243.4$$



Problem 7.143

7.140 through 7.143 For the given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$\epsilon_x = +300 \mu, \quad \epsilon_y = +60 \mu, \quad \gamma_{xy} = +100 \mu$$

$$X: (300 \mu, -50 \mu) \quad Y: (60 \mu, 50 \mu)$$

$$C: (180 \mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{100}{300 - 60} = 22.62^\circ$$

$$\theta_a = 11.31^\circ \quad \theta_b = 101.31^\circ$$

$$R = \sqrt{(120 \mu)^2 + (50 \mu)^2} = 130 \mu$$

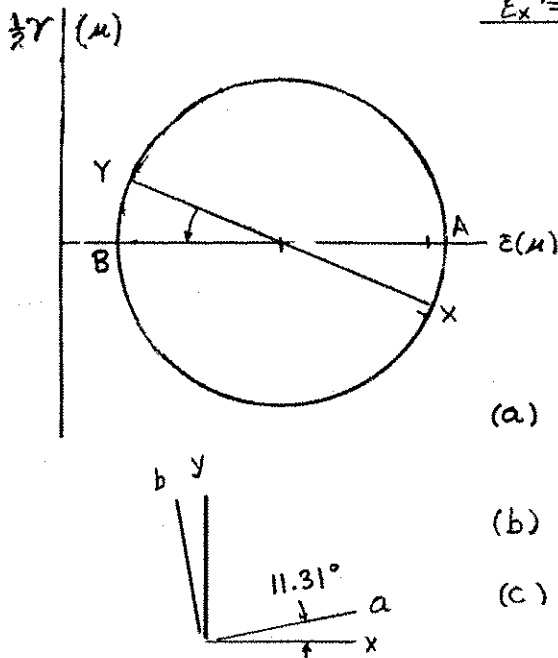
$$(a) \quad \epsilon_a = \epsilon_{ave} + R = 180 \mu + 130 \mu = 310 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = 180 \mu - 130 \mu = 50 \mu$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 260 \mu$$

$$(c) \quad \epsilon_c = 0 \quad \epsilon_{max} = 310 \mu \quad \epsilon_{min} = 0$$

$$\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 310 \mu$$



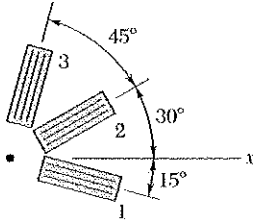
Problem 7.144

7.144 Determine the strain ϵ_x knowing that the following strains have been determined by use of the rosette shown:

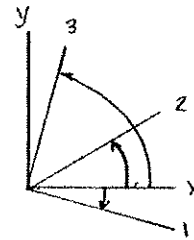
$$\epsilon_1 = +480\mu$$

$$\epsilon_2 = -120\mu$$

$$\epsilon_3 = +80\mu$$



$$\begin{aligned}\theta_1 &= -15^\circ \\ \theta_2 &= 30^\circ \\ \theta_3 &= 75^\circ\end{aligned}$$



$$\begin{aligned}E_x \cos^2 \theta_1 + E_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 &= \epsilon_1 \\ 0.9330 E_x + 0.06699 E_y - 0.25 \gamma_{xy} &= 480\mu\end{aligned}\quad (1)$$

$$\begin{aligned}E_x \cos^2 \theta_2 + E_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 &= \epsilon_2 \\ 0.75 E_x + 0.25 E_y + 0.4330 \gamma_{xy} &= -120\mu\end{aligned}\quad (2)$$

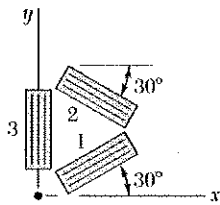
$$\begin{aligned}E_x \cos^2 \theta_3 + E_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 &= \epsilon_3 \\ 0.06699 E_x + 0.9330 E_y + 0.25 \gamma_{xy} &= 80\mu\end{aligned}\quad (3)$$

Solving (1), (2), and (3) simultaneously,

$$E_x = 253\mu \quad E_y = 307\mu \quad \gamma_{xy} = -893\mu$$

$$E_x = 253\mu$$

Problem 7.145



7.145 The strains determined by the use of the rosette shown during the test of a machine element are

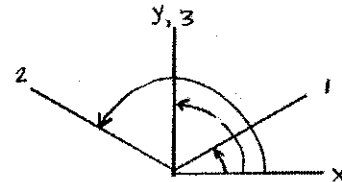
$$\epsilon_1 = +600\mu$$

$$\epsilon_2 = +450\mu$$

$$\epsilon_3 = -75\mu$$

Determine (a) the in-plane principal strains, (b) the in-plane maximum shearing strain.

$$\begin{aligned}\theta_1 &= 30^\circ \\ \theta_2 &= 150^\circ \\ \theta_3 &= 90^\circ\end{aligned}$$



$$\epsilon_x \cos^2 \theta_1 + \epsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \epsilon_1$$

$$0.75 \epsilon_x + 0.25 \epsilon_y + 0.43301 \gamma_{xy} = 600\mu \quad (1)$$

$$\epsilon_x \cos^2 \theta_2 + \epsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \epsilon_2$$

$$0.75 \epsilon_x + 0.25 \epsilon_y - 0.43301 \gamma_{xy} = 450\mu \quad (2)$$

$$\epsilon_x \cos^2 \theta_3 + \epsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \epsilon_3$$

$$0 + \epsilon_y + 0 = -75\mu \quad (3)$$

Solving (1), (2), and (3) simultaneously,

$$\epsilon_x = 725\mu, \quad \epsilon_y = -75\mu, \quad \gamma_{xy} = 173.21\mu$$

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 325\mu$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725 - (-75)}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3\mu$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = 734\mu$$

$$\epsilon_a = 734\mu \quad \blacktriangleleft$$

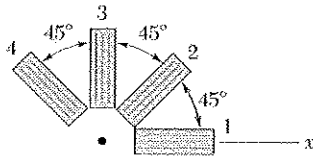
$$\epsilon_b = \epsilon_{ave} - R = -84.3\mu$$

$$\epsilon_b = -84.3\mu \quad \blacktriangleleft$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 819\mu$$

$$\gamma_{max} = 819\mu \quad \blacktriangleleft$$

Problem 7.146



7.146 The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$$\epsilon_1 = +420 \mu$$

$$\epsilon_2 = -45 \mu$$

$$\epsilon_3 = +165 \mu$$

(a) What should be the reading of gage 3? (b) Determine the principal strains and the maximum in-plane shearing strain.

(a) Gages 2 and 4 are 90° apart $E_{ave} = \frac{1}{2} (\epsilon_2 + \epsilon_4)$

$$E_{ave} = \frac{1}{2} (-45 \mu + 165 \mu) = 60 \mu$$

Gages 1 and 3 are also 90° apart $E_{ave} = \frac{1}{2} (\epsilon_1 + \epsilon_3)$

$$\epsilon_3 = 2E_{ave} - \epsilon_1 = (2)(60 \mu) - 420 \mu = -300 \mu$$

(b) $E_x = \epsilon_1 = 420 \mu$ $E_y = \epsilon_3 = -300 \mu$

$$\begin{aligned} \gamma_{xy} &= 2\epsilon_2 - E_x - E_y = (2)(-45 \mu) - 420 \mu + 300 \mu \\ &= -210 \mu \end{aligned}$$

$$\begin{aligned} R &= \sqrt{\left(\frac{E_x - E_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420 \mu + 300 \mu}{2}\right)^2 + \left(\frac{-210 \mu}{2}\right)^2} \\ &= 375 \mu \end{aligned}$$

$$\epsilon_a = E_{ave} + R = 60 \mu + 375 \mu = 435 \mu$$

$$\epsilon_b = E_{ave} - R = 60 \mu - 375 \mu = -315 \mu$$

$$\gamma_{max(in-plane)} = 2R = 750 \mu$$

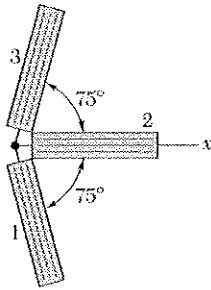
Problem 7.147

7.147 The strains determined by the use of a rosette attached as shown to the surface of a machine element are

$$\epsilon_1 = -93.1 \times 10^{-6} \text{ mm/mm} \quad \epsilon_2 = +385 \times 10^{-6} \text{ mm/mm}$$

$$\epsilon_3 = +210 \times 10^{-6} \text{ mm/mm}$$

Determine (a) the orientation and magnitude of the principal strains in the plane of the rosette, (b) the maximum in-plane shearing stress.



Use $\epsilon_{x'} = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$
 where $\theta = -75^\circ$ for gage 1, $\theta = 0$ for gage 2,
 and $\theta = +75^\circ$ for gage 3.

$$\epsilon_1 = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos(-150^\circ) + \frac{\gamma_{xy}}{2} \sin(-150^\circ) \quad (1)$$

$$\epsilon_2 = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos 0 + \frac{\gamma_{xy}}{2} \sin 0 \quad (2)$$

$$\epsilon_3 = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \cos(150^\circ) + \frac{\gamma_{xy}}{2} \sin(150^\circ) \quad (3)$$

From Eq. (2) $\epsilon_x = \epsilon_2 = 385 \times 10^{-6} \text{ mm/mm}$.

Adding Eq's (1) and (3)

$$\begin{aligned} \epsilon_1 + \epsilon_3 &= (\epsilon_x + \epsilon_y) + (\epsilon_x - \epsilon_y) \cos 150^\circ \\ &= \epsilon_x (1 + \cos 150^\circ) + \epsilon_y (1 - \cos 150^\circ) \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \frac{\epsilon_1 + \epsilon_3 - \epsilon_x (1 + \cos 150^\circ)}{(1 - \cos 150^\circ)} = \frac{-93.1 \times 10^{-6} + 210 \times 10^{-6} - 385 \times 10^{-6} (1 + \cos 150^\circ)}{1 - \cos 150^\circ} \\ &= 35.0 \times 10^{-6} \text{ mm/mm} \end{aligned}$$

Subtracting Eq (1) from Eq (3)

$$\epsilon_3 - \epsilon_1 = \gamma_{xy} \sin 150^\circ$$

$$\gamma_{xy} = \frac{\epsilon_3 - \epsilon_1}{\sin 150^\circ} = \frac{210 \times 10^{-6} - (-93.1 \times 10^{-6})}{\sin 150^\circ} = 606.2 \times 10^{-6} \text{ mm/mm}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{606.2 \times 10^{-6}}{385 \times 10^{-6} - 35.0 \times 10^{-6}} = 1.732 \quad (\alpha) \quad \theta_a = 30.0^\circ, \theta_b = 120.0^\circ$$

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = \frac{1}{2}(385 \times 10^{-6} + 35.0 \times 10^{-6}) = 210 \times 10^{-6} \text{ mm/mm}$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{385 \times 10^{-6} - 35.0 \times 10^{-6}}{2}\right)^2 + \left(\frac{606.2}{2}\right)^2} = 350.0 \times 10^{-6}$$

$$\epsilon_a = \epsilon_{ave} + R = 210 \times 10^{-6} + 350.0 \times 10^{-6} = 560 \times 10^{-6} \text{ mm/mm}$$

$$\epsilon_b = \epsilon_{ave} - R = 210 \times 10^{-6} + 350.0 \times 10^{-6} = -140.0 \times 10^{-6} \text{ mm/mm}$$

$$(b) \quad \frac{\gamma_{max(in plane)}}{2} = R = 350.0 \times 10^{-6} \text{ mm/mm}$$

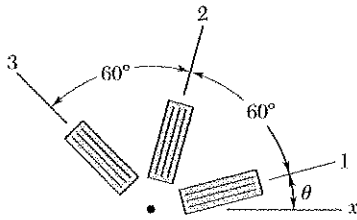
$$\gamma_{max(in plane)} = 700 \times 10^{-6} \text{ mm/mm}$$

Problem 7.148

7.148 Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{avg}$$

where ϵ_{avg} is the abscissa of the center of the corresponding Mohr's circle.



$$\epsilon_1 = \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \quad (1)$$

$$\begin{aligned} \epsilon_2 &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos(2\theta + 120^\circ) + \frac{\gamma_{xy}}{2} \sin(2\theta + 120^\circ) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} (\cos 120^\circ \cos 2\theta - \sin 120^\circ \sin 2\theta) \\ &\quad + \frac{\gamma_{xy}}{2} (\cos 120^\circ \sin 2\theta + \sin 120^\circ \cos 2\theta) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \left(-\frac{1}{2} \cos 2\theta - \frac{\sqrt{3}}{2} \sin 2\theta\right) \\ &\quad + \frac{\gamma_{xy}}{2} \left(-\frac{1}{2} \sin 2\theta + \frac{\sqrt{3}}{2} \cos 2\theta\right) \end{aligned} \quad (2)$$

$$\begin{aligned} \epsilon_3 &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos(2\theta + 240^\circ) + \frac{\gamma_{xy}}{2} \sin(2\theta + 240^\circ) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} (\cos 240^\circ \cos 2\theta - \sin 240^\circ \sin 2\theta) \\ &\quad + \frac{\gamma_{xy}}{2} (\cos 240^\circ \sin 2\theta + \sin 240^\circ \cos 2\theta) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \left(-\frac{1}{2} \cos 2\theta + \frac{\sqrt{3}}{2} \sin 2\theta\right) \\ &\quad + \frac{\gamma_{xy}}{2} \left(-\frac{1}{2} \sin 2\theta - \frac{\sqrt{3}}{2} \cos 2\theta\right) \end{aligned} \quad (3)$$

Adding (1), (2), and (3),

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{ave} + 0 + 0$$

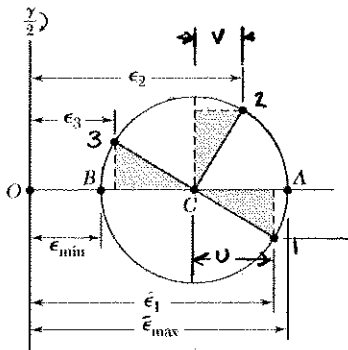
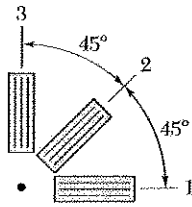
$$3\epsilon_{ave} = \epsilon_1 + \epsilon_2 + \epsilon_3$$

Problem 7.149

7.149 Using a 45° rosette, the strains ϵ_1 , ϵ_2 , and ϵ_3 have been determined at a given point. Using Mohr's circle, show that the principal strains are:

$$\epsilon_{\max, \min} = \frac{1}{2}(\epsilon_1 + \epsilon_3) \pm \frac{1}{\sqrt{2}}[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2]^{\frac{1}{2}}$$

(Hint: The shaded triangles are congruent.)



Since gage directions 1 and 3 are 90° apart,

$$\epsilon_{\text{ave}} = \frac{1}{2}(\epsilon_1 + \epsilon_3)$$

$$\text{Let } U = \epsilon_1 - \epsilon_{\text{ave}} = \frac{1}{2}(\epsilon_1 - \epsilon_3)$$

$$V = \epsilon_2 - \epsilon_{\text{ave}} = \epsilon_2 - \frac{1}{2}(\epsilon_1 + \epsilon_3)$$

$$\begin{aligned} R^2 &= U^2 + V^2 \\ &= \frac{1}{4}(\epsilon_1 - \epsilon_3)^2 + \epsilon_2^2 - \epsilon_2(\epsilon_1 + \epsilon_3) + \frac{1}{4}(\epsilon_1 + \epsilon_3)^2 \\ &= \frac{1}{4}\epsilon_1^2 - \frac{1}{2}\epsilon_1\epsilon_3 + \frac{1}{4}\epsilon_3^2 \\ &\quad + \epsilon_2^2 - \epsilon_2\epsilon_1 - \epsilon_2\epsilon_3 \\ &\quad + \frac{1}{4}\epsilon_1^2 + \frac{1}{2}\epsilon_1\epsilon_3 + \frac{1}{4}\epsilon_3^2 \end{aligned}$$

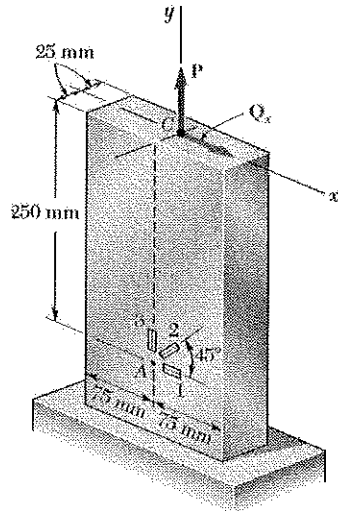
$$= \frac{1}{2}\epsilon_1^2 - \epsilon_2\epsilon_1 + \epsilon_2^2 - \epsilon_2\epsilon_3 + \frac{1}{2}\epsilon_3^2$$

$$= \frac{1}{2}(\epsilon_1 - \epsilon_2)^2 + \frac{1}{2}(\epsilon_2 - \epsilon_3)^2$$

$$R = \frac{1}{\sqrt{2}} [(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2]^{\frac{1}{2}}$$

$$\epsilon_{\max, \min} = \epsilon_{\text{ave}} \pm R \quad \text{gives the required formula.}$$

Problem 7.150



7.150 A centric axial force P and a horizontal force Q are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$$\begin{aligned}\epsilon_1 &= -75 \times 10^{-6} \text{ mm/mm} & \epsilon_2 &= +300 \times 10^{-6} \text{ mm/mm} \\ \epsilon_3 &= +250 \times 10^{-6} \text{ mm/mm}\end{aligned}$$

Knowing that $E = 200 \text{ GPa}$ and $\nu = 0.30$, determine the magnitudes of P and Q .

$$\epsilon_x = \epsilon_1 = -75 \times 10^{-6} \qquad \epsilon_y = \epsilon_3 = 250 \times 10^{-6}$$

$$\gamma_{xy} = 2\epsilon_2 - \epsilon_1 - \epsilon_3 = 425 \times 10^{-6}$$

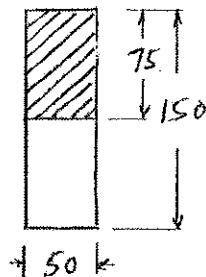
$$\begin{aligned}\sigma_x &= \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{200 \times 10^9}{1-0.3^2} [-75 + (0.3)(250)] (10^{-6}) \\ &= 0\end{aligned}$$

$$\begin{aligned}\sigma_y &= \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{200 \times 10^9}{1-0.3^2} [250 + (0.3)(-75)] (10^{-6}) \\ &= 50 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\frac{P}{A} &= \sigma_y & P &= A\sigma_y = (50)(150)(50^3) \\ & & &= 375 \text{ kN}\end{aligned}$$

$$G = \frac{E}{2(1+\nu)} = \frac{200 \times 10^9}{2(1+0.3)} = 76.92 \text{ GPa}$$

$$\tau_{xy} = G\gamma_{xy} = (76.92 \times 10^9)(425)(10^{-6}) = 32.69 \text{ MPa}$$



$$I = \frac{1}{12} b h^3 = \frac{1}{12} (50)(150)^3 = 14062500 \text{ mm}^4$$

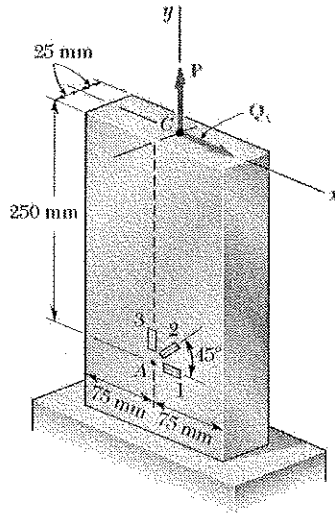
$$Q = A\bar{y} = (75)(50)\left(\frac{75}{2}\right) = 140625 \text{ mm}^3 \quad t = 50 \text{ mm}$$

$$\tau_{xy} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{xy}}{Q} = \frac{(14062500)(50)(32.69)}{140625} = 163.45 \text{ kN}$$

$$Q_x = V = 163.5 \text{ kN}$$

Problem 7.151



7.151 Solve Prob. 7.150, assuming that the rosette at point A indicates the following strains:

$$\epsilon_1 = -60 \times 10^{-6} \text{ mm/mm} \quad \epsilon_2 = +410 \times 10^{-6} \text{ mm/mm} \\ \epsilon_3 = +200 \times 10^{-6} \text{ mm/mm}$$

7.150 A centric axial force P and a horizontal force Q are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$$\epsilon_1 = -75 \times 10^{-6} \text{ mm/mm} \quad \epsilon_2 = +300 \times 10^{-6} \text{ mm/mm} \\ \epsilon_3 = +250 \times 10^{-6} \text{ mm/mm}$$

Knowing that $E = 200 \text{ GPa}$ and $\nu = 0.30$, determine the magnitudes of P and Q .

$$\epsilon_x = \epsilon_1 = -60 \times 10^{-6} \quad \epsilon_y = \epsilon_3 = 200 \times 10^{-6}$$

$$\gamma_{xy} = 2\epsilon_2 - \epsilon_1 - \epsilon_3 = 680 \times 10^{-6}$$

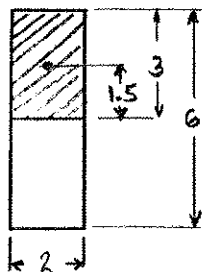
$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{200 \times 10^9}{1-0.3^2} [-60 + (0.3)(200)] (10^{-6}) \\ = 0$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{200 \times 10^9}{1-0.3^2} [200 + (0.3)(-60)] (10^{-6}) \\ = 40 \text{ MPa}$$

$$\frac{P}{A} = \sigma_y \quad P = A\sigma_y = (50)(150)(40) \\ = 300 \text{ kN}$$

$$G = \frac{E}{2(1+\nu)} = \frac{200 \times 10^9}{2(1.3)} = 76.92 \text{ GPa}$$

$$\tau_{xy} = G\gamma_{xy} = (76.92 \times 10^9)(680 \times 10^{-6}) = 52.306 \text{ MPa}$$



$$I = \frac{1}{12} b h^3 = \frac{1}{12} (50)(150)^3 = 14062500 \text{ mm}^4$$

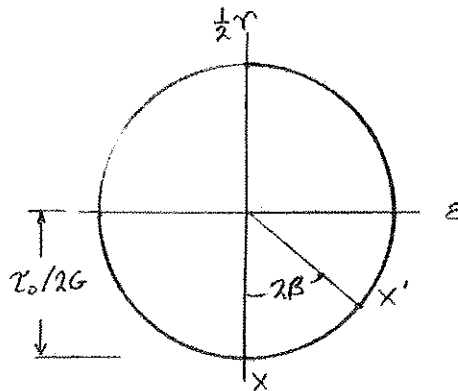
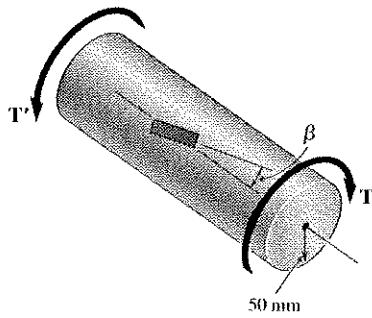
$$Q = A\bar{y} = (75)(50)\left(\frac{75}{2}\right) = 140625 \text{ mm}^3 \quad \bar{y} = 50 \text{ mm}$$

$$\tau_{xy} = \frac{VQ}{I\bar{t}}$$

$$V = \frac{I\bar{t}\tau_{xy}}{Q} = \frac{(14062500)(50)(52.306)}{140625} = 261.53 \text{ kN}$$

$$Q_x = V = 261.5 \text{ kN}$$

Problem 7.152



7.152 A single gage is cemented to a solid 100-mm-diameter steel shaft at an angle $\beta = 25^\circ$ with a line parallel to the axis of the shaft. Knowing that $G = 79 \text{ GPa}$, determine the torque T indicated by a gage reading of $300 \times 10^{-6} \text{ mm/mm}$.

For torsion, $\sigma_x = \sigma_y = 0$, $\tau = \tau_0$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = 0$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_0}{G} \quad \frac{1}{2} \gamma_{xy} = \frac{\tau_0}{2G}$$

Draw the Mohr's circle for strain

$$R = \frac{\tau_0}{2G}$$

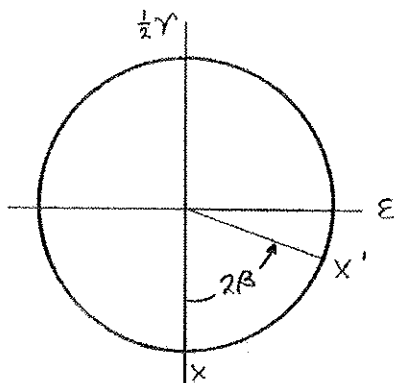
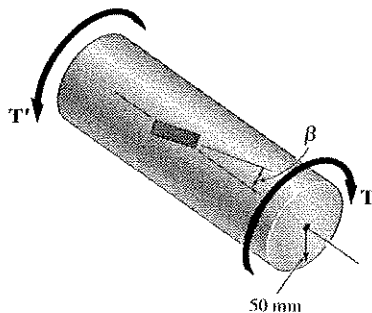
$$\epsilon_{x'} = R \sin 2\beta = \frac{\tau_0}{2G} \sin 2\beta$$

$$\text{But } \tau_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\epsilon_{x'}}{\sin 2\beta}$$

$$T = \frac{\pi c^3 G \epsilon_{x'}}{\sin 2\beta} = \frac{\pi (0.05)^3 (79 \times 10^9) (300 \times 10^{-6})}{\sin 50^\circ}$$

$$= 12.2 \text{ kNm}$$

Problem 7.153



7.153 Solve Prob. 7.152, assuming that the gage forms an angle $\beta = 35^\circ$ with a line parallel to the axis of the shaft.

7.152 A single gage is cemented to a solid 100-mm-diameter steel shaft at an angle $\beta = 25^\circ$ with a line parallel to the axis of the shaft. Knowing that $G = 79 \text{ GPa}$, determine the torque T indicated by a gage reading of $300 \times 10^{-6} \text{ mm/mm}$.

For torsion $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = \tau_0$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = 0$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_0}{G} \quad \frac{1}{2} \gamma_{xy} = \frac{\tau_0}{2G}$$

Draw Mohr's circle for strain

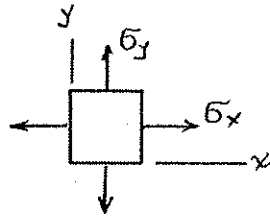
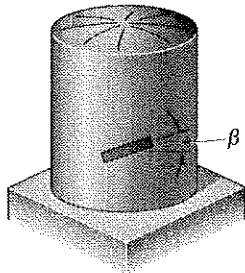
$$R = \frac{\tau_0}{2G} \quad \epsilon_{x'} = R \sin 2\beta = \frac{\tau_0}{2G} \sin 2\beta$$

$$\text{But } \tau_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\epsilon_{x'}}{\sin 2\beta}$$

$$T = \frac{\pi c^3 G \epsilon_{x'}}{\sin 2\beta} = \frac{\pi (0.05)^3 (79 \times 10^9) (300 \times 10^{-6})}{\sin 70^\circ}$$

$$= 9.9 \text{ kNm}$$

Problem 7.154



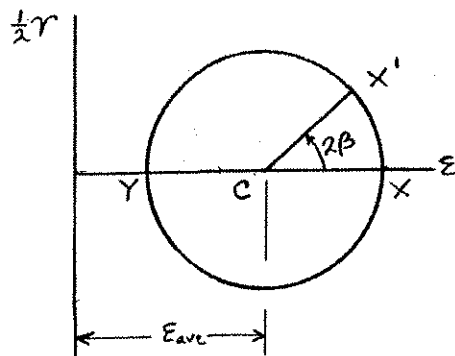
7.154 A single strain gage forming an angle $\beta = 18^\circ$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ .

$$\sigma_x = \sigma_r = \frac{pr}{t}, \quad \sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right) \frac{\sigma_x}{E} = 0.85 \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right) \frac{\sigma_x}{E} = 0.20 \frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



Draw Mohr's circle for strain.

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0.525 \frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\epsilon_x - \epsilon_y) = 0.325 \frac{\sigma_x}{E}$$

$$\epsilon_{x'} = \epsilon_{ave} + R \cos 2\beta = (0.525 + 0.325 \cos 2\beta) \frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\epsilon_{x'}}{r(0.525 + 0.325 \cos 2\beta)}$$

Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

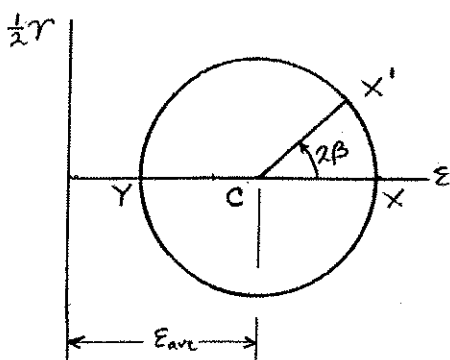
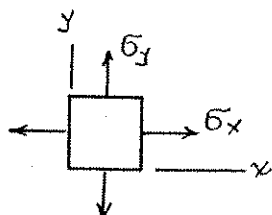
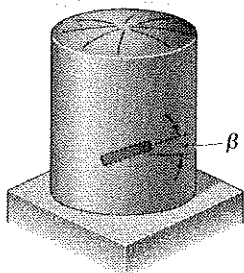
$t = 6 \times 10^{-3} \text{ mm}$ $E = 200 \times 10^9 \text{ Pa}$, $\epsilon_{x'} = 280 \times 10^{-6}$

$\beta = 18^\circ$

$$p = \frac{(6 \times 10^{-3})(200 \times 10^9)(280 \times 10^{-6})}{(0.300)(0.525 + 0.325 \cos 36^\circ)} = 1.421 \times 10^6 \text{ Pa}$$

$p = 1.421 \text{ MPa}$ ◀

Problem 7.155



7.155 Solve Prob. 7.154, assuming that the gage forms an angle $\beta = 35^\circ$ with a horizontal plane.

7.154 A single strain gage forming an angle $\beta = 18^\circ$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200 \text{ GPa}$ and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ .

$$\sigma_x = \sigma_t = \frac{pr}{t}, \quad \sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right) \frac{\sigma_x}{E} = 0.85 \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right) \frac{\sigma_x}{E} = 0.20 \frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$

Draw Mohr's circle for strain.

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0.525 \frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\epsilon_x - \epsilon_y) = 0.325 \frac{\sigma_x}{E}$$

$$\epsilon_{x'} = \epsilon_{ave} + R \cos 2\beta = (0.525 + 0.325 \cos 2\beta) \frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\epsilon_{x'}}{r(0.525 + 0.325 \cos 2\beta)}$$

Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

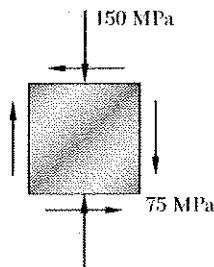
$$t = 6 \times 10^{-3} \text{ m} \quad E = 200 \times 10^9 \text{ Pa} \quad \epsilon_{x'} = 280 \times 10^{-6}$$

$$\beta = 35^\circ$$

$$p = \frac{(6 \times 10^{-3})(200 \times 10^9)(280 \times 10^{-6})}{(0.300)(0.525 + 0.325 \cos 70^\circ)} = 1.761 \times 10^6 \text{ Pa}$$

$$p = 1.761 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.156



7.156 The given state of plane strain is known to exist on the surface of a machine component. Knowing that $E = 200 \text{ GPa}$ and $G = 77.2 \text{ GPa}$, determine the direction and magnitude of the three principal strains (a) by determining the corresponding state of strain [use Eq. (2.43) and Eq. (2.38)] and then using Mohr's circle for strain, (b) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

$$(a) \quad \sigma_x = 0, \quad \sigma_y = -150 \times 10^6 \text{ Pa}, \quad \tau_{xy} = -75 \times 10^6 \text{ Pa}$$

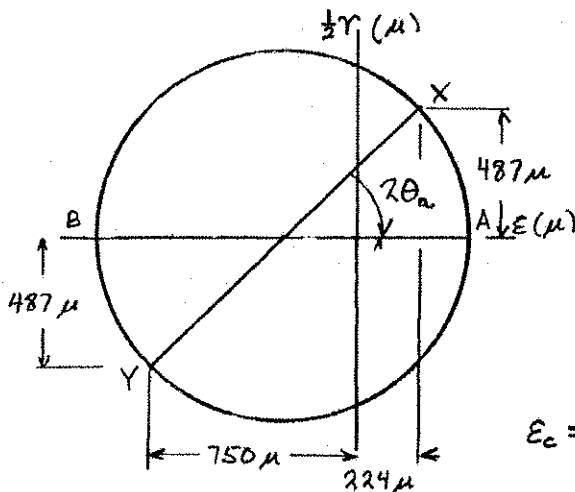
$$E = 200 \times 10^9 \text{ Pa} \quad G = 77.2 \times 10^9 \text{ Pa}$$

$$G = \frac{E}{2(1+\nu)} \quad \nu = \frac{E}{2G} - 1 = 0.2987$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{200 \times 10^9} [0 + (0.2987)(150 \times 10^6)] = 224 \mu$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{200 \times 10^9} [(-150 \times 10^6) - 0] = -750 \mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-75 \times 10^6}{77.2 \times 10^9} = -974 \mu \quad \frac{\gamma_{xy}}{2} = -487.0 \mu$$



$$\epsilon_{ave} = \frac{1}{2} (\epsilon_x + \epsilon_y) = -263 \mu$$

$$\epsilon_x - \epsilon_y = 974 \mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-974}{974} = -1.000$$

$$2\theta_a = -45.0^\circ \quad \theta_a = -22.5^\circ$$

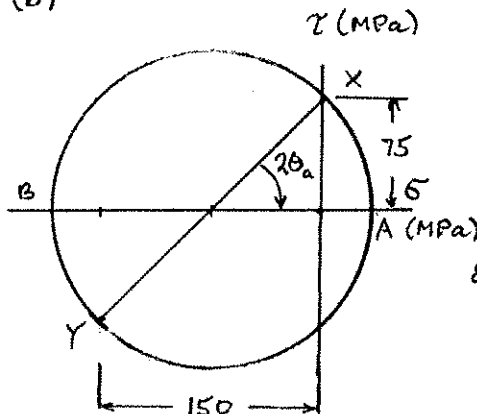
$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 689 \mu$$

$$\epsilon_a = \epsilon_{ave} + R = 426 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -952 \mu$$

$$\epsilon_c = -\frac{\nu}{E} (\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 150 \times 10^6)}{200 \times 10^9} = -224 \mu$$

(b)



$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = -75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 + 150}{2}\right)^2 + 75^2} = 106.07 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 31.07 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -181.07 \text{ MPa}$$

$$\epsilon_a = \frac{1}{E} (\sigma_a - \nu \sigma_b)$$

$$= \frac{1}{200 \times 10^9} [31.07 \times 10^6 - (0.2987)(-181.07 \times 10^6)]$$

$$= 426 \times 10^{-6}$$

$$\epsilon_a = 426 \mu$$

$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} = -1.000$$

$$2\theta_a = -45^\circ$$

$$\theta_a = -22.5^\circ$$

Problem 7.157

The 3rd principal stress is $\sigma_3 = 0$.

7.157 The following state of strain has been determined on the surface of a cast-iron machine part:

$$\epsilon_x = -720 \mu \quad \epsilon_y = -400 \mu \quad \gamma_{xy} = +660 \mu$$

Knowing that $E = 69 \text{ GPa}$ and $G = 28 \text{ GPa}$, determine the principal planes and principal stresses (a) by determining the corresponding state of plane stress [use Eq. (2.36), Eq. (2.43), and the first two equations of Prob. 2.73] and then using Mohr's circle for stress, (b) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and then determine the corresponding stresses.

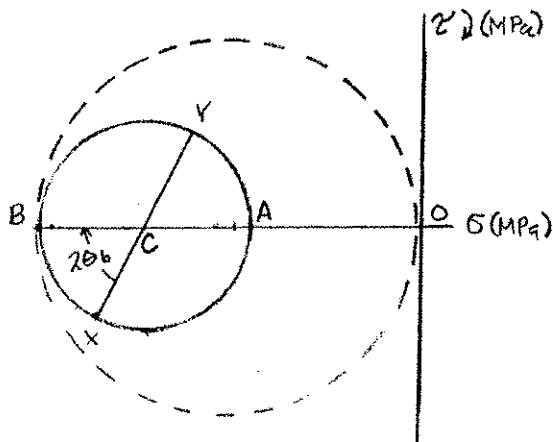
$$G = \frac{E}{2(1+\nu)} \quad \nu = \frac{E}{2G} - 1 = \frac{69}{56} - 1 = 0.2321$$

$$\frac{E}{1-\nu^2} = \frac{69}{1-(0.2321)^2} = 72.93 \text{ GPa}$$

$$(a) \quad \sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = (72.93 \times 10^9) [-720 \times 10^{-6} + (0.2321)(-400 \times 10^{-6})] = -59.28 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = (72.93 \times 10^9) [-400 \times 10^{-6} + (0.2321)(-720 \times 10^{-6})] = -41.36 \text{ MPa}$$

$$\tau_{xy} = G \gamma_{xy} = (28 \times 10^9)(660 \times 10^{-6}) = 18.48 \text{ MPa}$$



$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = -50.32 \text{ MPa}$$

$$\tan 2\theta_b = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -2.0625$$

$$2\theta_b = -64.1^\circ, \quad \theta_b = -32.1^\circ, \quad \theta_a = 57.9^\circ$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 20.54 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = -29.8 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -70.9 \text{ MPa}$$

$$(b) \quad \epsilon_{ave} = \frac{1}{2} (\epsilon_x + \epsilon_y) = -560 \mu$$

$$\tan 2\theta_b = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = -2.0625$$

$$2\theta_b = -64.1^\circ, \quad \theta_b = -32.1^\circ, \quad \theta_a = 57.9^\circ$$

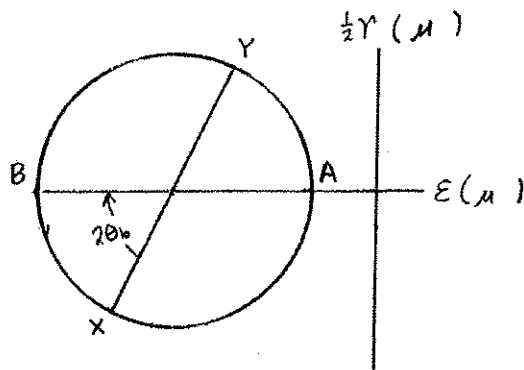
$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 366.74 \mu$$

$$\epsilon_a = \epsilon_{ave} + R = -193.26 \mu$$

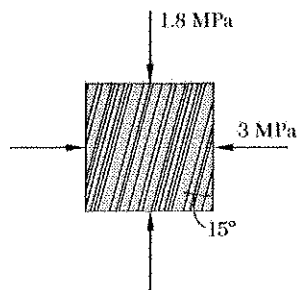
$$\epsilon_b = \epsilon_{ave} - R = -926.74 \mu$$

$$\sigma_a = \frac{E}{1-\nu^2} (\epsilon_a + \nu \epsilon_b) = -29.8 \text{ MPa}$$

$$\sigma_b = \frac{E}{1-\nu^2} (\epsilon_b + \nu \epsilon_a) = -70.9 \text{ MPa}$$



Problem 7.158



7.158 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

$$\begin{aligned}\sigma_x &= -3 \text{ MPa} & \sigma_y &= -1.8 \text{ MPa} & \tau_{xy} &= 0 \\ \sigma_{ave} &= \frac{\sigma_x + \sigma_y}{2} = -2.4 \text{ MPa}\end{aligned}$$

Plotted points for Mohr's circle:

$$X: (\sigma_x, \tau_{xy}) = (-3 \text{ MPa}, 0)$$

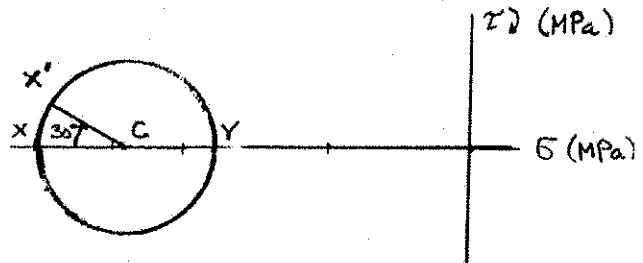
$$Y: (\sigma_y, \tau_{xy}) = (-1.8 \text{ MPa}, 0)$$

$$C: (\sigma_{ave}, 0) = (-2.4 \text{ MPa}, 0)$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$\overline{CX} = 0.6 \text{ MPa}$$

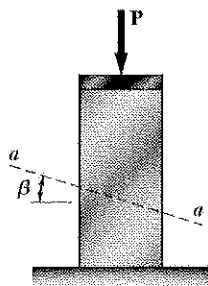
$$R = 0.6 \text{ MPa}$$



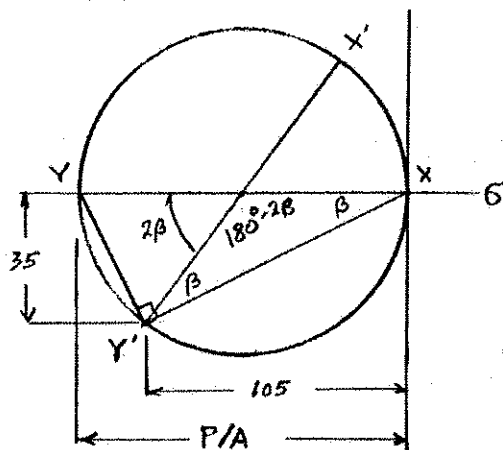
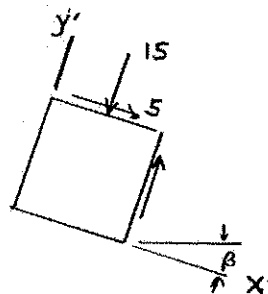
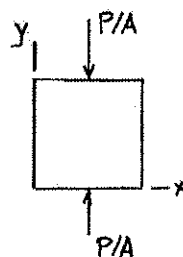
$$(a) \tau_{x'y'} = -\overline{CX'} \sin 30 = -R \sin 30^\circ = -0.6 \sin 30^\circ \quad \tau_{x'y'} = -0.30 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \sigma_{x'} = \sigma_{ave} - \overline{CX'} \cos 30^\circ = -2.4 - 0.6 \cos 30^\circ \quad \sigma_{x'} = -2.92 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.159



$$\begin{aligned}\sigma_x &= 0 \\ \tau_{xy} &= 0 \\ \sigma_y &= -P/A\end{aligned}$$



(a) From the Mohr's circle,

$$\tan \beta = \frac{35}{105} = 0.3333 \quad \beta = 18.4^\circ \quad \blacktriangleleft$$

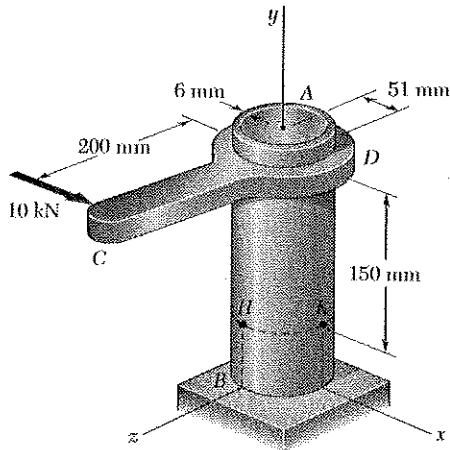
$$-\sigma = \frac{P}{2A} + \frac{P}{2A} \cos 2\beta$$

$$(b) \frac{P}{A} = \frac{2(-\sigma)}{1 + \cos 2\beta} = \frac{(2)(105)}{1 + \cos 2\beta}$$

$$\frac{P}{A} = 116.6 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.160

7.160 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point H.



$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2} J = 2.0927 \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K.

$$F_x = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

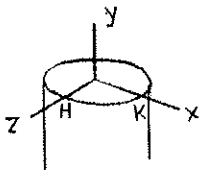
$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$

Torsion

$$T = M_y = 2000 \text{ N}\cdot\text{m}$$

$$C = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{TC}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6 \text{ Pa}$$



Transverse Shear

For semicircle

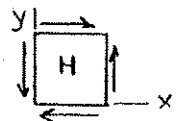
$$A = \frac{\pi}{2} r^2 \quad \bar{y} = \frac{4}{3\pi} r$$

$$Q = A\bar{y} = \frac{2}{3} r^3$$

$$Q = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = 27.684 \times 10^3 \text{ mm}^3 = 27.684 \times 10^{-6} \text{ m}^3$$

$$V = F_x = 10 \times 10^3 \text{ N} \quad t = (2)(6 \text{ mm}) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \times 10^6 \text{ Pa}$$



Bending: Point H lies on neutral axis. $\sigma_y = 0$

Total stresses at point H: $\sigma_x = 0, \sigma_y = 0$

$$\tau_{xy} = 24.37 \times 10^6 + 11.02 \times 10^6 = 35.39 \times 10^6 \text{ Pa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 35.39 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = \sigma_{ave} + R = 35.39 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 35.4 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -35.39 \times 10^6 \text{ Pa}$$

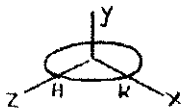
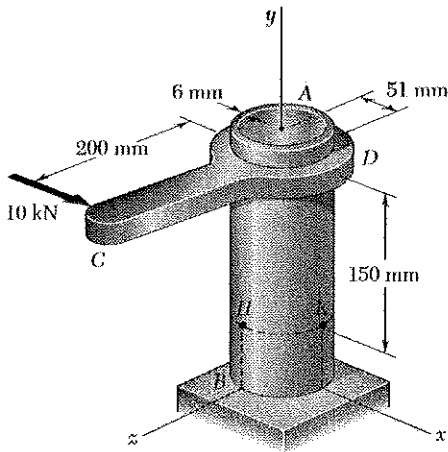
$$\sigma_{min} = -35.4 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max} = R$$

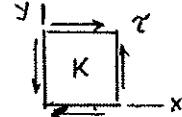
$$\tau_{max} = 35.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.161

7.161 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



Torsion:



$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K

$$F_x = 10 \text{ kN} = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$

At point K, place local x-axis in negative global z-direction

$$T = M_y = 2000 \text{ N}\cdot\text{m} \quad C = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^6} = 24.37 \times 10^6 \text{ Pa} = 24.37 \text{ MPa}$$

Transverse Shear: Stress due to transverse shear $V = F_x$ is zero at pt. K.

$$\text{Bending: } |\sigma_y| = \frac{M_z c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \times 10^6 \text{ Pa} = 36.56 \text{ MPa}$$

Point K lies on compression side of neutral axis: $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K: $\sigma_x = 0 \quad \sigma_y = -36.56 \text{ MPa}, \tau_{xy} = 24.37 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46$$

$$\sigma_{max} = 12.18 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46$$

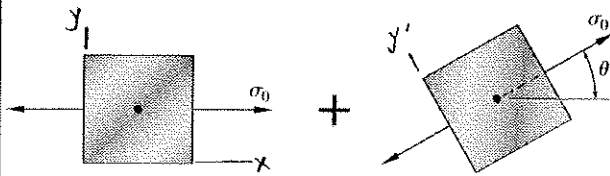
$$\sigma_{min} = -48.7 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max} = R$$

$$\tau_{max} = 30.5 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.162

7.162 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

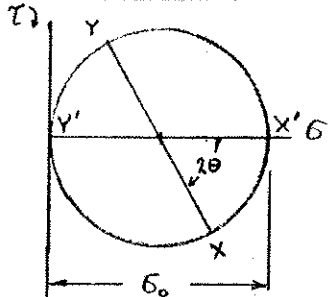


Mohr's circle for 2nd stress state.

$$\sigma_x = \frac{1}{2} \sigma_0 + \frac{1}{2} \sigma_0 \cos 2\theta$$

$$\sigma_y = \frac{1}{2} \sigma_0 - \frac{1}{2} \sigma_0 \cos 2\theta$$

$$\tau_{xy} = \frac{1}{2} \sigma_0 \sin 2\theta$$



Resultant stresses

$$\sigma_x = \sigma_0 + \frac{1}{2} \sigma_0 + \frac{1}{2} \sigma_0 \cos 2\theta = \frac{3}{2} \sigma_0 + \frac{1}{2} \sigma_0 \cos 2\theta$$

$$\sigma_y = 0 + \frac{1}{2} \sigma_0 - \frac{1}{2} \sigma_0 \cos 2\theta = \frac{1}{2} \sigma_0 - \frac{1}{2} \sigma_0 \cos 2\theta$$

$$\tau_{xy} = 0 + \frac{1}{2} \sigma_0 \sin 2\theta = \frac{1}{2} \sigma_0 \sin 2\theta$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = \sigma_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0 \cos 2\theta}$$

$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$$

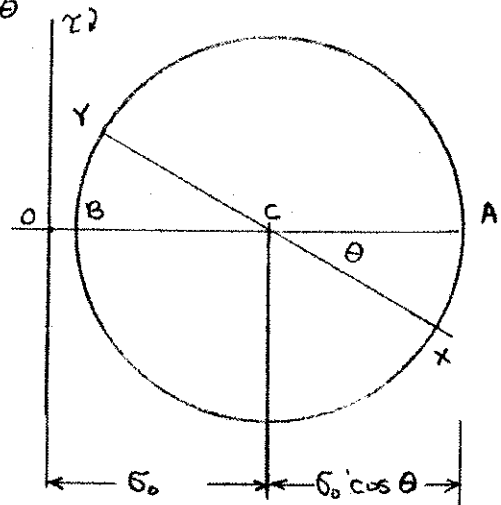
$$\theta_p = \frac{1}{2} \theta$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{1}{2} \sigma_0 + \frac{1}{2} \sigma_0 \cos 2\theta\right)^2 + \left(\frac{1}{2} \sigma_0 \sin 2\theta\right)^2}$$

$$= \frac{1}{2} \sigma_0 \sqrt{1 + 2 \cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}$$

$$= \frac{\sqrt{2}}{2} \sigma_0 \sqrt{1 + \cos 2\theta} = \sigma_0 |\cos \theta|$$

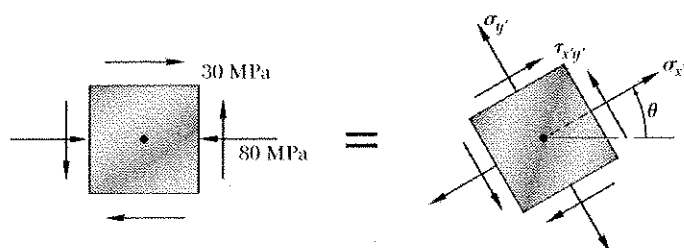


$$\sigma_a = \sigma_{ave} + R = \sigma_0 + \sigma_0 \cos \theta$$

$$\sigma_b = \sigma_{ave} - R = \sigma_0 - \sigma_0 \cos \theta$$

Problem 7.163

7.163 For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is equal to or less than 40 MPa.



$$\sigma_x = -80 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = 30 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = -40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-40)^2 + 30^2} = 50 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(30)}{-80} = -0.75$$

$$2\theta_p = -36.87^\circ$$

$$\theta_b = -18.435^\circ$$

$|\tau_{xy}| \leq 40 \text{ MPa}$ for states of stress corresponding to arcs HBK and UAV of Mohr's circle. The angle ϕ is calculated from

$$R \sin 2\phi = 40$$

$$\sin 2\phi = \frac{40}{50} = 0.8$$

$$2\phi = 53.130^\circ \quad \phi = 26.565^\circ$$

$$\theta_H = \theta_b - \phi = -18.435^\circ - 26.565^\circ = -45^\circ$$

$$\theta_K = \theta_b + \phi = -18.435^\circ + 26.565^\circ = 8.13^\circ$$

$$\theta_U = \theta_H + 90^\circ = 45^\circ$$

$$\theta_V = \theta_K + 90^\circ = 98.13^\circ$$

Permissible ranges of θ : $\theta_H \leq \theta \leq \theta_K$

$$-45^\circ \leq \theta \leq 8.13^\circ$$

$$\theta_U \leq \theta \leq \theta_V$$

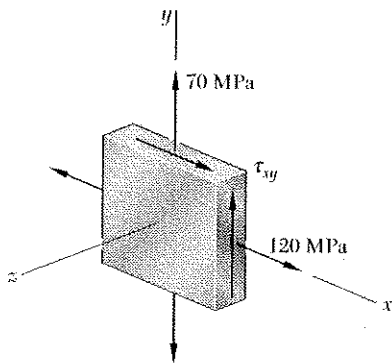
$$45^\circ \leq \theta \leq 98.13^\circ$$

$$\text{Also, } 135^\circ \leq \theta \leq 188.13^\circ$$

$$225^\circ \leq \theta \leq 278.13^\circ$$

Problem 7.164

7.164 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is 80 MPa.



$$\sigma_x = 120 \text{ MPa} \quad \sigma_y = 70 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 95 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{120 - 70}{2} = 25 \text{ MPa}$$

$$\text{Assume } \sigma_{min} = 0, \quad \sigma_{max} = 2\tau_{max} = 160 \text{ MPa}$$

$$\sigma_a = \sigma_{max} = \sigma_{ave} + R$$

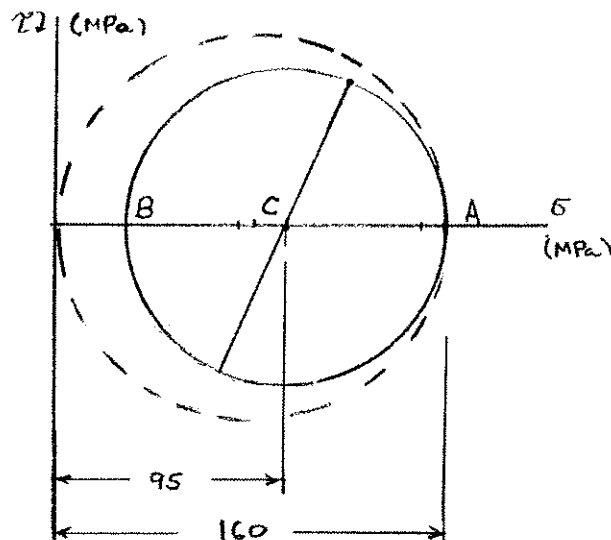
$$R = \sigma_{max} - \sigma_{ave} = 160 - 95 = 65 \text{ MPa}$$

$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\tau_{xy}^2 = R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 = 65^2 - 25^2 = 60^2$$

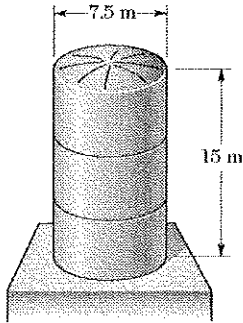
$$\tau_{xy} = \pm 60 \text{ MPa}$$

$$\sigma_b = \sigma_a - 2R = 160 - 130 = 30 \text{ MPa} \geq 0 \quad \text{O.K.}$$



$$\tau_{xy} = \pm 60.0 \text{ MPa}$$

Problem 7.165



7.165 When filled to capacity, the unpressurized storage tank shown contains water to a height of 14.6 m above its base. Knowing that the lower portion of the tank has a wall thickness of 16 mm, determine the maximum normal stress and the maximum shearing stress in the tank. (Density of water = 9810 N/m³.)

$$p_{max} = (1000)(9.81)(14.6) = 143.2 \text{ kPa}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(7500) - 16 = 3734 \text{ mm}$$

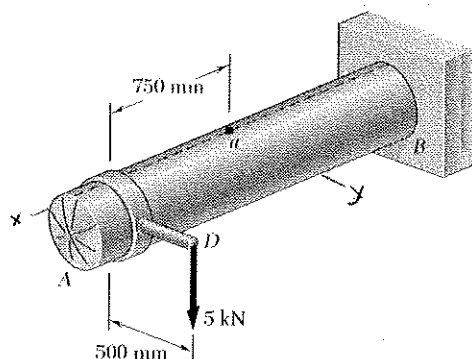
$$\sigma_r = \frac{pr}{t} = \frac{(143.2 \times 10^3)(3734)}{16} = 33.4 \text{ MPa}$$

$$\sigma_{max} = \sigma_r = 33.4 \text{ MPa}$$

$$\sigma_{max} = 33.4 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} \approx -p = -143.2 \text{ kPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 16.8 \text{ MPa} \quad \tau_{max} = 16.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.166



7.166 The compressed-air tank AB has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point a on the top of the tank.

$$r = \frac{1}{2}d = 225 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5 \text{ MPa}$$

Torsion: $C_1 = 225 \text{ mm}$, $C_2 = 225 + 6 = 231 \text{ mm}$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6 \text{ Pa} = 1.292 \text{ MPa}$$

Transverse shear: $\tau = 0$ at point a .

Bending: $I = \frac{1}{2}J = 223.45 \times 10^6 \text{ mm}^4$, $c = 231 \times 10^{-3} \text{ m}$.

At point a $M = (5 \times 10^3)(750 \times 10^{-3}) = 3750 \text{ N}\cdot\text{m}$

$$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88 \text{ MPa}$$

Total stresses (MPa)

Longitudinal: $\sigma_x = 22.5 + 3.88 = 26.38$

Circumferential: $\sigma_y = 45$

Shear: $\tau_{xy} = 1.292$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40 \text{ MPa}$$

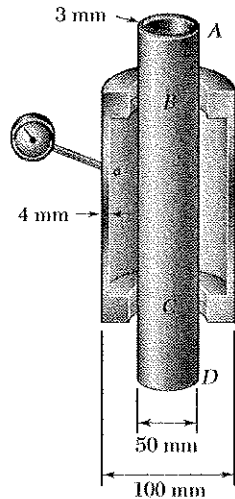
$$\sigma_{max} = \sigma_{ave} + R = 35.69 + 9.40$$

$$\sigma_{max} = 45.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max(\text{in-plane})} = R$$

$$\tau_{max(\text{in-plane})} = 9.40 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.167



7.167 The brass pipe AD is fitted with a jacket used to apply a hydrostatic pressure of 3.5 MPa to portion BC of the pipe. Knowing that the pressure inside the pipe is 0.7 MPa, determine the maximum normal stress in the pipe.

Let r be the mean radius of the pipe.

$$r = \frac{1}{2}(50 - 3) = 23.5 \text{ mm}$$

Do not distinguish among inner, mean, and outer radius.

For pipe AB , $t = 3 \text{ mm}$

longitudinal stress $\sigma_1 = \frac{P_i r}{2t}$

$$\sigma_1 = \frac{(0.7)(23.5)}{(2)(3)}$$

$$\sigma_1 = 2.74 \text{ MPa (tension)}$$

Hoop stress

$$\sigma_2 = \frac{P_i r}{t} - \frac{P_o r}{t}$$

$$\sigma_2 = \frac{(0.7)(23.5)}{3} - \frac{(3.5)(23.5)}{3}$$

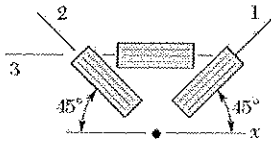
$$\sigma_2 = -21.9 \text{ MPa}$$

$$\sigma_2 = 21.9 \text{ MPa (compression)}$$

Problem 7.168

7.168 Determine the largest in-plane normal strain, knowing that the following strains have been obtained by the use of the rosette shown:

$$\begin{aligned}\epsilon_1 &= -50 \times 10^{-6} \text{ mm/mm} & \epsilon_2 &= +360 \times 10^{-6} \text{ mm/mm} \\ \epsilon_3 &= +315 \times 10^{-6} \text{ mm/mm}\end{aligned}$$



$$\theta_1 = 45^\circ \quad \theta_2 = -45^\circ \quad \theta_3 = 0$$

$$\begin{aligned}E_x \cos^2 \theta_1 + E_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 &= E_1 \\ 0.5 E_x + 0.5 E_y + 0.5 \gamma_{xy} &= -50 \times 10^{-6} \quad (1)\end{aligned}$$

$$\begin{aligned}E_x \cos^2 \theta_2 + E_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 &= E_2 \\ 0.5 E_x + 0.5 E_y - 0.5 \gamma_{xy} &= 360 \times 10^{-6} \quad (2)\end{aligned}$$

$$\begin{aligned}E_x \cos^2 \theta_3 + E_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 &= E_3 \\ E_x + 0 + 0 &= 315 \times 10^{-6} \quad (3)\end{aligned}$$

From (3) $E_x = 315 \times 10^{-6} \text{ mm/mm}$

Eq. (1) - Eq. (2) $\gamma_{xy} = -50 \times 10^{-6} - 360 \times 10^{-6} = -410 \times 10^{-6} \text{ mm/mm}$

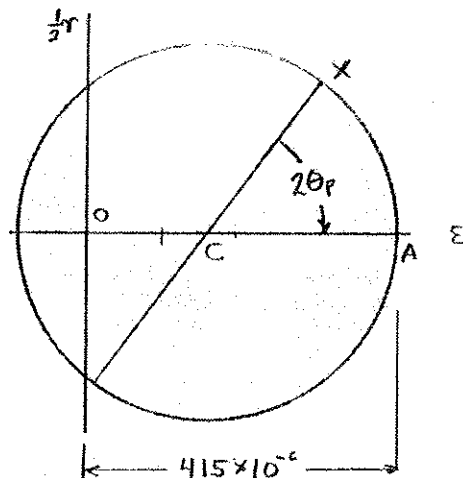
Eq. (1) + Eq. (2) $E_x + E_y = E_1 + E_2$

$$E_y = E_1 + E_2 - E_x = -50 \times 10^{-6} + 360 \times 10^{-6} - 315 \times 10^{-6} = -5 \times 10^{-6} \text{ mm/mm}$$

$$E_{ave} = \frac{1}{2} (E_x + E_y) = 155 \times 10^{-6} \text{ mm/mm}$$

$$\begin{aligned}R &= \sqrt{\left(\frac{E_x - E_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{315 \times 10^{-6} + 5 \times 10^{-6}}{2}\right)^2 + \left(\frac{-410 \times 10^{-6}}{2}\right)^2} \\ &= 260 \times 10^{-6} \text{ mm/mm}\end{aligned}$$

$$E_{max} = E_{ave} + R = 415 \times 10^{-6} \text{ mm/mm}$$

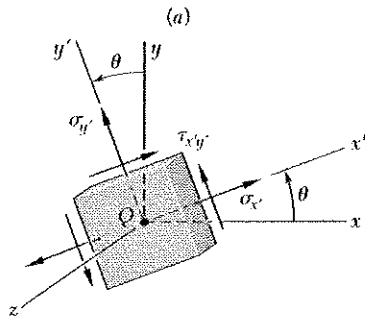
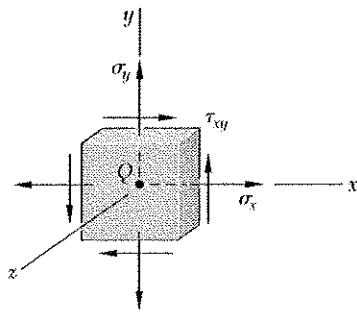


$$\tan 2\theta_p = \frac{\gamma_{xy}}{E_x - E_y} = -1.323$$

$$2\theta_p = -52.0^\circ$$

$$\theta_p = -26.0^\circ$$

PROBLEM 7.C1



7.C1 A state of plane stress is defined by the stress components σ_x , σ_y , and τ_{xy} associated with the element shown in Fig. P7.C1a. (a) Write a computer program that can be used to calculate the stress components $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ associated with the element after it has rotated through an angle θ about the z axis (Fig. P7.C1b). (b) Use this program to solve Probs. 7.13 through 7.16.

SOLUTION PROGRAM FOLLOWING EQUATIONS

$$\text{EQ (7.5), p 427: } \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\text{EQ (7.7), p 427: } \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\text{EQ (7.6), p 427: } \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

ENTER σ_x , σ_y , τ_{xy} AND θ

PRINT VALUES OBTAINED FOR $\sigma_{x'}$, $\sigma_{y'}$ and $\tau_{x'y'}$

Problem 7.13a

Sigma x = -60 MPa
Sigma y = 90 MPa
Tau xy = 30 MPa

Rotation of element
(+ counterclockwise)
theta = -25 degrees

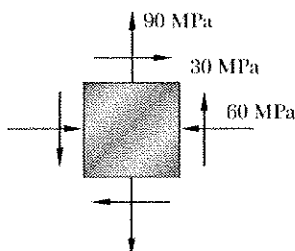
Sigma x' = -56.19 MPa
Sigma y' = 86.19 MPa
Tau x'y' = -38.17 MPa

Problem 7.13b

Sigma x = -60 MPa
Sigma y = 90 MPa
Tau xy = 30 MPa

Rotation of element
(+ counterclockwise)
theta = 10 degrees

Sigma x' = -45.22 MPa
Sigma y' = 75.22 MPa
Tau x'y' = 53.84 MPa



Problem 7.14a

Sigma x = 0 MPa
Sigma y = -80 MPa
Tau xy = -50 MPa

Rotation of element
(+ counterclockwise)
theta = -25 degrees

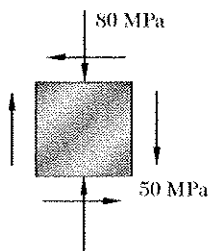
Sigma x' = 24.01 MPa
Sigma y' = -104.01 MPa
Tau x'y' = -1.50 MPa

Problem 7.14b

Sigma x = 0 MPa
Sigma y = -80 MPa
Tau xy = -50 MPa

Rotation of element
(+ counterclockwise)
theta = 10 degrees

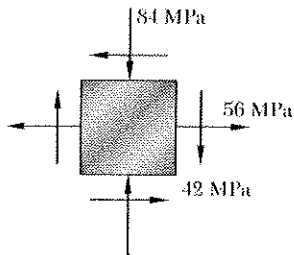
Sigma x' = -19.51 MPa
Sigma y' = -60.49 MPa
Tau x'y' = -60.67 MPa



CONTINUED

PROBLEM 7.C1 - CONTINUED

PROGRAM OUTPUT



Problem 7.15a

$$\begin{aligned}\sigma_x &= 56 \text{ MPa} \\ \sigma_y &= -84 \text{ MPa} \\ \tau_{xy} &= -42 \text{ MPa}\end{aligned}$$

Rotation of element
(+ counterclockwise)
 $\theta = -25 \text{ degrees}$

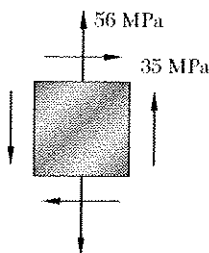
$$\begin{aligned}\sigma_{x'} &= 63.1 \text{ MPa} \\ \sigma_{y'} &= -91.1 \text{ MPa} \\ \tau_{x'y'} &= 26.6 \text{ MPa}\end{aligned}$$

Problem 7.15b

$$\begin{aligned}\sigma_x &= 56 \text{ MPa} \\ \sigma_y &= -84 \text{ MPa} \\ \tau_{xy} &= -42 \text{ MPa}\end{aligned}$$

Rotation of element
(+ counterclockwise)
 $\theta = 10 \text{ degrees}$

$$\begin{aligned}\sigma_{x'} &= 37.4 \text{ MPa} \\ \sigma_{y'} &= -65.4 \text{ MPa} \\ \tau_{x'y'} &= -63.4 \text{ MPa}\end{aligned}$$



Problem 7.16a

$$\begin{aligned}\sigma_x &= 0 \text{ MPa} \\ \sigma_y &= 56 \text{ MPa} \\ \tau_{xy} &= 35 \text{ MPa}\end{aligned}$$

Rotation of element
(+ counterclockwise)
 $\theta = -25 \text{ degrees}$

$$\begin{aligned}\sigma_{x'} &= -16.8 \text{ MPa} \\ \sigma_{y'} &= 72.8 \text{ MPa} \\ \tau_{x'y'} &= 1.05 \text{ MPa}\end{aligned}$$

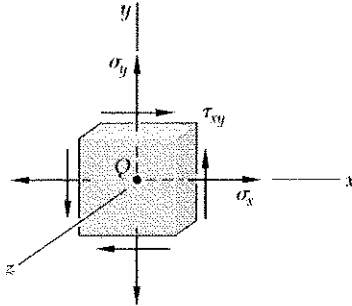
Problem 7.16b

$$\begin{aligned}\sigma_x &= 0 \text{ MPa} \\ \sigma_y &= 56 \text{ MPa} \\ \tau_{xy} &= 35 \text{ MPa}\end{aligned}$$

Rotation of element
(+ counterclockwise)
 $\theta = 10 \text{ degrees}$

$$\begin{aligned}\sigma_{x'} &= 13.7 \text{ MPa} \\ \sigma_{y'} &= 42.4 \text{ MPa} \\ \tau_{x'y'} &= 42.5 \text{ MPa}\end{aligned}$$

PROBLEM 7.C2



7.C2 A state of plane stress is defined by the stress components σ_x , σ_y , and τ_{xy} associated with the element shown in Fig. P7.C1a. (a) Write a computer program that can be used to calculate the principal axes, the principal stresses, the maximum in-plane shearing stress, and the maximum shearing stress. (b) Use this program to solve Probs. 7.5, 7.9, 7.68 and 7.69.

SOLUTION

PROGRAM FOLLOWING EQUATIONS

$$\text{EQ. (7.10)} \quad \bar{\sigma}_{ave} = \frac{\sigma_x + \sigma_y}{2} ; R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\text{EQ. (7.14)} \quad \bar{\sigma}_{max} = \bar{\sigma}_{ave} + R$$

$$\bar{\sigma}_{min} = \bar{\sigma}_{ave} - R$$

$$\text{EQ. (7.12)} \quad \theta_p = \tan^{-1} \frac{2 \tau_{xy}}{\sigma_x - \sigma_y}$$

$$\text{EQ. (7.15)} \quad \theta_s = \tan^{-1} - \frac{\sigma_x - \sigma_y}{2 \tau_{xy}}$$

SHEARING STRESS IF $\bar{\sigma}_{max} > 0$ and $\bar{\sigma}_{min} < 0$:

THEN $\tau_{max}(\text{in-plane}) = R$; $\tau_{max}(\text{out-of-plane}) = R$

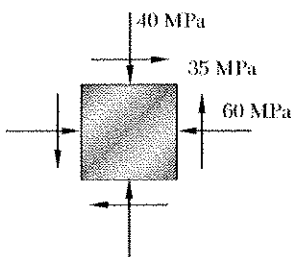
IF $\bar{\sigma}_{max} > 0$ and $\bar{\sigma}_{min} > 0$:

THEN $\tau_{max}(\text{in-plane}) = R$; $\tau_{max}(\text{out-of-plane}) = \frac{1}{2} \bar{\sigma}_{min}$

IF $\bar{\sigma}_{max} < 0$ and $\bar{\sigma}_{min} < 0$:

THEN $\tau_{max}(\text{in-plane}) = R$; $\tau_{max}(\text{out-of-plane}) = \frac{1}{2} |\bar{\sigma}_{min}|$

PROGRAM OUTPUT



Problems 7.5 and 7.9

Sigma x = -60.00 MPa

Sigma y = -40.00 MPa

Tau xy = 35.00 MPa

Angle between xy axes and principal axes
(+ counterclockwise)

Theta p = -37.03 deg. and 52.97 deg.

Sigma max = -13.60 MPa

Sigma min = -86.40 MPa

Angle between xy axis and planes of maximum in-plane shearing stress
(+ counterclockwise)

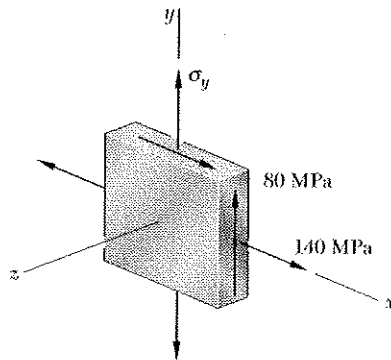
Theta s = 7.97 deg. and 97.97 deg.

Tau max (in plane) = 36.40 MPa

Tau max = 43.20 MPa

CONTINUED

PROBLEM 7.C2 - CONTINUED



Problem 7.68

Sigma x = 140.00 MPa
Sigma y = 40.00 MPa
Tau xy = 80.00 MPa

Angle between xy axes and principal axes
(+ counterclockwise)

Theta p = 29.00 deg. and 119.00 deg.

Sigma max = 184.34 MPa

Sigma min = -4.34 MPa

Angle between xy axis and planes of maximum in-plane
in-plane shearing stress (+ counterclockwise)

Theta s = 74.00 deg. and 164.00 deg.

Tau max (in-plane) = 94.34 MPa

Tau max (out-of-plane) = 94.34 MPa

Sigma x = 140.00 MPa
Sigma y = 120.00 MPa
Tau xy = 80.00 MPa

Angle between xy axes and principal axes
(+ counterclockwise)

Theta p = 41.44 deg. and 131.44 deg.

Sigma max = 210.62 MPa

Sigma min = 49.38 MPa

Angle between xy axis and planes of maximum in-plane
in-plane shearing stress (+ counterclockwise)

Theta s = 86.44 deg. and 176.44 deg.

Tau max (in-plane) = 80.62 MPa

Tau max (out-of-plane) = 105.31 MPa

Problem 7.69

Sigma x = 140.00 MPa
Sigma y = 20.00 MPa
Tau xy = 80.00 MPa

Angle between xy axes and principal axes
(+ counterclockwise)

Theta p = 26.57 deg. and 116.57 deg.

Sigma max = 180.00 MPa

Sigma min = -20.00 MPa

Angle between xy axis and planes of maximum in-plane
in-plane shearing stress (+ counterclockwise)

Theta s = 71.57 deg. and 161.57 deg.

Tau max (in-plane) = 100.00 MPa

Tau max (out-of-plane) = 100.00 MPa

Sigma x = 140.00 MPa
Sigma y = 140.00 MPa
Tau xy = 80.00 MPa

Angle between xy axes and principal axes
(+ counterclockwise)

Theta p = 45.00 deg. and 135.00 deg.

Sigma max = 220.00 MPa

Sigma min = 60.00 MPa

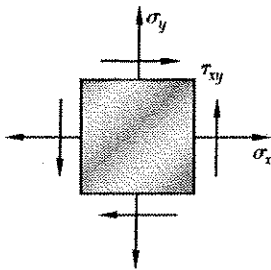
Angle between xy axis and planes of maximum in-plane
in-plane shearing stress (+ counterclockwise)

Theta s = 90.00 deg. and 180.00 deg.

Tau max (in-plane) = 80.00 MPa

Tau max (out-of-plane) = 110.00 MPa

PROBLEM 7.C3



7.C3 (a) Write a computer program that, for a given state of plane stress and a given yield strength of a ductile material, can be used to determine whether the material will yield. The program should use both the maximum shearing-stress criterion and the maximum-distortion-energy criterion. It should also print the values of the principal stresses and, if the material does not yield, calculate the factor of safety. (b) Use this program to solve Probs. 7.81, 7.82 and 7.165.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} ; R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MAXIMUM-SHEARING-STRESS CRITERION $\tau_y = \frac{1}{2} \sigma_y$

IF σ_a AND σ_b HAVE SAME SIGN, $\tau_{max} = \frac{1}{2} \sigma_a$

IF $\tau_{max} > \tau_y$, YIELDING OCCURS

IF $\tau_{max} < \tau_y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\tau_y}{\tau_{max}}$$

MAXIMUM-DISTORTION-ENERGY CRITERION

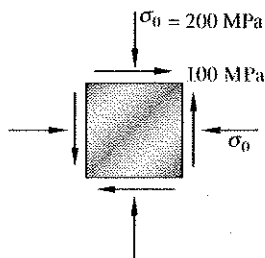
$$\text{COMPUTE RADICAL} = \sqrt{\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2}$$

IF RADICAL $> \sigma_y$, YIELDING OCCURS

IF RADICAL $< \sigma_y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\sigma_y}{\text{RADICAL}}$$

PROGRAM OUTPUT



Problems 7.81a and 7.82a Yield strength = 325 MPa

Sigma x = -200.00 MPa

Sigma y = -200.00 MPa

Tau xy = 100.00 MPa

Sigma max = -100.00 MPa

Sigma min = -300.00 MPa

Using the maximum-shearing-stress criterion:

Material will not yield

F.S. = 1.083

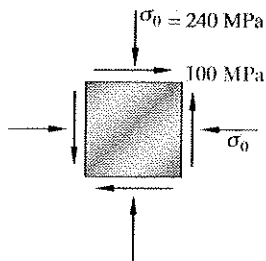
Using the maximum-distortion-energy criterion:

Material will not yield

F.S. = 1.228

CONTINUED

PROBLEM 7.C3 - CONTINUED



Problems 7.81b and 7.82b Yield strength = 325 MPa

$$\sigma_x = -240.00 \text{ MPa}$$

$$\sigma_y = -240.00 \text{ MPa}$$

$$\tau_{xy} = 100.00 \text{ MPa}$$

$$\sigma_{\max} = -140.00 \text{ MPa}$$

$$\sigma_{\min} = -340.00 \text{ MPa}$$

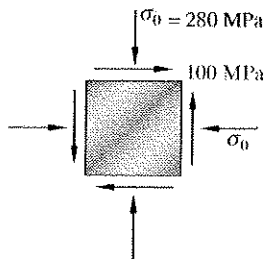
Using the maximum-shearing-stress criterion:

Material will yield

Using the maximum-distortion-energy criterion:

Material will not yield

$$\text{F.S.} = 1.098$$



Problems 7.81c and 7.82c Yield strength = 325 MPa

$$\sigma_x = -280.00 \text{ MPa}$$

$$\sigma_y = -280.00 \text{ MPa}$$

$$\tau_{xy} = 100.00 \text{ MPa}$$

$$\sigma_{\max} = -180.00 \text{ MPa}$$

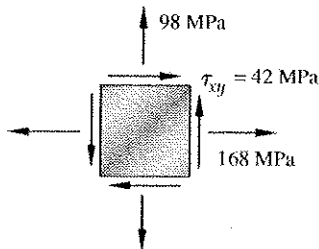
$$\sigma_{\min} = -380.00 \text{ MPa}$$

Using the maximum-shearing-stress criterion:

Material will yield

Using the maximum-distortion-energy criterion:

Material will yield



Problem 7.165a

Yield strength = 210 MPa

$$\sigma_x = 168 \text{ MPa}$$

$$\sigma_y = 98 \text{ MPa}$$

$$\tau_{xy} = 42 \text{ MPa}$$

$$\sigma_{\max} = 187.7 \text{ MPa}$$

$$\sigma_{\min} = 78.3 \text{ MPa}$$

(a) Using the maximum-shearing-stress criterion:

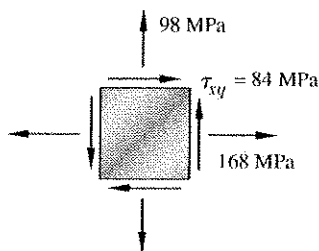
Material will not yield

$$\text{F.S.} = 1.119$$

(b) Using the maximum-distortion-energy criterion:

Material will not yield

$$\text{F.S.} = 1.286$$



Problem 7.165b

Yield strength = 210 MPa

$$\sigma_x = 168 \text{ MPa}$$

$$\sigma_y = 98 \text{ MPa}$$

$$\tau_{xy} = 84 \text{ MPa}$$

$$\sigma_{\max} = 224 \text{ MPa}$$

$$\sigma_{\min} = 42 \text{ MPa}$$

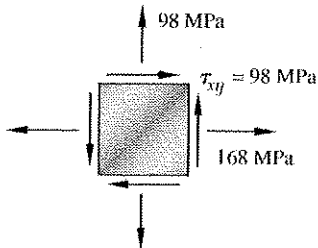
(a) Using the maximum-shearing-stress criterion:

Material will yield

(b) Using the maximum-distortion-energy criterion:

Material will not yield

$$\text{F.S.} = 1.018$$



Problem 7.165c

Yield strength = 210 MPa

$$\sigma_x = 168 \text{ MPa}$$

$$\sigma_y = 98 \text{ MPa}$$

$$\tau_{xy} = 98 \text{ MPa}$$

$$\sigma_{\max} = 237.1 \text{ MPa}$$

$$\sigma_{\min} = 28.9 \text{ MPa}$$

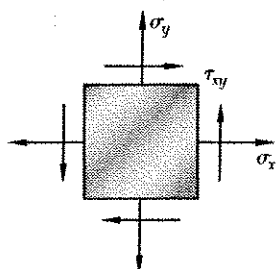
(a) Using the maximum-shearing-stress criterion:

Material will yield

(b) Using the maximum-distortion-energy criterion:

Material will yield

PROBLEM 7.C4



7.C4 (a) Write a computer program based on Mohr's fracture criterion for brittle materials that, for a given state of plane stress and given values of the ultimate strength of the material in tension and compression, can be used to determine whether rupture will occur. The program should also print the values of the principal stresses. (b) Use this program to solve Probs. 7.89 and 7.90 and to check the answers to Probs. 7.93 and 7.94.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MOHR'S FRACTURE CRITERION

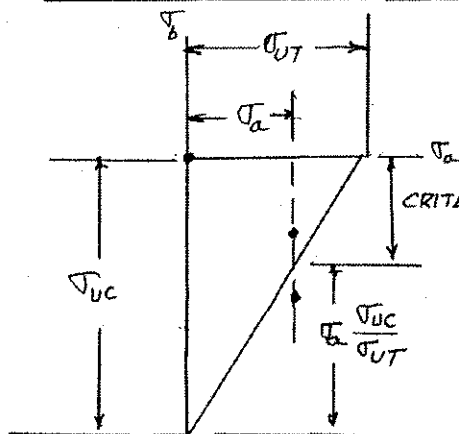
IF σ_a AND σ_b HAVE SAME SIGN, AND

$\sigma_a < \sigma_{UT}$ AND $\sigma_b < \sigma_{UC}$, NO FAILURE

$\sigma_a > \sigma_{UT}$ OR $\sigma_b > \sigma_{UC}$, FAILURE

IF $\sigma_a > 0$ AND $\sigma_b < 0$:

CONSIDER FOURTH QUADRANT OF FIG. 7.47



$$\text{CRITERION} = \sigma_{UC} - \sigma_a \frac{\sigma_{UC}}{\sigma_{UT}}$$

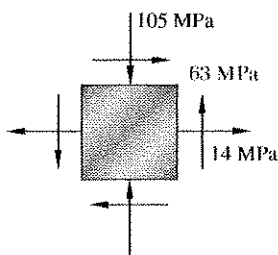
FOR NO RUPTURE TO OCCUR:

POINT (σ_a, τ) MUST LIE WITHIN
MOHR'S ENVELOPE (FIG. 7.47)

IF $\tau > \text{CRITERION}$,
THEN RUPTURE OCCURS

IF $\tau < \text{CRITERION}$,
THEN NO RUPTURE OCCURS

PROGRAM OUTPUT



Problem 7.89

Sigma x = 14 MPa

Sigma y = -105 MPa

Tau xy = 63 MPa

Ultimate strength in tension = 70 MPa

Ultimate strength in compression = 210 MPa

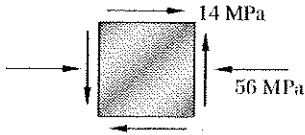
Sigma max = Sigma a = 41.16 MPa

Sigma min = Sigma b = -132.16 MPa

Rupture will occur

CONTINUED

PROBLEM 7.C4 - CONTINUED



Problem 7.90

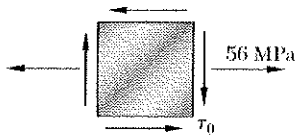
$$\begin{aligned}\sigma_x &= -56 \text{ MPa} \\ \sigma_y &= 0.00 \text{ MPa} \\ \tau_{xy} &= 49 \text{ MPa}\end{aligned}$$

Ultimate strength in tension = 70 MPa
Ultimate strength in compression = 210 MPa

$$\begin{aligned}\sigma_{\max} &= \sigma_a = 28.42 \text{ MPa} \\ \sigma_{\min} &= \sigma_b = -84.42 \text{ MPa}\end{aligned}$$

Rupture will not occur

TO CHECK ANSWERS TO FOLLOWING PROBLEMS, WE CHECK FOR RUPTURE USING GIVEN ANSWERS AND AN ADJACENT VALUE.



ANSWER:
RUPTURE OCCURS FOR $\tau_0 = 25.7 \text{ MPa}$

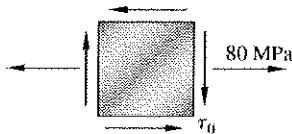
Problem 7.93

$$\begin{aligned}\sigma_x &= 56 \text{ MPa} \\ \sigma_y &= 0.00 \text{ MPa} \\ \tau_{xy} &= 25.7 \text{ MPa}\end{aligned}$$

Ultimate strength in tension = 70 MPa
Ultimate strength in compression = 175 MPa
 $\sigma_{\max} = \sigma_a = 66.01 \text{ MPa}$
 $\sigma_{\min} = \sigma_b = -10.01 \text{ MPa}$
Rupture will not occur

$$\begin{aligned}\sigma_x &= 56 \text{ MPa} \\ \sigma_y &= 0.00 \text{ MPa} \\ \tau_{xy} &= 25.7 \text{ MPa}\end{aligned}$$

Ultimate strength in tension = 70 MPa
Ultimate strength in compression = 175 MPa
 $\sigma_{\max} = \sigma_a = 66.08 \text{ MPa}$
 $\sigma_{\min} = \sigma_b = -10.08 \text{ MPa}$
Rupture will occur



ANSWER:
RUPTURE OCCURS FOR $\tau_0 = 49.1 \text{ MPa}$

Problem 7.94

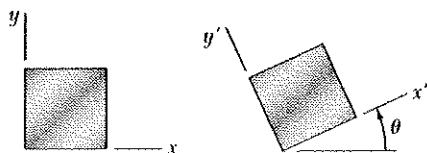
$$\begin{aligned}\sigma_x &= -80.00 \text{ MPa} \\ \sigma_y &= 0.00 \text{ MPa} \\ \tau_{xy} &= 49.10 \text{ MPa}\end{aligned}$$

Ultimate strength in tension = 75 MPa
Ultimate strength in compression = 150 MPa
 $\sigma_{\max} = \sigma_a = 23.33 \text{ MPa}$
 $\sigma_{\min} = \sigma_b = -103.33 \text{ MPa}$
Rupture will not occur

$$\begin{aligned}\sigma_x &= -80.00 \text{ MPa} \\ \sigma_y &= 0.00 \text{ MPa} \\ \tau_{xy} &= 49.20 \text{ MPa}\end{aligned}$$

Ultimate strength in tension = 75 MPa
Ultimate strength in compression = 150 MPa
 $\sigma_{\max} = \sigma_a = 23.41 \text{ MPa}$
 $\sigma_{\min} = \sigma_b = -103.41 \text{ MPa}$
Rupture will occur

PROBLEM 7.C5



7.C5 A state of plane strain is defined by the strain components ϵ_x , ϵ_y , and γ_{xy} associated with the x and y axes. (a) Write a computer program that can be used to calculate the strain components $\epsilon_{x'}$, $\epsilon_{y'}$, and $\gamma_{x'y'}$ associated with the frame of reference $x'y'$ obtained by rotating the x and y axes through an angle θ . (b) Use this program to solve Probs. 7.129 and 7.131.

SOLUTION PROGRAM FOLLOWING EQUATIONS

$$\text{EQ. (7.44)} \quad \epsilon'_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{1}{2} \gamma_{xy} \sin 2\theta$$

$$\text{EQ. (7.45)} \quad \epsilon'_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \sin 2\theta - \frac{1}{2} \gamma_{xy} \cos 2\theta$$

$$\text{EQ. (7.46)} \quad \gamma'_{x'y'} = -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta$$

ENTER ϵ_x , ϵ_y , γ_{xy} , AND θ

PRINT VALUES OBTAINED FOR $\epsilon'_{x'}$, $\epsilon'_{y'}$, AND $\gamma'_{x'y'}$

PROGRAM OUTPUT

Problem 7.129 Epsilon x = 240 micro meters
 Epsilon y = 160 micro meters
 Gamma xy = 150 micro radians
 Rotation of element, in degrees (+ counterclockwise)
 Theta = -60 degrees

Epsilon x' = 115.05 micro meters
 Epsilon y' = 284.95 micro meters
 Gamma x'y' = -5.72 micro radians

Problem 7.131 Epsilon x = 0 micro meters
 Epsilon y = 320 micro meters
 Gamma xy = -100 micro radians
 Rotation of element, in degrees (+ counterclockwise)
 Theta = 30 degrees

Epsilon x' = 36.70 micro meters
 Epsilon y' = 283.30 micro meters
 Gamma x'y' = 227.13 micro radians

PROBLEM 7.C6

7.C6 A state of strain is defined by the strain components ϵ_x , ϵ_y , and γ_{xy} associated with the x and y axes. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.136 through 7.139.

SOLUTION PROGRAM FOLLOWING EQUATIONS

$$\text{EQ (7.50)} \quad \epsilon_{\text{ave}} = \frac{\epsilon_x + \epsilon_y}{2} \quad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\text{EQ (7.51)} \quad \epsilon_{\text{max}} = \epsilon_{\text{ave}} + R \quad \epsilon_{\text{min}} = \epsilon_{\text{ave}} - R$$

$$\text{EQ (7.52)} \quad \theta_p = \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINSMAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{\text{max (in-plane)}} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

$$\text{LET } \epsilon_a = \epsilon_{\text{max}}$$

$$\epsilon_b = \epsilon_{\text{min}}$$

$$\text{CALCULATE } \epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b)$$

$$\text{IF } \epsilon_a > \epsilon_b > \epsilon_c : \gamma_{\text{out-of-plane}} = \epsilon_a - \epsilon_c$$

$$\text{IF } \epsilon_a > \epsilon_c > \epsilon_b : \gamma_{\text{out-of-plane}} = \epsilon_a - \epsilon_b = 2R$$

$$\text{IF } \epsilon_c > \epsilon_a > \epsilon_b : \gamma_{\text{out-of-plane}} = \epsilon_c - \epsilon_b$$

PROGRAM PRINTOUT

Problem 7.136

Epsilon x = 30 micro meters
Epsilon y = 570 micro meters
Gamma xy = 720 micro radians
nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = -26.57 degrees
Epsilon a = 750.00 micro meters
Epsilon b = -150.00 micro meters
Epsilon c = -300.00 micro meters

Gamma max (in plane) = 900.00 micro radians
Gamma max = 1050.00 micro radians

CONTINUED

PROBLEM 7.C6 - CONTINUED

Problem 7.137

Epsilon x = 160 micro meters
Epsilon y = -480 micro meters
Gamma xy = -600 micro radians
nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = -21.58 degrees
Epsilon a = 278.63 micro meters
Epsilon b = -598.63 micro meters
Epsilon c = 159.98 micro meters

Gamma max (in plane) = 877.27 micro radians
Gamma max = 877.27 micro radians

Problem 7.138

Epsilon x = -600 micro meters
Epsilon y = -400 micro meters
Gamma xy = 350 micro radians
nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = -30.13 degrees
Epsilon a = -298.44 micro meters
Epsilon b = -701.56 micro meters
Epsilon c = 500.00 micro meters

Gamma max (in plane) = 403.11 micro radians
Gamma max = 1201.56 micro radians

Problem 7.139

Epsilon x = -260 micro meters
Epsilon y = -60 micro meters
Gamma xy = 480 micro radians
nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = -33.69 degrees
Epsilon a = 100.00 micro meters
Epsilon b = -420.00 micro meters
Epsilon c = 159.98 micro meters

Gamma max (in plane) = 520.00 micro radians
Gamma max = 579.98 micro radians

PROBLEM 7.C7

7.C7 A state of plane strain is defined by the strain components ϵ_x , ϵ_y , and γ_{xy} measured at a point. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the magnitude of the shearing strain. (b) Use this program to solve Probs. 7.140 through 7.143.

SOLUTIONPROGRAM FOLLOWING EQUATIONS

$$\text{EQ(7.50)} \quad \epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2} \quad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\text{EQ(7.51)} \quad \epsilon_{max} = \epsilon_{ave} + R \quad \epsilon_{min} = \epsilon_{ave} - R$$

$$\text{EQ(7.52)} \quad \theta_p = \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINSMAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{xy}(\text{in-plane}) = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK
WHETHER IT IS THE MAXIMUM SHEARING STRAIN

$$\text{LET } \epsilon_a = \epsilon_{max}$$

$$\epsilon_b = \epsilon_{min}$$

$$\epsilon_c = 0 \quad (\text{PLAIN STRAIN})$$

$$\text{IF } \epsilon_a > \epsilon_b > \epsilon_c : \gamma_{\text{OUT-OF-PLANE}} = \epsilon_a - \epsilon_c$$

$$\text{IF } \epsilon_a > \epsilon_c > \epsilon_b : \gamma_{\text{OUT-OF-PLANE}} = \epsilon_a - \epsilon_b = 2R$$

$$\text{IF } \epsilon_c > \epsilon_a > \epsilon_b : \gamma_{\text{OUT-OF-PLANE}} = \epsilon_c - \epsilon_b$$

PROGRAM PRINTOUT

Problem 7.140

Epsilon x = 400 micro meters
Epsilon y = 200 micro meters
Gamma xy = 375 micro radians
nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 30.96 and -59.04 degrees
Epsilon a = 512.50 micro meters
Epsilon b = 87.50 micro meters
Epsilon c = 0.00 micro meters

Gamma max (in plane) = 425.00 micro radians
Gamma max = 512.50 micro radians

CONTINUED

PROBLEM 7.C7 - CONTINUED

Problem 7.141

Epsilon x = -180 micro meters
Epsilon y = -260 micro meters
Gamma xy = 315 micro radians
nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 37.87 and -52.13 degrees
Epsilon a = -57.50 micro meters
Epsilon b = -382.50 micro meters
Epsilon c = 0.00 micro meters

Gamma max (in plane) = 325.00 micro radians
Gamma max = 382.50 micro radians

Problem 7.142

Epsilon x = 60 micro meters
Epsilon y = 240 micro meters
Gamma xy = -50 micro radians
nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 7.76 and -82.24 degrees
Epsilon a = 243.41 micro meters
Epsilon b = 56.59 micro meters
Epsilon c = 0.00 micro meters

Gamma max (in plane) = 186.82 micro radians
Gamma max = 243.41 micro radians

Problem 7.143

Epsilon x = 300 micro meters
Epsilon y = 60 micro meters
Gamma xy = 100 micro radians
nu = 0.000

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 11.31 and -78.69 degrees
Epsilon a = 310.00 micro meters
Epsilon b = 50.00 micro meters
Epsilon c = 0.00 micro meters

Gamma max (in plane) = 260.00 micro radians
Gamma max = 310.00 micro radians

PROBLEM 7.C8

7.C8 A rosette consisting of three gages forming, respectively, angles of θ_1 , θ_2 , and θ_3 with the x axis is attached to the free surface of a machine component made of a material with a given Poisson's ratio ν . (a) Write a computer program that, for given readings ϵ_1 , ϵ_2 , and ϵ_3 of the gages, can be used to calculate the strain components associated with the x and y axes and determine the orientation and magnitude of the three principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.144, 7.145, 7.146 and 7.169.

SOLUTION

FOR $n=1$ TO 3, ENTER θ_n and ϵ_n

ENTER: $\nu = \nu$

SOLVE EQS. (7.60) FOR ϵ_x , ϵ_y , AND γ_{xy} USING METHOD OF DETERMINATES OR ANY OTHER METHOD,

$$\text{ENTER } \epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2}; \quad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \gamma_{xy}^2}$$

$$\epsilon_a = \epsilon_{max} = \epsilon_{ave} + R$$

$$\epsilon_b = \epsilon_{min} = \epsilon_{ave} - R$$

$$\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b)$$

$$\theta_p = \frac{1}{2} \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max(in-plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN,
AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN,

$$\text{IF } \epsilon_c < \epsilon_b: \gamma_{OUT-OF-PLANE} = \epsilon_a - \epsilon_c$$

$$\text{IF } \epsilon_c > \epsilon_a: \gamma_{OUT-OF-PLANE} = \epsilon_c - \epsilon_b$$

$$\text{OTHERWISE: } \gamma_{OUT-OF-PLANE} = 2R$$

PROGRAM OUTPUT

Problem 7.144

Gage	theta degrees	epsilon micro meters
1	-15	480
2	30	-120
3	75	80

Epsilon x = 253.21 micro meters

Epsilon y = 306.79 micro meters

Gamma xy = -892.82 micro radians

Epsilon a = 727.21 micro meters

Epsilon b = -167.21 micro meters

Gamma max (in plane) = 894.43 micro radians

CONTINUED

PROBLEM 7.C8 - CONTINUED

Problem 7.145

	Gage	theta degrees	epsilon micro meters
	1	30	600
	2	-30	450
	3	90	-75
Epsilon x = 725.000 micro meters			
Epsilon y = -75.000 micro meters			
Gamma xy = 173.205 micro radians			
Epsilon a = 734.268 micro meters			
Epsilon b = -84.268 micro meters			
Gamma max (in plane) = 818.535 micro radians			

Problem 7.146

OBSERVE THAT GAGE 3 IS ORIENTATED ALONG THE y AXIS. THEREFORE
ENTER ϵ_4 AND ϵ_4 AS ϵ_3 AND ϵ_3
THE VALUE OF ϵ_4 THAT IS OBTAINED
IS ALSO THE EXPECTED READING OF GAGE 3.

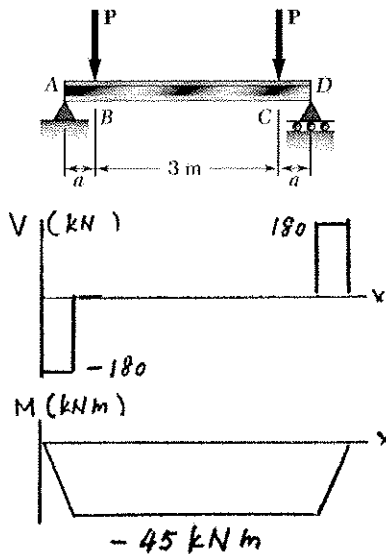
	Gage	theta degrees	epsilon mm/mm
	1	0	420
	2	45	-45
	4	135	165
Epsilon x = 420.00 mm/mm			
Epsilon y = -300.00 mm/mm			
Gamma xy = -210.00 micro radians			
Epsilon a = 435.00 mm/mm			
Epsilon b = -315.00 mm/mm			
Gamma max (in plane) = 750.00 micro radians			

Problem 7.169

	Gage	theta degrees	epsilon mm/mm
	1	45	-50
	2	-45	360
	3	0	315
Epsilon x = 315.000 mm/mm			
Epsilon y = -5.000 mm/mm			
Gamma xy = -410.000 micro radians			
Epsilon a = 415.048 mm/mm			
Epsilon b = -105.048 mm/mm			
Gamma max (in plane) = 520.096 micro radians			

Chapter 8

Problem 8.1



8.1 A W250 × 101 rolled-steel beam supports a load P as shown. Knowing that $P = 180 \text{ kN}$, $a = 0.25 \text{ m}$, and $\sigma_{\text{all}} = 126 \text{ MPa}$, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

$$|V|_{\text{max}} = 180 \text{ kN}$$

$$|M|_{\text{max}} = (180)(0.25) = 45 \text{ kNm}$$

For W250 × 101 rolled steel section,

$$d = 264 \text{ mm} \quad b_f = 257 \text{ mm} \quad t_f = 19.6 \text{ mm}$$

$$t_w = 11.9 \text{ mm} \quad I_x = 164 \times 10^6 \text{ mm}^4 \quad S_x = 1240 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 132 \text{ mm} \quad y_b = c - t_f = 112.4 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{\text{max}}}{S_x} = \frac{45 \times 10^3}{1240 \times 10^{-6}} \quad \sigma_m = 36.3 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{112.4}{132} \right) (36.3) = 30.9 \text{ MPa}$$

$$A_f = b_f t_f = 5037 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 122.2 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 615521 \text{ mm}^3$$

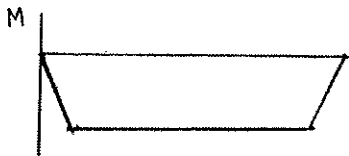
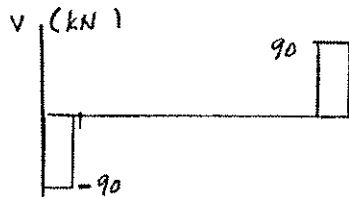
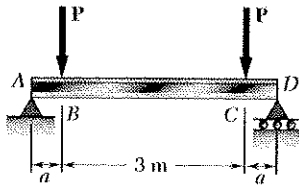
$$\tau_{xy} = \frac{|V|_{\text{max}} Q_b}{I_x t_w} = \frac{(180 \times 10^3)(615521 \times 10^{-9})}{(164 \times 10^6)(0.0119)} = 56.77 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2} \right)^2 + \tau_{xy}^2} = 58.83 \text{ MPa}$$

$$(b) \quad \sigma_{\text{max}} = \frac{\sigma_b}{2} + R = 74.29 \text{ MPa} \quad \sigma_{\text{max}} = 74.3 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \text{Since } \sigma_{\text{max}} < \sigma_{\text{all}} (= 126 \text{ MPa}), \text{ W250} \times 101 \text{ is acceptable.} \quad \blacktriangleleft$$

Problem 8.2



8.2 Solve Prob. 8.1, assuming that $P = 90 \text{ kN}$, $a = 0.5 \text{ m}$.

8.1 A W250 $\times$ 101 rolled-steel beam supports a load P as shown. Knowing that $P = 180 \text{ kN}$, $a = 0.25 \text{ m}$, and $\sigma_{\text{all}} = 126 \text{ MPa}$, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

$$|V|_{\text{max}} = 90 \text{ kN}$$

$$|M|_{\text{max}} = (90 \times 0.5) = 45 \text{ kNm}$$

For W250 $\times$ 101 rolled steel section,

$$d = 264 \text{ mm} \quad b_f = 257 \text{ mm} \quad t_f = 19.6 \text{ mm}$$

$$t_w = 11.9 \text{ mm} \quad I_x = 164 \times 10^6 \text{ mm}^4 \quad S_x = 1240 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 132 \text{ mm} \quad y_b = c - t_f = 112.4 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{\text{max}}}{S_x} = \frac{45 \times 10^3}{1240 \times 10^{-6}} \quad \sigma_m = 36.3 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{112.4}{132} \right) (36.3) = 30.9 \text{ MPa}$$

$$A_f = b_f t_f = 5037 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 122.2 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 615521 \text{ mm}^3$$

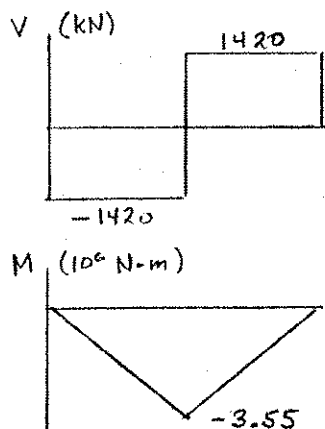
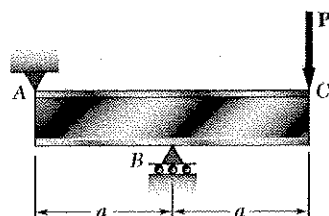
$$\tau_{xy} = \frac{|V|_{\text{max}} Q_b}{I_x t_w} = \frac{(45 \times 10^3)(615521 \times 10^{-9})}{(164 \times 10^6)(0.0119)} = 4.12 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2} \right)^2 + \tau_{xy}^2} = 15.99 \text{ MPa}$$

$$(b) \quad \sigma_{\text{max}} = \frac{\sigma_b}{2} + R = 31.44 \text{ MPa} \quad \sigma_{\text{max}} = 31.4 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \text{Since } \sigma_{\text{max}} < \sigma_{\text{all}} (= 126 \text{ MPa}), \text{ W } 250 \times 101 \text{ is acceptable.} \quad \blacktriangleleft$$

Problem 8.3



8.3 An overhanging W920 × 446 rolled-steel beam supports a load P as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{all} = 100$ MPa, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

$$|V|_{max} = 700 \text{ kN} = 700 \times 10^3 \text{ N}$$

$$|M|_{max} = (700 \times 10^3)(2.5) = 1.75 \times 10^6 \text{ N}\cdot\text{m}$$

For W920 × 446 rolled steel beam

$$d = 933 \text{ mm} \quad b_f = 423 \text{ mm} \quad t_f = 42.70 \text{ mm}$$

$$t_w = 24.0 \text{ mm} \quad I_x = 8470 \times 10^6 \text{ mm}^4 \quad S_x = 18200 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 466.5 \text{ mm} \quad y_b = c - t_f = 423.8 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.75 \times 10^6}{18200 \times 10^3} \quad \sigma_m = 96.2 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{423.8}{466.5} (96.2) = 87.4 \text{ MPa}$$

$$A_f = b_f t_f = 18.062 \times 10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 445.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8040.3 \times 10^3 \text{ mm}^3 = 8040.3 \times 10^{-6} \text{ m}^3$$

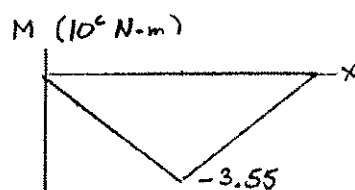
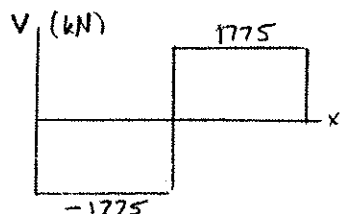
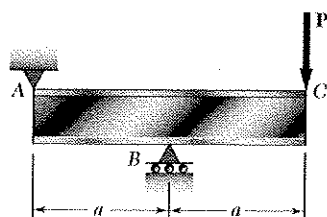
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(700 \times 10^3)(8040.3 \times 10^{-6})}{(8470 \times 10^6)(24.0 \times 10^{-3})} = 27.7 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{87.4}{2}\right)^2 + 27.7^2} = 51.7 \text{ MPa}$$

$$(b) \quad \sigma_{max} = \frac{\sigma_b}{2} + R = 95.4 \text{ MPa} \quad \sigma_{max} = 95.4 \text{ MPa}$$

$$(c) \quad \text{Since } 51.7 \text{ MPa} < \sigma_{all} (= 100 \text{ MPa}), \text{ W920} \times 446 \text{ is acceptable}$$

Problem 8.4



8.4 Solve Prob. 8.3, assuming that $P = 850 \text{ kN}$ and $a = 2.0 \text{ m}$.

8.3 An overhanging W920 $\times$ 446 rolled-steel beam supports a load P as shown. Knowing that $P = 700 \text{ kN}$, $a = 2.5 \text{ m}$, and $\sigma_{\text{all}} = 100 \text{ MPa}$, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

$$|V|_{\text{max}} = 850 \text{ kN} = 850 \times 10^3 \text{ N}$$

$$|M|_{\text{max}} = (850 \times 10^3)(2.0) = 1.70 \times 10^6 \text{ N}\cdot\text{m}$$

For W920 $\times$ 446 rolled steel section

$$d = 933 \text{ mm} \quad b_f = 423 \text{ mm} \quad t_f = 42.70 \text{ mm}$$

$$t_w = 24.0 \text{ mm} \quad I_x = 8470 \times 10^6 \text{ mm}^4 \quad S_x = 18200 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 466.5 \text{ mm} \quad y_b = c - t_f = 423.8 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{\text{max}}}{S_x} = \frac{1.70 \times 10^6}{18200 \times 10^3} \quad \sigma_m = 93.4 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{423.8}{466.5} (93.4) = 84.86 \text{ MPa}$$

$$A_f = b_f t_f = 18.062 \times 10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 445.5 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8040.3 \times 10^3 \text{ mm}^3 = 8040.3 \times 10^{-6} \text{ m}^3$$

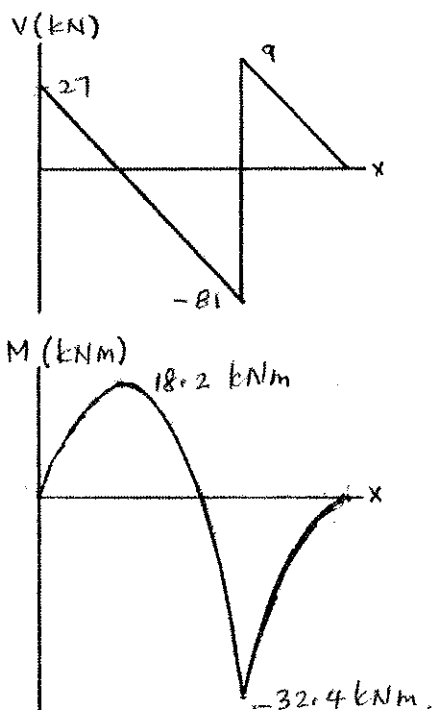
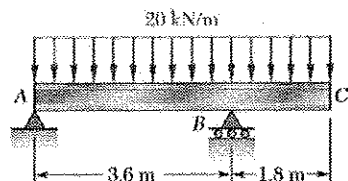
$$\tau_{xy} = \frac{|V|_{\text{max}} Q_b}{I_x t_w} = \frac{(850 \times 10^3)(8040.3 \times 10^{-6})}{(8470 \times 10^6)(24.0 \times 10^{-3})} = 33.62 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{93.4}{2}\right)^2 + 33.62^2} = 54.1 \text{ MPa}$$

$$(b) \quad \sigma_{\text{max}} = \frac{\sigma_m}{2} + R = 96.6 \text{ MPa} \quad \sigma_{\text{max}} = 96.6 \text{ MPa}$$

$$(c) \quad \text{Since } 96.6 \text{ MPa} < \sigma_{\text{all}} (= 100 \text{ MPa}), \text{ W920} \times 446 \text{ is acceptable}$$

Problem 8.5



8.5 (a) Knowing that $\sigma_{all} = 165 \text{ MPa}$ and $\tau_{all} = 100 \text{ MPa}$, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

$$\textcircled{1} \sum M_B = 0 \quad -3.6 R_A + (20)(5.4)(0.9) = 0 \quad R_A = 27 \text{ kN} \uparrow$$

$$\textcircled{2} \sum M_A = 0 \quad 3.6 R_B + (20)(5.4)(2.7) = 0 \quad R_B = 81 \text{ kN} \uparrow$$

$$|V|_{max} = 81 \text{ kN}$$

$$|M|_{max} = 32.4 \text{ kNm}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{32.4 \times 10^6}{165} = 196.4 \times 10^3 \text{ mm}^3$$

Shape	$S (\times 10^3 \text{ mm}^3)$
W 310 $\times$ 23.8	280
W 250 $\times$ 22.3	229
W 260 $\times$ 26.6	249
W 150 $\times$ 29.8	219

(a) Use

W 250 $\times$ 22.3

$$d = 254 \text{ mm}$$

$$t_f = 6.9 \text{ mm}$$

$$t_w = 5.8 \text{ mm}$$

$$(b) \quad \sigma_m = \frac{|M|_{max}}{S} = \frac{32.4 \times 10^6}{229 \times 10^3} = 141.5 \text{ MPa}$$

$$\tau_m = \frac{|V|_{max}}{d t_w} = \frac{81000}{(254)(5.8)} = 55 \text{ MPa}$$

$$c = \frac{1}{2}d = \frac{254}{2} = 127 \text{ mm}$$

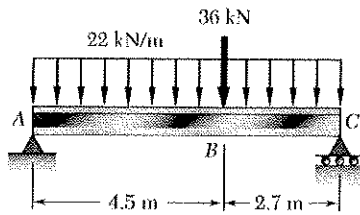
$$y_b = c - t_f = 127 - 6.9 = 120.1 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{120.1}{127} \right) (141.5) = 133.8 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2} \right)^2 + \tau_m^2} = \sqrt{\left(\frac{133.8}{2} \right)^2 + (55)^2} = 86.6 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{133.8}{2} + 86.6 = 153.5 \text{ MPa}$$

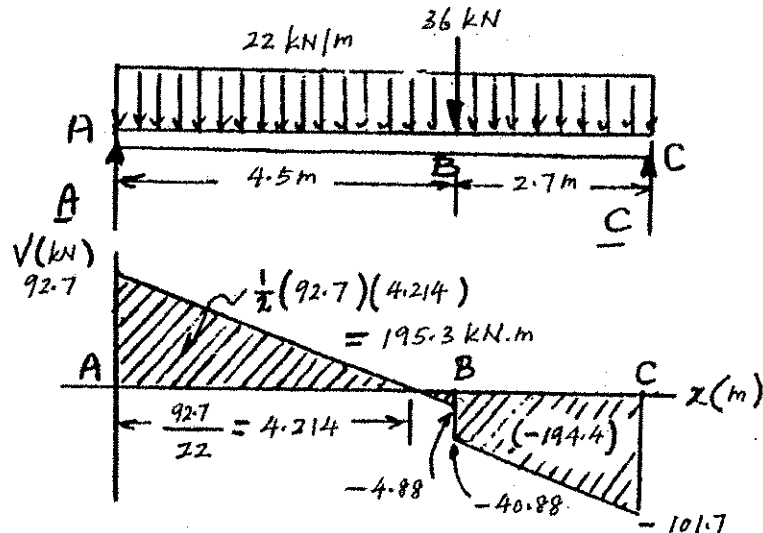
Problem 8.6



$$\begin{aligned}
 +\circlearrowleft \sum M_C &= 0: \\
 (22 \times 7.2)(3.6) &+ (36)(2.7) \\
 - A(7.2) &= 0 \\
 A &= 92.7 \text{ kN} \\
 B &= 101.7 \text{ kN}
 \end{aligned}$$

$$|V|_{\max} = 101.7 \text{ kN}$$

$$|M|_{\max} = 195.3 \text{ kN}\cdot\text{m}$$



Shape	$S(10^3 \text{ mm})$
W 530x66	1340
W 460x74	1460
W 410x85	1510
W 360x79	1280
W 310x107	1590
W 250x101	1240

Section Modulus

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{195.3 \times 10^6}{165} = 1183.64 \times 10^3 \text{ mm}^3$$

(a) Select

W 530x66

$$S = 1340 \times 10^3 \text{ mm}^3$$

$$d = 525 \text{ mm}$$

$$t_f = 11.4 \text{ mm}$$

$$t_w = 8.9 \text{ mm}$$

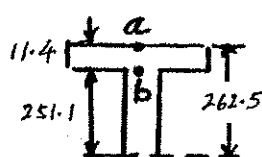
$$\text{Stress } \sigma_m: \sigma_m = \frac{|M|_{\max}}{S} = \frac{195.3 \times 10^3}{1340 \times 10^{-6}} = 145.7 \text{ MPa}$$

$$\sigma_m = 145.7 \text{ MPa}$$

$$\text{Shearing stress } \tau_m: \tau_m = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{|V|_{\max}}{t_w d} = \frac{101.7 \times 10^3}{(8.9)(525)} = 21.8 \text{ MPa}$$

$$\tau_m = 21.8 \text{ MPa}$$

Max. Normal Stress in Web at B



$$\sigma_a = \frac{|M_B|}{S} = \frac{194.4 \times 10^3}{1340 \times 10^{-6}} = 145.1 \text{ MPa}$$

$$\sigma_b = (145.1)(251.1/262.5) = 138.8 \text{ MPa}$$

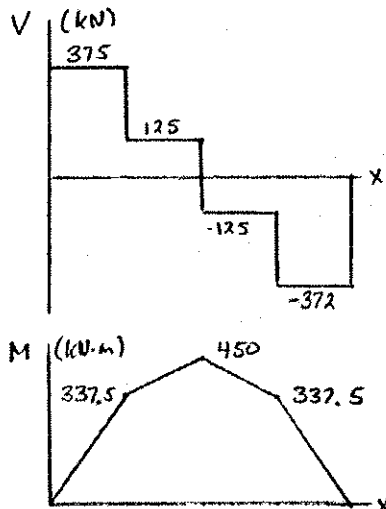
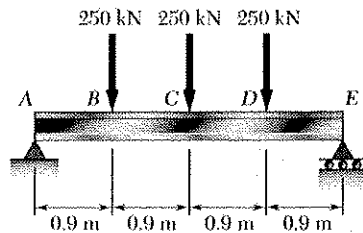
$$\tau_b = \frac{40.88 \times 10^3}{(8.9)(525)} = 26.3 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{\left(\frac{138.8}{2}\right)^2 + (26.3)^2} = 69.4 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R = \frac{138.8}{2} + 69.4 = 138.8 \text{ MPa}$$

$$\sigma_{\max} = 138.8 \text{ MPa}$$

Problem 8.7



8.7 and 8.8 (a) Knowing that $\sigma_{all} = 160 \text{ MPa}$ and $\tau_{all} = 100 \text{ MPa}$, select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

Reactions: $R_A = 375 \text{ kN} \uparrow$, $R_E = 375 \text{ kN} \uparrow$

$|V|_{max} = 375 \text{ kN}$

$|M|_{max} = 450 \text{ kN-m}$

$|V|$ at point C: 125 kN

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{450 \times 10^3}{160 \times 10^6} = 2.8125 \times 10^{-3} \text{ m}^3$$

$$= 2812.5 \times 10^3 \text{ mm}^3$$

Shape	$S_x (10^3 \text{ mm}^3)$
W 840 x 176	5890
W 760 x 147	4410
W 690 x 125	3510
W 610 x 155	4220
W 530 x 150	3720
W 460 x 158	3340
W 360 x 216	3800

(a) Use
W 690 x 125. $\blacktriangleleft$

$d = 678 \text{ mm}$

$t_f = 16.30 \text{ mm}$

$t_w = 11.7 \text{ mm}$

(b) $\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450 \times 10^3}{3510 \times 10^{-6}} = 128.2 \times 10^6 \text{ Pa}$

$\sigma_m = 128.2 \text{ MPa} \blacktriangleleft$

$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d t_w} = \frac{375 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})}$

$\tau_m = 47.3 \text{ MPa} \blacktriangleleft$

At point C, $\tau_w = \frac{V}{A_w} = \frac{125 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 15.76 \times 10^6 \text{ Pa}$
 $= 15.76 \text{ MPa}$

$c = \frac{1}{2}d = \frac{678}{2} = 339 \text{ mm}$

$y_b = c - t_f = 339 - 16.30 = 322.7 \text{ mm}$

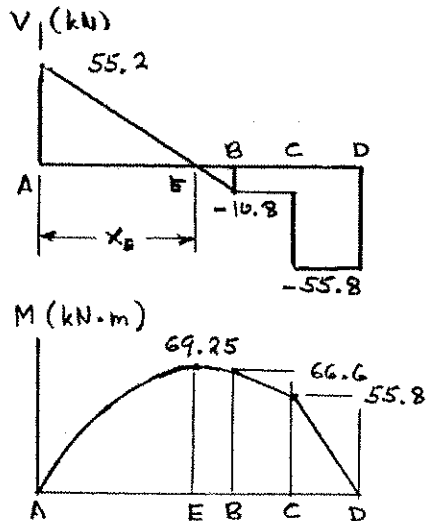
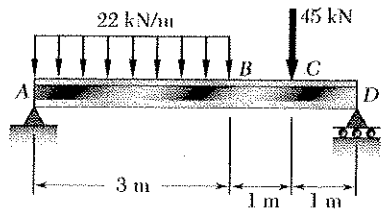
$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{322.7}{339}\right)(128.2) = 122.0 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(61.0)^2 + (15.76)^2} = 63.0 \text{ MPa}$

$\sigma_{max} = \frac{\sigma_b}{2} + R = 61.0 + 63.0$

$\sigma_{max} = 124.0 \text{ MPa} \blacktriangleleft$

Problem 8.8



8.7 and 8.8 (a) Knowing that $\sigma_{all} = 160 \text{ MPa}$ and $\tau_{all} = 100 \text{ MPa}$, select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

$$+\circlearrowleft \sum M_D = 0: -5R_A + (3.5)(3)(22) + (1)(45) = 0$$

$$R_A = 55.2 \text{ kN} \uparrow \quad R_D = 55.8 \text{ kN} \uparrow$$

Draw shear and bending moment diagrams.

Locate point where $V = 0$ and M_E .

$$55.2 - 22x_E = 0 \quad x_E = 2.5091 \text{ m}$$

$$M_E = \frac{1}{2}(55.2)(2.5091) = 69.25 \text{ kN-m}$$

$$S_{min} = \frac{|M_{max}|}{\sigma_{all}} = \frac{69.25 \times 10^3}{160 \times 10^6} = 432.8 \times 10^{-6} \text{ m}^3$$

$$= 432.8 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
W 410 $\times$ 38.8	637
$\rightarrow$ W 360 $\times$ 32.9	474
W 310 $\times$ 38.7	549
W 250 $\times$ 44.8	535
W 200 $\times$ 46.1	448

(a) Use

W 360 $\times$ 32.9 $\blacktriangleleft$

$$\text{For W 360} \times 32.9 \quad S = 474 \times 10^3 \text{ mm}^3 = 474 \times 10^{-6} \text{ m}^3$$

$$A_{web} = d t_w = (349)(5.8) = 2024.2 \text{ mm}^2 = 2024.2 \times 10^{-6} \text{ m}^2$$

$$\text{At point E: } \sigma_m = \frac{M_E}{S} = \frac{69.25 \times 10^3}{474 \times 10^{-6}}$$

$$\sigma_m = 146.1 \text{ MPa} \blacktriangleleft$$

$$\text{At point C: } \sigma_m = \frac{M_C}{S} = \frac{55.8 \times 10^3}{474 \times 10^{-6}}$$

$$\sigma_m = 117.7 \text{ MPa}$$

$$\tau_m = \frac{|V|}{A_{web}} = \frac{55.8 \times 10^3}{2024.2 \times 10^{-6}}$$

$$\tau_m = 27.6 \text{ MPa} \blacktriangleleft$$

$$c = \frac{1}{2}d = \frac{1}{2}(349) = 174.5 \text{ mm} \quad y_b = c - t_f = 166 \text{ mm}$$

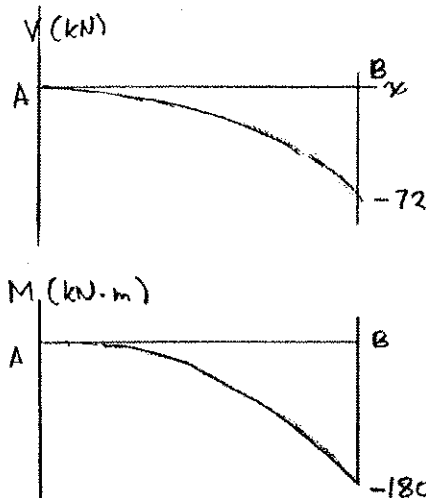
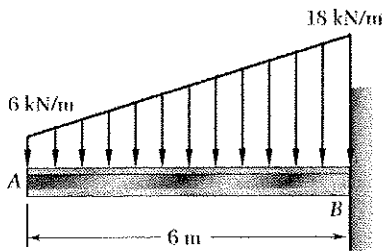
$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{166}{174.5}\right)(117.7) = 112.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = 62.4 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R$$

$$\sigma_{max} = 118.4 \text{ MPa} \blacktriangleleft$$

Problem 8.9



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.9 Loading of Prob. 5.74 and selected W530 x 66 shape.

From Problem 5.74 $\sigma_{all} = 160 \text{ MPa}$

$|M_{max}| = 180 \text{ kN-m}$ at section B.

$|V| = 72 \text{ kN}$ at section B

For W530 x 66 rolled steel section

$$d = 525 \text{ mm} \quad b_f = 165 \text{ mm} \quad t_f = 11.40 \text{ mm}$$

$$t_w = 8.9 \text{ mm} \quad I = 357 \times 10^6 \text{ mm}^4 \quad S = 1340 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 262.5 \text{ mm}$$

(a) $|M|_{max} = 180 \times 10^3 \text{ N-m} \quad S = 1340 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{max}}{S} = 134.33 \times 10^6 \quad \sigma_m = 134.3 \text{ MPa} \quad \leftarrow$$

(b) $y_b = c - t_f = 251.1 \text{ mm} \quad \bar{y} = \frac{1}{2}(c + y_b) = 256.8 \text{ mm}$

$$A_f = b_f t_f = 1881 \text{ mm}^2$$

At section B $V = 72 \times 10^3 \text{ N}$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f \bar{y}}{It} = \frac{(72 \times 10^3)(1881 \times 10^{-6})(256.8 \times 10^{-3})}{(357 \times 10^{-6})(8.9 \times 10^{-3})}$$

$$= 11.523 \times 10^6 = 11.523 \text{ MPa}$$

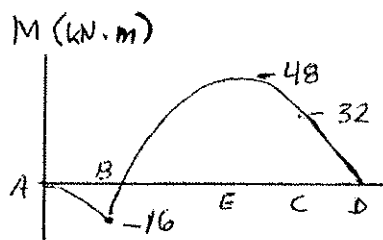
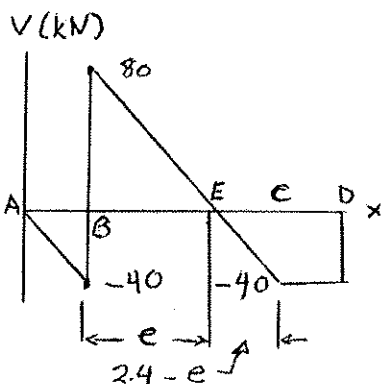
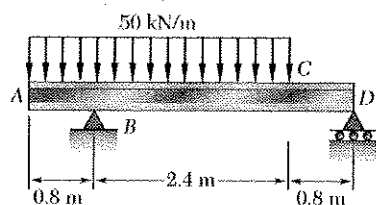
$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{251.1}{262.5} (134.33) = 128.50 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 65.226 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{128.50}{2} + 65.226$$

$$\sigma_{max} = 129.5 \text{ MPa} \quad \leftarrow$$

Problem 8.10



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.10 Loading of Prob. 5.73 and selected W250 x 28.4 shape.

From Problem 5.73 $\sigma_{all} = 160 \text{ MPa}$

$|M|_{max} = 48 \text{ kN}\cdot\text{m}$ at section E, which lies 1.6 m to the right of B.

$$|V| = 0$$

For W 250 x 28.4 rolled steel section

$$d = 260 \text{ mm} \quad b_f = 102 \text{ mm} \quad t_f = 10.0 \text{ mm}$$

$$t_w = 6.4 \text{ mm} \quad I = 40.0 \times 10^6 \text{ mm}^4 \quad S = 308 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 130 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{max}}{S} = \frac{48 \times 10^3}{308 \times 10^3} \quad \sigma_m = 155.8 \text{ MPa}$$

$$y_b = c - t_f = 120 \text{ mm}$$

$$A_f = b_f t_f = 1020 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 125 \text{ mm}$$

$$Q = A_f \bar{y} = 127.5 \times 10^3 \text{ mm}^3 = 127.5 \times 10^{-6} \text{ m}^3$$

At section E $V = 0$ $M = M_{max}$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 143.8 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It_w} = 0 \quad (b) \quad \sigma_{max} = \sigma_b \quad \sigma_{max} = 143.8 \text{ MPa}$$

At section C $|V| = 40 \text{ kN}$ $M = 32 \text{ kN}\cdot\text{m}$

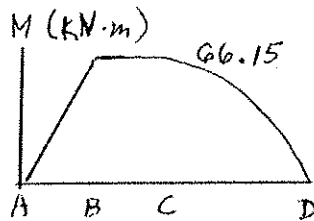
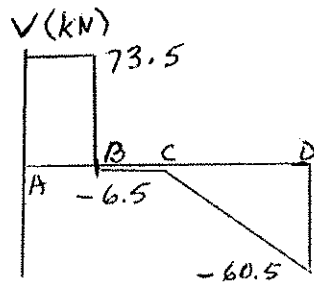
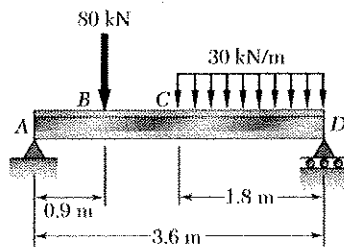
$$\sigma_b = \frac{My_b}{I} = \frac{(32 \times 10^3)(120 \times 10^{-3})}{40.0 \times 10^6} = 96 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It_w} = \frac{(40 \times 10^3)(127.5 \times 10^{-6})}{(40.0 \times 10^6)(6.4 \times 10^{-3})} = 19.92 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{48^2 + 19.92^2} = 51.97 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 100.0 \text{ MPa (less than value at section E)}$$

Problem 8.11



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.11 Loading of Prob. 5.78 and selected S310 $\times$ 47.3 shape.

From Problem 5.78 $\sigma_{all} = 160 \text{ MPa}$

$|M|_{max} = 66.15 \text{ kN}\cdot\text{m}$ at section B.

$|V| = 73.5 \text{ kN}$ at B

For S 310 $\times$ 47.3 rolled steel section

$$d = 305 \text{ mm} \quad b_f = 127 \text{ mm} \quad t_f = 13.8 \text{ mm}$$

$$t_w = 8.9 \text{ mm} \quad I = 90.6 \times 10^6 \text{ mm}^4 \quad S = 593 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 152.5 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{max}}{S} = \frac{66.15 \times 10^3}{593 \times 10^3} \quad \sigma_m = 111.6 \text{ MPa}$$

$$y_b = c - t_f = 138.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 101.5 \text{ MPa}$$

$$A_f = b_f t_f = 1752.6 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 145.6 \text{ mm}$$

$$Q = A_f \bar{y} = 255.18 \times 10^3 \text{ mm}^3 = 255.18 \times 10^{-6} \text{ m}^3$$

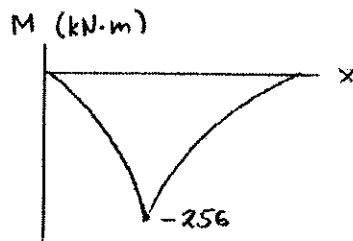
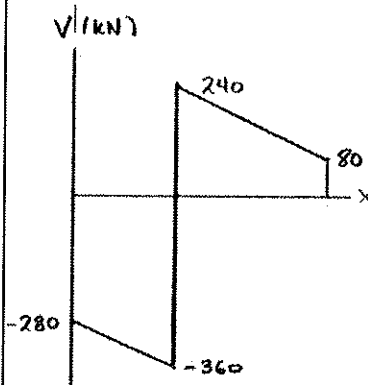
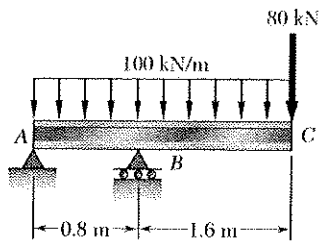
$$\tau_b = \frac{VQ}{It_w} = \frac{(73.5 \times 10^3)(255.18 \times 10^{-6})}{(90.6 \times 10^6)(8.9 \times 10^{-3})} = 23.26 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{50.75^2 + 23.26^2} = 55.83 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 106.6 \text{ MPa}$$

$$\sigma_{max} = 106.6 \text{ MPa}$$

Problem 8.12



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.12 Loading of Prob. 5.75 and selected S20 × 66 shape.

From Problem 5.75 $\sigma_{all} = 160 \text{ MPa}$

$|M|_{max} = 256 \text{ kN}\cdot\text{m}$ at point B

$|V| = 360 \text{ kN}$ at B

For S 20 × 66 rolled steel section

$$d = 508 \text{ mm}, \quad b_f = 159 \text{ mm}, \quad t_f = 20.2 \text{ mm}$$

$$t_w = 12.8 \text{ mm}, \quad I_x = 495 \times 10^6 \text{ mm}^4, \quad S_x = 1950 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 254 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{max}}{S_x} = \frac{256 \times 10^3}{1950 \times 10^3} = 131.3 \text{ MPa}$$

$$y_b = c - t_f = 233.8$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 120.9 \text{ MPa} \quad \frac{\sigma_b}{2} = 60.45 \text{ MPa}$$

$$A_f = b_f t_f = 3212 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 243.9 \text{ mm}$$

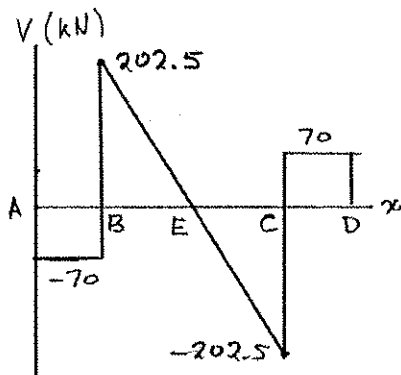
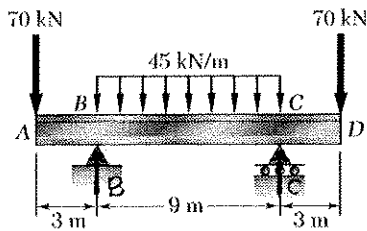
$$Q = A_f \bar{y} = 783.4 \times 10^3 \text{ mm}^3$$

$$\tau_b = \frac{VQ}{It_w} = \frac{(360 \times 10^3)(783.4 \times 10^3)}{(495 \times 10^6)(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

$$(b) \quad \sigma_{max} = \frac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5 \text{ MPa}$$

Problem 8.13



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.13 Loading of Prob. 5.77 and selected S510 $\times$ 98.3 shape.

From Problem 5.77, $\sigma_{all} = 160 \text{ MPa}$

Bending moments:

- 210 kN·m at sections B and C.

245.625 kN·m at midspan E.

$|V|_{max} = 202.5 \text{ kN}$ at sections B and C.

For S510 $\times$ 98.3 rolled steel section

$$d = 508 \text{ mm} \quad b_f = 159 \text{ mm} \quad t_f = 20.2 \text{ mm}$$

$$t_w = 12.8 \text{ mm} \quad I = 495 \times 10^6 \text{ mm}^4 \quad S = 1950 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 254 \text{ mm}$$

$$(a). \quad |M|_{max} = 245.625 \times 10^3 \text{ N·m} \quad S = 1960 \times 10^{-6} \text{ m}^3$$

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{245.625 \times 10^3}{1950 \times 10^{-6}} = 125.962 \times 10^6 \text{ Pa} \quad \sigma_m = 126.0 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad y_b = c - t_f = 233.8 \text{ mm} \quad A_f = b_f t_f = 3.212 \times 10^3 \text{ mm}^2 \quad \bar{y} = \frac{1}{2}(c + y_b) = 243.9 \text{ mm}$$

$$\text{At midspan.} \quad V = 0 \quad \tau_b = 0$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{233.8}{254} (125.962) = 115.9 \text{ MPa} \quad \sigma_{max} = 115.9 \text{ MPa} \quad \blacktriangleleft$$

At sections B and C.

$$\sigma_m = \frac{M}{S} = \frac{210 \times 10^3}{1950 \times 10^{-6}} = 107.69 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{233.8}{254} (107.69) = 99.128 \text{ MPa}$$

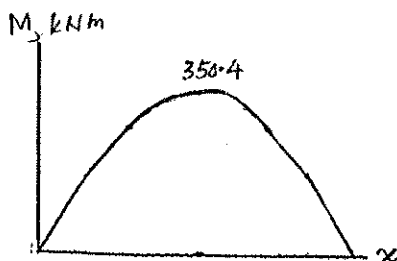
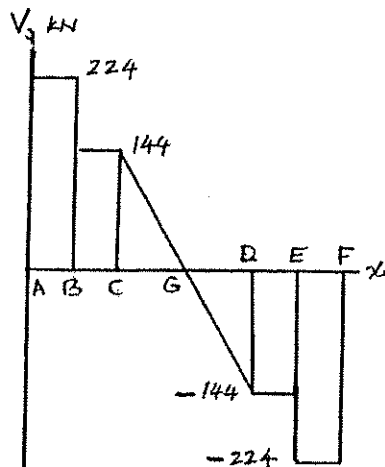
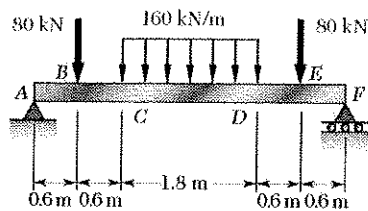
$$\tau_b = \frac{VQ}{It} = \frac{VA_f \bar{y}}{I t_w} = \frac{(202.5 \times 10^3)(3.212 \times 10^3)(243.9 \times 10^{-3})}{(495 \times 10^{-6})(12.8 \times 10^{-3})}$$

$$= 25.04 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 55.53 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \quad \sigma_{max} = 105.1 \text{ MPa} \quad \blacktriangleleft$$

Problem 8.14



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.14 Loading of Prob. 5.76 and selected S510 $\times$ 98.3 shape.

From Problem 5.76, $\sigma_{all} = 165 \text{ MPa}$

Bending moments. $M_A = 0$

$$M_B = 0 + 134.4 = 134.4 \text{ kNm}$$

$$M_C = 134.4 + 86.4 = 220.8 \text{ kNm}$$

$$M_G = 220.8 + 129.6 = 350.4 \text{ kNm}$$

$$M_D = 350.4 - 129.6 = 220.8 \text{ kNm}$$

$$M_E = 220.8 - 86.4 = 134.4 \text{ kNm}$$

$$M_F = 134.4 - 134.4 = 0$$

For S510 $\times$ 98.3 rolled steel section

$$d = 508 \text{ mm} \quad b_f = 159 \text{ mm} \quad t_f = 20.2 \text{ mm}$$

$$t_w = 12.8 \text{ mm} \quad I = 495 \times 10^6 \text{ mm}^4 \quad S = 1.95 \times 10^6 \text{ mm}^3$$

$$c = \frac{1}{2}d = 254 \text{ mm}$$

(a) $|M|_{max} = 350.4 \text{ kNm}$

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{350.4 \times 10^3}{1.95 \times 10^6} \quad \sigma_m = 179.7 \text{ MPa}$$

(b) $y_b = c - t_f = 233.8 \text{ mm} \quad \bar{y} = \frac{1}{2}(c + y_b) = 243.8 \text{ mm}$

$$A_f = b_f t_f = 3211.8 \text{ mm}^2$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f \bar{y}}{It_w} = \frac{V(3211.8 \times 10^{-6})(0.2438)}{(495 \times 10^6)(0.0128)} = 123.59 \text{ V}$$

$$\sigma_b = \frac{y_b}{c} \frac{M}{S} = \frac{0.2338}{0.254} \frac{M}{1.95 \times 10^3} = 472.04 \text{ M}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} \quad \sigma_{max} = \frac{\sigma_b}{2} + R$$

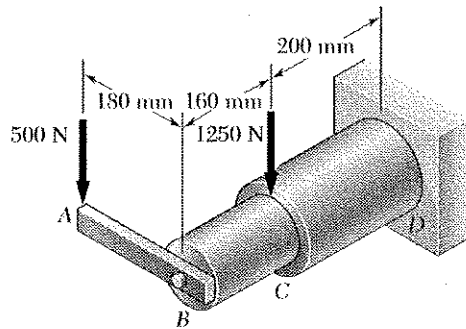
Section	τ_b (MPa)	σ_b (MPa)	R (MPa)	σ_{max} (MPa)
B & E	27.68	63.44	42.1	73.82
C & D	17.8	104.23	55.08	107.2
G	0	165.4	82.7	165.4

At C & D $\sigma_{max} = 107.2 \text{ MPa}$

At midspan, $\sigma_{max} = 165.4 \text{ MPa}$

Problem 8.15

8.15 Neglecting the effect of fillets and of stress concentrations, determine the smallest permissible diameters of the solid rods BC and CD. Use $\tau_{all} = 60 \text{ MPa}$.



$$\tau_{all} = 60 \times 10^6 \text{ Pa}$$

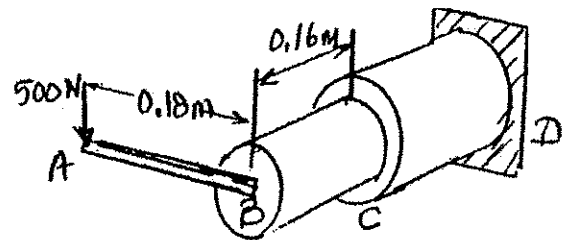
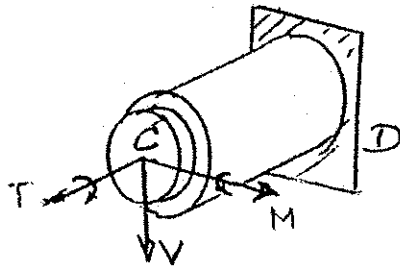
$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$C^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$d = 2C$$

Bending Moments and Torques

Just to the left of C:

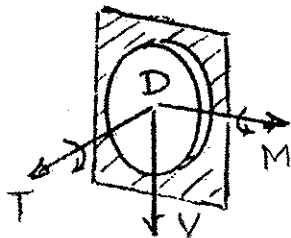


$$M = (500)(0.16) = 80 \text{ N}\cdot\text{m}$$

$$T = (500)(0.18) = 90 \text{ N}\cdot\text{m}$$

$$\sqrt{M^2 + T^2} = 120.416 \text{ N}\cdot\text{m}$$

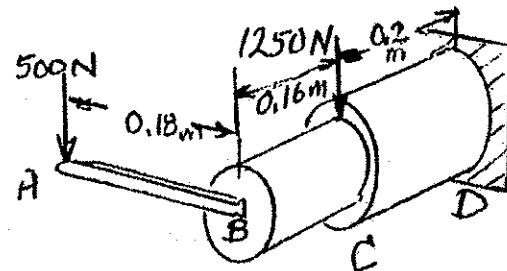
Just to the left of D:



$$T = 90 \text{ N}\cdot\text{m}$$

$$M = (500)(0.36) + (1250)(0.2) = 430 \text{ N}\cdot\text{m}$$

$$\sqrt{M^2 + T^2} = 439.32 \text{ N}\cdot\text{m}$$



Smallest permissible diameter d_{BC} .

$$C^3 = \frac{(2)(120.416)}{\pi(60 \times 10^6)} = 1.27765 \times 10^{-6} \text{ m}^3$$

$$C = 0.01085 \text{ m} = 10.85 \text{ mm}$$

$$d_{BC} = 21.7 \text{ mm} \blacktriangleleft$$

Smallest permissible diameter d_{CD} .

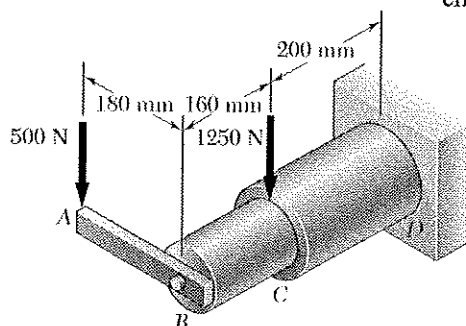
$$C^3 = \frac{(2)(439.32)}{\pi(60 \times 10^6)} = 4.6613 \times 10^{-6} \text{ m}^3$$

$$C = 0.01670 \text{ m} = 16.7 \text{ mm}$$

$$d_{CD} = 33.4 \text{ mm} \blacktriangleleft$$

Problem 8.16

8.16 Knowing that rods BC and CD are of diameter 24 mm and 36 mm, respectively, determine the maximum shearing stress in each rod. Neglect the effect of fillets and of stress concentrations.



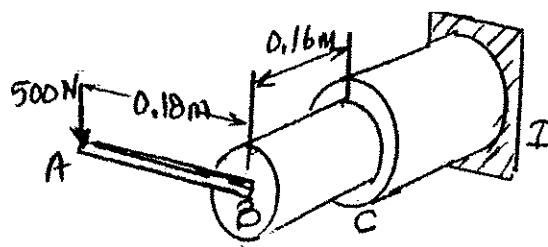
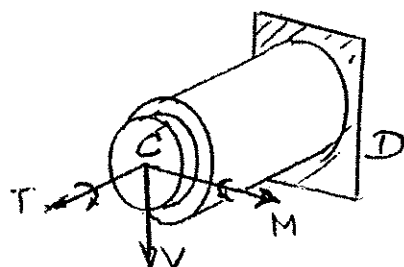
Over BC : $c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$

Over CD : $c = \frac{1}{2}d = 18 \text{ mm} = 0.018 \text{ m}$

$$\tau = \frac{\sqrt{M^2 + T^2} c}{J} = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{c^3}$$

Bending Moments and Torques

Just to the left of C:

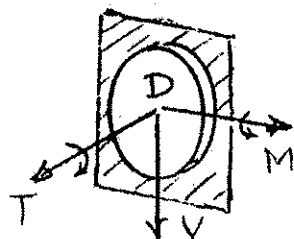


$$M = (500)(0.16) = 80 \text{ N}\cdot\text{m}$$

$$T = (500)(0.18) = 90 \text{ N}\cdot\text{m}$$

$$\sqrt{M^2 + T^2} = 120.416 \text{ N}\cdot\text{m}$$

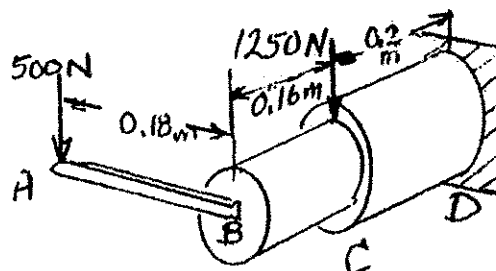
Just to the left of D:



$$T = 90 \text{ N}\cdot\text{m}$$

$$M = (500)(0.36) + (1250)(0.2) \\ \approx 430 \text{ N}\cdot\text{m}$$

$$\sqrt{M^2 + T^2} = 439.32 \text{ N}\cdot\text{m}$$



Maximum shearing stress in portion BC.

$$\tau_{\max} = \frac{(2)(120.416)}{\pi (0.012)^3} = 44.36 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 44.4 \text{ MPa} \leftarrow$$

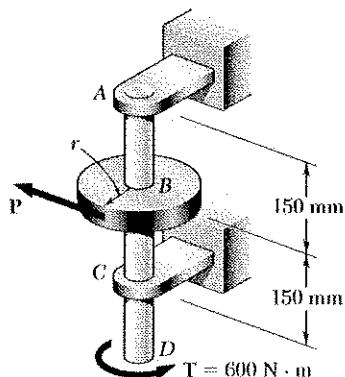
Maximum shearing stress in portion CD.

$$\tau_{\max} = \frac{(2)(439.32)}{\pi (0.018)^3} = 47.96 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 48.0 \text{ MPa} \leftarrow$$

Problem 8.17

8.17 Determine the smallest allowable diameter of the solid shaft $ABCD$, knowing that $\tau_{all} = 60 \text{ MPa}$ and that the radius of disk B is $r = 80 \text{ mm}$.



$$\sum M_{axis} = 0$$

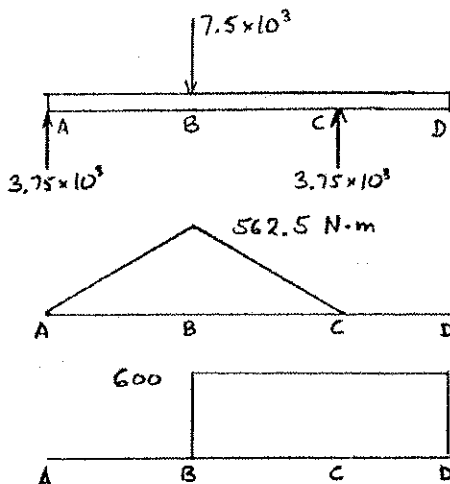
$$T - Pr = 0$$

$$P = \frac{T}{r} = \frac{600}{80 \times 10^{-3}} = 7.5 \times 10^3 \text{ N}$$

$$R_A = R_C = \frac{1}{2} P = 3.75 \times 10^3 \text{ N}$$

$$M_B = (3.75 \times 10^3)(150 \times 10^{-3}) = 562.5 \text{ N}\cdot\text{m}$$

Bending moment.



Torque.

Critical section lies at point B.

$$M = 562.5 \text{ N}\cdot\text{m}, T = 600 \text{ N}\cdot\text{m}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{(\sqrt{M^2 + T^2})_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi} \frac{\sqrt{(562.5)^2 + (600)^2}}{60 \times 10^6} = 8.726 \times 10^{-6} \text{ m}^3$$

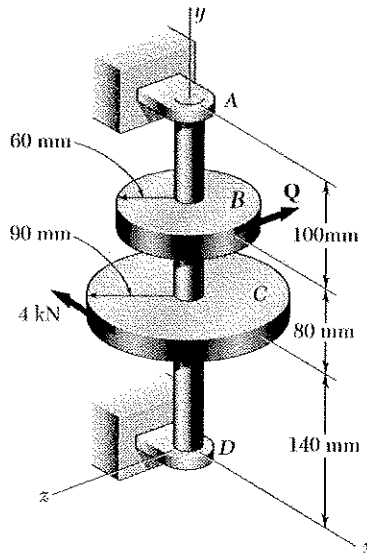
$$c = 20.58 \times 10^{-3} \text{ m}$$

$$d = 2c = 41.2 \times 10^{-3} \text{ m}$$

$$d = 41.2 \text{ mm} \blacktriangleleft$$

Problem 8.18

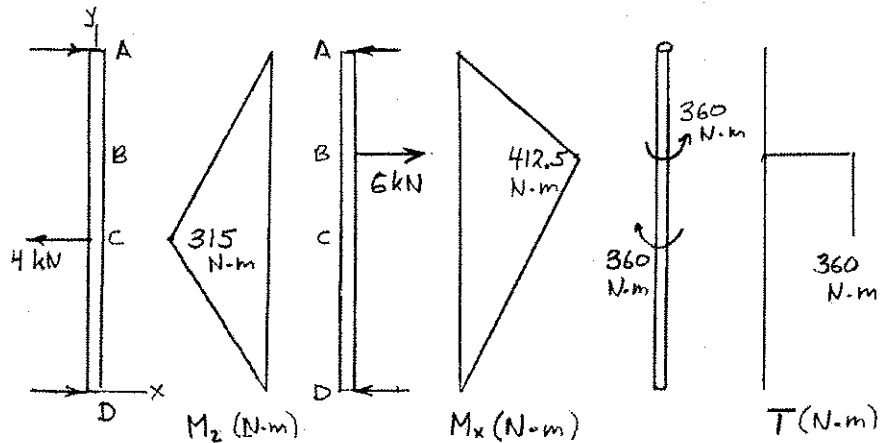
8.18 The 4-kN force is parallel to the x axis, and the force Q is parallel to the z axis. The shaft AD is hollow. Knowing that the inner diameter is half the outer diameter and that $\tau_{all} = 60 \text{ MPa}$, determine the smallest permissible outer diameter of the shaft.



$$\sum M_y = 0: 60 \times 10^{-3} Q - (90 \times 10^{-3})(4 \times 10^3) = 0$$

$$Q = 6 \times 10^3 \text{ N} = 6 \text{ kN}$$

Bending moment and torque diagrams.



In xy plane $(M_z)_{max} = 315 \text{ N.m}$ at C.

In yz plane $(M_x)_{max} = 412.5 \text{ N.m}$ at B.

About z-axis $T_{max} = 360 \text{ N.m}$ between B and C.

$$\text{At B: } M_z = \left(\frac{100}{180}\right)(315) = 175 \text{ N.m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{175^2 + 412.5^2 + 360^2} = 574.79 \text{ N.m}$$

$$\text{At C: } M_x = \left(\frac{140}{220}\right)(412.5) = 262.5 \text{ N.m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{315^2 + 262.5^2 + 360^2} = 545.65 \text{ N.m}$$

Largest value is 574.79 N.m

$$\tau_{max} = \frac{\max \sqrt{M_x^2 + M_z^2 + T^2}}{J} c$$

$$\frac{J}{c} = \frac{\max \sqrt{M_x^2 + M_z^2 + T^2}}{\tau_{max}} = \frac{574.79}{60 \times 10^6} = 9.5798 \times 10^{-6} \text{ m}^3 = 9.5798 \times 10^3 \text{ mm}^3$$

$$\frac{J}{c} = \frac{\frac{\pi}{2}(c_o^4 - c_i^4)}{c_o} = \frac{\pi}{2} c_o^3 \left(1 - \frac{c_i^4}{c_o^4}\right) = \frac{\pi}{2} c_o^3 \left[1 - \left(\frac{1}{2}\right)^4\right] = 1.47262 c_o^3$$

$$1.47262 c_o^3 = 9.5798 \times 10^3$$

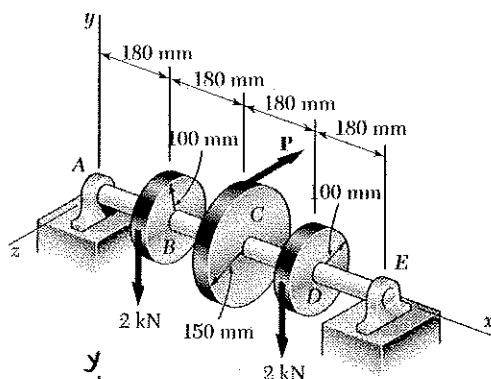
$$c_o = 18.67 \text{ mm}$$

$$d_o = 2c_o$$

$$d_o = 37.3 \text{ mm}$$

Problem 8.19

8.19 The two 2-kN forces are vertical and the force P is parallel to the z axis. Knowing that $\tau_{all} = 55 \text{ MPa}$, determine the smallest permissible diameter of the solid shaft AE .



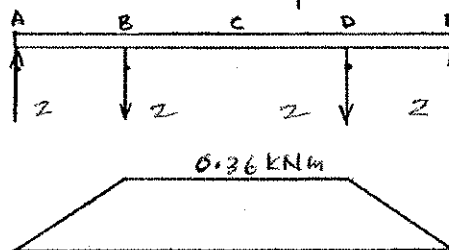
$$\sum M_x = 0 \quad (2)(100) - 150P + (2)(100) = 0$$

$$P = 2.67 \text{ kN}$$

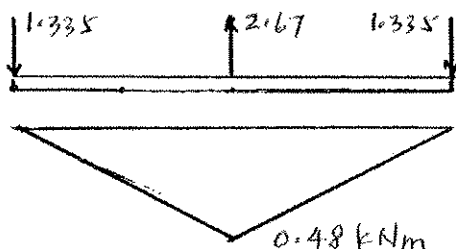
Torques:

$$\begin{aligned} AB: T &= 0 \\ BC: T &= -(2)(0.1) = -0.2 \text{ kNm} \\ CD: T &= (2)(0.1) = 0.2 \text{ kNm} \\ DE: T &= 0 \end{aligned}$$

Forces in vertical plane:



Forces in horizontal plane:



Critical sections are either side of disk C.

$$T = 0.2 \text{ kNm} \quad M_z = 0.36 \text{ kNm}$$

$$M_y = 0.48 \text{ kNm}$$

$$\tau_{all} = \frac{C}{J} \sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi C^3}{2} = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}} = \frac{\sqrt{0.48^2 + 0.36^2 + 0.2^2}}{55 \times 10^3} = 11.5 \times 10^{-6} \text{ m}^3$$

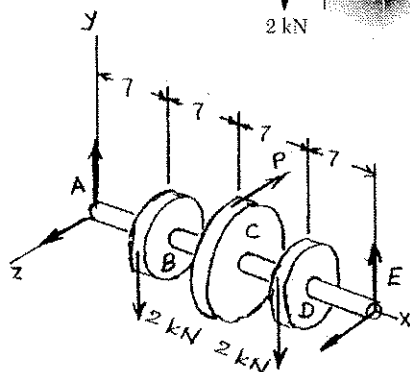
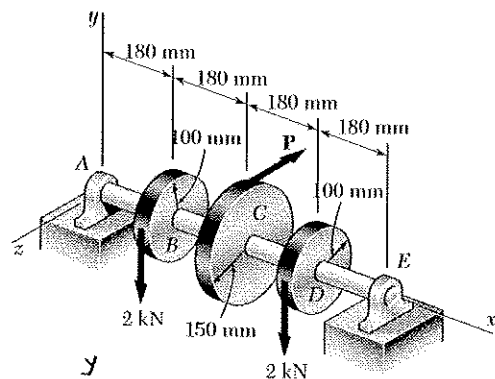
$$C = 0.0194 \text{ m}$$

$$d = 2C = 38.8 \text{ mm}$$

Problem 8.20

8.20 For the gear-and-shaft system and loading of Prob. 8.19, determine the smallest permissible diameter of shaft AE, knowing that the shaft is hollow and has an inner diameter that is $\frac{2}{3}$ the outer diameter.

8.19 The two 2-kN forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{\text{all}} = 55 \text{ MPa}$, determine the smallest permissible diameter of the solid shaft AE.



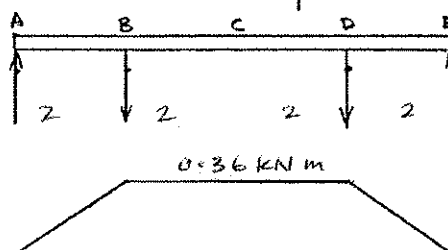
$$\sum M_x = 0 \quad (2)(100) - 150P + (2)(100) = 0$$

$$P = 2.67 \text{ kN}$$

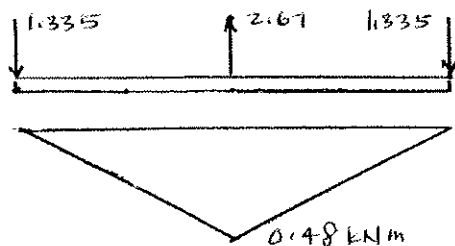
Torques:

$$\begin{aligned} \text{AB: } T &= 0 \\ \text{BC: } T &= -(2)(0.1) = -0.2 \text{ kNm} \\ \text{CD: } T &= (2)(0.1) = 0.2 \text{ kNm} \\ \text{DE: } T &= 0 \end{aligned}$$

Forces in vertical plane:



Forces in horizontal plane:



Critical sections are either side of disk C.

$$\begin{aligned} T &= 0.2 \text{ kNm} & M_z &= 0.36 \text{ kNm} \\ M_y &= 0.48 \text{ kNm} \end{aligned}$$

$$\tau_{\text{all}} = \frac{C}{J} \sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi}{2C} [C_o^4 - C_i^4] = \frac{\pi}{2C} \left[C^4 - \left(\frac{2}{3}C\right)^4 \right] = \frac{\pi}{2C} \frac{65C^4}{81} = \frac{65\pi}{162} C^3$$

$$\frac{65\pi}{162} C^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\text{all}}} = \frac{\sqrt{(0.48)^2 + (0.36)^2 + (0.2)^2}}{55 \times 10^3} = 11.5 \times 10^{-6} \text{ m}^3$$

$$C = 0.0209 \text{ m}$$

$$d = 2C = 41.8 \text{ mm}$$

Problem 8.21

8.21 Using the notation of Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a circular shaft can be expressed as follows:

$$\sigma_{\max} = \frac{C}{J} [(M_y^2 + M_z^2)^{1/2} + (M_y^2 + M_z^2 + T^2)^{1/2}]_{\max}$$

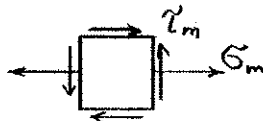
Maximum bending stress

$$\sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2} c}{I}$$

Maximum torsional stress

$$\tau_m = \frac{Tc}{J}$$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2} c}{2I} = \frac{C}{J} \sqrt{M_y^2 + M_z^2}$$



Using Mohr's circle,

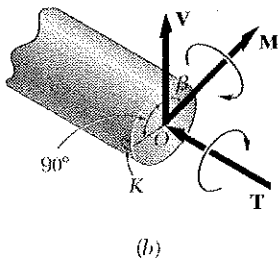
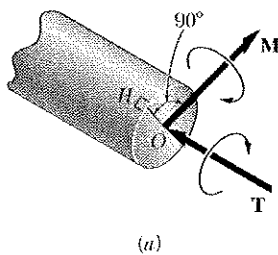
$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{C^2}{J^2} (M_y^2 + M_z^2) + \frac{T^2 c^2}{J^2}}$$

$$= \frac{C}{J} \sqrt{M_y^2 + M_z^2 + T^2}$$

$$\sigma_{\max} = \frac{\sigma_m}{2} + R = \frac{C}{J} \sqrt{M_y^2 + M_z^2} + \frac{C}{J} \sqrt{M_y^2 + M_z^2 + T^2}$$

$$= \frac{C}{J} \left[(M_y^2 + M_z^2)^{1/2} + (M_y^2 + M_z^2 + T^2)^{1/2} \right]$$

Problem 8.22



8.22 It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point H (Fig. P8.22a) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J} \sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point K (Fig. P8.22b), where the effect of the shear V is greatest, can be expressed as

$$\tau_K = \frac{c}{J} \sqrt{(M \cos \beta)^2 + \left(\frac{2}{3} c V + T\right)^2}$$

where β is the angle between the vectors $\mathbf{V}$ and $\mathbf{M}$. It is clear that the effect of the shear V cannot be ignored when $\tau_K \geq \tau_H$. (Hint: Only the component of $\mathbf{M}$ along $\mathbf{V}$ contributes to the shearing stress at K.)

Shearing stress at point K.

Due to V: For a semicircle $Q = \frac{2}{3} c^3$
 For a circle cut across its diameter $t = d = 2c$
 For a circular section $I = \frac{1}{2} J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)(\frac{2}{3} c^3)}{(\frac{1}{2} J)(2c)} = \frac{2}{3} \frac{Vc^2}{J}$$

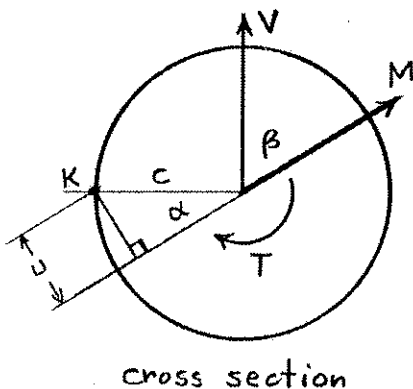
Due to T: $\tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation,

$$\tau_{xy} = \frac{c}{J} \left(\frac{2}{3} Vc + T \right)$$

Bending stress at point K: $\sigma_x = \frac{Mu}{I} = \frac{2Mu}{J}$

where u is distance between point K and the neutral axis,



cross section

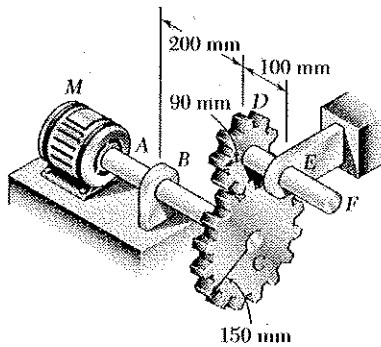
$$u = c \sin \alpha = c \sin \left(\frac{\pi}{2} - \beta \right) = c \cos \beta$$

$$\sigma_x = \frac{2Mc \cos \beta}{J}$$

Using Mohr's circle.

$$\begin{aligned} \tau_K = R &= \sqrt{\left(\frac{\sigma_x}{2} \right)^2 + \tau_{xy}^2} \\ &= \frac{c}{J} \sqrt{(M \cos \beta)^2 + \left(\frac{2}{3} Vc + T \right)^2} \end{aligned}$$

Problem 8.23



8.23 The solid shafts ABC and DEF and the gears shown are used to transmit 15 kW from the motor M to a machine tool connected to shaft DEF . Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 50$ MPa, determine the smallest permissible diameter of (a) shaft ABC , (b) shaft DEF .

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

$$(a) \text{ Shaft } ABC: T = \frac{P}{2\pi f} = \frac{30}{(2\pi)(4)} = 1.194 \text{ kNm}$$

$$\text{Gear C} \quad F_{cd} = \frac{T}{r_c} = \frac{1.194}{0.15} = 7.96 \text{ kN}$$

$$\text{Bending moment at B: } M_B = (0.2)(7.96) = 1.592 \text{ kN}$$

$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi C^3}{2} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(1.194)^2 + (1.592)^2}}{50 \times 10^3} = 39.8 \times 10^{-6} \text{ m}^3$$

$$C = 0.0294 \text{ m}$$

$$d = 2C = 58.8 \text{ mm}$$

$$(b) \text{ Shaft } DEF: T = r_d F_{cd} = (0.09)(7.96) = 0.7164 \text{ kNm}$$

$$\text{Bending moment at E: } M_E = (0.1)(7.96) = 0.796 \text{ kNm}$$

$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi C^3}{2} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(0.796)^2 + (0.7164)^2}}{50 \times 10^3} = 21.418 \times 10^{-6} \text{ m}^3$$

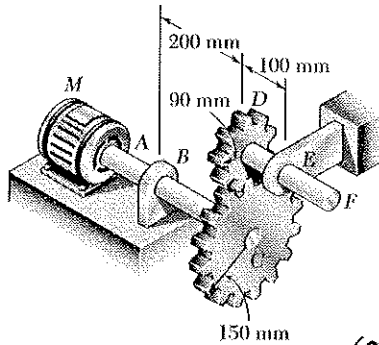
$$C = 0.0239 \text{ m}$$

$$d = 2C = 47.8 \text{ mm}$$

Problem 8.24

8.24 Solve Prob. 8.23, assuming that the motor rotates at 360 rpm.

8.23 The solid shafts ABC and DEF and the gears shown are used to transmit 15 kW from the motor M to a machine tool connected to shaft DEF . Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 50$ MPa, determine the smallest permissible diameter of (a) shaft ABC , (b) shaft DEF .



$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$(a) \text{ Shaft } ABC: T = \frac{P}{2\pi f} = \frac{30}{(2\pi)(6)} = 0.796 \text{ kNm}$$

$$\text{Gear C} \quad F_{co} = \frac{T}{r_c} = \frac{0.796}{0.15} = 5.31 \text{ kN}$$

$$\text{Bending moment at B: } M_B = (0.2)(5.31) = 1.062 \text{ kNm}$$

$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{1.062^2 + 0.796^2}}{50 \times 10^3} = 26.544 \times 10^{-6} \text{ m}^3$$

$$C = 0.0257 \text{ m} \quad d = 2C = 51.4 \text{ mm}$$

$$(b) \text{ Shaft } DEF: T = r_D F_{co} = (0.09)(5.31) = 0.478 \text{ kNm}$$

$$\text{Bending moment at E: } M_E = (0.1)(5.31) = 0.531 \text{ kNm}$$

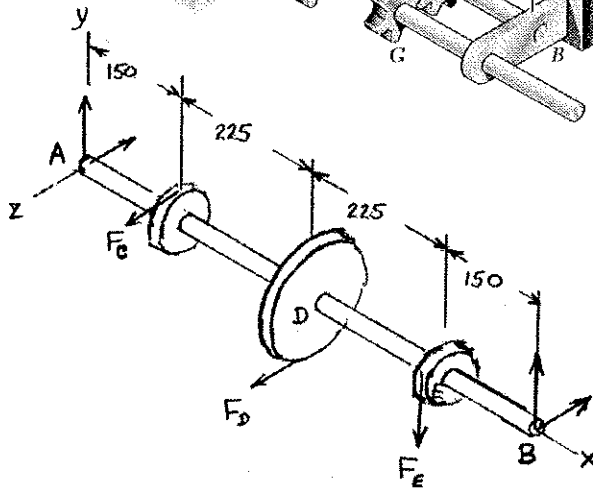
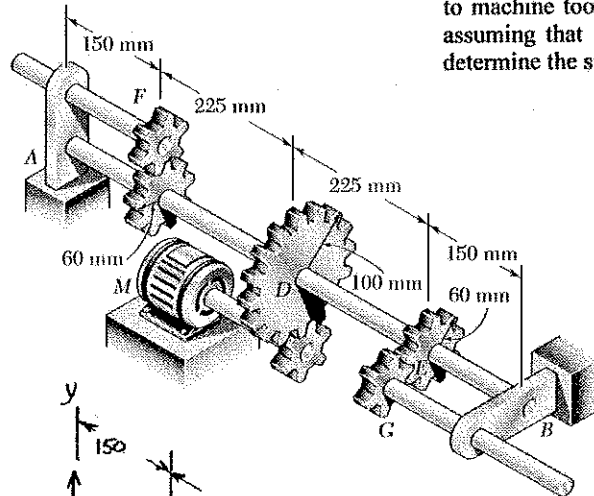
$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{0.531^2 + 0.478^2}}{50 \times 10^3} = 14.29 \times 10^{-6}$$

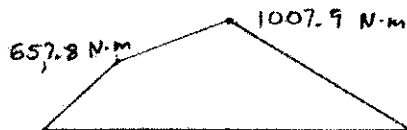
$$C = 0.0209 \text{ m} \quad d = 2C = 41.8 \text{ mm}$$

Problem 8.25

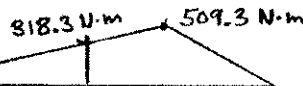
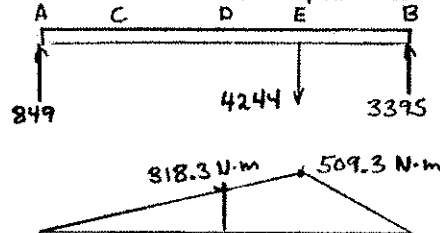
8.25 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G . Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G , determine the smallest permissible diameter of shaft AB .



Forces in horizontal plane.



Forces in vertical plane.



$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D:

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)} = 424.41 \text{ N·m}$$

Torques on gears C and E:

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N·m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N·m}$$

Forces on gears:

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60 \times 10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60 \times 10^{-3}} = 4244 \text{ N}$$

Torques in various parts

$$AC: T = 0$$

$$CD: T = 169.76 \text{ N·m}$$

$$DE: T = 254.65 \text{ N·m}$$

$$EB: T = 0$$

Critical point lies just the right of D.

$$T = 254.65 \text{ N·m}$$

$$M_y = 1007.9 \text{ N·m}$$

$$M_z = 318.3 \text{ N·m}$$

$$(\sqrt{M_y^2 + M_z^2 + T^2})_{max} = 1087.2 \text{ N·m}$$

$$\tau_{all} = \frac{c}{J} (\sqrt{M_y^2 + M_z^2 + T^2})_{max}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{(\sqrt{M_y^2 + M_z^2 + T^2})_{max}}{\tau_{all}} = \frac{1087.2}{55 \times 10^6}$$

$$= 19.767 \times 10^{-3} \text{ m}$$

$$c = 23.26 \times 10^{-3} \text{ m}$$

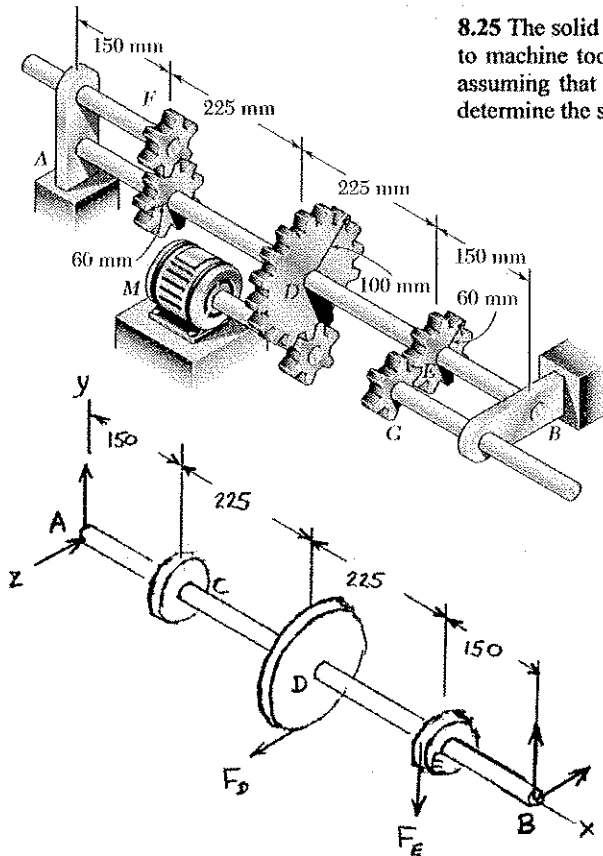
$$d = 2c = 46.5 \times 10^{-3} \text{ m}$$

$$d = 46.5 \text{ mm}$$

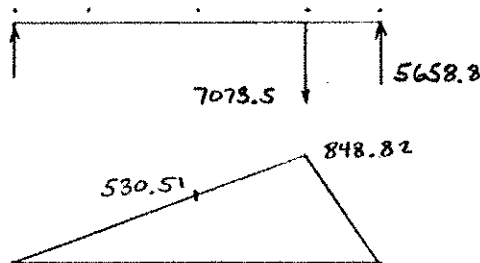
Problem 8.26

8.26 Solve Prob. 8.25, assuming that the entire 20 kW is taken off at gear G.

8.25 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G . Knowing that $\tau_{all} = 55 \text{ MPa}$ and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G , determine the smallest permissible diameter of shaft AB .



Forces in vertical plane



$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)} = 424.41 \text{ N}\cdot\text{m}$$

Torque on gear E

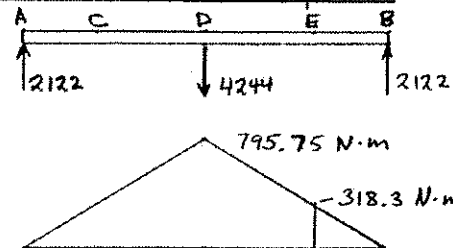
$$T_E = T_D = 424.41 \text{ N}\cdot\text{m}$$

Forces on gears D and E

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{424.41}{60 \times 10^{-3}} = 7073.5 \text{ N}$$

Forces in horizontal plane



Bending moments

$$M_D = \sqrt{530.51^2 + 795.75^2} = 956.4 \text{ N}\cdot\text{m}$$

$$M_E = \sqrt{848.82^2 + 318.3^2} = 906.5 \text{ N}\cdot\text{m}$$

$$(\sqrt{M^2 + T^2})_{max} = \sqrt{956.4^2 + 424.41^2} = 1046.3 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{C}{J} (\sqrt{M^2 + T^2})_{max}$$

$$\frac{J}{C} = \frac{\pi C^3}{32} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{1046.3}{55 \times 10^6} = 19.024 \times 10^{-6} \text{ m}^3$$

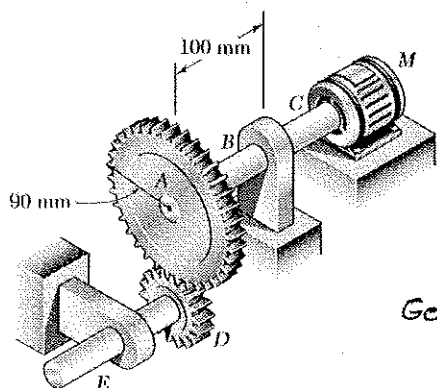
$$C = 22.96 \times 10^{-3} \text{ m}$$

$$d = 2C = 45.9 \times 10^{-3} \text{ m}$$

$$d = 45.9 \text{ mm}$$

Problem 8.27

8.27 The solid shaft ABC and the gears shown are used to transmit 10 kW from the motor M to a machine tool connected to gear D . Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 60 \text{ MPa}$, determine the smallest permissible diameter of shaft ABC .



$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A.



$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B:

$$M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$$

$$\tau_{\text{all}} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}}$$

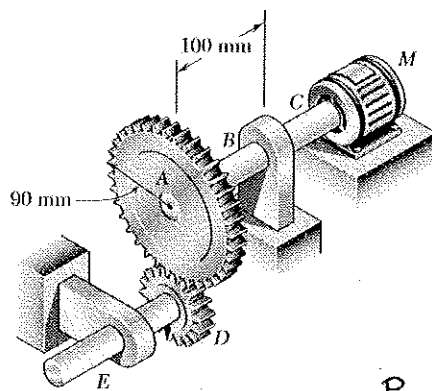
$$C^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{(2) \sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)} = 6.3108 \times 10^{-6} \text{ m}^3$$

$$C = 18.479 \times 10^{-3} \text{ m}$$

$$d = 2C = 37.0 \times 10^{-3} \text{ m}$$

$$d = 37.0 \text{ mm} \blacktriangleleft$$

Problem 8.28



8.28 Assuming that shaft *ABC* of Prob. 8.27 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.

8.27 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A,



$$Fr_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B:

$$M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{C_o}{J} \sqrt{M^2 + T^2}$$

$$C_o = \frac{1}{2} d_o = 25 \times 10^{-3} \text{ m}$$

$$\frac{J}{C_o} = \frac{\pi}{2} \frac{(C_o^4 - C_i^4)}{C_o} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$C_i^4 = C_o^4 - \frac{2C_o \sqrt{M^2 + T^2}}{\pi \tau_{all}} = (25 \times 10^{-3})^4 - \frac{(2)(25 \times 10^{-3}) \sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)}$$

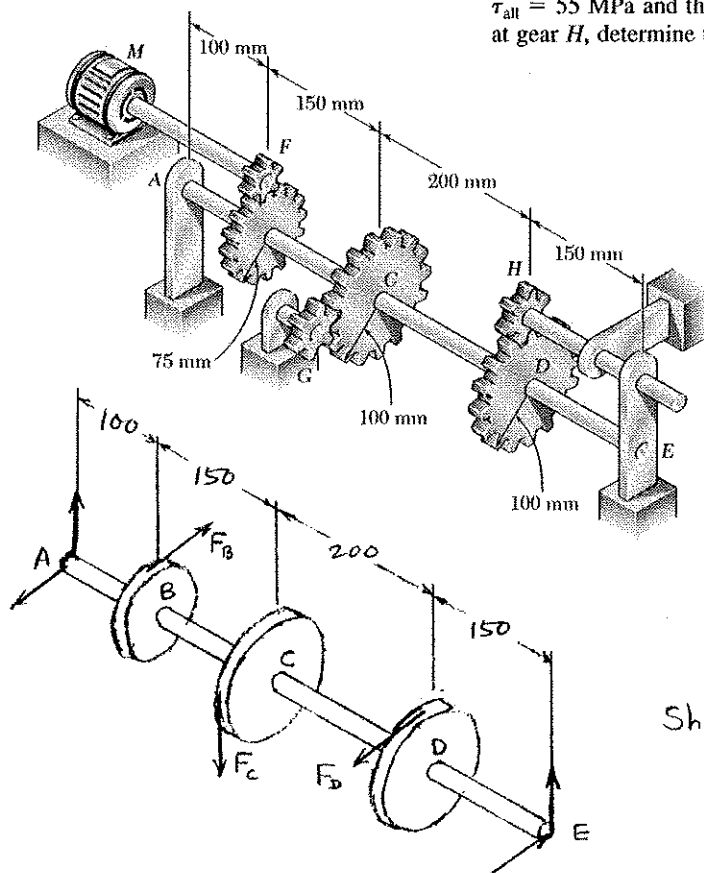
$$= 390.625 \times 10^{-9} - 157.772 \times 10^{-9} = 232.85 \times 10^{-9}$$

$$C_i = 21.967 \times 10^{-3} \text{ m}$$

$$d_i = 2C_i = 43.93 \times 10^{-3} \text{ m} \quad d = 43.9 \text{ mm} \quad \blacktriangleleft$$

Problem 8.29

8.29 The solid shaft AE rotates at 600 rpm and transmits 45 kW from the motor M to machine tools connected to gears G and H . Knowing that $\tau_{all} = 55 \text{ MPa}$ and that 30 kW is taken off at gear G and 15 kW is taken off at gear H , determine the smallest permissible diameter of shaft AE .



$$f = \frac{600 \text{ rpm}}{60 \text{ s/min}} = 10 \text{ Hz}$$

Torque on gear B:

$$T_B = \frac{P}{2\pi f} = \frac{(45000)}{2\pi(10)} = 716.2 \text{ Nm}$$

Torques on gears C and D:

$$T_C = \frac{30}{45} T_B = 477.5 \text{ Nm}$$

$$T_D = \frac{15}{45} T_B = 238.7 \text{ Nm}$$

Shaft torques:

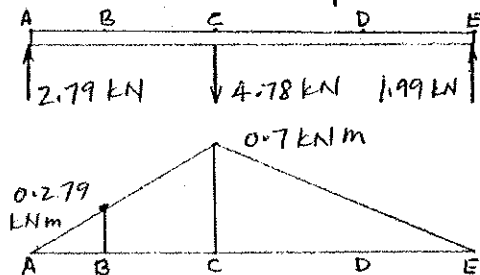
$$AB: T_{AB} = 0$$

$$BC: T_{BC} = 716.2 \text{ Nm}$$

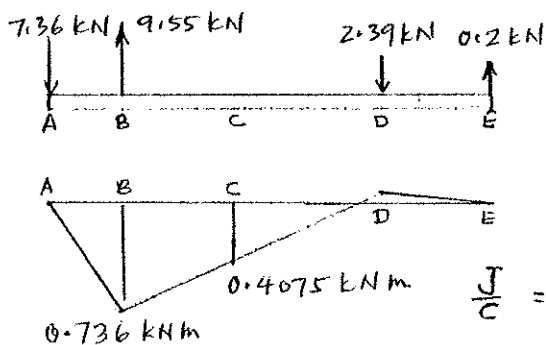
$$CD: T_{CD} = 238.7 \text{ Nm}$$

$$DE: T_{DE} = 0$$

Forces in vertical plane



Forces in horizontal plane



Gear forces:

$$F_B = \frac{T_B}{r_B} = \frac{716.2}{0.075} = 9.55 \text{ kN}$$

$$F_C = \frac{T_C}{r_C} = \frac{477.5}{0.1} = 4.78 \text{ kN}$$

$$F_D = \frac{T_D}{r_D} = \frac{238.7}{0.1} = 2.39 \text{ kN}$$

At B, $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{0.279^2 + 0.736^2 + 0.7162^2} = 1.064 \text{ kNm}$$

At C, $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{0.7^2 + 0.4075^2 + 0.7162^2} = 1.081 \text{ kNm (maximum)}$$

$$\tau_{all} = \frac{C}{J} (\sqrt{M_z^2 + M_y^2 + T^2})_{max}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{(\sqrt{M_z^2 + M_y^2 + T^2})_{max}}{\tau_{all}} = \frac{1.081}{55 \times 10^3} = 19.655 \times 10^{-6} \text{ m}^3$$

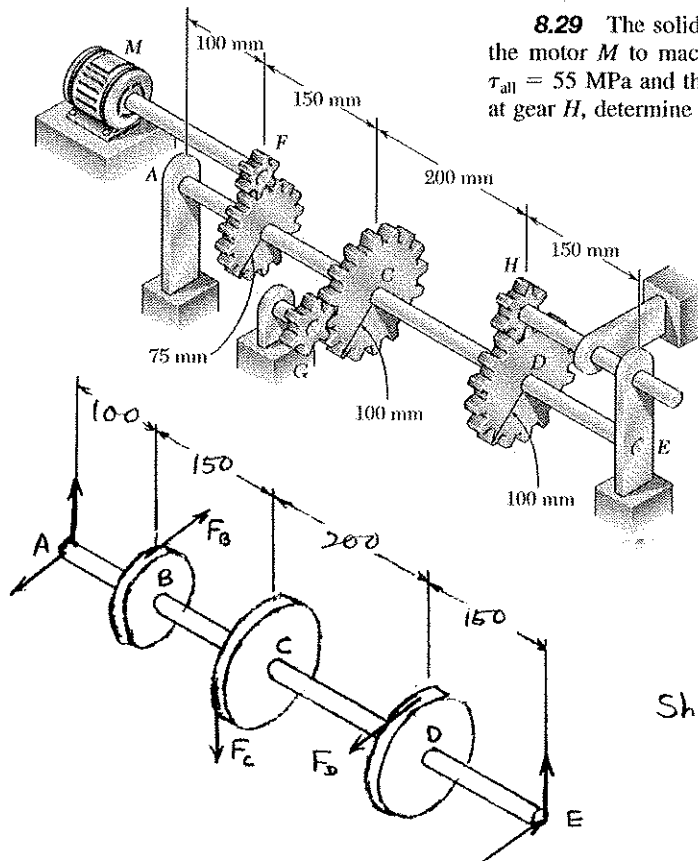
$$C = 0.0232 \text{ m}$$

$$d = 2C = 46.4 \text{ mm}$$

Problem 8.30

8.30 Solve Prob. 8.29, assuming that 22 kW is taken off at gear G and 22 kW is taken off at gear H.

8.29 The solid shaft AE rotates at 600 rpm and transmits 45 kW from the motor M to machine tools connected to gears G and H. Knowing that $\tau_{all} = 55 \text{ MPa}$ and that 30 kW is taken off at gear G and 15 kW is taken off at gear H, determine the smallest permissible diameter of shaft AE.



$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear B

$$T_B = \frac{P}{2\pi f} = \frac{45000}{2\pi(10)} = 716.2 \text{ Nm}$$

Torques on gears C and D

$$T_C = \frac{22}{45} T_B = 350 \text{ Nm}$$

$$T_D = \frac{22}{45} T_B = 350 \text{ Nm}$$

Shaft torques

$$AB: T_{AB} = 0$$

$$BC: T_{BC} = 716.2 \text{ Nm}$$

$$CD: T_{CD} = 350 \text{ Nm}$$

$$DE: T_{DE} = 0$$

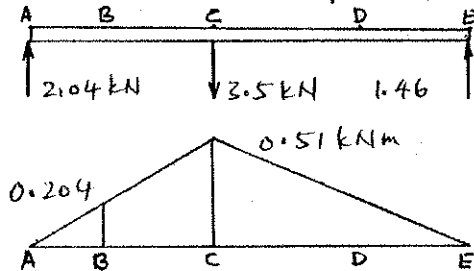
Gear forces

$$F_B = \frac{T_B}{r_B} = \frac{716.2}{0.075} = 9.55 \text{ kN}$$

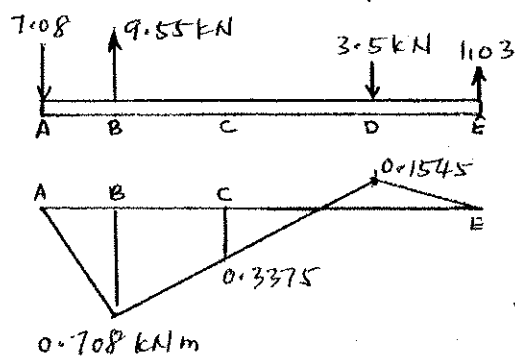
$$F_C = \frac{T_C}{r_C} = \frac{350}{0.1} = 3.5 \text{ kN}$$

$$F_D = \frac{T_D}{r_D} = \frac{350}{0.1} = 3.5 \text{ kN}$$

Forces in vertical plane



Forces in horizontal plane



$$\begin{aligned} \text{At B}^+ & \sqrt{M_z^2 + M_y^2 + T^2} \\ &= \sqrt{0.204^2 + 0.708^2 + 0.7162^2} \\ &= 1.0275 \text{ kNm (maximum)} \end{aligned}$$

$$\begin{aligned} \text{At C}^- & \sqrt{M_z^2 + M_y^2 + T^2} \\ &= \sqrt{0.51^2 + 0.3375^2 + 0.7162^2} \\ &= 0.942 \text{ kNm} \end{aligned}$$

$$\tau_{all} = \frac{C}{J} (\sqrt{M_z^2 + M_y^2 + T^2})_{max}$$

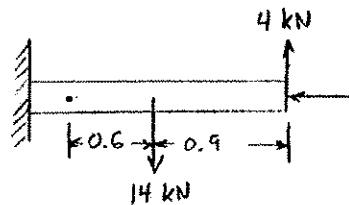
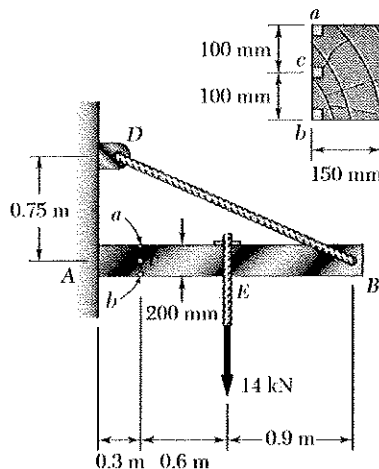
$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{(\sqrt{M_z^2 + M_y^2 + T^2})_{max}}{\tau_{all}} = \frac{1.0275}{55 \times 10^3} = 18.682 \times 10^{-6} \text{ m}^3$$

$$C = 0.0228$$

$$d = 2C = 45.6 \text{ mm}$$

Problem 8.31

8.31 The cantilever beam AB has a rectangular cross section of 150×200 mm. Knowing that the tension in cable BD is 10.4 kN and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.



$$\overline{DB} = \sqrt{0.75^2 + 1.8^2} = 1.95 \text{ m}$$

$$\text{Vertical component of } T_{DB} = \left(\frac{0.75}{1.95}\right)(10.4) = 4 \text{ kN}$$

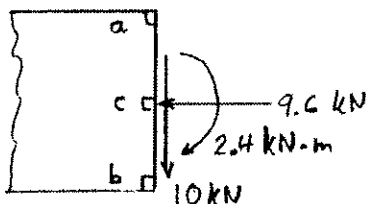
$$\text{Horizontal component of } T_{DB} = \left(\frac{1.8}{1.95}\right)(10.4) = 9.6 \text{ kN}$$

At section containing points a , b , and c

$$P = -9.6 \text{ kN}$$

$$V = 14 - 4 = 10 \text{ kN}$$

$$M = (1.5)(4) - (0.6)(14) = -2.4 \text{ kN}\cdot\text{m}$$



Section properties

$$A = (0.150)(0.200) = 0.030 \text{ m}^2$$

$$I = \frac{1}{12}(0.150)(0.200)^3 = 100 \times 10^{-6} \text{ m}^4$$

$$c = 0.100 \text{ m}$$

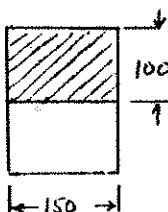
$$\text{At point } a \quad \sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} - \frac{(2.4 \times 10^3)(0.100)}{100 \times 10^{-6}} = -2.08 \text{ MPa}$$

$$\tau_{xy} = 0$$

$$\text{At point } b \quad \sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} + \frac{(2.4 \times 10^3)(0.100)}{100 \times 10^{-6}} = -2.72 \text{ MPa}$$

$$\tau_{xy} = 0$$

$$\text{At point } c \quad \sigma_x = -\frac{P}{A} = -\frac{9.6 \times 10^3}{0.030} = -0.320 \text{ MPa}$$

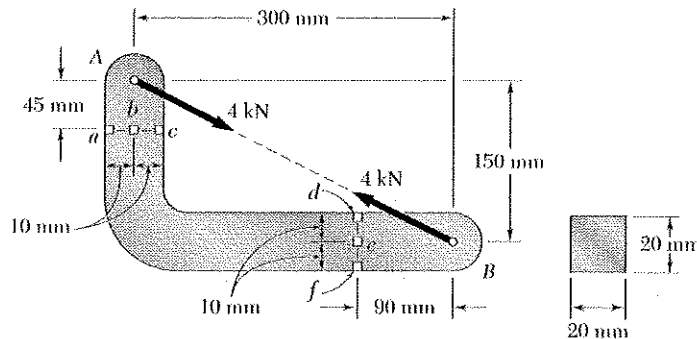


$$Q = (150)(100)(50) = 750 \times 10^3 \text{ mm}^3 = 750 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(10 \times 10^3)(750 \times 10^{-6})}{(100 \times 10^{-6})(0.150)} = -0.500 \text{ MPa}$$

Problem 8.32

8.32 Two 4-kN forces are applied to an L-shaped machine element AB as shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.

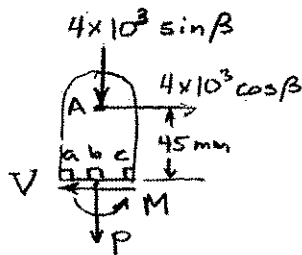


Let β be the slope angle of line AB.

$$\tan \beta = \frac{150}{300} \quad \beta = 26.565^\circ$$

Draw free body sketch of the portion of the machine element lying above section abc.

$$P = -(4 \times 10^3) \sin \beta = -1.78885 \times 10^3 \text{ N}$$



$$V = (4 \times 10^3) \cos \beta = 3.5777 \times 10^3 \text{ N}$$

$$M = (45 \times 10^{-3})(4 \times 10^3) \cos \beta = 160.997 \text{ N}\cdot\text{m}$$

$$\text{Section properties: } A = (20)(20) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(20)(20)^3 = 13.3333 \times 10^3 \text{ mm}^4 = 13.3333 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ Point a. } \sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885 \times 10^3}{400 \times 10^{-6}} - \frac{(160.997)(-10 \times 10^{-3})}{13.3333 \times 10^{-9}}$$

$$= 116.3 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = 0 \quad \blacktriangleleft$$

$$(b) \text{ Point b. } \sigma = \frac{P}{A} = -\frac{1.78885 \times 10^3}{400 \times 10^{-6}}$$

$$= -4.47 \text{ MPa} \quad \blacktriangleleft$$

$$Q = (20)(10)(5) = 1000 \text{ mm}^3 = 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{IL} = \frac{(3.5777)(10^{-6})}{(13.3333 \times 10^{-9})(20 \times 10^{-3})} = 13.42 \text{ MPa} \quad \blacktriangleleft$$

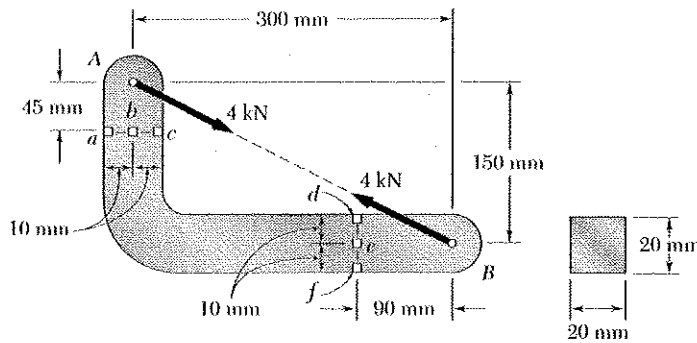
$$(c) \text{ Point c. } \sigma = \frac{P}{A} - \frac{Mx}{I} = -\frac{1.78885 \times 10^3}{400 \times 10^{-6}} - \frac{(160.997)(10 \times 10^{-3})}{13.3333 \times 10^{-9}}$$

$$= -125.2 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = 0 \quad \blacktriangleleft$$

Problem 8.33

8.33 Two 4-kN forces are applied to an L-shaped machine element AB as shown. Determine the normal and shearing stresses at (a) point d, (b) point e, (c) point f.



Let β be the slope angle of line AB.

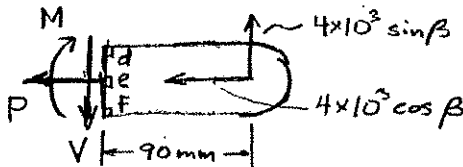
$$\tan \beta = \frac{150}{300} \quad \beta = 26.565^\circ$$

Draw free body sketch of the portion of the machine element lying to the right of section def.

$$P = -(4 \times 10^3) \cos \beta = -3.5777 \times 10^3 \text{ N}$$

$$V = (4 \times 10^3) \sin \beta = 1.78885 \times 10^3 \text{ N}$$

$$M = (90 \times 10^{-3})(4 \times 10^3) \sin \beta = 160.997 \text{ N}\cdot\text{m}$$



Section properties. $A = (20)(20) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$
 $I = \frac{1}{12}(20)(20)^3 = 13.3333 \times 10^3 \text{ mm}^4 = 13.3333 \times 10^{-9} \text{ m}^4$

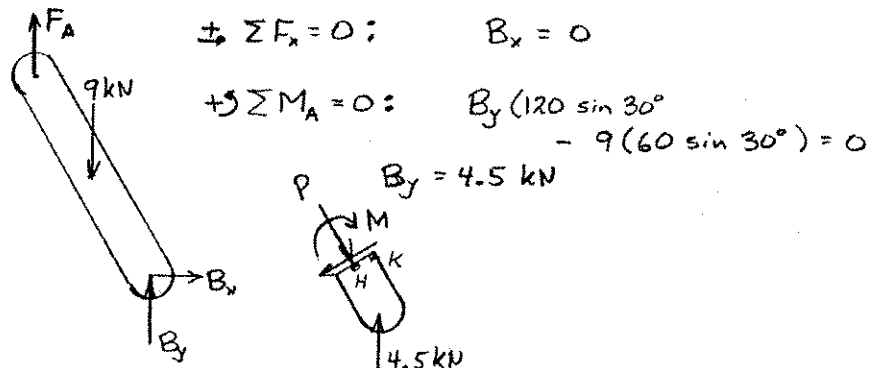
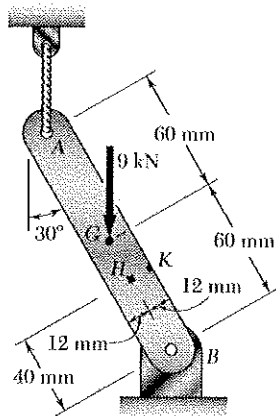
(a) Point d. $\sigma = \frac{P}{A} - \frac{My}{I} = \frac{-3.5777 \times 10^3}{400 \times 10^{-6}} - \frac{(160.997)(10 \times 10^{-3})}{13.3333 \times 10^{-9}}$
 $= -129.7 \text{ MPa} \quad \leftarrow$
 $\tau = 0 \quad \leftarrow$

(b) Point e. $\sigma = \frac{P}{A} = \frac{-3.5777 \times 10^3}{400 \times 10^{-6}} = -8.94 \text{ MPa} \quad \leftarrow$
 $Q = (20)(10)(5) = 1000 \text{ mm}^3 = 10^{-6} \text{ m}^3$
 $\tau = \frac{VQ}{It} = \frac{(1.78885 \times 10^3)(10^{-6})}{(13.3333 \times 10^{-9})(20 \times 10^{-3})} = 6.71 \text{ MPa} \quad \leftarrow$

(c) Point f. $\sigma = \frac{P}{A} - \frac{My}{I} = \frac{-3.5777 \times 10^3}{400 \times 10^{-6}} - \frac{(160.997)(-10 \times 10^{-3})}{13.3333 \times 10^{-9}}$
 $= 111.8 \text{ MPa} \quad \leftarrow$
 $\tau = 0 \quad \leftarrow$

Problem 8.34

8.34 through 8.36 Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (a) point H, (b) point K.



At the section containing points H and K,

$$P = 4.5 \cos 30^\circ = 3.897 \text{ kN}$$

$$V = 4.5 \sin 30^\circ = 2.25 \text{ kN}$$

$$M = (4.5 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) = 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

(a) At point H: $\sigma_x = -\frac{P}{A} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} = \sigma = -16.24 \text{ MPa} \leftarrow$

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2.25 \times 10^3}{240 \times 10^{-6}} \quad \tau = 14.06 \text{ MPa} \leftarrow$$

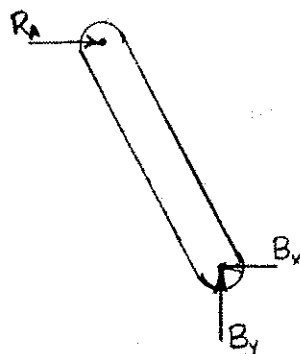
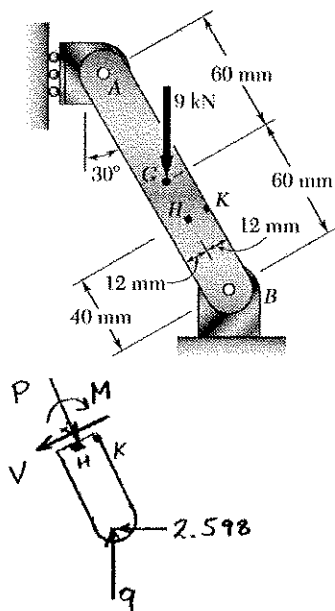
(b) At point K: $\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$

$$\sigma = 110.0 \text{ MPa} \leftarrow$$

$$\tau = 0 \quad \leftarrow$$

Problem 8.35

8.34 through 8.36 Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (a) point H, (b) point K.



$$\sum M_B = 0:$$

$$(120 \cos 30^\circ) R_A - (60 \sin 30^\circ)(9) = 0$$

$$R_A = 2.598 \text{ kN}$$

$$\uparrow \sum F_y = 0: B_y - 9 = 0 \quad B_y = 9 \text{ kN} \uparrow$$

$$\rightarrow \sum F_x = 0: 2.598 - B_x = 0 \quad B_x = 2.598 \text{ kN} \leftarrow$$

At the section containing points H and K,

$$P = 9 \cos 30^\circ + 2.598 \sin 30^\circ = 9.093 \text{ kN}$$

$$V = 9 \sin 30^\circ - 2.598 \cos 30^\circ = 2.25 \text{ kN}$$

$$M = (9 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) - (2.598 \times 10^3)(40 \times 10^{-3} \cos 30^\circ) = 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 240 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

(a) At point H: $\sigma_x = -\frac{P}{A} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}}$

$$\sigma = -37.9 \text{ MPa} \leftarrow$$

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2.25 \times 10^3}{240 \times 10^{-6}}$$

$$\tau = 14.06 \text{ MPa} \leftarrow$$

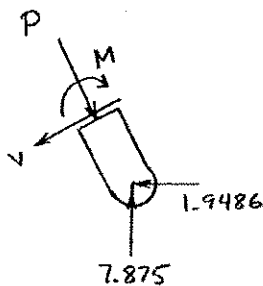
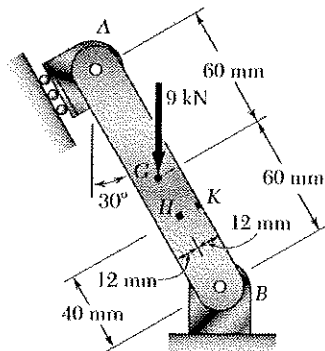
(b) At point K: $\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$

$$\sigma = -131.6 \text{ MPa} \leftarrow$$

$$\tau = 0 \leftarrow$$

Problem 8.36

8.34 through 8.36 Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (a) point H, (b) point K.



$$\sum M_B = 0;$$

$$(9)(60 \sin 30^\circ) - 120 R_A = 0$$

$$R_A = 2.25 \text{ kN}$$

$$\sum F_x = 0: 2.25 \cos 30^\circ - B_x = 0$$

$$B_x = 1.9486 \text{ kN} \leftarrow$$

$$\sum F_y = 0:$$

$$2.25 \sin 30^\circ - 9 + B_y = 0$$

$$B_y = 7.875 \text{ kN} \uparrow$$

At the section containing points H and K,

$$P = 7.875 \cos 30^\circ + 1.9486 \sin 30^\circ = 7.794 \text{ kN}$$

$$V = 7.875 \sin 30^\circ - 1.9486 \cos 30^\circ = 2.25 \text{ kN}$$

$$M = (7.875 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) - (1.9486 \times 10^3)(40 \times 10^{-3} \cos 30^\circ) = 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

$$(a) \text{ At point H: } \sigma_x = -\frac{P}{A} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}}$$

$$\sigma_x = -32.5 \text{ MPa} \leftarrow$$

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2.25 \times 10^3}{240 \times 10^{-6}}$$

$$\tau_{xy} = 14.06 \text{ MPa} \leftarrow$$

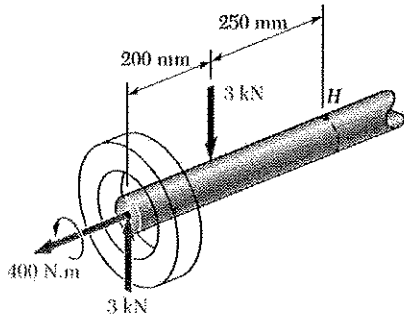
$$(b) \text{ At point K: } \sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

$$\sigma_x = -126.2 \text{ MPa} \leftarrow$$

$$\tau_{xy} = 0 \leftarrow$$

Problem 8.37

8.37 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 36 mm, determine the normal and shearing stresses at point *H* located on the top of the axle.



The bending moment causing normal stress at point *H* is

$$M = (0.2)(3) = 0.6 \text{ kNm.}$$

$$c = \frac{1}{2}d = 18 \text{ mm.}$$

$$I = \frac{\pi}{4}c^4 = 82448 \text{ mm}^4, \quad J = 2I = 164896 \text{ mm}^4$$

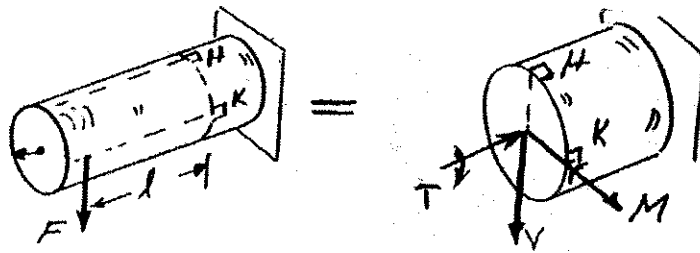
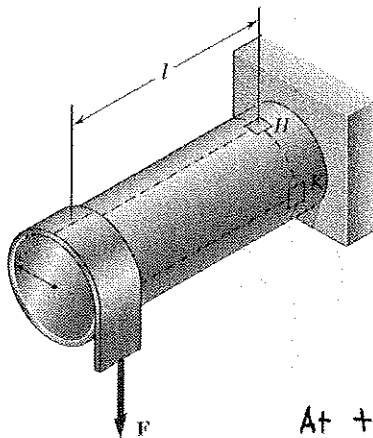
Normal stress at *H*: $\sigma_H = -\frac{Mc}{I} = -\frac{(0.6 \times 10^6)(18)}{82448} = -131 \text{ MPa.}$ $\blacktriangleleft$

At the section containing point *H*, $V = 0$, $T = 400 \text{ Nm.}$

$$\tau_H = \frac{TC}{J} = \frac{(400000)(18)}{164896} = 43.7 \text{ MPa}$$
 $\blacktriangleleft$

Problem 8.38

8.38 A thin strap is wrapped around a solid rod of radius $c = 20$ mm as shown. Knowing that $l = 100$ mm and $F = 5$ kN, determine the normal and shearing stresses at (a) point H, (b) point K.



At the section containing points H and K,

$$T = Fc \quad M = Fl \quad V = F$$

$$J = \frac{\pi}{2} c^4 \quad I = \frac{\pi}{4} c^4$$

Point H: $\sigma = \frac{Mc}{I} = \frac{Flc}{\frac{\pi}{4} c^4} \quad \sigma = \frac{4Fl}{\pi c^3}$

$$\tau = \frac{Tc}{J} = \frac{Fc^2}{\frac{\pi}{2} c^4} \quad \tau = \frac{2F}{\pi c^2}$$

Point K: Point K lies on the neutral axis. $\sigma = 0$

Due to torque: $\tau = \frac{Tc}{J} = \frac{2F}{\pi c^2}$

Due to shear: For a semicircle $Q = \frac{2}{3} c^3$, $t = d = 2c$

$$\tau = \frac{VQ}{It} = \frac{F \frac{2}{3} c^3}{\frac{\pi}{4} c^4 (2c)} = \frac{4F}{3\pi c^2}$$

Combined $\tau = \frac{2F}{\pi c^2} + \frac{4F}{3\pi c^2} \quad \tau = \frac{10F}{3\pi c^2}$

Data: $F = 5 \text{ kN} = 5 \times 10^3 \text{ N}$, $l = 100 \text{ mm} = 0.100 \text{ m}$

$c = 20 \text{ mm} = 0.020 \text{ m}$

(a) Point H: $\sigma = \frac{(4)(5 \times 10^3)(0.100)}{\pi (0.020)^3} \quad \sigma = 79.6 \text{ MPa} \quad \blacktriangleleft$

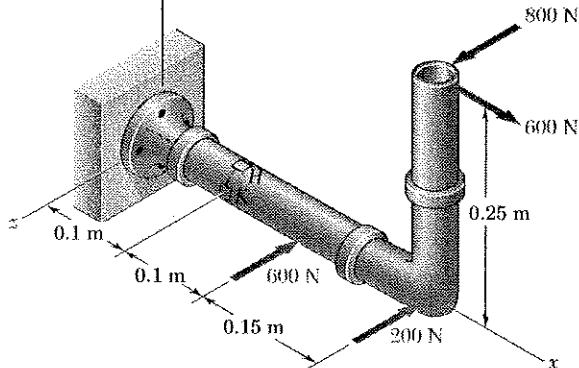
$$\tau = \frac{(2)(5 \times 10^3)}{\pi (0.020)^2} \quad \tau = 7.96 \text{ MPa} \quad \blacktriangleleft$$

(b) Point K: $\sigma = 0 \quad \blacktriangleleft$

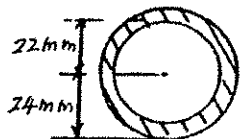
$$\tau = \frac{(40)(5 \times 10^3)}{3\pi (0.020)^2} \quad \tau = 13.26 \text{ MPa} \quad \blacktriangleleft$$

Problem 8.39

8.39 Several forces are applied to the pipe assembly shown. Knowing that the pipe has inner and outer diameters equal to 40 mm and 48 mm, respectively, determine the normal and shearing stresses at (a) point H, (b) point K.

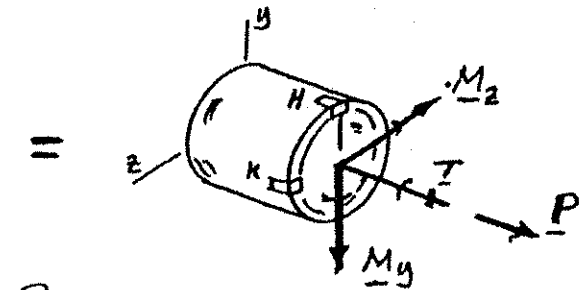


Section properties



$$A = \pi(24^2 - 22^2) = 289 \text{ mm}^2$$

$$I = \frac{\pi}{4}(24^4 - 22^4) = 76592 \text{ mm}^4$$



$$P = 600 \text{ N}$$

$$T = (800 \text{ N})(0.25 \text{ m}) = 200 \text{ N}\cdot\text{m}$$

$$M_z = (600 \text{ N})(0.25 \text{ m}) = 150 \text{ N}\cdot\text{m}$$

$$M_y = (800 \text{ N} - 200 \text{ N})(0.25 \text{ m}) - (600 \text{ N})(0.1 \text{ m}) = 90 \text{ N}\cdot\text{m}$$

$$V_z = 800 - 600 - 200 = 0$$

$$V_y = 0$$

$$J = 2I = 153184 \text{ mm}^4$$

(a) POINT H: $\sigma_H = \frac{P}{A} + \frac{M_z c}{I} = \frac{600}{289 \times 10^{-6}} + \frac{(150)(0.024)}{76592 \times 10^{-12}} = 49.078 \text{ MPa}$

$$\sigma_H = 49.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_H = \frac{Tc}{J} = \frac{(200)(0.024)}{153184 \times 10^{-12}} = 31.33 \text{ MPa}$$

$$\tau_H = 31.3 \text{ MPa} \quad \blacktriangleleft$$

(b) POINT K: $\sigma_K = \frac{P}{A} + \frac{M_y c}{I} = \frac{600}{289 \times 10^{-6}} - \frac{(90)(0.024)}{76592 \times 10^{-12}} = -26.125 \text{ MPa}$

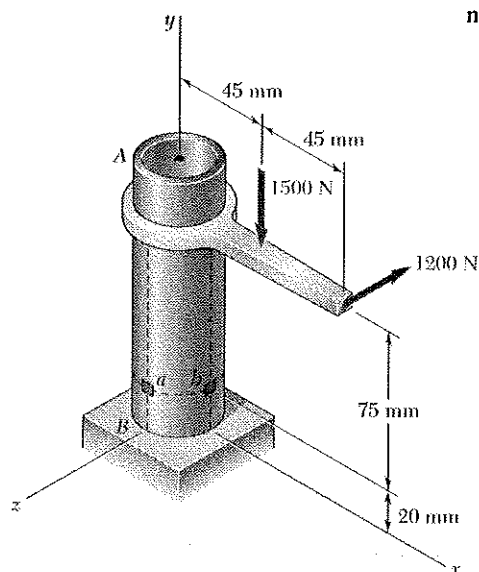
$$\sigma_K = -26.1 \text{ MPa} \quad \blacktriangleright$$

$$\tau_K = \frac{Tc}{J} = \text{SAME AS FOR } \tau_H$$

$$\tau_K = 31.3 \text{ MPa} \quad \blacktriangleright$$

Problem 8.40

8.40 Two forces are applied to the pipe AB as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (a) point a, (b) point b.

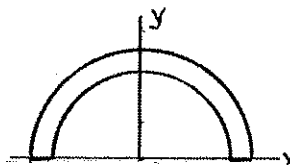


$$c_o = \frac{d_o}{2} = 21 \text{ mm}, \quad c_i = \frac{d_i}{2} = 17.5 \text{ mm}$$

$$A = \pi(c_o^2 - c_i^2) = 423.33 \text{ mm}^2$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 158.166 \times 10^3 \text{ mm}^4$$

$$I = \frac{1}{2}J = 79.083 \times 10^3 \text{ mm}^4$$



For semicircle with semicircular cutout

$$Q = \frac{2}{3}(c_o^3 - c_i^3)$$

$$Q = 2.6011 \times 10^3 \text{ mm}^3$$

At the section containing points a and b

$$P = -1500 \text{ N}$$

$$V_z = -1200 \text{ N}$$

$$V_x = 0$$

$$M_z = -(45 \times 10^{-3})(1500) = -67.5 \text{ N}\cdot\text{m}$$

$$M_x = -(75 \times 10^{-3})(1200) = -90 \text{ N}\cdot\text{m}$$

$$T = (90 \times 10^{-3})(1200) = 108 \text{ N}\cdot\text{m}$$

$$(a) \quad \sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-90)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$

$$\sigma = 20.4 \text{ MPa} \quad \leftarrow$$

$$\tau = \frac{Tc}{J} + \frac{V_z Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0$$

$$\tau = 14.34 \text{ MPa} \quad \leftarrow$$

$$(b) \quad \sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$

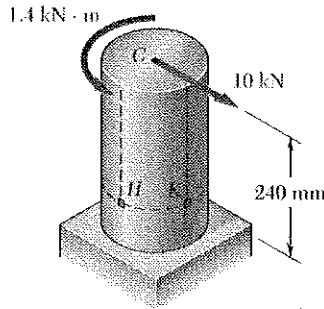
$$\sigma = -21.5 \text{ MPa} \quad \leftarrow$$

$$\tau = \frac{Tc}{J} + \frac{|V_z|Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^{-3})}{(79.083 \times 10^{-9})(7 \times 10^{-3})}$$

$$\tau = 19.98 \text{ MPa} \quad \leftarrow$$

Problem 8.41

8.41 A 10-kN force and a 1.4-kN · m couple are applied at the top of the 65-mm diameter cast-iron post shown. Determine the principal stresses and maximum shearing stress at (a) point H, (b) point K.



At the section containing points H and K,

$$V = 10 \text{ kN} = 10 \times 10^3 \text{ N}$$

$$M = (10 \times 10^3)(240 \times 10^{-3}) = 2.4 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 1.4 \times 10^3 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 32.5 \text{ mm} = 0.0325 \text{ m}$$

$$J = \frac{\pi}{2} c^4 = 1.75248 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2}J = 0.87624 \times 10^{-6} \text{ m}^4$$

For a semicircle, $Q = \frac{2}{3}c^3 = \frac{2}{3}(0.0325)^3 = 22.885 \times 10^{-6} \text{ m}^3$

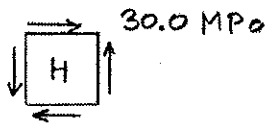
(a) Stresses at point H.

H lies on the neutral axis: $\sigma = 0$

Due to torque: $\tau = \frac{Tc}{J} = \frac{(1.4 \times 10^3)(0.0325)}{1.75248 \times 10^{-6}} = 25.963 \text{ MPa}$

Due to shear: $\tau = \frac{VQ}{It} = \frac{(10 \times 10^3)(22.885 \times 10^{-6})}{(0.87624 \times 10^{-6})(0.065)} = 4.018 \text{ MPa}$

Total at H: $\tau = 30.0 \text{ MPa}$



$$\sigma_c = 0, R = 30.0 \text{ MPa}$$

$$\sigma_{\max} = \sigma_c + R$$

$$\sigma_{\min} = \sigma_c - R$$

$$\tau_{\max} = R$$

$$\sigma_{\max} = 30.0 \text{ MPa} \leftarrow$$

$$\sigma_{\min} = -30.0 \text{ MPa} \leftarrow$$

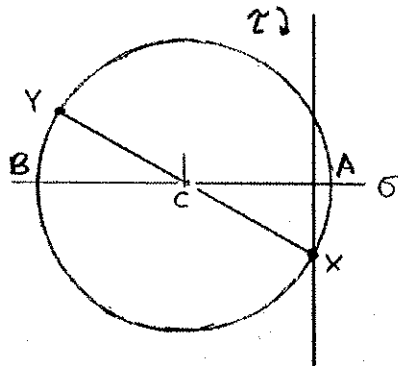
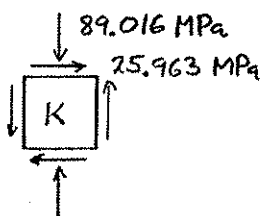
$$\tau_{\max} = 30.0 \text{ MPa} \leftarrow$$

(b) Stresses at point K

Due to shear: $\tau = 0$

Due to torque: $\tau = \frac{Tc}{J} = 25.963 \text{ MPa}$

Due to bending: $\sigma = -\frac{Mc}{I} = -\frac{(2.4 \times 10^3)(0.0325)}{(0.87624 \times 10^{-6})} = -89.016 \text{ MPa}$



$$\sigma_c = \frac{-89.016}{2} = -44.508 \text{ MPa}$$

$$R = \sqrt{\left(\frac{89.016}{2}\right)^2 + (25.963)^2} = 51.527 \text{ MPa}$$

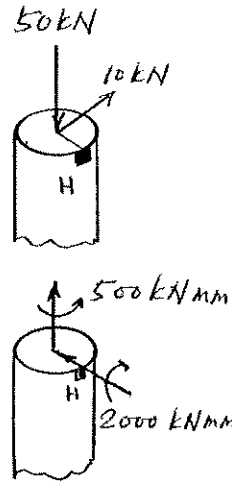
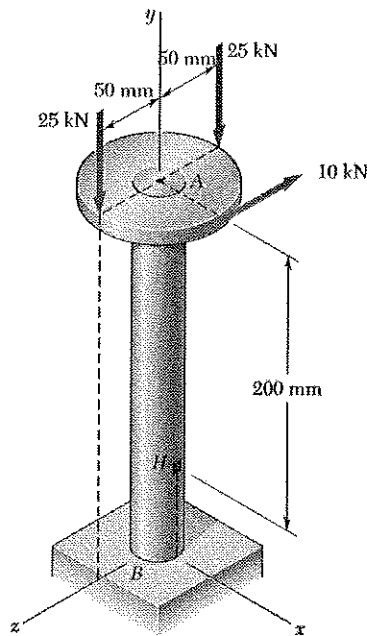
$$\sigma_{\max} = \sigma_c + R \quad \sigma_{\max} = 7.02 \text{ MPa} \leftarrow$$

$$\sigma_{\min} = \sigma_c - R \quad \sigma_{\min} = -96.0 \text{ MPa} \leftarrow$$

$$\tau_{\max} = R \quad \tau_{\max} = 51.5 \text{ MPa} \leftarrow$$

Problem 8.42

8.42 Three forces are applied to a 100-mm-diameter plate that is attached to the solid 45-mm diameter shaft AB. At point H, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



At the section containing point H,

$$P = 50 \text{ kN (compression)}$$

$$V = 10 \text{ kN}$$

$$T = (10)(50) = 500 \text{ kNmm}$$

$$M = (10)(200) = 2000 \text{ kNmm}$$

$$d = 45 \text{ mm} \quad c = \frac{1}{2}d = 22.5 \text{ mm}$$

$$A = \pi c^2 = 1590 \text{ mm}^2$$

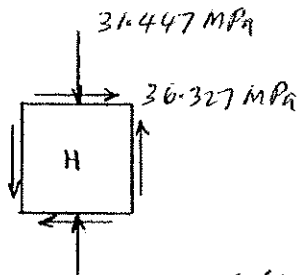
$$I = \frac{\pi}{4} c^4 = 0.2013 \times 10^6 \text{ mm}^4$$

$$J = 2I = 0.4026 \times 10^6 \text{ mm}^4$$

For a semicircle, $Q = \frac{2}{3} c^3 = 7.594 \times 10^3 \text{ mm}^3$

Point H lies on neutral axis of bending. $\sigma_H = \frac{P}{A} = -\frac{50000}{1590} = -31.447 \text{ MPa}$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5 \times 10^3)(22.5)}{0.4026 \times 10^6} + \frac{(10 \times 10^3)(7.594 \times 10^3)}{(0.2013 \times 10^6)(45)} = 36.327 \text{ MPa}$$



Use Mohr's circle.

$$\sigma_c = \frac{1}{2}(-31.447) = -15.7235 \text{ MPa}$$

$$R = \sqrt{\left(\frac{15.7235}{2}\right)^2 + 36.327^2} = 37.168 \text{ MPa}$$

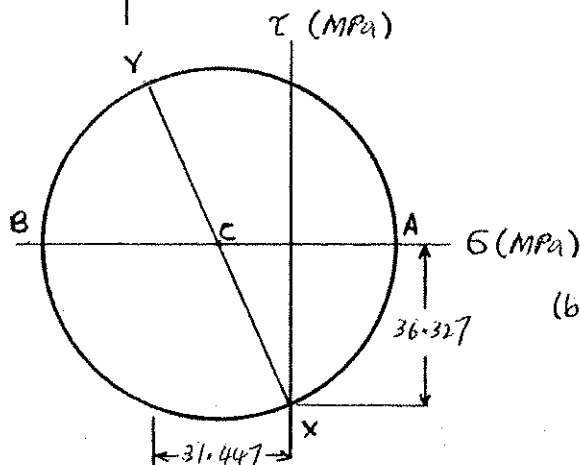
$$(a) \sigma_a = \sigma_c + R = 21.4 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -52.9 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2(36.327)}{31.447} = 2.3164$$

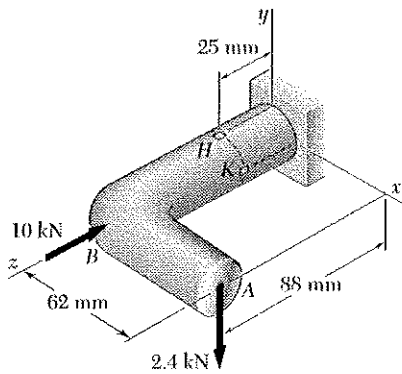
$$\theta_a = 33.3^\circ \quad \theta_b = 123.3^\circ$$

$$(b) \tau_{max} = R = 37.2 \text{ MPa}$$



Problem 8.43

8.43 Forces are applied at points A and B of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 20 mm, determine the principal stresses and the maximum shearing stress (a) point H, (b) point K.



At the section containing points H and K

$$P = 10 \text{ kN} \quad (\text{compression})$$

$$V_y = -2.4 \text{ kN}$$

$$V_x = 0$$

$$M_x = (0.088 - 0.025)(2400) = 151.2 \text{ Nm}$$

$$M_y = 0 \quad M_z = -(0.062)(2400) = -148.8 \text{ Nm}$$

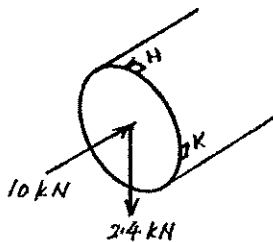
$$c = \frac{1}{2}d = 10 \text{ mm}$$

$$A = \pi c^2 = 314.16 \text{ mm}^2$$

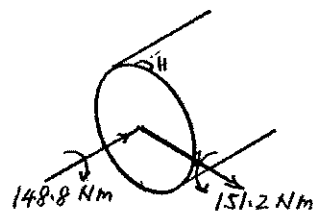
$$I = \frac{\pi}{4} c^4 = 7854 \text{ mm}^4$$

$$J = 2I = 15708 \text{ mm}^4$$

$$\text{For semi circle } Q = \frac{2}{3}c^3 = 666.7 \text{ mm}^3$$



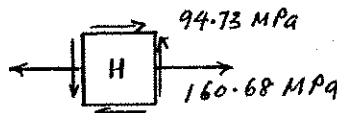
Forces



Couples

(a) At point H: $\sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{10000}{314.16 \times 10^{-6}} + \frac{(151.2)(0.01)}{7854 \times 10^{-12}} = 160.68 \text{ MPa}$

$$\tau_H = \frac{Tc}{J} = \frac{(148.8)(0.01)}{15708 \times 10^{-12}} = 94.73 \text{ MPa}$$



$$\sigma_{ave} = \frac{160.68}{2} = 80.34 \text{ MPa}$$

$$R = \sqrt{\left(\frac{160.68}{2}\right)^2 + (94.73)^2} = 124.2 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R$$

$$\sigma_{max} = 204.5 \text{ MPa} \quad \leftarrow$$

$$\sigma_{min} = \sigma_{ave} - R$$

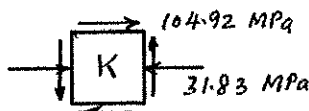
$$\sigma_{min} = -43.9 \text{ MPa} \quad \leftarrow$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

$$\tau_{max} = 124.2 \text{ MPa} \quad \leftarrow$$

(b) At point K: $\sigma_K = \frac{P}{A} = -\frac{10000}{314.16 \times 10^{-6}} = -31.83 \text{ MPa}$

$$\tau_K = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(148.8)(0.01)}{15708 \times 10^{-12}} + \frac{(2400)(666.7 \times 10^{-9})}{(7854 \times 10^{-12})(0.02)} = 104.92 \text{ MPa}$$



$$\sigma_{ave} = -\frac{31.83}{2} = -15.915 \text{ MPa}$$

$$R = \sqrt{\left(-\frac{31.83}{2}\right)^2 + (104.92)^2} = 106.12 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R$$

$$\sigma_{max} = 90.2 \text{ MPa} \quad \leftarrow$$

$$\sigma_{min} = \sigma_{ave} - R$$

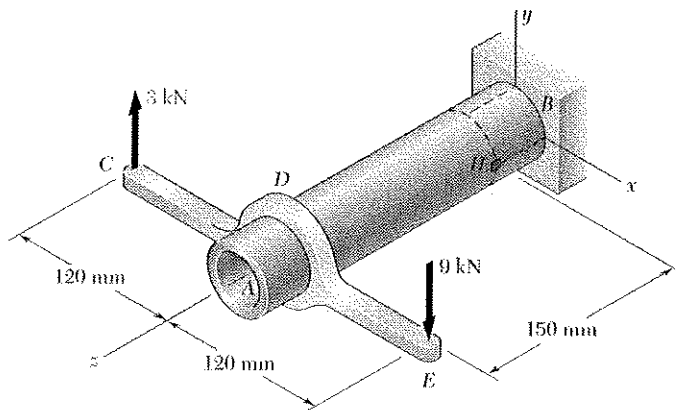
$$\sigma_{min} = -122 \text{ MPa} \quad \leftarrow$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

$$\tau_{max} = 106.1 \text{ MPa} \quad \leftarrow$$

Problem 8.44

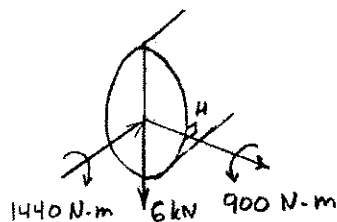
8.44 The steel pipe AB has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that the arm CDE is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shearing stress at point H .



Replace the forces at C and E by an equivalent force-couple system at D .

$$F_D = 9 - 3 = 6 \text{ kN} \downarrow$$

$$T_D = (9 \times 10^3)(120 \times 10^{-3}) + (3 \times 10^3)(120 \times 10^{-3}) = 1440 \text{ N}\cdot\text{m}$$



At the section containing point H ,

$$P = 0, \quad V = 6 \text{ kN}, \quad T = 1440 \text{ N}\cdot\text{m}$$

$$M = (6 \times 10^3)(150 \times 10^{-3}) = 900 \text{ N}\cdot\text{m}$$

Section properties: $d_o = 72 \text{ mm}$, $c_o = \frac{1}{2}d_o = 36 \text{ mm}$, $c_2 = c_o - t = 31 \text{ mm}$

$$A = \pi(c_o^2 - c_i^2) = 1.0524 \times 10^3 \text{ mm}^2 = 1.0524 \times 10^{-3} \text{ m}^2$$

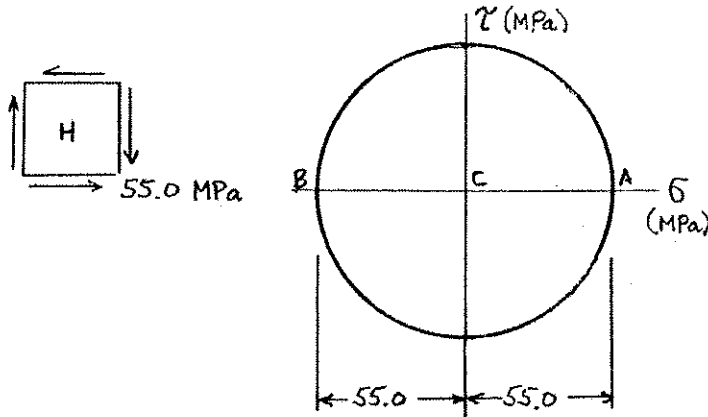
$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84 \times 10^{-3} \text{ mm}^4 = 593.84 \times 10^{-9} \text{ m}^4$$

$$J = 2I = 1.1877 \times 10^{-6} \text{ m}^4$$

$$\text{For half-pipe, } Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243 \times 10^3 \text{ mm}^3 = 11.243 \times 10^{-6} \text{ m}^3$$

At point H . Point H lies on the neutral axis of bending. $\sigma_H = 0$.

$$\tau_H = \frac{TC}{J} + \frac{VQ}{It} = \frac{(1440)(36 \times 10^{-3})}{1.1877 \times 10^{-6}} + \frac{(6 \times 10^3)(11.243 \times 10^{-6})}{(593.84 \times 10^{-9})(10 \times 10^{-3})} = 55.0 \text{ MPa}$$



Use Mohr's circle.

$$\sigma_c = 0$$

$$R = 55.0 \text{ MPa}$$

$$\sigma_a = \sigma_c + R \quad \sigma_a = 55.0 \text{ MPa} \quad \leftarrow$$

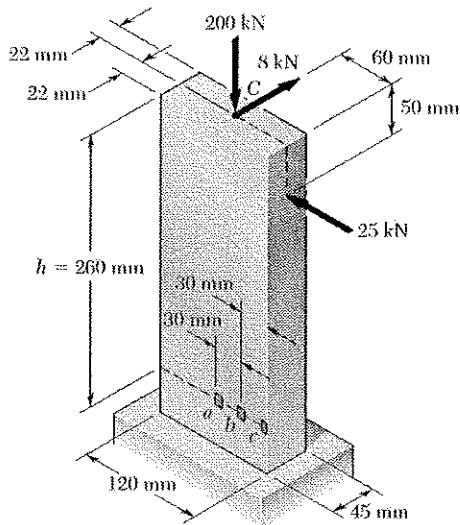
$$\sigma_b = \sigma_c - R \quad \sigma_b = -55.0 \text{ MPa} \quad \leftarrow$$

$$\theta_a = -45^\circ, \quad \theta_b = +45^\circ \quad \leftarrow$$

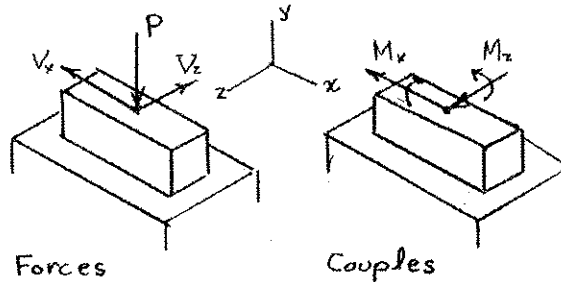
$$\tau_{\min} = R \quad \tau_{\max} = 55.0 \text{ MPa} \quad \leftarrow$$

Problem 8.45

8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.



Calculate forces and couples at section containing points a, b, and c. $h = 260 \text{ mm}$



$$P = 200 \text{ kN} \quad V_x = 25 \text{ kN} \quad V_z = 8 \text{ kN}$$

$$M_z = (260 - 50)(25) = 5250 \text{ kN}\cdot\text{mm}$$

$$M_x = (260)(8) = 2080 \text{ kN}\cdot\text{mm}$$

Section properties. $A = (120)(44) = 5280 \text{ mm}^2$

$$I_x = \frac{1}{12}(120)(44)^3 = 851840$$

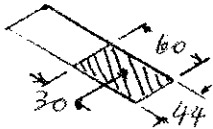
$$I_z = \frac{1}{12}(44)(120)^3 = 6.336 \times 10^6 \text{ mm}^4$$

Stresses

$$\sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x}$$

$$\tau = \frac{V_x Q}{I_x t}$$

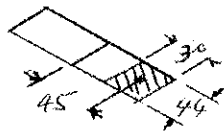
(a) Point a: $x = 0, z = 22 \text{ mm}, Q = (44)(60)(\frac{60}{2}) = 79200 \text{ mm}^3$



$$\sigma = -\frac{200 \times 10^3}{5280} + 0 + \frac{(2.08 \times 10^6)(22)}{0.85184 \times 10^6} = 15.8 \text{ MPa}$$

$$\tau = \frac{(25 \times 10^3)(79200)}{(6.336 \times 10^6)(44)} = 7.1 \text{ MPa}$$

(b) Point b: $x = 30 \text{ mm}, z = 22 \text{ mm}, Q = (44)(30)(45) = 59400 \text{ mm}^3$



$$\sigma = -\frac{200 \times 10^3}{5280} + \frac{(5.25 \times 10^6)(30)}{6.336 \times 10^6} + \frac{(2.08 \times 10^6)(22)}{0.85184 \times 10^6} = 40.7 \text{ MPa}$$

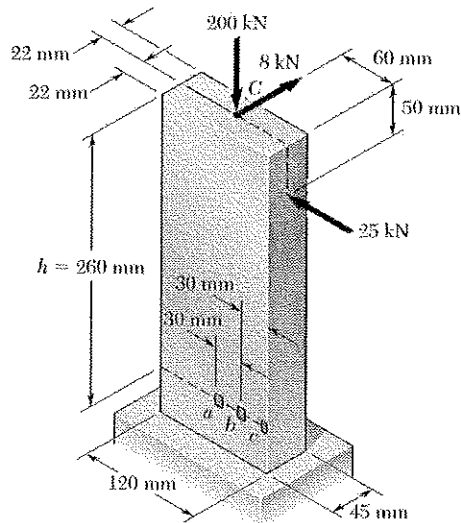
$$\tau = \frac{(25 \times 10^3)(59400)}{[(6.336 \times 10^6)(44)]} = 5.3 \text{ MPa}$$

(c) Point c: $x = 60 \text{ mm}, z = 22 \text{ mm}, Q = 0$

$$\sigma = -\frac{200 \times 10^3}{5280} + \frac{(5.25 \times 10^6)(60)}{6.336 \times 10^6} + \frac{(2.08 \times 10^6)(22)}{0.85184 \times 10^6} = 65.6 \text{ MPa}$$

$$\tau = 0$$

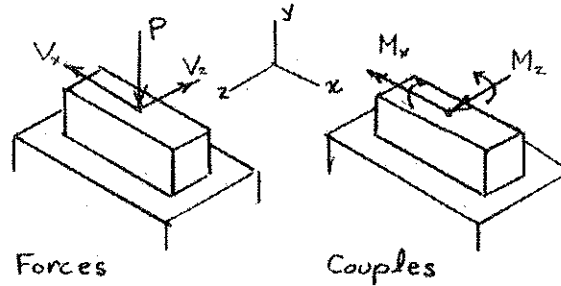
Problem 8.46



8.46 Solve Prob. 8.45, assuming that $h = 300$ mm.

8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.

Calculate forces and couples at section containing points a, b, and c. $h = 300$ mm.



$$P = 200 \text{ kN} \quad V_x = 25 \text{ kN} \quad V_z = 8 \text{ kN}$$

$$M_z = (300 - 50)(25) = 6250 \text{ kN}\cdot\text{mm}$$

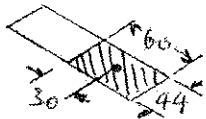
$$M_x = (300)(8) = 2400 \text{ kN}\cdot\text{mm}$$

Section properties. $A = (120)(44) = 5280 \text{ mm}^2$

$$I_x = \frac{1}{12}(120)(44)^3 = 851840 \text{ mm}^4 \quad I_z = \frac{1}{12}(44)(120)^3 = 6.336 \times 10^6 \text{ mm}^4$$

$$\text{Stresses} \quad \sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \quad \tau = \frac{V_x Q}{I_z t}$$

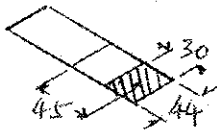
(a) Point a: $x = 0$, $z = 22 \text{ mm}$, $Q = (44)(60)(30) = 79200 \text{ mm}^3$



$$\sigma = -\frac{200 \times 10^3}{5280} + 0 + \frac{(2.4 \times 10^6)(22)}{0.85184 \times 10^6} = 24.1 \text{ MPa}$$

$$\tau = \frac{(25 \times 10^3)(79200)}{(6.336 \times 10^6)(44)} = 7.1 \text{ MPa}$$

(b) Point b: $x = 30 \text{ mm}$, $z = 22 \text{ mm}$, $Q = (30)(44)(45) = 59400 \text{ mm}^3$



$$\sigma = -\frac{200 \times 10^3}{5280} + \frac{(6.25 \times 10^6)(30)}{6.336 \times 10^6} + \frac{(2.4 \times 10^6)(22)}{0.85184 \times 10^6} = 53.7 \text{ MPa}$$

$$\tau = \frac{(25 \times 10^3)(59400)}{(6.336 \times 10^6)(44)} = 5.3 \text{ MPa}$$

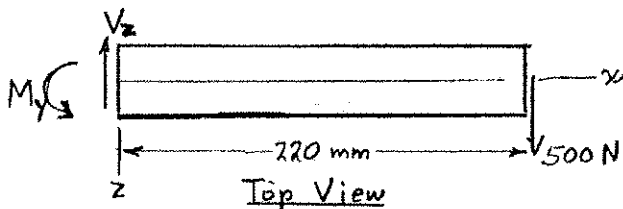
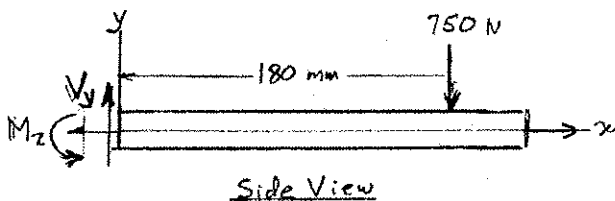
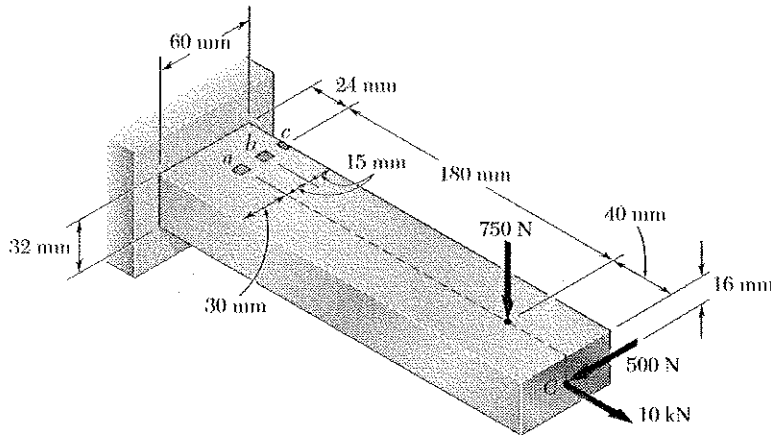
(c) Point c: $x = 60 \text{ mm}$, $z = 22 \text{ mm}$, $Q = 0$

$$\sigma = -\frac{200 \times 10^3}{5280} + \frac{(6.25 \times 10^6)(60)}{(6.336 \times 10^6)} + \frac{(2.4 \times 10^6)(22)}{0.85184 \times 10^6} = 83.3 \text{ MPa}$$

$$\tau = 0$$

Problem 8.47

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.



$$A = (60)(32) = 1920 \text{ mm}^2 = 1920 \times 10^{-6} \text{ m}^2$$

$$I_z = \frac{1}{12} (60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4 = 163.84 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12} (32)(60)^3 = 576 \times 10^3 \text{ mm}^4 = 576 \times 10^{-9} \text{ m}^4$$

At the section containing points a, b, and c

$$P = 10 \text{ kN}$$

$$V_y = 750 \text{ N}, \quad V_z = 500 \text{ N}$$

$$M_z = (180 \times 10^{-3})(750) = 135 \text{ N}\cdot\text{m}$$

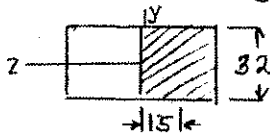
$$M_y = (220 \times 10^{-3})(500) = 110 \text{ N}\cdot\text{m}$$

$$T = 0$$

$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$

$$\tau = \frac{V_y Q}{I_y t}$$

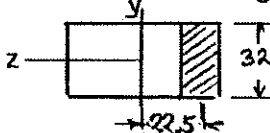
(a) Point a. $y = 16 \text{ mm}, z = 0, \quad Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3 \text{ mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - 0 \quad \sigma = 18.39 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = \frac{(500)(14.4 \times 10^{-6})}{(576 \times 10^{-9})(32 \times 10^{-3})} \quad \tau = 0.391 \text{ MPa} \quad \blacktriangleleft$$

(b) Point b. $y = 16 \text{ mm}, z = -15 \text{ mm}, \quad Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3 \text{ mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}} \quad \sigma = 21.3 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = \frac{(500)(10.8 \times 10^{-6})}{(576 \times 10^{-9})(32 \times 10^{-3})} \quad \tau = 0.293 \text{ MPa} \quad \blacktriangleleft$$

(c) Point c. $y = 16 \text{ mm}, z = -30 \text{ mm}, \quad Q = 0$

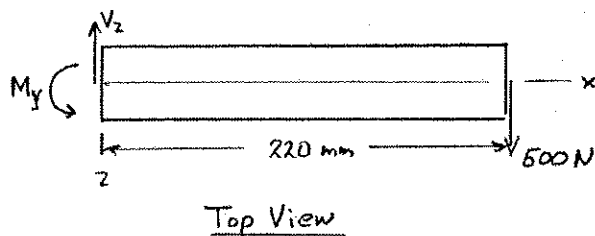
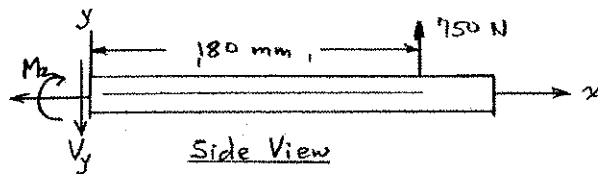
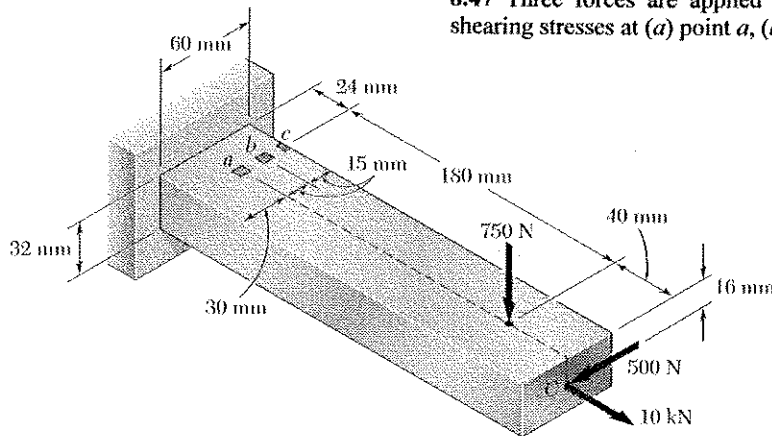
$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} + \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-3})}{576 \times 10^{-9}} \quad \sigma = 24.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = 0 \quad \blacktriangleleft$$

Problem 8.48

8.48 Solve Prob. 8.47, assuming that the 750-N force is directed vertically upward.

8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.



$$A = (60)(32) = 1920 \text{ mm}^2 = 1920 \times 10^{-6} \text{ m}^2$$

$$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4 = 163.84 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12}(32)(60)^3 = 576 \times 10^3 \text{ mm}^4 = 576 \times 10^{-9} \text{ m}^4$$

At the section containing points a, b, and c

$$P = 10 \text{ kN}, \quad T = 0$$

$$V_y = 750 \text{ N}, \quad V_z = 500 \text{ N}$$

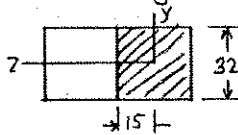
$$M_z = (180 \times 10^{-3})(750) = 135 \text{ N}\cdot\text{m}$$

$$M_y = (220 \times 10^{-3})(500) = 110 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{P}{A} - \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$

$$\tau = \frac{V_y Q}{I_z t}$$

(a) Point a. $y = 16 \text{ mm}, z = 0, Q = A\bar{z} = (32)(30)(15) = 14.4 \times 10^3 \text{ mm}^3$



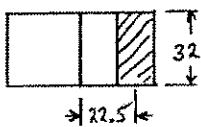
$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - 0$$

$$\sigma = -7.98 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = \frac{(500)(14.4 \times 10^{-6})}{(163.84 \times 10^{-9})(32 \times 10^{-3})}$$

$$\tau = 0.391 \text{ MPa} \quad \blacktriangleleft$$

(b) Point b. $y = 16 \text{ mm}, z = -15 \text{ mm}, Q = A\bar{z} = (32)(15)(22.5) = 10.8 \times 10^3 \text{ mm}^3$



$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-15 \times 10^{-3})}{576 \times 10^{-9}}$$

$$\sigma = -5.11 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = \frac{(500)(10.8 \times 10^{-6})}{(163.84 \times 10^{-9})(32 \times 10^{-3})}$$

$$\tau = 0.293 \text{ MPa} \quad \blacktriangleleft$$

(c) Point c. $y = 16 \text{ mm}, z = -30 \text{ mm}, Q = 0$

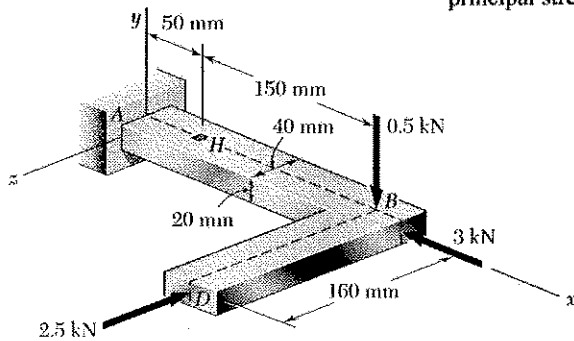
$$\sigma = \frac{10 \times 10^3}{1920 \times 10^{-6}} - \frac{(135)(16 \times 10^{-3})}{163.84 \times 10^{-9}} - \frac{(110)(-30 \times 10^{-3})}{576 \times 10^{-9}}$$

$$\sigma = -2.25 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = 0 \quad \blacktriangleleft$$

Problem 8.49

8.49 Three forces are applied to the machine component ABD as shown. Knowing that the cross section containing point H is a 20×40 -mm rectangle, determine the principal stresses and the maximum shearing stress at point H .

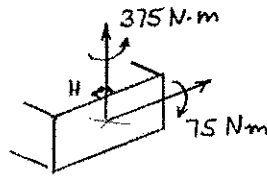
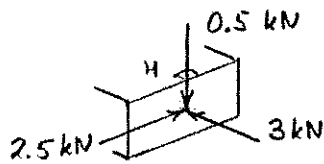


Equivalent force-couple system at section containing point H :

$$F_x = -3 \text{ kN}, F_y = -0.5 \text{ kN}, F_z = -2.5 \text{ kN}$$

$$M_x = 0, M_y = (0.150)(2500) = 375 \text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75 \text{ N}\cdot\text{m}$$



$$A = (20)(40) = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$$

$$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3 \text{ mm}^4 = 26.667 \times 10^{-9} \text{ m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375 \text{ MPa}$$

$$\tau_H = \frac{3}{2} \frac{|V_z|}{A} = \frac{3}{2} \frac{2500}{800 \times 10^{-6}} = 4.6875 \text{ MPa}$$

Use Mohr's circle.

$$\sigma_c = \frac{1}{2} \sigma_H = 12.1875 \text{ MPa}$$

$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2} = 13.0579 \text{ MPa}$$

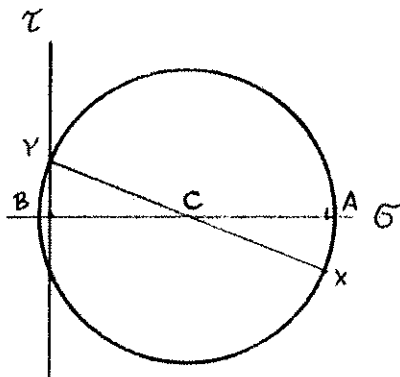
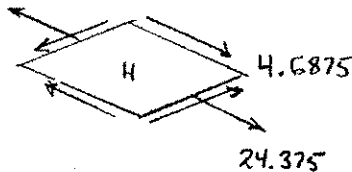
$$\sigma_a = \sigma_c + R = 25.2 \text{ MPa} \quad \leftarrow$$

$$\sigma_b = \sigma_c - R = -0.87 \text{ MPa} \quad \leftarrow$$

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$$

$$\theta_a = 10.5^\circ, \theta_b = 100.5^\circ$$

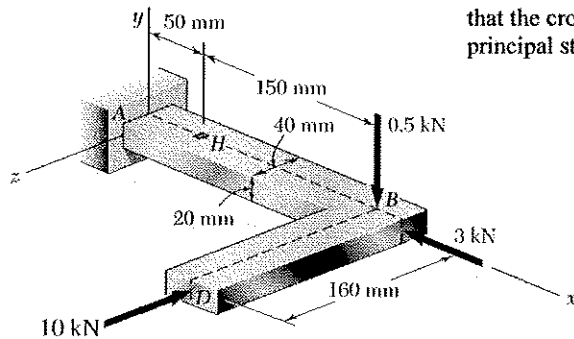
$$\tau_{\max} = R = 13.06 \text{ MPa} \quad \leftarrow$$



Problem 8.50

8.50 Solve Prob. 8.49, assuming that the magnitude of the 2.5-kN force is increased to 10 kN.

8.49 Three forces are applied to the machine component ABD as shown. Knowing that the cross section containing point H is a 20×40 -mm rectangle, determine the principal stresses and the maximum shearing stress at point H.



Equivalent force-couple system at section containing point H.

$$F_x = -3 \text{ kN}, \quad F_y = -0.5 \text{ kN}, \quad F_z = -10 \text{ kN}$$

$$M_x = 0, \quad M_y = (0.150)(10000) = 1500 \text{ N}\cdot\text{m}$$

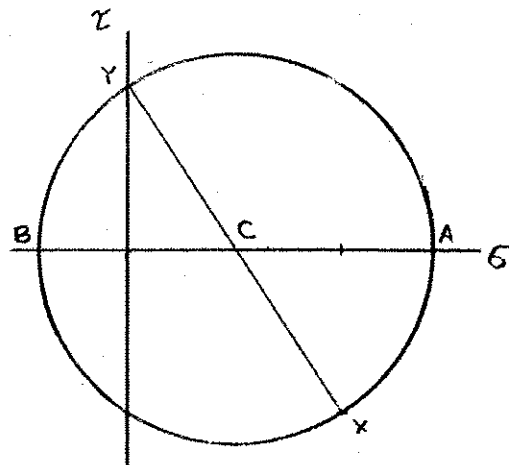
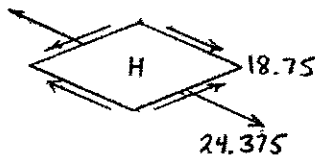
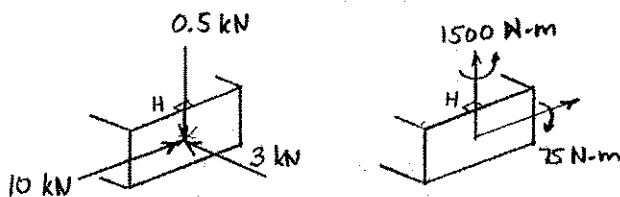
$$M_z = -(0.150)(500) = -75 \text{ N}\cdot\text{m}$$

$$A = (20)(40) = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$$

$$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3 \text{ mm}^4 = 26.667 \times 10^{-9} \text{ m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375 \text{ MPa}$$

$$\tau_H = \frac{3}{2} \frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{10000}{800 \times 10^{-6}} = 18.75 \text{ MPa}$$



$$\sigma_c = \frac{1}{2} \sigma_H = 12.1875 \text{ MPa}$$

$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (18.75)^2} = 22.363 \text{ MPa}$$

$$\sigma_a = \sigma_c + R = 34.6 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -10.18 \text{ MPa}$$

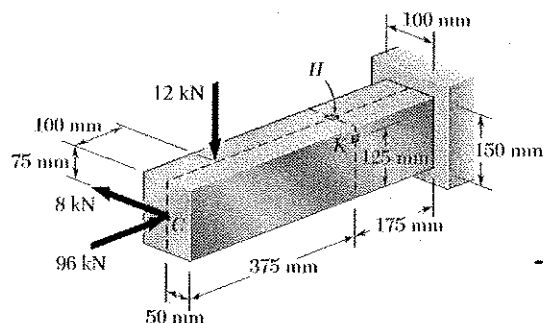
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(18.75)}{24.375} = 1.5385$$

$$\theta_a = 28.5^\circ, \quad \theta_b = 118.5^\circ$$

$$\tau_{max} = R = 22.4 \text{ MPa}$$

Problem 8.51

8.51 Three forces are applied to the cantilever beam shown. Determine the principal stresses and the maximum shearing stress at point H.



At the section containing points H and K, the axial and shearing forces are:

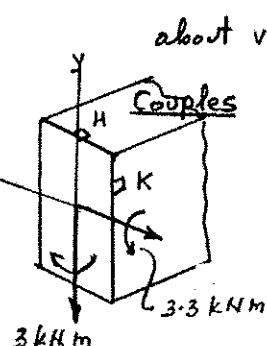
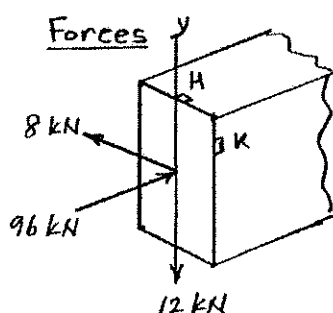
$$P = 96 \text{ kN}, \quad V = \begin{matrix} 12 \text{ kN vertical} \\ 8 \text{ kN horizontal} \end{matrix}$$

The bending moment components are:

$$\text{about horizontal axis} = M = (0.375 - 0.1)(12000) = 3.3 \text{ kNm}$$

$$\text{about vertical axis} = M = (0.375)(8000) = 3 \text{ kNm}$$

Forces



Section properties

$$A = (0.1)(0.15) = 0.015 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.15)^3 = 28.125 \times 10^{-6} \text{ m}^4$$

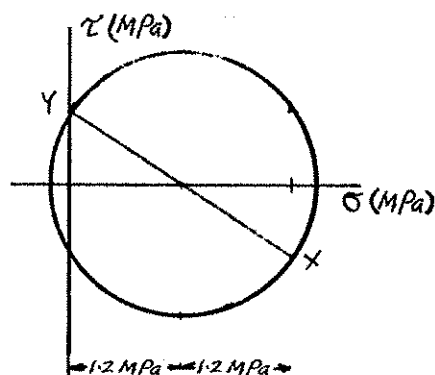
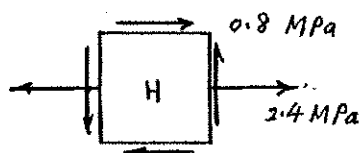
$$I_y = \frac{1}{12}(0.15)(0.1)^3 = 12.5 \times 10^{-6} \text{ m}^4$$

$$\text{At point H: } \sigma = -\frac{P}{A} + \frac{Mc}{I} = -\frac{96000}{0.015} + \frac{(3300)(0.075)}{28.125 \times 10^{-6}}$$

$$\sigma = 2.4 \text{ MPa}$$

$$\tau = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{12000}{0.015}$$

$$\tau = 0.8 \text{ MPa}$$



$$\sigma_c = \frac{2.4}{2} = 1.2 \text{ MPa}$$

$$R = \sqrt{\left(\frac{2.4}{2}\right)^2 + (0.8)^2} = 1.44 \text{ MPa}$$

$$\sigma_a = \sigma_c + R$$

$$\sigma_a = 2.64 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_c - R$$

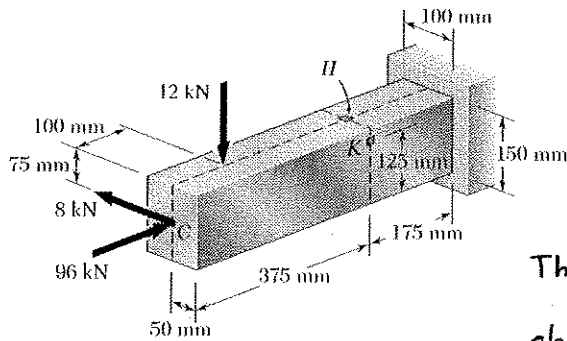
$$\sigma_b = -0.24 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{\max} = R$$

$$\tau_{\max} = 1.44 \text{ MPa} \quad \blacktriangleleft$$

Problem 8.52

8.52 For the beam and loading of Prob. 8.51, determine the principal stresses and maximum shearing stress at point K.



At the section containing points H and K the axial and shearing forces are

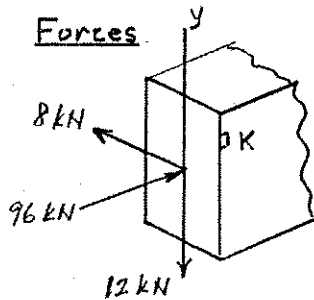
$$P = 96 \text{ kN}, \quad V = \begin{matrix} 12 \text{ kN vertical} \\ 8 \text{ kN horizontal} \end{matrix}$$

The bending moment components are

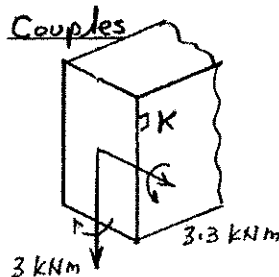
$$\text{about horizontal axis: } M = (0.375 - 0.1)(12000) = 3300 \text{ Nm}$$

$$\text{about vertical axis: } M = (0.375)(8000) = 3000 \text{ Nm}$$

Forces



Couples



Section properties

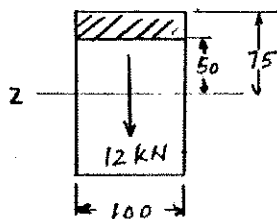
$$A = (0.1)(0.15) = 0.015 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.15)^3 = 28.125 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(0.15)(0.1)^3 = 12.5 \times 10^{-6} \text{ m}^4$$

At point K:

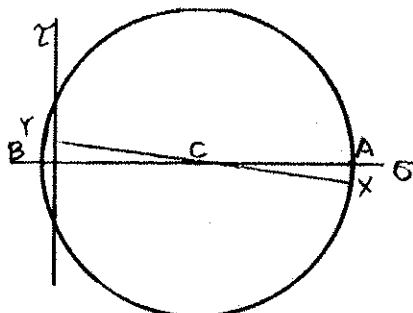
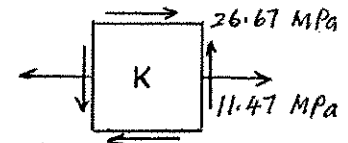
$$\sigma = -\frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{96000}{0.015} - \frac{(-3300)(0.05)}{28.125 \times 10^{-6}} + \frac{(-3000)(-0.05)}{12.5 \times 10^{-6}} = 11.47 \text{ MPa}$$



$$A^* = (0.025)(0.1) = 0.0025 \text{ m}^2 \quad \bar{y} = 62.5 \text{ mm}$$

$$Q = A^* \bar{y} = (0.1)(0.0625) = 0.00625 \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(12000)(0.00625)}{(28.125 \times 10^{-6})(0.1)} = 26.67 \text{ MPa}$$



$$\sigma_c = \frac{11.47}{2} = 5.735 \text{ MPa}$$

$$R = \sqrt{\left(\frac{11.47}{2}\right)^2 + (26.67)^2} = 27.28 \text{ MPa}$$

$$\sigma_a = \sigma_c + R$$

$$\sigma_a = 33.02 \text{ MPa}$$

$$\sigma_b = \sigma_c - R$$

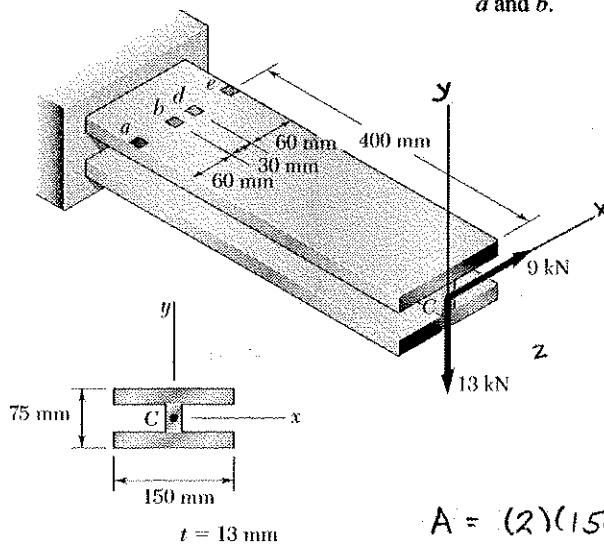
$$\sigma_b = -21.55 \text{ MPa}$$

$$\tau_{max} = R$$

$$\tau_{max} = 27.28 \text{ MPa}$$

Problem 8.53

8.53 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.



Equivalent force-couple system at section containing points *a* and *b*.

$$F_x = 9 \text{ kN}, F_y = -13 \text{ kN}, F_z = 0$$

$$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N}\cdot\text{m}$$

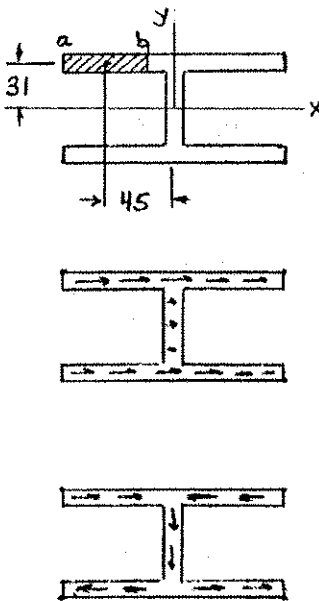
$$M_y = 0.400(9 \times 10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75 - 26) = 4537 \text{ mm}^2 = 4537 \times 10^{-6} \text{ m}^2$$

$$I_x = 2 \left[\frac{1}{12} (150)(13)^3 + (150)(13)(37.5 - 6.5)^2 \right] + \frac{1}{12} (13)(75 - 26)^3 = 3.9303 \times 10^6 \text{ mm}^4 = 3.9303 \times 10^{-6} \text{ m}^4$$

$$I_y = 2 \cdot \frac{1}{12} (13)(150)^3 + \frac{1}{12} (75 - 26)(13)^3 = 7.3215 \times 10^6 \text{ mm}^4 = 7.3215 \times 10^{-6} \text{ m}^4$$



For point *a*, $Q_x = 0$ $Q_y = 0$

For point *b*, $A^* = (60)(13) = 780 \text{ mm}^2$
 $\bar{x} = -45 \text{ mm}$ $\bar{y} = 31 \text{ mm}$

$$Q_x = A^* \bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$$

$$Q_y = A^* \bar{x} = -35.1 \times 10^3 \text{ mm}^3 = -35.1 \times 10^{-6} \text{ m}^3$$

At point *a*: $\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$
 $= \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(-75 \times 10^{-3})}{7.3215 \times 10^{-6}}$ $\sigma = 86.5 \text{ MPa}$

$$\tau = 0$$

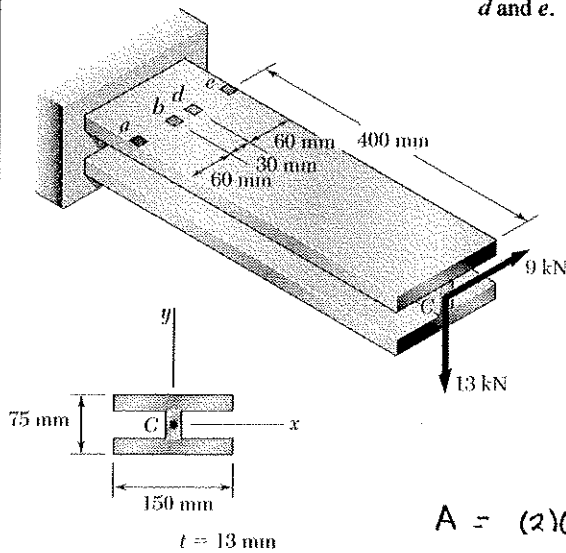
At point *b*: $\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$
 $= \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(-15 \times 10^{-3})}{7.3215 \times 10^{-6}}$ $\sigma = 57.0 \text{ MPa}$

$$\tau = \frac{|V_x| Q_y}{I_y t} + \frac{|V_y| Q_x}{I_x t} = \frac{(9 \times 10^3)(35.1 \times 10^{-6})}{(7.3215 \times 10^{-6})(13 \times 10^{-3})} + \frac{(13 \times 10^3)(24.18 \times 10^{-6})}{(3.9303 \times 10^{-6})(13 \times 10^{-3})}$$

$$= 3.32 \text{ MPa} + 6.15 \text{ MPa} \quad \tau = 9.47 \text{ MPa}$$

Problem 8.54

8.54 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.



Equivalent force-couple system at section containing points *a* and *b*.

$$F_x = 9 \text{ kN}, F_y = -13 \text{ kN}, F_z = 0$$

$$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N}\cdot\text{m}$$

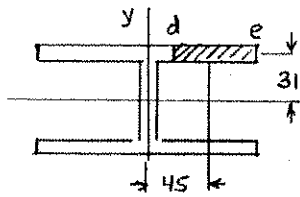
$$M_y = (0.400)(9 \times 10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75 - 26) = 4537 \text{ mm}^2 = 4537 \times 10^{-6} \text{ m}^2$$

$$I_x = 2 \left[\frac{1}{12} (150)(13)^3 + (150)(13)(37.5 - 6.5)^2 \right] + \frac{1}{12} (13)(75 - 26)^3 = 3.9303 \times 10^6 \text{ mm}^4 = 3.9303 \times 10^{-6} \text{ m}^4$$

$$I_y = 2 \left[\frac{1}{12} (13)(150)^3 \right] + \frac{1}{12} (75 - 26)(13)^3 = 7.3215 \times 10^6 \text{ mm}^4 = 7.3215 \times 10^{-6} \text{ m}^4$$



$$\text{For point } d, A^* = (60)(13) = 780 \text{ mm}^2$$

$$\bar{x} = 45 \text{ mm}, \bar{y} = 31 \text{ mm}$$

$$Q_x = A^* \bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$$

$$Q_y = A^* \bar{x} = 35.1 \times 10^3 \text{ mm}^3 = 35.1 \times 10^{-6} \text{ m}^3$$

$$\text{For point } e, Q_x = 0, Q_y = 0$$

$$\text{At point } d: \sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(15 \times 10^{-3})}{7.3215 \times 10^{-6}} \quad \sigma = 42.2 \text{ MPa} \leftarrow$$

$$\text{Due to } V_x: \tau = \frac{V_x Q_y}{I_y t} = \frac{(9000)(35.1 \times 10^{-6})}{(7.3215 \times 10^{-6})(13 \times 10^{-3})} = 3.32 \text{ MPa} \rightarrow$$

$$\text{Due to } V_y: \tau = \frac{V_y Q_x}{I_x t} = \frac{(13000)(24.18 \times 10^{-6})}{(3.9303 \times 10^{-6})(13 \times 10^{-3})} = 6.15 \text{ MPa} \leftarrow$$

$$\text{By superposition the net value is } \tau = 2.83 \text{ MPa} \leftarrow$$

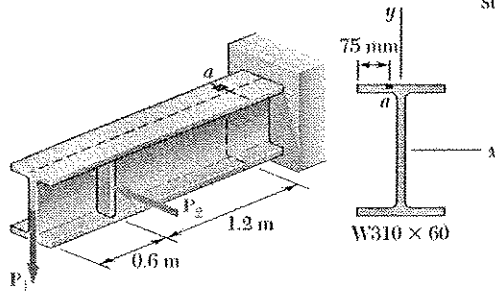
$$\text{At point } e: \sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(75 \times 10^{-3})}{7.3215 \times 10^{-6}}$$

$$\sigma = 12.74 \text{ MPa} \leftarrow$$

$$\tau = 0 \leftarrow$$

Problem 8.55

8.55 Two forces P_1 and P_2 are applied as shown in directions perpendicular to the longitudinal axis of a W310 $\times$ 60 beam. Knowing that $P_1 = 20$ kN and $P_2 = 12$ kN, determine the principal stresses and the maximum shearing stress at point a .



At the section containing point a

$$M_x = (20)(1.3) = 26 \text{ kNm}$$

$$M_y = -(12)(0.6) = -7.2 \text{ kNm}$$

$$V_x = -12 \text{ kN}$$

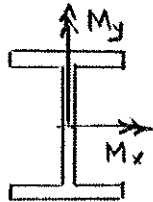
$$V_y = -20 \text{ kN}$$

For W310 $\times$ 60 rolled steel section

$$d = 303 \text{ mm} \quad b_f = 203 \text{ mm} \quad t_f = 13.1 \text{ mm} \quad t_w = 7.5 \text{ mm}$$

$$I_x = 129 \times 10^6 \text{ mm}^4 \quad I_y = 18.3 \times 10^6 \text{ mm}^4$$

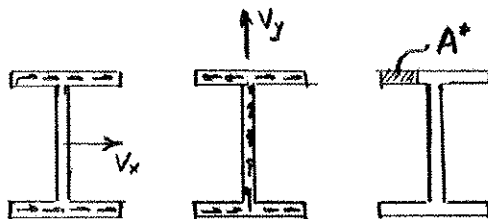
Normal stress at point a . $x = -\frac{b_f}{2} + 75 = -26.5 \text{ mm}$ $y = \frac{d}{2} = 151.5 \text{ mm}$



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(26 \times 10^3)(151.5)}{129 \times 10^6} - \frac{(-7.2 \times 10^3)(-26.5)}{18.3 \times 10^6}$$

$$= 42.28 - 20.85 = 21.43 \text{ MPa}$$

Shearing stress at point a .



$$\tau_{xz} = \frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$

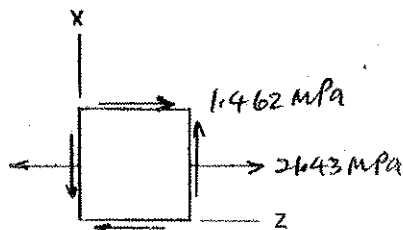
$$A^* = (75)(13.1) = 982.5 \text{ mm}^2$$

$$\bar{x} = -\frac{203}{2} + \frac{75}{2} = -64 \text{ mm}$$

$$\bar{y} = \frac{303}{2} - \frac{13.1}{2} = 144.95 \text{ mm}$$

$$\tau_{xz} = \frac{(-12 \times 10^3)(982.5)(-64)}{(18.3 \times 10^6)(13.1)} - \frac{(-20 \times 10^3)(982.5)(144.95)}{(129 \times 10^6)(13.1)}$$

$$= 3.1475 - 1.6855 = 1.462 \text{ MPa}$$



$$\sigma_{ave} = \frac{21.43 + 0}{2} = 10.715 \text{ MPa}$$

$$R = \sqrt{\left(\frac{21.43 - 0}{2}\right)^2 + 1.462^2} = 10.814 \text{ MPa}$$

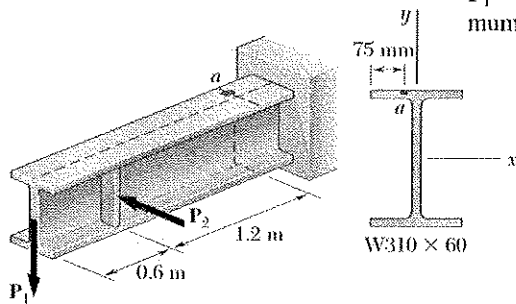
$$\sigma_{max} = \sigma_{ave} + R = 10.715 + 10.814 = 21.5 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = 10.715 - 10.814 = -0.099 \text{ MPa}$$

$$\tau_{max} = R = 10.8 \text{ MPa}$$

Problem 8.56

8.56 Two forces P_1 and P_2 are applied as shown in directions perpendicular to the longitudinal axis of a W310 $\times$ 60 beam. Knowing that $P_1 = 20$ kN and $P_2 = 12$ kN, determine the principal stresses and the maximum shearing stress at point b .



At the section containing point b ,

$$M_x = (20)(1.8) = 36 \text{ kNm}$$

$$M_y = -(12)(1.2) = -14.4 \text{ kNm}$$

$$V_x = -12 \text{ kN}$$

$$V_y = -20 \text{ kN}$$

For W 310 $\times$ 60 rolled steel section

$$d = 303 \text{ mm}$$

$$b_f = 203 \text{ mm}$$

$$t_f = 13.1 \text{ mm}$$

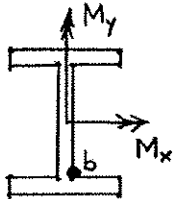
$$t_w = 7.5 \text{ mm}$$

$$I_x = 129 \times 10^6 \text{ mm}^4$$

$$I_y = 18.3 \times 10^6 \text{ mm}^4$$

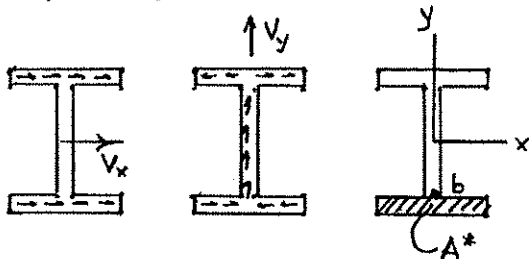
Normal stress at point b

$$x \approx 0 \quad y = -\frac{1}{2}d + t_f = -164.6 \text{ mm}$$



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(36000)(-164.6 \times 10^{-3})}{129 \times 10^{-6}} - 0$$

$$= -45.93 \text{ MPa}$$



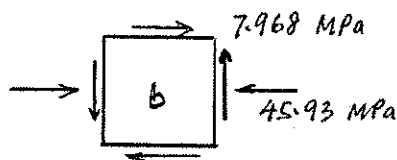
Shearing stress at point b

$$\tau_{yz} = \frac{V_y A^* \bar{y}}{I_x t_w}$$

$$A^* = A_f = b_f t_f = 2659.3 \text{ mm}^2$$

$$\bar{x} = 0, \quad \bar{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.95 \text{ mm}$$

$$\tau_{yz} = \frac{(-20000)(2659.3 \times 10^{-6})(-0.14495)}{(129 \times 10^{-6})(0.0075)} = 7.968 \text{ MPa}$$



$$\sigma_c = -\frac{45.93}{2} = -22.965 \text{ MPa}$$

$$R = \sqrt{\left(\frac{45.93}{2}\right)^2 + (7.968)^2} = 24.308 \text{ MPa}$$

$$\sigma_{\max} = \sigma_c + R$$

$$\sigma_{\max} = 1.34 \text{ MPa}$$

$$\sigma_{\min} = \sigma_c - R$$

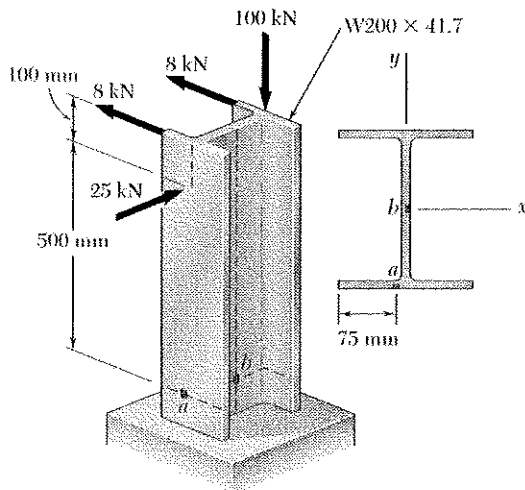
$$\sigma_{\min} = -47.27 \text{ MPa}$$

$$\tau_{\max} = R$$

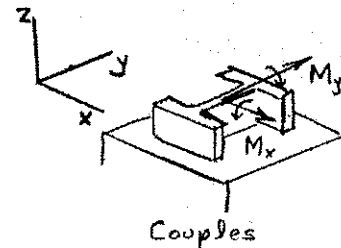
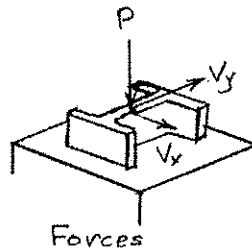
$$\tau_{\max} = 24.31 \text{ MPa}$$

Problem 8.57

8.57 Four forces are applied to a W 200 × 41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point a.



Calculate forces and couples at section containing point a.



$$P = 100 \text{ kN} \quad V_x = -16 \text{ kN} \quad V_y = 25 \text{ kN}$$

$$M_x = -(500 \times 10^{-3})(25 \times 10^3) - \left(\frac{205}{2} \times 10^{-3}\right)(100 \times 10^3) = -122.75 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = (500 + 100)(16 \times 10^3) = 9.6 \times 10^3 \text{ N}\cdot\text{m}$$

Section properties.

$$A = 5310 \text{ mm}^2 \quad d = 205 \text{ mm}$$

$$b_f = 166 \text{ mm} \quad t_f = 11.8 \text{ mm}$$

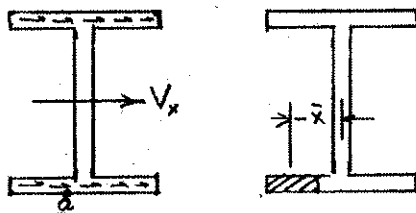
$$t_w = 7.2 \text{ mm}$$

$$I_x = 40.9 \times 10^6 \text{ mm}^4 = 40.9 \times 10^{-6} \text{ m}^4, \quad I_y = 9.01 \times 10^6 \text{ mm}^4 = 9.01 \times 10^{-6} \text{ m}^4$$

$$\text{Point a: } x_a = -\frac{166}{2} + 75 = -8 \text{ mm}, \quad y_a = -\frac{205}{2} = -102.5 \text{ mm}$$

$$\begin{aligned} \sigma_a &= -\frac{P}{A} + \frac{M_x y_a}{I_x} + \frac{M_y x_a}{I_y} \\ &= -\frac{100 \times 10^3}{5310 \times 10^{-6}} + \frac{(-122.75 \times 10^3)(-102.5 \times 10^{-3})}{40.9 \times 10^{-6}} + \frac{(9.6 \times 10^3)(-8 \times 10^{-3})}{9.01 \times 10^{-6}} \\ &= -18.83 \times 10^6 + 57.01 \times 10^6 - 8.52 \times 10^6 = 29.66 \text{ MPa} \end{aligned}$$

Shearing stress at point a due to V_x



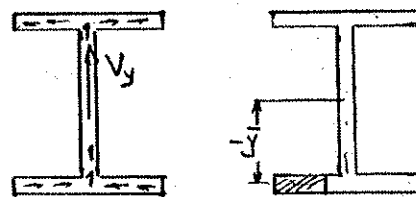
$$A = (75)(11.8) = 885 \text{ mm}^2$$

$$\bar{x} = -\frac{166}{2} + \frac{75}{2} = -45.5 \text{ mm}$$

$$Q = -A\bar{x} = 40.2675 \times 10^3 \text{ mm}^3$$

$$\begin{aligned} \tau_{xz} &= \frac{V_x Q}{I_y t} = \frac{(-16 \times 10^3)(40.2675 \times 10^{-6})}{(9.01 \times 10^{-6})(11.8 \times 10^{-3})} \\ &= -6.060 \text{ MPa} \end{aligned}$$

Shearing stress at point a due to V_y



$$A = (75)(11.8) = 885 \text{ mm}^2$$

$$\bar{y} = -\frac{205}{2} + \frac{11.8}{2} = -96.6 \text{ mm}$$

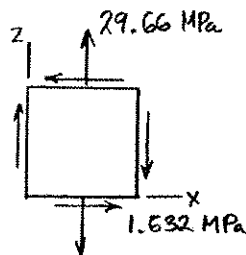
$$Q = -A\bar{y} = -85.491 \times 10^3 \text{ mm}^3$$

$$\begin{aligned} \tau_{yz} &= \frac{V_y Q}{I_x t} = \frac{(25 \times 10^3)(-85.491 \times 10^{-6})}{(40.9 \times 10^{-6})(11.8 \times 10^{-3})} \\ &= 4.428 \text{ MPa} \end{aligned}$$

Continued

Problem 8.57 continued

Combined shearing stress $\tau_a = -6.060 + 4.428 = -1.632 \text{ MPa}$



$$\sigma_{ave} = \frac{29.66 + 0}{2} = 14.83 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.66 - 0}{2}\right)^2 + (1.632)^2} = 14.92 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R$$

$$\sigma_{max} = 29.8 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R$$

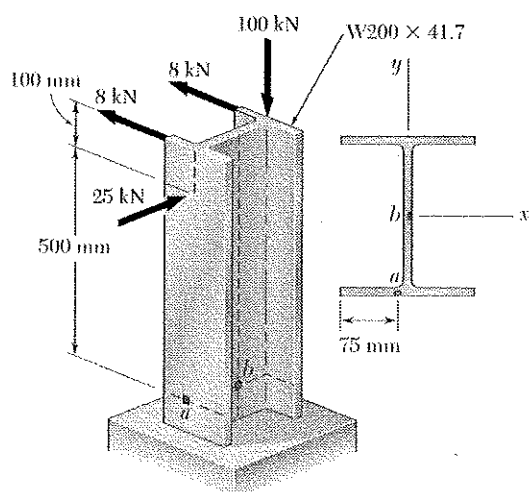
$$\sigma_{min} = -0.09 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max} = R$$

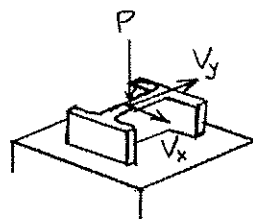
$$\tau_{max} = 14.92 \text{ MPa} \quad \blacktriangleleft$$

Problem 8.58

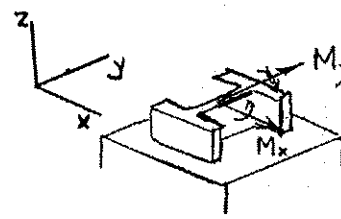
8.58 Four forces are applied to a W 200 $\times$ 41.7 rolled beam as shown. Determine the principal stresses and maximum shearing stress at point b .



Calculate forces and couples at section containing point b .



Forces



Couples

$$P = 100 \text{ kN} \quad V_x = -16 \text{ kN} \quad V_y = 25 \text{ kN}$$

$$M_x = -(500 \times 10^{-3})(25 \times 10^3) - \left(\frac{205}{2} \times 10^{-3}\right)(100 \times 10^3) = -22.75 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = (500 + 100)(10^{-3})(16 \times 10^3) = 9.6 \times 10^3 \text{ N}\cdot\text{m}$$

Section properties.

$$A = 5310 \text{ mm}^2 \quad d = 205 \text{ mm}$$

$$b_f = 166 \text{ mm} \quad t_f = 11.8 \text{ mm}$$

$$t_w = 7.2 \text{ mm}$$

$$I_x = 40.9 \times 10^6 \text{ mm}^4 = 40.9 \times 10^{-6} \text{ m}^4, \quad I_y = 9.01 \times 10^6 \text{ mm}^4 = 9.01 \times 10^{-6} \text{ m}^4$$

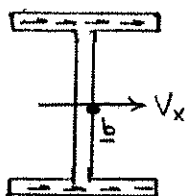
Point b : $x_b = 0, y_b = 0$

$$\sigma_b = -\frac{P}{A} + \frac{M_x y_b}{I_x} + \frac{M_y x_b}{I_y} = -\frac{100 \times 10^3}{5310 \times 10^{-6}} = -18.83 \text{ MPa}$$

continued

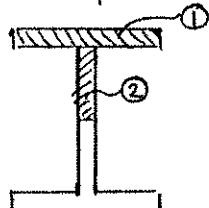
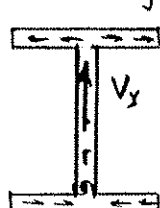
Problem 8.58 continued

Shearing stress at point b due to V_x .



$$\tau_{xz} = 0$$

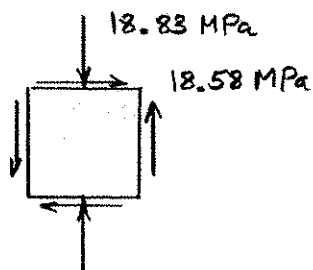
Shearing stress at point b due to V_y .



	$A \text{ (mm}^2\text{)}$	$\bar{y} \text{ (mm)}$	$A\bar{y} \text{ (10}^3\text{ mm}^3\text{)}$
①	1958.8	96.6	189.220
②	653.04	45.35	29.615
Σ			218.835

$$Q = \Sigma A\bar{y} = 218.835 \times 10^3 \text{ mm}^3 = 218.835 \times 10^{-6} \text{ m}^3$$

$$\tau_b = \frac{V_y Q}{I_x t_w} = \frac{(25 \times 10^3)(218.835 \times 10^{-6})}{(40.9 \times 10^{-6})(7.2 \times 10^{-3})} = 18.58 \text{ MPa}$$



$$\sigma_{ave} = \frac{-18.83 + 0}{2} = -9.415 \text{ MPa}$$

$$R = \sqrt{\left(\frac{-18.83 - 0}{2}\right)^2 + (18.58)^2} = 20.829 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R$$

$$\sigma_{max} = 11.41 \text{ MPa} \quad \blacktriangleleft$$

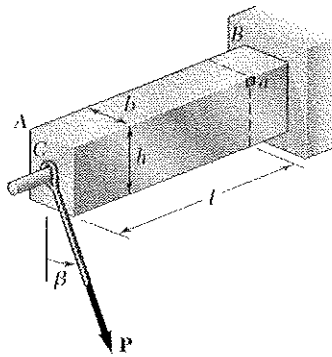
$$\sigma_{min} = \sigma_{ave} - R$$

$$\sigma_{min} = -30.2 \text{ MPa} \quad \blacktriangleleft$$

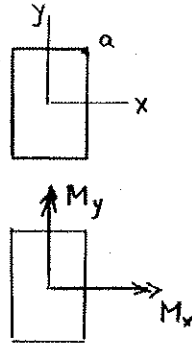
$$\tau_{max} = R$$

$$\tau_{max} = 20.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 8.59



8.59 A force P is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that P acts in a direction perpendicular to the longitudinal axis of the beam, determine (a) the normal stress at point a in terms of P , b , h , l , and β , (b) the values of β for which the normal stress at a is zero.



$$I_x = \frac{1}{12} b h^3 \quad I_y = \frac{1}{12} h b^3$$

$$\begin{aligned} \sigma &= \frac{M_x (h/2)}{I_x} - \frac{M_y (b/2)}{I_y} \\ &= \frac{6 M_x}{b h^2} - \frac{6 M_y}{h b^2} \end{aligned}$$

$$\vec{P} = P \sin \beta \vec{i} - P \cos \beta \vec{j} \quad \vec{r} = l \vec{k}$$

$$\vec{M} = \vec{r} \times \vec{P} = l \vec{k} \times (P \sin \beta \vec{i} - P \cos \beta \vec{j}) = P l \cos \beta \vec{i} + P l \sin \beta \vec{j}$$

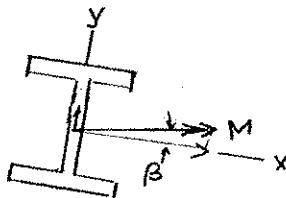
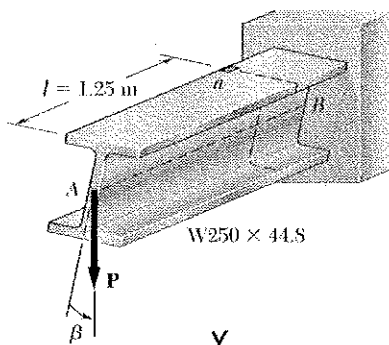
$$M_x = P l \cos \beta \quad M_y = P l \sin \beta$$

$$(a) \quad \sigma = \frac{6 P l \cos \beta}{b h^2} - \frac{6 P l \sin \beta}{h b^2} = \frac{6 P l}{b h} \left[\frac{\cos \beta}{h} - \frac{\sin \beta}{b} \right]$$

$$(b) \quad \sigma = 0 \quad \frac{\cos \beta}{h} - \frac{\sin \beta}{b} = 0 \quad \tan \beta = \frac{b}{h}$$

$$\beta = \tan^{-1} \left(\frac{b}{h} \right)$$

Problem 8.60



8.60 A vertical force P is applied at the center of the free end of cantilever beam AB . (a) If the beam is installed with the web vertical ($\beta = 0$) and with its longitudinal axis AB horizontal, determine the magnitude of the force P for which the normal stress at point a is $+120$ MPa. (b) Solve part a, assuming that the beam is installed with $\beta = 3^\circ$.

For $W250 \times 44.8$ rolled steel section

$$S_x = 535 \times 10^3 \text{ mm}^3 = 535 \times 10^{-6} \text{ m}^3$$

$$S_y = 95.0 \times 10^3 \text{ mm}^3 = 95.0 \times 10^{-6} \text{ m}^3$$

At the section containing point a .

$$M_x = Pl \cos \beta, \quad M_y = Pl \sin \beta$$

Stress at a

$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl \cos \beta}{S_x} + \frac{Pl \sin \beta}{S_y}$$

Allowable load. $P_{all} = \frac{\sigma_{all}}{l} \left[\frac{\cos \beta}{S_x} + \frac{\sin \beta}{S_y} \right]^{-1}$

(a) $\beta = 0$: $P_{all} = \frac{120 \times 10^6}{1.25} \left[\frac{1}{535 \times 10^{-6}} + 0 \right]^{-1} = 51.4 \times 10^3 \text{ N}$

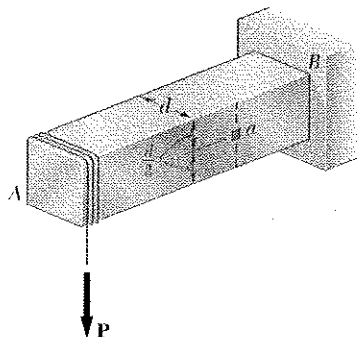
$$P_{all} = 51.4 \text{ kN} \quad \blacktriangleleft$$

(b) $\beta = 3^\circ$: $P_{all} = \frac{120 \times 10^6}{1.25} \left[\frac{\cos 3^\circ}{535 \times 10^{-6}} + \frac{\sin 3^\circ}{95.0 \times 10^{-6}} \right]^{-1} = 39.7 \times 10^3 \text{ N}$

$$P_{all} = 39.7 \text{ kN} \quad \blacktriangleleft$$

Problem 8.61

*8.61 A 5-kN force P is applied to a wire that is wrapped around bar AB as shown. Knowing that the cross section of the bar is a square of side $d = 40$ mm, determine the principal stresses and the maximum shearing stress at point a .



Bending: Point a lies on the neutral axis.

$$\sigma = 0$$

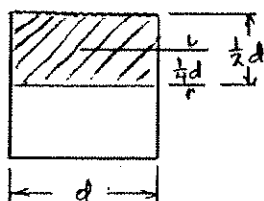
Torsion: $\tau = \frac{T}{C_1 a b^2}$ where $a = b = d$ and

$C_1 = 0.208$ for a square section.

$$\text{Since } T = \frac{Pd}{2}, \quad \tau = \frac{P}{0.416 d^2} = 2.404 \frac{P}{d^2}$$

Transverse shear: $V = P$

$$I = \frac{1}{12} d^4$$



$$A = \frac{1}{2} d^2 \quad \bar{y} = \frac{1}{4} d \quad Q = A\bar{y} = \frac{1}{8} d^3$$

$$t = d$$

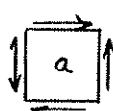
$$\tau_v = \frac{VQ}{It} = 1.5 \frac{P}{d^2}$$

By superposition

$$\tau = \tau_T + \tau_v = 3.904 \frac{P}{d^2}$$

$$\tau = \frac{(3.904)(5 \times 10^3)}{(40 \times 10^{-3})^2} = 12.2 \times 10^6 \text{ Pa}$$

$$12.2 \text{ MPa.}$$



$$12.2 \text{ MPa}$$

By Mohr's circle

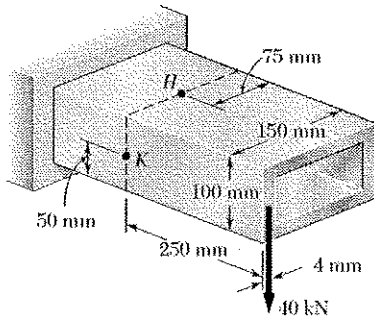
$$\sigma_{\max} = 12.2 \text{ MPa}$$

$$\sigma_{\min} = -12.2 \text{ MPa}$$

$$\tau_{\max} = 12.2 \text{ MPa}$$

Problem 8.62

*8.62 Knowing that the structural tube shown has a uniform wall thickness of 8 mm, determine the principal stresses, principal planes, and maximum shearing stress at (a) point H, (b) point K.



At the section containing points H and K

$$V = 40 \text{ kN} \quad M = (40)(0.25) = 10 \text{ kNm}$$

$$T = (40)(0.075 - 0.004) = 2.84 \text{ kNm}$$

Torsion:

$$Q = (142)(92) = 13064 \text{ mm}^2$$

$$\gamma = \frac{T}{2tQ} = \frac{(2.84 \times 10^6)}{(2)(8)(13064)} = 13.59 \text{ MPa}$$

Transverse shear:

$$Q_H = 0$$

$$Q_K = (75)(50)(25) - (67)(42)(21) = 34656 \text{ mm}^3$$

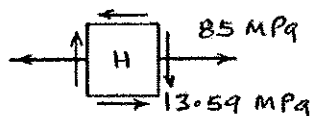
$$I = \frac{1}{12}(150)(100)^3 - \frac{1}{12}(134)(84)^3 = 5.88 \times 10^6 \text{ mm}^4$$

$$\gamma_H = 0$$

$$\gamma_K = \frac{VQ_K}{It} = \frac{(40 \times 10^3)(34656)}{(5.88 \times 10^6)(8)} = 29.47 \text{ MPa}$$

Bending: $\sigma_H = \frac{Mc}{I} = \frac{(10 \times 10^6)(50)}{5.88 \times 10^6} = 85 \text{ MPa}, \quad \sigma_K = 0$

(a) Point H:



$$\sigma_c = \frac{85}{2} = 42.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{85}{2}\right)^2 + (13.59)^2} = 44.62 \text{ MPa}$$

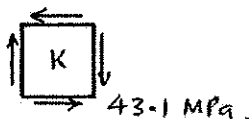
$$\sigma_{\max} = \sigma_c + R = 87.1 \text{ MPa}$$

$$\sigma_{\min} = \sigma_c - R = -2.1 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\gamma}{\sigma} = -0.32 \quad \theta_p = -8.9^\circ, 81.1^\circ$$

$$\gamma_{\max} = R = 44.6 \text{ MPa}$$

(b) Point K: $\sigma = 0 \quad \gamma = 13.59 + 29.47 = 43.06 \text{ MPa}$



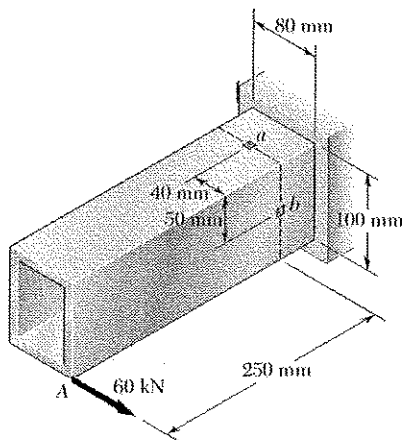
$$\sigma_{\max} = 43.1 \text{ MPa}$$

$$\sigma_{\min} = -43.1 \text{ MPa}$$

$$\theta_p = \pm 45^\circ$$

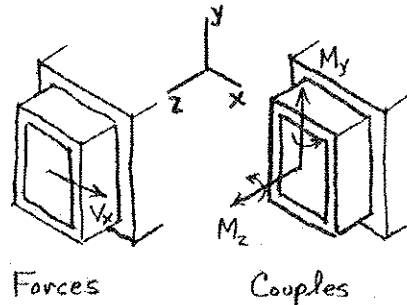
$$\gamma_{\max} = 43.1 \text{ MPa}$$

Problem 8.63



*8.63 The structural tube shown has a uniform wall thickness of 8 mm. Knowing that the 60-kN load is applied 4 mm above the base of the tube, determine the shearing stress at (a) point a, (b) point b.

Calculate forces and couples at section containing points a and b.



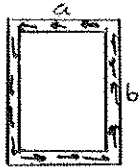
$$V_x = 60 \text{ kN}$$

$$M_z = (50 - 4)(60) = 2760 \text{ kN}\cdot\text{mm}$$

$$M_y = (60)(250) = 15000 \text{ kN}\cdot\text{mm}$$

Shearing stresses due to torque $T = M_z$

$$Q = [80 - (2)(4)][100 - (2)(4)] = 6624 \text{ mm}^2$$



$$q = \frac{M_z}{2Q} = \frac{2760 \times 10^3}{(2)(6624)} = 208.3 \text{ N/mm}$$

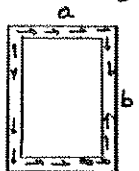
At point a. $t = 8 \text{ mm}$

$$\tau_a = \frac{q}{t} = \frac{208.3}{8} = 26.04 \text{ MPa} \leftarrow$$

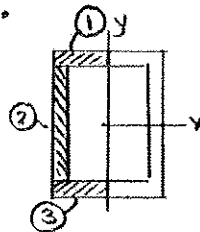
At point b. $t = 8 \text{ mm}$

$$\tau_b = \frac{q}{t} = \frac{208.3}{8} = 26.04 \text{ MPa} \uparrow$$

Shearing stresses due to V_x .



At point a



Part	A (mm ²)	$\bar{x}$ (mm)	A $\bar{x}$ (mm ³)
①	320	-20	-6400
②	672	36	24192
③	320	-20	-6400
Σ			-36992

$$Q = |\Sigma A \bar{x}| = 36992 \text{ mm}^3$$

$$t = (2)(8) = 16 \text{ mm}$$

$$I_y = \frac{1}{12}(100)(80)^3 - \frac{1}{12}(84)(64)^3 = 2.4317 \times 10^6 \text{ mm}^4$$

$$\tau_a = \frac{V_x Q}{I_y t} = \frac{(60000)(36992)}{(2.4317 \times 10^6)(16)} = 57.05 \text{ MPa}$$

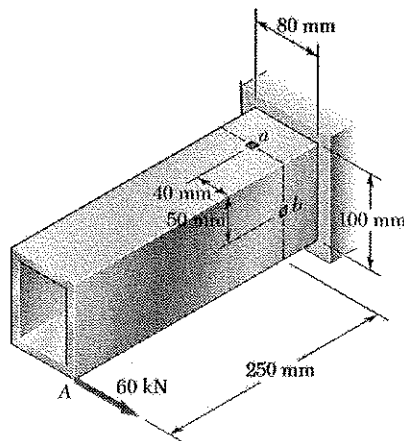
At point b. $\tau_b = 0$

Combined shearing stresses.

At point a. $\tau_a = 26.04 \leftarrow + 57.05 \rightarrow = 31 \text{ MPa} \rightarrow$

At point b. $\tau_b = 26.04 \uparrow + 0 = 26 \text{ MPa} \uparrow$

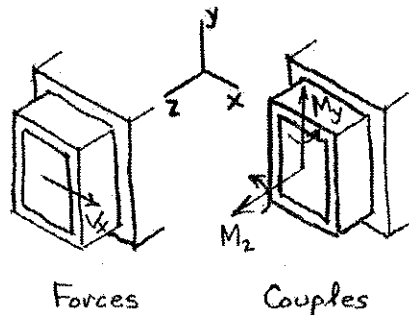
Problem 8.64



*8.64 For the tube and loading of Prob. 8.63, determine the principal stresses and the maximum shearing stress at point *b*.

*8.63 The structural tube shown has a uniform wall thickness of 8 mm. Knowing that the 60-kN load is applied 4 mm above the base of the tube, determine the shearing stress at (a) point *a*, (b) point *b*.

Calculate forces and couples at section containing point *b*.



$$V_x = 60 \text{ kN}$$

$$M_z = (50 - 4)(60) = 2760 \text{ kN mm}$$

$$M_y = (60)(250) = 15000 \text{ kN mm}$$

$$I_y = \frac{1}{12} (100)(80)^3 - \frac{1}{12} (84)(64)^3 = 2.4317 \times 10^6 \text{ mm}^4$$

$$\sigma_b = -\frac{M_y x_b}{I_y} = -\frac{(15 \times 10^6)(40)}{2.4317 \times 10^6} = -246.7 \text{ MPa}$$

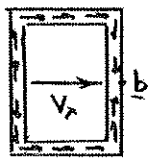
Shearing stress at point *b* due to torque M_z .

$$Q = [80 - (2)(4)][100 - (2)(4)] = 6624 \text{ mm}^2$$

$$q = \frac{M_z}{2Q} = \frac{2760 \times 10^3}{(2)(6624)} = 208.3 \text{ N/mm}$$

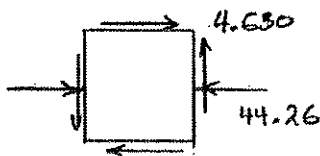
$$\tau = \frac{q}{t} = \frac{208.3}{8} = 26.04 \text{ MPa}$$

Shearing at point *b* due to V_x .



$$\tau = 0$$

Calculation of principal stresses and maximum shearing stress.



$$\sigma_{ave} = \frac{-246.7 + 0}{2} = -123.35 \text{ MPa}$$

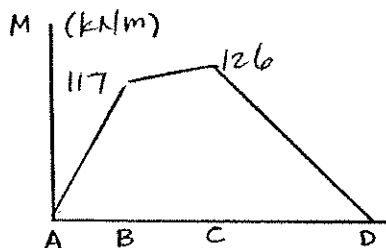
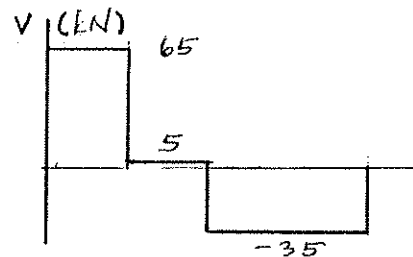
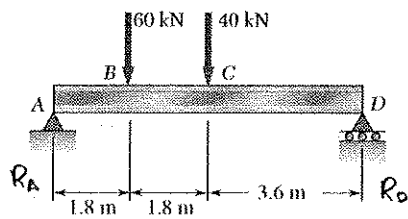
$$R = \sqrt{\left(\frac{-246.7 - 0}{2}\right)^2 + (26.04)^2} = 126.07 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = 2.72 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -249.4 \text{ MPa}$$

$$\tau_{max} = R = 126 \text{ MPa}$$

Problem 8.65



8.65 (a) Knowing that $\sigma_{all} = 165 \text{ MPa}$ and $\tau_{all} = 100 \text{ MPa}$, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m , τ_m , and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

$$+\circlearrowleft \sum M_o = 0: -7.2 R_A + (60)(5.4) + (40)(3.6) = 0$$

$$R_A = 65 \text{ kN} \uparrow$$

$$|V|_{max} = 65 \text{ kN}$$

$$|M|_{max} = 126 \text{ kN}\cdot\text{m}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{126 \times 10^3}{165 \times 10^6} = 763.6 \times 10^{-6} \text{ m}^3 = 763.6 \times 10^3 \text{ mm}^3$$

Shape	$S (\times 10^3 \text{ mm}^3)$
W 460x52	942
W 410x46.1	774
W 360x57.8	899
W 310x60	851
W 250x67	809
W 200x86	853

(a) Use

W 410x46.1

$d = 403 \text{ mm}$

$t_f = 11.2 \text{ mm}$

$t_w = 7 \text{ mm}$

(a) Use W 410x46.1 with $S = 774 \times 10^3 \text{ mm}^3$

$$c = \frac{1}{2}d = 201.5 \quad y_b = c - t_f = 190.3 \text{ mm}$$

At C: $M = (35)(3.6) = 126 \text{ kN}\cdot\text{m}$

$$\sigma_m = \frac{M}{S} = \frac{126 \times 10^6}{774 \times 10^3} = 162.8 \text{ MPa}$$

$$|V| = 35 \text{ kN}$$

$$\tau_m = \frac{V}{dt_w} = \frac{35000}{(403)(7)} = 12.4 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{190.3}{201.5}\right)(162.8) = 153.8 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{153.8}{2}\right)^2 + 12.4^2} = 77.9 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 154.8 \text{ MPa}$$

At B: $M = (65)(1.8) = 117 \text{ kN}\cdot\text{m}$

$$\sigma_m = \frac{117 \times 10^6}{774 \times 10^3} = 151.1 \text{ MPa}$$

$$|V| = 65 \text{ kN}$$

$$\tau_m = \frac{V}{dt_w} = \frac{65000}{(403)(7)} = 23 \text{ MPa}$$

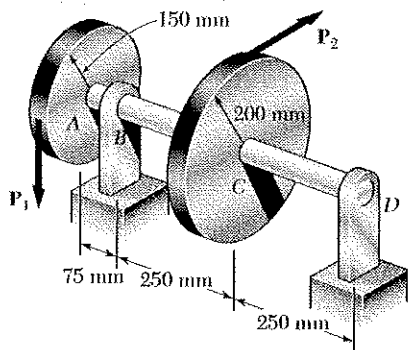
$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{190.3}{201.5}\right)(151.1) = 142.7 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = 75 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 146.4 \text{ MPa}$$

Problem 8.66

8.66 The vertical force P_1 and the horizontal force P_2 are applied as shown to disks welded to the solid shaft AD . Knowing that the diameter of the shaft is 40 mm and that $\tau_{\text{all}} = 55 \text{ MPa}$, determine the largest permissible magnitude of the force P_2 .

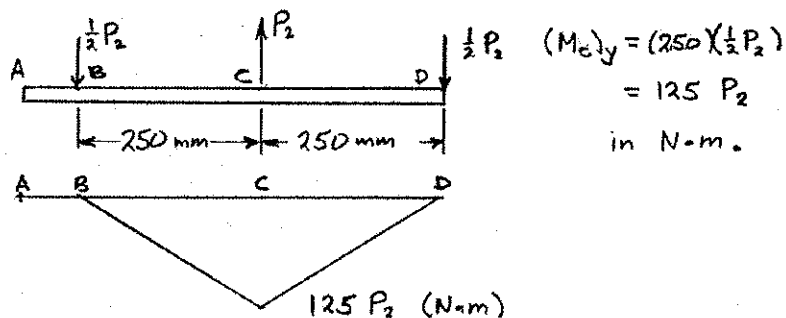


Let the dimensions of P_2 be kN.

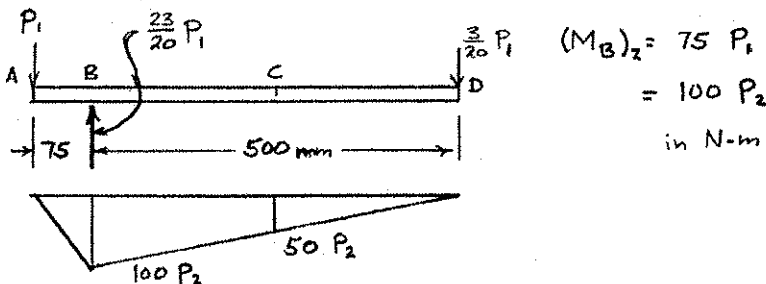
$$\sum M_{\text{shaft}} = 0: 150 P_1 - 200 P_2 = 0 \quad P_1 = \frac{4}{3} P_2$$

Torque over portion ABC: $T = 200 P_2$ in N·m.

Bending in horizontal plane.



Bending in vertical plane.



$$\text{At B: } \sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{(100 P_2)^2 + 0 + (200 P_2)^2} = 223.61 P_2$$

$$\text{At C: } \sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{(125 P_2)^2 + (50 P_2)^2 + (200 P_2)^2} = 241.09 P_2$$

$$d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} \quad c = \frac{1}{2} d = 20 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (20 \times 10^{-3})^4 = 251.33 \times 10^{-9} \text{ m}^4$$

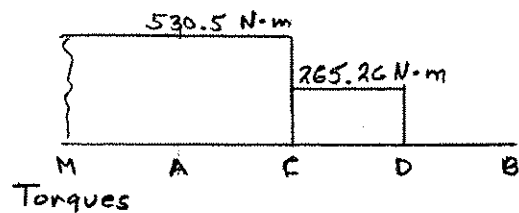
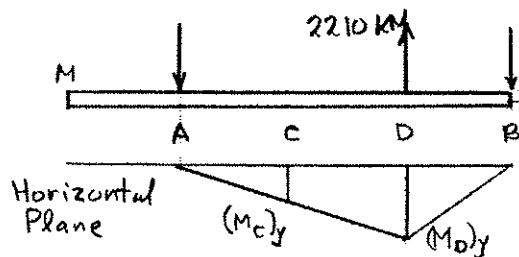
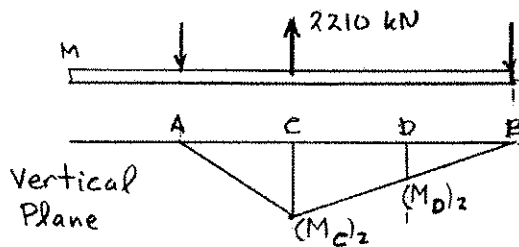
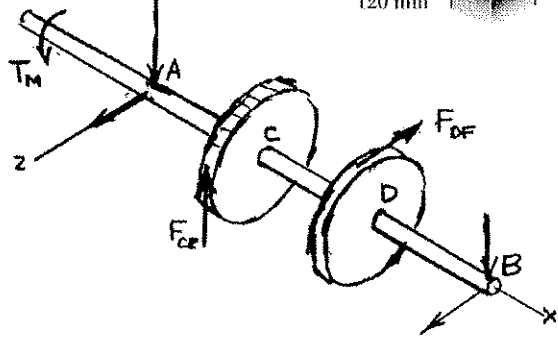
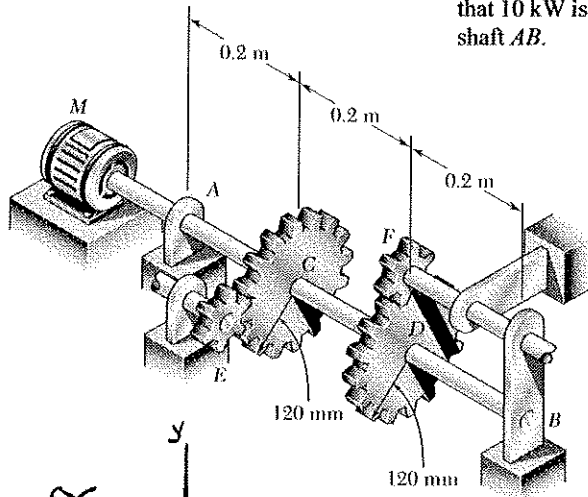
$$\tau_{\text{max}} = \frac{\max \sqrt{M_y^2 + M_z^2 + T^2} c}{J}$$

$$55 \times 10^6 = \frac{(241.09 P_2)(20 \times 10^{-3})}{251.33 \times 10^{-9}}$$

$$P_2 = 2.87 \text{ kN} \quad \blacktriangleleft$$

Problem 8.67

8.67 The solid shaft AB rotates at 360 rpm and transmits 20 kW from the motor M to machine tools connected to gears E and F . Knowing that $\tau_{all} = 45 \text{ MPa}$ and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft AB .



$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$T_M = \frac{P_M}{2\pi f} = \frac{20 \times 10^3}{2\pi(6)} = 530.5 \text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{2\pi f} = \frac{10 \times 10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$T_D = \frac{P_D}{2\pi f} = \frac{10 \times 10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$F_{CE} = \frac{T_C}{r_C} = \frac{265.26}{120 \times 10^{-3}} = 2.210 \times 10^3 \text{ N}$$

$$F_{DF} = \frac{T_D}{r_D} = \frac{265.26}{120 \times 10^{-3}} = 2.210 \times 10^3 \text{ N}$$

$$(M_C)_z = \frac{(0.2)(2.210 \times 10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

$$(M_D)_z = \frac{1}{2}(M_C)_z = 147.37 \text{ N}\cdot\text{m}$$

$$(M_C)_y = \frac{(0.4)(0.2)(2.210 \times 10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

$$(M_D)_y = \frac{1}{2}(M_C)_y = 147.37 \text{ N}\cdot\text{m}$$

Torques in shaft.

$$T_{MAC} = 530.5 \text{ N}\cdot\text{m}$$

$$T_{CD} = 265.26 \text{ N}\cdot\text{m}$$

$$T_{DB} = 0$$

Just to the left of gear C.

$$\max \sqrt{M_y^2 + M_z^2 + T^2} = 624.5 \text{ N}\cdot\text{m}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\max \sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}}$$

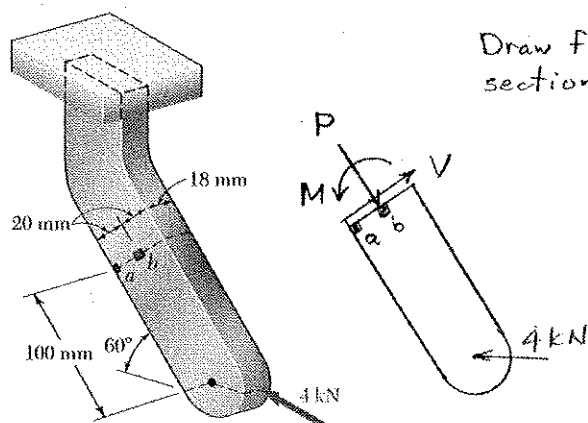
$$= \frac{624.5}{45 \times 10^6} = 13.878 \times 10^{-6} \text{ m}^3$$

$$C = 20.67 \times 10^{-3} \text{ m} = 20.67 \text{ mm}$$

$$d = 2C = 41.3 \text{ mm}$$

Problem 8.68

8.68 For the bracket and loading shown, determine the normal and shear stresses at (a) point a, (b) point b.



Draw free body diagram of portion below section ab.

From statics,

$$P = 4000 \cos 60^\circ = 2000 \text{ N}$$

$$V = 4000 \sin 60^\circ = 3464 \text{ N}$$

$$M = (0.1)(4000) \sin 60^\circ = 346.4 \text{ Nm}$$

Section properties.

$$A = (18)(40) = 720 \text{ mm}^2$$

$$I = \frac{1}{12} (18)(40)^3 = 96000 \text{ mm}^4$$

(a) Point a $\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2000}{720} - \frac{(346.4)(20)}{96000} = -74.9 \text{ MPa}$

$$\tau = 0$$

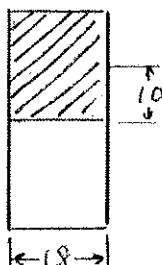
(b) Point b $\sigma = -\frac{P}{A} = -\frac{2000}{720} = -2.8 \text{ MPa}$

$$A = (20)(18) = 360 \text{ mm}^2$$

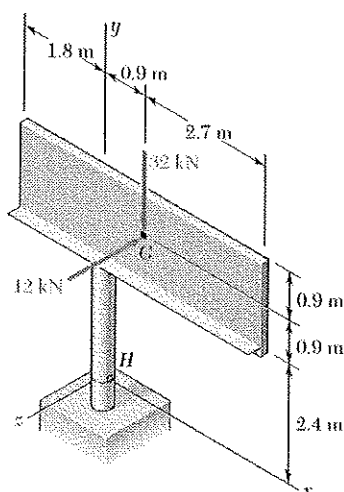
$$\bar{y} = 10 \text{ mm}$$

$$Q = A\bar{y} = (360)(10) = 3600 \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{(3464)(3600)}{(96000)(18)} = 7.2 \text{ MPa}$$



Problem 8.69



8.69 The billboard shown weighs 32 kN and is supported by a structural tube that has a 380-mm outer diameter and a 12-mm wall thickness. At a time when the resultant of the wind pressure is 12 kN located at the center C of the billboard, determine the normal and shearing stresses at point H.

At section containing point H,

$$P = 32 \text{ kN (compression)}$$

$$T = (12)(0.9) = 10.8 \text{ kNm}$$

$$M_x = -(12)(3.3) = -39.6 \text{ kNm}$$

$$M_z = -(0.9)(32) = -28.8 \text{ kNm}$$

$$V = 12 \text{ kN}$$

Section properties.

$$d_o = 380 \text{ mm} \quad c_o = \frac{1}{2} d_o = 190 \text{ mm} \quad c_i = c_o - t = 178 \text{ mm}$$

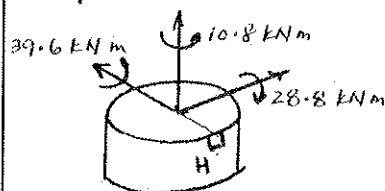
$$A = \pi (c_o^2 - c_i^2) = 13873 \text{ mm}^2$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = 235.1 \times 10^6 \text{ mm}^4$$

$$J = 2I = 470.2 \times 10^6 \text{ mm}^4$$

$$Q = \frac{2}{3} (c_o^3 - c_i^3) = 812832 \text{ mm}^3$$

Couples

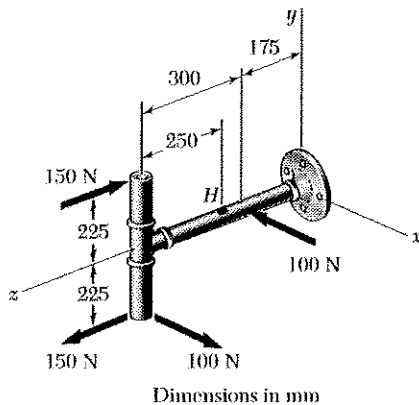


$$\sigma = -\frac{P}{A} - \frac{M_z}{I} = -\frac{32 \times 10^3}{13873} - \frac{(28.8 \times 10^3)(190)}{235.1 \times 10^6} = -2.307 - 23.275 = -25.6 \text{ MPa}$$

$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(10.8 \times 10^3)(190)}{470.2 \times 10^6} + \frac{(12 \times 10^3)(812832)}{(235.1 \times 10^6)(24)} = 4.364 + 1.729 = 6.1 \text{ MPa}$$

Problem 8.70

8.70 Several forces are applied to the pipe assembly shown. Knowing that each section of pipe has inner and outer diameters equal to 36 and 42 mm, respectively, determine the normal and shearing stresses at point H located at the top of the outer surface of the pipe.



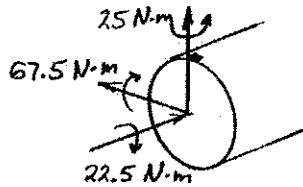
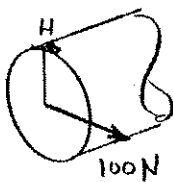
At the section containing point H ,

$$P = 0, \quad V_x = 100 \text{ N}, \quad V_y = 0$$

$$M_x = -(0.450)(150) = -67.5 \text{ N}\cdot\text{m}$$

$$M_y = (0.250)(100) = 25 \text{ N}\cdot\text{m}$$

$$M_z = -(0.225)(100) = -22.5 \text{ N}\cdot\text{m}$$



$$d_o = 42 \text{ mm} \quad d_i = 32 \text{ mm}$$

$$c_o = 21 \text{ mm} \quad c_i = 18 \text{ mm}$$

$$t = c_o - c_i = 3 \text{ mm}$$

$$A = \pi(c_o^2 - c_i^2) = 367.57 \text{ mm}^2 = 367.57 \times 10^{-6} \text{ m}^2$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 70.30 \times 10^3 \text{ mm}^4 = 70.30 \times 10^{-9} \text{ m}^4, \quad J = 2I = 140.59 \times 10^{-9} \text{ m}^4$$

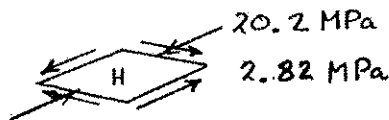
$$\text{For half-pipe, } Q = \frac{2}{3}(c_o^3 - c_i^3) = 2.286 \times 10^3 \text{ mm}^3 = 2.286 \times 10^{-6} \text{ m}^3$$

$$\sigma_H = \frac{M_x y}{I_x} = \frac{(-67.5)(21 \times 10^{-3})}{70.30 \times 10^{-9}} \quad \sigma = -20.2 \text{ MPa} \quad \blacktriangleleft$$

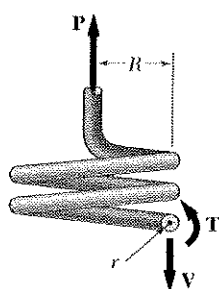
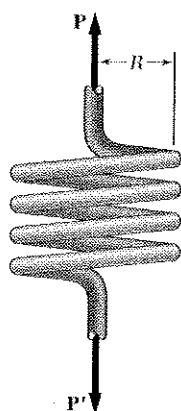
$$\text{Due to torque: } (\tau_H)_T = \frac{Tc}{J} = \frac{(22.5)(21 \times 10^{-3})}{140.59 \times 10^{-9}} = 3.36 \text{ MPa}$$

$$\text{Due to shear: } (\tau_H)_V = \frac{VQ}{It} = \frac{(100)(2.286 \times 10^{-6})}{(70.30 \times 10^{-9})(6 \times 10^{-3})} = 0.54 \text{ MPa}$$

$$\text{Net: } \tau_H = 3.36 - 0.54 \quad \tau = 2.82 \text{ MPa} \quad \blacktriangleleft$$



Problem 8.71



8.71 A close-coiled spring is made of a circular wire of radius r that is formed into a helix of radius R . Determine the maximum shearing stress produced by the two equal and opposite forces P and P' . (Hint: First determine the shear V and the torque T in a transverse cross section.)

$$+\uparrow \Sigma F_y = 0: \quad P - V = 0 \quad V = P$$

$$+\circlearrowleft \Sigma M_c = 0: \quad T - PR = 0 \quad T = PR$$

Shearing stress due to T .

$$\tau_T = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2PR}{\pi r^3}$$

Shearing stress due to V .

$$\text{For semicircle, } Q = \frac{2}{3}r^3, \quad t = d = 2r$$

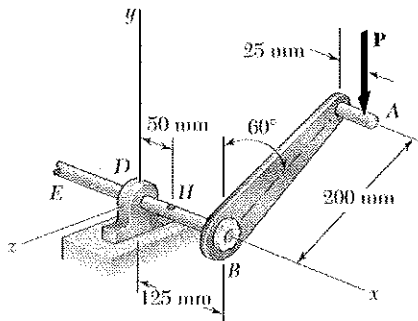
$$\text{For solid circular section, } I = \frac{1}{2}J = \frac{\pi}{4}r^4$$

$$\tau_V = \frac{VQ}{It} = \frac{V(\frac{2}{3}r^3)}{\frac{\pi}{4}r^4(2r)} = \frac{4V}{3\pi r^2} = \frac{4P}{3\pi r^2}$$

$$\text{By superposition, } \tau_{\max} = \tau_T + \tau_V \quad \tau_{\max} = P(2R + 4r/3)/\pi r^3 \quad \blacktriangleleft$$

Problem 8.72

8.72 A vertical force P of magnitude 250 N is applied to the crank at point A . Knowing that the shaft BDE has a diameter of 18 mm, determine the principal stresses and the maximum shearing stress at point H located at the top of the shaft, 50 mm to the right of support D .

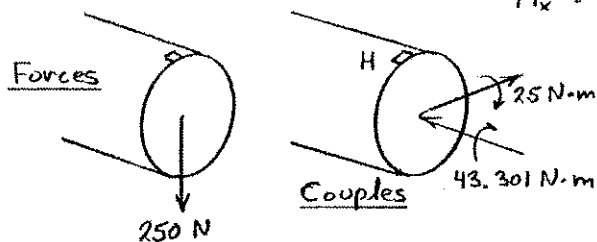


Force-couple system at the centroid of the section containing point H .

$$F_x = 0, \quad V_y = -250 \text{ N}, \quad V_z = 0$$

$$M_z = -(125 - 50 + 25)(10^{-3})(250) = -25 \text{ N}\cdot\text{m}$$

$$M_x = -(200 \sin 60^\circ)(10^{-3})(250) = -43.301 \text{ N}\cdot\text{m}$$



$$d = 18 \text{ mm} \quad c = \frac{1}{2}d = 9 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$$

$$J = 2I = 10.306 \times 10^{-9} \text{ m}^4$$

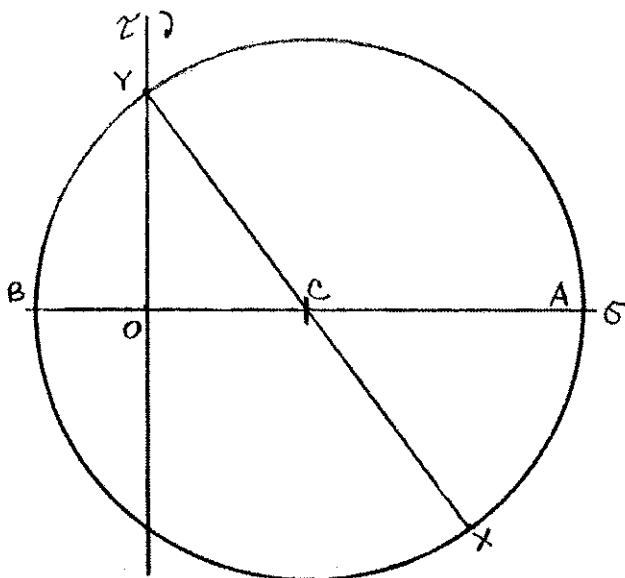
At point H ,
$$\sigma_H = -\frac{M_z y}{I_x} = -\frac{(-25)(9 \times 10^{-3})}{5.153 \times 10^{-9}} = 43.66 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} = \frac{(43.301)(9 \times 10^{-3})}{10.306 \times 10^{-9}} = 37.81 \text{ MPa}$$

Use Mohr's circle.

$$\sigma_c = \frac{1}{2}\sigma_H = 21.83 \text{ MPa}$$

$$R = \sqrt{\left(\frac{43.66}{2}\right)^2 + (37.81)^2} = 43.66 \text{ MPa}$$



$$\sigma_a = \sigma_c + R \quad \sigma_{max} = 65.5 \text{ MPa}$$

$$\sigma_b = \sigma_c - R \quad \sigma_{min} = -21.8 \text{ MPa}$$

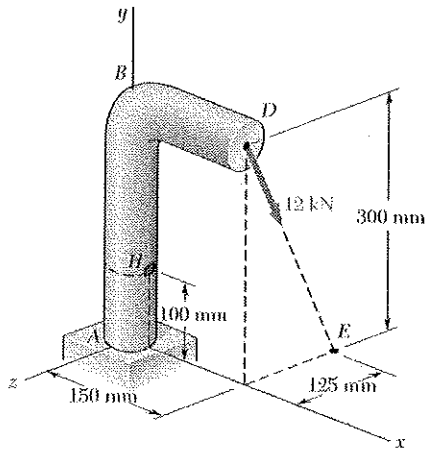
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{75.62}{43.66} = 1.7320$$

$$\theta_a = 30^\circ, \quad \theta_b = 120^\circ$$

$$\tau_{max} = R \quad \tau_{max} = 43.7 \text{ MPa}$$

Problem 8.73

8.73 A 12-kN force is applied as shown to the 60-mm-diameter cast-iron post ABD. At point H, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



$$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$$

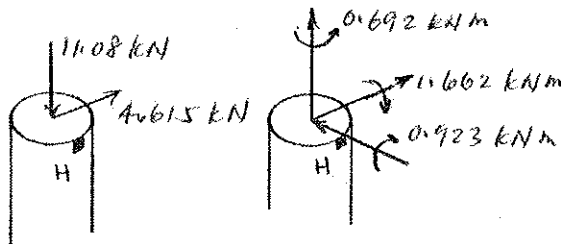
At point D $F_x = 0$

$$F_y = -\left(\frac{300}{325}\right)(12) = -11.08 \text{ kN}$$

$$F_z = -\left(\frac{125}{325}\right)(12) = -4.615 \text{ kN}$$

Moment of equivalent force-couple system at C, the centroid of the section containing point H

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.15 & 0.2 & 0 \\ 0 & -11.08 & -4.615 \end{vmatrix} = 0.923 \vec{i} + 0.692 \vec{j} - 1.662 \vec{k} \text{ kN}\cdot\text{m}$$



Section properties

$$d = 60 \text{ mm} \quad c = \frac{1}{2}d = 30 \text{ mm}$$

$$A = \pi c^2 = 2827 \text{ mm}^2$$

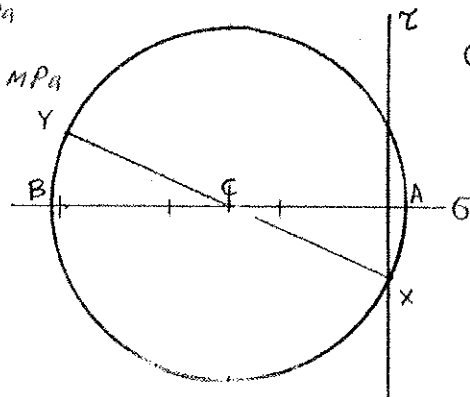
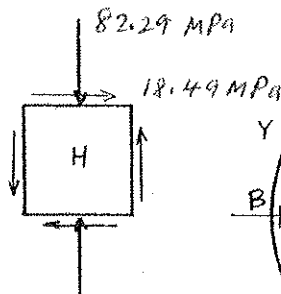
$$I = \frac{\pi}{4} c^4 = 0.6362 \times 10^6 \text{ mm}^4$$

$$J = 2I = 1.2724 \times 10^6 \text{ mm}^4$$

For a semicircle $Q = \frac{2}{3} c^3 = 18 \times 10^3 \text{ mm}^3$

$$\text{At point H } \sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{(11.08 \times 10^3)}{(2827)} - \frac{(1.662 \times 10^6)(30)}{(0.6362 \times 10^6)} = -82.29 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.692 \times 10^6)(30)}{1.2724 \times 10^6} + \frac{(4.615 \times 10^3)(18 \times 10^3)}{(0.6362 \times 10^6)(60)} = 18.49 \text{ MPa}$$



$$(a) \sigma_c = \frac{\sigma_H}{2} = -41.145 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 45.109 \text{ MPa}$$

$$\sigma_a = \sigma_c + R = 4 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -86.3 \text{ MPa}$$

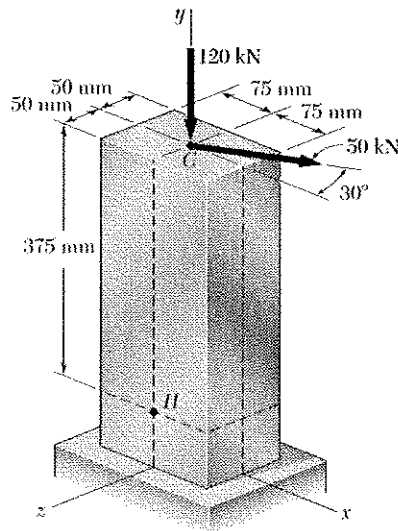
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = 0.4494$$

$$\theta_a = 12.1^\circ, \theta_b = 102.1^\circ$$

$$(b) \tau_{max} = R = 45.1 \text{ MPa}$$

Problem 8.74

8.74 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point H.



Components of force at point C.

$$F_x = 50 \cos 30^\circ = 43.301 \text{ kN}$$

$$F_z = -50 \sin 30^\circ = -25 \text{ kN}, \quad F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points H and K.

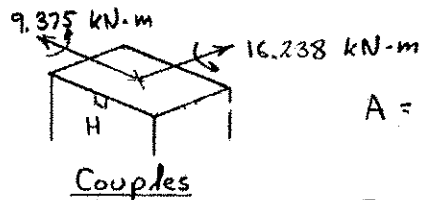
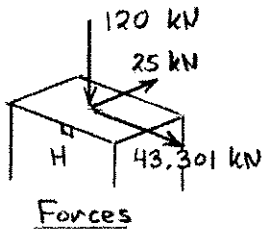
$$P = 120 \text{ kN (compression)}$$

$$V_x = 43.301 \text{ kN}, \quad V_z = -25 \text{ kN}$$

$$M_x = -(25)(0.375) = -9.375 \text{ kN}\cdot\text{m}$$

$$M_y = 0$$

$$M_z = -(43.301)(0.375) = -16.238 \text{ kN}\cdot\text{m}$$



$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 = 15 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12} (150)(100)^3 = 12.5 \times 10^6 \text{ mm}^4 = 12.5 \times 10^{-6} \text{ m}^4$$

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120 \times 10^3)}{15 \times 10^{-3}} - \frac{(-9.375 \times 10^3)(50 \times 10^{-3})}{12.5 \times 10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2} \frac{V_x}{A} = \frac{3}{2} \frac{43.301 \times 10^3}{15 \times 10^{-3}} = 4.33 \text{ MPa}$$

Use Mohr's circle.

$$\sigma_c = \frac{1}{2} \sigma_H = 14.75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37 \text{ MPa}$$

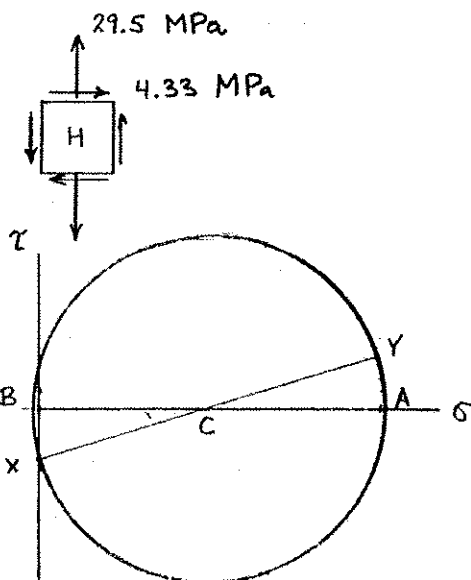
$$\sigma_a = \sigma_c + R = \sigma_a = 30.1 \text{ MPa} \rightarrow$$

$$\sigma_b = \sigma_c - R = \sigma_b = -0.62 \text{ MPa} \rightarrow$$

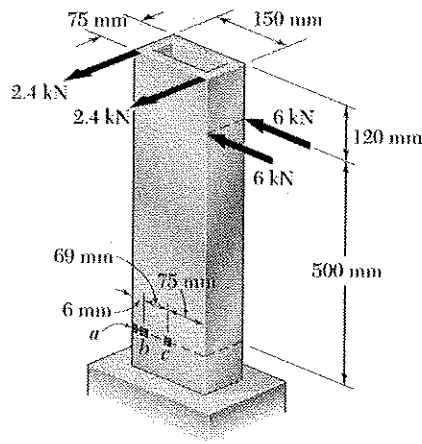
$$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$$

$$\theta_a = -8.2^\circ \quad \theta_b = 81.8^\circ \rightarrow$$

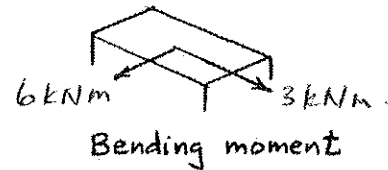
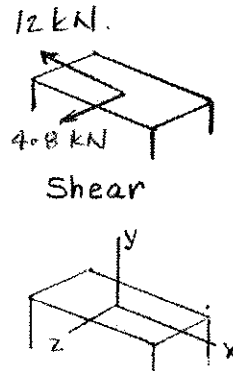
$$\tau_{max} = R \quad \tau_{max} = 15.37 \text{ MPa} \rightarrow$$



Problem 8.75



8.75 Knowing that the structural tube shown has a uniform wall thickness of 6 mm, determine the normal and shearing stresses at the three points indicated.



$$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 2.4 \times 10^6 \text{ mm}^4$$

$$I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 7.3 \times 10^6 \text{ mm}^4$$

Normal stresses

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

$$\frac{(6 \times 10^6)(-75)}{(7.3 \times 10^6)} - \frac{(3 \times 10^6)(37.5)}{2.4 \times 10^6} = -108.5 \text{ MPa}$$

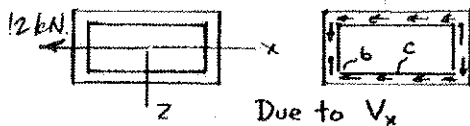
$$\frac{(6 \times 10^6)(69)}{(7.3 \times 10^6)} - \frac{(3 \times 10^6)(37.5)}{2.4 \times 10^6} = -103.6 \text{ MPa}$$

$$\frac{(6 \times 10^6)(0)}{(7.3 \times 10^6)} - \frac{(3 \times 10^6)(37.5)}{2.4 \times 10^6} = -46.9 \text{ MPa}$$

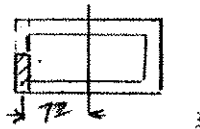
Shearing stresses

Point a is an outside corner; $\tau_a = 0$

Direction of shearing stresses



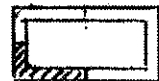
At point b



$$Q_{zb} = (37.5)(6)(72) = 16200 \text{ mm}^3$$

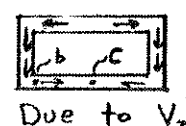
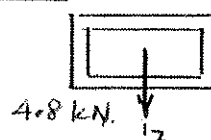
$$\tau_{b, V_x} = \frac{V_x Q_z}{I_z t} = \frac{(12000)(16200)}{(7.3 \times 10^6)(6)} = 4.44 \text{ MPa}$$

At point c



$$Q_{zc} = Q_{zb} + (69)(6)(34.5) = 30483 \text{ mm}^3$$

$$\tau_{c, V_x} = \frac{V_x Q_z}{I_z t} = \frac{(12000)(30483)}{(7.3 \times 10^6)(6)} = 8.4 \text{ MPa}$$



At point b



$$Q_{xb} = (69)(6)(34.5) = 14283 \text{ mm}^3$$

$$\tau_{b, V_z} = \frac{V_z Q_x}{I_x t} = \frac{(4800)(14283)}{(2.4 \times 10^6)(6)} = 4.76 \text{ MPa}$$

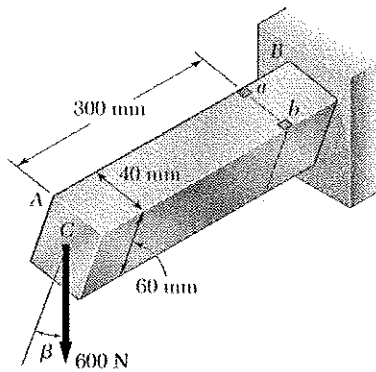
At point c (symmetry axis)
 $\tau_{c, V_z} = 0$

Net shearing stress at points b and c

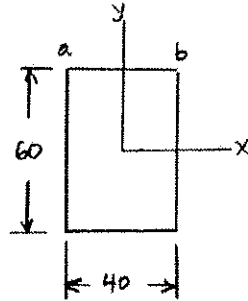
$$\tau_b = 4.76 - 4.44 = 0.32 \text{ MPa}$$

$$\tau_c = 8.4 \text{ MPa}$$

Problem 8.76

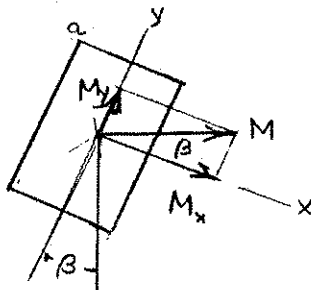


8.76 The cantilever beam AB will be installed so that the 60-mm side forms an angle β between 0 and 90° with the vertical. Knowing that the 600-N vertical force is applied at the center of the free end of the beam, determine the normal stress at point a when (a) $\beta = 0$, (b) $\beta = 90^\circ$. (c) Also, determine the value of β for which the normal stress at point a is a maximum and the corresponding value of that stress.



$$S_x = \frac{1}{6}(40)(60)^2 = 24 \times 10^3 \text{ mm}^3 \\ = 24 \times 10^{-6} \text{ m}^3$$

$$S_y = \frac{1}{6}(60)(40)^2 = 16 \times 10^3 \text{ mm}^3 \\ = 16 \times 10^{-6} \text{ m}^3$$



$$M = Pl = (600)(300 \times 10^{-3}) = 180 \text{ N} \cdot \text{m}$$

$$M_x = M \cos \beta = 180 \cos \beta$$

$$M_y = M \sin \beta = 180 \sin \beta$$

$$\sigma_a = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{180 \cos \beta}{24 \times 10^{-6}} + \frac{180 \sin \beta}{16 \times 10^{-6}} \\ = (7.5 \times 10^6)(\cos \beta + \frac{3}{2} \sin \beta) \text{ Pa} \\ = 7.5 (\cos \beta + \frac{3}{2} \sin \beta) \text{ MPa}$$

(a) $\beta = 0$ $\sigma_a = 7.50 \text{ MPa}$ $\sigma = 7.50 \text{ MPa} \leftarrow$

(b) $\beta = 90^\circ$ $\sigma_a = 11.25 \text{ MPa}$ $\sigma = 11.25 \text{ MPa} \leftarrow$

(c) $\frac{d\sigma_a}{d\beta} = 7.5 (-\sin \beta + \frac{3}{2} \cos \beta) = 0$

$$\sin \beta = \frac{3}{2} \cos \beta \quad \tan \beta = \frac{3}{2}$$

$$\beta = 56.3^\circ \leftarrow$$

$$\sigma_a = 7.5 (\cos 56.3^\circ + \frac{3}{2} \sin 56.3^\circ)$$

$$\sigma = 13.52 \text{ MPa} \leftarrow$$

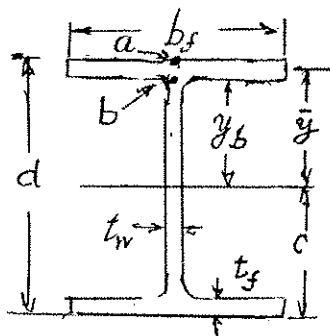
PROBLEM 8.C1

8.C1 Let us assume that the shear V and the bending moment M have been determined in a given section of a rolled-steel beam. Write a computer program to calculate in that section, from the data available in Appendix C, (a) the maximum normal stress σ_m , (b) the principal stress σ_{max} at the junction of a flange and the web. Use this program to solve parts a and b of the following problems:

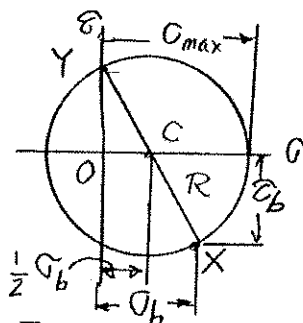
- (1) Prob. 8.1 (Use $V = 180 \text{ kN}$ and $M = 45 \text{ kN} \cdot \text{m}$)
- (2) Prob. 8.2 (Use $V = 90 \text{ kN}$ and $M = 45 \text{ kN} \cdot \text{m}$)
- (3) Prob. 8.3 (Use $V = 700 \text{ kN}$ and $M = 1750 \text{ kN} \cdot \text{m}$)
- (4) Prob. 8.4 (Use $V = 850 \text{ kN}$ and $M = 1700 \text{ kN} \cdot \text{m}$)

SOLUTION

We enter the given values of V and M obtain from Appendix C the values of d , b_f , t_f , t_w , I , and S for the given WF shape.



We compute $c = d/2$, $y_b = c - t_f$
 $\bar{y} = c - \frac{1}{2}t_f$, $\sigma_a = M/S$, $\sigma_b = \sigma_a(y_b/c)$
 $Q = b_f t_f \bar{y}$, $\tau_b = \frac{VQ}{It_w}$



From Mohr's circle:

$$\sigma_{max} = \frac{1}{2} \sigma_b + R$$

$$\sigma_{max} = \frac{1}{2} \sigma_b + \sqrt{\left(\frac{1}{2} \sigma_b\right)^2 + \tau_b^2}$$

PROGRAM OUTPUTS

Problem 8.1

Given Data:

$$V = 180 \text{ kN}, M = 45 \text{ kNm}$$

$$d = 264 \text{ mm}, b_f = 257 \text{ mm}$$

$$t_f = 19.6 \text{ mm}, t_w = 11.9 \text{ mm}$$

$$I = 164 \times 10^6 \text{ mm}^4, S = 1240 \times 10^3 \text{ mm}^3$$

Answers:

$$(a) \text{ SIGA} = 36.29 \text{ MPa}$$

$$(b) \text{ SIGM} = 74.29 \text{ MPa}$$

Problem 8.2

Given Data:

$$V = 90 \text{ kN}, M = 45 \text{ kNm}$$

$$d = 264 \text{ mm}, b_f = 257 \text{ mm}$$

$$t_f = 19.6 \text{ mm}, t_w = 11.9 \text{ mm}$$

$$I = 164 \times 10^6 \text{ mm}^4, S = 1240 \times 10^3 \text{ mm}^3$$

Answers:

$$(a) \text{ SIGA} = 36.29 \text{ MPa}$$

$$(b) \text{ SIGM} = 31.44 \text{ MPa}$$

Problem 8.3

Given Data:

$$V = 700 \text{ kN}, M = 1750 \text{ kNm}$$

$$d = 930 \text{ mm}, b_f = 423 \text{ mm}$$

$$t_f = 43 \text{ mm}, t_w = 24 \text{ mm}$$

$$I = 8470 (10^6 \text{ mm}^4)$$

$$S = 18200 (10^3 \text{ mm}^3)$$

Answers:

$$(a) \text{ SIGA} = 96.15 \text{ MPa}$$

$$(b) \text{ SIGM} = 95.39 \text{ MPa}$$

Problem 8.4

Given Data:

$$V = 850 \text{ kN}, M = 1700 \text{ kNm}$$

$$d = 930 \text{ mm}, b_f = 423 \text{ mm}$$

$$t_f = 43 \text{ mm}, t_w = 24 \text{ mm}$$

$$I = 8470 (10^6 \text{ mm}^4)$$

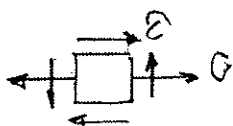
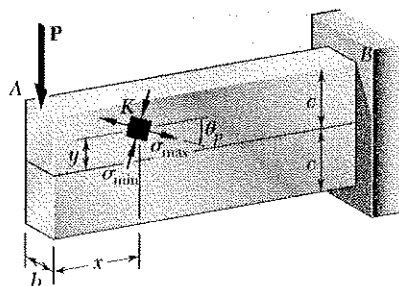
$$S = 18200 (10^3 \text{ mm}^3)$$

Answers:

$$(a) \text{ SIGA} = 93.40 \text{ MPa}$$

$$(b) \text{ SIGM} = 96.56 \text{ MPa}$$

PROBLEM 8.C2



8.C2 A cantilever beam AB with a rectangular cross section of width b and depth $2c$ supports a single concentrated load P at its end A . Write a computer program to calculate, for any values of x/c and y/c , (a) the ratios $\sigma_{\max}/\sigma_m$ and $\sigma_{\min}/\sigma_m$, where $\sigma_{\max}$ and $\sigma_{\min}$ are the principal stresses at point $K(x, y)$ and σ_m the maximum normal stress in the same transverse section, (b) the angle θ_p that the principal planes at K form with a transverse and a horizontal plane through K . Use this program to check the values shown in Fig. 8.8 and to verify that $\sigma_{\max}$ exceeds σ_m if $x \leq 0.544c$, as indicated in the second footnote on page 499.

SOLUTION

Since the distribution of the normal stresses is linear, we have $\sigma = \sigma_m (y/c)$ (1)

$$\text{where } \sigma_m = \frac{Mc}{I} = \frac{Px c}{I} \quad (2)$$

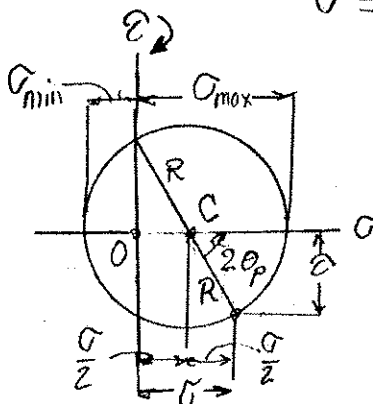
$$\text{We use Eq. (8.4), page 498: } \sigma = \frac{3}{2} \frac{P}{A} \left(1 - \frac{y^2}{c^2}\right) \quad (3)$$

$$\text{Dividing (3) by (2): } \frac{\sigma}{\sigma_m} = \frac{3}{2} \frac{I}{A} \frac{1 - (y/c)^2}{xc}$$

$$\text{or, since } \frac{I}{A} = \frac{\frac{1}{12} b (2c)^3}{b (2c)} = \frac{1}{3} c^2; \quad \frac{\sigma}{\sigma_m} = \frac{1}{2} \frac{1 - (y/c)^2}{x/c} \quad (4)$$

Letting $X = x/c$ and $Y = y/c$, Eqs. (1) and (4) yield

$$\sigma = \sigma_m Y \quad \sigma = \sigma_m \frac{1 - Y^2}{2X}$$



Using Mohr's circle, we calculate

$$R = \sqrt{\left(\frac{1}{2} \sigma\right)^2 + \tau^2}$$

$$= \frac{1}{2} \sigma_m \sqrt{Y^2 + \left(\frac{1 - Y^2}{X}\right)^2}$$

$$\frac{\sigma_{\max}}{\sigma_m} = \frac{1}{2} Y + R \quad \frac{\sigma_{\min}}{\sigma_m} = \frac{1}{2} Y - R$$

$$\tan 2\theta_p = \frac{\tau}{\sigma/2} = \frac{1 - Y^2}{2X(Y/2)} = \frac{1 - Y^2}{XY} \quad \theta_p = \frac{1}{2} \tan^{-1} \left(\frac{1 - Y^2}{XY} \right)$$

NOTE

For $y > 0$, the angle θ_p is $\searrow$, which is opposite to what was arbitrarily assumed in Fig. P8.C2.

(CONTINUED)

PROBLEM 8.C2 CONTINUED

PROGRAM OUTPUTS

For $x/c = 2$:

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta °
1.0	0.000	1.000	0.00
0.8	-0.010	0.810	6.34
0.6	-0.040	0.640	14.04
0.4	-0.090	0.490	23.20
0.2	-0.160	0.360	33.69
0.0	-0.250	0.250	45.00
-0.2	-0.360	0.160	-33.69
-0.4	-0.490	0.090	-23.20
-0.6	-0.640	0.040	-14.04
-0.8	-0.810	0.010	-6.34
-1.0	-1.000	0.000	-0.00

For $x/c = 8$:

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta °
1.0	0.000	1.000	0.00
0.8	-0.001	0.801	1.61
0.6	-0.003	0.603	3.80
0.4	-0.007	0.407	7.35
0.2	-0.017	0.217	15.48
0.0	-0.062	0.063	45.00
-0.2	-0.217	0.017	-15.48
-0.4	-0.407	0.007	-7.35
-0.6	-0.603	0.003	-3.80
-0.8	-0.801	0.001	-1.61
-1.0	-1.000	0.000	-0.00

To check that $\sigma_{max} > \sigma_m$ if $x \leq 0.544c$, we run the program for $x/c = 0.544$ and for $x/c = 0.545$ and observe that σ_{max}/σ_m exceeds 1 for several values of y/c in the first case, but does not exceed 1 in the second case.

For $x/c = 0.544$:

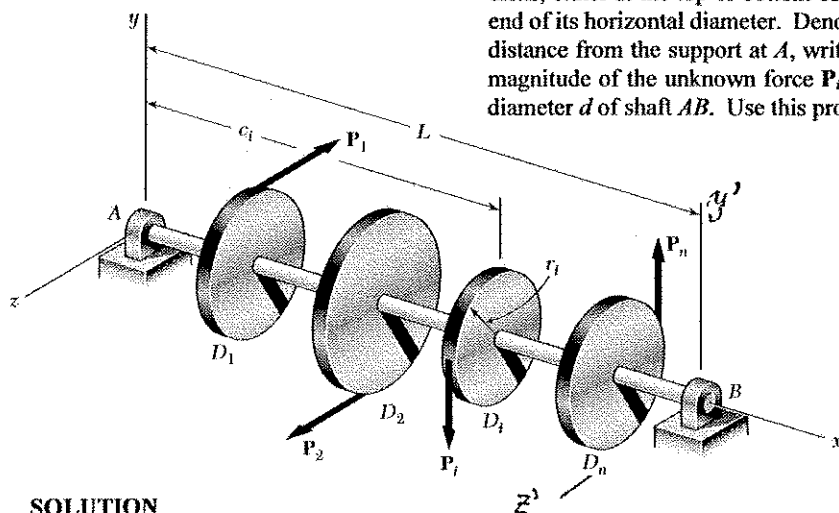
y/c	Sigmin/Sigm	Sigmax/Sigm	Theta °
0.30	-0.700	0.9997	39.92
0.31	-0.690	1.0001	39.72
0.32	-0.680	1.0004	39.51
0.33	-0.670	1.0005	39.30
0.34	-0.660	1.0005	39.09
0.35	-0.650	1.0003	38.88
0.36	-0.640	1.0000	38.66
0.37	-0.630	0.9996	38.44
0.38	-0.619	0.9990	38.21
0.39	-0.608	0.9983	37.98
0.40	-0.598	0.9975	37.74

For $x/c = 0.545$:

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta °
0.30	-0.698	0.9982	39.91
0.31	-0.689	0.9986	39.71
0.32	-0.679	0.9989	39.50
0.33	-0.669	0.9990	39.29
0.34	-0.659	0.9990	39.08
0.35	-0.649	0.9988	38.87
0.36	-0.639	0.9986	38.65
0.37	-0.628	0.9982	38.42
0.38	-0.618	0.9976	38.20
0.39	-0.607	0.9970	37.96
0.40	-0.596	0.9962	37.73

PROBLEM 8.C3

8.C3 Disks $D_1, D_2, \dots, D_n$ are attached as shown in Fig. 8.C3 to the solid shaft AB of length L , uniform diameter d , and allowable shearing stress τ_{all} . Forces $P_1, P_2, \dots, P_n$ of known magnitude (except for one of them) are applied to the disks, either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. Denoting by r_i the radius of disk D_i and by c_i its distance from the support at A , write a computer program to calculate (a) the magnitude of the unknown force P_i , (b) the smallest permissible value of the diameter d of shaft AB . Use this program to solve Prob. 8.19.



SOLUTION

1. Determine the unknown force P_i by equating to zero the sum of their torques T_i about the x axis.

2. Determine the components $(F_y)_i$ and $(F_z)_i$ of all forces.

3. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:

$$\sum M_{z'} = 0: -A_y L - \sum (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L} \sum (F_y)_i (L - c_i)$$

$$\sum M_{y'} = 0: A_z L + \sum (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L} \sum (F_z)_i (L - c_i)$$

4. Determine $(M_y)_i, (M_z)_i$, and torque T_i just to the left of disk D_i :

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle'$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle'$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

Where $\langle \rangle$ indicates a singularity function.

5. The minimum diameter d required to the left of D_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{\text{all}}}$$

(CONTINUED)

Problem 8.19

Length of shaft = 720 mm

TAU (ksi) = 55 MPa

For Disk 1

Force = 2 kN

Radius of disk = 100 mm

Distance from A = 180 mm

For Disk 2

Force = 0.000 kN

Radius of disk = 150 mm

Distance from A = 360 mm

For Disk 3

Force = 2 kN

Radius of disk = 100 mm

Distance from A = 540 mm

Unknown force = -2.67 kN

AY = 2 kN, AZ = 1.35 kN

BY = 2 kN, BZ = 1.35 kN

Just to the left of Disk 1

MY = 0.24 kNm

MZ = -0.36 kNm

T = 0.0000 kNm

Diameter must be at least 34.15 mm

Just to the right of Disk 1

T = 0.2 kNm

Diameter must be at least 35.33 mm

Just to the left of Disk 2

MY = 0.48 kNm

MZ = -0.36 kNm

T = 0.2 kNm

Diameter must be at least 38.8 mm 

Just to the right of Disk 2

T = -0.2 kNm

Diameter must be at least 30.8 mm

Just to the left of Disk 3

MY = 0.24 kNm

MZ = -0.36 kNm

T = -0.2 kNm

Diameter must be at least 35.33 mm

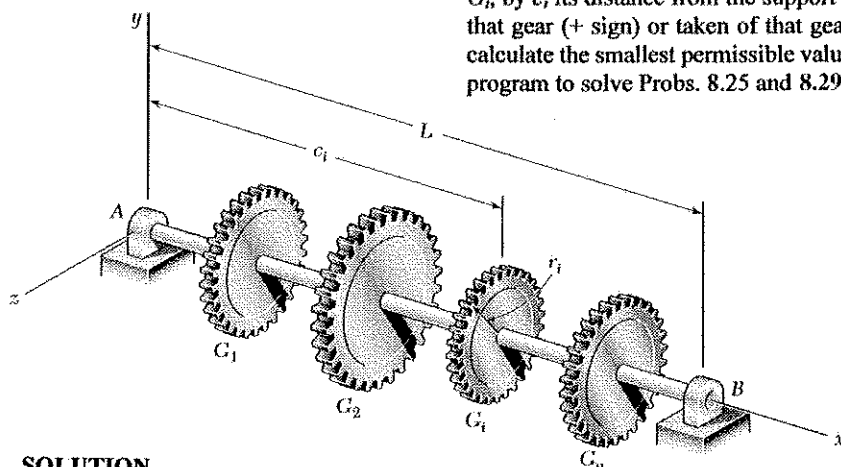
Just to the right of Disk 3

T = 0.00 kNm

Diameter must be at least 34.15 mm

PROBLEM 8.C4

8.C4 The solid shaft AB of length L , uniform diameter d , and allowable shearing stress τ_{all} rotates at a given speed expressed in rpm (Fig. 8.C4). Gears $G_1, G_2, \dots, G_n$ are attached to the shaft and each of these gears meshes with another gear (not shown), either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. One of these gears is connected to a motor and the rest of them to various machine tools. Denoting by r_i the radius of disk G_i , by c_i its distance from the support at A , and by P_i the power transmitted to that gear (+ sign) or taken of that gear (- sign), write a computer program to calculate the smallest permissible value of the diameter d of shaft AB . Use this program to solve Probs. 8.25 and 8.29.



SOLUTION

1. Enter ω in rpm and determine frequency $f = \omega/60$.
2. For each gear, determine the torque $T_i = P_i / 2\pi f$, where P_i is the power input (+) or output (-) at the gear.
3. For each gear, determine the force $F_i = T_i / r_i$ exerted on the gear and its components $(F_y)_i$ and $(F_z)_i$.
4. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:

$$\sum M_{z'} = 0: -A_y L - \sum (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L} \sum (F_y)_i (L - c_i)$$

$$\sum M_{y'} = 0: A_z L + \sum (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L} \sum (F_z)_i (L - c_i)$$
5. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of gear G_i :

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle'$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle'$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

Where $\langle \rangle$ indicates a singularity function.

(CONTINUED)

PROBLEM 8.C4 CONTINUED

6. The minimum diameter d required to the left of G_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

7. Recalling that $J = \frac{1}{2} \pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2} \pi c_i^3$, we have $c_i = \frac{2}{\pi} \left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi} \left(\frac{J}{c}\right)_i^{1/3}$ ◀

This is the required diameter just to the left of gear G_i .

8. The required diameter just to the right of gear G_i is obtained by replacing T_i with T_{i+1} in the above computation.

9. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for d_i .

(CONTINUED)

PROBLEM 8.C4 CONTINUED

PROGRAM OUTPUTS

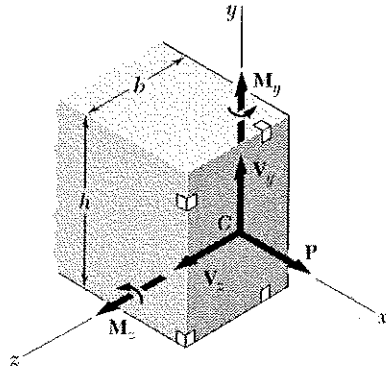
Problem 8.25

Omega = 600 rpm
 Number of Gears: 3
 Length of shaft = 600 mm
 Tau = 55 MPa
 For Gear 1
 Power input = 45 kW
 Radius of gear = 75 mm
 Distance from A in mm = 100 mm
 FY = 0
 FZ = 9.55 kN
 For Gear 2
 Power input = -30 kW
 Radius of gear = 100 mm
 Distance from A in inches = 250 mm
 FY = 1.70 kN
 FZ = 0
 For Gear 3
 Power input = -15 kW
 Radius of gear = 100 mm
 Distance from A in inches = 450 mm
 FY = 0
 FZ = -2.39 kN
 AY = -2.79 kN, AZ = -7.36 kN
 BY = -2 kN, BZ = 0.2 kN
 Just to the left of Gear 1
 MY = -0.736 kNm
 MZ = 0.03 kNm
 T = 0.000 kNm
 Diameter must be at least 41.8 mm
 Just to the right of Gear 1
 T = 716.2 kNm
 Diameter must be at least 46.2 mm
 Just to the left of Gear 2
 MY = -0.4075 kNm
 MZ = 0.701 kNm
 T = 716.25 kNm
 Diameter must be at least 46.4 mm
 Just to the right of Gear 2
 T = 238.7 kNm
 Diameter must be at least 42.7 mm
 Just to the left of Gear 3
 MY = 0.0299 kNm
 MZ = 0.298 kNm
 T = 238.72 kNm
 Diameter must be at least 32.9 mm
 Just to the right of Gear 3
 T = 0.0000 kNm
 Diameter must be at least 30.3 mm

Problem 8.29

Omega = 450 rpm
 Number of Gears: 3
 Length of shaft = 750 mm
 Tau = 55 MPa
 For Gear 1
 Power input = -8.00 kW
 Radius of gear = 60 mm
 Distance from A in mm = 150
 For Gear 2
 Power input = 20.00 kW
 Radius of gear = 100 mm
 Distance from A in mm = 375
 For Gear 3
 Power input = -12.00 kW
 Radius of gear = 60 mm
 Distance from A in mm = 600
 AY = -0.849 kN, AZ = 4.386
 BY = -3.395 kN, BZ = 2.688
 Just to the left of Gear 1
 MY = 657.84 Nm
 MZ = 127.32 Nm
 T = 0.00 Nm
 Diameter must be at least 39.59 mm
 Just to the right of Gear 1
 T = -169.77 Nm
 Diameter must be at least 40.00 mm
 Just to the left of Gear 2
 MY = 1007.98 Nm
 MZ = 318.31 Nm
 T = -169.77 Nm
 Diameter must be at least 46.28 mm
 Just to the right of Gear 2
 T = 254.65 Nm
 Diameter must be at least 46.52 mm
 Just to the left of Gear 3
 MY = 403.19 Nm
 MZ = 509.30 Nm
 T = 254.65 Nm
 Diameter must be at least 40.13 mm
 Just to the right of Gear 3
 T = 0.00 Nm
 Diameter must be at least 39.18 mm

PROBLEM 8.C5



8.C5 Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates y and z located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (a) Prob. 8.45b, (b) Prob. 8.47a.

SOLUTION

ENTER: b and h

PROGRAM: $A = bh$ $I_y = b^3 h / 12$ $I_z = b h^3 / 12$

FOR POINT ON SURFACE, ENTER y AND z

NOTE y AND z MUST SATISFY ONE OF FOLLOWING:

$$y^2 = h^2/4 \text{ AND } z^2 \leq b^2/4 \quad (1)$$

$$\text{OR } z^2 = b^2/4 \text{ AND } y^2 \leq h^2/4 \quad (2)$$

IF EITHER (1) OR (2) ARE SATISFIED, COMPUTE

$$\sigma = \frac{P}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

IF $z^2 = b^2/4$, THEN POINT IS ON VERTICAL SURFACE AND

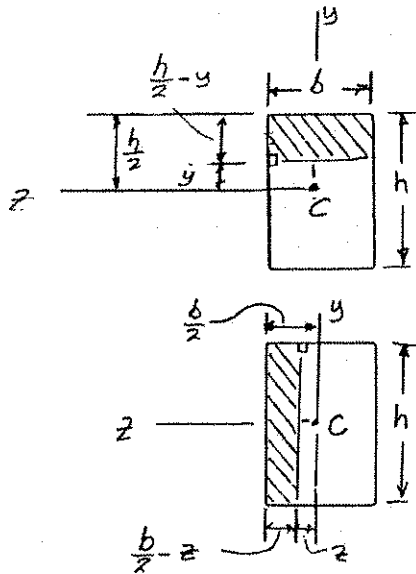
$$Q_z = b \left(\frac{h}{2} - y \right) \left(\frac{h}{2} + y \right) \frac{1}{2} = b \left(\frac{h^2}{8} - \frac{y^2}{2} \right)$$

$$\tau = \frac{V_y Q_z}{I_z b}$$

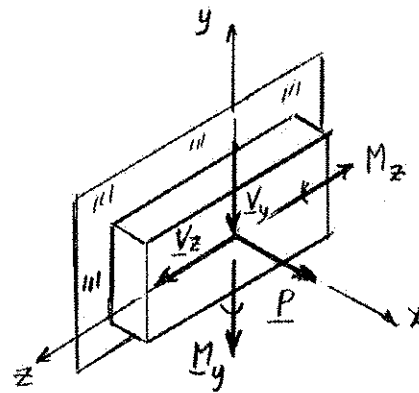
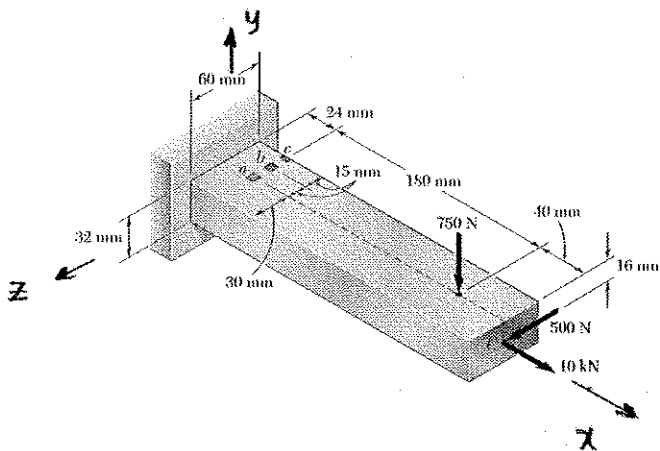
IF $y^2 = h^2/4$, THE POINT IS ON HORIZONTAL SURFACE, AND

$$Q_y = h \left(\frac{b}{2} - z \right) \left(\frac{b}{2} + z \right) \frac{1}{2} = h \left(\frac{b^2}{8} - \frac{z^2}{2} \right)$$

$$\tau = \frac{V_z Q_y}{I_y h}$$



PROBLEM 8.C5 CONTINUED



FORCE-COUPLE SYSTEM

$$P = 10 \text{ kN} \quad V_y = -750 \text{ N} \quad V_z = 500 \text{ N}$$

$$M_y = (500 \text{ N})(220 \text{ mm}) = 110 \text{ N}\cdot\text{m} \quad M_z = (750 \text{ N})(180 \text{ mm}) = 135 \text{ N}\cdot\text{m}$$

Problem 8.45 a

Force-Couple at Centroid

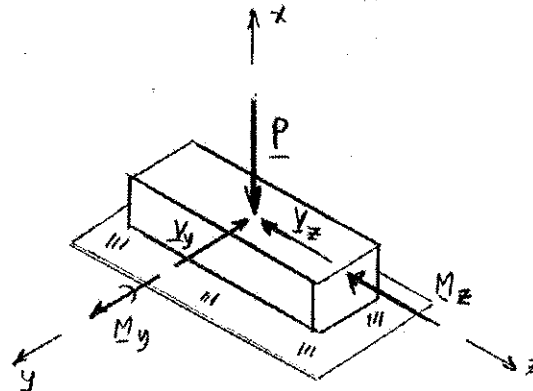
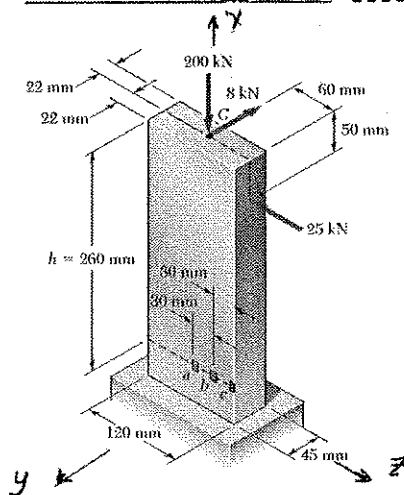
$$\begin{aligned} P &= 10000.00 \text{ N} \\ M_y &= 110.00 \text{ N}\cdot\text{m} & M_z &= -135.00 \text{ N}\cdot\text{m} \\ V_y &= 750.00 \text{ N} & V_z &= 500.00 \text{ N} \end{aligned}$$

+++++

At point of coordinates: $y = 16.00 \text{ mm} \quad z = 0.00 \text{ mm}$

$\sigma = 18.392 \text{ MPa}$

$\tau = 0.391 \text{ MPa}$



FORCE-COUPLE SYSTEM

$$P = -200 \text{ kN} \quad V_z = -25 \text{ kN} \quad V_y = -8 \text{ kN}$$

$$M_y = (25 \text{ kN})(0.252 \text{ m}) = 5.25 \text{ kN}\cdot\text{m}$$

$$M_z = (8 \text{ kN})(0.26 \text{ m}) = 2.08 \text{ kN}\cdot\text{m}$$

Problem 8.47 b

Force-Couple at Centroid

$$\begin{aligned} P &= -200 \text{ kN} \\ M_y &= 5.25 \text{ kN}\cdot\text{m} & M_z &= -2.08 \text{ kN}\cdot\text{m} \\ V_y &= -8 \text{ kN} & V_z &= -25 \text{ kN} \end{aligned}$$

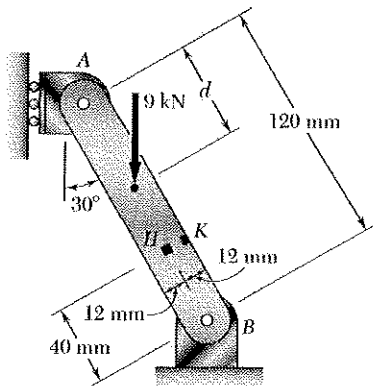
+++++

At point of coordinates: $y = 45 \text{ mm} \quad z = 30 \text{ mm}$

$\sigma = 40.7 \text{ MPa}$

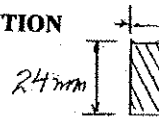
$\tau = -5.3 \text{ MPa}$

PROBLEM 8.C6



8.C6 Member AB has a rectangular cross section of 10 × 24 mm. For the loading shown, write a computer program that can be used to determine the normal and shearing stresses at points H and K for values of d from 0 to 120 mm, using 15-mm increments. Use this program to solve Prob. 8.35.

SOLUTION



CROSS SECTION

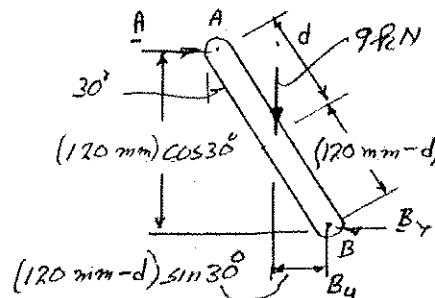
ENTER

$$A = (0.010 \text{ m})(0.024 \text{ m}) = 240 \times 10^{-6} \text{ m}^2$$

$$I = (0.010 \text{ m})(0.024 \text{ m})^3 / 12 = 138.24 \times 10^{-9} \text{ m}^4$$

$$c = 0.5(0.024 \text{ m}) = 12 \times 10^{-3} \text{ m}$$

COMPUTE REACTION AT A.

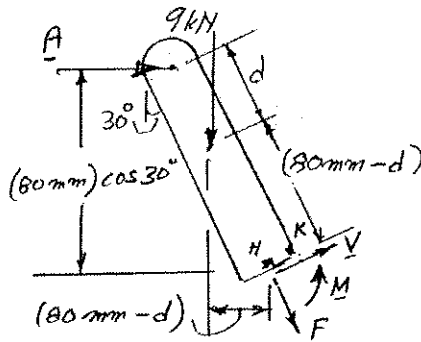


$$+\uparrow \Sigma M_B = 0:$$

$$(9 \text{ kN})(120 - d) \sin 30^\circ - A(120) \cos 30^\circ = 0$$

$$A = (9 \text{ kN}) \frac{(120 \text{ mm} - d)}{120 \text{ mm}} \tan 30^\circ$$

FREE BODY FROM A TO SECTION CONTAINING POINTS H AND K.



$$\text{DEFINE: IF } d < 80 \text{ mm THEN STP} = 1 \text{ ELSE STP} = 0$$

PROGRAM FORCE-COUPLE SYSTEM

$$F = -A \sin 30^\circ - (9 \text{ kN}) \cos 30^\circ (\text{STP})$$

$$V = -A \cos 30^\circ + (9 \text{ kN}) \sin 30^\circ (\text{STP})$$

$$M = A(60 \text{ mm}) \cos 30^\circ - (9 \text{ kN})(60 \text{ mm} - d) \sin 30^\circ (\text{STP})$$

AT POINT H:

$$\sigma_H = +F/A \quad \tau_H = \frac{3}{2} V/A$$

AT POINT K:

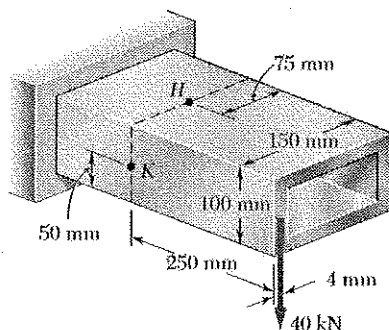
$$\sigma_K = +F/A - Mc/I \quad \tau_K = 0$$

PROGRAM OUTPUT

Problem 8.35

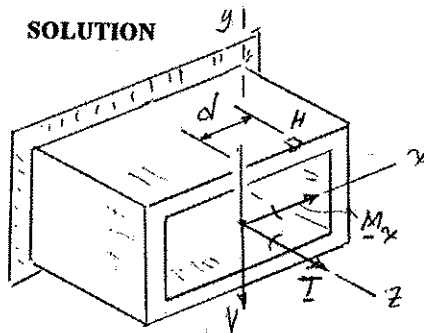
d mm	Stresses in MPa			
	SigmaH	TauH	SigmaK	TauK
0.0	-43.30	0.00	-43.30	0.00
15.0	-41.95	3.52	-65.39	0.00
30.0	-40.59	7.03	-87.47	0.00
45.0	-39.24	10.55	-109.55	0.00
60.0	-37.89	14.06	-131.64	0.00
75.0	-36.54	17.58	-153.72	0.00
90.0	-2.71	-7.03	-96.46	0.00
105.0	-1.35	-3.52	-48.23	0.00
120.0	0.00	0.00	0.00	0.00

PROBLEM 8.C7



***8.C7** The structural tube shown has a uniform wall thickness of 8 mm. A 40-kN force is applied to a bar (not shown) that is welded to the end of the tube. Write a computer program that can be used to determine, for any given value of c , the principal stresses, principal planes, and maximum shearing stress at point H for values of d from -75 mm to 75 mm, using 25-mm increments. Use this program to solve Prob. 8.62a.

SOLUTION



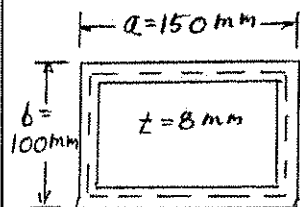
FORCE-COUPLE SYSTEM

ENTER:

$$V = 40 \text{ kN} \downarrow$$

$$M_x = (40 \text{ kN})(0.25 \text{ m}) = 10 \text{ kNm}$$

$$T = (40 \text{ kN})$$

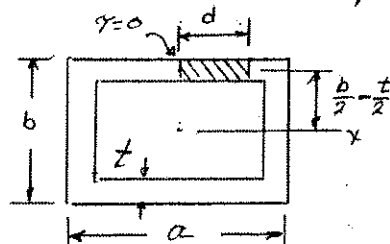
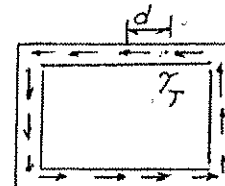


AREA ENCLOSED

$$A = (a-t)(b-t)$$

$$\tau = \frac{T}{A} = \frac{9c}{2tA}$$

τ_T = SHEARING STRESS DUE TO TORSION

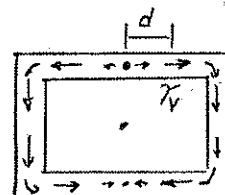


$$Q = dt \left(\frac{b}{2} - \frac{t}{2} \right)$$

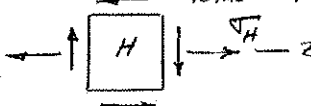
$$\bar{I} = ab^3/12 - (a-2t)(b-2t)^3/12$$

$$\tau_v = \frac{VQ}{I t}$$

τ_v = SHEARING STRESS DUE TO V



$$\tau_{TOTAL} = \tau_T + \tau_v$$



BENDING: $\sigma_H = \frac{M_x (b/2)}{I}$

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{1}{2} \sigma_H ; R = \sqrt{\left(\frac{\sigma_H}{2} \right)^2 + \tau_{TOTAL}^2}$$

$$\sigma_{max} = \sigma_{ave} + R ; \sigma_{min} = \sigma_{ave} - R ; \theta_p = \frac{1}{2} \tan^{-1} \left(\frac{\tau_{TOTAL}}{\sigma_{ave}} \right) ; \tau_{max} = \sqrt{\left(\frac{\sigma_H}{2} \right)^2 + \tau_{TOTAL}^2}$$

Rectangular tube of uniform thickness $t = 8 \text{ mm}$.

Outside dimensions

Horizontal width $a = 150 \text{ mm}$

Vertical depth $b = 100 \text{ mm}$.

Vertical load $P = 40 \text{ kN}$; line of action at $x = -c$

Find normal and shearing stresses at

Point H ($x = d$, $y = b/2$)

Problem 8.64a Program Output for Value of $c = 71 \text{ mm}$.

d mm	sigma MPa	tauV MPa	tauT MPa	tauTotal MPa	sigmaMax MPa	sigmaMin MPa	tauMax MPa	theta p degrees
-75	85.03	-23.36	-13.59	-36.96	98.08	-13.95	35.97	-123.78
-50	85.03	-15.60	-13.59	-29.12	93.32	-9.11	37.21	-107.11
-25	85.03	-7.77	-13.59	-21.36	89.31	-5.09	47.20	-85.56
0	85.03	0	-13.59	-13.59	87.12	-2.12	44.62	-8.87
25	85.03	7.77	-13.59	-5.76	84.55	-0.402	42.51	-26.04
50	85.03	15.60	-13.59	2.01	84.22	-0.067	42.18	9.11
75	85.03	23.36	-13.59	9.77	85.29	-1.14	43.25	43.25

Chapter 9

Problem 9.1

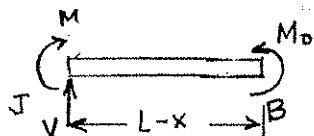


9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.

$$+\circlearrowleft \sum M_J = 0: \quad M_0 - M = 0 \quad M = M_0$$

$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



$$EI \frac{d^2 y}{dx^2} = M_0$$

$$EI \frac{dy}{dx} = M_0 x + C_1$$

$$EI y = \frac{1}{2} M_0 x^2 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 = 0 + C_1$$

$$C_1 = 0$$

$$EI y = \frac{1}{2} M_0 x^2 + C_2$$

$$[x=0, y=0]$$

$$0 = 0 + C_2$$

$$C_2 = 0$$

(a) Elastic curve.

$$y = \frac{M_0 x^2}{2EI}$$

$$\frac{dy}{dx} = \frac{M_0 x}{EI}$$

(b) y @ x = L.

$$y_B = \frac{M_0 L^2}{2EI}$$

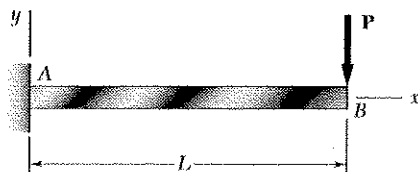
$$y_B = \frac{M_0 L^2}{2EI} \uparrow$$

(c) $\frac{dy}{dx}$ @ x = L.

$$\left. \frac{dy}{dx} \right|_B = \frac{M_0 L}{EI}$$

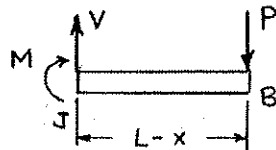
$$\theta_B = \frac{M_0 L}{EI} \nearrow$$

Problem 9.2



$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



$$[x=0, y=0]$$

(a) Elastic curve.

(b) y @ x = L.

(c) $\frac{dy}{dx}$ @ x = L.

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB; (b) the deflection at the free end, (c) the slope at the free end.

$$\sum M_J = 0: -M - P(L-x) = 0$$

$$M = -P(L-x)$$

$$EI \frac{d^2y}{dx^2} = -P(L-x) = -PL + Px$$

$$EI \frac{dy}{dx} = -PLx + \frac{1}{2}Px^2 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 = -0 + 0 + C_1$$

$$C_1 = 0$$

$$EI y = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_1x + C_2$$

$$0 = -0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$y = -\frac{Px^2}{6EI} (3L-x)$$

$$\frac{dy}{dx} = -\frac{Px}{2EI} (2L-x)$$

$$y_B = -\frac{PL^2}{6EI} (3L-L) = -\frac{PL^3}{3EI}$$

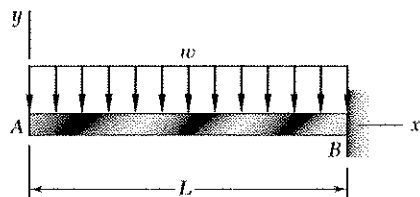
$$y_B = \frac{PL^3}{3EI} \downarrow$$

$$\left. \frac{dy}{dx} \right|_B = -\frac{PL}{2EI} (2L-L) = -\frac{PL^2}{2EI}$$

$$\theta_B = \frac{PL^2}{2EI} \swarrow$$

Problem 9.3

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.



$$\sum M_J = 0: (wx)\frac{x}{2} + M = 0$$

$$M = -\frac{1}{2}wx^2$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$$[x=L, \frac{dy}{dx}=0] \quad 0 = -\frac{1}{6}wL^3 + C_1$$

$$C_1 = \frac{1}{6}wL^3$$

$$EI \frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EI y = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$$[x=L, y=0]$$

$$0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$$

$$C_2 = (\frac{1}{24} - \frac{1}{6})wL^4 = -\frac{3}{24}wL^4$$

(a) Elastic curve.

$$y = -\frac{w}{24EI} (x^4 - 4L^3x + 3L^4)$$

(b) y @ x=0:

$$y_A = -\frac{3wL^4}{24EI} = -\frac{wL^4}{8EI}$$

$$y_A = \frac{wL^4}{8EI} \downarrow$$

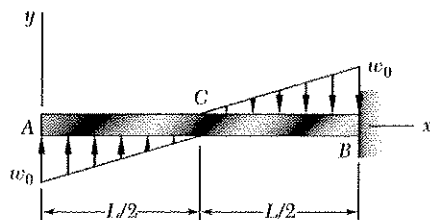
(c) $\frac{dy}{dx}$ @ x=0:

$$\left. \frac{dy}{dx} \right|_A = \frac{wL^3}{6EI}$$

$$\theta_A = \frac{wL^3}{6EI} \swarrow$$

Problem 9.4

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB , (b) the deflection at the free end, (c) the slope at the free end.



$$w(x) = \frac{2w_0}{L}x - w_0$$

$$\begin{aligned} V(x) &= -\int w(x) dx = -\int \left(\frac{2w_0}{L}x - w_0 \right) dx \\ &= -\frac{w_0}{L}x^2 + w_0x + C_1 \end{aligned}$$

$$[x=0, V=0] \quad 0 = 0 + 0 + C_1 \quad \therefore C_1 = 0$$

$$\begin{aligned} M(x) &= \int V(x) dx = \int \left(-\frac{w_0}{L}x^2 + w_0x \right) dx \\ &= -\frac{w_0}{3L}x^3 + \frac{w_0}{2}x^2 + C_2 \end{aligned}$$

$$[x=0, M=0] \quad 0 = 0 + 0 + C_2 \quad \therefore C_2 = 0$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{w_0}{3L}x^3 + \frac{w_0}{2}x^2$$

$$EI \frac{dy}{dx} = -\frac{w_0}{12L}x^4 + \frac{w_0}{6}x^3 + C_3$$

$$[x=L, \frac{dy}{dx}=0] \quad 0 = -\frac{w_0L^3}{12} + \frac{w_0L^3}{6} + C_3$$

$$\therefore C_3 = -\frac{w_0L^3}{12}$$

$$EI y = -\frac{w_0}{60L}x^5 + \frac{w_0}{24}x^4 - \frac{w_0L^3}{12}x + C_4$$

$$[x=L, y=0] \quad 0 = -\frac{w_0L^4}{60} + \frac{w_0L^4}{24} - \frac{w_0L^4}{12} + C_4$$

$$\therefore C_4 = \frac{7w_0L^4}{120}$$

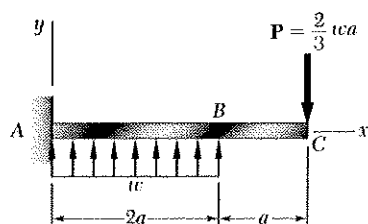
(a) ELASTIC CURVE: $y = -\frac{w_0}{120EI} (2x^5 - 5Lx^4 + 10L^4x - 7L^5)$

(b) y @ $x=0$: $y_A = +\frac{7w_0L^4}{120EI}$ $y_A = \frac{7w_0L^4}{120EI} \uparrow$

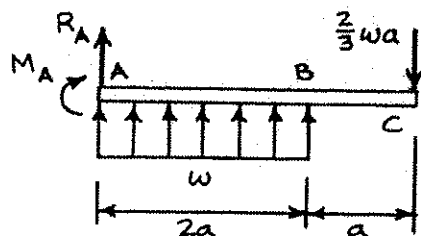
(c) $\frac{dy}{dx}$ @ $x=0$: $\frac{dy}{dx}\bigg|_A = -\frac{w_0L^3}{12EI}$ $\theta_A = \frac{w_0L^3}{12EI} \searrow$

Problem 9.5

9.5 and 9.6 For the cantilever beam and loading shown, determine (a) the equation of the elastic curve for portion AB of the beam, (b) the deflection at B, (c) the slope at B.



FBD ABC:



USING ABC AS A FREE BODY,

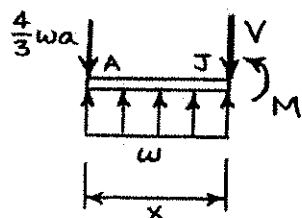
$$+\uparrow \Sigma F_y = 0: R_A + 2wa - \frac{2}{3}wa = 0$$

$$R_A = -\frac{4}{3}wa = \frac{4}{3}wa \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: -M_A + (2wa)(a) - \left(\frac{2}{3}wa\right)(3a) = 0$$

$$M_A = 0$$

FBD AJ:



USING AJ AS A FREE BODY,

$$+\circlearrowleft \Sigma M_J = 0: M + \left(\frac{4}{3}wa\right)(x) - (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI \frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{2}{3}wax^2 + C_1$$

$$[x=0, \frac{dy}{dx}=0]: 0 = 0 - 0 + C_1 \quad \therefore C_1 = 0$$

$$EI y = \frac{1}{24}wx^4 - \frac{2}{9}wax^3 + C_2$$

$$[x=0, y=0]: 0 = 0 - 0 + C_2 \quad \therefore C_2 = 0$$

(a) ELASTIC CURVE OVER AB:

$$y = \frac{w}{72EI} (3x^4 - 16ax^3)$$

$$\frac{dy}{dx} = \frac{w}{6EI} (x^3 - 4ax^2)$$

(b) y AT x = 2a:

$$y_B = -\frac{10wa^4}{9EI}$$

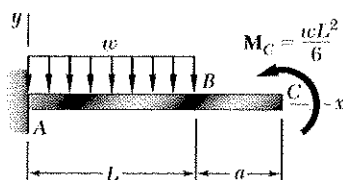
$$y_B = \frac{10wa^4}{9EI} \downarrow$$

(c) $\frac{dy}{dx}$ AT x = 2a:

$$\left(\frac{dy}{dx}\right)_B = -\frac{4wa^3}{3EI}$$

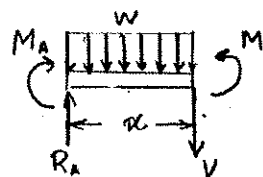
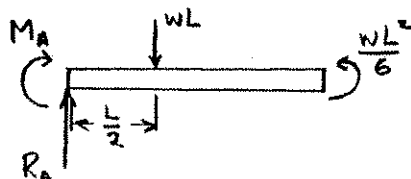
$$\theta_B = \frac{4wa^3}{3EI} \swarrow$$

Problem 9.6



$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



Using ABC as a free body,

$$+\uparrow \Sigma F_y = 0: \quad R_A - wL = 0 \quad R_A = wL$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (wL)(\frac{L}{2}) + \frac{wL^2}{6} = 0$$

$$M_A = -\frac{1}{3} wL^2$$

Using AJ as a free body, (Portion AB only)

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)(\frac{x}{2}) - R_A x - M_A = 0$$

$$M = -\frac{1}{2} wx^2 + R_A x + M_A$$

$$= -\frac{1}{2} wx^2 + wLx - \frac{1}{3} wL^2$$

$$EI \frac{d^2 y}{dx^2} = -\frac{1}{2} wx^2 + wLx - \frac{1}{3} wL^2$$

$$EI \frac{dy}{dx} = -\frac{1}{6} wx^3 + \frac{1}{2} wLx^2 - \frac{1}{3} wLx + C_1$$

$$[x=0, \frac{dy}{dx}=0]: \quad -0 + 0 - 0 + C_1 = 0 \quad C_1 = 0$$

$$EI y = -\frac{1}{24} wx^4 + \frac{1}{6} wLx^3 - \frac{1}{6} wLx^2 + C_2$$

$$[x=0, y=0] \quad -0 + 0 - 0 + C_2 = 0 \quad C_2 = 0$$

(a) Elastic curve over AB. $y = \frac{w}{24EI} (-x^4 + 4Lx^3 - 4L^2x^2)$ ▶

$$\frac{dy}{dx} = \frac{w}{6EI} (-x^3 + 2Lx^2 - L^2x)$$

(b) y at x = L:

$$y_B = -\frac{wL^4}{24EI}$$

$$y_B = \frac{wL^4}{24EI} \downarrow$$
 ▶

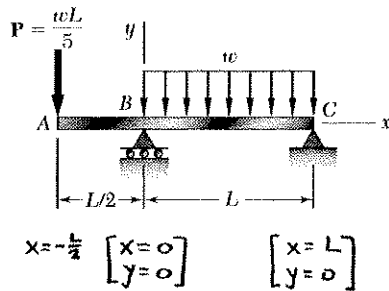
(c) $\frac{dy}{dx}$ at x = L:

$$\frac{dy}{dx} \Big|_B = 0$$

$$\theta_B = 0$$
 ▶

Problem 9.7

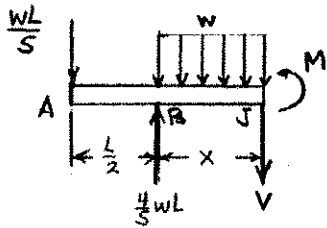
9.7 For the beam and loading shown, determine (a) the equation of the elastic curve for portion BC of the beam, (b) the deflection at midspan, (c) the slope at B.



Using ABC as a free body,

$$\uparrow \sum M_C = 0: \left(\frac{wL}{5}\right)\left(\frac{3L}{2}\right) + R_B L + (wL)\left(\frac{L}{2}\right) = 0$$

$$R_B = \frac{4}{5}wL$$



For portion BC only, ($0 < x < L$)

$$\uparrow \sum M_J = 0: \frac{wL}{5}\left(\frac{L}{2} + x\right) - \frac{4}{5}wLx + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{3}{5}wLx - \frac{1}{2}wx^2 - \frac{1}{10}wL^2$$

$$EI \frac{d^2 y}{dx^2} = \frac{3}{5}wLx - \frac{1}{2}wx^2 - \frac{1}{10}wL^2$$

$$EI \frac{dy}{dx} = \frac{3}{10}wLx^2 - \frac{1}{6}wx^3 - \frac{1}{10}wL^2x + C_1$$

$$EI y = \frac{1}{10}wLx^3 - \frac{1}{24}wx^4 - \frac{1}{20}wL^2x^2 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \left(\frac{1}{10} - \frac{1}{24} - \frac{1}{20}\right)wL^4 + C_1L + 0 \quad C_1 = -\frac{1}{120}wL^3$$

(a) Elastic curve. $y = \frac{w}{EI} \left(\frac{1}{10}Lx^3 - \frac{1}{24}x^4 - \frac{1}{20}L^2x^2 - \frac{1}{120}L^3x \right)$

$$\frac{dy}{dx} = \frac{w}{EI} \left(\frac{3}{10}Lx^2 - \frac{1}{6}x^3 - \frac{1}{10}L^2x - \frac{1}{120}L^3 \right)$$

(b) y @ x = L/2: $y_M = \frac{w}{EI} \left\{ \frac{1}{10}L\left(\frac{L}{2}\right)^3 - \frac{1}{24}\left(\frac{L}{2}\right)^4 - \frac{1}{20}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{120}L^3\left(\frac{L}{2}\right) \right\}$

$$= \frac{wL^4}{EI} \left\{ \frac{1}{80} - \frac{1}{384} - \frac{1}{80} - \frac{1}{240} \right\} = -\frac{13wL^4}{1920EI}$$

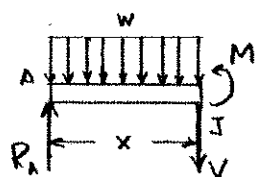
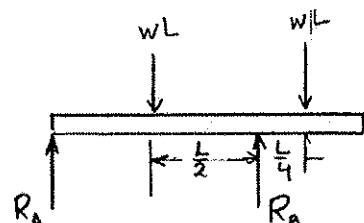
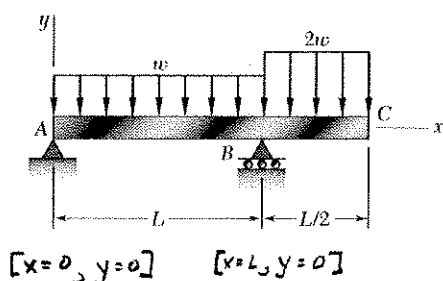
$$y_M = \frac{13wL^4}{1920EI} \downarrow$$

(c) dy/dx @ x = 0: $\left. \frac{dy}{dx} \right|_B = \frac{w}{EI} \left(0 - 0 - 0 - \frac{1}{120}L^3 \right) = -\frac{wL^3}{120EI}$

$$\theta_B = \frac{wL^3}{120EI} \swarrow$$

Problem 9.8

9.8 For the beam and loading shown, determine (a) the equation of the elastic curve for portion AB of the beam, (b) the slope at A, (c) the slope at B.



Using free body ABC,

$$+\circlearrowleft \sum M_B = 0: -R_A L + (WL)(\frac{L}{2}) - (wL)(\frac{L}{4}) = 0$$

$$R_A = \frac{1}{4} WL$$

For portion AB, $(0 \leq x \leq L)$

$$+\circlearrowleft \sum M_x = 0 \quad M - R_A x + (wx)(\frac{x}{2}) = 0$$

$$M = \frac{1}{4} WLx - \frac{1}{2} wx^2$$

$$EI \frac{d^2 y}{dx^2} = \frac{1}{4} WLx - \frac{1}{2} wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{8} WLx^2 - \frac{1}{6} wx^3 + C_1$$

$$EI y = \frac{1}{24} WLx^3 - \frac{1}{24} wx^4 + C_1 x + C_2$$

$$[x=0, y=0]: \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{1}{24} WL^4 - \frac{1}{24} wL^4 + C_1 L + 0 = 0 \quad C_1 = 0$$

(a) Elastic curve $(0 \leq x \leq L)$. $y = \frac{w}{24EI} (Lx^3 - x^4)$

$$\frac{dy}{dx} = \frac{w}{24EI} (3Lx^2 - 4x^3)$$

(b) $\frac{dy}{dx}$ at $x=0$.

$$\frac{dy}{dx} \Big|_A = 0$$

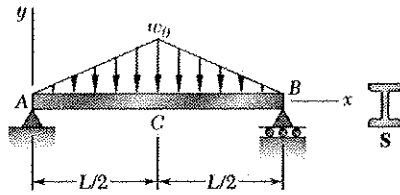
$$\theta_A = 0$$

(c) $\frac{dy}{dx}$ at $x=L$.

$$\frac{dy}{dx} \Big|_B = -\frac{wL^3}{24EI}$$

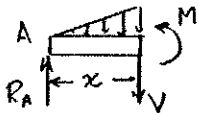
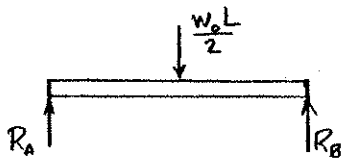
$$\theta_B = \frac{wL^3}{24EI}$$

Problem 9.9



$$[x=0, y=0] \quad [x=L, y=0]$$

$$[x=L/2, \frac{dy}{dx}=0]$$



$$[x=L/2, \frac{dy}{dx}=0]$$

$$[x=0, y=0]$$

Elastic curve.

9.9 Knowing that beam AB is an S200 × 27.4 rolled shape and that $w_0 = 60 \text{ kN/m}$, $L = 2.7 \text{ m}$, and $E = 200 \text{ GPa}$, determine (a) the slope at A, (b) the deflection at C.

Use symmetry boundary condition at C.

Using free body ACB and symmetry

$$R_A = R_B = \frac{1}{4} w_0 L$$

$$\text{For } 0 < x < \frac{L}{2} \quad w = \frac{2w_0 x}{L}$$

$$\frac{dV}{dx} = -w = -\frac{2w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{L} + R_A = \frac{w_0}{L} (\frac{1}{4} L^2 - x^2)$$

$$M = \frac{w_0}{L} (\frac{1}{4} L^2 x - \frac{1}{3} x^3) + C_m$$

But $M = 0$ at $x=0$; hence $C_m = 0$

$$EI \frac{d^2 y}{dx^2} = \frac{w_0}{L} (\frac{1}{4} L^2 x - \frac{1}{3} x^3)$$

$$EI \frac{dy}{dx} = \frac{w_0}{L} (\frac{1}{8} L^2 x^2 - \frac{1}{12} x^4) + C_1$$

$$0 = \frac{w_0}{L} (\frac{1}{32} L^4 - \frac{1}{192} L^4) + C_1 = 0 \quad C_1 = -\frac{5}{192} w_0 L^3$$

$$EI y = \frac{w_0}{L} (\frac{1}{24} L^2 x^3 - \frac{1}{120} x^5) - \frac{5}{192} w_0 L^3 x + C_2$$

$$0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$y = \frac{w_0}{EIL} (\frac{1}{24} L^2 x^3 - \frac{1}{60} x^5 - \frac{5}{192} L^4 x)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL} (\frac{1}{8} L^2 x^2 - \frac{1}{12} x^4 - \frac{5}{192} L^4)$$

$$\text{Data: } w_0 = 60 \text{ kN/m} \quad E = 200 \text{ GPa}, \quad I = 23.9 \times 10^6 \text{ mm}^4$$

$$EI = (200 \times 10^9)(23.9 \times 10^{-6}) = 4.78 \times 10^6 \text{ N m}^2$$

$$L = 2.7 \text{ m}$$

$$(a) \text{ Slope at } x=0. \quad \frac{dy}{dx} = \frac{60000}{(4.78 \times 10^6)(2.7)} \left[-\left(\frac{5}{192}\right)(2.7)^4 \right] = -6.434 \times 10^{-3}$$

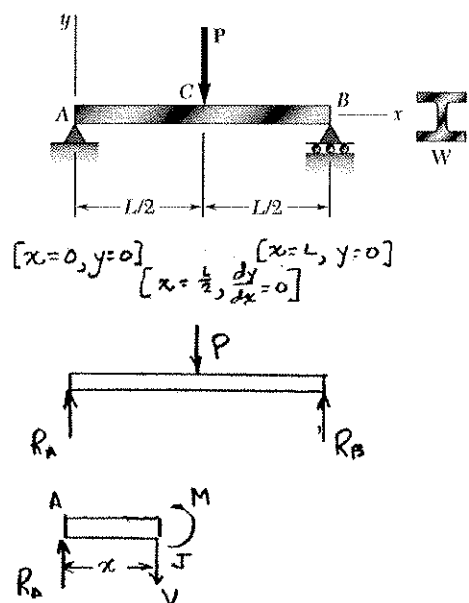
$$(b) \text{ Deflection at } x=1.35 \text{ m} \quad \theta_A = 6.43 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

$$y = \frac{60000}{(4.78 \times 10^6)(2.7)} \left[\left(\frac{1}{24}\right)(2.7)^2(1.35)^3 - \frac{1}{60}(1.35)^5 - \frac{5}{192}(2.7)^4(1.35) \right] = -0.0056 \times 10^{-3} \text{ m}$$

$$= 5.6 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.10

9.10 Knowing that beam AB is a $W130 \times 23.8$ rolled shape and that $P = 50 \text{ kN}$, $L = 1.25 \text{ m}$, and $E = 200 \text{ GPa}$, determine (a) the slope at A , (b) the deflection at C .



$$[x=0, y=0] \quad [x=L, y=0]$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0]$$

Use symmetry boundary condition at C .

By symmetry, $R_A = R_B = \frac{1}{2}P$

Using free body AJ , $0 \leq x \leq \frac{L}{2}$

$$+\circlearrowleft \sum M_J = 0: \quad M - R_A x = 0$$

$$M = R_A x = \frac{1}{2}Px$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{2}Px$$

$$EI \frac{dy}{dx} = \frac{1}{4}Px^2 + C_1$$

$$EI y = \frac{1}{12}Px^3 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0] \quad 0 = \frac{1}{4}P(\frac{L}{2})^2 + C_1 \quad C_1 = -\frac{1}{16}PL^2$$

Elastic curve.

$$y = \frac{PL}{48EI} (4x^3 - 3L^2x)$$

$$\frac{dy}{dx} = \frac{PL}{16EI} (4x^2 - L^2)$$

Slope at $x=0$.

$$\left. \frac{dy}{dx} \right|_A = -\frac{PL^2}{16EI}$$

$$\theta_A = \frac{PL^2}{16EI} \quad \swarrow$$

Deflection at $x=\frac{L}{2}$.

$$y_C = -\frac{PL^3}{48EI}$$

$$y_C = \frac{PL^3}{48EI} \quad \downarrow$$

$$\text{Data: } P = 50 \times 10^3 \text{ N}, \quad I = 8.80 \times 10^6 \text{ mm}^4 = 8.80 \times 10^{-6} \text{ m}^4$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$EI = 1.76 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$L = 1.25 \text{ m}$$

$$(a) \quad \theta_A = \frac{(50 \times 10^3)(1.25)^2}{(16)(1.76 \times 10^6)}$$

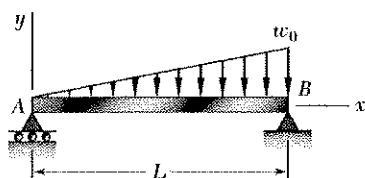
$$\theta_A = 2.77 \times 10^{-3} \text{ rad} \quad \swarrow$$

$$(b) \quad y_C = \frac{(50 \times 10^3)(1.25)^3}{(48)(1.76 \times 10^6)} = \pm 1.56 \times 10^{-3} \text{ m}$$

$$y_C = 1.56 \text{ mm} \quad \downarrow$$

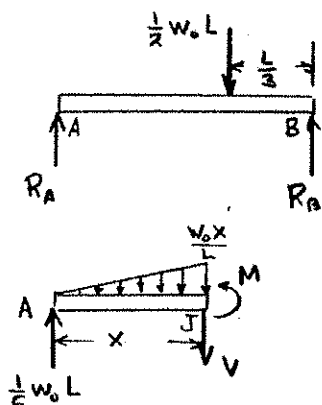
Proprietary Material. © 2009 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

Problem 9.11



9.11 For the beam and loading shown, (a) express the magnitude and location of the maximum deflection in terms of w_0 , L , E , and I . (b) Calculate the value of the maximum deflection, assuming that beam AB is a W460 $\times$ 74 rolled shape and that $w_0 = 60 \text{ kN/m}$, $L = 6 \text{ m}$, and $E = 200 \text{ GPa}$.

$[x=0, y=0]$ $[x=L, y=0]$



Using entire beam as a free body,

$$\begin{aligned} \sum M_B = 0: & \quad -R_A L + \left(\frac{1}{2} w_0 L\right) \left(\frac{L}{3}\right) = 0 \\ R_A &= \frac{1}{6} w_0 L \end{aligned}$$

Using AJ as a free body, $\sum M_J = 0$:

$$\begin{aligned} -\frac{1}{6} w_0 L x + \left(\frac{1}{2} \frac{w_0 x^2}{L}\right) \left(\frac{x}{3}\right) + M &= 0 \\ M &= \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3 \end{aligned}$$

$$EI \frac{d^2 y}{dx^2} = \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3$$

$$EI \frac{dy}{dx} = \frac{1}{12} w_0 L x^2 - \frac{1}{24} \frac{w_0}{L} x^4 + C_1$$

$$EI y = \frac{1}{36} w_0 L x^3 - \frac{1}{120} \frac{w_0}{L} x^5 + C_1 x + C_2$$

$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$

$[x=L, y=0] \quad 0 = \frac{1}{36} w_0 L^4 - \frac{1}{120} w_0 L^4 + C_1 L + 0 \quad C_1 = -\frac{7 w_0 L^3}{360}$

Elastic curve.
$$y = \frac{w_0}{EI} \left\{ \frac{1}{36} L x^3 - \frac{1}{120} \frac{x^5}{L} - \frac{7}{360} L^3 x \right\}$$

$$\frac{dy}{dx} = \frac{w_0}{EI} \left\{ \frac{1}{12} L x^2 - \frac{1}{24} \frac{x^4}{L} - \frac{7}{360} L^3 \right\}$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$.

$$15x_m^4 - 30L^2x_m^2 + 7L^4 = 0 \quad x_m^2 = \frac{30L^2 - \sqrt{900L^4 - 420L^4}}{30}$$

$$x_m^2 = \left(1 - \sqrt{\frac{8}{15}}\right) L^2 = 0.2697 L^2 \quad x_m = 0.5193 L$$

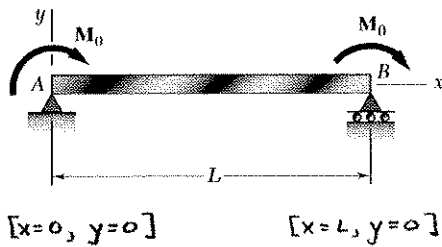
$$\begin{aligned} y_m &= \frac{w_0}{EI} \left\{ \frac{1}{36} L (0.5193 L)^3 - \frac{1}{120} \frac{(0.5193 L^5)}{L} - \frac{7}{360} L^3 (0.5193 L) \right\} \\ &= -0.00652 \frac{w_0 L^4}{EI} \quad \text{or} \quad 0.00652 \frac{w_0 L^4}{EI} \downarrow \end{aligned}$$

Data: $w_0 = 60 \text{ kN/m} = 60 \times 10^3 \text{ N/m}$ $L = 6 \text{ m}$

For W460 $\times$ 74, $I = 333 \times 10^6 \text{ mm}^4 = 333 \times 10^{-6} \text{ m}^4$

$$y_m = \frac{(0.00652)(60 \times 10^3)(6)^4}{(200 \times 10^9)(333 \times 10^{-6})} = 7.61 \times 10^{-3} \text{ m} \quad y_m = 7.61 \text{ mm} \downarrow$$

Problem 9.12



9.12 (a) Determine the location and magnitude of the maximum absolute deflection in AB between A and the center of the beam. (b) Assuming that beam AB is a W 460 $\times$ 113, $M_0 = 224 \text{ kN}\cdot\text{m}$ and $E = 200 \text{ GPa}$, determine the maximum allowable length L so that the maximum deflection does not exceed 1.2 mm .

Using AB as a free body

$$\sum M_B = 0: -2M_0 - R_A L = 0$$

$$R_A = -\frac{2M_0}{L}$$

Using portion AJ as a free body

$$\sum M_J = 0: -M_0 + \frac{2M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(L - 2x)$$

$$EI \frac{d^2 y}{dx^2} = \frac{M_0}{L}(L - 2x)$$

$$EI \frac{dy}{dx} = \frac{M_0}{L}(Lx - x^2) + C_1$$

$$EI y = \frac{M_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3\right) + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{M_0}{L}\left(\frac{1}{2}L^3 - \frac{1}{3}L^3\right) + C_1 L + 0 \quad C_1 = -\frac{1}{6}M_0 L^2$$

$$y = \frac{M_0}{EIL}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3 - \frac{1}{6}L^2 x\right) \quad \frac{dy}{dx} = \frac{M_0}{EIL}\left(Lx - x^2 - \frac{1}{6}L^2\right)$$

To find location of maximum deflection set $\frac{dy}{dx} = 0$

$$x_m^2 - Lx_m - \frac{1}{6}L^2 = 0 \quad x_m = \frac{L - \sqrt{L^2 - 4\left(\frac{1}{6}L^2\right)}}{2} = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right)L = 0.21132 L$$

$$y_m = \frac{M_0 L^2}{EI} \left\{ \left(\frac{1}{2}\right)(0.21132)^2 - \left(\frac{1}{3}\right)(0.21132)^3 - \left(\frac{1}{6}\right)(0.21132) \right\} = -0.0160375 \frac{M_0 L^2}{EI}$$

$$|y_m| = 0.0160375 \frac{M_0 L^2}{EI}$$

$$\text{Solving for } L, \quad L = \left\{ \frac{EI |y_m|}{0.0160375 M_0} \right\}^{1/2}$$

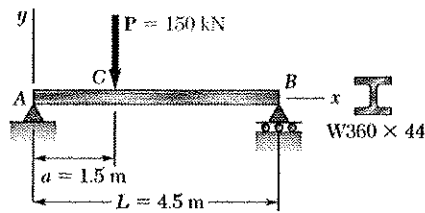
$$\text{Data: } E = 200 \times 10^9 \text{ Pa}, \quad I = 556 \times 10^6 \text{ mm}^4 = 556 \times 10^{-6} \text{ m}^4$$

$$|y_m| = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}, \quad M_0 = 224 \times 10^3 \text{ N}\cdot\text{m}$$

$$L = \left\{ \frac{(200 \times 10^9)(556 \times 10^{-6})(1.2 \times 10^{-3})}{(0.0160375)(224 \times 10^3)} \right\}^{1/2} = 6.09 \text{ m}$$

Problem 9.13

9.13 For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.



$$[x=0, y=0] \quad [x=L, y=0]$$

$$[x=a, y=y]$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$\text{Let } b = L - a$$

$$\text{Reactions: } R_A = \frac{Pb}{L} \uparrow, \quad R_B = \frac{Pa}{L} \uparrow$$

Bending moments

$$0 < x < a \quad M = \frac{Pb}{L}x$$

$$a < x < L \quad M = \frac{P}{L}[bx - L(x-a)]$$

$$0 < x < a$$

$$EI \frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI \frac{dy}{dx} = \frac{P}{L}(\frac{1}{2}bx^2) + C_1 \quad (1)$$

$$EI y = \frac{P}{L}(\frac{1}{6}bx^3) + C_1x + C_2 \quad (2)$$

$$a < x < L$$

$$EI \frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x-a)]$$

$$EI \frac{dy}{dx} = \frac{P}{L}[\frac{1}{2}bx^2 - \frac{1}{2}L(x-a)^2] + C_3 \quad (2)$$

$$EI y = \frac{P}{L}[\frac{1}{6}bx^3 - \frac{1}{6}L(x-a)^3] + C_3x + C_4 \quad (4)$$

$$[x=0, y=0] \quad \text{Eq. (2)} \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad \text{Eqs. (1) and (3)} \quad \frac{P}{L}(\frac{1}{2}ba^2) + C_1 = \frac{P}{L}[\frac{1}{2}ba^2 + 0] + C_3 \therefore C_3 = C_1$$

$$[x=a, y=y] \quad \text{Eqs. (2) and (4)} \quad \frac{P}{L}(\frac{1}{6}ba^3) + C_1a + C_2 = \frac{P}{L}[\frac{1}{6}ba^3 + 0] + C_1a + C_4 \quad C_4 = C_2 = 0$$

$$[x=L, y=0] \quad \text{Eq. (4)} \quad \frac{P}{L}[\frac{1}{6}bL^3 - \frac{1}{6}L(L-a)^3] + C_3L = 0$$

$$C_1 = C_3 = \frac{P}{L}[\frac{1}{6}(L-a)^3 - \frac{1}{6}bL^2] = \frac{P}{L}(\frac{1}{6}b^3 - \frac{1}{6}bL^2)$$

Make $x = a$ in Eq. (2).

$$y_c = \frac{P}{EIL}[\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

$$\text{Data: } P = 150 \text{ kN}, \quad E = 200 \text{ GPa}$$

$$L = 4.5 \text{ m}, \quad a = 1.5 \text{ m}, \quad b = 3 \text{ m}$$

$$I = 122 \times 10^{-6} \text{ m}^4, \quad EI = 24.4 \times 10^6 \text{ N m}^2$$

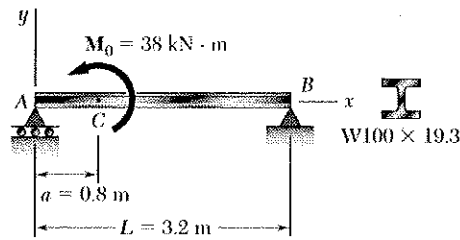
$$y_c = \frac{150 \times 10^3}{(24.4 \times 10^6)(4.5)} [(3)(1.5)^3 + (3)^3(1.5) - (4.5)^2(1.5)(3)]$$

$$= -9.22 \times 10^{-3} \text{ m}$$

$$y_c = 9.2 \text{ mm} \quad \downarrow$$

Problem 9.14

9.14 For the beam and loading shown, determine the deflection at point C. Use $E = 200 \text{ GPa}$.



$$[x=0, y=0] \quad [x=L, y=0]$$

$$[x=a, y=y] \\ [x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

Reactions: $R_A = M_0/L \uparrow, R_B = M_0/L \downarrow$

$$0 < x < a; \quad \sum M_f = 0:$$

$$\begin{array}{c} \text{A} \quad \text{C} \quad \text{B} \\ \text{M}_0/L \quad \text{M} \quad \text{V} \end{array} \quad -\frac{M_0}{L}x + M = 0 \\ M = \frac{M_0}{L}x$$

$$a < x < L; \quad \sum M_k = 0:$$

$$\begin{array}{c} \text{A} \quad \text{C} \quad \text{B} \\ \text{M}_0/L \quad \text{M}_0 \quad \text{M} \quad \text{V} \end{array} \quad -\frac{M_0}{L}x + M_0 + M = 0 \\ M = \frac{M_0}{L}(x-L)$$

$$0 < x < a$$

$$EI \frac{d^2y}{dx^2} = \frac{M_0}{L}x$$

$$EI \frac{dy}{dx} = \frac{M_0}{L}(\frac{1}{2}x^2) + C_1 \quad (1)$$

$$EI y = \frac{M_0}{L}(\frac{1}{6}x^3) + C_1x + C_2 \quad (2)$$

$$a < x < L$$

$$EI \frac{d^2y}{dx^2} = \frac{M_0}{L}(x-L)$$

$$EI \frac{dy}{dx} = \frac{M_0}{L}(\frac{1}{2}x^2 - Lx) + C_3 \quad (3)$$

$$EI y = \frac{M_0}{L}(\frac{1}{6}x^3 - \frac{1}{2}Lx^2) + C_3x + C_4 \quad (4)$$

$$[x=0, y=0] \quad \text{Eq. (2)}: \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad \text{Eqs. (1) \& (3)}: \quad \frac{M_0}{L}(\frac{1}{2}a^2) + C_1 = \frac{M_0}{L}(\frac{1}{2}a^2 - La) + C_3$$

$$C_3 = C_1 + M_0a$$

$$[x=a, y=y] \quad \text{Eqs. (2) \& (4)}: \quad \frac{M_0}{L}(\frac{1}{6}a^3) + C_1a = \frac{M_0}{L}(\frac{1}{6}a^3 - \frac{1}{2}La^2) + (C_1 + M_0a)a + C_4$$

$$C_4 = -\frac{1}{2}M_0a^2$$

$$[x=L, y=0] \quad \text{Eq. (4)}: \quad \frac{M_0}{L}(\frac{1}{6}L^3 - \frac{1}{2}L^3) + (C_1 + M_0a)L - \frac{1}{2}M_0a^2 = 0$$

$$C_1 = \frac{M_0}{L}(\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)$$

Elastic curve for $0 < x < a$. $y = \frac{M_0}{EIL} \left[\frac{1}{6}x^3 + (\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)x \right]$

Make $x = a$. $y_c = \frac{M_0}{EIL} \left[\frac{1}{6}a^3 + \frac{1}{3}L^2a + \frac{1}{2}a^3 - a^2L \right] = \frac{M_0}{EIL} \left[\frac{2}{3}a^3 + \frac{1}{3}L^2a - La^2 \right]$

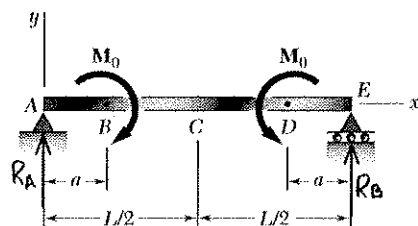
Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 4.77 \times 10^6 \text{ mm}^4 = 4.77 \times 10^{-6} \text{ m}^4$, $M_0 = 38 \times 10^3 \text{ N}\cdot\text{m}$

$$y_c = \frac{(38 \times 10^3) \left[\frac{2}{3}(0.8)^3 + \frac{1}{3}(3.2)^2(0.8) - (3.2)(0.8)^2 \right]}{(200 \times 10^9)(4.77 \times 10^{-6})(3.2)} = 12.75 \times 10^{-3} \text{ m}$$

$$y_c = 12.75 \text{ mm} \uparrow$$

Problem 9.15

9.15 Knowing that beam AE is a $W360 \times 101$ rolled shape and that $M_0 = 310 \text{ kN}\cdot\text{m}$, $L = 2.4 \text{ m}$, $a = 0.5 \text{ m}$, and $E = 200 \text{ GPa}$, determine (a) the equation of the elastic curve for portion BD , (b) the deflection at point C .



Use continuity boundary condition at B and symmetry boundary condition at C .

From Statics, $R_A = R_E = 0$

$$[x=0, y=0]$$

$$[x=a, y=y]$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = 0]$$

$$0 \leq x \leq a$$

$$M = 0$$

$$EI \frac{d^2y}{dx^2} = 0$$

$$EI \frac{dy}{dx} = C_1$$

$$EI y = C_1 x + C_2$$

$$a \leq x \leq L-a$$

$$M = M_0$$

$$EI \frac{d^2y}{dx^2} = M_0$$

$$EI \frac{dy}{dx} = M_0 x + C_3$$

$$EI y = \frac{1}{2} M_0 x^2 + C_3 x + C_4$$

$$[x=0, y=0]$$

$$0 = 0 + C_2 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = 0]$$

$$0 = \frac{1}{2} M_0 L + C_3$$

$$C_3 = -\frac{1}{2} M_0 L$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$C_1 = M_0 a + C_3 = -M_0 (\frac{1}{2} L - a)$$

$$[x=a, y=y]$$

$$-M_0 (\frac{1}{2} L - a) a + 0 = \frac{1}{2} M_0 a^2 - \frac{1}{2} M_0 L a + C_4$$

$$C_4 = \frac{1}{2} M_0 a^2$$

(a) Elastic curve ($a \leq x \leq L-a$), $EI y = \frac{1}{2} M_0 x^2 + \frac{1}{2} M_0 L x + \frac{1}{2} M_0 a^2$

$$y = \frac{M_0}{2EI} (x^2 - Lx + a^2)$$

(b) Deflection at $x = \frac{L}{2}$.

$$y_c = \frac{M_0}{2EI} \left[\left(\frac{L}{2} \right)^2 - (L) \left(\frac{L}{2} \right) + a^2 \right] = -\frac{M_0}{8EI} (L^2 - 4a^2)$$

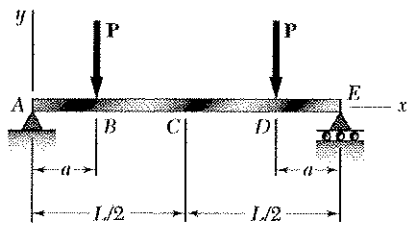
Data: $M_0 = 310 \times 10^3 \text{ N}\cdot\text{m}$, $L = 2.4 \text{ m}$, $a = 0.5 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$

$$I = 302 \times 10^6 \text{ mm}^4 = 302 \times 10^{-6} \text{ m}^4 \quad EI = 60.4 \times 10^6 \text{ N}\cdot\text{m}^2$$

$$y_c = -\frac{310 \times 10^3}{(8)(60.4 \times 10^6)} [(2.4)^2 - (4)(0.5)^2] = -3.05 \times 10^{-3}$$

$$y_c = 3.05 \text{ mm} \downarrow$$

Problem 9.16



9.16. Knowing that beam AE is an $S200 \times 27.4$ rolled shape and that $P = 17.5 \text{ kN}$, $L = 2.5 \text{ m}$, $a = 0.8 \text{ m}$ and $E = 200 \text{ GPa}$, determine (a) the equation of the elastic curve for portion BD , (b) the deflection at the center C of the beam.

Consider portion ABC only. Apply symmetry about C .

Reactions: $R_A = R_E = P$

Boundary conditions: $[x=0, y=0]$, $[x=a, y=y]$, $[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$, $[x=\frac{L}{2}, \frac{dy}{dx} = 0]$

$0 < x < a$

$$EI \frac{d^2y}{dx^2} = M = Px$$

$$EI \frac{dy}{dx} = \frac{1}{2}Px^2 + C_1 \quad (1)$$

$$EI y = \frac{1}{6}Px^3 + C_1x + C_2 \quad (2)$$

$$[x=0, y=0] \rightarrow C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \quad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$$

$$[x=\frac{L}{2}, y=y] \quad \frac{1}{6}Pa^3 + (\frac{1}{2}Pa^2 - \frac{1}{2}PaL)a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L + C_4$$

$$C_4 = \frac{1}{6}Pa^3$$

$a < x < L-a$

$$EI \frac{d^2y}{dx^2} = M = Pa$$

$$EI \frac{dy}{dx} = Pax + C_3$$

$$EI y = \frac{1}{2}Pax^2 + C_3x + C_4$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = 0] \rightarrow C_3 = -\frac{1}{2}PaL$$

(a) Elastic curve for portion BD .

$$y = \frac{1}{EI} \left(\frac{1}{2}Pax^2 + C_3x + C_4 \right)$$

$$= \frac{P}{EI} \left(\frac{1}{2}ax^2 - \frac{1}{2}aLx + \frac{1}{6}a^3 \right)$$

For deflection at C , set $x = \frac{L}{2}$.

$$y_c = \frac{P}{EI} \left(\frac{1}{8}aL^2 - \frac{1}{4}aL^2 + \frac{1}{6}a^3 \right) = -\frac{Pa}{EI} \left(\frac{1}{8}L^2 - \frac{1}{6}a^2 \right)$$

Data: $I = 23.9 \times 10^6 \text{ mm}^4 = 23.9 \times 10^{-6} \text{ m}^4$, $E = 200 \times 10^9 \text{ Pa}$

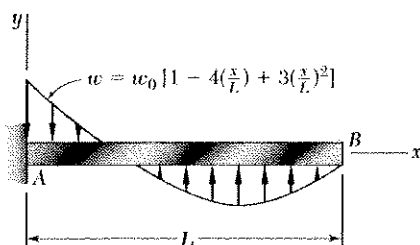
$P = 17.5 \times 10^3 \text{ N}$, $L = 2.5 \text{ m}$, $a = 0.8 \text{ m}$

$$(b) \quad y_c = -\frac{(17.5 \times 10^3)(0.8)}{(200 \times 10^9)(23.9 \times 10^{-6})} \left\{ \frac{2.5^2}{8} - \frac{0.8^2}{6} \right\} = -1.976 \times 10^{-3} \text{ m}$$

$$y_c = 1.976 \text{ mm} \downarrow$$

Problem 9.17

9.17 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at the free end.



Boundary conditions are shown at left.

$$\begin{aligned} [x=0, y=0] \\ [x=0, \frac{dy}{dx}=0] \end{aligned}$$

$$\begin{aligned} [x=L, V=0] \\ [x=L, M=0] \end{aligned}$$

$$\frac{dV}{dx} = -W = -w_0 \left[1 - 4\left(\frac{x}{L}\right) + 3\left(\frac{x}{L}\right)^2 \right]$$

$$V = -w_0 \left[x - \frac{2x^2}{L} + \frac{x^3}{L^2} \right] + C_v$$

$$[x=L, V=0]:$$

$$0 = -w_0 [L - 2L + L] + C_v = 0 \quad C_v = 0$$

$$\frac{dM}{dx} = V = -w_0 \left[x - \frac{2x^2}{L} + \frac{x^3}{L^2} \right]$$

$$M = -w_0 \left[\frac{x^2}{2} - \frac{2x^3}{3L} + \frac{x^4}{4L^2} \right] + C_m$$

$$[x=L, M=0]: \quad 0 = -w_0 \left[\frac{1}{2}L^2 - \frac{2}{3}L^2 + \frac{1}{4}L^2 \right] + C_m$$

$$C_m = \frac{1}{12} w_0 L^2$$

$$EI \frac{d^2y}{dx^2} = M = -w_0 \left[\frac{1}{2}x^2 - \frac{2}{3}\frac{x^3}{L} + \frac{1}{4}\frac{x^4}{L^2} - \frac{1}{12}L^2 \right]$$

$$EI \frac{dy}{dx} = -w_0 \left[\frac{1}{6}x^3 - \frac{1}{6}\frac{x^4}{L} + \frac{1}{20}\frac{x^5}{L^2} - \frac{1}{12}L^2x \right] + C_1$$

$$[x=0, \frac{dy}{dx}=0] \rightarrow C_1 = 0$$

$$EI y = -w_0 \left[\frac{1}{24}x^4 - \frac{1}{30}\frac{x^5}{L} + \frac{1}{120}\frac{x^6}{L^2} - \frac{1}{24}L^2x^2 \right] + C_2$$

$$[x=0, y=0] \rightarrow C_2 = 0$$

(a) Elastic curve. $y = -\frac{w_0}{EI L^2} \left(\frac{1}{24}L^2x^4 - \frac{1}{30}Lx^5 + \frac{1}{120}x^6 - \frac{1}{24}L^4x^2 \right)$ $\blacktriangleleft$

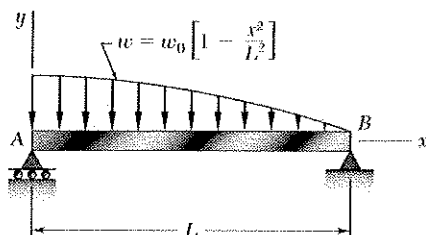
(b) Deflection at $x=L$.

$$y_B = -\frac{w_0}{EI L^2} \left(\frac{1}{24}L^6 - \frac{1}{30}L^6 + \frac{1}{120}L^6 - \frac{1}{24}L^6 \right) = -\frac{w_0 L^4}{40 EI}$$

$$y_B = \frac{w_0 L^4}{40 EI} \downarrow \blacktriangleleft$$

Problem 9.18

9.18 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection at the midpoint of the span.



$$w = w_0 \left[1 - \frac{x^2}{L^2} \right] = \frac{w_0}{L^2} [L^2 - x^2]$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} [x^2 - L^2]$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2} \left[\frac{x^3}{3} - L^2 x \right] + C_1$$

$$M = \frac{w_0}{L^2} \left[\frac{1}{12} x^4 - \frac{1}{2} L^2 x^2 \right] + C_1 x + C_2$$

$$[x=0, M=0]: 0 = 0 - 0 + 0 + C_2 \quad \therefore C_2 = 0$$

$$[x=L, M=0]: 0 = \frac{w_0}{L^2} \left[\frac{1}{12} L^4 - \frac{1}{2} L^4 \right] + C_1 L \quad \therefore C_1 = \frac{5}{12} w_0 L$$

$$EI \frac{d^2 y}{dx^2} = M = \frac{w_0}{L^2} \left[\frac{1}{12} x^4 - \frac{1}{2} L^2 x^2 + \frac{5}{12} L^3 x \right]$$

$$EI \frac{dy}{dx} = \frac{w_0}{L^2} \left[\frac{1}{60} x^5 - \frac{1}{6} L^2 x^3 + \frac{5}{24} L^3 x^2 \right] + C_3$$

$$EI y = \frac{w_0}{L^2} \left[\frac{1}{360} x^6 - \frac{1}{24} L^2 x^4 + \frac{5}{72} L^3 x^3 \right] + C_3 x + C_4$$

$$[x=0, y=0]: 0 = 0 - 0 + 0 + 0 + C_4 \quad \therefore C_4 = 0$$

$$[x=L, y=0]: 0 = \frac{w_0}{L^2} \left[\frac{1}{360} L^6 - \frac{1}{24} L^6 + \frac{5}{72} L^6 \right] + C_3 L \quad \therefore C_3 = -\frac{11}{360} w_0 L^3$$

(a) ELASTIC CURVE. $y = w_0 (x^6 - 15L^2 x^4 + 25L^3 x^3 - 11L^5 x) / 360 EIL^2$

$$\frac{dy}{dx} = w_0 (6x^5 - 60L^2 x^3 + 75L^3 x^2 - 11L^5) / 360 EIL^2$$

(b) SLOPE AT END A. SET $x=0$ IN $\frac{dy}{dx}$. $\frac{dy}{dx} \Big|_A = -\frac{11}{360} \frac{w_0 L^3}{EI}$

$$\theta_A = \frac{11}{360} \frac{w_0 L^3}{EI}$$

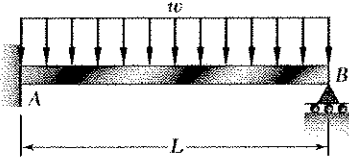
(c) DEFLECTION AT MIDPOINT (SAY POINT C). SET $x = \frac{L}{2}$ IN y .

$$\begin{aligned} y_c &= w_0 \left(\frac{1}{64} L^6 - \frac{15}{16} L^6 + \frac{25}{8} L^6 - \frac{11}{2} L^6 \right) / 360 EIL^2 \\ &= w_0 \left(\frac{1}{64} L^6 - \frac{60}{64} L^6 + \frac{200}{64} L^6 - \frac{352}{64} L^6 \right) / 360 EIL^2 \\ &= -\frac{211}{23040} \frac{w_0 L^4}{EI} \end{aligned}$$

$$y_c = 0.00916 \frac{w_0 L^4}{EI} \downarrow$$

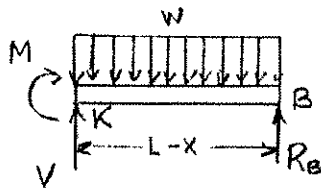
Problem 9.19

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



$$[x=0, y=0] \quad [x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body KB,

$$+\circlearrowleft M_K = 0: R_B(L-x) - w(L-x)\left(\frac{L-x}{2}\right) - M = 0$$

$$M = R_B(L-x) - \frac{1}{2}w(L-x)^2$$

$$EI \frac{d^2y}{dx^2} = R_B(L-x) - \frac{1}{2}w(L-x)^2$$

$$EI \frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + \frac{1}{6}w(L-x)^3 + C_1$$

$$[x=0, \frac{dy}{dx}=0]: \quad 0 = -\frac{1}{2}R_B L^2 + \frac{1}{6}wL^3 + C_1, \quad C_1 = \frac{1}{2}R_B L^2 - \frac{1}{6}wL^3$$

$$EI y = \frac{1}{6}R_B(L-x)^3 - \frac{1}{24}w(L-x)^4 + C_1 x + C_2$$

$$[x=0, y=0]: \quad 0 = \frac{1}{6}R_B L^3 - \frac{1}{24}wL^4 + C_2 \quad C_2 = -\frac{1}{6}R_B L^3 + \frac{1}{24}wL^4$$

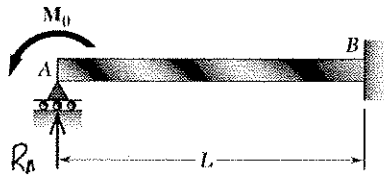
$$[x=L, y=0] \quad 0 = 0 - 0 + C_1 L + C_2$$

$$\frac{1}{2}R_B L^3 - \frac{1}{6}wL^4 - \frac{1}{6}R_B L^3 + \frac{1}{24}wL^4 = 0$$

$$R_B = \frac{3}{8}wL \uparrow \quad \blacktriangleleft$$

Problem 9.20

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$

Using free body AJ,

$$+\circlearrowleft \sum M_J = 0: M_0 - R_A x + M = 0$$

$$M = R_A x - M_0$$

$$EI \frac{d^2 y}{dx^2} = R_A x - M_0$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_0 x + C_1$$

$$[x=L, \frac{dy}{dx}=0]$$

$$0 = \frac{1}{2} R_A L^2 - M_0 L + C_1$$

$$C_1 = M_0 L - \frac{1}{2} R_A L^2$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_0 x^2 + C_1 x + C_2$$

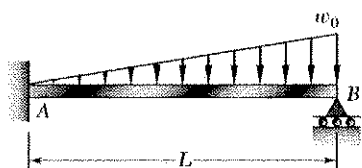
$$[x=0, y=0] \rightarrow C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{1}{6} R_A L^3 - \frac{1}{2} M_0 L^2 + (M_0 L - \frac{1}{2} R_A L^2) L + 0$$

$$R_A = \frac{3}{2} \frac{M_0}{L} \uparrow$$

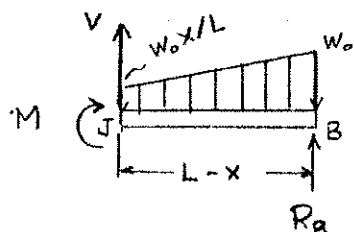
Problem 9.21

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



$$\begin{aligned} [x=0, y=0] \\ [x=0, \frac{dy}{dx}=0] \end{aligned}$$

$$[x=L, y=0]$$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body JB, $\sum M_J = 0$:

$$\begin{aligned} -M + R_B(L-x) + \frac{1}{2}w_0(L-x)\frac{2}{3}(L-x) \\ + \frac{1}{2}\frac{w_0 x}{L}(L-x)\frac{1}{3}(L-x) = 0 \end{aligned}$$

$$M = R_B(L-x) - \frac{w_0}{6L} [2L(L-x)^2 + x(L-x)^2]$$

$$\begin{aligned} = R_B(L-x) - \frac{w_0}{6L} [2L^3 - 4L^2x + 2Lx^2 \\ + xL^2 - 2Lx^2 + x^3] \end{aligned}$$

$$= R_B(L-x) - \frac{w_0}{6L} (x^3 - 3L^2x + 2L^3)$$

$$EI \frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L} (x^3 - 3L^2x + 2L^3)$$

$$EI \frac{dy}{dx} = R_B(Lx - \frac{1}{2}x^2) - \frac{w_0}{6L} (\frac{1}{4}x^4 - \frac{3}{2}L^2x^2 + 2L^2x) + C_1$$

$$EI y = R_B(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) - \frac{w_0}{6L} (\frac{1}{20}x^5 - \frac{1}{2}L^2x^3 + L^2x^2) + C_1x + C_2$$

$$[x=0, y=0] \rightarrow C_2 = 0$$

$$[x=0, \frac{dy}{dx}=0] \rightarrow C_1 = 0$$

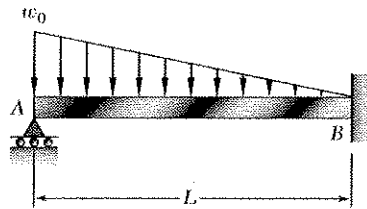
$$[x=L, y=0] \quad 0 = R_B L^3 (\frac{1}{2} - \frac{1}{6}) - \frac{w_0 L^4}{6} (\frac{1}{20} - \frac{1}{2} + 1)$$

$$\frac{1}{3} R_B = (\frac{1}{6}) (\frac{11}{20}) w_0 L$$

$$R_B = \frac{11}{40} w_0 L \uparrow$$

Problem 9.22

9.19 through 9.22 For the beam and loading shown, determine the reaction at the roller support.



Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$

$$w = \frac{w_0}{L} (L-x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L} (L-x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L} (Lx - \frac{1}{2}x^2) + R_A$$

$$M = -\frac{w_0}{L} (\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + R_A x$$

$$EI \frac{d^2y}{dx^2} = -\frac{w_0}{L} (\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + R_A x$$

$$EI \frac{dy}{dx} = -\frac{w_0}{L} (\frac{1}{6}Lx^3 - \frac{1}{24}x^4) + \frac{1}{2}R_A x^2 + C_1$$

$$EI y = -\frac{w_0}{L} (\frac{1}{24}Lx^4 - \frac{1}{120}x^5) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$$[x=0, y=0]$$

$$0 = 0 + 0 + 0 + C_2$$

$$C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0]$$

$$-\frac{w_0}{L} (\frac{1}{6}L^4 - \frac{1}{24}L^4) + \frac{1}{2}R_A L^2 + C_1 = 0$$

$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

$$[x=L, y=0]$$

$$-\frac{w_0}{L} (\frac{1}{24}L^4 - \frac{1}{120}L^4) + \frac{1}{6}R_A L^3 + (\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2)L = 0$$

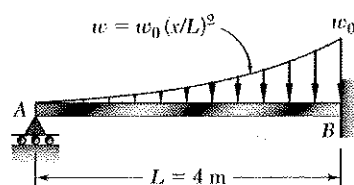
$$(\frac{1}{2} - \frac{1}{6})R_A = (\frac{1}{8} - \frac{1}{24} + \frac{1}{120})w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L$$

$$R_A = \frac{11}{40}w_0 L \uparrow$$

Problem 9.23

9.23 For the beam shown, determine the reaction at the roller support when $w_0 = 65$ kN/m.



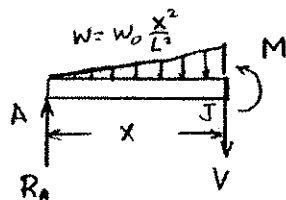
Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2} \frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2} \frac{x^4}{12} + R_A x$$

$$EI \frac{d^2y}{dx^2} = -\frac{w_0}{L^2} \frac{x^4}{12} + R_A x$$

$$EI \frac{dy}{dx} = -\frac{w_0}{L^2} \frac{x^5}{60} + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = -\frac{w_0}{L^2} \frac{x^6}{360} + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$[x=0, y=0]$$

$$0 = 0 + 0 + 0 + C_2$$

$$C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0]$$

$$-\frac{1}{60} w_0 L^3 + \frac{1}{2} R_A L^2 + C_1 = 0$$

$$C_1 = \frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2$$

$$[x=L, y=0]$$

$$-\frac{1}{360} w_0 L^4 + \frac{1}{6} R_A L^3 + \left(\frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2 \right) L = 0$$

$$\left(\frac{1}{2} - \frac{1}{6} \right) R_A = \left(\frac{1}{60} - \frac{1}{360} \right) w_0 L$$

$$\frac{1}{3} R_A = \frac{1}{72} w_0 L$$

$$R_A = \frac{1}{18} w_0 L$$

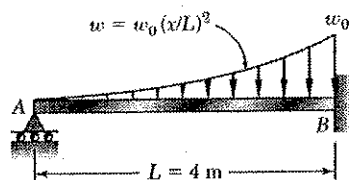
Data: $w_0 = 65$ kN/m, $L = 4$ m

$$R_A = \frac{1}{18} (65)(4)$$

$$R_A = 14.44 \text{ kN} \uparrow$$

Problem 9.24

9.24 For the beam shown determine the reaction at the roller support when $w_0 = 65$ kN/m.



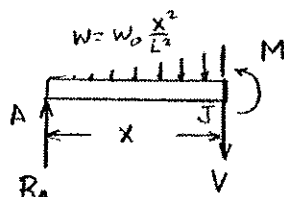
Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2} \frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2} \frac{x^4}{12} + R_A x$$

$$EI \frac{d^2y}{dx^2} = -\frac{w_0}{L^2} \frac{x^4}{12} + R_A x$$

$$EI \frac{dy}{dx} = -\frac{w_0}{L^2} \frac{x^5}{60} + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = -\frac{w_0}{L^2} \frac{x^6}{360} + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad -\frac{1}{60} w_0 L^3 + \frac{1}{2} R_A L^2 + C_1 = 0 \quad C_1 = \frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2$$

$$[x=L, y=0] \quad -\frac{1}{360} w_0 L^4 + \frac{1}{6} R_A L^3 + (\frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2) L = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_A = (\frac{1}{60} - \frac{1}{360}) w_0 L$$

$$\frac{1}{3} R_A = \frac{1}{72} w_0 L$$

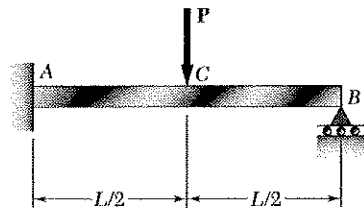
$$R_A = \frac{1}{18} w_0 L$$

$$\text{Data: } w_0 = 65 \text{ kN/m, } L = 4 \text{ m}$$

$$R_A = \frac{1}{18} (65)(4) = 14.44 \text{ kN} \uparrow$$

Problem 9.25

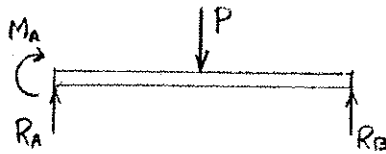
9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=L, y=0]$$



Reactions are statically indeterminate.

$$+\uparrow \sum F_y = 0: R_A + R_B - P = 0 \quad R_A = P - R_B$$

$$+\circlearrowleft \sum M_A = 0: -M_A + \frac{1}{2}PL - R_B L = 0$$

$$M_A = R_B L - \frac{1}{2}PL$$

$$0 < x < \frac{1}{2}L: M = M_A + R_A x$$

$$EI \frac{d^2 y}{dx^2} = M_A + R_A x$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$\frac{1}{2}L < x < L: M = M_A + R_A x - P(x - \frac{1}{2}L)$$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x - P(x - \frac{1}{2}L)$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P(x - \frac{1}{2}L)^2 + C_3$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P(x - \frac{1}{2}L)^3 + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x = \frac{1}{2}L, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2} M_A L + \frac{1}{8} R_A L^2 + 0 = \frac{1}{2} M_A + \frac{1}{8} R_A L - 0 + C_3 \quad C_3 = 0$$

$$[x = \frac{1}{2}L, y = y] \quad \frac{1}{8} M_A L^2 + \frac{1}{48} R_A L^3 + 0 + 0 = \frac{1}{8} M_A L^2 + \frac{1}{48} R_A L^3 - 0 + 0 + C_4 \quad C_4 = 0$$

$$[x=L, y=0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{48} PL^3 + 0 + 0 = 0$$

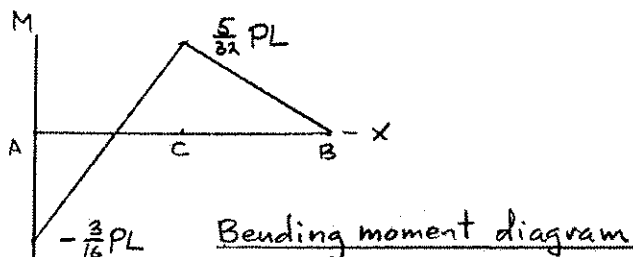
$$\frac{1}{2} (R_B L - \frac{1}{2}P) L^2 + \frac{1}{6} (P - R_B) L^3 - \frac{1}{48} PL^3 = 0 \quad R_B = \frac{5}{16} P \uparrow$$

$$R_A = P - \frac{5}{16} P \quad R_A = \frac{7}{16} P \uparrow$$

$$M_A = \frac{5}{16} PL - \frac{1}{2} PL \quad M_A = -\frac{3}{16} PL$$

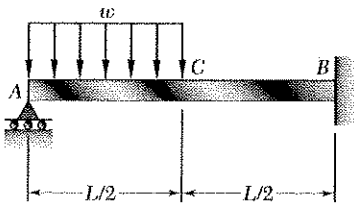
$$M_C = R_B (\frac{1}{2}L) = (\frac{5}{16} P) (\frac{1}{2}L) \quad M_C = \frac{5}{32} PL$$

$$M_B = 0$$



Problem 9.26

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



Reactions are statically indeterminate.

$$\uparrow \Sigma F_y = 0: R_A + R_B - \frac{1}{2}WL = 0 \quad R_B = \frac{1}{2}WL - R_A$$

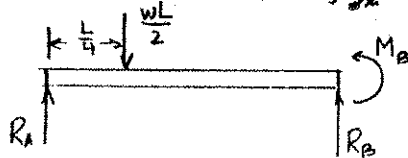
$$\rightarrow \Sigma M_A = 0: -R_A L + \left(\frac{WL}{2}\right)\left(\frac{3L}{4}\right) + M_B = 0$$

$$M_B = R_A L - \frac{3}{8}WL^2$$

$$[x=0, y=0]$$

$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$



$$0 < x \leq \frac{L}{2}: M = R_A x - \frac{1}{2}wx^2$$

$$EI \frac{d^2 y}{dx^2} = R_A x - \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$$EI y = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$$\frac{L}{2} \leq x < L: M = R_A x - \frac{WL}{2}\left(x - \frac{L}{4}\right)$$

$$EI \frac{d^2 y}{dx^2} = R_A x - WL\left(\frac{1}{2}x - \frac{1}{8}L\right)$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - WL\left(\frac{1}{4}x^2 - \frac{1}{8}Lx\right) + C_3$$

$$EI y = \frac{1}{6}R_A x^3 - WL\left(\frac{1}{12}x^3 - \frac{1}{16}Lx^2\right) + C_3 x + C_4$$

$$[x=0, y=0]$$

$$0 - 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$\frac{1}{8}R_A L^2 - \frac{1}{48}WL^3 + C_1 = \frac{1}{8}R_A L^2 - 0 + C_3$$

$$C_3 = C_1 - \frac{1}{48}WL^3$$

$$[x=\frac{L}{2}, y=y]$$

$$\frac{1}{48}R_A L^3 - \frac{1}{384}WL^4 + \frac{1}{2}C_1 L + 0$$

$$= \frac{1}{48}R_A L^3 - WL\left(\frac{1}{96}L^3 - \frac{1}{64}L^3\right) + \left(C_1 - \frac{1}{48}WL^3\right)\left(\frac{L}{2}\right) + C_4$$

$$C_4 = \frac{1}{384}WL^4$$

$$[x=L, \frac{dy}{dx}=0]$$

$$\frac{1}{2}R_A L^2 - WL\left(\frac{1}{4}L^2 - \frac{1}{8}L^2\right) + C_3 = 0$$

$$C_3 = \frac{1}{8}WL^3 - \frac{1}{2}R_A L^2$$

$$[x=L, y=0]$$

$$\frac{1}{6}R_A L^3 - WL\left(\frac{1}{12}L^3 - \frac{1}{16}L^3\right) + \left(\frac{1}{8}WL^3 - \frac{1}{2}R_A L^2\right)L + \frac{1}{384}WL^4 = 0$$

$$R_A = \frac{41}{128}WL \uparrow$$

$$R_B = \frac{1}{2}WL - \frac{41}{128}WL$$

$$R_B = \frac{23}{128}WL \uparrow$$

$$M_B = \frac{41}{128}WL^2 - \frac{3}{8}WL^2$$

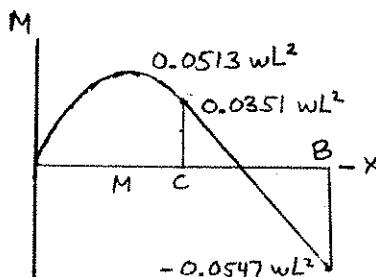
$$M_B = -\frac{7}{128}WL^2 = -0.0547WL^2$$

$$\text{Over } 0 < x < \frac{L}{2}, \quad V = R_A - wx = \frac{41}{128}WL - wx$$

$$V = 0 \text{ at } x = x_m = \frac{41}{128}L$$

$$M = \frac{41}{128}WLx - \frac{1}{2}wx^2$$

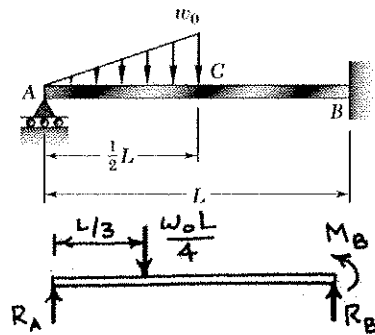
$$M_C = \frac{41}{128}WL^2 - \frac{1}{8}WL^2 = 0.0351WL^2, \quad M_m = \frac{41}{128}WL\left(\frac{41}{128}L\right) - \frac{1}{2}w\left(\frac{41}{128}L\right)^2 = 0.0513WL^2$$



Bending moment diagram

Problem 9.27

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



REACTIONS ARE STATICALLY INDETERMINATE.

$$+\uparrow \Sigma F_y = 0: R_A + R_B - \frac{w_0 L}{4} = 0 \quad R_B = \frac{w_0 L}{4} - R_A$$

$$+\circlearrowleft \Sigma M_B = 0: -R_A L + \left(\frac{w_0 L}{4}\right)\left(\frac{2L}{3}\right) + M_B = 0 \quad M_B = R_A L - \frac{w_0 L^2}{6}$$

$$0 \leq x \leq \frac{L}{2}: \quad w = \frac{2w_0}{L}x \quad V = R_A - \frac{w_0}{L}x^2 \quad M = R_A x - \frac{w_0}{3L}x^3$$

$$EI \frac{d^2 y}{dx^2} = R_A x - \frac{w_0}{3L}x^3$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{12}\frac{w_0}{L}x^4 + C_1$$

$$EI y = \frac{1}{6}R_A x^3 - \frac{1}{60}\frac{w_0}{L}x^5 + C_1 x + C_2$$

$$\frac{L}{2} \leq x \leq L: \quad M = R_A x - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right)$$

$$EI \frac{d^2 y}{dx^2} = R_A x - w_0 L \left(\frac{1}{4}x - \frac{1}{12}L\right)$$

$$EI \frac{dy}{dx} = \frac{1}{2}R_A x^2 - w_0 L \left(\frac{1}{8}x^2 - \frac{1}{12}Lx\right) + C_3$$

$$EI y = \frac{1}{6}R_A x^3 - w_0 L \left(\frac{1}{24}x^3 - \frac{1}{24}Lx^2\right) + C_3 x + C_4$$

$$[x=0, y=0]: \quad 0 = 0 - 0 + 0 + C_2 \quad \therefore C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}]: \quad \frac{1}{8}R_A L^2 - \frac{1}{192}w_0 L^3 + C_1 = \frac{1}{8}R_A L^2 + \frac{1}{96}w_0 L^3 + C_3 \quad \therefore C_3 = C_1 - \frac{1}{64}w_0 L^3$$

$$[x=\frac{L}{2}, y=y]: \quad \frac{1}{48}R_A L^3 - \frac{1}{1920}w_0 L^4 + \frac{1}{24}C_1 L = 0$$

$$= \frac{1}{48}R_A L^3 - w_0 L \left(\frac{1}{192}L^3 - \frac{1}{96}L^3\right) + \left(C_1 - \frac{1}{64}w_0 L^3\right)\left(\frac{L}{2}\right) + C_4 \quad \therefore C_4 = \frac{1}{480}w_0 L^4$$

$$[x=L, \frac{dy}{dx}=0]: \quad \frac{1}{2}R_A L^2 - w_0 L \left(\frac{1}{8}L^2 - \frac{1}{12}L^2\right) + C_3 = 0 \quad \therefore C_3 = -\frac{1}{2}R_A L^2 + \frac{1}{24}w_0 L^3$$

$$[x=L, y=0]: \quad \frac{1}{6}R_A L^3 - w_0 L \left(\frac{1}{24}L^3 - \frac{1}{24}L^3\right) + \left(-\frac{1}{2}R_A L^2 + \frac{1}{24}w_0 L^3\right)(L) + \frac{1}{480}w_0 L^4 = 0$$

$$R_A = \frac{21}{160}w_0 L \uparrow$$

$$R_B = \frac{w_0 L}{4} - \frac{21}{160}w_0 L$$

$$R_B = \frac{19}{160}w_0 L \uparrow$$

$$M_B = \frac{21}{160}w_0 L^2 - \frac{w_0 L^2}{6}$$

$$M_B = -\frac{17}{480}w_0 L^2 = -0.0354w_0 L^2$$

$$\text{OVER } 0 < x < \frac{L}{2}, \quad V = R_A - \frac{w_0}{L}x^2 = \frac{21}{160}w_0 L - \frac{w_0}{L}x^2$$

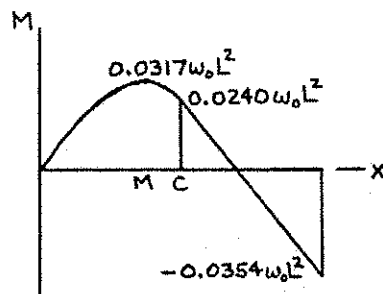
$$V=0 \text{ AT } x=x_m=0.36228L$$

$$M = \frac{21}{160}w_0 L x - \frac{w_0}{3L}x^3$$

$$M_A = M(x=0) = 0$$

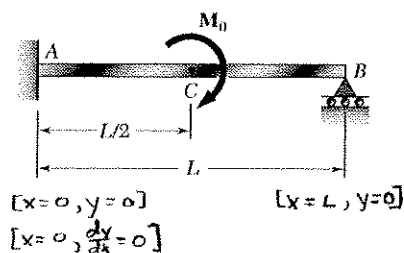
$$M_C = M(x=\frac{L}{2}) = 0.0240w_0 L^2$$

$$M_m = M(x_m=0.36228L) = 0.0317w_0 L^2$$



Problem 9.28

9.25 through 9.28 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

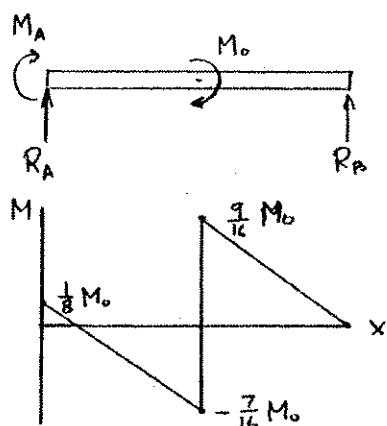


Reactions are statically indeterminate.

$$\uparrow \sum F_y = 0: R_A + R_B = 0 \quad R_A = -R_B$$

$$\circlearrowleft \sum M_A = 0: -M_A - M_o + R_B L = 0$$

$$M_A = R_B L - M_o$$



$$0 < x < \frac{L}{2}$$

$$M = R_o x + M_A = -M_o + R_B L - R_B x$$

$$EI \frac{d^2 y}{dx^2} = -M_o + R_B (L - x)$$

$$EI \frac{dy}{dx} = -M_o x + R_B (Lx - \frac{1}{2} x^2) + C_1$$

$$EI y = -\frac{1}{2} M_o x^2 + R_B (\frac{1}{2} L x^2 - \frac{1}{6} x^3) + C_1 x + C_2$$

$$\frac{L}{2} < x < L \quad M = R_B (L - x)$$

$$EI \frac{d^2 y}{dx^2} = R_B (L - x)$$

$$EI \frac{dy}{dx} = R_B (Lx - \frac{1}{2} x^2) + C_3$$

$$EI y = R_B (\frac{1}{2} L x^2 - \frac{1}{6} x^3) + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \quad -M_o \frac{L}{2} + R_B (\frac{1}{2} L^2 - \frac{1}{6} L^2) = R_B (\frac{1}{2} L^2 - \frac{1}{6} L^2) + C_3 \quad C_3 = -\frac{M_o L}{2}$$

$$[x=\frac{L}{2}, y=y] \quad -\frac{1}{2} M_o (\frac{L}{2})^2 + R_B (\frac{1}{8} L^3 - \frac{1}{48} L^3) = R_B (\frac{1}{8} L^3 - \frac{1}{48} L^3) + C_3 \frac{L}{2} + C_4$$

$$C_4 = -\frac{1}{8} M_o L^2 - \frac{1}{2} C_3 L = (-\frac{1}{8} + \frac{1}{4}) M_o L^2 = \frac{1}{8} M_o L^2$$

$$[x=L, y=0] \quad R_B (\frac{1}{2} L^3 - \frac{1}{6} L^3) + \frac{M_o L}{2} L + \frac{1}{8} M_o L^2 = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_B L^3 = (\frac{1}{2} - \frac{1}{8}) M_o L^2 \quad \frac{1}{3} R_B = \frac{3}{8} \frac{M_o}{L}$$

$$R_B = \frac{9}{8} \frac{M_o}{L} \uparrow$$

$$M_A = \frac{9}{8} M_o - M_o = \frac{1}{8} M_o$$

$$M_A = \frac{1}{8} M_o$$

$$M_{c-} = -M_o + \frac{9}{8} \frac{M_o}{L} \frac{L}{2} = -\frac{7}{16} M_o$$

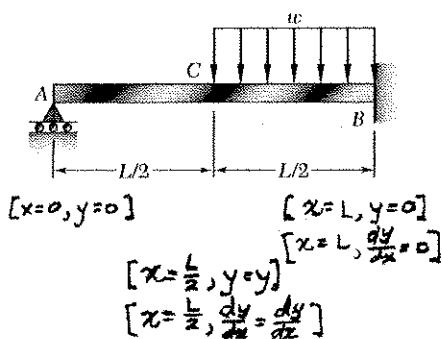
$$M_{c-} = -\frac{7}{16} M_o$$

$$M_{c+} = R_B (L - \frac{L}{2}) = \frac{9}{8} \frac{M_o}{L} (\frac{L}{2}) = \frac{9}{16} M_o$$

$$M_{c+} = \frac{9}{16} M_o$$

Problem 9.29

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point C.



Reactions are statically indeterminate.

$$0 < x \leq \frac{L}{2} \quad M = R_A x$$

$$EI \frac{d^2 y}{dx^2} = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$\frac{L}{2} \leq x < L$$

$$M = R_A x - w(x - \frac{L}{2}) \frac{1}{2}(x - \frac{L}{2}) = R_A x - \frac{1}{2} w(x - \frac{L}{2})^2$$

$$EI \frac{d^2 y}{dx^2} = R_A x - \frac{1}{2} w(x - \frac{L}{2})^2$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} w(x - \frac{L}{2})^3 + C_3$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} w(x - \frac{L}{2})^4 + C_3 x + C_4$$

$$[x=0, y=0] \quad 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x = \frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{8} R_A L^2 + C_1 = \frac{1}{8} R_A x^2 + C_3 \quad C_3 = C_1$$

$$[x = \frac{L}{2}, y = y] \quad \frac{1}{48} R_A L^3 + \frac{1}{2} C_1 L + 0 = \frac{1}{48} R_A x^3 - 0 + \frac{1}{2} C_1 L + C_4 \quad C_4 = 0$$

$$[x=L, \frac{dy}{dx} = 0] \quad \frac{1}{2} R_A L^2 - \frac{1}{24} w L^3 + C_3 = 0 \quad C_3 = -(\frac{1}{2} R_A L^2 - \frac{1}{24} w L^3)$$

$$[x=L, y=0] \quad \frac{1}{6} R_A L^3 - \frac{1}{384} w L^4 - (\frac{1}{2} R_A - \frac{1}{48} w L^3) L + 0$$

$$R_A = \frac{7}{128} w L \uparrow$$

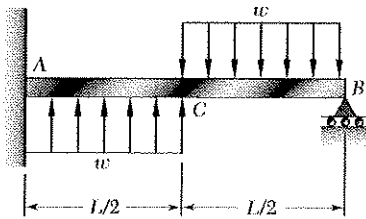
$$C_1 = C_3 = -(\frac{7}{256} w L^3 - \frac{1}{48} w L^3) = -\frac{5}{768} w L^3$$

$$\text{At } x = \frac{L}{2} \quad EI y_c = \frac{1}{6} R_A (\frac{L}{2})^3 + (-\frac{5}{768} w L^3) \frac{L}{2} + 0 = -\frac{13}{6144} w L^4$$

$$y_c = \frac{13}{6144} \frac{w L^4}{EI} \downarrow$$

Problem 9.30

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point C.



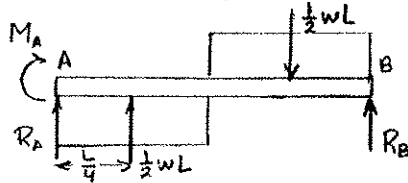
$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=L, y=0]$$

$$[x=\frac{L}{2}, y=y]$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}]$$



Reactions are statically indeterminate.

$$+\uparrow \sum F_y = 0: R_A + \frac{1}{2}wL - \frac{1}{2}wL + R_B = 0 \quad R_A = -R_B$$

$$\circlearrowleft \sum M_A = 0: -M_A - (\frac{1}{2}wL)(\frac{L}{2}) + R_B L = 0$$

$$M_A = R_B L - \frac{1}{4}wL^2$$

From A to C: $0 < x \leq \frac{L}{2}$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}wx^3 + C_1$$

$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wx^4 + C_1 x + C_2$$

From C to B: $\frac{L}{2} \leq x < L$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wL(x - \frac{L}{4}) - \frac{1}{2}w(x - \frac{L}{2})^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{4}wL(x - \frac{L}{4}) - \frac{1}{6}w(x - \frac{L}{2})^3 + C_3$$

$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wL(x - \frac{L}{4})^2 - \frac{1}{24}(x - \frac{L}{2})^4 + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \quad M_A \frac{L}{2} - \frac{1}{2}R_A (\frac{L}{2})^2 + \frac{1}{6}wL(\frac{L}{2})^3 = M_A \frac{L}{2} + \frac{1}{2}R_A (\frac{L}{2})^2 + \frac{1}{4}wL(\frac{L}{4})^2 - 0 + C_3$$

$$C_3 = (\frac{1}{48} - \frac{1}{64})wL^3 = \frac{1}{192}wL^3$$

$$[x=\frac{L}{2}, y=y] \quad \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{6}R_A (\frac{L}{2})^3 + \frac{1}{24}wL(\frac{L}{4})^3 - \frac{1}{24}(x - \frac{L}{2})^4 = \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{6}R_A (\frac{L}{2})^3 + \frac{1}{12}wL(\frac{L}{4})^3 - 0 + \frac{1}{192}wL^3(\frac{L}{2}) + C_4$$

$$C_4 = (\frac{1}{384} - \frac{1}{768} + \frac{1}{384})wL^4 = -\frac{1}{768}wL^4$$

$$[x=L, y=0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 + \frac{1}{12}wL(\frac{3L}{4})^3 - \frac{1}{24}w(\frac{L}{2})^4 + \frac{1}{192}wL^3(L) - \frac{1}{768}wL^4 = 0$$

$$\frac{1}{2}(R_B L - \frac{1}{4}wL^2)L^2 + \frac{1}{6}(-R_B)L^3 + (\frac{27}{768} - \frac{1}{384} + \frac{1}{192} - \frac{1}{768})wL^4 = 0$$

$$(\frac{1}{2} - \frac{1}{6})R_B L^3 = (\frac{1}{8} - \frac{7}{192})wL^4 \quad \frac{1}{3}R_B = \frac{17}{192}wL \quad R_B = \frac{17}{64}wL \uparrow$$

$$R_A = -R_B = -\frac{17}{64}wL$$

$$M_A = R_B L - \frac{1}{4}wL^2 = (\frac{17}{64} - \frac{1}{4})wL^2 = \frac{1}{64}wL^2$$

(b) Deflection at C. (y at $x = \frac{L}{2}$)

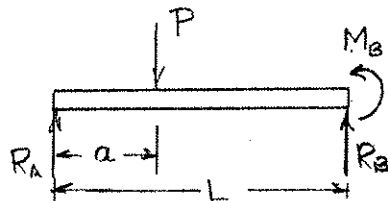
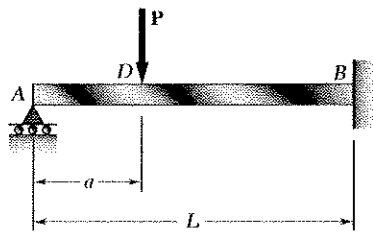
$$EI y_C = \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{6}R_A (\frac{L}{2})^3 + \frac{1}{24}wL(\frac{L}{4})^3 = \frac{1}{2}(\frac{1}{64}wL^2)(\frac{L}{2})^2 + \frac{1}{6}(-\frac{17}{64}wL)(\frac{L}{2})^3 + \frac{1}{24}wL(\frac{L}{4})^3$$

$$= (\frac{1}{512} - \frac{17}{3072} + \frac{1}{384})wL^4 = -\frac{1}{1024}wL^4$$

$$y_C = \frac{1}{1024} \frac{wL^4}{EI} \downarrow$$

Problem 9.31

9.31 and 9.32 Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.



$$0 \leq x \leq a:$$

$$M = R_A x$$

$$EI \frac{d^2 y}{dx^2} = M = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$a \leq x \leq L:$$

$$M = R_A x - P(x-a)$$

$$EI \frac{d^2 y}{dx^2} = M = R_A x - P(x-a)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P(x-a)^2 + C_3$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{6} P(x-a)^3 + C_3 x + C_4$$

$$[x=0, y=0]: 0 + 0 + C_2 = 0$$

$$\therefore C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]: \frac{1}{2} R_A a^2 + C_1 = \frac{1}{2} R_A a^2 - 0 + C_3$$

$$\therefore C_1 = C_3$$

$$[x=a, y=y]: \frac{1}{6} R_A a^3 + C_1 a + 0 = \frac{1}{6} R_A a^3 - 0 + C_1 a + C_4$$

$$\therefore C_4 = 0$$

$$[x=L, \frac{dy}{dx} = 0]: \frac{1}{2} R_A L^2 - \frac{1}{2} P(L-a)^2 + C_3 = 0$$

$$\therefore C_3 = \frac{1}{2} P(L-a)^2 - \frac{1}{2} R_A L^2$$

$$[x=L, y=0]: \frac{1}{6} R_A L^3 - \frac{1}{6} P(L-a)^3 + [\frac{1}{2} P(L-a)^2 - \frac{1}{2} R_A L^2](L) + 0 = 0$$

$$R_A = \frac{P}{2L^3} (2L^3 - 3aL^2 + a^3) = \frac{P}{2L^3} (2L^3 - L^3 + \frac{L^3}{9})$$

$$R_A = \frac{14}{27} P \uparrow$$

DEFLECTION AT D. (y AT $x = a = \frac{L}{3}$)

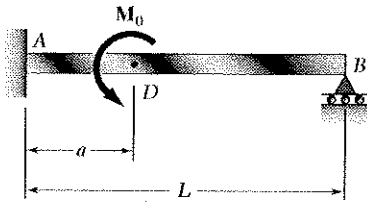
$$y_D = \frac{1}{EI} \left\{ \frac{1}{6} R_A \left(\frac{L}{3} \right)^3 + C_1 \left(\frac{L}{3} \right) \right\}$$

$$= \frac{1}{EI} \left\{ \frac{1}{6} \left(\frac{14}{27} P \right) \left(\frac{L}{3} \right)^3 + \left[\frac{1}{2} P \left(L - \frac{L}{3} \right)^2 - \frac{1}{2} \left(\frac{14}{27} P \right) L^2 \right] \left(\frac{L}{3} \right) \right\} = - \frac{20}{2187} \frac{PL^3}{EI}$$

$$y_D = \frac{20}{2187} \frac{PL^3}{EI} \downarrow$$

Problem 9.32

9.31 and 9.32 Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.



$$+\uparrow \Sigma F_y = 0: R_A + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: M_o - M_A + R_B L = 0 \quad M_A = R_B L + M_o$$

$$0 \leq x \leq a: M = M_A + R_A x$$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$a \leq x \leq L: M = M_A + R_A x - M_o$$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x - M_o$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - M_o x + C_3$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{2} M_o x^2 + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0]: 0+0+C_1=0 \quad \therefore C_1=0$$

$$[x=0, y=0]: 0+0+0+C_2=0 \quad \therefore C_2=0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]: M_A a + \frac{1}{2} R_A a^2 = M_A a + \frac{1}{2} R_A a^2 - M_o a + C_3 \quad \therefore C_3 = M_o a$$

$$[x=a, y=y]: \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 = \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 - \frac{1}{2} M_o a^2 + (M_o a)(a) + C_4 \quad \therefore C_4 = -\frac{1}{2} M_o a^2$$

$$[x=L, y=0]: \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{2} M_o L^2 + (M_o a)(L) - \frac{1}{2} M_o a^2 = 0$$

$$\frac{1}{2} (R_B L + M_o) L^2 + \frac{1}{6} (-R_B) L^3 - \frac{1}{2} M_o L^2 + M_o a L - \frac{1}{2} M_o a^2 = 0$$

$$R_B = \frac{3M_o a}{2L^3} (a - 2L) = \frac{3M_o}{2L^3} \left(\frac{L}{3}\right) \left(\frac{L}{3} - 2L\right) = -\frac{5M_o}{6L}$$

$$R_B = \frac{5M_o}{6L} \downarrow$$

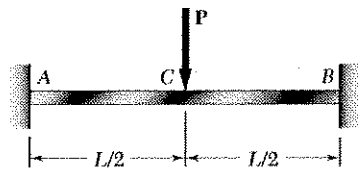
DEFLECTION AT D. (y AT $x=a=\frac{L}{3}$)

$$y_D = \frac{1}{EI} \left\{ \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 \right\} = \frac{1}{EI} \left\{ \frac{1}{2} \left(-\frac{5M_o}{6L} L + M_o \right) \left(\frac{L}{3} \right)^2 + \frac{1}{6} \left(+\frac{5M_o}{6L} \right) \left(\frac{L}{3} \right)^3 \right\}$$

$$= \frac{7M_o L^2}{486EI}$$

$$y_D = \frac{7M_o L^2}{486EI} \uparrow$$

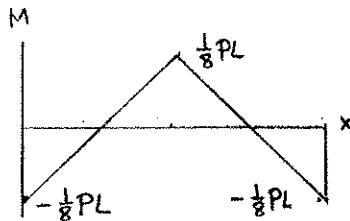
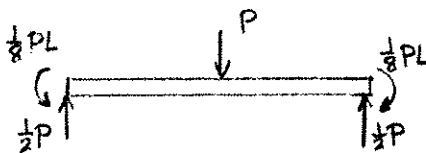
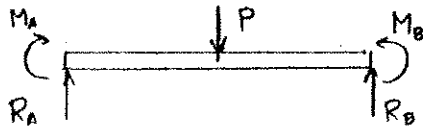
Problem 9.33



$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0]$$



9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

By symmetry, $R_A = R_B$ and $\frac{dy}{dx} = 0$ at $x = \frac{L}{2}$.

$$+\uparrow \Sigma F_y = 0: R_A + R_B - P = 0 \quad R_A = R_B = \frac{1}{2}P \quad \blacktriangleleft$$

Moment reaction is statically indeterminate.

$$0 < x < \frac{L}{2}$$

$$M = M_A + R_A x = M_A + \frac{1}{2}Px$$

$$EI \frac{d^2 y}{dx^2} = M_A + \frac{1}{2}Px$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$$

$$[x=0, \frac{dy}{dx}=0]$$

$$0 - 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=0]$$

$$M_A \frac{L}{2} + \frac{1}{4}P(\frac{L}{2})^2 + 0 = 0$$

$$M_A = -\frac{1}{8}PL$$

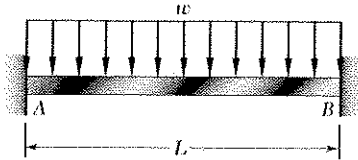
$$M_A = \frac{1}{8}PL \quad \blacktriangleleft$$

$$\text{By symmetry, } M_B = M_A = \frac{1}{8}PL \quad \blacktriangleleft$$

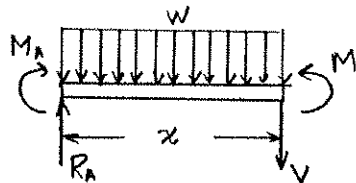
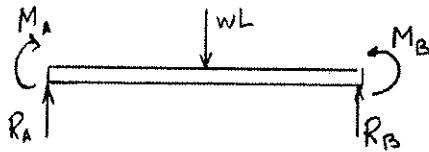
$$M_C = M_A + \frac{1}{2}P \frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL = \frac{1}{8}PL \quad \blacktriangleleft$$

Problem 9.34

9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



$$\begin{aligned} [x=0, y=0] & \quad [x=L, y=0] \\ [x=0, \frac{dy}{dx}=0] & \quad [x=L, \frac{dy}{dx}=0] \end{aligned}$$



Reactions are statically indeterminate.

$$\text{By symmetry, } R_B = R_A; \quad M_B = M_A$$

$$\frac{dy}{dx} = 0 \quad \text{at } x = \frac{L}{2}$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - WL = 0$$

$$R_B = R_A = \frac{1}{2}WL \quad \uparrow$$

$$\text{Over entire beam, } M = M_A + R_A x - \frac{1}{2}wx^2$$

$$EI \frac{d^2y}{dx^2} = M_A + \frac{1}{2}WLx - \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{4}WLx^2 - \frac{1}{6}wx^3 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 - 0 + C_1 = 0$$

$$C_1 = 0$$

$$[x = \frac{L}{2}, \frac{dy}{dx} = 0] \quad \frac{1}{2}M_A L + \frac{1}{16}WL^3 - \frac{1}{48}wL^3 + 0 = 0$$

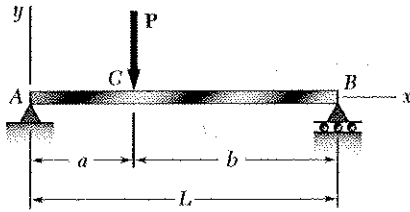
$$M_A = -\frac{1}{12}wL^2 \quad \blacktriangleleft$$

$$M = -\frac{1}{12}wL^2 + \frac{1}{2}WLx - \frac{1}{2}wx^2$$

$$M = w \cdot [6x(L-x) - L^2] / 12 \quad \blacktriangleleft$$

Problem 9.35

9.35 and 9.36 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection of point C.

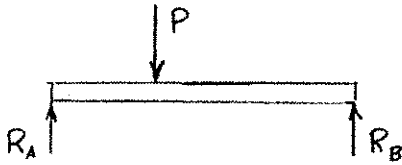


$$[x=0, M=0]$$

$$[x=0, y=0]$$

$$[x=L, M=0]$$

$$[x=L, y=0]$$



$$+\circlearrowleft \sum M_B = 0: -R_A L + Pb = 0 \quad R_A = \frac{Pb}{L}$$

$$\frac{dM}{dx} = V = R_A - P\langle x-a \rangle^0 = \frac{Pb}{L} - P\langle x-a \rangle^0$$

$$M = \frac{Pb}{L}x - P\langle x-a \rangle^1 + M_A$$

$$EI \frac{d^2y}{dx^2} = \frac{Pb}{L}x - P\langle x-a \rangle^0$$

$$EI \frac{dy}{dx} = \frac{Pb}{2L}x^2 - \frac{1}{2}P\langle x-a \rangle^2 + C_1$$

$$EI y = \frac{Pb}{6L}x^3 - \frac{1}{6}P\langle x-a \rangle^3 + C_1x + C_2$$

$$[x=0, y=0] \quad C_2 = 0$$

$$[x=L, y=0] \quad \frac{Pb}{6L}L^3 - \frac{1}{6}P(L-a)^3 + C_1L = 0$$

$$C_1 = -\frac{1}{6}P \left(bL^2 - b^3 \right) = -\frac{1}{6} \frac{Pb}{L} (L^2 - b^2)$$

(a) Elastic curve.

$$y = \frac{P}{EI} \left\{ \frac{b}{6L}x^3 - \frac{1}{6}\langle x-a \rangle^3 - \frac{1}{6} \frac{b}{L}(L^2 - b^2)x \right\}$$

$$y = \frac{P}{6EIL} \left\{ bx^3 - L\langle x-a \rangle^3 - b(L^2 - b^2)x \right\} \quad \blacktriangleleft$$

(b) Slope at end A.

$$EI \left. \frac{dy}{dx} \right|_{x=0} = C_1 = -\frac{Pb}{6L} (L^2 - b^2)$$

$$\theta_A = -\frac{Pb}{6EIL} (L^2 - b^2)$$

$$\theta_A = \frac{Pb}{6EIL} (L^2 - b^2) \quad \blacktriangleleft \quad \blacktriangleleft$$

(c) Deflection at C.

$$EI y_c = \frac{Pb}{6L} a^3 + C_1 a = \frac{Pba^3}{6L} - \frac{Pb(L^2 - b^2)a}{6L}$$

$$= \frac{Pba}{6L} (a^2 - L^2 + b^2)$$

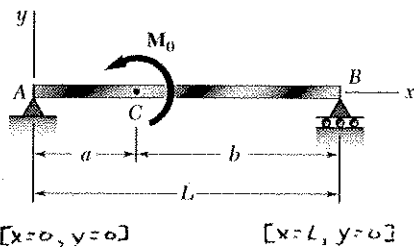
$$y_c = -\frac{Pab}{6EIL} (L^2 - a^2 - b^2) = -\frac{Pab}{6EIL} \{ a^2 + 2ab + b^2 - a^2 - b^2 \}$$

$$= -\frac{Pa^2b^2}{3EIL}$$

$$y_c = \frac{Pa^2b^2}{3EIL} \quad \downarrow \quad \blacktriangleleft$$

Problem 9.36

9.35 and 9.36 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection of point C.



Reactions $R_A = \frac{M_o}{L} \uparrow$, $R_B = \frac{M_o}{L} \downarrow$

$0 < x < a$ $M = R_A x$

$a < x < L$ $M = R_A x - M_o$

Using singularity functions,

$$EI \frac{d^2 y}{dx^2} = M = R_A x - M_o \langle x-a \rangle^0$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_o \langle x-a \rangle' + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_o \langle x-a \rangle^2 + C_1 x + C_2$$

$[x=0, y=0]$ $0 = 0 - 0 + 0 + C_2$ $C_2 = 0$

$[x=L, y=0]$ $\frac{1}{6} R_A L^3 - \frac{1}{2} M_o (L-a)^2 + C_1 L + 0 = 0$

$$C_1 L = -\frac{1}{6} \frac{M_o}{L} L^3 + \frac{1}{2} M_o b^2 \quad C_1 = \frac{M_o}{6L} (3b^2 - L^2)$$

(a) Elastic curve. $y = \frac{1}{EI} \left\{ \frac{1}{6} \frac{M_o}{L} x^3 - \frac{1}{2} M_o \langle x-a \rangle^2 + \frac{M_o}{6L} (3b^2 - L^2) x \right\}$

$$= \frac{M_o}{6EIL} \left\{ x^3 - 3L \langle x-a \rangle^2 + (3b^2 - L^2) x \right\}$$

$$\frac{dy}{dx} = \frac{M_o}{6EIL} \left\{ 3x^2 - 6L \langle x-a \rangle' + (3b^2 - L^2) \right\}$$

(b) Slope at A. $\left(\frac{dy}{dx} \text{ at } x=0 \right)$

$$\theta_A = \frac{M_o}{6EIL} \left\{ 0 - 0 + 3Lb^2 - L^3 \right\}$$

$$\theta_A = \frac{M_o}{6EIL} (3b^2 - L^2)$$

(c) Deflection at C. $(y \text{ at } x=a)$

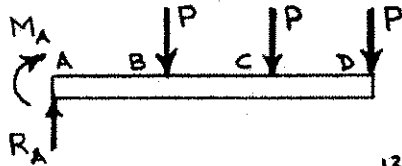
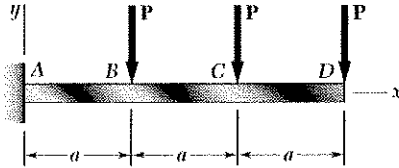
$$y_c = \frac{M_o}{6EIL} \left\{ a^3 - 0 + (3b^2 - L^2) a \right\} = \frac{M_o a}{6EIL} \left\{ a^2 + 3b^2 - (a+b)^2 \right\}$$

$$= \frac{M_o a}{6EIL} \left\{ a^2 + 3b^2 - a^2 - 2ab - b^2 \right\} = \frac{M_o a}{6EIL} \left\{ 2b^2 - 2ab \right\}$$

$$y_c = \frac{M_o ab}{3EIL} (b-a) \uparrow$$

Problem 9.37

9.37 and 9.38 For the beam and loading shown, determine the deflection at (a) point B, (b) point C, (c) point D.



$$+\uparrow \Sigma F_y = 0: R_A - P - P - P = 0 \quad R_A = 3P$$

$$+\circlearrowleft \Sigma M_A = 0: -M_A - Pa - P(2a) - P(3a) = 0 \quad M_A = -6Pa$$

$$\frac{dM}{dx} = V = 3P - P\langle x-a \rangle^0 - P\langle x-2a \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = 3Px - P\langle x-a \rangle^1 - P\langle x-2a \rangle^1 - 6Pa$$

$$EI \frac{dy}{dx} = \frac{3}{2}Px^2 - \frac{1}{2}P\langle x-a \rangle^2 - \frac{1}{2}P\langle x-2a \rangle^2 - 6Pax + C_1$$

$$[x=0, \frac{dy}{dx}=0]: 0 - 0 - 0 - 0 + C_1 = 0 \quad \therefore C_1 = 0$$

$$EI y = \frac{1}{2}Px^3 - \frac{1}{6}P\langle x-a \rangle^3 - \frac{1}{6}P\langle x-2a \rangle^3 - 3Pax^2 + C_2$$

$$[x=0, y=0]: 0 - 0 - 0 - 0 + C_2 = 0 \quad \therefore C_2 = 0$$

$$\text{ELASTIC CURVE: } y = \frac{P}{6EI} [3x^3 - \langle x-a \rangle^3 - \langle x-2a \rangle^3 - 18ax^2]$$

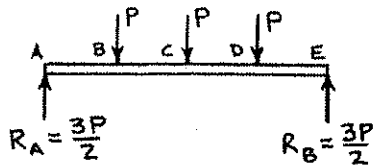
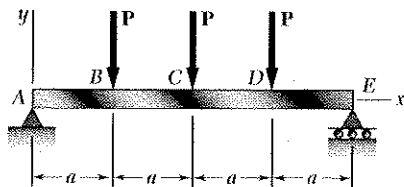
$$(a) \ x = a: \quad y_B = \frac{Pa^3}{6EI} [3 - 0 - 0 - 18] \quad y_B = \frac{5Pa^3}{2EI} \downarrow$$

$$(b) \ x = 2a: \quad y_C = \frac{Pa^3}{6EI} [24 - 1 - 0 - 72] \quad y_C = \frac{49Pa^3}{6EI} \downarrow$$

$$(c) \ x = 3a: \quad y_D = \frac{Pa^3}{6EI} [81 - 8 - 1 - 162] \quad y_D = \frac{15Pa^3}{EI} \downarrow$$

Problem 9.38

9.37 and 9.38 For the beam and loading shown, determine the deflection at (a) point B, (b) point C, (c) point D.



$$\frac{dM}{dx} = V = \frac{3P}{2} - P\langle x-a \rangle^0 - P\langle x-2a \rangle^0 - P\langle x-3a \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = \frac{3P}{2}x - P\langle x-a \rangle^1 - P\langle x-2a \rangle^1 - P\langle x-3a \rangle^1$$

$$EI \frac{dy}{dx} = \frac{3P}{4}x^2 - \frac{1}{2}P\langle x-a \rangle^2 - \frac{1}{2}P\langle x-2a \rangle^2 - \frac{1}{2}P\langle x-3a \rangle^2 + C_1$$

$$EI y = \frac{P}{4}x^3 - \frac{1}{6}P\langle x-a \rangle^3 - \frac{1}{6}P\langle x-2a \rangle^3 - \frac{1}{6}P\langle x-3a \rangle^3 + C_1x + C_2$$

$$[x=0, y=0]: 0 - 0 - 0 - 0 + 0 + C_2 = 0 \quad \therefore C_2 = 0$$

$$[x=4a, y=0]: 16Pa^3 - \frac{9}{2}Pa^3 - \frac{4}{3}Pa^3 - \frac{1}{6}Pa^3 + 4aC_1 = 0$$

$$\therefore C_1 = -\frac{5}{2}Pa^2$$

ELASTIC CURVE:

$$y = \frac{P}{12EI} [3x^3 - 2\langle x-a \rangle^3 - 2\langle x-2a \rangle^3 - 2\langle x-3a \rangle^3 - 30a^2x]$$

$$(a) \ x=a: y_B = \frac{Pa^3}{12EI} [3 - 0 - 0 - 0 - 30]$$

$$y_B = \frac{9Pa^3}{4EI} \downarrow$$

$$(b) \ x=2a: y_C = \frac{Pa^3}{12EI} [24 - 2 - 0 - 0 - 60]$$

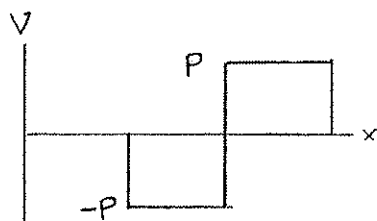
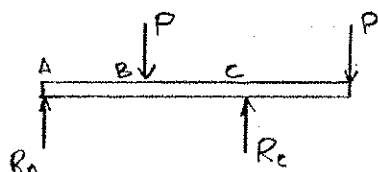
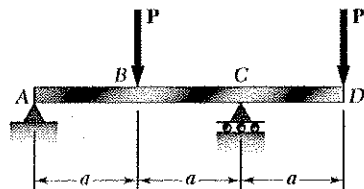
$$y_C = \frac{19Pa^3}{6EI} \downarrow$$

$$(c) \ x=3a: y_D = \frac{Pa^3}{12EI} [81 - 16 - 2 - 0 - 90]$$

$$y_D = \frac{9Pa^3}{4EI} \downarrow$$

Problem 9.39

9.39 and 9.40 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point B, (c) the deflection at end D.



$$+\circlearrowleft \Sigma M_c = 0: -2aR_A + aP - aP = 0 \quad R_A = 0$$

$$+\uparrow \Sigma F_y = 0: R_A + R_C - P - P = 0 \quad R_C = 2P \uparrow$$

$$\frac{dM}{dx} = V = -P\langle x-a \rangle^0 + 2P\langle x-2a \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = -P\langle x-a \rangle^1 + 2P\langle x-2a \rangle^1$$

$$EI \frac{dy}{dx} = -\frac{1}{2}P\langle x-a \rangle^2 + P\langle x-2a \rangle^2 + C_1$$

$$EI y = -\frac{1}{6}P\langle x-a \rangle^3 + \frac{1}{3}P\langle x-2a \rangle^3 + C_1 x + C_2$$

$$[x=0, y=0] \quad -0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=2a, y=0] \quad -\frac{1}{6}Pa^3 + 0 + 0 + C_1(2a) + 0 = 0$$

$$C_1 = \frac{1}{12}Pa^2$$

$$EI y = -\frac{1}{6}P\langle x-a \rangle^3 + \frac{1}{3}P\langle x-2a \rangle^3 + \frac{1}{12}Pa^2 x$$

Elastic curve.

$$y = \frac{P}{EI} \left\{ -\frac{1}{6}\langle x-a \rangle^3 + \frac{1}{3}\langle x-2a \rangle^3 + \frac{1}{12}a^2 x \right\}$$

$$\frac{dy}{dx} = \frac{P}{EI} \left\{ -\frac{1}{2}\langle x-a \rangle^2 + \langle x-2a \rangle^2 + \frac{1}{12}a^2 \right\}$$

(a) Slope at end A. $\left(\frac{dy}{dx} \text{ at } x=0 \right)$

$$\left. \frac{dy}{dx} \right|_A = \frac{P}{EI} \left\{ -0 + 0 + \frac{1}{12}a^2 \right\} = \frac{Pa^2}{12EI}$$

$$\theta_A = \frac{Pa^2}{12EI} \quad \swarrow \quad \blacktriangle$$

(b) Deflection at point B. $(y \text{ at } x=a)$

$$y_B = \frac{P}{EI} \left\{ -0 + 0 + \frac{1}{12}a^3 \right\} = \frac{Pa^3}{12EI}$$

$$y_B = \frac{Pa^3}{12EI} \quad \uparrow \quad \blacktriangle$$

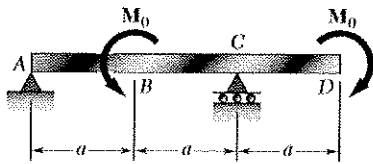
(c) Deflection at end D. $(y \text{ at } x=3a)$

$$y_D = \frac{P}{EI} \left\{ -\frac{1}{6}(2a)^3 + \frac{1}{3}a^3 + \left(\frac{1}{12}a^2 \right)(3a) \right\} = -\frac{3Pa^3}{4EI}$$

$$y_D = \frac{3Pa^3}{4EI} \quad \downarrow \quad \blacktriangle$$

Problem 9.40

9.39 and 9.40 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point B, (c) the deflection at end D.



Loading is self-equilibrated. $R_A = R_C = 0$

$$EI \frac{d^2 y}{dx^2} = M = -M_0 \langle x-a \rangle^0$$

$$EI \frac{dy}{dx} = -M_0 \langle x-a \rangle' + C_1$$

$$EI y = -\frac{1}{2} M_0 \langle x-a \rangle^2 + C_1 x + C_2$$

$$[x=0, y=0]$$

$$0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=2a, y=0]$$

$$-\frac{1}{2} M_0 a^2 + 0 + C_1 (2a)$$

$$C_1 = \frac{1}{4} M_0 a$$

$$EI y = -\frac{1}{2} M_0 \langle x-a \rangle^2 + \frac{1}{4} M_0 a x$$

Elastic curve.

$$y = \frac{M_0}{EI} \left\{ -\frac{1}{2} \langle x-a \rangle^2 + \frac{1}{4} a x \right\}$$

$$\frac{dy}{dx} = \frac{M_0}{EI} \left\{ -\langle x-a \rangle' + \frac{1}{4} a \right\}$$

(a) Slope at end A. ($\frac{dy}{dx}$ at $x=0$)

$$\left. \frac{dy}{dx} \right|_A = \frac{M_0 a}{4EI}$$

$$\theta_A = \frac{M_0 a}{4EI} \nearrow$$

(b) Deflection at point B. (y at $x=a$)

$$y_B = \frac{M_0}{EI} \left\{ -0 + \left(\frac{1}{4} a \right) (a) \right\}$$

$$y_B = \frac{M_0 a^2}{4EI} \uparrow$$

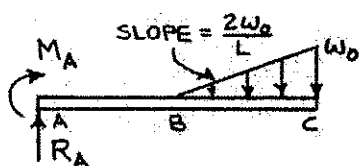
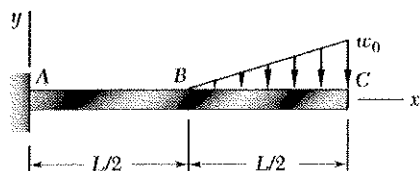
(c) Deflection at end D. (y at $x=3a$)

$$y_D = \frac{M_0}{EI} \left\{ -\frac{1}{2} (2a)^2 + \left(\frac{1}{4} a \right) (3a) \right\} = -\frac{5M_0 a^2}{4EI}$$

$$y_D = \frac{5M_0 a^2}{4EI} \downarrow$$

Problem 9.41

9.41 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at point B, (c) the deflection at point C.



$$+\uparrow \Sigma F_y = 0: R_A - \frac{1}{2}(w_0)\left(\frac{L}{2}\right) = 0 \quad R_A = \frac{1}{4}w_0L$$

$$+\circlearrowleft \Sigma M_A = 0: -M_A - \frac{1}{2}(w_0)\left(\frac{L}{2}\right)\left(\frac{5L}{6}\right) = 0 \quad M_A = -\frac{5}{24}w_0L^2$$

USING SINGULARITY FUNCTIONS

$$w = \frac{2w_0}{L} \left\langle x - \frac{L}{2} \right\rangle'$$

$$\frac{dV}{dx} = -w = -\frac{2w_0}{L} \left\langle x - \frac{L}{2} \right\rangle'$$

$$\frac{dM}{dx} = V = \frac{1}{4}w_0L - \frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^2$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{5}{24}w_0L^2 + \frac{1}{4}w_0Lx - \frac{1}{3}\frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^3$$

$$EI \frac{dy}{dx} = -\frac{5}{24}w_0L^2x + \frac{1}{8}w_0Lx^2 - \frac{1}{12}\frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^4 + C_1$$

$$[x=0, \frac{dy}{dx}=0]: -0 + 0 - 0 + C_1 = 0 \quad \therefore C_1 = 0$$

$$EI y = -\frac{5}{48}w_0L^2x^2 + \frac{1}{24}w_0Lx^3 - \frac{1}{60}\frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^5 + C_2$$

$$[x=0, y=0]: -0 + 0 - 0 + C_2 = 0 \quad \therefore C_2 = 0$$

(a) ELASTIC CURVE. $y = \frac{w_0}{EIL} \left[-\frac{5}{48}L^3x^2 + \frac{1}{24}L^2x^3 - \frac{1}{60} \left\langle x - \frac{L}{2} \right\rangle^5 \right]$

$$\frac{dy}{dx} = \frac{w_0}{EIL} \left[-\frac{5}{24}L^3x + \frac{1}{8}L^2x^2 - \frac{1}{12} \left\langle x - \frac{L}{2} \right\rangle^4 \right]$$

(b) DEFLECTION AT B. (y AT $x = \frac{L}{2}$)

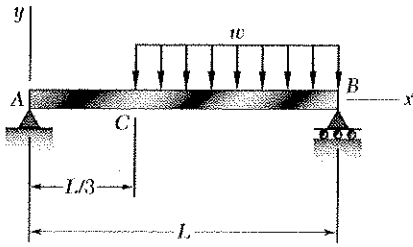
$$y_B = \frac{w_0L^4}{EI} \left[-\frac{5}{48} \left(\frac{1}{2}\right)^2 + \frac{1}{24} \left(\frac{1}{2}\right)^3 - 0 \right] = -\frac{4}{192} \frac{w_0L^4}{EI}, \quad y_B = \frac{1}{48} \frac{w_0L^4}{EI} \downarrow$$

(c) DEFLECTION AT C. (y AT $x = L$)

$$y_C = \frac{w_0L^4}{EI} \left[-\frac{5}{48}(1)^2 + \frac{1}{24}(1)^3 - \frac{1}{60} \left(\frac{1}{2}\right)^5 \right] = -\frac{121}{1920} \frac{w_0L^4}{EI}, \quad y_C = \frac{121}{1920} \frac{w_0L^4}{EI} \downarrow$$

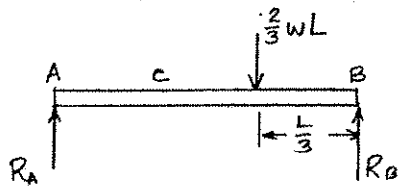
Problem 9.42

9.42 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at point A, (c) the deflection at point C.



Use free body ACB with the distributed load replaced by equivalent concentrated load.

$$\begin{aligned} \sum M_B = 0: & -R_A L + \left(\frac{2}{3} w L\right) \left(\frac{L}{3}\right) = 0 \\ R_A &= \frac{2}{9} w L \end{aligned}$$



$$\frac{dV}{dx} = -w = -w \left\langle x - \frac{L}{3} \right\rangle^0$$

$$\frac{dM}{dx} = V = \frac{2}{9} w L - w \left\langle x - \frac{L}{3} \right\rangle^1$$

$$EI \frac{d^2 y}{dx^2} = M = \frac{2}{9} w L x - \frac{1}{2} w \left\langle x - \frac{L}{3} \right\rangle^2$$

$$EI \frac{dy}{dx} = \frac{1}{9} w L x^2 - \frac{1}{6} w \left\langle x - \frac{L}{3} \right\rangle^3 + C_1$$

$$EI y = \frac{1}{27} w L x^3 - \frac{1}{24} w \left\langle x - \frac{L}{3} \right\rangle^4 + C_1 x + C_2$$

$$[x = 0, y = 0]$$

$$0 - 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x = L, y = 0]$$

$$\left(\frac{1}{27} w L\right) L^3 - \frac{1}{24} \left(\frac{2L}{3}\right)^4 + C_1 L + 0 = 0$$

$$C_1 = -\frac{7}{243} w L^3$$

$$EI y = \frac{1}{27} w L x^3 - \frac{1}{24} w \left\langle x - \frac{L}{3} \right\rangle^4 - \frac{7}{243} w L^3 x$$

(a) Elastic curve.

$$y = \frac{w}{EI} \left\{ \frac{1}{27} L x^3 - \frac{1}{24} \left\langle x - \frac{L}{3} \right\rangle^4 - \frac{7}{243} L^3 x \right\}$$

$$\frac{dy}{dx} = \frac{w}{EI} \left\{ \frac{1}{9} L x^2 - \frac{1}{6} \left\langle x - \frac{L}{3} \right\rangle^3 - \frac{7}{243} L^3 \right\}$$

(b) Slope at point A. $\left(\frac{dy}{dx} \text{ at } x = 0\right)$

$$\left. \frac{dy}{dx} \right|_A = \frac{w}{EI} \left\{ 0 - 0 - \frac{7}{243} L^3 \right\} = -\frac{7 w L^3}{243 EI}$$

$$\theta_A = \frac{7 w L^3}{243 EI}$$

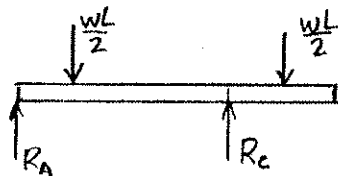
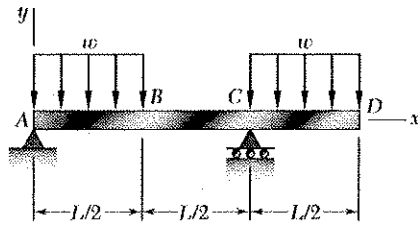
(c) Deflection at point C. $(y \text{ at } x = \frac{L}{3})$

$$y_C = \frac{w}{EI} \left\{ \left(\frac{1}{27} L\right) \left(\frac{L}{3}\right)^3 - 0 - \left(\frac{7}{243} L^3\right) \left(\frac{L}{3}\right) \right\} = -\frac{2 w L^4}{243 EI}$$

$$y_C = \frac{2 w L^4}{243 EI}$$

Problem 9.43

9.43 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at point B, (c) the deflection at point D.



Use free body ABCD with the distributed loads replaced by equivalent concentrated loads.

$$+\circlearrowleft \sum M_C = 0: -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{1}{4}wL$$

$$+\circlearrowleft \sum M_A = 0: R_C L - \left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) - \left(\frac{wL}{2}\right)\left(\frac{5L}{4}\right) = 0$$

$$R_C = \frac{3}{4}wL$$

$$\frac{dV}{dx} = -w = -w + w\langle x - \frac{L}{2} \rangle^0 - w\langle x - L \rangle^0$$

Integrating and adding terms to account for the reactions,

$$\frac{dM}{dx} = V = -wx + w\langle x - \frac{L}{2} \rangle' - w\langle x - L \rangle' + R_A + R_C\langle x - L \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2 + \frac{1}{2}w\langle x - \frac{L}{2} \rangle^2 - \frac{1}{2}w\langle x - L \rangle^2 + R_A x + R_C\langle x - L \rangle^1$$

$$EI \frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}w\langle x - \frac{L}{2} \rangle^3 - \frac{1}{6}w\langle x - L \rangle^3 + \frac{1}{2}R_A x^2 + \frac{1}{2}R_C\langle x - L \rangle^2 + C_1$$

$$EI y = -\frac{1}{24}wx^4 + \frac{1}{24}w\langle x - \frac{L}{2} \rangle^4 - \frac{1}{24}w\langle x - L \rangle^4 + \frac{1}{6}R_A x^3 + \frac{1}{6}R_C\langle x - L \rangle^3 + C_1 x + C_2$$

$$[x=0, y=0] \quad -0 + 0 - 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, y=0] \quad -\frac{1}{24}wL^4 + \frac{1}{24}w\left(\frac{L}{2}\right)^4 - 0 + \frac{1}{6}\left(\frac{wL}{4}\right)L^3 + 0 + C_1 L + 0 = 0 \quad C_1 = -\frac{1}{384}wL^3$$

$$EI y = -\frac{1}{24}wx^4 + \frac{1}{24}w\langle x - \frac{L}{2} \rangle^4 + \frac{1}{24}w\langle x - L \rangle^4 + \frac{1}{6}\left(\frac{wL}{4}\right)x^3 + \frac{1}{6}\left(\frac{3wL}{4}\right)\langle x - L \rangle^3 - \frac{1}{384}wL^3 x$$

(a) Elastic curve.

$$y = \frac{w}{24EI} \left\{ -x^4 + \langle x - \frac{L}{2} \rangle^4 - \langle x - L \rangle^4 + Lx^3 + 3L\langle x - L \rangle^3 - \frac{1}{16}L^3 x \right\}$$

(b) Deflection at B. (y at $x = \frac{L}{2}$)

$$y_B = \frac{w}{24EI} \left\{ -\left(\frac{L}{2}\right)^4 + 0 - 0 + (L)\left(\frac{L}{2}\right)^3 + 0 - \left(\frac{1}{16}L^3\right)\left(\frac{L}{2}\right) \right\} \quad y_B = \frac{wL^4}{768EI} \uparrow$$

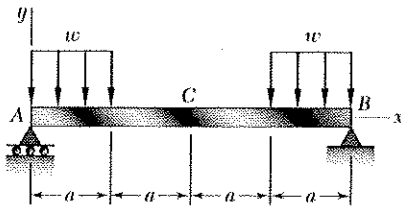
(c) Deflection at D. (y at $x = \frac{3L}{2}$)

$$y_D = \frac{w}{24EI} \left\{ -\left(\frac{3L}{2}\right)^4 + L^4 - \left(\frac{L}{2}\right)^4 + (L)\left(\frac{3L}{2}\right)^3 + (3L)\left(\frac{L}{2}\right)^3 - \left(\frac{1}{16}L\right)\left(\frac{3L}{2}\right) \right\}$$

$$= -\frac{5wL^4}{256EI} \quad y_D = \frac{5wL^4}{256EI} \downarrow$$

Problem 9.44

9.44 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at the midpoint C.



BY SYMMETRY, $R_A = R_B = wa$

$$w(x) = w - w\langle x-a \rangle^0 + w\langle x-3a \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w\langle x-a \rangle^0 - w\langle x-3a \rangle^0$$

$$\frac{dM}{dx} = V = R_A - wx + w\langle x-a \rangle^1 - w\langle x-3a \rangle^1$$

$$M = M_A + R_A x - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a \rangle^2 - \frac{1}{2}w\langle x-3a \rangle^2 \quad (\text{with } M_A = 0)$$

$$EI \frac{d^2y}{dx^2} = M = wax - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a \rangle^2 - \frac{1}{2}w\langle x-3a \rangle^2$$

$$EI \frac{dy}{dx} = \frac{1}{2}wax^2 - \frac{1}{6}wx^3 + \frac{1}{6}w\langle x-a \rangle^3 - \frac{1}{6}w\langle x-3a \rangle^3 + C_1$$

$$EI y = \frac{1}{6}wax^3 - \frac{1}{24}wx^4 + \frac{1}{24}w\langle x-a \rangle^4 - \frac{1}{24}w\langle x-3a \rangle^4 + C_1 x + C_2$$

$$[x=0, y=0]: 0-0+0-0+0+C_2=0 \quad \therefore C_2=0$$

$$[x=4a, y=0]: \frac{1}{6}wa(4a)^3 - \frac{1}{24}w(4a)^4 + \frac{1}{24}w(3a)^4 - \frac{1}{24}w(a)^4 + C_1(4a) = 0$$

$$\therefore C_1 = -\frac{5}{6}wa^3$$

(a) EQUATION OF ELASTIC CURVE.

$$y = \frac{w}{EI} \left[\frac{1}{6}ax^3 - \frac{1}{24}x^4 + \frac{1}{24}\langle x-a \rangle^4 - \frac{1}{24}\langle x-3a \rangle^4 - \frac{5}{6}a^3x \right] \quad \blacktriangleleft$$

(b) DEFLECTION AT C. (y AT $x=2a$)

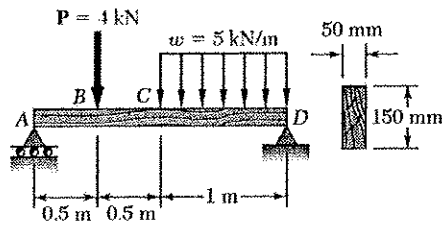
$$y_c = \frac{wa^4}{EI} \left[\frac{1}{6}(2)^3 - \frac{1}{24}(2)^4 + \frac{1}{24}(1)^4 - 0 - \frac{5}{6}(2) \right]$$

$$= -\frac{23}{24} \frac{wa^4}{EI}$$

$$y_c = \frac{23}{24} \frac{wa^4}{EI} \downarrow \quad \blacktriangleleft$$

Problem 9.45

9.45 For the timber beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 12 \text{ GPa}$.



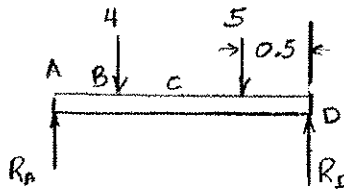
Units: Forces in kN, lengths in meters.

$$I = \frac{1}{12}(50)(150)^3 = 14.0625 \times 10^6 \text{ mm}^4 = 14.0625 \times 10^{-6} \text{ m}^4$$

$$EI = (12 \times 10^9)(14.0625 \times 10^{-6}) = 168.75 \times 10^3 \text{ N}\cdot\text{m}^2 = 168.75 \text{ kN}\cdot\text{m}^2$$

$$+\circlearrowleft M_D = 0: -2R_A + (1.5)(4) + (0.5)(5) = 0$$

$$R_A = 4.25 \text{ kN}$$



$$w(x) = 5\langle x-1 \rangle^0 \quad \text{kN}\cdot\text{m}$$

$$\frac{dV}{dx} = -w = -5\langle x-1 \rangle^0 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = -5\langle x-1 \rangle^1 + 4.25 - 4\langle x-0.5 \rangle^0 \quad \text{kN}$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{5}{2}\langle x-1 \rangle^2 + 4.25x - 4\langle x-0.5 \rangle^1 \quad \text{kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = -\frac{5}{6}\langle x-1 \rangle^3 + 2.125x^2 - 2\langle x-0.5 \rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI y = -\frac{5}{24}\langle x-1 \rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5 \rangle^3 + C_1x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$$[x=0, y=0] \quad -0 + 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=2\text{m}, y=0] \quad -\left(\frac{5}{24}\right)(1)^4 + \left(\frac{2.125}{3}\right)(2)^3 - \left(\frac{2}{3}\right)(1.5)^3 + 2C_1 = 0$$

$$C_1 = -1.60417 \text{ kN}\cdot\text{m}^2$$

(a) Slope at end A. $\left(\frac{dy}{dx}\right)_A$ at $x=0$

$$EI \left(\frac{dy}{dx}\right)_A = -0 + 0 - 0 + C_1$$

$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-1.60417}{168.75} = -9.51 \times 10^{-3} \quad \theta_A = 9.51 \times 10^{-3} \text{ rad} \searrow$$

(b) Deflection at midpoint C. $(y \text{ at } x=1\text{m})$

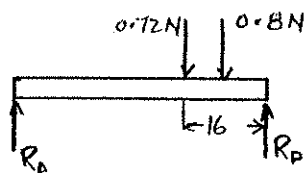
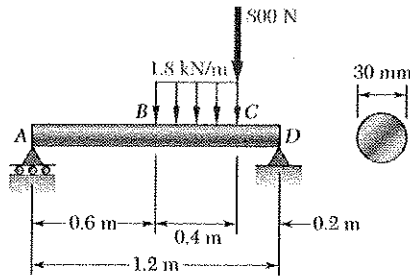
$$EI y_C = -0 + \left(\frac{2.125}{3}\right)(1)^3 - \left(\frac{2}{3}\right)(0.5)^3 + (-1.60417)(1) = -979.17 \times 10^{-3} \text{ kN}\cdot\text{m}^3$$

$$y_C = \frac{-979.17 \times 10^{-3}}{168.75} = -5.80 \times 10^{-3} \text{ m}$$

$$y_C = 5.80 \text{ mm} \downarrow$$

Problem 9.46

9.46 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point B. Use $E = 200 \text{ GPa}$.



Units: Forces in kN Lengths in metres

$$c = \frac{1}{2}d = \left(\frac{1}{2}\right)(30) = 15 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(15)^4 = 39761 \text{ mm}^4$$

$$EI = (200 \times 10^6)(39761 \times 10^{-12}) = 7.95 \text{ kN m}^2$$

Use entire beam ABCD as free body.

$$+\circlearrowleft \sum M_D = 0: -1.2 R_A + (0.72)(0.4) + (0.8)(0.2) = 0$$

$$R_A = 0.373 \text{ kN} \uparrow$$

$$w(x) = 1.8 \langle x - 0.6 \rangle^0 - 1.8 \langle x - 1 \rangle^0 \text{ kN/m}$$

$$\frac{dV}{dx} = -w = -1.8 \langle x - 0.6 \rangle^0 + 1.8 \langle x - 1 \rangle^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = -1.8 \langle x - 0.6 \rangle^1 + 1.8 \langle x - 1 \rangle^1 + 0.373 - 0.8 \langle x - 1 \rangle^0 \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = -0.9 \langle x - 0.6 \rangle^2 + 0.9 \langle x - 1 \rangle^2 + 0.373x - 0.8 \langle x - 1 \rangle^1 \text{ kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = -0.3 \langle x - 0.6 \rangle^3 + 0.3 \langle x - 1 \rangle^3 + 0.1865x^2 - 0.4 \langle x - 1 \rangle^2 + C_1 \text{ kN}\cdot\text{m}^2$$

$$EI y = -0.075 \langle x - 0.6 \rangle^4 + 0.075 \langle x - 1 \rangle^4 + 0.0622x^3 - 0.133 \langle x - 1 \rangle^3 + C_1 x + C_2 \text{ kN}\cdot\text{m}^3$$

$$[x=0, y=0] -0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=1.2, y=0] -0.075(0.6)^4 + 0.075(0.2)^4 + 0.0622(1.2)^3 - 0.133(0.2)^3 + 1.2 C_1 = 0$$

$$C_1 = -0.0807 \text{ kN}\cdot\text{m}^2$$

(a) Slope at end A. $\left(\frac{dy}{dx} \text{ at } x=0\right)$

$$EI \left(\frac{dy}{dx}\right)_A = -0 + 0 + 0 + C_1$$

$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-0.0807}{7.95} = -0.01015 \quad \theta_A = 0.01015 \text{ rad} \swarrow$$

(b) Deflection at point B. $(y \text{ at } x=0.6 \text{ m})$

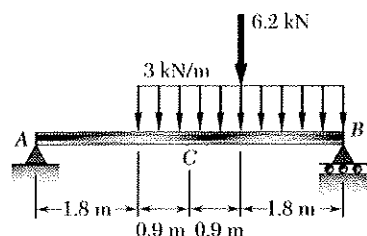
$$EI y_B = -0 + 0 + 0.0622(0.6)^3 - 0 + (-0.0807)(0.6) = -0.035 \text{ kN}\cdot\text{m}^3$$

$$y_B = -\frac{0.035}{7.95} = 4.402 \times 10^{-3} \text{ m}$$

$$y_B = 4.4 \text{ mm} \downarrow$$

Problem 9.47

9.47 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 200$ GPa.

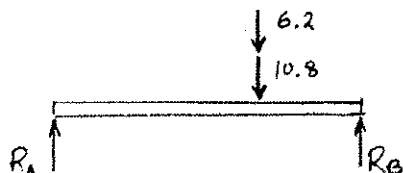


Units: Forces in kN, lengths in meters.

$$+\circlearrowleft \sum M_B = 0:$$

$$-5.4 R_A - (1.8)(6.2 + 10.8) = 0$$

$$R_A = 5.6667 \text{ kN}$$



$$w(x) = 3 \langle x - 1.8 \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -3 \langle x - 1.8 \rangle^0$$

$$\frac{dM}{dx} = V = 5.6667 - 3 \langle x - 1.8 \rangle^1 - 6.2 \langle x - 3.6 \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = 5.6667x - \frac{3}{2} \langle x - 1.8 \rangle^2 - 6.2 \langle x - 3.6 \rangle^1 \quad \text{kN} \cdot \text{m}$$

$$EI \frac{dy}{dx} = 2.8333x^2 - \frac{1}{2} \langle x - 1.8 \rangle^3 - 3.1 \langle x - 3.6 \rangle^2 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EI y = 0.9444x^3 - \frac{1}{8} \langle x - 1.8 \rangle^4 - 1.0333 \langle x - 3.6 \rangle^3 + C_1 x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x=0, y=0] \quad 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=5.4, y=0] \quad (0.9444)(5.4)^3 - \frac{1}{8}(3.6)^4 - 1.0333(1.8)^3 + C_1(5.4) + 0 = 0$$

$$C_1 = -22.535 \text{ kN} \cdot \text{m}^2$$

$$\text{Data: } E = 200 \times 10^9 \text{ Pa}, \quad I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N} \cdot \text{m}^2 = 25.8 \times 10^3 \text{ kN} \cdot \text{m}^2$$

(a) Slope at A. ($\frac{dy}{dx}$ at $x = 0$)

$$EI \frac{dy}{dx} = 0 - 0 - 0 - 22.535 \text{ kN} \cdot \text{m}^2$$

$$\theta_A = -\frac{22.535}{25.8 \times 10^3} = -873 \times 10^{-6} \quad \theta_A = 0.873 \times 10^{-3} \text{ rad} \quad \leftarrow$$

(b) Deflection at C. (y at $x = 2.7 \text{ m}$)

$$EI y_C = (0.9444)(2.7)^3 - \frac{1}{8}(0.9)^4 - 0 + (22.535)(2.7) + 0$$

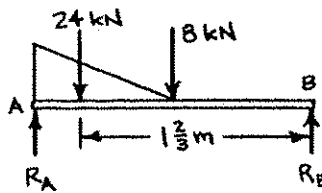
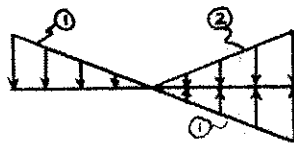
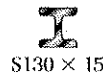
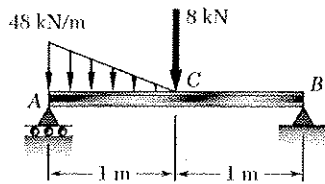
$$= -42.337 \text{ kN} \cdot \text{m}^3$$

$$y_C = -\frac{42.337}{25.8 \times 10^3} = -1.641 \times 10^{-3} \text{ m}$$

$$y_C = 1.641 \text{ mm} \quad \downarrow \quad \leftarrow$$

Problem 9.48

9.48 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 200$ GPa.



DISTRIBUTED LOADS: ① $w_1(x) = w_0 - kx$ ② $w_2 = k(x-1)^1$

$$+\circlearrowleft \sum M_B = 0: -2R_A + (24)(\frac{1}{3}) + (8)(1) = 0 \quad R_A = 24 \text{ kN} \uparrow$$

$$w(x) = w_0 - kx + k(x-1)^1 = 48 - 48x + 48(x-1)^1$$

$$\frac{dV}{dx} = -w = -48 + 48x - 48(x-1)^1 \text{ kN/m}$$

$$\frac{dM}{dx} = V = 24 - 48x + 24x^2 - 24(x-1)^2 - 8(x-1)^0 \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = 24x - 24x^2 + 8x^3 - 8(x-1)^3 - 8(x-1)^1 \text{ kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2(x-1)^4 - 4(x-1)^2 + C_1 \text{ kN}\cdot\text{m}^2$$

$$EI y = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}(x-1)^5 - \frac{4}{3}(x-1)^3 + C_1x + C_2 \text{ kN}\cdot\text{m}^3$$

$$[x=0, y=0]: 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \quad \therefore C_2 = 0$$

$$[x=2, y=0]: 4(2)^3 - 2(2)^4 + \frac{2}{5}(2)^5 - \frac{2}{5}(1)^5 - \frac{4}{3}(1)^3 + C_1(2) = 0 \quad \therefore C_1 = -\frac{83}{15} \text{ kN}\cdot\text{m}^2$$

DATA: $E = 200(10^6) \text{ kN/m}^2$ $I = 5.07(10^6) \text{ mm}^4 = 5.07(10^{-6}) \text{ m}^4$

$$EI = (200 \times 10^6)(5.07 \times 10^{-6}) = 1014 \text{ kN}\cdot\text{m}^2$$

(a) **SLOPE AT A.** ($\frac{dy}{dx}$ AT $x=0$)

$$EI \theta_A = 0 - 0 + 0 - 0 - 0 - \frac{83}{15} \text{ kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{83/15}{1014} = -5.4569 \times 10^{-3} \text{ rad} \quad 5.46 \times 10^{-3} \text{ rad} \searrow$$

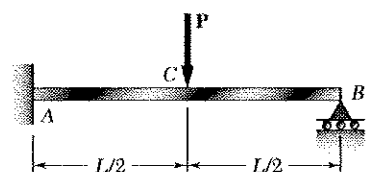
(b) **DEFLECTION AT C.** (y AT $x=1$ m)

$$EI y_C = 4(1)^3 - 2(1)^4 + \frac{2}{5}(1)^5 - 0 - 0 - \frac{83}{15}(1) = -3.1333 \text{ kN}\cdot\text{m}^3$$

$$y_C = -\frac{3.1333}{1014} = -3.0900 \times 10^{-3} \text{ m} \quad 3.09 \text{ mm} \downarrow$$

Problem 9.49

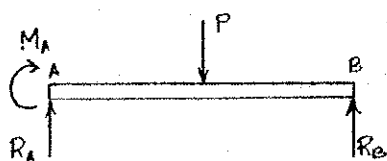
9.49 and 9.50 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point C.



$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=L, y=0]$$



$$+\uparrow \sum F_y = 0: R_A + R_B - P = 0$$

$$R_A = P - R_B$$

$$+\circlearrowleft \sum M_A = 0: -M_A - P\left(\frac{L}{2}\right) + R_B L = 0$$

$$M_A = R_B L - \frac{1}{2} PL$$

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{2} \right\rangle^0$$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{2} \right\rangle^1$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P\left\langle x - \frac{L}{2} \right\rangle^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P\left\langle x - \frac{L}{2} \right\rangle^3 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, y=0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P\left(\frac{L}{2}\right)^3 + 0 + 0 = 0$$

$$\frac{1}{2} (R_B L - \frac{1}{2} PL) L^2 + \frac{1}{6} (P - R_B) L^3 - \frac{1}{48} PL^3 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_B L^3 = \left(\frac{1}{4} - \frac{1}{6} + \frac{1}{48}\right) PL^3 \quad \frac{1}{3} R_B = \frac{5}{48} P \quad (a) \quad R_B = \frac{5}{16} P \uparrow$$

$$R_A = P - \frac{5}{16} P = \frac{11}{16} P$$

$$M_A = \frac{5}{16} PL - \frac{1}{2} PL = -\frac{3}{16} PL$$

(b) Deflection at C. (y at $x = \frac{L}{2}$)

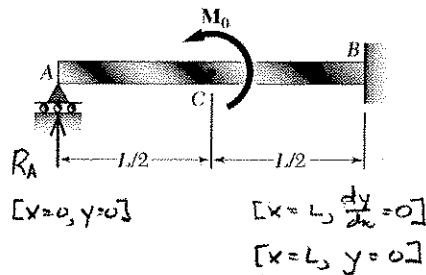
$$y_C = \frac{1}{EI} \left\{ \frac{1}{2} M_A \left(\frac{L}{2}\right)^2 + \frac{1}{6} R_A \left(\frac{L}{2}\right)^3 + 0 + 0 + 0 \right\}$$

$$= \frac{PL^3}{EI} \left\{ \left(\frac{1}{2}\right) \left(-\frac{3}{16}\right) \left(\frac{1}{4}\right) + \left(\frac{1}{6}\right) \left(\frac{11}{16}\right) \left(\frac{1}{8}\right) \right\} = -\frac{7}{168} \frac{PL^3}{EI}$$

$$y_C = \frac{7}{168} \frac{PL^3}{EI} \downarrow$$

Problem 9.50

9.49 and 9.50 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point C.



$$\text{For } 0 \leq x \leq \frac{L}{2}, \quad M = R_A x$$

$$\text{For } \frac{L}{2} \leq x \leq L, \quad M = R_A x - M_o$$

$$\text{Then } EI \frac{d^2 y}{dx^2} = M = R_A x - M_o \langle x - \frac{L}{2} \rangle^0$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_o \langle x - \frac{L}{2} \rangle^1 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_o \langle x - \frac{L}{2} \rangle^2 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad \frac{1}{2} R_A L^2 - M_o (\frac{L}{2}) + C_1 = 0 \quad C_1 = \frac{1}{2} (M_o L - R_A L^2)$$

$$[x=L, y=0] \quad \frac{1}{6} R_A L^3 - \frac{1}{2} M_o (\frac{L}{2})^2 + \frac{1}{2} (M_o L - R_A L^2) L + 0 = 0$$

$$-\frac{1}{3} R_A L^3 + \frac{3}{8} M_o L^2 = 0$$

(a) Reaction at A.

$$M_A = 0, \quad R_A = \frac{9 M_o}{8 L} \uparrow$$

$$C_1 = \frac{1}{2} [M_o L - (\frac{9 M_o}{8 L})(L^2)] = -\frac{1}{16} M_o L$$

$$EI y = \frac{1}{6} (\frac{9 M_o}{8 L}) x^3 - \frac{1}{2} M_o \langle x - \frac{L}{2} \rangle^2 - \frac{1}{16} M_o L x + 0$$

Elastic curve. $y = \frac{M_o}{EIL} \left\{ \frac{9}{8} x^3 - \frac{1}{2} L \langle x - \frac{L}{2} \rangle^2 - \frac{1}{16} L^2 x \right\}$

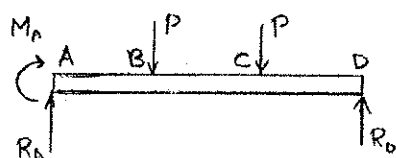
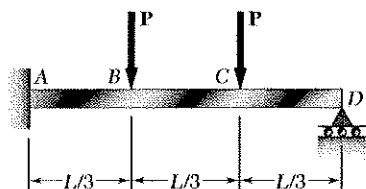
(b) Deflection at point C. (y at $x = \frac{L}{2}$)

$$y_c = \frac{M_o}{EIL} \left\{ (\frac{1}{6})(\frac{9}{8})(\frac{L}{2})^3 - 0 - (\frac{1}{16} L^2)(\frac{L}{2}) \right\} = -\frac{M_o L^2}{128 EI}$$

$$y_c = \frac{M_o L^2}{128 EI} \downarrow$$

Problem 9.51

9.51 and 9.52 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point B.



$$+\uparrow \sum F_y = 0; \quad R_A - P - P + R_D = 0 \quad R_A = 2P - R_D$$

$$+\circlearrowleft \sum M_A = 0; \quad -M_A - \frac{PL}{3} - \frac{2PL}{3} + R_D L = 0$$

$$M_A = R_D L - PL$$

$$\frac{dM}{dx} = V = R_A - P\langle x - \frac{L}{3} \rangle^0 - P\langle x - \frac{2L}{3} \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - P\langle x - \frac{L}{3} \rangle^1 - P\langle x - \frac{2L}{3} \rangle^1$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P \langle x - \frac{L}{3} \rangle^2 - \frac{1}{2} P \langle x - \frac{2L}{3} \rangle^2 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0+0-0-0+C_1=0 \quad C_1=0$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P \langle x - \frac{L}{3} \rangle^3 - \frac{1}{6} P \langle x - \frac{2L}{3} \rangle^3 + C_2$$

$$[x=0, y=0] \quad 0+0-0-0+C_2=0 \quad C_2=0$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P \langle x - \frac{L}{3} \rangle^3 - \frac{1}{6} P \langle x - \frac{2L}{3} \rangle^3$$

$$[x=L, y=0] \quad \frac{1}{2} (R_D L - PL) L^2 + \frac{1}{6} (2P - R_D) L^3 - \frac{1}{6} P \left(\frac{2L}{3}\right)^3 - \frac{1}{6} P \left(\frac{L}{3}\right)^3 = 0$$

$$\frac{1}{3} R_D L^3 - \frac{2}{9} PL^3 = 0$$

(a) Reaction at D.

$$R_D = \frac{2}{3} P \uparrow \quad \blacktriangleleft$$

$$M_A = \frac{2}{3} PL - PL = -\frac{1}{3} PL$$

$$R_A = 2P - \frac{2}{3} P = \frac{4}{3} P$$

$$EI y = \frac{1}{2} \left(-\frac{1}{3} PL\right) x^2 + \frac{1}{6} \left(\frac{4}{3} P\right) x^3 - \frac{1}{6} P \langle x - \frac{L}{3} \rangle^3 - \frac{1}{6} P \langle x - \frac{2L}{3} \rangle^3$$

Elastic curve.

$$y = \frac{P}{EI} \left\{ -\frac{1}{6} L x^2 + \frac{2}{9} x^3 - \frac{1}{6} \langle x - \frac{L}{3} \rangle^3 - \frac{1}{6} \langle x - \frac{2L}{3} \rangle^3 \right\}$$

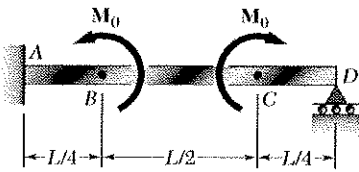
(b) Deflection at B. (y at $x = \frac{L}{3}$)

$$y_B = \frac{P}{EI} \left\{ -\frac{1}{6} L \left(\frac{L}{3}\right)^2 + \frac{2}{9} \left(\frac{L}{3}\right)^3 - 0 - 0 \right\} = -\frac{5PL^3}{486 EI}$$

$$y_B = \frac{5PL^3}{486 EI} \downarrow \quad \blacktriangleleft$$

Problem 9.52

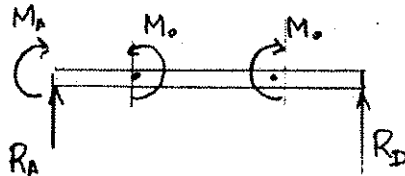
9.51 and 9.52 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point B.



$$+\uparrow \sum F_y = 0: R_A + R_D = 0 \quad R_A = -R_D$$

$$+\circlearrowleft \sum M_A = 0: -M_A + M_0 - M_0 + R_D L = 0$$

$$M_A = R_D L$$



$$M(x) = M_A + R_A x - M_0 \langle x - \frac{L}{4} \rangle^0 + M_0 \langle x - \frac{3L}{4} \rangle^0$$

$$EI \frac{d^2 y}{dx^2} = R_D L - R_D x - M_0 \langle x - \frac{L}{4} \rangle^0 + M_0 \langle x - \frac{3L}{4} \rangle^0$$

$$EI \frac{dy}{dx} = R_D L x - \frac{1}{2} R_D x^2 - M_0 \langle x - \frac{L}{4} \rangle^1 + M_0 \langle x - \frac{3L}{4} \rangle^1 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 - 0 - 0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$EI y = \frac{1}{2} R_D L x^2 - \frac{1}{6} R_D x^3 - \frac{1}{2} M_0 \langle x - \frac{L}{4} \rangle^2 + \frac{1}{2} M_0 \langle x - \frac{3L}{4} \rangle^2 + C_2$$

$$[x=0, y=0] \quad 0 - 0 - 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=L, y=0] \quad \frac{1}{2} R_D L^3 - \frac{1}{6} R_D L^3 - \frac{1}{2} M_0 \left(\frac{3L}{4}\right)^2 + \frac{1}{2} M_0 \left(\frac{L}{4}\right)^2 = 0$$

(a) Reaction at D.

$$R_D = \frac{3M_0}{4L} \uparrow$$

$$EI y = \frac{1}{2} \left(\frac{3M_0}{4L}\right) L x^2 - \frac{1}{6} \left(\frac{3M_0}{4L}\right) x^3 - \frac{1}{2} M_0 \langle x - \frac{L}{4} \rangle^2 + \frac{1}{2} M_0 \langle x - \frac{3L}{4} \rangle^2$$

Elastic curve.

$$y = \frac{M_0}{EIL} \left\{ \frac{3}{8} L x^2 - \frac{1}{8} x^3 - \frac{1}{2} L \langle x - \frac{L}{4} \rangle^2 + \frac{1}{2} L \langle x - \frac{3L}{4} \rangle^2 \right\}$$

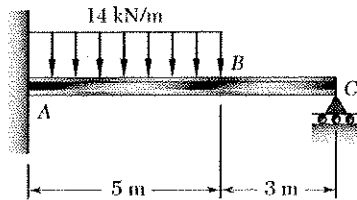
(b) Deflection at point B. (y at $x = \frac{L}{4}$)

$$y_B = \frac{M_0}{EIL} \left\{ \left(\frac{3}{8} L\right) \left(\frac{L}{4}\right)^2 - \left(\frac{1}{8} \left(\frac{L}{4}\right)^3\right) - 0 + 0 \right\}$$

$$y_B = \frac{11 M_0 L^2}{512 EI} \uparrow$$

Problem 9.53

9.53 For the beam and loading shown, determine (a) the reaction at point C, (b) the deflection at point B. Use $E = 200 \text{ GPa}$.



I
W410 x 60

Units: Forces in kN, lengths in m.

$$+\uparrow \Sigma F_y = 0: R_A - 70 + R_C = 0$$

$$R_A = 70 - R_C \quad \text{kN}$$

$$+\circlearrowleft \Sigma M_A = 0: -M_A - (70)(2.5) + 8R_C = 0$$

$$M_A = 8R_C - 175 \quad \text{kN}\cdot\text{m}$$

Reactions are statically indeterminate.

$$w(x) = 14 - 14\langle x-5 \rangle^0 \quad \text{kN/m}$$

$$\frac{dV}{dx} = -w = -14 + 14\langle x-5 \rangle^0 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = R_A - 14x + 14\langle x-5 \rangle^1 \quad \text{kN}$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7\langle x-5 \rangle^2 \quad \text{kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{7}{3} x^3 + \frac{7}{3} \langle x-5 \rangle^3 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{7}{12} x^4 + \frac{7}{12} \langle x-5 \rangle^4 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=8, y=0] \quad \frac{1}{2} M_A (8)^2 + \frac{1}{6} R_A (8)^3 - \frac{7}{12} (8)^4 + \frac{7}{12} (3)^4 + 0 + 0 = 0$$

$$32(8R_C - 175) + \frac{512}{6}(70 - R_C) - \frac{28105}{12} = 0$$

$$170.667 R_C = 5600 - \frac{35840}{6} + \frac{28105}{12} = 1968.75 \quad R_C = 11.536 \text{ kN} \uparrow$$

$$M_A = (8)(11.536) - 175 = -82.715 \text{ kN}\cdot\text{m}$$

$$R_A = 70 - 11.536 = 58.464 \text{ kN}$$

Data: $E = 200 \times 10^9 \text{ Pa}$ $I = 216 \times 10^6 \text{ mm}^4 = 216 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(216 \times 10^{-6}) = 43.2 \times 10^6 \text{ N}\cdot\text{m}^2 = 43200 \text{ kN}\cdot\text{m}^2$$

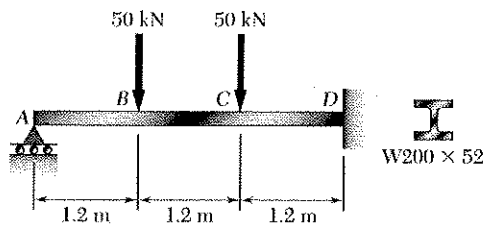
(b) Deflection at B. (y at $x = 5 \text{ m}$)

$$EI y_B = \frac{1}{2}(-82.715)(5)^2 + \frac{1}{6}(58.464)(5)^3 - \frac{7}{12}(5)^4 = -180.52 \text{ kN}\cdot\text{m}^3$$

$$y_B = -\frac{180.52}{43200} = -4.18 \times 10^{-3} \text{ m} \quad y_B = 4.18 \text{ mm} \downarrow$$

Problem 9.54

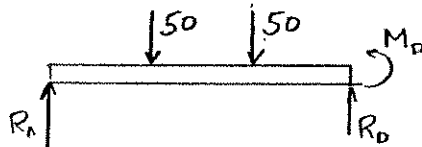
9.54 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at point B. Use $E = 200$ GPa.



Units: Forces in kN, lengths in meters.

$$I = 52.7 \times 10^6 \text{ mm}^4 = 52.7 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(52.7 \times 10^{-6}) = 10.54 \times 10^6 \text{ N} \cdot \text{m}^2 = 10540 \text{ kN} \cdot \text{m}^2$$



$$\frac{dM}{dx} = V = R_A - 50\langle x - 1.2 \rangle^0 - 50\langle x - 2.4 \rangle^0 \quad \text{kN}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - 50\langle x - 1.2 \rangle^1 - 50\langle x - 2.4 \rangle^1 \quad \text{kN} \cdot \text{m}$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - 25\langle x - 1.2 \rangle^2 - 25\langle x - 2.4 \rangle^2 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{25}{3}\langle x - 1.2 \rangle^3 - \frac{25}{3}\langle x - 2.4 \rangle^3 + C_1 x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x=0, y=0] \quad 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=3.6, \frac{dy}{dx} = 0] \quad \frac{1}{2} R_A (3.6)^2 - (25)(2.4)^2 - (25)(1.2)^2 + C_1 = 0$$

$$C_1 = 180 - 6.48 R_A \quad \text{kN} \cdot \text{m}^2$$

$$[x=3.6, y=0] \quad \frac{1}{6} R_A (3.6)^3 - \frac{25}{3}(2.4)^3 - \frac{25}{3}(1.2)^3 + (180 - 6.48 R_A)(3.6) = 0$$

$$-15.552 R_A + 518.4 = 0$$

(a) Reaction at A. $R_A = \frac{100}{3} \text{ kN}$ $R_A = 33.3 \text{ kN} \uparrow$

$$C_1 = 180 - (6.48)(33.3333) = -36 \text{ kN} \cdot \text{m}^2$$

$$EI y = \frac{1}{6} \left(\frac{100}{3} \right) x^3 - \frac{25}{3}\langle x - 1.2 \rangle^3 - \frac{25}{3}\langle x - 2.4 \rangle^3 - 36 x \quad \text{kN} \cdot \text{m}^3$$

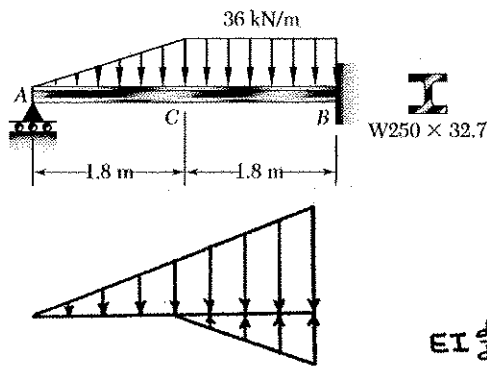
(b) Deflection at point B. (y at $x = 1.2 \text{ m}$)

$$EI y_B = \left(\frac{1}{6} \right) \left(\frac{100}{3} \right) (1.2)^3 - 0 - 0 - (36)(1.2) = -33.6 \text{ kN} \cdot \text{m}^3$$

$$y_B = \frac{-33.6}{10540} = -3.19 \times 10^{-5} \text{ m} \quad y_B = 3.19 \text{ mm} \downarrow$$

Problem 9.55

9.55 and 9.56 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at point C. Use $E = 200$ GPa.



$$k = \frac{36 \text{ kN/m}}{1.8 \text{ m}} = 20 \text{ kN/m}^2$$

$$w(x) = 20x - 20 \langle x - 1.8 \rangle^1 \text{ kN/m}$$

$$\frac{dV}{dx} = -w(x) = -20x + 20 \langle x - 1.8 \rangle^1 \text{ kN/m}$$

$$\frac{dM}{dx} = V = R_A - 10x^2 + 10 \langle x - 1.8 \rangle^2 \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - \frac{10}{3} x^3 + \frac{10}{3} \langle x - 1.8 \rangle^3 \text{ kN m}$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{5}{6} x^4 + \frac{5}{6} \langle x - 1.8 \rangle^4 + C_1 \text{ kN m}^2$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{6} x^5 + \frac{1}{6} \langle x - 1.8 \rangle^5 + C_1 x + C_2 \text{ kN m}^3$$

$$[x=0, y=0]: 0 - 0 + 0 + 0 + C_2 = 0 \quad \therefore C_2 = 0$$

$$[x=3.6 \text{ m}, \frac{dy}{dx}=0]: \frac{1}{2} R_A (3.6)^2 - \frac{5}{6} (3.6)^4 + \frac{5}{6} (1.8)^4 + C_1 = 0$$

$$\therefore C_1 = 131.22 - 6.48 R_A \text{ kN m}^2$$

$$[x=3.6 \text{ m}, y=0]: \frac{1}{6} R_A (3.6)^3 - \frac{1}{6} (3.6)^5 + \frac{1}{6} (1.8)^5 + (131.22 - 6.48 R_A)(3.6) = 0$$

$$(7.776 - 23.328) R_A = -374.76 \quad R_A = 24.097 \text{ kN}$$

(a) REACTION AT A.

$$R_A = 24.1 \text{ kN} \uparrow$$

$$C_1 = 131.22 - 6.48(24.097) = -24.929 \text{ kN m}^2$$

$$\text{DATA: } E = 200 \times 10^6 \text{ kPa} \quad I = 48.9 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^6)(48.9 \times 10^{-6}) = 9780 \text{ kN m}^2$$

(b) DEFLECTION AT C. (y AT $x = 1.8 \text{ m}$)

$$EI y_c = \frac{1}{6} (24.097)(1.8)^3 - \frac{1}{6} (1.8)^5 + 0 - 24.929(1.8)$$

$$= -24.599 \text{ kN m}^3$$

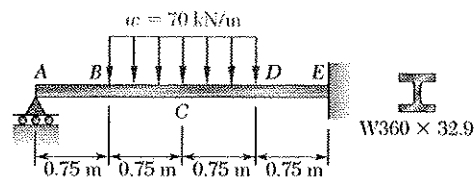
$$y_c = -\frac{24.599}{9780} = -2.515 \times 10^{-3} \text{ m}$$

$$y_c = 2.5 \text{ mm} \downarrow$$

Proprietary Material. © 2009 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

Problem 9.56

9.55 and 9.56 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at point C. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m

$$\begin{aligned} [x=0, y=0] \\ [x=0, M=0] \end{aligned}$$

$$\begin{aligned} [x=3, y=0] \\ [x=3, \frac{dy}{dx}=0] \end{aligned}$$

$$w(x) = 70 \langle x - 0.75 \rangle^0 - 70 \langle x - 2.25 \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -70 \langle x - 0.75 \rangle^0 + 70 \langle x - 2.25 \rangle^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = R_A + 70 \langle x - 0.75 \rangle^1 - 70 \langle x - 2.25 \rangle^1 \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - 35 \langle x - 0.75 \rangle^2 + 35 \langle x - 2.25 \rangle^2 \text{ kN m}$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{35}{3} \langle x - 0.75 \rangle^3 + \frac{35}{3} \langle x - 2.25 \rangle^3 + C_1 \text{ kN m}^2$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{35}{12} \langle x - 0.75 \rangle^4 + \frac{35}{12} \langle x - 2.25 \rangle^4 + C_1 x + C_2 \text{ kN m}^3$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=3, \frac{dy}{dx}=0] \quad \frac{1}{2} R_A (3)^2 - \frac{35}{3} (2.25)^3 + \frac{35}{3} \langle 0.75 \rangle^3 + C_1 = 0$$

$$C_1 = 128 - 4.5 R_A \text{ kN m}^2$$

$$[x=3, y=0] \quad \frac{1}{6} R_A (3)^3 - \frac{35}{12} (2.25)^4 + \frac{35}{12} (0.75)^4 + (128 - 4.5 R_A)(3) + 0 = 0$$

$$9 R_A = 310.2 \quad R_A = 34.5 \text{ kN} \uparrow$$

$$C_1 = 128 - 4.5 (34.5) = -27.25 \text{ kN m}^2$$

Data: $E = 200 \text{ GPa}$, $I = 82.7 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^6)(82.7 \times 10^{-6}) = 16540 \text{ kN m}^2$$

(b) Deflection at C. (y at $x = 1.5 \text{ m}$)

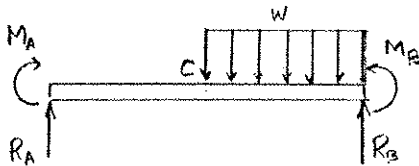
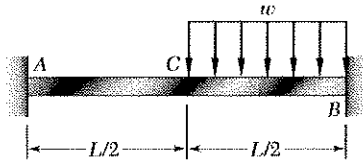
$$EI y_c = \frac{1}{6} (34.5)(1.5)^3 - \frac{35}{12} (0.75)^4 + 0 - (27.25)(1.5) = -22.39$$

$$y_c = -\frac{22.39}{16540} = -1.35 \times 10^{-3} \text{ m}$$

$$y_c = 1.35 \text{ mm} \downarrow$$

Problem 9.57

9.57 and 9.58 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at midpoint C.



$$w(x) = w \left\langle x - \frac{L}{2} \right\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w \left\langle x - \frac{L}{2} \right\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w \left\langle x - \frac{L}{2} \right\rangle^1$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{1}{2} w \left\langle x - \frac{L}{2} \right\rangle^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{6} w \left\langle x - \frac{L}{2} \right\rangle^3 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{24} w \left\langle x - \frac{L}{2} \right\rangle^4 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0]$$

$$0 + 0 - 0 + C_1 = 0$$

$$C_1 = 0$$

$$[x=0, y=0]$$

$$0 + 0 - 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0]$$

$$M_A L + \frac{1}{2} R_A L^2 - \frac{1}{6} w \left(\frac{L}{2} \right)^3 = 0$$

$$(1)$$

$$[x=L, y=0]$$

$$\frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^2 - \frac{1}{24} w \left(\frac{L}{2} \right)^4 = 0$$

$$(2)$$

Solving equations (1) and (2) simultaneously,

$$(a) \quad R_A = \frac{3wL}{32} \quad M_A = -\frac{5wL^2}{192}$$

$$R_A = \frac{3wL}{32} \uparrow$$

$$M_A = \frac{5wL^2}{192} \curvearrowright$$

$$EI y = -\frac{5}{384} wL^2 x^2 + \frac{3}{192} wL x^3 - \frac{1}{24} w \left\langle x - \frac{L}{2} \right\rangle^4$$

Elastic curve.

$$y = \frac{w}{EI} \left\{ -\frac{5}{384} L^2 x^2 + \frac{3}{192} L x^3 - \frac{1}{24} \left\langle x - \frac{L}{2} \right\rangle^4 \right\}$$

(b) Deflection at midpoint C. (y at $x = \frac{L}{2}$)

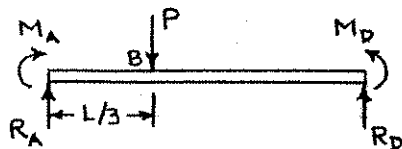
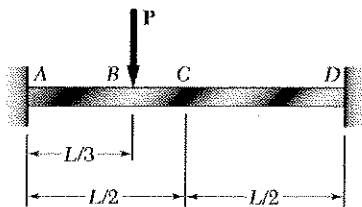
$$y_C = \frac{w}{EI} \left\{ \left(-\frac{5}{384} L^2 \right) \left(\frac{L}{2} \right)^2 + \left(\frac{3}{192} L \right) \left(\frac{L}{2} \right)^3 - 0 \right\}$$

$$= -\frac{wL^4}{768 EI}$$

$$y_B = \frac{wL^4}{768 EI} \downarrow$$

Problem 9.58

9.57 and 9.58 For the beam and loading shown, determine (a) the reaction at point A, (b) the deflection at midpoint C.



$$\frac{dM}{dx} = V = R_A - P\langle x - \frac{L}{3} \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - P\langle x - \frac{L}{3} \rangle^1$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P \langle x - \frac{L}{3} \rangle^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P \langle x - \frac{L}{3} \rangle^3 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0]: 0 + 0 - 0 + C_1 = 0 \quad \therefore C_1 = 0$$

$$[x=0, y=0]: 0 + 0 - 0 + C_2 = 0 \quad \therefore C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0]: M_A L + \frac{1}{2} R_A L^2 - \frac{1}{2} P (\frac{2L}{3})^2 = 0 \quad (1)$$

$$[x=L, y=0]: \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P (\frac{2L}{3})^3 = 0 \quad (2)$$

(a) SOLVING EQ'S (1) & (2) SIMULTANEOUSLY,

$$R_A = \frac{20}{27} P \quad M_A = -\frac{4}{27} PL$$

$$R_A = \frac{20}{27} P \uparrow$$

$$M_A = \frac{4}{27} PL \curvearrowright$$

ELASTIC CURVE.

$$y = \frac{P}{EI} \left[-\frac{2}{27} L x^2 + \frac{10}{81} x^3 - \frac{1}{6} \langle x - \frac{L}{3} \rangle^3 \right]$$

(b) DEFLECTION AT MIDPOINT C. (y AT $x = \frac{L}{2}$)

$$y_C = \frac{P}{EI} \left[-\frac{2}{27} L (\frac{L}{2})^2 + \frac{10}{81} (\frac{L}{2})^3 - \frac{1}{6} (\frac{L}{6})^3 \right]$$

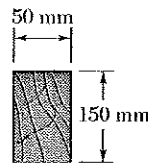
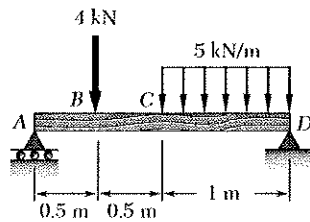
$$= -\frac{5PL^3}{1296EI}$$

$$y_C = \frac{5PL^3}{1296EI} \downarrow$$

Problem 9.59

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.59 Beam and loading of Prob. 9.45.



See solution to Prob. 9.45 for the derivation of the equations used in the following.

$$EI = 168.75 \text{ kN}\cdot\text{m}^2$$

$$C_1 = -1.60417 \text{ kN}\cdot\text{m}^2$$

$$C_2 = 0$$

$$EI \frac{dy}{dx} = -\frac{5}{6} \langle x-1 \rangle^3 + 2.125x^2 - 2 \langle x-0.5 \rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI y = -\frac{5}{24} \langle x-1 \rangle^4 + \frac{2.125}{3} x^3 - \frac{2}{3} \langle x-0.5 \rangle^3 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$$

Compute slope at C. ($\frac{dy}{dx}$ at $x = 1 \text{ m}$)

$$EI \left(\frac{dy}{dx} \right)_c = 0 + (2.125)(1)^2 - 2(0.5)^2 - 1.60417 = 20.83 \times 10^{-3} \text{ kN}\cdot\text{m}^2$$

Since the slope at C is positive, the largest deflection occurs in portion BC, where

$$EI \frac{dy}{dx} = 2.125x^2 - 2(x-0.5)^2 - 1.60417$$

$$EI y = \frac{2.125}{3} x^3 - \frac{2}{3} (x-0.5)^3 - 1.60417 x$$

To find the location of the largest downward deflection, set $\frac{dy}{dx} = 0$.

$$\begin{aligned} 2.125x_m^2 - 2(x_m^2 - x_m + 0.25) - 1.60417 \\ = 0.125x_m^2 + 2x_m - 2.10417 = 0 \end{aligned}$$

$$x = 1.0521 - 0.0625x^2$$

Solve by iteration. $x_m = 1, 0.989, 0.991, x_m = 0.991 \text{ m}$ ◀

$$\begin{aligned} EI y_m &= \left(\frac{2.125}{3} \right) (0.991)^3 - \frac{2}{3} (0.991 - 0.5)^3 - (1.60417)(0.991) \\ &= -0.97927 \text{ kN}\cdot\text{m}^3 \end{aligned}$$

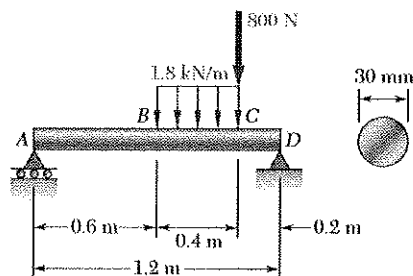
$$y_m = \frac{-0.97927}{168.75} = -5.80 \times 10^{-3} \text{ m}$$

$$y_m = 5.80 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.60

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.60 Beam and loading of Prob. 9.46.



See solution to Prob. 9.46 for the derivation of equations used in the following.

$$EI = 7.95 \text{ kNm}^2$$

$$C_1 = -0.0807 \text{ kNm}^2, \quad C_2 = 0$$

$$EI \frac{dy}{dx} = -0.3 \langle x - 0.6 \rangle^3 + 0.3 \langle x - 1 \rangle^3 + 0.1865 x^2 - 0.4 \langle x - 1 \rangle^2 + C_1 \text{ kNm}^2$$

$$EI y = -0.075 \langle x - 0.6 \rangle^4 + 0.075 \langle x - 1 \rangle^4 + 0.0622 x^3 - 0.133 \langle x - 1 \rangle^3 + C_1 x + C_2 \text{ kNm}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0.6 \text{ m} < x_m < 1 \text{ m}$

$$EI \frac{dy}{dx} = -0.3(x_m - 0.6)^3 + 0 + 0.1865 x_m^2 - 0 - 0.0807 = 0$$

$$f = -0.3(x_m - 0.6)^3 + 0.1865 x_m^2 - 0.0807$$

$$\frac{df}{dx_m} = -0.9(x_m - 0.6)^2 + 0.373$$

Solve by iteration: $x_m = 0.6 \quad 0.6615 \quad 0.65875$

$$f_i = -0.01356 \quad 8.393 \times 10^{-4} \quad 1.711 \times 10^{-4}$$

$$df/dx_m = 0.2238 \quad 0.2264$$

$$x_m = 0.66 \text{ m} \quad \blacktriangleleft$$

$$\begin{aligned} EI y_m &= -0.075(0.06)^4 + 0 + 0.0622(0.66)^3 - (0.0807)(0.66) \\ &= -0.0354 \text{ kNm}^3 \end{aligned}$$

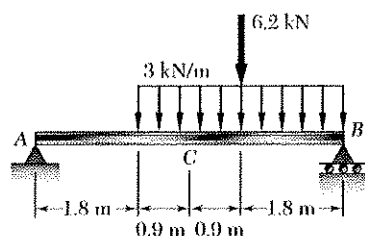
$$y_m = \frac{-0.0354}{7.95} = -4.45 \times 10^{-3} \text{ m}$$

$$y_m = 4.45 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.61

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.61 Beam and loading of Prob. 9.47.



See solution to Prob. 9.47 for the derivation of the equations used in the following.

$$EI = 25.8 \times 10^3 \text{ kN}\cdot\text{m}^2$$

$$EI \frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}(x-1.8)^2 - 3.1(x-3.6)^2 - 22.535$$

$$EI y = 0.9444x^3 - \frac{1}{8}(x-1.8)^3 - 1.03333(x-3.6)^3 - 22.535x$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $1.8 < x_m < 3.6$

$$EI \frac{dy}{dx} = 2.8333x_m^2 - \frac{1}{2}(x_m-1.8)^2 - 22.535 = 0$$

Solving by iteration: $x_m = 3, 2.86, 2.855$ $x_m = 2.855 \text{ m}$
 $dF/dx = 15.8, 15.15$

$$EI y_m = 0.9444x_m^3 - \frac{1}{8}(x_m-1.8)^3 - 22.535x_m$$

$$= (0.9444)(2.855)^3 - \frac{1}{8}(2.855-1.8)^3 - (22.535)(2.855) = -42.507 \text{ kN}\cdot\text{m}^2$$

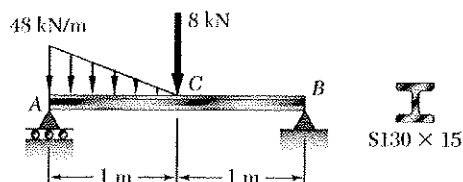
$$y_m = -\frac{42.507}{25.8 \times 10^3} = -1.648 \times 10^{-3} \text{ m}$$

$$y_m = 1.648 \text{ mm} \downarrow$$

Problem 9.62

9.59 through 9.62 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.62 Beam and loading of Prob. 9.48.



SEE SOLUTION TO PROB. 9.48 FOR THE DERIVATION OF THE EQUATIONS USED IN THE FOLLOWING.

$$EI = 1014 \text{ kN}\cdot\text{m}^2$$

$$EI \frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1 \rangle^4 - 4\langle x-1 \rangle^2 - \frac{83}{15} \text{ kN}\cdot\text{m}^2$$

$$EI y = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}\langle x-1 \rangle^5 - \frac{4}{3}\langle x-1 \rangle^3 - \frac{83}{15}x \text{ kN}\cdot\text{m}^3$$

TO FIND LOCATION OF MAXIMUM $|y|$, SET $\frac{dy}{dx} = 0$.
ASSUME $0 < x_m < 1 \text{ m}$

$$EI \frac{dy}{dx} = 12x_m^2 - 8x_m^3 + 2x_m^4 - \frac{83}{15} = 0$$

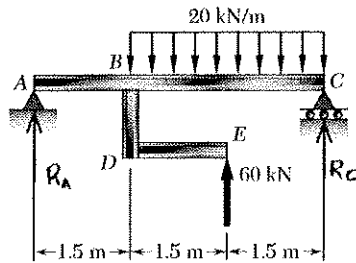
$$\text{SOLVING: } x_m = 0.94166 \text{ m} \quad x_m = 0.942 \text{ m}$$

$$EI y_m = 4(0.94166)^3 - 2(0.94166)^4 + \frac{2}{5}(0.94166)^5 - \frac{83}{15}(0.94166) \\ = -3.1469 \text{ kN}\cdot\text{m}^3$$

$$y_m = -\frac{3.1469}{1014} = -3.1035 \times 10^{-3} \text{ m} \quad y_m = 3.10 \text{ mm} \downarrow$$

Problem 9.63

9.63 The rigid bar BDE is welded at point B to the rolled-steel beam AC . For the loading shown, determine (a) the slope at point A , (b) the deflection at point B . Use $E = 200 \text{ GPa}$.



$$\sum M_C = 0:$$

$$-4.5 R_A + (20)(3)(1.5) - (60)(1.5) = 0$$

$$R_A = 0$$

Units: Forces in kN; lengths in m.

$$EI \frac{d^2 y}{dx^2} = M = 60(x-1.5)' - 90(x-1.5)^0 - \frac{1}{2}(20)(x-1.5)^2$$

$$EI \frac{dy}{dx} = 30(x-1.5)^2 - 90(x-1.5)' - \left(\frac{1}{6}\right)(20)(x-1.5)^3 + C_1$$

$$EI y = 10(x-1.5)^3 - 45(x-1.5)^2 - \frac{1}{24}(20)(x-1.5)^4 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=4.5, y=0] \quad (10)(3)^3 - (45)(3)^2 - \frac{1}{24}(20)(3)^4 + 4.5 C_1 + 0 = 0$$

$$C_1 = 45 \text{ kN}\cdot\text{m}^2$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 315 \times 10^6 \text{ mm}^4 = 315 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(315 \times 10^{-6}) = 63 \times 10^6 \text{ N}\cdot\text{m}^2 = 63000 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A. $\left(\frac{dy}{dx} \text{ at } x=0\right)$

$$EI \theta_A = C_1 = 45 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = \frac{45}{63000} = 0.714 \times 10^{-3} \text{ rad}$$

$$\theta_A = 0.714 \times 10^{-3} \text{ rad} \quad \swarrow$$

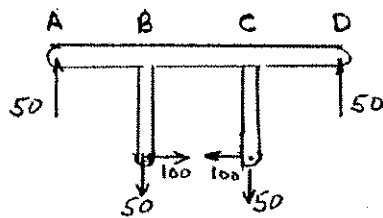
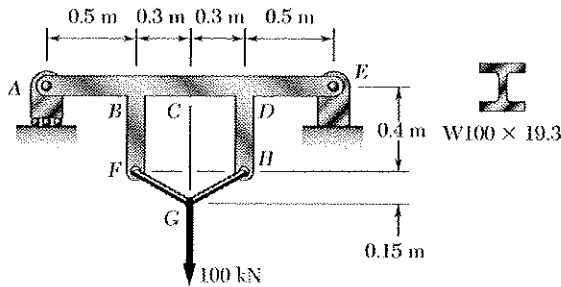
(b) Deflection at B. $(y \text{ at } x=1.5)$

$$EI y_B = (C_1)(1.5) = (45)(1.5) = 67.5 \text{ kN}\cdot\text{m}^3$$

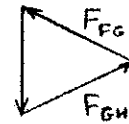
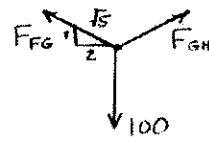
$$y_B = \frac{67.5}{63000} = 1.071 \times 10^{-3} \text{ m} = 1.071 \text{ mm} \quad \uparrow$$

Problem 9.64

9.64 The rigid bars BF and DH are welded to the rolled-steel beam AE as shown. Determine for the loading shown (a) the deflection at point B , (b) the deflection at midpoint C of the beam. Use $E = 200$ GPa.



Using joint G as a free body,



By symmetry
 $F_{GH} = F_{FG}$

$$2 F_{GH} - 100 = 0 \quad F_{GH} = 50 \text{ kN}$$

$$F_{GHx} = 2 F_{GH} = 100 \text{ kN}$$

Forces in kN. Lengths in m.

$$V = 50 - 50 \langle x - 0.5 \rangle^0 - 50 \langle x - 1.1 \rangle^0 \quad \text{kN}$$

$$M = 50x - 50 \langle x - 0.5 \rangle^1 - 50 \langle x - 1.1 \rangle^1 + 40 \langle x - 0.5 \rangle^0 - 40 \langle x - 1.1 \rangle^0 \quad \text{kN} \cdot \text{m}$$

$$EI \frac{dy}{dx} = 25x^2 - 25 \langle x - 0.5 \rangle^2 - 25 \langle x - 1.1 \rangle^2 - 40 \langle x - 0.5 \rangle^1 + 40 \langle x - 1.1 \rangle^1 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EI y = \frac{25}{3} x^3 - \frac{25}{3} \langle x - 0.5 \rangle^3 - \frac{25}{3} \langle x - 1.1 \rangle^3 - 20 \langle x - 0.5 \rangle^2 + 20 \langle x - 1.1 \rangle^2 + C_1 x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x = 0, y = 0] \quad C_2 = 0$$

$$[x = 1.6, y = 0]$$

$$\left(\frac{25}{3}\right)(1.6)^3 - \left(\frac{25}{3}\right)(1.1)^3 - \left(\frac{25}{3}\right)(0.5)^3 - (20)(1.1)^2 + (20)(0.5)^2 + C_1(1.6) + 0 = 0$$

$$C_1 = -1.75 \text{ kN} \cdot \text{m}^2$$

$$\text{For } EI y_B, \quad x = 0.5 \text{ m}$$

$$EI y_B = \left(\frac{25}{3}\right)(0.5)^3 - 0 - 0 + 0 - 0 - (1.75)(0.5) = 0.1667 \text{ kN} \cdot \text{m}^3$$

$$\text{For } EI y_C, \quad x = 0.8 \text{ m}$$

$$EI y_C = \left(\frac{25}{3}\right)(0.8)^3 - \left(\frac{25}{3}\right)(0.3)^3 - 0 - (20)(0.3)^2 - 0 - (1.75)(0.8) + 0 = -0.8417 \text{ kN} \cdot \text{m}^3$$

$$\text{For } W 100 \times 19.3 \text{ rolled steel section } I = 4.77 \times 10^6 \text{ mm}^4 = 4.77 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(4.77 \times 10^{-6}) = 954 \times 10^3 \text{ N} \cdot \text{m}^2 = 954 \text{ kN} \cdot \text{m}^2$$

$$(a) \quad y_B = \frac{0.1667}{954} = 0.175 \times 10^{-3} \text{ m}$$

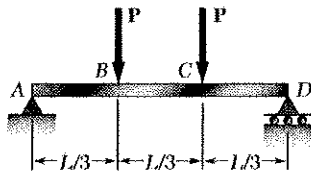
$$y_B = 0.175 \text{ mm} \uparrow$$

$$(b) \quad y_C = \frac{0.8417}{954} = 0.882 \times 10^{-3} \text{ m}$$

$$y_C = 0.882 \text{ mm} \uparrow$$

Problem 9.65

9.65 through 9.68 For the beam and loading shown, determine (a) the deflection at C, (b) the slope at end A.



LOADING I: LOAD AT B. CASE 5 OF APPENDIX D.

$$a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad x = \frac{2L}{3}$$

FOR $x > a$, REPLACE x BY $L-x$ & INTERCHANGE a & b

$$y = \frac{Pa}{6EI} [(L-x)^3 - (L^2 - a^2)(L-x)]$$

$$y_c = \frac{P(L/3)}{6EI} \left[\left(L - \frac{2L}{3} \right)^3 - \left(L^2 - \frac{L^2}{9} \right) \left(L - \frac{2L}{3} \right) \right] = -\frac{7}{486} \frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EI} = -\frac{P(2L/3)(L^2 - 4L^2/9)}{6EI} = -\frac{5}{81} \frac{PL^2}{EI}$$

LOADING II: LOAD AT C. CASE 5 OF APPENDIX D.

$$a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad x = \frac{2L}{3}$$

$$y_c = \frac{Pb}{6EI} [x^3 - (L^2 - b^2)x] = \frac{P(L/3)}{6EI} \left[\frac{8L^3}{27} - \left(L^2 - \frac{L^2}{9} \right) \left(\frac{2L}{3} \right) \right]$$

$$= -\frac{8}{486} \frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EI} = -\frac{P(L/3)(L^2 - L^2/9)}{6EI} = -\frac{4}{81} \frac{PL^2}{EI}$$

(a) DEFLECTION AT C. $y_c = -\frac{7}{486} \frac{PL^3}{EI} - \frac{8}{486} \frac{PL^3}{EI} = -\frac{15}{486} \frac{PL^3}{EI}$

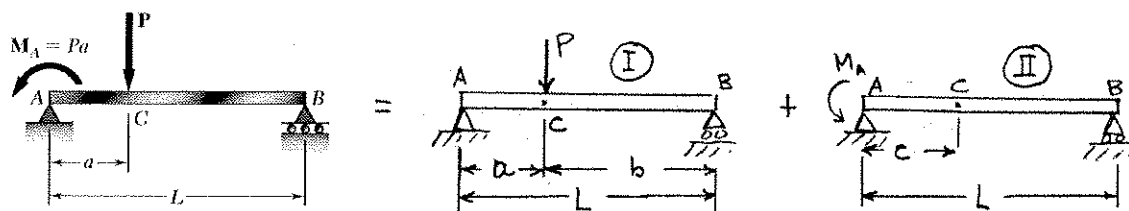
$$y_c = \frac{5}{162} \frac{PL^3}{EI} \downarrow$$

(b) SLOPE AT A. $\theta_A = -\frac{5}{81} \frac{PL^2}{EI} - \frac{4}{81} \frac{PL^2}{EI} = -\frac{9}{81} \frac{PL^2}{EI}$

$$\theta_A = \frac{1}{9} \frac{PL^2}{EI} \swarrow$$

Problem 9.66

For the beam and loading shown, determine (a) the deflection at the C, (b) the slope at end A.



Loading I: Case 5 of Appendix D. $b = L - a$

$$\text{For } x = a: (y_c)_1 = -\frac{Pa^2b^2}{3EIL} = -\frac{Pa^2(L-a)^2}{3EIL}$$

$$\begin{aligned} \text{For end A: } (\theta_A)_1 &= -\frac{Pb(L^2-b^2)}{6EIL} = -\frac{P(L-a)(2La-a^2)}{6EIL} \\ &= -\frac{Pa(2L^2-3La+a^2)}{6EIL} \end{aligned}$$

Loading II: Mirror image of Case 5 of Appendix D.

Replace x by $(L-x)$ in expression for y .

$$y = -\frac{M_A x}{6EIL} [(L-x)^3 - L^2(L-x)] = -\frac{M_A x}{6EIL} (3Lx - 2L^2 - x^2)$$

$$\text{With } x = a, \text{ and } M_A = Pa, (y_c)_2 = -\frac{Pa^2}{6EIL} (3La - 2L^2 - a^2)$$

$$(\theta_A)_2 = \frac{M_A L}{3EI} = \frac{PaL}{3EI}$$

(a) Deflection at C. $y_c = (y_c)_1 + (y_c)_2$

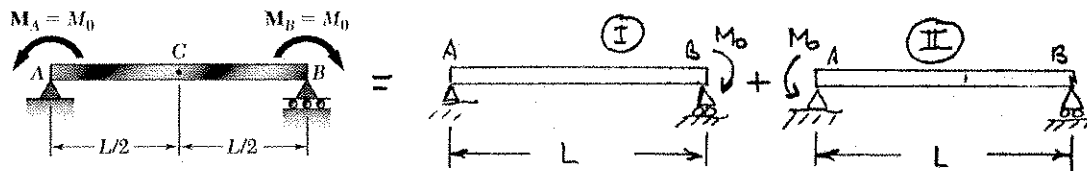
$$y_c = -\frac{Pa^2}{6EIL} [2(L-a)^2 + 3La - 2L^2 - a^2] \quad y_c = \frac{Pa^3(L-a)}{6EIL} \uparrow \blacktriangleleft$$

(b) Slope at end A. $\theta_A = (\theta_A)_1 + (\theta_A)_2$

$$\begin{aligned} \theta_A &= -\frac{Pa}{6EIL} [2L^2 - 3La + a^2 - 2L^2] = +\frac{Pa^2(3L-a)}{6EIL} \\ \theta_A &= \frac{Pa^2(3L-a)}{6EIL} \curvearrowright \blacktriangleleft \end{aligned}$$

Problem 9.67

For the beam and loading shown, determine (a) the deflection at the C, (b) the slope at end A.



Loading I: Case 7 of Appendix D.

$$y = -\frac{M_0}{6ETL} (x^3 - L^2x)$$

At C, $x = \frac{L}{2}$

$$(y_C)_1 = -\frac{M_0}{6EI} \left[\left(\frac{L}{2} \right)^3 - L^2 \left(\frac{L}{2} \right) \right] = \frac{M_0 L^2}{16EI}$$

$$(\theta_A)_1 = \frac{M_0 L}{6EI}$$

$$(\theta_B)_1 = -\frac{M_0 L}{3EI}$$

Loading II: Mirror image of Loading I.

$$(y_C)_2 = (y_C)_1; \quad (\theta_A)_2 = -(\theta_B)_1$$

(a) Deflection at C.

By superposition, $y_C = (y_C)_1 + (y_C)_2 = \frac{M_0 L^2}{8EI}$

$$y_C = \frac{M_0 L^2}{8EI} \uparrow$$

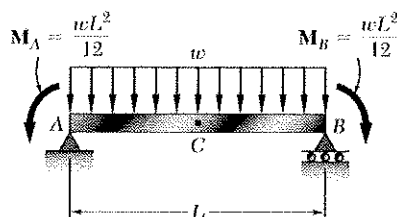
(b) Slope at A.

By superposition, $\theta_A = (\theta_A)_1 + (\theta_A)_2 = -\frac{M_0 L}{2EI}$

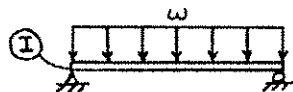
$$\theta_A = \frac{M_0 L}{2EI} \searrow$$

Problem 9.68

9.65 through 9.68 For the beam and loading shown, determine (a) the deflection at C, (b) the slope at end A.



LOADING I: CASE 6 IN APPENDIX D.



$$y_C = -\frac{5wL^4}{384EI} \quad \theta_A = -\frac{wL^3}{24EI}$$

LOADING II: CASE 7 IN APPENDIX D.



$$y_C = -\frac{M_A}{6EI} \left[\left(\frac{L}{2} \right)^3 - L^2 \left(\frac{L}{2} \right) \right] = \frac{1}{16} \frac{M_A L^2}{EI} \quad \theta_A = \frac{M_A L}{3EI}$$



$$\text{WITH } M_A = \frac{wL^2}{12} \quad y_C = \frac{1}{192} \frac{wL^4}{EI} \quad \theta_A = \frac{1}{36} \frac{wL^3}{EI}$$

LOADING III: CASE 7 IN APPENDIX D.

$$y_C = \frac{1}{16} \frac{M_B L^2}{EI} \quad (\text{USING LOADING II RESULT}) \quad \theta_A = \frac{M_B L}{6EI}$$

$$\text{WITH } M_B = \frac{wL^2}{12} \quad y_C = \frac{1}{192} \frac{wL^4}{EI} \quad \theta_A = \frac{1}{72} \frac{wL^3}{EI}$$

(a) DEFLECTION AT C.

$$y_C = -\frac{5}{384} \frac{wL^4}{EI} + \frac{1}{192} \frac{wL^4}{EI} + \frac{1}{192} \frac{wL^4}{EI} = -\frac{1}{384} \frac{wL^4}{EI}$$

$$y_C = \frac{1}{384} \frac{wL^4}{EI} \downarrow$$

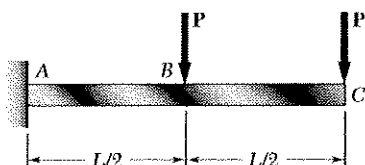
(b) SLOPE AT A.

$$\theta_A = -\frac{1}{24} \frac{wL^3}{EI} + \frac{1}{36} \frac{wL^3}{EI} + \frac{1}{72} \frac{wL^3}{EI} = 0$$

$$\theta_A = 0$$

Problem 9.69

9.69 and 9.70 For the cantilever beam and loading shown, determine the slope and deflection at the free end.

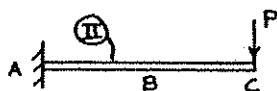


LOADING I: P DOWNWARD AT B.

CASE 1 OF APPENDIX D APPLIED TO PORTION AB.

$$\theta_B' = -\frac{P(L/2)^2}{2EI} = -\frac{1}{8} \frac{PL^2}{EI}$$

$$y_B' = -\frac{P(L/2)^3}{3EI} = -\frac{1}{24} \frac{PL^3}{EI}$$



BC REMAINS STRAIGHT.

$$\theta_C' = \theta_B' = -\frac{1}{8} \frac{PL^2}{EI}$$

$$y_C' = y_B' - \left(\frac{L}{2}\right)\theta_B' = -\frac{1}{24} \frac{PL^3}{EI} - \frac{1}{16} \frac{PL^3}{EI} = -\frac{5}{48} \frac{PL^3}{EI}$$

LOADING II: P DOWNWARD AT C. CASE 1 OF APPENDIX D.

$$\theta_C'' = -\frac{PL^2}{2EI}$$

$$y_C'' = -\frac{PL^3}{3EI}$$

BY SUPERPOSITION,

$$\theta_C = \theta_C' + \theta_C'' = -\frac{1}{8} \frac{PL^2}{EI} - \frac{1}{2} \frac{PL^2}{EI} = -\frac{5}{8} \frac{PL^2}{EI}$$

$$\frac{5PL^2}{8EI} \nearrow$$



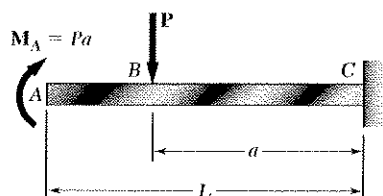
$$y_C = y_C' + y_C'' = -\frac{5}{48} \frac{PL^3}{EI} - \frac{1}{3} \frac{PL^3}{EI} = -\frac{21}{48} \frac{PL^3}{EI}$$

$$\frac{7PL^3}{16EI} \downarrow$$



Problem 9.70

9.69 and 9.70 For the cantilever beam and loading shown, determine the slope and deflection at the free end.

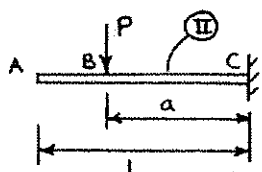


LOADING I: M_A AT A. CASE 3 OF APPENDIX D.



$$\theta_A' = -\frac{M_A L}{EI} \quad y_A' = \frac{M_A L^2}{2EI}$$

WITH $M_A = Pa$



$$\theta_A' = -\frac{PaL}{EI} \quad y_A' = \frac{PaL^2}{2EI}$$

LOADING II: P DOWNWARD AT B.

CASE I OF APPENDIX D APPLIED TO PORTION BC.

$$\theta_B'' = \frac{Pa^2}{2EI} \quad y_B'' = -\frac{Pa^3}{3EI}$$

AB REMAINS STRAIGHT

$$\theta_A'' = \theta_B'' = \frac{Pa^2}{2EI}$$

$$\begin{aligned} y_A'' &= y_B'' - (L-a)\theta_B'' \\ &= -\frac{Pa^3}{3EI} - (L-a)\frac{Pa^2}{2EI} = -\frac{Pa^2L}{2EI} + \frac{Pa^3}{6EI} \end{aligned}$$

BY SUPERPOSITION,

$$\theta_A = \theta_A' + \theta_A'' = -\frac{PaL}{EI} + \frac{Pa^2}{2EI} = -\frac{Pa}{2EI}(2L-a)$$

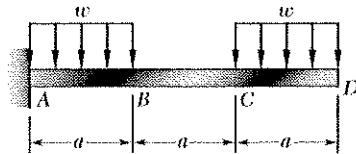
$$\frac{Pa}{2EI}(2L-a) \searrow$$

$$\begin{aligned} y_A &= y_A' + y_A'' = \frac{PaL^2}{2EI} - \frac{Pa^2L}{2EI} + \frac{Pa^3}{6EI} \\ &= \frac{Pa}{6EI}(3L^2 - 3aL + a^2) \end{aligned}$$

$$\frac{Pa}{6EI}(3L^2 - 3aL + a^2) \uparrow$$

Problem 9.71

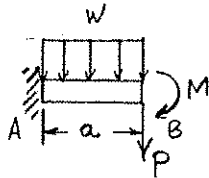
9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at point B.



Consider portion AB as a cantiver beam subjected to the three loadings shown.

By statics: $P = wa$

$$M = (wa)\left(a + \frac{a}{2}\right) = \frac{3}{2}wa^2$$



Slope at B. $\theta_B = (\theta_B)_w + (\theta_B)_P + (\theta_B)_M$

Case 2 of App. D. $(\theta_B)_w = -\frac{wa^3}{6EI}$

Case 1 of App. D. $(\theta_B)_P = -\frac{(wa)a^2}{2EI} = -\frac{wa^3}{2EI}$

Case 3 of App. D $(\theta_B)_M = -\frac{(\frac{3}{2}wa^2)a}{EI} = -\frac{3wa^3}{2EI}$

$$\theta_B = -\frac{13wa^3}{6EI}$$

$$\theta_B = \frac{13wa^3}{6EI} \quad \blacktriangleleft$$

Deflection at B.

$$y_B = (y_B)_w + (y_B)_P + (y_B)_M$$

Case 2 of App. D. $(y_B)_w = -\frac{wa^4}{8EI}$

Case 1 of App. D. $(y_B)_P = -\frac{(wa)a^3}{3EI} = -\frac{wa^4}{3EI}$

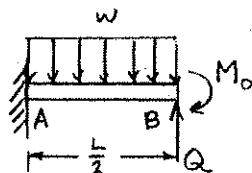
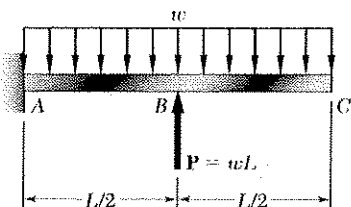
Case 3 of App. D. $(y_B)_M = -\frac{(\frac{3}{2}wa^2)a^2}{2EI} = -\frac{3wa^4}{4EI}$

$$y_B = -\frac{29wa^4}{24EI}$$

$$y_B = \frac{29wa^4}{24EI} \quad \downarrow \quad \blacktriangleleft$$

Problem 9.72

9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at point B.



Consider portion AB as a cantilever beam subjected to the three loadings shown.

By statics: $Q = P - \frac{wL}{2} = \frac{wL}{2}$

$$M_o = \left(\frac{wL}{2}\right)\left(\frac{1}{2} - \frac{L}{2}\right) = -\frac{wL^2}{8}$$

Slope at B. $\theta_B = (\theta_B)_w + (\theta_B)_Q + (\theta_B)_M$

Case 2 of App. D. $(\theta_B)_w = -\frac{w}{6EI} \left(\frac{L}{2}\right)^3 = -\frac{wL^3}{48EI}$

Case 1 of App. D. $(\theta_B)_Q = \frac{+(wL/2)}{2EI} \left(\frac{L}{2}\right)^2 = \frac{wL^3}{16EI}$

Case 3 of App. D. $(\theta_B)_M = -\frac{(wL^2/8)}{EI} \left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$

$$\theta_B = -\frac{wL^3}{48EI}$$

$$\theta_B = \frac{wL^3}{48EI} \quad \swarrow$$

Deflection at B.

$$y_B = (y_B)_w + (y_B)_Q + (y_B)_M$$

Case 2 of App. D.

$$(y_B)_w = -\frac{w}{8EI} \left(\frac{L}{2}\right)^4 = -\frac{wL^4}{128EI}$$

Case 1 of App. D.

$$(y_B)_Q = \frac{+(wL/2)}{3EI} \left(\frac{L}{2}\right)^3 = \frac{wL^4}{48EI}$$

Case 3 of App. D.

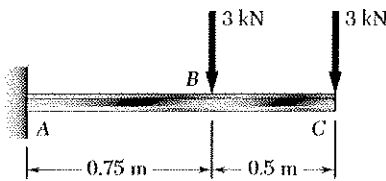
$$(y_B)_M = -\frac{(wL^2/8)}{2EI} \left(\frac{L}{2}\right)^2 = -\frac{wL^4}{64EI}$$

$$y_B = -\frac{wL^4}{384EI}$$

$$y_B = \frac{wL^4}{384EI} \quad \downarrow$$

Problem 9.73

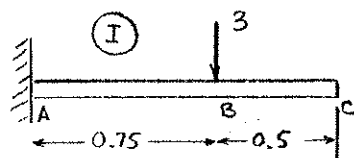
9.73 For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200 \text{ GPa}$.



I
S100 × 11.5

Units: Forces in kN, lengths in m.

Loading I: Concentrated load at B.



Case 1 of Appendix D applied to portion AB.

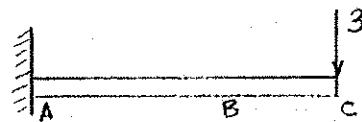
$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight.

$$\theta'_C = \theta'_B = -\frac{0.84375}{EI}$$

$$y'_C = y'_B - (0.5)\theta'_B = -\frac{0.84375}{EI}$$



Loading II: Concentrated load at C.

Case 1 of Appendix D.

$$\theta_A'' = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y_A'' = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition, $\theta_A = \theta_A' + \theta_A'' = -\frac{3.1875}{EI}$

$$y_A = y_A' + y_A'' = -\frac{2.796875}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

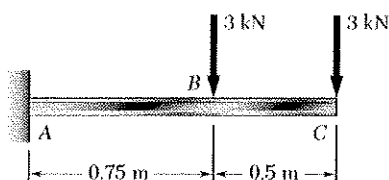
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N}\cdot\text{m}^2 = 506 \text{ kN}\cdot\text{m}^2$$

Slope at C. $\theta_C = -\frac{3.1875}{506} = -6.30 \times 10^{-3} \text{ rad} = 6.30 \times 10^{-3} \text{ rad} \swarrow$

Deflection at C. $y_C = -\frac{2.796875}{506} = -5.53 \times 10^{-3} \text{ m} = 5.53 \text{ mm} \downarrow$

Problem 9.74

9.74 For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200 \text{ GPa}$.

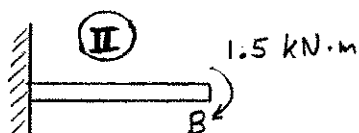
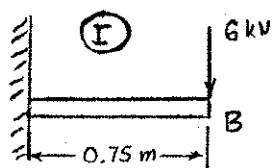


I
S100 × 11.5

Units: Forces in kN, lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.

Reduce the force at C to an equivalent force-couple system at B and add the force already at B to obtain the loadings I and II shown.



Loading I: Case 1 of Appendix D.

$$\theta_B' = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y_B' = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D.

$$\theta_B'' = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y_B'' = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{2EI} = -\frac{0.421875}{EI}$$

By superposition,

$$\theta_B = \theta_B' + \theta_B'' = -\frac{2.8125}{EI}$$

$$y_B = y_B' + y_B'' = -\frac{1.265625}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 2.53 \times 10^{-6} \text{ m}^4 = 2.53 \times 10^{-6} \text{ m}^4$

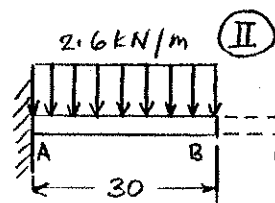
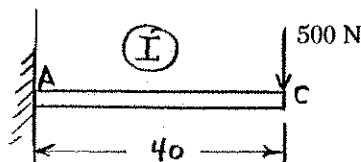
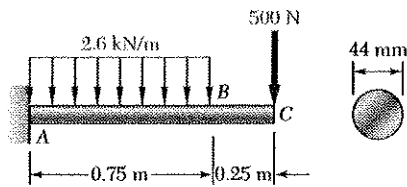
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N} \cdot \text{m}^2 = 506 \text{ kN} \cdot \text{m}^2$$

Slope at B. $\theta_B = -\frac{2.8125}{506} = -5.56 \times 10^{-3} \text{ rad} = 5.56 \times 10^{-3} \text{ rad} \swarrow$

Deflection at B. $y_B = -\frac{1.265625}{506} = -2.50 \times 10^{-3} \text{ m} = 2.50 \text{ mm} \downarrow$

Problem 9.75

9.75 For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200 \text{ GPa}$.



Loading I. Case 1 of Appendix D.

$$P = 0.5 \text{ kN}, \quad L = 1 \text{ m}$$

$$(y_c)_1 = -\frac{PL^3}{3EI} = -\frac{(0.5)(1)^3}{(3)(36.8)} = -4.529 \times 10^{-3} \text{ m}$$

$$(\theta_c)_1 = -\frac{PL^2}{2EI} = -\frac{(0.5)(1)^2}{(2)(36.8)} = -6.793 \times 10^{-3}$$

Loading II. Treat portion AB as a cantilever beam. (Case 2)

$$w = 2.6 \text{ kN/m}, \quad L = 0.75 \text{ m}$$

$$(y_B)_2 = -\frac{wL^4}{8EI} = -\frac{(2.6)(0.75)^4}{(8)(36.8)} = -2.794 \times 10^{-3} \text{ m}$$

$$(\theta_B)_2 = -\frac{wL^3}{6EI} = -\frac{(2.6)(0.75)^3}{(6)(36.8)} = -4.968 \times 10^{-3}$$

Portion BC remains straight for loading II.

$$L_{BC} = 0.25 \text{ m}$$

$$(y_c)_2 = (y_B)_2 + L_{BC}(\theta_B)_2 = -4.036 \times 10^{-3} \text{ m}$$

$$(\theta_c)_2 = (\theta_B)_2 = -4.968 \times 10^{-3}$$

Slope at end C. By superposition, $\theta_c = (\theta_c)_1 + (\theta_c)_2$

$$\theta_c = -11.761 \times 10^{-3}$$

$$\theta_c = 11.76 \times 10^{-3} \text{ rad} \swarrow$$

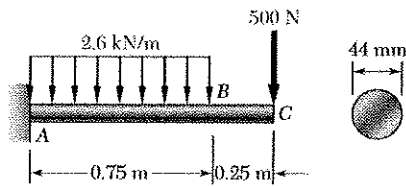
Deflection at end C. By superposition, $y_c = (y_c)_1 + (y_c)_2$

$$y_c = -8.565 \times 10^{-3} \text{ m}$$

$$y_c = 8.57 \text{ mm} \downarrow \swarrow$$

Problem 9.76

9.76 For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200 \text{ GPa}$.



$$c = \frac{1}{2}d = \frac{1}{2}(44) = 22 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(22)^4 = 183984 \text{ mm}^4$$

$$EI = (200 \times 10^6)(183984 \times 10^{-6}) = 36.8 \text{ kNm}^2$$

loading I: Case 1 of Appendix D.

$$P = 0.5 \text{ kN} \quad L = 1 \text{ m} \quad x = 0.75 \text{ m}$$

$$y_1 = \frac{P}{6EI}(x^3 - 3Lx^2)$$

$$\theta_1 = \frac{dy}{dx} = \frac{P}{2EI}(x^2 - 2Lx)$$

$$(y_B)_1 = \frac{0.5}{(6)(36.8)}[(0.75)^3 - (3)(1)(0.75)^2] = -2.866 \times 10^{-3} \text{ m}$$

$$(\theta_B)_1 = \frac{0.5}{(2)(36.8)}[(0.75)^2 - (2)(1)(0.75)] = -6.369 \times 10^{-3}$$

Loading II. Case 2 of Appendix D.

$$W = 2.6 \text{ kN/m}, \quad L = 0.75 \text{ m}$$

$$(y_B)_2 = -\frac{WL^4}{8EI} = -\frac{(2.6)(0.75)^4}{(8)(36.8)} = -2.794 \times 10^{-3} \text{ m}$$

$$(\theta_B)_2 = -\frac{WL^3}{6EI} = -\frac{(2.6)(0.75)^3}{(6)(36.8)} = -4.968 \times 10^{-3}$$

Slope at point B.

$$\theta_B = (\theta_B)_1 + (\theta_B)_2$$

$$\theta_B = -11.337 \times 10^{-3}$$

$$\theta_B = 11.34 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

Deflection at point B.

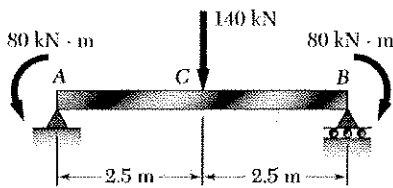
$$y_B = (y_B)_1 + (y_B)_2$$

$$y_B = -5.66 \times 10^{-3} \text{ m}$$

$$y_B = 5.66 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.77

9.77 and 9.78 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point C. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m.

Loading I: Moment at B.

Case 7 of Appendix D. $M = 80 \text{ kN}\cdot\text{m}$, $L = 5.0 \text{ m}$, $x = 2.5 \text{ m}$

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EIL} (x^3 - L^2x) = -\frac{80}{6EI(5.0)} [2.5^3 - (5.0)^2(2.5)] = \frac{125}{EI}$$

Loading II: Moment at A. Case 7 of Appendix D.

$M = 80 \text{ kN}\cdot\text{m}$, $L = 5.0 \text{ m}$, $x = 2.5 \text{ m}$

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \quad (\text{Same as loading I})$$

Loading III: 140 kN concentrated load at C. $P = 140 \text{ kN}$

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = 156 \times 10^6 \text{ mm}^4 = 156 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(156 \times 10^{-6}) = 31.2 \times 10^6 \text{ N}\cdot\text{m}^2 = 31200 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A. $\theta_A = \frac{66.667 + 133.333 - 218.75}{31200} = -0.601 \times 10^{-3} \text{ rad}$

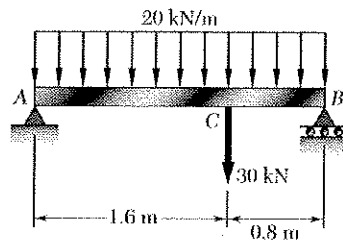
$$\theta_A = 0.601 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

(b) Deflection at C. $y_C = \frac{125 + 125 - 364.583}{31200} = -3.67 \times 10^{-3} \text{ m}$

$$y_C = 3.67 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.78

9.77 and 9.78 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point C. Use $E = 200 \text{ GPa}$.

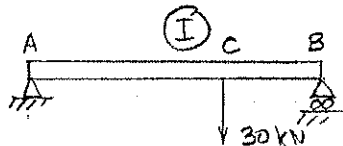


Units: Forces in kN. Lengths in meters.

For W150 x 24 $I = 13.4 \times 10^6 \text{ mm}^4$
 $= 13.4 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^4 \text{ N} \cdot \text{m}$$

$$= 2680 \text{ kN} \cdot \text{m}^2$$



Loading I: Case 5 of Appendix D.

$$P = 30 \text{ kN}, L = 2.4 \text{ m}, a = 1.6 \text{ m}, b = 0.8 \text{ m}$$

$$(\theta_A)_1 = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(30)(0.8)(2.4^2 - 0.8^2)}{(6)(2680)(2.4)}$$

$$= -3.1841 \times 10^{-3}$$

$$(y_C)_1 = -\frac{Pa^2b^2}{3EIL} = -\frac{(30)(1.6)^2(0.8)^2}{(3)(2680)(2.4)}$$

$$= -2.54726 \times 10^{-3} \text{ m}$$

Loading II: Case 6 of Appendix D.

$$W = 20 \text{ kN/m}, L = 2.4 \text{ m}, x = 1.6 \text{ m at point C.}$$

$$(\theta_A)_2 = -\frac{WL^3}{24EI} = -\frac{(20)(2.4)^3}{(24)(2680)} = -4.2985 \times 10^{-3}$$

$$y_2 = -\frac{W}{24EI} (x^4 - 2Lx^3 + L^3x)$$

$$(y_C)_2 = -\frac{20}{(24)(2680)} [(1.6)^4 - (2)(2.4)(1.6)^3 + (2.4)^3(1.6)] = -2.80199 \times 10^{-3} \text{ m}$$

(a) Slope at end A. $\theta_A = (\theta_A)_1 + (\theta_A)_2$

$$\theta_A = -7.4826 \times 10^{-3}$$

$$\theta_A = 7.48 \times 10^{-3} \text{ rad} \quad \nwarrow$$

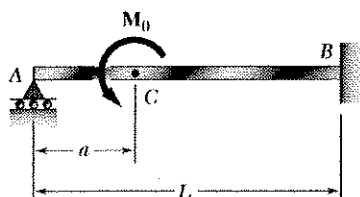
(b) Deflection at point C. $y_C = (y_C)_1 + (y_C)_2$

$$y_C = -5.34925 \times 10^{-3} \text{ m}$$

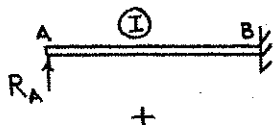
$$y_C = 5.35 \text{ mm} \quad \downarrow$$

Problem 9.79

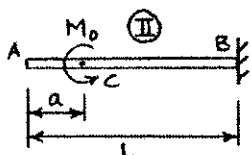
9.79 and 9.80 For the uniform beam shown, determine (a) the reaction at A, (b) the reaction at B.



CONSIDER R_A AS REDUNDANT AND REPLACE LOADING SYSTEM BY I AND II.



LOADING I. (CASE 1 OF APPENDIX D.) $y_A' = \frac{R_A L^3}{3EI}$



LOADING II. PORTION BC. (CASE 3 OF APPENDIX D.)

$$y_c'' = -\frac{M_o(L-a)^2}{2EI} \quad \theta_c'' = \frac{M_o(L-a)}{EI}$$

PORTION AC IS STRAIGHT. $y_A'' = y_c'' - (a)\theta_c''$

$$= -\frac{M_o(L-a)^2}{2EI} - \frac{a M_o(L-a)}{EI}$$

(a) SUPERPOSITION AND CONSTRAINT: $y_A = y_A' + y_A'' = 0$

$$\frac{R_A L^3}{3EI} - \frac{M_o(L-a)^2}{2EI} - \frac{a M_o(L-a)}{EI} = 0$$

$$\frac{2}{3} R_A L^3 - M_o(L-a)(L-a+2a) = 0$$

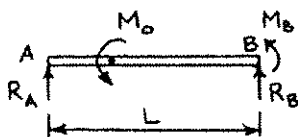
$$\frac{2}{3} R_A L^3 = M_o(L^2 - a^2)$$

$$R_A = \frac{3M_o}{2L^3}(L^2 - a^2) \uparrow$$

(b)

$$+\uparrow \Sigma F_y = 0: R_A + R_B = 0 \quad R_B = -R_A$$

$$R_B = \frac{3M_o}{2L^3}(L^2 - a^2) \downarrow$$



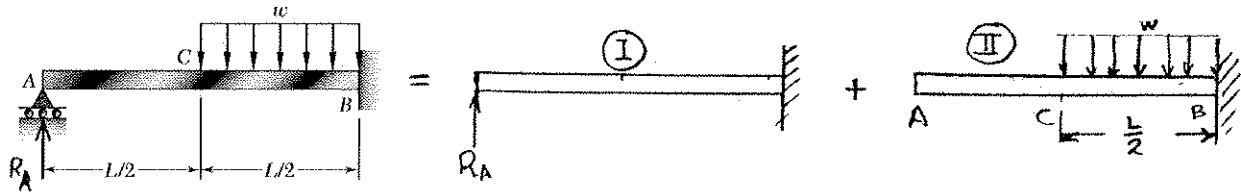
$$+\circlearrowleft \Sigma M_B = 0: M_B + M_o - R_A L = 0 \quad M_B + M_o - \frac{3M_o}{2L^2}(L^2 - a^2) = 0$$

$$M_B = \frac{3M_o}{2L^2}(L^2 - a^2 - \frac{2}{3}L^2)$$

$$M_B = \frac{M_o}{2L^2}(L^2 - 3a^2) \curvearrowright$$

Prolem 9.80

9.79 and 9.80 For the uniform beam shown, determine (a) the reaction at A, (b) the reaction at B.



Consider R_A as redundant and replace loading system by I and II.

Loading I. (Case 1 of Appendix D) $(y_A)_1 = \frac{R_A L^3}{3EI}$

Loading II. Portion CB. (Case 2 of Appendix D)

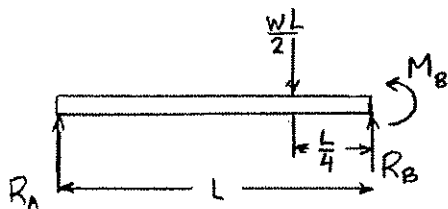
$$(y_B)_2 = -\frac{w(L/2)^4}{8EI} = -\frac{wL^4}{128EI}, \quad (\theta_B)_2 = \frac{w(L/2)^3}{6EI} = \frac{wL^3}{48EI}$$

Portion AC is straight. $(y_A)_2 = (y_B)_2 - \frac{L}{2}(\theta_B)_2 = -\frac{7wL^4}{384EI}$

Superposition and constraint. $y_A = (y_A)_1 + (y_A)_2 = 0$

$$(a) \quad \frac{R_A L^3}{3EI} - \frac{7wL^4}{384EI} = 0$$

$$R_A = \frac{7wL}{128} \uparrow$$



Using entire beam as a free body,

$$+\uparrow \Sigma F_y = 0: R_A + R_B - \frac{wL}{2} = 0$$

$$R_B = \frac{wL}{2} - \frac{7wL}{128} \quad R_B = \frac{57wL}{128} \uparrow$$

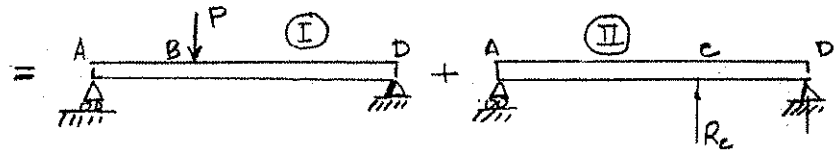
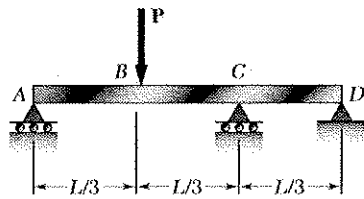
$$+\circlearrowleft \Sigma M_B = 0: M_B - R_A L - \frac{wL}{2} \cdot \frac{L}{4} = 0$$

$$M_B = R_A L - \frac{wL^2}{8} = -\frac{9wL^2}{128}$$

$$M_B = \frac{9wL^2}{128} \curvearrowright$$

Problem 9.81

9.81 and 9.82 For the uniform beam shown, determine the reaction at each of the three supports.



Consider R_c as redundant and replace loading system by I and II.

Loading I. (Case 5 of Appendix D) $a = \frac{2L}{3}$, $b = \frac{L}{3}$, $x = \frac{L}{3}$ at C.

$$(y_c)_1 = \frac{Pb}{6EIL} [x^3 - (L^2 - b^2)x] = \frac{P(L/3)}{6EIL} \left[\left(\frac{L}{3}\right)^3 - \left\{ L^2 - \left(\frac{L}{3}\right)^2 \right\} \frac{L}{3} \right]$$

$$= -\frac{7PL^3}{486EI}$$

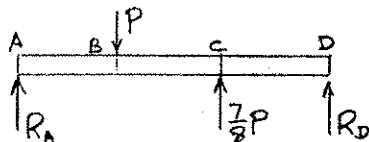
Loading II. (Case 5 of Appendix D) $a = \frac{2L}{3}$, $b = \frac{L}{3}$

$$(y_c)_2 = \frac{R_c a^2 b^2}{3EIL} = \frac{R_c (L/3)^2 (2L/3)^2}{3EIL} = \frac{4R_c L^3}{243EI}$$

Superposition and constraint. $y_c = (y_c)_1 + (y_c)_2 = 0$

$$-\frac{7PL^3}{486EI} + \frac{4R_c}{243EI} = 0$$

$$R_c = \frac{7}{8}P \uparrow$$



$$+\circlearrowleft \sum M_D = 0:$$

$$-R_A L + P\left(\frac{2L}{3}\right) - \left(\frac{7}{8}P\right)\left(\frac{L}{3}\right) = 0$$

$$R_A = \frac{3}{8}P \uparrow$$

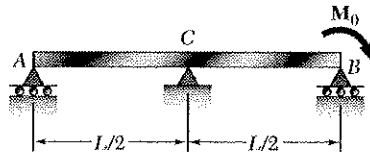
$$+\uparrow \sum F_y = 0: R_A + R_D - P + \frac{7}{8}P = 0$$

$$R_D = P - \frac{7}{8}P - \frac{3}{8}P = -\frac{1}{4}P$$

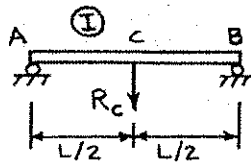
$$R_D = \frac{1}{4}P \downarrow$$

Problem 9.82

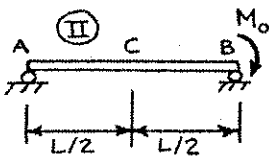
9.81 and 9.82 For the uniform beam shown, determine the reaction at each of the three supports.



CONSIDER R_C AS REDUNDANT AND REPLACE LOADING SYSTEM BY I AND II.



LOADING I. (CASE 4 OF APPENDIX D.) $y'_C = -\frac{R_c L^3}{48EI}$



LOADING II. (CASE 7 OF APPENDIX D.)

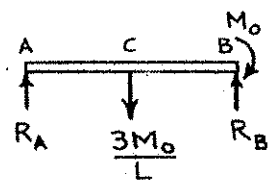
$$y''_C = -\frac{M_o}{6EI} \left[\left(\frac{L}{2} \right)^3 - L^2 \left(\frac{L}{2} \right) \right] = \frac{M_o L^2}{16EI}$$

SUPERPOSITION AND CONSTRAINT.

$$y_C = y'_C + y''_C = 0$$

$$-\frac{R_c L^3}{48EI} + \frac{M_o L^2}{16EI} = 0$$

$$R_c = \frac{3M_o}{L} \downarrow$$



$$+\circlearrowleft \sum M_B = 0: -R_A L + \left(\frac{3M_o}{L} \right) \left(\frac{L}{2} \right) - M_o = 0$$

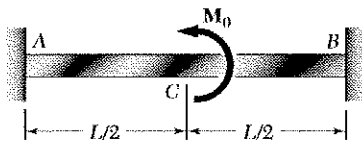
$$R_A = \frac{M_o}{2L} \uparrow$$

$$+\uparrow \sum F_y = 0: \frac{M_o}{2L} - \frac{3M_o}{L} + R_B = 0$$

$$R_B = \frac{5M_o}{2L} \uparrow$$

Problem 9.83

9.83 and 9.84 For the beam shown, determine the reaction at B.



Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.



loading I. Case 1 of Appendix D.

$$(y_B)_I = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_I = -\frac{R_B L^2}{2EI}$$



loading II. Case 3 of Appendix D.

$$(y_B)_{II} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{II} = \frac{M_B L}{EI}$$



loading III. Case 3 applied to portion AC.

$$(y_C)_{III} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{III} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{3}{8} \frac{M_0 L^2}{EI}$$

$$(\theta_B)_{III} = (\theta_C)_{III} = \frac{1}{2} \frac{M_0 L}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$-\frac{L^3}{3EI} R_B + \frac{L^2}{2EI} M_B + \frac{3}{8} \frac{M_0 L^2}{EI} = 0 \quad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$-\frac{L^2}{2EI} R_B + \frac{L}{EI} M_B + \frac{1}{2} \frac{M_0 L}{EI} = 0 \quad (2)$$

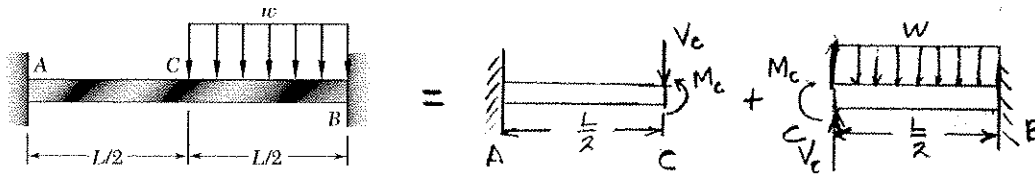
Solving (1) and (2) simultaneously,

$$R_B = \frac{3}{2} \frac{M_0}{L} \downarrow \quad \blacktriangleleft$$

$$M_B = \frac{1}{4} M_0 \curvearrowright \quad \blacktriangleleft$$

Problem 9.84

9.83 and 9.84 For the beam shown, determine the reaction at B.



Portion AC: Superposition of Cases 3 and 1 of Appendix D.

$$y_c = \frac{M_c(L/2)^2}{2EI} - \frac{V_c(L/2)^3}{3EI} = \frac{M_c L^2}{8EI} - \frac{V_c L^3}{24EI}$$

$$\theta_c = \frac{M_c(L/2)}{EI} - \frac{V_c(L/2)^2}{2EI} = \frac{M_c L}{2EI} - \frac{V_c L^2}{8EI}$$

Portion CB: Superposition of Cases 3, 1, and 2 of Appendix D.

$$y_c = \frac{M_c(L/2)^2}{2EI} + \frac{V_c(L/2)^3}{3EI} - \frac{w(L/2)^4}{8EI}$$

$$= \frac{M_c L^2}{8EI} + \frac{V_c L^3}{24EI} - \frac{w L^4}{128EI}$$

$$\theta_c = -\frac{M_c(L/2)}{EI} - \frac{V_c(L/2)^2}{2EI} + \frac{w(L/2)^3}{6EI}$$

$$= -\frac{M_c L}{2EI} - \frac{V_c L^2}{8EI} + \frac{w L^3}{48EI}$$

Matching expressions for y_c ,

$$\frac{M_c L^2}{8EI} - \frac{V_c L^3}{24EI} = \frac{M_c L^2}{8EI} + \frac{V_c L^3}{24EI} - \frac{w L^4}{128EI}$$

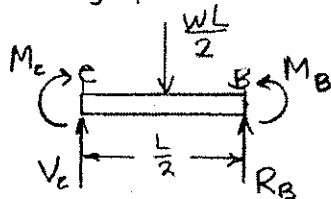
$$V_c = \frac{3}{32} w L$$

Matching expressions for θ_c ,

$$\frac{M_c L}{2EI} - \frac{V_c L^2}{8EI} = -\frac{M_c L}{2EI} - \frac{V_c L^2}{8EI} + \frac{w L^3}{48EI}$$

$$M_c = \frac{1}{48} w L^2$$

Using portion CB as a free body,



$$+\uparrow \sum F_y = 0: R_B + V_c - \frac{wL}{2} = 0$$

$$R_B = \frac{wL}{2} - \frac{3}{32} w L$$

$$R_B = \frac{13}{32} w L \uparrow$$

$$+\circlearrowleft \sum M_B = 0: M_B - M_c - V_c \frac{L}{2} + \frac{wL}{2} \cdot \frac{L}{4} = 0$$

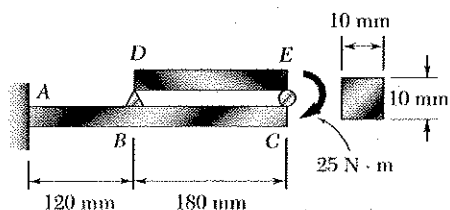
$$M_B = \frac{1}{48} w L^2 + \left(\frac{3}{32} w L\right) \left(\frac{L}{2}\right) - \frac{w L^2}{8}$$

$$M_B = -\frac{11}{192} w L^2$$

$$M_B = \frac{11}{192} w L^2 \curvearrowright$$

Problem 9.85

9.85 Beam DE rests on the cantilever beam AC as shown. Knowing that a square rod of side 10 mm is used for each beam, determine the deflection at end C if the 25-N-m couple is applied (a) to end E of beam DE , (b) to end C of beam AC . Use $E = 200$ GPa.



$$E = 200 \times 10^9 \text{ Pa}$$

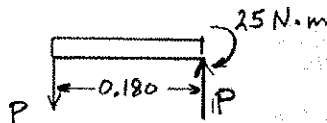
$$I = \frac{1}{12}(10)(10)^3 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = 166.667 \text{ N}\cdot\text{m}^2$$

(a) Couple applied to beam DE .

Free body DE . $+\circlearrowleft \Sigma M = 0$:

$$0.180 P - 25 = 0 \quad P = 138.889 \text{ N}$$



Loads on cantilever beam ABC are $P \uparrow$ at point B and $P \downarrow$ at point C as shown.

Due to $P \uparrow$ at point B .

Using portion AB and applying Case 1 of Appendix D,

$$(y_B)_1 = \frac{PL^3}{3EI} = \frac{(138.889)(0.120)^3}{(3)(166.667)} = 0.480 \times 10^{-3} \text{ m}$$

$$(\theta_B)_1 = \frac{PL^2}{2EI} = \frac{(138.889)(0.120)^2}{(2)(166.667)} = 6.00 \times 10^{-3}$$

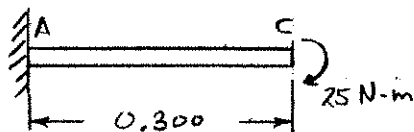
$$(y_C)_1 = (y_B)_1 + L_{BC}(\theta_B)_1 = 0.480 \times 10^{-3} + (0.180)(6.00 \times 10^{-3}) = 1.56 \times 10^{-3} \text{ m}$$

Due to load $P \downarrow$ at point C . Case 1 of App. D applied to ABC .

$$(y_C)_2 = -\frac{PL^3}{3EI} = -\frac{(138.889)(0.120 + 0.180)^3}{(3)(166.667)} = -7.50 \times 10^{-3} \text{ m}$$

Total deflection at point C . $y_C = (y_C)_1 + (y_C)_2 = -5.94 \times 10^{-3} \text{ m}$

$$y_C = 5.94 \text{ mm} \downarrow$$



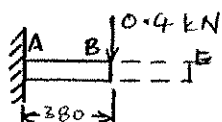
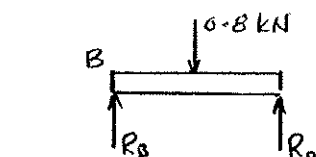
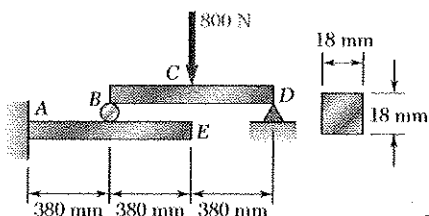
(b) Couple applied to beam AC .

Case 3 of Appendix D.

$$y_C = -\frac{ML^2}{2EI} = -\frac{(25)(0.300)^2}{(2)(166.667)} = -6.75 \times 10^{-3} \text{ m}$$

$$y_C = 6.75 \text{ mm} \downarrow$$

Problem 9.86



9.86 Beam BD rests on the cantilever beam AE , as shown. Knowing that a square rod of side 18 mm is used for each beam, determine for the loading shown (a) the deflection at point C , (b) the deflection at point E . Use $E = 200$ GPa.

$$I = \frac{1}{12}(18)(18)^3 = 8748 \text{ mm}^4$$

$$EI = (200 \times 10^6)(8748 \times 10^{-12}) = 1.75 \text{ kNm}^2$$

For free body BCD , $\sum M_D = 0$:

$$-760R_B + (380)(0.8) = 0 \quad R_B = 0.4 \text{ kN}$$

Cantilever ABE . Portion AB .

Case 1 of Appendix D.

$$y_B = -\frac{PL^3}{3EI} = -\frac{(0.4)(0.38)^3}{(3)(1.75)} = -4.181 \times 10^{-3} \text{ m}$$

$$\theta_B = -\frac{PL^2}{2EI} = -\frac{(0.4)(0.38)^2}{(2)(1.75)} = -16.5 \times 10^{-3} \text{ m}$$

Deflection at point C if point B does not move.

Apply Case 4 of Appendix D to beam BCD .

$$(y_C)_1 = -\frac{PL^3}{48EI} = -\frac{(0.8)(0.76)^3}{(48)(1.75)} = -4.181 \times 10^{-3} \text{ m}$$

Addition deflection at point C due to movement of point B .

$$(y_C)_2 = \frac{380}{760} y_B = -2.091 \times 10^{-3} \text{ m}$$

(a) Total deflection at point C . $y_C = (y_C)_1 + (y_C)_2 = -6.272 \times 10^{-3} \text{ m}$

$$y_C = 6.27 \text{ mm} \downarrow$$

(b) Deflection at point E . $y_E = y_B + L_{BE}\theta_B$

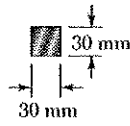
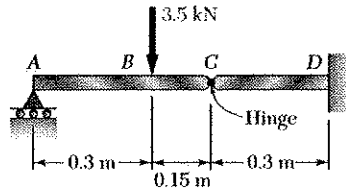
$$= -4.181 \times 10^{-3} + (0.38)(-16.5 \times 10^{-3})$$

$$= -10.451 \times 10^{-3} \text{ m}$$

$$y_E = 10.45 \text{ mm} \downarrow$$

Problem 9.87

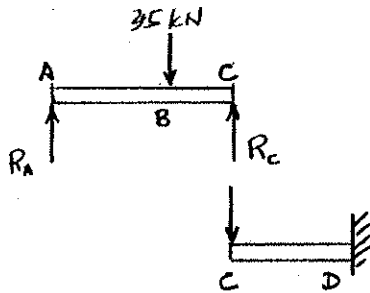
9.87 The two beams shown have the same cross section and are joined by a hinge at C. For the loading shown, determine (a) the slope at point A, (b) the deflection at point B. Use $E = 200 \text{ GPa}$.



Using free body ABC,

$$\sum M_A = 0: 0.45 R_C - (0.3)(35) = 0$$

$$R_C = 2.33 \text{ kN}$$



$$E = 200 \text{ GPa}$$

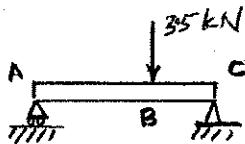
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (30)(30)^3 = 67500 \text{ mm}^4$$

$$EI = (200 \times 10^9)(67500 \times 10^{-12}) = 13.5 \text{ kNm}^2$$

Using cantilever beam CD with load R_C ,

Case 1 of Appendix D.

$$y_C = -\frac{R_C L_{CD}^3}{3EI} = -\frac{(2.33)(0.3)^3}{(3)(13.5)} = -0.001555 \text{ m}$$



Calculation of θ_A' and y_B' assuming that point C does not move.

Case 5 of Appendix D

$$P = 35 \text{ kN} \quad L = 0.45 \text{ m}, \quad a = 0.3 \text{ m}, \quad b = 0.15 \text{ m}$$

$$\theta_A' = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(35)(0.15)(0.45^2 - 0.15^2)}{(6)(13.5)(0.45)} = -0.00259 \text{ rad}$$

$$y_B' = -\frac{Pb^2a^2}{3EIL} = -\frac{(35)(0.15)^2(0.3)^2}{(3)(13.5)(0.45)} = -0.000389 \text{ m}$$

Additional slope and deflection due to movement of point C.

$$\theta_A'' = \frac{y_C}{L_{AC}} = -\frac{0.001555}{0.45} = -0.003456 \text{ rad}$$

$$y_B'' = \frac{a}{L} y_C = -\frac{(0.3)(0.001555)}{0.45} = -0.0010367 \text{ m}$$

(a) Slope at A. $\theta_A = \theta_A' + \theta_A'' = -0.00259 - 0.003456$

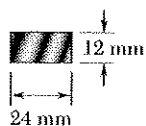
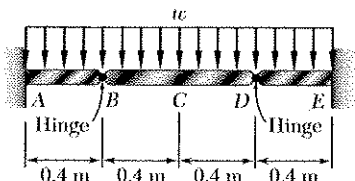
$$= -0.006046 \text{ rad} = 0.00605 \text{ rad} \quad \swarrow \quad \blacktriangleleft$$

(b) Deflection at B. $y_B = y_B' + y_B'' = -0.000389 - 0.0010367$

$$= -1.4257 \times 10^{-3} \text{ m} = 1.43 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.88

9.88 A central beam BD is joined at hinges to two cantilever beams AB and DE . All beams have the cross section shown. For the loading shown, determine the largest w so that the deflection at C does not exceed 3 mm. Use $E = 200$ GPa.



$$\text{Let } a = 0.4 \text{ m}$$

Cantilever beams AB and CD .

Cases 1 and 2 of Appendix D; $y_c = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{11}{24} \frac{wa^4}{EI}$

Beam BCD , with $L = 0.8$ m, assuming that points B and D do not move.

Case 6 of Appendix D.

$$y_c' = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points B and D .

$$y_c'' = y_B = y_D = -\frac{11}{24} \frac{wa^4}{EI}$$

Total deflection at C .

$$y_c = y_c' + y_c''$$

$$y_c = -\frac{w}{EI} \left\{ \frac{5L^4}{384} + \frac{11a^4}{24} \right\}$$

Data: $E = 200 \times 10^9 \text{ Pa}$, $I = \frac{1}{12}(24)(12)^3 = 3.456 \times 10^{-3} \text{ mm}^4 = 3.456 \times 10^{-9} \text{ m}^4$

$$EI = (200 \times 10^9)(3.456 \times 10^{-9}) = 691.2 \text{ N} \cdot \text{m}^2$$

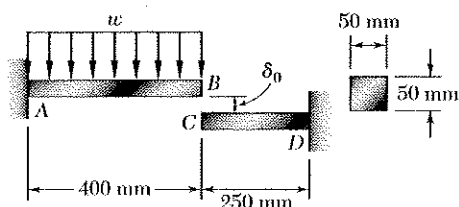
$$y_c = -3 \times 10^{-3} \text{ m}$$

$$-3 \times 10^{-3} = -\frac{w}{691.2} \left\{ \frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24} \right\} = -24.69 \times 10^{-6} w$$

$$w = 121.5 \text{ N/m} \quad \blacktriangleleft$$

Problem 9.89

9.89 Before the uniformly distributed load w is applied, a gap, $\delta_0 = 1.2 \text{ mm}$, exists between the ends of the cantilever bars AB and CD . Knowing that $E = 105 \text{ GPa}$ and $w = 30 \text{ kN/m}$, determine (a) the reaction at A , (b) the reaction at D .



$$I = \frac{1}{12}(50)(50)^3 = 520.833 \times 10^3 \text{ mm}^4 = 520.833 \times 10^{-7} \text{ m}^4$$

$$EI = (105 \times 10^9)(520.833 \times 10^{-7}) = 54.6875 \times 10^3 \text{ N}\cdot\text{m}^2$$

$$= 54.6875 \text{ kN}\cdot\text{m}^2$$

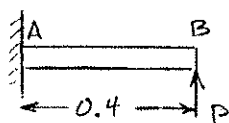
Units: Forces in kN. Lengths in meters.

Compute deflection at B due to w . Case 8 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(30)(0.400)^4}{(8)(54.6875)} = -1.75543 \times 10^{-3} = -1.7553 \text{ mm}$$

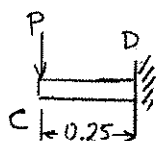
The displacement is more than δ_0 , the gap closes.

Let P be the contact force between points B and C .



Compute deflection of B due to P . Use Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(0.4)^3}{(3)(54.6875)} = 390.095 \times 10^{-6} P \text{ m}$$



Compute deflection of C due to P .

$$y_C = -\frac{PL^3}{3EI} = -\frac{P(0.25)^3}{(3)(54.6875)} = -95.238 \times 10^{-6} P \text{ m}$$

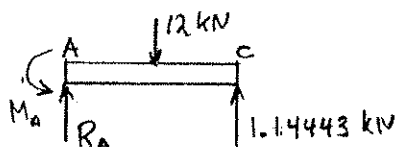
Displacement condition. $y_B + \delta_0 = y_C$

Using superposition, $(y_B)_1 + (y_B)_2 - \delta_0 = y_C$

$$-1.75543 \times 10^{-3} + 390.095 \times 10^{-6} P + 1.2 \times 10^{-3} = -95.238 \times 10^{-6} P$$

$$485.333 \times 10^{-6} P = 0.55543 \times 10^{-3} \quad P = 1.14443 \text{ kN.}$$

(a) Reaction at A.



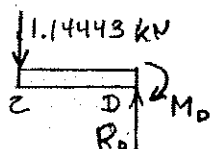
$$+\uparrow \Sigma F_y = 0: R_A - 12 + 1.14443 = 0$$

$$R_A = 10.86 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: M_A - (0.2)(12) + (0.4)(1.14443) = 0$$

$$M_A = 1.942 \text{ kN}\cdot\text{m} \circlearrowleft$$

(b) Reaction at D.



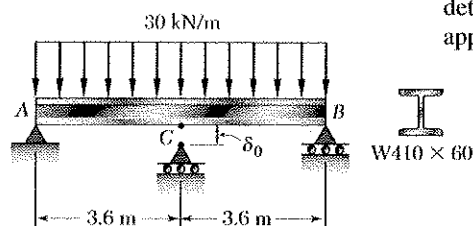
$$+\uparrow \Sigma F_y = 0: R_D - 1.14443 = 0$$

$$R_D = 1.144 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_D = 0: -M_D + (0.25)(1.14443) = 0$$

$$M_D = 0.286 \text{ kN}\cdot\text{m} \circlearrowleft$$

Problem 9.90



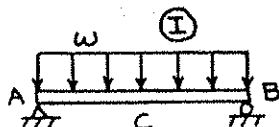
9.90 Before the 30 kN/m load is applied, a gap, $\delta_0 = 20$ mm, exists between the W410 $\times$ 60 beam and the support at C. Knowing that $E = 200$ GPa, determine the reaction at each support after the uniformly distributed load is applied.

DATA: $\delta_0 = 0.02$ m

$$E = 200 \times 10^6 \text{ kPa}$$

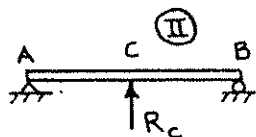
$$I = 216 \times 10^{-6} \text{ m}^4$$

$$EI = 43.2 \times 10^3 \text{ kN m}^2$$



LOADING I. CASE 6 OF APPENDIX D.

$$\begin{aligned} y'_C &= -\frac{5wL^4}{384EI} = -\frac{5(30)(7.2)^4}{384(43.2 \times 10^3)} \\ &= -24.3 \times 10^{-3} \text{ m} \end{aligned}$$

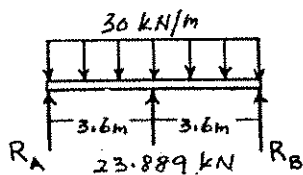


LOADING II. CASE 4 OF APPENDIX D.

$$\begin{aligned} y''_C &= \frac{R_C L^3}{48EI} = \frac{R_C (7.2)^3}{48(43.2 \times 10^3)} \\ &= 0.18 \times 10^{-3} R_C \end{aligned}$$

DEFLECTION AT C.

$$\begin{aligned} y_C &= y'_C + y''_C = -\delta_0 \\ -24.3 \times 10^{-3} + 0.18 \times 10^{-3} R_C &= -20 \times 10^{-3} \end{aligned}$$



$$R_C = 23.889 \text{ kN}$$

$$R_C = 23.9 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: (30)(7.2)(3.6) - R_A(7.2) - (23.889)(3.6) = 0$$

$$R_A = 96.055$$

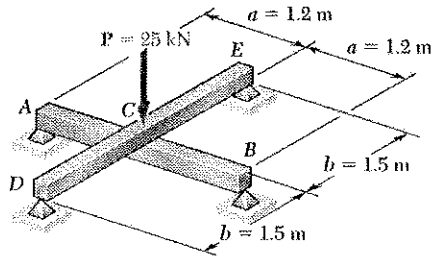
$$R_A = 96.1 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: 96.055 - 30(7.2) + 23.889 + R_B = 0$$

$$R_B = 96.1 \text{ kN} \uparrow$$

Problem 9.91

9.91 For the loading shown, and knowing that beams AB and DE have the same flexural rigidity, determine the reaction (a) at B , (b) at E .



Units: Forces in kN; lengths in m.

For beam ACB , using Case 4 of Appendix D.

$$(y_c)_1 = -\frac{R_c(2a)^3}{48EI}$$

For beam DCE , using Case 4 of Appendix D.

$$(y_c)_2 = \frac{(R_c - P)(2b)^3}{48EI}$$

Matching deflections at C ,

$$-\frac{R_c(2a)^3}{48EI} = \frac{(R_c - P)(2b)^3}{48EI}$$

$$R_c = \frac{Pb^3}{a^3 + b^3} = \frac{(25)(1.5)^3}{1.2^3 + 1.5^3} = 16.53 \text{ kN}$$

$$P - R_c = 25 - 16.53 = 8.47 \text{ kN}$$

Using free body ACB , $\sum M_A = 0$: $2aR_B - aR_c = 0$

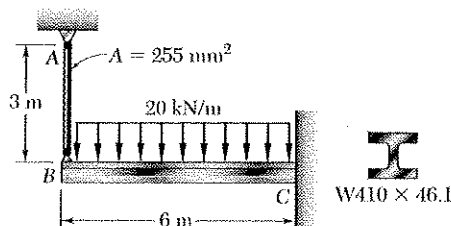
$$(a) \quad R_B = \frac{1}{2}R_c = 8.265 \text{ kN} \uparrow$$

Using free body DCE , $\sum M_D = 0$: $2bR_E - b(P - R_c) = 0$

$$(b) \quad R_E = \frac{1}{2}(P - R_c) = 4.235 \text{ kN} \uparrow$$

Problem 9.92

9.92 The cantilever beam BC is attached to the steel cable AB as shown. Knowing that the cable is initially taut, determine the tension in the cable caused by the distributed load shown. Use $E = 200 \text{ GPa}$.



Let P be the tension developed in member AB and S_B be the elongation of that member.

Cable AB:

$$A = 255 \text{ mm}^2 = 255 \times 10^{-6} \text{ m}^2$$

$$S_B = \frac{PL}{EA} = \frac{(P)(3)}{(200 \times 10^9)(255 \times 10^{-6})} = 58.82 \times 10^{-9} P$$

Beam BC:

$$I = 156 \times 10^6 \text{ mm}^4 = 156 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(156 \times 10^{-6}) = 31.2 \times 10^6 \text{ N} \cdot \text{m}^2$$

Loading I. 20 kN/m downward.

Refer to Case 2 of Appendix D.

$$(y_B)_1 = -\frac{WL^4}{8EI} = -\frac{(20 \times 10^3)(6)^4}{8(31.2 \times 10^6)} = -103.846 \times 10^{-3} \text{ m}$$

Loading II. Upward force P at point B.

Refer to Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(6)^3}{3(31.2 \times 10^6)} = 2.3077 \times 10^{-6} P$$

By superposition, $y_B = (y_B)_1 + (y_B)_2$

Also, matching the deflection at B, $y_B = -S_B$

$$-103.846 \times 10^{-3} + 2.3077 \times 10^{-6} P = -58.82 \times 10^{-9} P$$

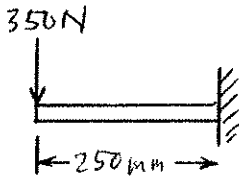
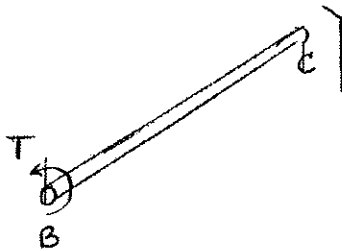
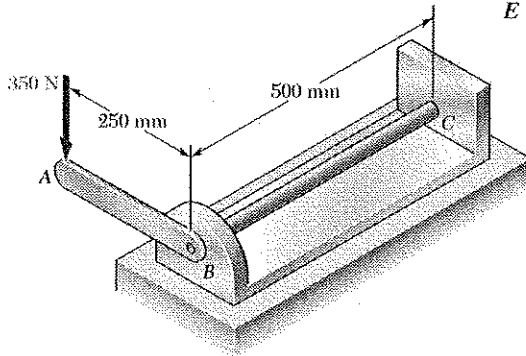
$$2.3666 \times 10^{-6} P = -103.846 \times 10^{-3}$$

$$P = 43.9 \times 10^3 \text{ N}$$

$$P = 43.9 \text{ kN}$$

Problem 9.93

9.93 A 22-mm-diameter rod BC is attached to the lever AB and to the fixed support at C . Lever AB has a uniform cross section 10 mm thick and 25 mm deep. For the loading shown, determine the deflection of point A . Use $E = 200 \text{ GPa}$ and $G = 77 \text{ GPa}$.



Deformation of rod BC . (Torsion)

$$c = \frac{1}{2}d = \frac{1}{2}(22) = 11 \text{ mm}$$

$$J = \frac{\pi}{2} c^4 = 22998 \text{ mm}^4$$

$$T = Pa = (350)(0.25) = 87.5 \text{ N}\cdot\text{m}$$

$$L = 0.5 \text{ m}$$

$$\phi_B = \frac{TL}{GJ} = \frac{(87.5)(0.5)}{(77 \times 10^9)(22998 \times 10^{-12})}$$

$$= 0.0247 \text{ rad}$$

Deflection of point A assuming lever AB to be rigid.

$$(y_A)_1 = a \phi_B = (0.25)(0.0247) = 0.006175 \text{ m} \downarrow$$

Additional deflection due to bending of lever AB .
Refer to Case 1 of Appendix D.

$$I = \frac{1}{12}(10)(25)^3 = 13021 \text{ mm}^4$$

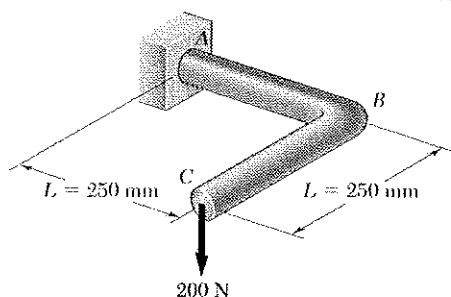
$$(y_A)_2 = \frac{PL^3}{3EI} = \frac{(350)(0.25)^3}{(3)(200 \times 10^9)(13021 \times 10^{-12})} = 2.14 \times 10^{-3} \text{ m} \downarrow$$

Total deflection at point A .

$$y_A = (y_A)_1 + (y_A)_2 = 8.28 \text{ mm} \downarrow$$

Problem 9.94

9.94 A 16-mm-diameter rod has been bent into the shape shown. Determine the deflection of end C after the 200-N force is applied. Use $E = 200$ GPa and $G = 80$ GPa.



$$\text{LET } 200 \text{ N} = P.$$

CONSIDER TORSION OF ROD AB.

$$\phi_B = \frac{TL}{JG} = \frac{(PL)L}{JG} = \frac{PL^2}{JG}$$

$$\gamma_C' = -L\phi_B = -\frac{PL^3}{JG}$$

CONSIDER BENDING OF AB. (CASE 1 OF APPENDIX D.)

$$\gamma_C'' = \gamma_B = -\frac{PL^3}{3EI}$$

CONSIDER BENDING OF BC. (CASE 1 OF APPENDIX D.)

$$\gamma_C''' = -\frac{PL^3}{3EI}$$

SUPERPOSITION:

$$\gamma_C = \gamma_C' + \gamma_C'' + \gamma_C''' = -\frac{PL^3}{JG} - \frac{PL^3}{3EI} - \frac{PL^3}{3EI} = -\frac{PL^3}{EI} \left(\frac{EI}{JG} + \frac{2}{3} \right)$$

$$\text{DATA: } G = 80(10^9) \text{ Pa} \quad J = \frac{1}{2} \pi (0.008)^4 = 6.4340(10^{-9}) \text{ m}^4$$

$$E = 200(10^9) \text{ Pa} \quad I = \frac{1}{2} J = 3.2170(10^{-9}) \text{ m}^4$$

$$EI = 643.40 \text{ N} \cdot \text{m}^2$$

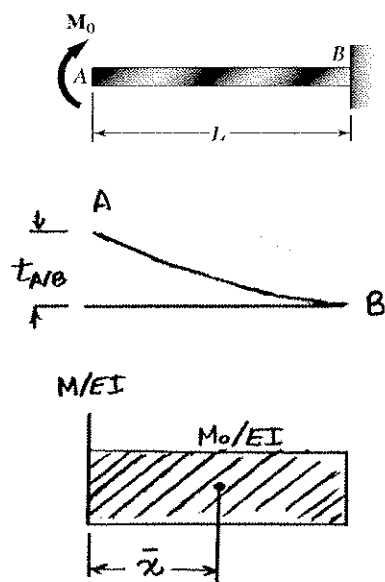
$$JG = 514.72 \text{ N} \cdot \text{m}^2$$

$$\gamma_C = -\frac{(200)(0.25)^3}{643.40} \left(\frac{643.40}{514.72} + \frac{2}{3} \right) = -9.3093(10^{-3}) \text{ m}$$

$$\gamma_C = 9.31 \text{ mm} \downarrow$$

Problem 9.95

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.



Place reference tangent at B.

Draw $\frac{M}{EI}$ diagram.

$$A = \left(\frac{M_0}{EI}\right)L = \frac{M_0 L}{EI}$$

$$\bar{x} = \frac{1}{2}L$$

$$\theta_{B/A} = A = \frac{M_0 L}{EI}$$

$$t_{B/A} = A\bar{x} = \left(\frac{M_0 L}{EI}\right)\left(\frac{1}{2}L\right) = \frac{M_0 L^2}{2EI}$$

(a) Slope at end A.

$$\theta_B = \theta_A + A$$

$$0 = \theta_A + \frac{M_0 L}{EI}$$

$$\theta_A = -\frac{M_0 L}{EI}$$

$$\theta_A = \frac{M_0 L}{EI} \quad \swarrow$$

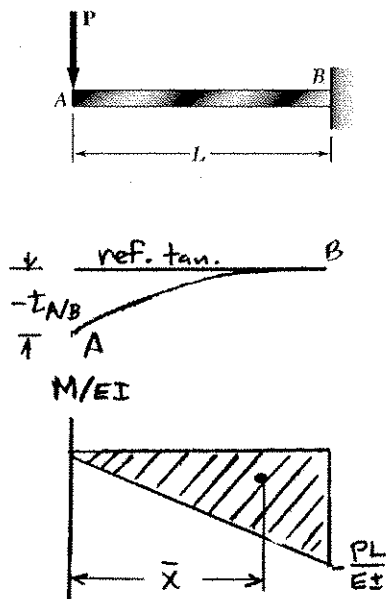
(b) Deflection at A.

$$y_A = t_{A/B} = \frac{M_0 L^2}{2EI}$$

$$y_A = \frac{M_0 L^2}{2EI} \quad \uparrow$$

Problem 9.96

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.



Place reference tangent at B.

Draw $\frac{M}{EI}$ diagram.

$$A = \frac{1}{2} \left(-\frac{PL}{EI}\right)L = -\frac{PL^2}{2EI}$$

$$\bar{x} = \frac{2}{3}L$$

$$\theta_{B/A} = A = -\frac{PL^2}{2EI}$$

$$t_{A/B} = A\bar{x} = \left(-\frac{PL^2}{2EI}\right)\left(\frac{2}{3}L\right) = -\frac{PL^3}{3EI}$$

(a) Slope at end A.

$$\theta_B = \theta_A + A$$

$$0 = \theta_A - \frac{PL^2}{2EI}$$

$$\theta_A = \frac{PL^2}{2EI} \quad \searrow$$

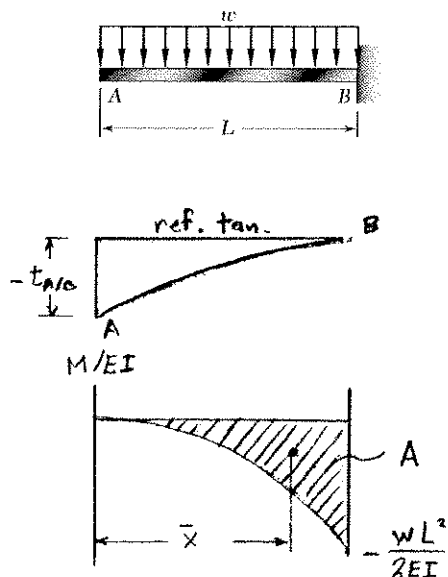
(b) Deflection at A.

$$y_A = t_{A/B} = -\frac{PL^3}{3EI}$$

$$y_A = \frac{PL^3}{3EI} \quad \downarrow$$

Problem 9.97

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.



Place reference tangent at B. $\theta_B = 0$

Draw $\frac{M}{EI}$ curve as parabola.

$$A = -\frac{1}{3} \left(\frac{wL^2}{2EI} \right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$

By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{6} \frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6} \frac{wL^3}{EI} = \frac{1}{6} \frac{wL^3}{EI}$$

By second moment-area theorem,

$$t_{A/B} = \bar{x} A = \left(\frac{3}{4}L \right) \left(-\frac{1}{6} \frac{wL^3}{EI} \right) = -\frac{1}{8} \frac{wL^4}{EI}$$

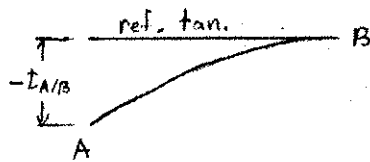
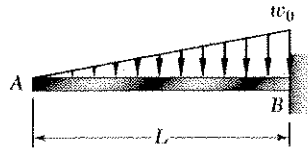
$$y_A = t_{A/B} = -\frac{1}{8} \frac{wL^4}{EI}$$

$$(a) \quad \theta_A = \frac{wL^3}{6EI} \quad \curvearrowright$$

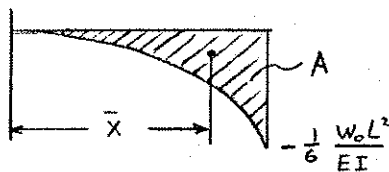
$$(b) \quad y_A = \frac{wL^4}{8EI} \quad \downarrow$$

Problem 9.98

9.95 through 9.98 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.



M/EI



Place reference tangent at B. $\theta_B = 0$

$$\sum M_B = 0: \left(\frac{1}{2}w_0L\right)\frac{L}{3} + M_B = 0$$

$$M_B = -\frac{1}{6}w_0L^2$$

Draw $\frac{M}{EI}$ curve as cubic parabola.

$$A = -\frac{1}{4}\left(\frac{1}{6}\frac{w_0L^2}{EI}\right)L = -\frac{1}{24}\frac{w_0L^3}{EI}$$

$$\bar{x} = L - \frac{1}{3}L = \frac{2}{3}L$$

By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{24}\frac{w_0L^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{24}\frac{w_0L^3}{EI} = \frac{1}{24}\frac{w_0L^3}{EI}$$

By second moment-area theorem,

$$t_{A/B} = \bar{x}A = \left(\frac{2}{3}L\right)\left(-\frac{1}{24}\frac{w_0L^3}{EI}\right) = -\frac{1}{30}\frac{w_0L^4}{EI}$$

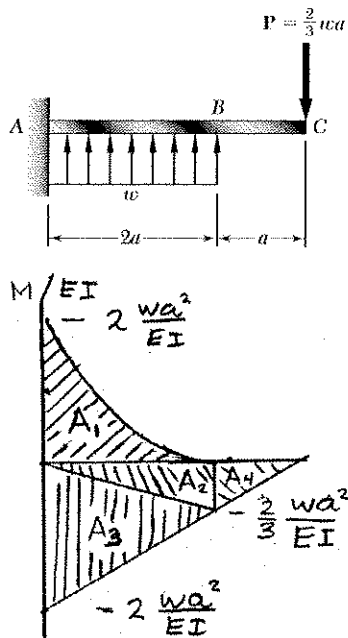
$$y_A = t_{A/B} = -\frac{1}{30}\frac{w_0L^4}{EI}$$

$$(a) \theta_A = \frac{w_0L^3}{24EI} \nearrow$$

$$(b) y_A = \frac{w_0L^4}{30EI} \downarrow$$

Problem 9.99

9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (a) the slope and deflection at (a) point B, (b) point C.



Draw $\frac{M}{EI}$ diagram by parts and divide into areas A_1, A_2, A_3 , and A_4 .

$$A_1 = \frac{1}{3} (2a) \left(2 \frac{wa^2}{EI} \right) = \frac{4}{3} \frac{wa^3}{EI}$$

$$A_2 = \frac{1}{2} (2a) \left(-\frac{2wa^2}{3EI} \right) = -\frac{2}{3} \frac{wa^3}{EI}$$

$$A_3 = \frac{1}{2} (2a) \left(-\frac{2wa^2}{EI} \right) = -2 \frac{wa^3}{EI}$$

$$A_4 = \frac{1}{2} (a) \left(-\frac{2}{3} \frac{wa^2}{EI} \right) = -\frac{1}{3} \frac{wa^3}{EI}$$

(a) Slope at B.

$$\theta_B = A_1 + A_2 + A_3 = -\frac{4}{3} \frac{wa^3}{EI} = \frac{4wa^3}{3EI} \quad \blacktriangleleft$$

Deflection at B. $y_B = t_{B/A}$

$$\begin{aligned} y_B = t_{B/A} &= A_1 \left(\frac{3}{4} \cdot 2a \right) + A_2 \left(\frac{1}{3} \cdot 2a \right) + A_3 \left(\frac{2}{3} \cdot 2a \right) \\ &= 2 \frac{wa^4}{EI} - \frac{4}{9} \frac{wa^4}{EI} - \frac{8}{3} \frac{wa^4}{EI} = -\frac{10}{9} \frac{wa^4}{EI} \end{aligned}$$

$$y_B = \frac{10}{9} \frac{wa^4}{EI} \downarrow \quad \blacktriangleleft$$

(b) Slope at C.

$$\theta_C = \theta_B + A_4 = -\frac{4}{3} \frac{wa^3}{EI} - \frac{1}{3} \frac{wa^3}{EI} = -\frac{5}{3} \frac{wa^3}{EI}$$

$$\theta_C = \frac{5}{3} \frac{wa^3}{EI} \quad \blacktriangleleft$$

Deflection at C.

$$y_C = y_B + \theta_B a + t_{C/B}$$

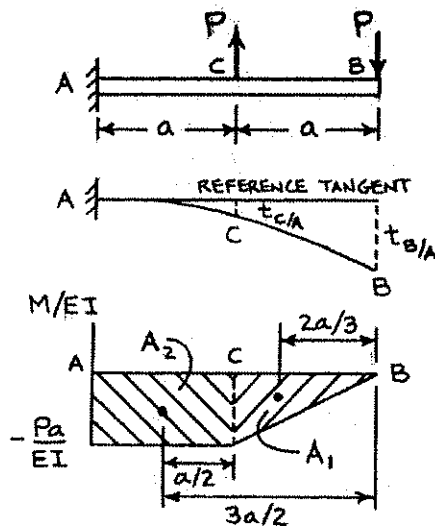
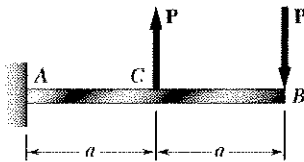
$$= -\frac{10}{9} \frac{wa^4}{EI} + \left(-\frac{4}{3} \frac{wa^3}{EI} \right) (a) + \left(-\frac{1}{3} \frac{wa^3}{EI} \right) \left(\frac{2a}{3} \right)$$

$$= -\frac{8}{3} \frac{wa^4}{EI}$$

$$y_C = \frac{8}{3} \frac{wa^4}{EI} \downarrow \quad \blacktriangleleft$$

Problem 9.100

9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (a) the slope and deflection at (a) point B, (b) point C.



$$A_1 = \frac{1}{2}(a)\left(-\frac{Pa}{EI}\right) = -\frac{Pa^2}{2EI}$$

$$A_2 = (a)\left(-\frac{Pa}{EI}\right) = -\frac{Pa^2}{EI}$$

(a) AT POINT B

$$\theta_B = \theta_{B/A} = A_1 + A_2 = -\frac{Pa^2}{2EI} - \frac{Pa^2}{EI} = -\frac{3Pa^2}{2EI}$$

$$\theta_B = \frac{3Pa^2}{2EI} \quad \swarrow$$

$$y_B = t_{B/A} = A_1\left(\frac{2a}{3}\right) + A_2\left(\frac{3a}{2}\right)$$

$$= \left(-\frac{Pa^2}{2EI}\right)\left(\frac{2a}{3}\right) + \left(-\frac{Pa^2}{EI}\right)\left(\frac{3a}{2}\right) = -\frac{11Pa^3}{6EI}$$

$$y_B = \frac{11Pa^3}{6EI} \quad \downarrow$$

(b) AT POINT C

$$\theta_C = \theta_{C/A} = A_2 = -\frac{Pa^2}{EI}$$

$$\theta_C = \frac{Pa^2}{EI} \quad \swarrow$$

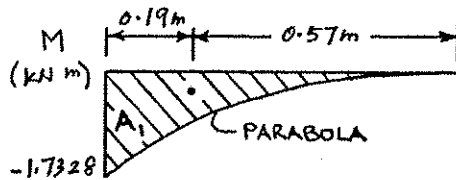
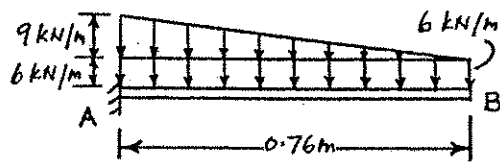
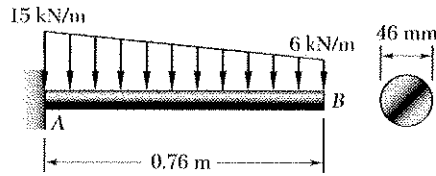
$$y_C = t_{C/A} = A_2\left(\frac{a}{2}\right)$$

$$= \left(-\frac{Pa^2}{EI}\right)\left(\frac{a}{2}\right) = -\frac{Pa^3}{2EI}$$

$$y_C = \frac{Pa^3}{2EI} \quad \downarrow$$

Problem 9.101

9.101 For the cantilever beam and loading shown, determine (a) the slope at point B, (b) the deflection at point B. Use $E = 200$ GPa.

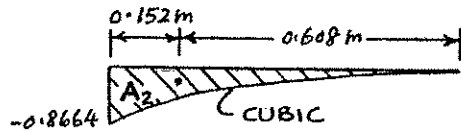


$$I = \frac{\pi}{4} (0.023)^4 = 219.8 \times 10^{-9} \text{ m}^4$$

$$EI = (200 \times 10^6) (219.8 \times 10^{-9}) = 43.96 \text{ kN m}^2$$

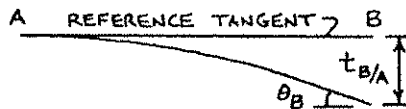
$$M_A = -\frac{wL^2}{2} = -\frac{(6)(0.76)^2}{2} = -1.7328 \text{ kNm}$$

$$A_1 = \frac{1}{3} (-1.7328)(0.76) = -0.439 \text{ kNm}^2$$



$$M_A = -\frac{wL^2}{6} = -\frac{(9)(0.76)^2}{6} = -0.8664 \text{ kNm}$$

$$A_2 = \frac{1}{4} (-0.8664)(0.76) = -0.1646 \text{ kNm}^2$$



(a) SLOPE AT B

$$EI \theta_{B/A} = A_1 + A_2 = -0.439 - 0.1646 = -0.6036 \text{ kNm}^2$$

$$\theta_B = \theta_{B/A} = \frac{-0.6036}{43.96} = -13.7307 \times 10^{-3} \text{ rad}$$

$$\theta_B = 13.73 \times 10^{-3} \text{ rad} \quad \curvearrowright$$

(b) DEFLECTION AT B

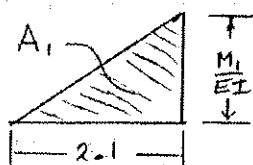
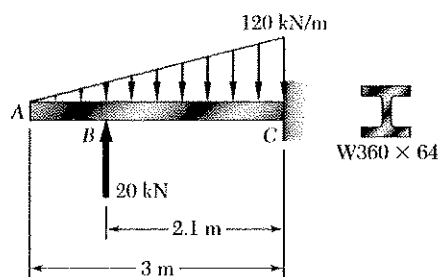
$$EI y_B = EI t_{B/A} = (-0.439)(0.57) + (-0.1646)(0.608) = -0.3503 \text{ kNm}^3$$

$$y_B = \frac{-0.3503}{43.96} = -0.00797 \text{ m}$$

$$y_B = 7.97 \text{ mm} \quad \downarrow$$

Problem 9.102

9.102 For the cantilever beam and loading shown, determine (a) the slope at point A, (b) the deflection at point A. Use $E = 200$ GPa.



Units: Forces in kN; lengths in meters.

$$I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6}$$

$$EI = (200 \times 10^9)(178 \times 10^{-6}) = 35600 \text{ kN} \cdot \text{m}^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{(20)(2.1)}{35600} = 1.17978 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \left(\frac{1}{2}\right)(2.1)(1.17978 \times 10^{-3}) = 1.23876 \times 10^{-3}$$

$$M_2 = -\frac{\left(\frac{1}{2}\right)(120)(3)(1)}{35600} = -5.0562 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \left(\frac{1}{4}\right)(3)(-5.0562 \times 10^{-3}) = -3.7921 \times 10^{-3}$$

Place reference tangent at C. $\theta_c = 0$

(a) Slope at A.

$$\theta_A = -\theta_{c/A} = -A_1 - A_2 = 2.55 \times 10^{-3} \text{ } \blacktriangleleft$$

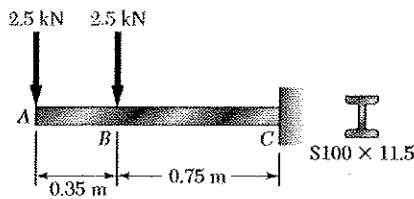
(b) Deflection at A.

$$y_A = t_{A/C}$$

$$y_C = A_1(3 - 0.7) + A_2\left(3 - \frac{3}{5}\right) = -6.25 \times 10^{-3} \text{ m} = 6.25 \text{ mm } \blacktriangleleft$$

Problem 9.103

9.103 For the cantilever beam and loading shown, determine the slope and deflection at (a) end A, (b) point B. Use $E = 200 \text{ GPa}$.



$$I = 2.53 \times 10^6 \text{ mm}^4$$

$$EI = (200 \times 10^6)(2.53 \times 10^{-6}) = 506 \text{ kN m}^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = -\frac{1}{2}(0.75) \frac{1.875}{EI} = -1.39 \times 10^{-3}$$

$$A_2 = -\frac{1}{2}(0.75) \frac{2.75}{EI} = -2.038 \times 10^{-3}$$

$$A_3 = -\frac{1}{2}(0.75) \frac{0.875}{EI} = -0.648 \times 10^{-3}$$

$$A_4 = -\frac{1}{2}(0.35) \frac{0.875}{EI} = -0.324 \times 10^{-3}$$

Place reference tangent at C.

$$\theta_C = 0 \quad y_C = 0$$

(a) Slope at A. $\theta_A = -\theta_{C/A} = -A_1 - A_2 - A_3 - A_4 = 4.4 \times 10^{-3}$ $\blacktriangleleft$

Deflection at A. $y_A = t_{A/C}$

$$y_A = A_1(0.35 + 0.5) + A_2(0.35 + 0.5) + A_3(0.35 + 0.25) + A_4(0.23) = -3.38 \text{ mm} \quad \blacktriangleleft$$

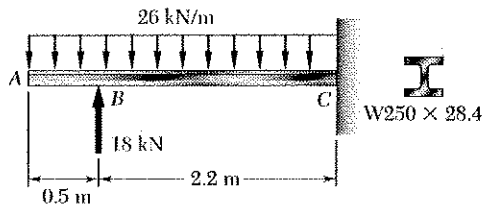
(b) Slope at B. $\theta_B = -\theta_{B/A} = -A_1 + A_2 - A_3 = 4.08 \times 10^{-3}$ $\blacktriangleleft$

Deflection at B. $y_B = t_{B/C}$

$$y_B = A_1(0.5) + (A_2 + A_3)(0.23) = -1.86 \times 10^{-3} \text{ m} = -1.86 \text{ mm} \quad \blacktriangleleft$$

Problem 9.104

9.104 For the cantilever beam and loading shown, determine (a) the slope at point A, (b) the deflection at point A. Use $E = 200 \text{ GPa}$.



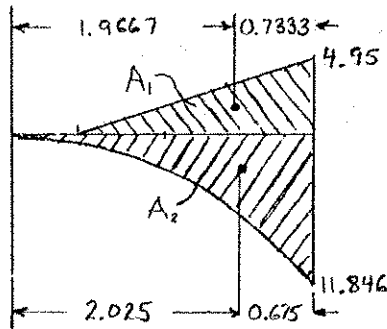
Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 40.0 \times 10^6 \text{ mm}^4 = 40.0 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(40.0 \times 10^{-6}) = 8.00 \times 10^6 \text{ N}\cdot\text{m}^2 = 8000 \text{ kN}\cdot\text{m}^2$$

10^3 M/EI



Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{(18)(2.2)}{8000} = 4.95 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(4.95 \times 10^{-3})(2.2) = 5.445 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(2.2) = 0.7333 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8000)} = -11.846 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \frac{1}{3}(-11.846 \times 10^{-3})(2.7) = -10.662 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.7) = 0.675 \text{ m}$$

Draw reference tangent at C.

$$\theta_c = \theta_A + \theta_{c/A} = \theta_A + A_1 + A_2 = 0$$

$$\theta_A = -A_1 - A_2 = -5.445 \times 10^{-3} + 10.662 \times 10^{-3} = 5.22 \times 10^{-3} \text{ rad}$$

(a) Slope at A.

$$\theta_A = 5.22 \times 10^{-3} \text{ rad} \quad \swarrow$$

$$y_A = y_c - \theta_c L + t_{A/C}$$

$$= 0 - 0 + A_1 \bar{x}_1 + A_2 \bar{x}_2$$

$$= 0 - 0 + (5.445 \times 10^{-3})(1.9667) - (10.662 \times 10^{-3})(2.025)$$

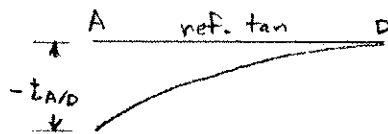
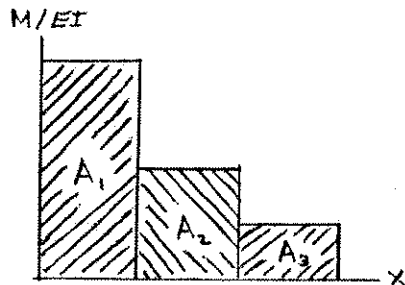
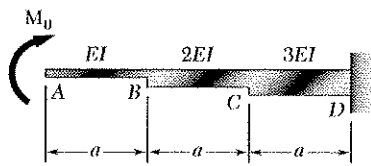
$$= -10.881 \times 10^{-3} \text{ m}$$

(b) Deflection at A.

$$y_A = 10.88 \text{ mm} \downarrow$$

Problem 9.105

9.105 For the cantilever beam and loading shown, determine the deflection and slope at end A caused by the moment M_0 .



Draw $\frac{M}{EI}$ diagram.

$$A_1 = + \frac{M_0 a}{EI}$$

$$A_2 = + \frac{M_0 a}{2EI}$$

$$A_3 = + \frac{M_0 a}{3EI}$$

$$\theta_D = 0, y_D = 0$$

Place reference tangent at D.

Deflection at A. $y_A = t_{A/D}$

$$y_A = A_1 \left(\frac{1}{2}a \right) + A_2 \left(\frac{3}{2}a \right) + A_3 \left(\frac{5}{2}a \right) = \frac{25M_0 a^2}{12EI} \uparrow$$

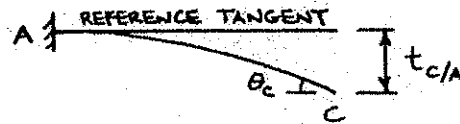
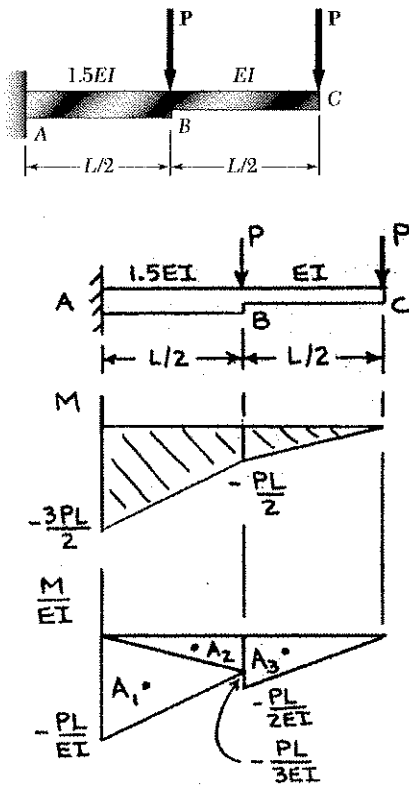
Slope at A. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 - A_3 = -\frac{11M_0 a}{6EI}$$

$$\theta_A = \frac{11M_0 a}{6EI} \swarrow$$

Problem 9.106

9.106 For the cantilever beam and loading shown, determine (a) the slope at point C, (b) the deflection at point C.



$$A_1 = \frac{1}{2} \left(-\frac{PL}{EI} \right) \left(\frac{L}{2} \right) = -\frac{PL^2}{4EI}$$

$$A_2 = \frac{1}{2} \left(-\frac{PL}{3EI} \right) \left(\frac{L}{2} \right) = -\frac{PL^2}{12EI}$$

$$A_3 = \frac{1}{2} \left(-\frac{PL}{2EI} \right) \left(\frac{L}{2} \right) = -\frac{PL^2}{8EI}$$

(a) SLOPE AT C

$$\theta_c = A_1 + A_2 + A_3$$

$$= -\frac{PL^2}{EI} \left(\frac{1}{4} + \frac{1}{12} + \frac{1}{8} \right) = -\frac{11PL^2}{24EI}$$

$$\theta_c = \frac{11PL^2}{24EI} \downarrow$$

(b) DEFLECTION AT C

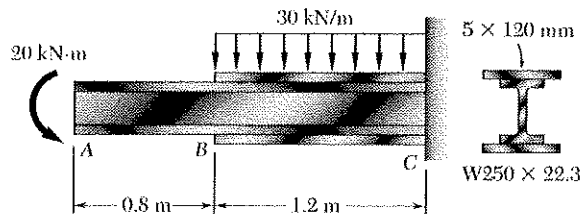
$$y_c = t_{C/A} = A_1 \left(\frac{5}{6}L \right) + A_2 \left(\frac{2}{3}L \right) + A_3 \left(\frac{1}{3}L \right)$$

$$= \left(-\frac{PL^2}{4EI} \right) \left(\frac{5}{6}L \right) + \left(-\frac{PL^2}{12EI} \right) \left(\frac{2}{3}L \right) + \left(-\frac{PL^2}{8EI} \right) \left(\frac{1}{3}L \right) = -\frac{22PL^3}{72EI}$$

$$y_c = \frac{11PL^3}{36EI} \downarrow$$

Problem 9.107

9.107 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine the (a) slope at end A, (b) the deflection at end A.



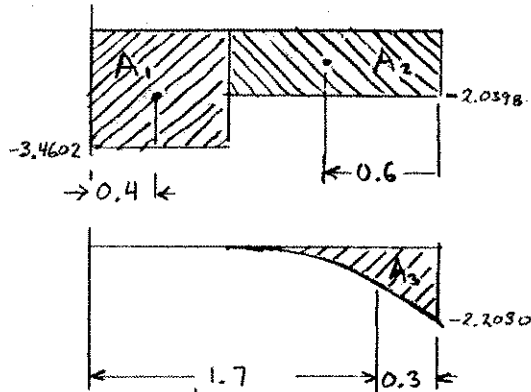
Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

From A to B $I = 28.9 \times 10^6 \text{ mm}^4 = 28.9 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6 \text{ N} \cdot \text{m}^2 = 5780 \text{ kN} \cdot \text{m}^2$$

10^3 M/EI



From B to C $I = I_w + 2A_p d^2 + 2\bar{I}_p$

$$A_p = 5 \times 120 = 600 \text{ mm}^2$$

$$d = \frac{254}{2} + \frac{5}{2} = 129.5 \text{ mm}$$

$$Ad^2 = 10.062 \times 10^6 \text{ mm}^4$$

$$\bar{I}_p = \frac{1}{12}(120)(5)^3 = 0.00125 \times 10^6 \text{ mm}^4$$

$$I = [28.9 + (2)(10.062) + (2)(0.00125)] \times 10^6 \text{ mm}^4 = 49.03 \times 10^6 \text{ mm}^4 = 49.03 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(49.03 \times 10^{-6}) = 9.805 \times 10^6 \text{ N} \cdot \text{m}^2 = 9805 \text{ kN} \cdot \text{m}^2$$

Draw M/EI diagram by parts.

A to B: $\frac{M_1}{EI} = -\frac{20}{5780} = -3.4602 \times 10^{-3} \text{ m}^{-1}$

B to C: $\frac{M_2}{EI} = -\frac{20}{9805} = -2.0398 \times 10^{-3} \text{ m}^{-1}$

$$\frac{M_3}{EI} = -\frac{(30)(1.2)^2}{(2)(9805)} = -2.2030 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = (-3.4602 \times 10^{-3})(0.8) = -2.7682 \times 10^{-3}$$

$$A_2 = (-2.0398 \times 10^{-3})(1.2) = -2.4478 \times 10^{-3}$$

$$A_3 = \frac{1}{3}(-2.2030 \times 10^{-3})(1.2) = -0.8812 \times 10^{-3}$$

Place reference tangent at C. $\theta_c = 0$

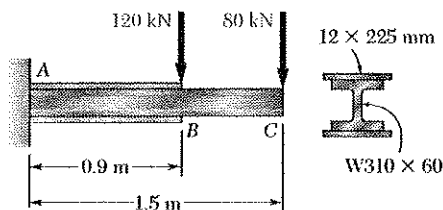
(a) Slope at A. $\theta_A = \theta_c - \theta_{A/C} = 0 - (A_1 + A_2 + A_3) = 6.10 \times 10^{-3} \text{ rad} \angle$

(b) Deflection at A.

$$y_A = t_{A/C} = (-2.7682 \times 10^{-3})(0.4) + (-2.4478 \times 10^{-3})(1.4) + (-0.8812 \times 10^{-3})(1.7) = -6.03 \times 10^{-3} \text{ m} = 6.03 \text{ mm} \downarrow$$

Problem 9.108

9.108 Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine the slope and deflection at end C.



Use units of kN and m

Over portion BC $I = 129 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^6)(129 \times 10^{-6}) = 25800 \text{ kN m}^2$$

Portion AB	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
Top plate	2700	145.5	57.16×10^6	32400
W310 x 60		0	0	129×10^6
Bot. plate	2700	145.5	57.16×10^6	32400
Σ			114.32×10^6	129.065×10^6

$$I = (10^6)(114.32 + 129.065) = 243.385 \times 10^6 \text{ mm}^4$$

$$EI = 48677 \text{ kN m}^2$$

Draw $\frac{M}{EI}$ diagram.

$$\frac{M_1}{EI} = -\frac{(120)(0.9)}{48677} = -2.219 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(80)(1.5)}{48677} = -2.465 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_4}{EI} = -\frac{(80)(0.6)}{25800} = -1.86 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \left(\frac{1}{2}\right)(0.9)(-0.002219) = -0.9986 \times 10^{-3}$$

$$A_2 = \left(\frac{1}{2}\right)(0.9)(0.002465) = -1.10925 \times 10^{-3}$$

$$A_3 = A_2\left(\frac{0.6}{1.5}\right) = -0.4437 \times 10^{-3}$$

$$A_4 = \left(\frac{1}{2}\right)(0.6)(-0.00186) = -0.558 \times 10^{-3}$$

Place reference tangent at A.

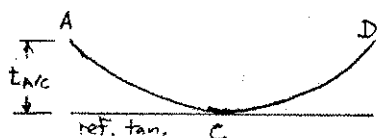
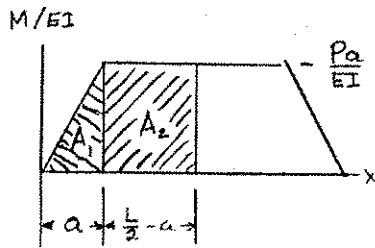
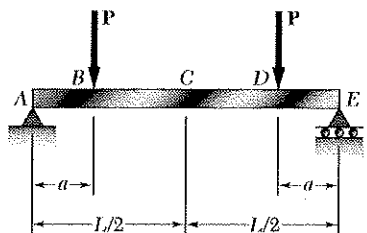
$$\text{Slope at C. } \theta_c = \theta_{c/A} = A_1 + A_2 + A_3 + A_4 = -3.11 \times 10^{-3}$$

$$\text{Deflection at C. } y_c = t_{c/A}$$

$$y_c = (1.2)(A_1) + (1.2)(A_2) + (0.9)(A_3) + (0.4)(A_4) = -3.152 \times 10^{-3} \text{ m} = 3.15 \text{ mm} \downarrow$$

Problem 9.109

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam with symmetric loading.

$\theta_C = 0$ Place reference tangent at C.

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2} a \left(\frac{Pa}{EI} \right) = \frac{Pa^2}{2EI}$$

$$A_2 = \left(\frac{L}{2} - a \right) \left(\frac{Pa}{EI} \right) = \frac{Pa(L-2a)}{2EI}$$

Slope at end A. $\theta_A = -\theta_{C/A}$

$$\theta_A = A_1 + A_2 = -\frac{Pa(L-a)}{2EI}$$

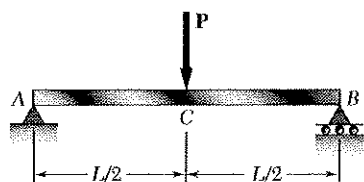
Deflection at C. $y_C = -t_{A/C}$

$$\begin{aligned} t_{A/C} &= \left(\frac{2}{3} a \right) A_1 + \left[\frac{1}{2} \left(\frac{L}{2} + a \right) \right] A_2 \\ &= \frac{2}{3} \frac{Pa^3}{2EI} + \left(\frac{L+2a}{4} \right) \frac{Pa(L-2a)}{2EI} \\ &= \frac{Pa}{EI} \left\{ \frac{1}{3} a^3 + \frac{1}{8} (L^2 - 4a^2) \right\} \\ &= \frac{Pa}{24EI} (3L^2 - 4a^2) \end{aligned}$$

$$y_C = -\frac{Pa}{24EI} (3L^2 - 4a^2)$$

Problem 9.110

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

Place reference tangent at C.

$$\theta_c = 0, \quad y_c = -t_{A/C}$$

$$\text{Reactions } R_A = R_B = \frac{1}{2}P$$

$$\text{Bending moment at C. } M_c = \frac{1}{4}PL$$

$$A = \frac{1}{2} \left(\frac{1}{4} \frac{PL}{EI} \right) \left(\frac{L}{2} \right) = \frac{1}{16} \frac{PL^2}{EI}$$

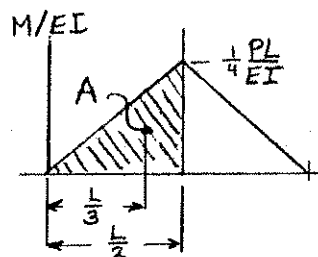
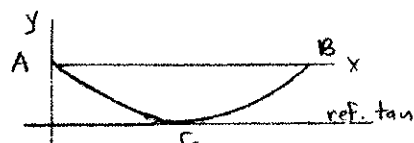
$$(a) \text{ Slope at A. } \theta_A = \theta_c - \theta_{c/A}$$

$$\theta_A = 0 - \frac{1}{16} \frac{PL^2}{EI} = -\frac{1}{16} \frac{PL^2}{EI}$$

$$(b) \text{ Deflection at C.}$$

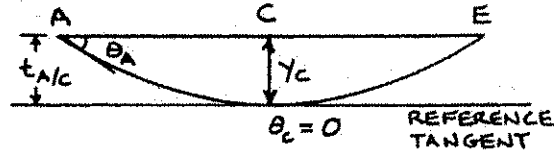
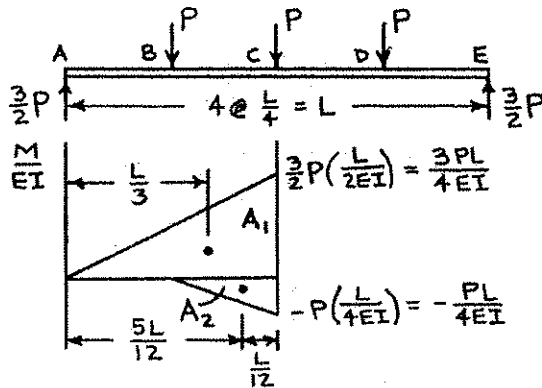
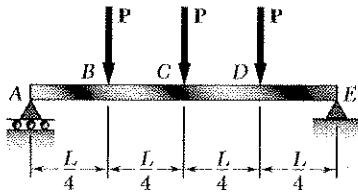
$$y_c = -t_{A/C} = -A \left(\frac{L}{3} \right) = - \left(\frac{1}{16} \frac{PL^2}{EI} \right) \left(\frac{L}{3} \right)$$

$$y_c = \frac{1}{48} \frac{PL^3}{EI}$$



Problem 9.111

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



$$A_1 = \frac{1}{2} \left(\frac{3PL}{4EI} \right) \left(\frac{L}{2} \right) = \frac{3PL^2}{16EI}$$

$$A_2 = \frac{1}{2} \left(-\frac{PL}{4EI} \right) \left(\frac{L}{4} \right) = -\frac{PL^2}{32EI}$$

(a) SLOPE AT A: $\theta_C = \theta_A + \theta_{C/A}$; $\theta_A = 0 - \theta_{C/A}$

$$\theta_A = -\theta_{C/A} = -(A_1 + A_2) = -\left[\frac{3PL^2}{16EI} - \frac{PL^2}{32EI} \right] = -\frac{5PL^2}{32EI}$$

$$\theta_A = \frac{5PL^2}{32EI} \quad \nwarrow$$

(b) DEFLECTION AT C:

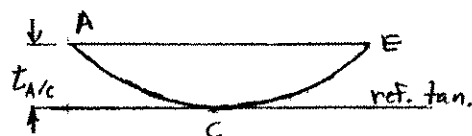
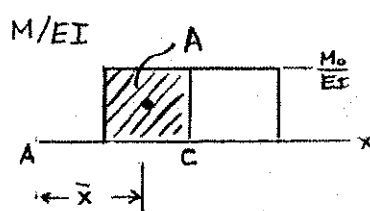
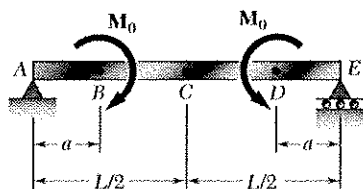
$$t_{A/C} = A_1 \left(\frac{L}{3} \right) + A_2 \left(\frac{5L}{12} \right) = \left(\frac{3PL^2}{16EI} \right) \left(\frac{L}{3} \right) + \left(-\frac{PL^2}{32EI} \right) \left(\frac{5L}{12} \right) = \frac{19PL^3}{384EI}$$

$$y_C = -t_{A/C} = -\frac{19PL^3}{384EI}$$

$$y_C = \frac{19PL^3}{384EI} \quad \downarrow$$

Problem 9.112

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

Place reference tangent at C. $\theta_c = 0$.

Draw $\frac{M}{EI}$ diagram.

(a) Slope at A.

θ_A

$$A = \frac{M_0}{EI} \left(\frac{L}{2} - a \right) = \frac{1}{2} \frac{M_0}{EI} (L - 2a)$$

$$\theta_A = \theta_c - \theta_{c/A} = 0 - A = -\frac{1}{2} \frac{M_0}{EI} (L - 2a)$$

$$\theta_A = \frac{1}{2} \frac{M_0}{EI} (L - 2a) \downarrow$$

(b) Deflection at C.

$$\bar{x} = a + \frac{1}{2} \left(\frac{L}{2} - a \right) = \frac{1}{4} (L + 2a)$$

$$y_C = -t_{c/A} = A \bar{x}$$

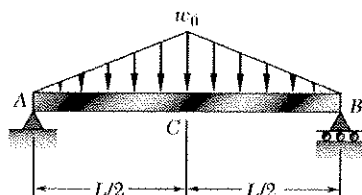
$$= -\frac{1}{2} \frac{M_0}{EI} (L - 2a) \frac{1}{4} (L + 2a)$$

$$= -\frac{1}{8} \frac{M_0}{EI} (L^2 - 4a^2)$$

$$y_C = \frac{1}{8} \frac{M_0}{EI} (L^2 - 4a^2) \downarrow$$

Problem 9.113

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



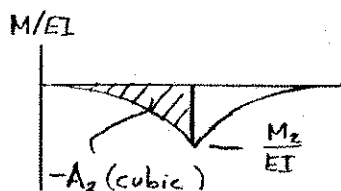
Symmetric beam and loading.

$$\theta_c = 0$$

Place reference tangent at C.

Reactions. $R_A = R_B = \frac{w_0 L}{4}$

Draw $\frac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{R_A L}{2} = \frac{w_0 L^2}{8 EI}$$

$$A_1 = \frac{1}{2} \left(\frac{L}{2} \right) \left(\frac{M_1}{EI} \right) = \frac{w_0 L^3}{32 EI}$$

$$\frac{M_2}{EI} = \frac{1}{EI} \cdot \frac{1}{2} \left(\frac{w_0 L}{2} \right) \left(\frac{1}{3} \cdot \frac{L}{2} \right) = -\frac{w_0 L^2}{24 EI}$$

$$A_2 = \frac{1}{4} \left(\frac{L}{2} \right) \left(-\frac{w_0 L^2}{24 EI} \right) = -\frac{w_0 L^3}{192 EI}$$

Slope at A. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 = \left(-\frac{1}{32} + \frac{1}{192} \right) \frac{w_0 L^3}{EI}$$

$$\theta_A = \frac{5 w_0 L^3}{192 EI}$$

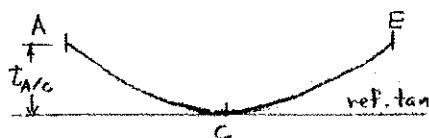
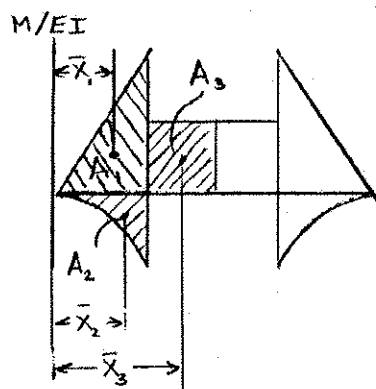
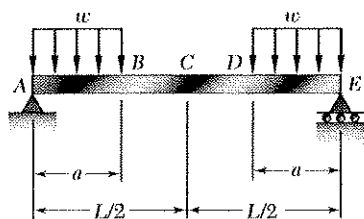
Deflection at C. $y_C = t_{A/C}$

$$t_{A/C} = A_1 \left[\left(\frac{2}{3} \right) \left(\frac{L}{2} \right) \right] + A_2 \left[\left(\frac{4}{5} \right) \left(\frac{L}{2} \right) \right] = \left[\left(\frac{1}{3} \right) \left(\frac{1}{32} \right) - \left(\frac{2}{5} \right) \left(\frac{1}{192} \right) \right] \frac{w_0 L^4}{EI} = \frac{w_0 L^4}{120 EI}$$

$$y_C = \frac{w_0 L^4}{120 EI}$$

Problem 9.114

9.109 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

Place reference tangent at C. $\theta_c = 0$

Reactions. $R_A = R_E = wa$

Bending moment

$$\text{Over AB} \quad M = wax - \frac{1}{2}wa^2$$

$$\text{Over BD} \quad M = \frac{1}{2}wa^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{wa^2}{EI} \quad \frac{M_2}{EI} = -\frac{1}{2} \frac{wa^2}{EI}$$

$$\frac{M_3}{EI} = \frac{1}{2} \frac{wa^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} a = \frac{1}{2} \frac{wa^3}{EI}$$

$$A_2 = \frac{1}{3} \frac{M_2}{EI} a = -\frac{1}{6} \frac{wa^3}{EI}$$

$$A_3 = \frac{M_3}{EI} \left(\frac{L}{2} - a\right) = \frac{1}{4} \frac{wa^2}{EI} (L - 2a)$$

$$(a) \text{ Slope at A. } \theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{1}{2} \frac{wa^3}{EI} + \frac{1}{6} \frac{wa^3}{EI} - \frac{1}{4} \frac{wa^2}{EI} (L - 2a) = -\frac{wa^2}{EI} \left(\frac{1}{4}L - \frac{1}{6}a\right)$$

$$= -\frac{1}{12} \frac{wa^2}{EI} (3L - 2a)$$

$$\theta_A = \frac{wa^2}{12EI} (3L - 2a) \quad \leftarrow$$

$$(b) \text{ Deflection at C. } y_c = -t_{c/A}$$

$$\bar{x}_1 = \frac{2}{3}a, \quad \bar{x}_2 = \frac{3}{4}a, \quad \bar{x}_3 = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

$$y_c = -t_{c/A} = -A_1\bar{x}_1 - A_2\bar{x}_2 - A_3\bar{x}_3$$

$$= -\left(\frac{1}{2} \frac{wa^3}{EI}\right)\left(\frac{2}{3}a\right) + \left(\frac{1}{6} \frac{wa^3}{EI}\right)\left(\frac{3}{4}a\right) - \frac{1}{4}\left(\frac{wa^2}{EI}\right)(L - 2a)\frac{1}{4}(L + 2a)$$

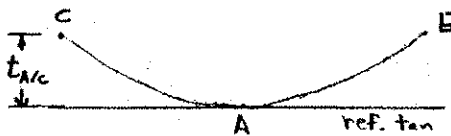
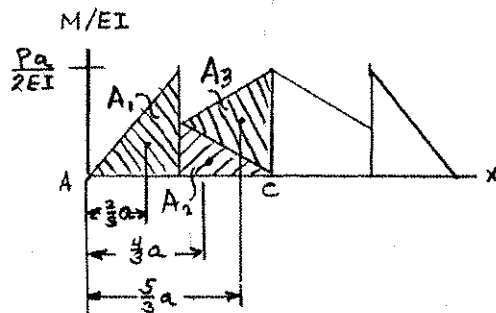
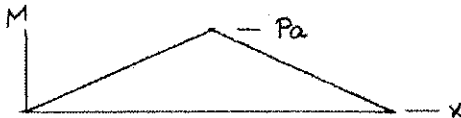
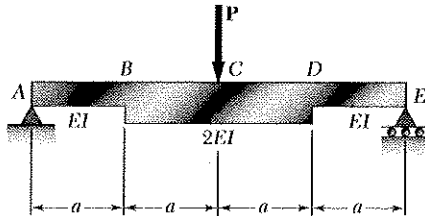
$$= -\frac{1}{3} \frac{wa^3}{EI} + \frac{1}{8} \frac{wa^3}{EI} - \frac{1}{16} \frac{wa^2}{EI} (L^2 - 4a^2)$$

$$= -\frac{wa^2}{EI} \left(\frac{1}{16}L^2 - \frac{1}{24}a^2\right) = -\frac{1}{48} \frac{wa^2}{EI} (3L^2 - 2a^2)$$

$$y_A = \frac{wa^2}{48EI} (3L^2 - 2a^2) \quad \leftarrow$$

Problem 9.115

9.115 and 9.116 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading. $R_A = R_E = \frac{1}{2}P$

$$M_{max} = (\frac{1}{2}P)(2a) = Pa$$

Draw M and $\frac{M}{EI}$ diagrams.

$$A_1 = \frac{1}{2}(\frac{Pa}{2EI})a = \frac{1}{4} \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}(\frac{Pa}{4EI})a = \frac{1}{8} \frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}(\frac{Pa}{2EI})a = \frac{1}{4} \frac{Pa^2}{EI}$$

Place reference tangent at C. $\theta_C = 0$

(a) Slope at A.

$$\theta_A = \theta_C - \theta_{CA} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{5}{8} \frac{Pa^2}{EI} \quad \theta_A = \frac{5Pa^2}{8EI} \quad \swarrow$$

(b) Deflection at C.

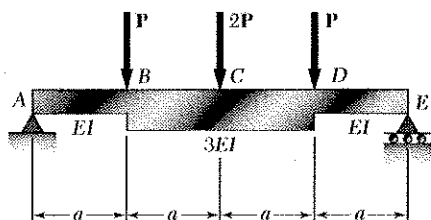
$$|y_C| = t_{A/C} = A_1(\frac{2}{3}a) + A_2(\frac{4}{3}a) + A_3(\frac{5}{3}a)$$

$$\frac{1}{6} \frac{Pa^3}{EI} + \frac{1}{6} \frac{Pa^3}{EI} + \frac{5}{12} \frac{Pa^3}{EI} = \frac{3}{4} \frac{Pa^3}{EI}$$

$$y_C = \frac{3Pa^3}{4EI} \quad \downarrow$$

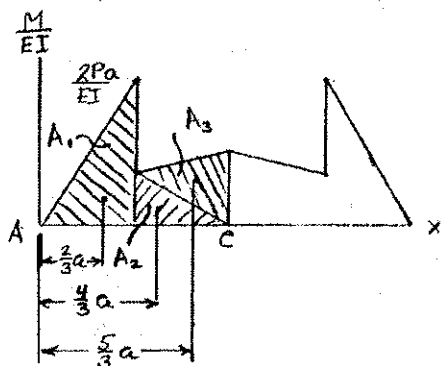
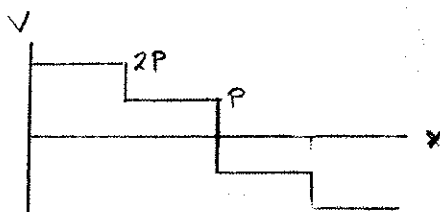
Problem 9.116

9.115 and 9.116 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading. $R_A = R_E = 2P$.

Draw V , M , and $\frac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2} \left(\frac{2Pa}{EI} \right) a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2} \left(\frac{2Pa}{3EI} \right) a = \frac{1}{3} \frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2} \left(\frac{Pa}{EI} \right) a = \frac{1}{2} \frac{Pa^2}{EI}$$

Place reference tangent at C. $\theta_C = 0$

(a) Slope at A.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{11}{6} \frac{Pa^2}{EI} \quad \theta_A = \frac{11Pa^2}{6EI}$$

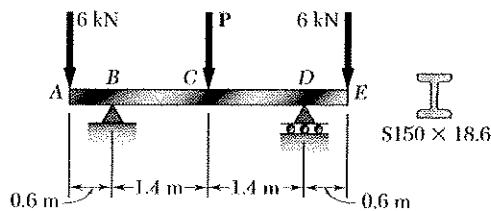
(b) Deflection at C.

$$|y_C| = t_{A/C} = A_1 \left(\frac{2}{3}a \right) + A_2 \left(\frac{4}{3}a \right) + A_3 \left(\frac{5}{3}a \right)$$

$$= \frac{35}{18} \frac{Pa^3}{EI} \quad y_C = \frac{35Pa^3}{18EI} \downarrow$$

Problem 9.117

9.117 Knowing that the magnitude of the load P is 30 kN, determine (a) the slope at end A, (b) the deflection at end A, (c) the deflection at midpoint C of the beam. Use $E = 200$ GPa.



Use units of kN and m $P = 30$ kN

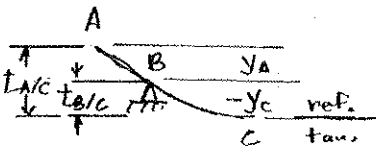
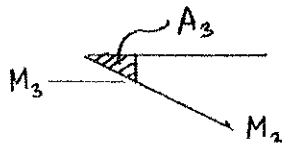
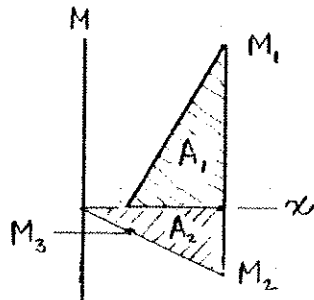
For S150 x 18.6 $I = 9.11 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(9.11 \times 10^{-6}) = 1822 \text{ kN m}^2$$

Symmetric beam with symmetric loading. Place reference tangent at midpoint C where $\theta_c = 0$.

$$R_B = R_D = \frac{1}{2}(6 + 30 + 6) = 21 \text{ kN} \uparrow$$

Draw the bending moment diagram by parts for the left half of the beam.



$$M_1 = (1.4)(21) = 29.4 \text{ kN m}$$

$$A_1 = \frac{1}{2}(1.4)(29.4) = 20.58 \text{ kN m}^2$$

$$M_2 = -(0.6 + 1.4)(6) = -12 \text{ kN m}$$

$$A_2 = \frac{1}{2}(2)(-12) = -12 \text{ kN m}^2$$

$$M_3 = -(0.6)(6) = -3.6 \text{ kN m}$$

$$A_3 = \frac{1}{2}(0.6)(-3.6) = -1.08 \text{ kN m}^2$$

Formulas. $\theta_A = -\theta_{C/A}$, $y_A - y_C = t_{A/C}$ $y_C = y_A - t_{A/C}$

$$y_B = y_A - 0.6\theta_A + t_{B/A} = 0 \quad y_A = -2\theta_A - t_{B/A}$$

$$\theta_{C/A} = \frac{1}{EI} \{A_1 + A_2\} = \frac{20.58 - 12}{1822} = 4.709 \times 10^{-3}$$

$$t_{A/C} = \frac{1}{EI} \left\{ (0.6 + 0.933)A_1 + \frac{2}{3}(2)A_2 \right\} = \frac{15.549}{1822} = 8.534 \times 10^{-3} \text{ m}$$

$$t_{B/A} = \frac{1}{EI} \left\{ \frac{1}{3}(0.6)A_3 \right\} = \frac{-0.216}{1822} = -0.1186 \times 10^{-3} \text{ m}$$

(a) Slope at end A.

$$\theta_A = -4.71 \times 10^{-3} \quad \blacktriangleleft$$

(b) Deflection at A.

$$y_A = -(0.6)(-4.709 \times 10^{-3}) - (-0.1186 \times 10^{-3})$$

$$= 2.944 \times 10^{-3} \text{ m}$$

$$y_A = 2.94 \text{ mm} \uparrow \quad \blacktriangleleft$$

(c) Deflection at C

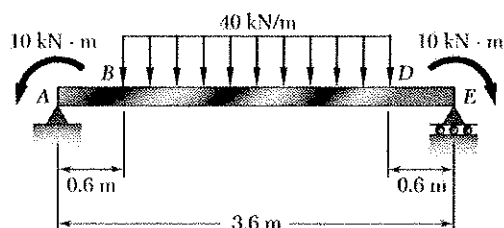
$$y_C = 2.944 \times 10^{-3} - 8.534 \times 10^{-3}$$

$$y_C = -5.59 \times 10^{-3} \text{ m}$$

$$y_C = 5.6 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.118

9.118 and 9.119 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint of the beam. Use $E = 200$ GPa.



Use units of kN and m.

For S250 x 37.8

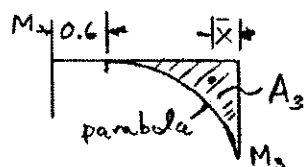
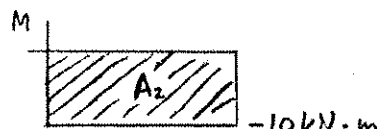
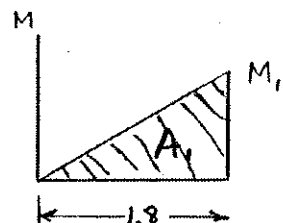
$$I = 51.1 \times 10^6 \text{ mm}^4 = 51.1 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(51.1 \times 10^{-6}) = 10.22 \times 10^6 \text{ N}\cdot\text{m}^2 = 10220 \text{ kN}\cdot\text{m}^2$$

Place reference tangent at midpoint C.

$$\text{Reactions. } R_A = R_E = \frac{1}{2}(40)(3.6 - 1.2) = 48 \text{ kN} \uparrow$$

Draw bending moment diagram of left half of beam by parts.



$$M_1 = (48)(1.8) = 86.4 \text{ kN}\cdot\text{m}$$

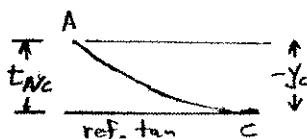
$$A_1 = \frac{1}{2}(1.8)(86.4) = 77.76 \text{ kN}\cdot\text{m}^2$$

$$A_2 = (1.8)(-10) = -18 \text{ kN}\cdot\text{m}^2$$

$$M_3 = \frac{1}{2}(40)(1.8 - 0.6)^2 = -28.8 \text{ kN}\cdot\text{m}$$

$$A_3 = \frac{1}{3}(1.2)(-28.8) = -11.52 \text{ kN}\cdot\text{m}^2$$

$$\bar{x} = \frac{1}{4}(1.2) = 0.30 \text{ m}$$



(a) Slope at end A.

$$\theta_A = -\theta_{A/C}$$

$$\theta_A = \frac{1}{EI} \{ -A_1 - A_2 - A_3 \} = \frac{-77.76 + 18 + 11.52}{10220}$$

$$= -4.72 \times 10^{-3} \text{ rad}$$

$$\theta_A = 4.72 \times 10^{-3} \text{ rad} \swarrow$$

(b) Deflection at midpoint C. $y_C = -t_{A/C}$

$$t_{A/C} = \frac{1}{EI} \{ 1.2 A_1 + 0.9 A_2 + (1.8 - 0.3) A_3 \}$$

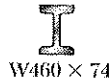
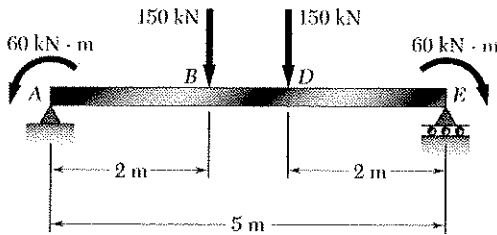
$$= \frac{(1.2)(77.76) - (0.9)(18) - (1.5)(11.52)}{10220} = 5.85 \times 10^{-3} \text{ m}$$

$$y_C = -5.85 \times 10^{-3} \text{ m}$$

$$y_C = 5.85 \text{ mm} \downarrow$$

Problem 9.119

9.118 and 9.119 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint of the beam. Use $E = 200 \text{ GPa}$.

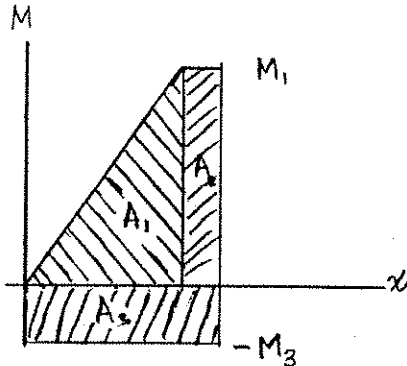


Use units of kN and m.

For W460 x 74

$$I = 333 \times 10^6 \text{ mm}^4 = 333 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(333 \times 10^{-6}) = 66.6 \times 10^6 \text{ N}\cdot\text{m}^2 = 66600 \text{ kN}\cdot\text{m}^2$$



Symmetric beam and loading. Place reference tangent at midpoint C where $\theta_C = 0$.

Reactions. $R_A = R_E = 150 \text{ kN} \uparrow$

Draw bending moment diagram of left half of beam by parts

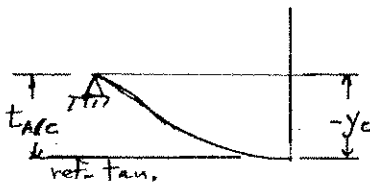
$$M_1 = (2)(150) = 300 \text{ kN}\cdot\text{m}$$

$$A_1 = \left(\frac{1}{2}\right)(2)(300) = 300 \text{ kN}\cdot\text{m}^2$$

$$A_2 = (0.5)(300) = 150 \text{ kN}\cdot\text{m}^2$$

$$M_3 = -60 \text{ kN}\cdot\text{m}$$

$$A_3 = (2.5)(-60) = -150 \text{ kN}\cdot\text{m}^2$$



(a) Slope at end A. $\theta_A = -\theta_{C/A}$

$$\theta_A = \frac{1}{EI} \{-A_1 - A_2 - A_3\} = \frac{-300 - 150 + 150}{66600}$$

$$= -4.50 \times 10^{-3} \text{ rad}$$

$$\theta_A = 4.50 \times 10^{-3} \text{ rad} \swarrow$$

(b) Deflection at midpoint C.

$$y_C = -t_{A/C}$$

$$t_{A/C} = \frac{1}{EI} \left\{ \left(\frac{2}{3} \cdot 2\right) A_1 + \left(2 + \frac{0.5}{2}\right) A_2 + \left(\frac{2.5}{2}\right) A_3 \right\}$$

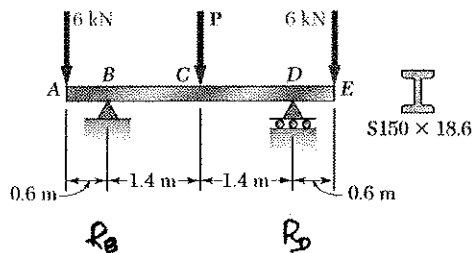
$$= \frac{400 + 337.5 - 187.5}{66600} = 8.26 \times 10^{-3} \text{ m}$$

$$y_C = -8.26 \times 10^{-3} \text{ m}$$

$$y_C = 8.26 \text{ mm} \downarrow$$

Problem 9.120

9.120 For the beam and loading shown, determine (a) the load P for which the deflection is zero at the midpoint C of the beam, (b) the corresponding deflection at end A . Use $E = 200$ GPa.



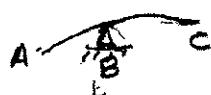
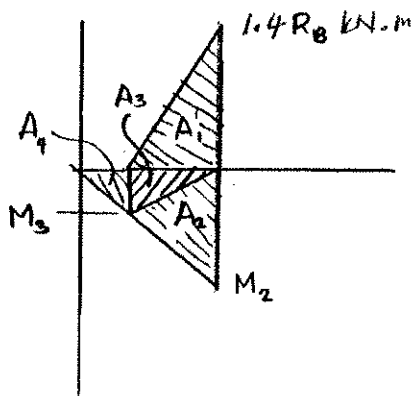
Use units of kN and m .

$$\text{For } S150 \times 18.6 \quad I = 9.11 \times 10^{-6} \text{ m}^4$$

$$EI = (200)(9.11) = 1822 \text{ kN m}^2$$

Symmetric beam with symmetric loading. Place reference tangent at midpoint C where $\theta_c = 0$.

Draw the bending moment diagram by parts for the left half of the beam.



$$A_1 = \frac{1}{2}(1.4)(1.4) R_B = 0.98 R_B \text{ kN m}$$

$$M_2 = -(0.6 + 1.4)(6) = -12 \text{ kN m}$$

$$A_2 = \frac{1}{2}(1.4)(-12) = -8.4 \text{ kN m}^2$$

$$M_3 = -(0.6)(6) = -3.6 \text{ kN m}$$

$$A_3 = \frac{1}{2}(1.4)(-3.6) = -2.52 \text{ kN m}^2$$

$$A_4 = \frac{1}{2}(0.6)(-3.6) = -1.08 \text{ kN m}^2$$

$$\begin{aligned} (a) \quad t_{B/C} = 0: \quad & \frac{1}{EI} \left\{ \frac{2}{3}(1.4)A_1 + \frac{2}{3}(1.4)A_2 + \frac{1}{3}(1.4)A_3 \right\} = \\ & = \frac{0.915 R_B - 9.016}{EI} = 0 \quad R_B = 9.85 \text{ kN} \end{aligned}$$

$$\text{By statics, } -(2)(6) + 2R_B - P = 0 \quad P = 7.7 \text{ kN}$$

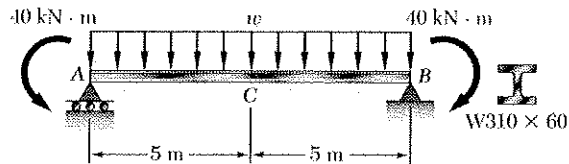
$$A_1 = (0.98)(9.85) = 9.653 \text{ kN m}$$

$$\begin{aligned} (b) \quad y_A = t_{A/C} &= \frac{1}{EI} \left\{ (0.6 + 0.93)A_1 + (0.6 + 0.93)A_2 + (0.6 + 0.47)A_3 + \left(\frac{2}{3} \cdot 0.6\right)A_4 \right\} \\ &= \frac{(1.53)(9.653) + (1.53)(-8.4) + (1.07)(-2.52) + (0.4)(-1.08)}{1822} \\ &= -0.665 \times 10^{-3} \text{ m} \end{aligned}$$

$$y_C = 0.67 \text{ mm} \downarrow$$

Problem 9.121

9.121 For the beam and loading shown and knowing that $w = 8 \text{ kN/m}$, determine (a) the slope at end A, (b) the deflection at midpoint C. Use $E = 200 \text{ GPa}$.



$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N}\cdot\text{m}^2 = 25800 \text{ kN}\cdot\text{m}^2$$

Symmetrical beam and loading.

$$R_A = R_B = \frac{1}{2}(8)(10) = 40 \text{ kN}$$

Bending moment

$$M = 40x - 40 - \frac{1}{2}(8)x^2$$

$$\text{At } x = 5$$

$$M = 200 - 40 - 100$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{200}{25800} = 7.7519 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = \frac{-40}{25800} = -1.5504 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_3}{EI} = \frac{-100}{25800} = -3.8760 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(7.7519 \times 10^{-3})(5) = 19.380 \times 10^{-3} \quad \bar{x}_1 = \left(\frac{2}{3}\right)(5) = 3.3333 \text{ m}$$

$$A_2 = -(1.5504)(5) = -7.7520 \times 10^{-3} \quad \bar{x}_2 = \left(\frac{1}{2}\right)(5) = 2.5 \text{ m}$$

$$A_3 = -\frac{1}{3}(3.8760)(5) = -6.4600 \times 10^{-3} \quad \bar{x}_3 = \left(\frac{3}{4}\right)(5) = 3.75 \text{ m}$$

Place reference tangent at C. $\theta_c = 0$

$$(a) \text{ Slope at A, } \theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3)$$

$$\theta_A = -(19.380 \times 10^{-3} - 7.7520 \times 10^{-3} - 6.4600 \times 10^{-3}) = -5.17 \times 10^{-3}$$

$$\theta_A = 5.17 \times 10^{-3} \text{ rad} \swarrow$$

$$(b) \text{ Deflection at C. } |y_c| = t_{A/C}$$

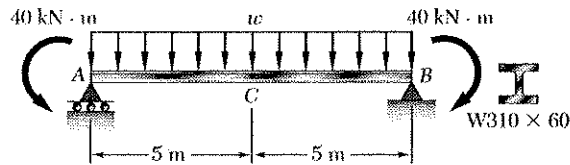
$$= (19.380 \times 10^{-3})(3.3333) - (7.7520 \times 10^{-3})(2.5) - (6.4600 \times 10^{-3})(3.75)$$

$$= 21.0 \times 10^{-3} \text{ m}$$

$$y_c = 21.0 \text{ mm} \downarrow$$

Problem 9.122

9.122 For the beam and loading shown, determine the value of w for which the deflection is zero at the midpoint C of the beam. Use $E = 200$ GPa.



Symmetric beam and loading.

$$R_A = R_B = 5w \quad (w \text{ in kN/m})$$

Bending moment in kN·m.

$$M = 5wx - 40 - \frac{1}{2}wx^2$$

$$\text{At } x = 5 \text{ m}$$

$$M = 25w - 40 - 12.5w$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{25w}{EI} \right) (5) = \frac{62.5w}{EI}$$

$$A_2 = - \frac{(40)(5)}{EI} = - \frac{200}{EI}$$

$$A_3 = - \frac{1}{3} \left(\frac{12.5w}{EI} \right) (5) = - \frac{20.833w}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(5) = 3.3333 \text{ m}$$

$$\bar{x}_2 = \frac{1}{2}(5) = 2.5 \text{ m}$$

$$\bar{x}_3 = \frac{3}{4}(5) = 3.75 \text{ m}$$

Place reference tangent at C .

Deflection at C is zero. $t_{A/C} = y_A - y_C = 0$

$$A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3 = 0$$

$$\left(\frac{62.5w}{EI} \right) (3.3333) - \left(\frac{200}{EI} \right) (2.5) - \left(\frac{20.833w}{EI} \right) (3.75) = 0$$

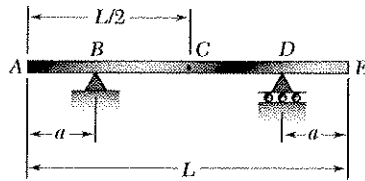
$$\frac{130.21w}{EI} - \frac{500}{EI} = 0$$

$$w = \frac{500}{130.21} = 3.84 \text{ kN/m}$$

$$w = 3.84 \text{ kN/m}$$

Problem 9.123

9.123 A uniform rod AE is to be supported at two points B and D . Determine the distance a for which the slope at ends A and E is zero



Let w = weight per unit length of rod.

Symmetric beam and loading.

$$R_B = R_D = \frac{1}{2} wL$$

Bending moment

Over AB $M = -\frac{1}{2} wx^2$

Over BCD $M = -\frac{1}{2} wx^2 + \frac{1}{2} wL(x-a)$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2} \frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4} \frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = \frac{1}{2} \frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8} \frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} (\frac{L}{2}-a) = \frac{1}{16} \frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3} (\frac{M_2}{EI}) \frac{L}{2} = -\frac{1}{48} \frac{wL^3}{EI}$$

Place reference tangent at C . $\theta_c = 0$

$$\theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2) = 0$$

$$-\frac{1}{16} \frac{wL(L-2a)^2}{EI} + \frac{1}{48} \frac{wL^3}{EI} = 0$$

Let $u = \frac{a}{L}$ and divide by $\frac{wL^3}{48EI}$.

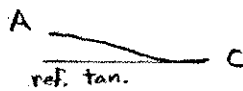
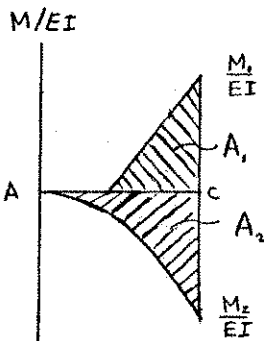
$$1 - 3(1-2u)^2 = 0$$

$$1 - 2u = \frac{\sqrt{3}}{3}$$

$$u = \frac{1}{2} (1 - \frac{\sqrt{3}}{3}) = 0.21132$$

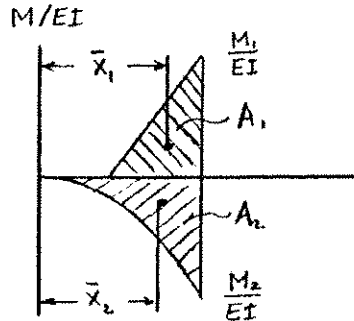
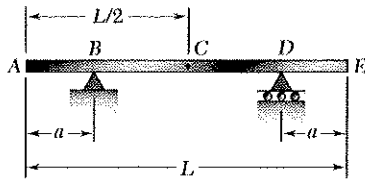
$$\frac{a}{L} = 0.211$$

$$a = 0.211 L$$



Problem 9.124

*9.124 A uniform rod AE is to be supported at two points B and D . Determine the distance a from the ends of the rod to the points of support, if the downward deflections of points A , C , and E are to be equal.



A C

Let w = weight per unit length of rod.

Symmetric beam and loading.

$$R_B = R_D = \frac{1}{2}WL$$

Bending moment:

Over AB $M = -\frac{1}{2}wx^2$

Over BCD $M = -\frac{1}{2}wx^2 + \frac{1}{2}WL(x-a)$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2} \frac{WL(\frac{L}{2}-a)}{EI} = \frac{1}{4} \frac{WL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2} \frac{W(\frac{L}{2})^2}{EI} = -\frac{1}{8} \frac{WL^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} \left(\frac{L}{2}-a\right) = \frac{1}{16} \frac{WL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3} \left(\frac{M_2}{EI}\right) \left(\frac{L}{2}\right) = -\frac{1}{48} \frac{WL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3} \left(\frac{L}{2}-a\right) = \frac{1}{3}(L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4} \left(\frac{L}{2}\right) = \frac{3}{8}L$$

Place reference tangent at C.

$$y_C - y_C = t_{A/C} = 0$$

$$A_1 \bar{x}_1 + A_2 \bar{x}_2 = 0$$

$$\frac{1}{16} \frac{WL(L-2a)^2}{EI} \frac{1}{3}(L+a) - \frac{1}{48} \frac{WL^3}{EI} \frac{3}{8}L = 0$$

Let $u = a/L$. Divide by $\frac{WL^4}{48EI}$.

$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

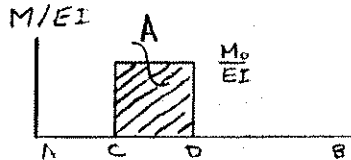
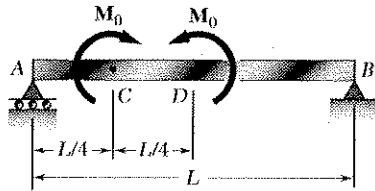
Solving for u , $u = 0.22315$

$$\frac{a}{L} = 0.223$$

$$a = 0.223L$$

Problem 9.125

9.125 through 9.128 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



From Statics $R_A = R_B = 0$.

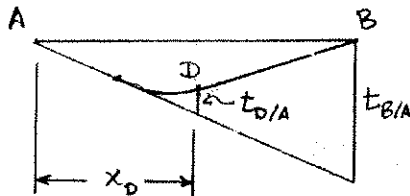
Draw $\frac{M}{EI}$ diagram.

$$A = \left(\frac{M_0}{EI} \right) \left(\frac{L}{4} \right) = \frac{1}{4} \frac{M_0 L}{EI}$$

Place reference tangent at A.

$$t_{B/A} = A \left(\frac{L}{2} + \frac{L}{8} \right) = \frac{5}{32} \frac{M_0 L^2}{EI}$$

$$t_{D/A} = A \left(\frac{L}{8} \right) = \frac{1}{32} \frac{M_0 L^2}{EI}$$



(a) Deflection at D.

$$\begin{aligned} y_D &= t_{D/A} - \frac{x_D}{L} t_{B/A} = t_{D/A} - \frac{1}{2} t_{B/A} \\ &= \frac{1}{32} \frac{M_0 L^2}{EI} - \frac{5}{64} \frac{M_0 L^2}{EI} = -\frac{3}{64} \frac{M_0 L^2}{EI} \end{aligned}$$

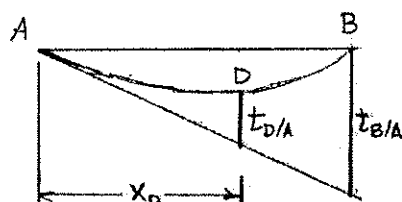
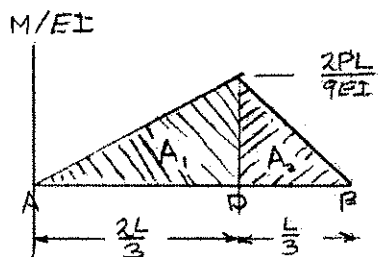
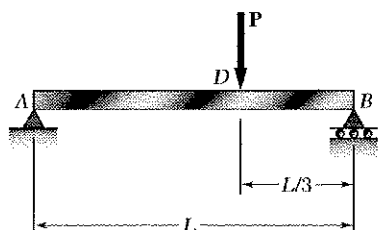
$$y_D = \frac{3M_0 L^2}{64EI} \downarrow$$

(b) Slope at A.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{32} \frac{M_0 L}{EI} \quad \theta_A = \frac{5M_0 L}{32EI} \swarrow$$

Problem 9.126

9.125 through 9.128 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



From statics, $R_A = \frac{1}{3}P \uparrow$, $R_B = \frac{2}{3}P \uparrow$

$$M_D = R_A \left(\frac{2}{3}L \right) = \frac{2}{9}PL$$

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2} \left(\frac{2L}{3} \right) \left(\frac{2PL}{9EI} \right) = \frac{2PL^2}{27EI}$$

$$A_2 = \frac{1}{2} \left(\frac{L}{3} \right) \left(\frac{2PL}{9EI} \right) = \frac{PL^2}{27EI}$$

Place reference tangent at A.

$$t_{B/A} = A_1 \left(\frac{L}{3} + \frac{1}{3} \cdot \frac{2L}{3} \right) + A_2 \left(\frac{2}{3} \cdot \frac{L}{3} \right) = \frac{4PL^3}{81EI}$$

$$t_{D/A} = A_1 \left(\frac{1}{3} \cdot \frac{2L}{3} \right) = \frac{4PL^3}{243EI}$$

(a) Deflection at D.

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$$

$$y_D = \frac{4PL^3}{243EI} - \left(\frac{2}{3} \right) \frac{4PL^3}{81EI}$$

$$= - \frac{4PL^3}{243EI}$$

$$y_D = \frac{4PL^3}{243EI} \downarrow$$

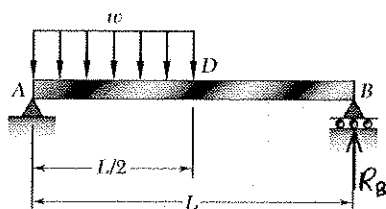
(b) Slope at end A.

$$\theta_A = - \frac{t_{B/A}}{L} = - \frac{4PL^2}{81EI}$$

$$\theta_D = \frac{4PL^2}{81EI} \curvearrowright$$

Problem 9.127

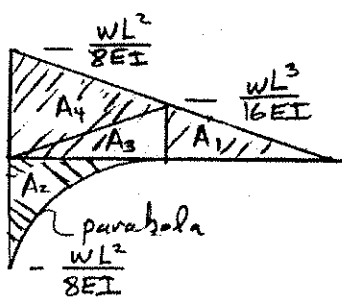
9.125 through 9.128 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



$$+\circlearrowleft \sum M_A = 0: R_B L - \frac{wL}{2} \left(\frac{L}{4} \right) = 0$$

$$R_B = \frac{1}{8} wL$$

Draw $\frac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{R_B L}{EI} = \frac{wL^2}{8EI}$$

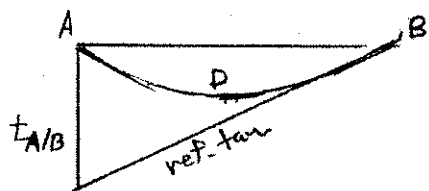
$$\frac{M_2}{EI} = -\frac{wL^3}{8EI}$$

$$A_1 = \left(\frac{1}{2} \right) \left(\frac{L}{2} \right) \frac{wL^2}{16EI} = \frac{wL^3}{32EI}$$

$$A_2 = \left(\frac{1}{3} \right) \left(\frac{L}{2} \right) \frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \left(\frac{1}{4} \right) \left(\frac{L}{2} \right) \frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_4 = \left(\frac{1}{2} \right) \left(\frac{L}{2} \right) \frac{wL^2}{8EI} = -\frac{wL^3}{32EI}$$



(a) Deflection at D.

Place reference tangent at B.

$$y_D = t_{D/B} - \frac{1}{2} t_{A/B}$$

$$t_{D/A} = \left(\frac{1}{3} - \frac{1}{2} \right) A_1 = \frac{wL^4}{384EI}$$

$$t_{B/A} = \frac{L}{3} (A_1 + A_3 + A_4) + \left(\frac{1}{4} - \frac{1}{2} \right) A_2 = \frac{7wL^4}{384EI}$$

$$y_D = \frac{wL^4}{384EI} - \frac{1}{2} \cdot \frac{7wL^4}{384EI} = -\frac{5wL^4}{768EI}$$

$$y_D = \frac{5wL^4}{768EI} \downarrow$$

(b) Slope at A.

Place reference tangent at A.

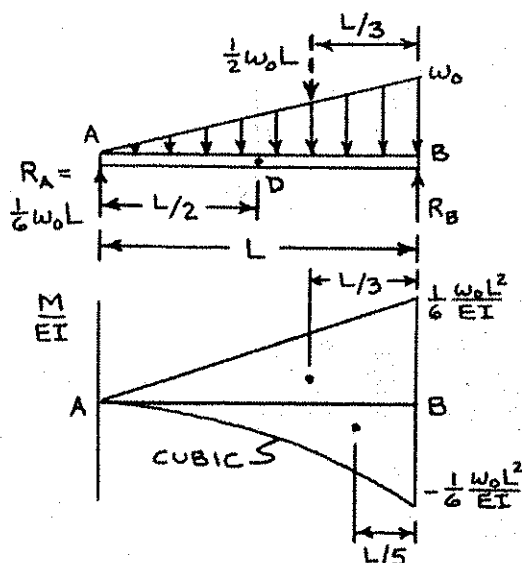
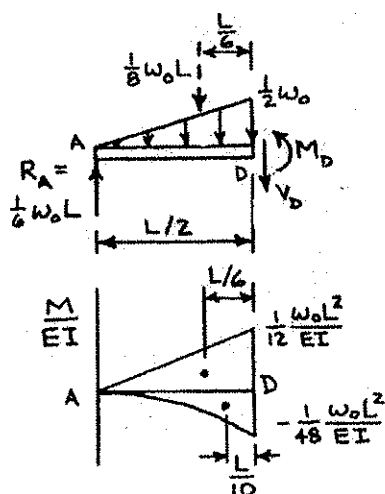
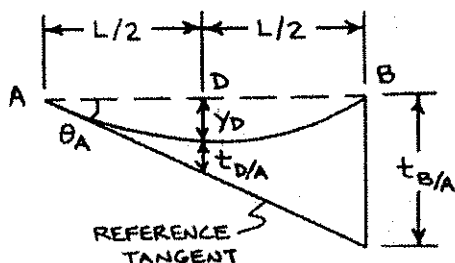
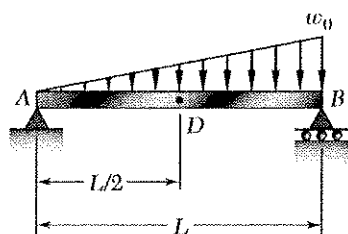
$$\theta_A = -\frac{1}{L} t_{B/A} = -\left(\frac{1}{L} \right) \left\{ \left(\frac{2L}{3} \right) (A_1 + A_3 + A_4) + \left(L - \frac{1}{4} - \frac{1}{2} \right) A_2 \right\}$$

$$= -\frac{3wL^3}{128EI}$$

$$\theta_A = \frac{3wL^3}{128EI} \curvearrowright$$

Problem 9.128

9.125 through 9.128 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



$$t_{B/A} = \frac{1}{2} \left(\frac{1}{6} \frac{w_0 L^2}{EI} \right) (L) \left(\frac{L}{3} \right) + \frac{1}{4} \left(-\frac{1}{6} \frac{w_0 L^2}{EI} \right) (L) \left(\frac{L}{5} \right)$$

$$= \frac{7w_0 L^4}{360EI}$$

$$t_{D/A} = \frac{1}{2} \left(\frac{1}{12} \frac{w_0 L^2}{EI} \right) \left(\frac{L}{2} \right) \left(\frac{L}{6} \right) + \frac{1}{4} \left(-\frac{1}{48} \frac{w_0 L^2}{EI} \right) \left(\frac{L}{2} \right) \left(\frac{L}{10} \right)$$

$$= \frac{37w_0 L^4}{11520EI}$$

(a) DEFLECTION AT D:

$$y_D = \frac{1}{2} t_{B/A} - t_{D/A} = \frac{1}{2} \left(\frac{7w_0 L^4}{360EI} \right) - \frac{37w_0 L^4}{11520EI} = \frac{75w_0 L^4}{11520EI}$$

$$y_D = \frac{5w_0 L^4}{768EI} \downarrow$$

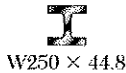
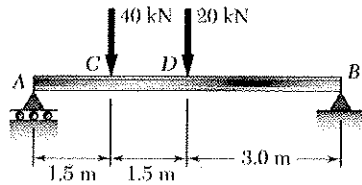
(b) SLOPE AT A:

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{7w_0 L^3}{360EI}$$

$$\theta_A = \frac{7w_0 L^3}{360EI} \nearrow$$

Problem 9.129

9.129 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D. Use $E = 200 \text{ GPa}$.

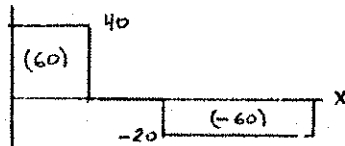


$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 71.1 \times 10^6 \text{ mm}^4 = 71.1 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(71.1 \times 10^{-6}) = 14.22 \times 10^6 \text{ N}\cdot\text{m}^2 = 14220 \text{ kN}\cdot\text{m}^2$$

$V \text{ (kN)}$

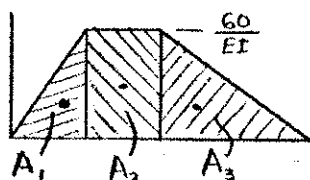


$$+\circlearrowleft \sum M_B = 0: -6R_A + (4.5)(40) + (3)(20) = 0$$

$$R_A = 40 \text{ kN.}$$

Draw shear and $\frac{M}{EI}$ diagrams.

$M/EI \text{ (m}^{-1}\text{)}$



$$A_1 = \frac{1}{2} \left(\frac{60}{EI} \right) (1.5) = \frac{45}{EI}$$

$$A_2 = \left(\frac{60}{EI} \right) (1.5) = \frac{90}{EI}$$

$$A_3 = \frac{1}{2} \left(\frac{60}{EI} \right) (3) = \frac{90}{EI}$$

Place reference tangent at A.



$$t_{B/A} = A_1(4.5 + 0.5) + A_2(3 + 0.75) + A_3(2.0) = \frac{742.5}{EI} \text{ m.}$$

$$t_{D/A} = A_1(1.5 + 0.5) + A_2(0.75) = \frac{157.5}{EI} \text{ m}$$

(a) Slope at A. $\theta_A = -\frac{t_{B/A}}{L} = -\frac{742.5}{6EI} = -\frac{123.75}{EI} = -\frac{123.75}{14220}$

$$= -8.70 \times 10^{-3}$$

$$\theta_A = 8.70 \times 10^{-3} \text{ rad} \swarrow$$

(b) Deflection at D.

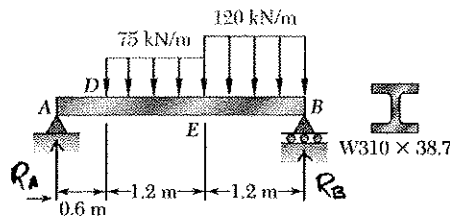
$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{157.5}{EI} - \left(\frac{3}{6} \right) \left(\frac{742.5}{EI} \right) = -\frac{213.75}{EI}$$

$$= -\frac{213.75}{14220} = -15.03 \times 10^{-3} \text{ m}$$

$$y_D = 15.03 \text{ mm} \downarrow$$

Problem 9.130

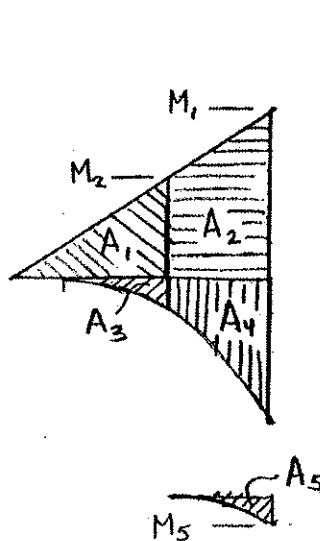
9.130 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point E. Use $E = 200$ GPa.



Units: Forces in kN, lengths in m.

For W310x38.7, $I = 85.1 \times 10^{-6} \text{ m}^4$

$EI = (200)(85.1) = 17020 \text{ kN m}^2$



$$+\circlearrowleft \sum M_B = 0:$$

$$-3R_A + (75)(1.2)(1.8) + (120)(1.2)(0.6) = 0 \quad \rightarrow R_A = 82.8 \text{ kN} \uparrow$$

Consider loading as 75 kN/m from D to B plus 45 kN/m from E to B. Draw bending moment diagram by parts.

$$M_1 = 3R_A = 248.4 \text{ kN m}$$

$$M_2 = 1.8R_A = 149 \text{ kN m}$$

$$M_3 = -\frac{1}{2}(75)(2.4)^2 = -216 \text{ kN m}$$

$$M_4 = -\frac{1}{2}(75)(1.2)^2 = -54 \text{ kN m}$$

$$M_5 = -\frac{1}{2}(45)(1.2)^2 = -32.4 \text{ kN m}$$

$$A_1 + A_2 = \frac{1}{2}(3)(248.4) = 372.3 \text{ kN m}^2$$

$$A_1 = \frac{1}{2}(1.8)(149) = 134 \text{ kN m}^2$$

$$A_3 + A_4 = \frac{1}{3}(2.4)(-216) = -172.8 \text{ kN m}^2$$

$$A_3 = \frac{1}{3}(1.2)(-54) = -21.6 \text{ kN m}^2$$

$$A_5 = \frac{1}{3}(1.2)(-32.4) = -13$$

(a) Slope at A. $y_B = y_A + \theta_A L + t_{B/A}$. $y_A = y_B = 0$
 $\theta_A = -t_{B/A}/L$

$$t_{B/A} = \frac{1}{EI} \left\{ (A_1 + A_2) \left(\frac{1}{3} \right) (3) + (A_3 + A_4) \left(\frac{1}{4} \right) (2.4) + (A_5) \left(\frac{1}{4} \right) (1.2) \right\}$$

$$= \frac{264.72}{17020} = 0.01555 \text{ m}$$

$$\theta_A = -\frac{0.01555}{3} = -5.183 \times 10^{-3}$$

$$\theta_A = 5.18 \times 10^{-3} \text{ rad} \quad \swarrow$$

(b) Deflection at E. $y_E = x_E \theta_A + t_{E/A}$

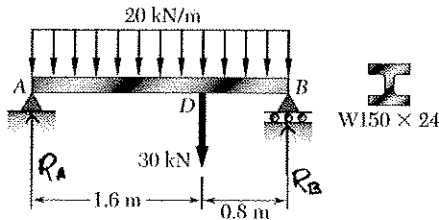
$$t_{E/A} = \frac{1}{EI} \left\{ (A_1) \left(\frac{1}{3} \right) (1.8) + (A_3) \left(\frac{1}{4} \right) (1.2) \right\} = \frac{73.92}{17020} = 4.343 \times 10^{-3} \text{ m}$$

$$y_E = (1.8)(-0.005183) + 0.004343 = -0.00499 \text{ m}$$

$$= 5 \text{ mm} \downarrow$$

Problem 9.131

9.131 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point D. Use $E = 200$ GPa.



Units: Forces in kN. Lengths in meters.

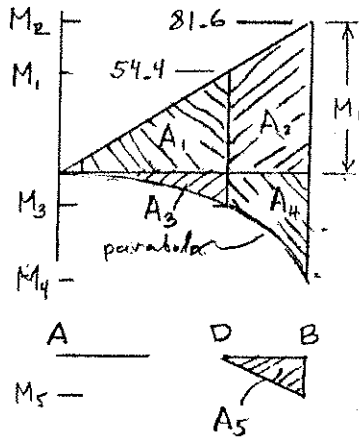
$$\text{For } W150 \times 24 \quad I = 13.4 \times 10^6 \text{ mm}^4 \\ = 13.4 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(13.4 \times 10^{-6}) = 2.68 \times 10^6 \text{ N} \cdot \text{m}^2 \\ = 2680 \text{ kN} \cdot \text{m}^2$$

$$+\circlearrowleft \sum M_B = 0: -2.4 R_A + (0.8)(30) + (1.2)(2.4)(20) = 0$$

$$R_A = 34 \text{ kN} \uparrow$$

Draw bending moment diagram by parts.



$$M_1 = (1.6)(34) = 54.4 \text{ kN} \cdot \text{m}$$

$$M_2 = (2.4)(34) = 81.6 \text{ kN} \cdot \text{m}$$

$$M_3 = -\frac{1}{2}(20)(1.6)^2 = -25.6 \text{ kN} \cdot \text{m}$$

$$M_4 = -\frac{1}{2}(20)(2.4)^2 = -57.6 \text{ kN} \cdot \text{m}$$

$$M_5 = -(0.8)(30) = -24 \text{ kN} \cdot \text{m}$$

$$A_1 = \frac{1}{2}(1.6)(54.4) = 43.52 \text{ kN} \cdot \text{m}^2$$

$$A_1 + A_2 = \frac{1}{2}(2.4)(81.6) = 97.92 \text{ kN} \cdot \text{m}^2$$

$$A_3 = \frac{1}{3}(1.6)(-25.6) = -13.6533 \text{ kN} \cdot \text{m}^2$$

$$A_3 + A_4 = \frac{1}{3}(2.4)(-57.6) = -46.08 \text{ kN} \cdot \text{m}^2$$

$$A_5 = \frac{1}{2}(0.8)(-24) = -9.6 \text{ kN} \cdot \text{m}^2$$

(a) Slope at A. Place reference tangent at A. $\theta_A = -\frac{1}{L} t_{B/A}$

$$t_{B/A} = \frac{1}{EI} \left\{ (A_1 + A_2) \left(\frac{1}{3} \right) (2.4) + (A_3 + A_4) \left(\frac{1}{4} \right) (2.4) + A_5 \left(\frac{1}{3} \right) (0.8) \right\}$$

$$= \frac{48.128}{2680} = 17.9582 \times 10^{-3} \text{ m}$$

$$\theta_A = -\frac{17.9582 \times 10^{-3}}{2.4} = -7.48258 \times 10^{-3} \quad \theta_A = 7.48 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

(b) Deflection at point D. $y_D = t_{D/A} + \theta_A x_D$

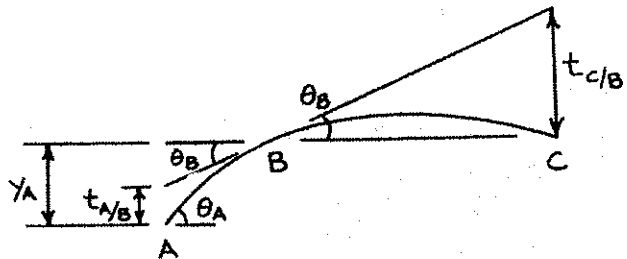
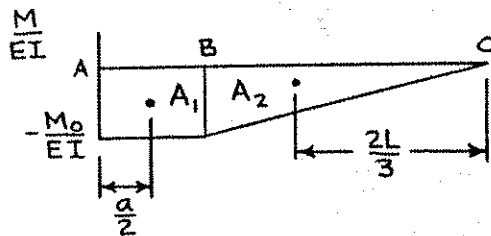
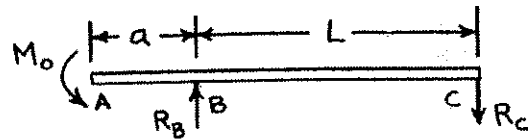
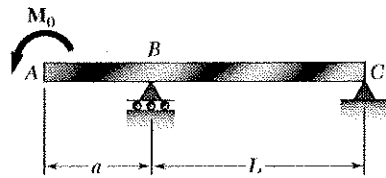
$$t_{D/A} = \frac{1}{EI} \left\{ A_1 \left(\frac{1}{3} \right) (1.6) + A_2 \left(\frac{1}{4} \right) (1.6) \right\} = \frac{17.7493}{2680} = 6.62289 \times 10^{-3} \text{ m}$$

$$y_D = 6.62289 \times 10^{-3} + (-7.48258 \times 10^{-3})(1.6) = -5.3492 \times 10^{-3} \text{ m}$$

$$y_D = 5.35 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.132

9.132 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point A.



$$A_1 = -\frac{M_o a}{EI}$$

$$A_2 = -\frac{M_o L}{2EI}$$

$$t_{C/B} = A_2 \left(\frac{2L}{3} \right) = \left(-\frac{M_o L}{2EI} \right) \left(\frac{2L}{3} \right) = -\frac{M_o L^2}{3EI}$$

(a) SLOPE AT A: $\theta_B = \frac{t_{C/B}}{L} = \frac{M_o L}{3EI}$

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1$$

$$\frac{M_o L}{3EI} = \theta_A - \frac{M_o a}{EI}$$

$$\theta_A = \frac{M_o}{3EI} (L + 3a) \nearrow$$

(b) DEFLECTION AT A:

$$t_{A/B} = A_1 \left(\frac{a}{2} \right) = -\frac{M_o a^2}{2EI}$$

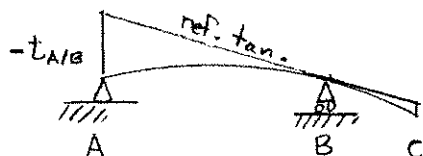
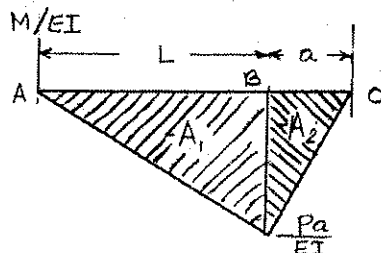
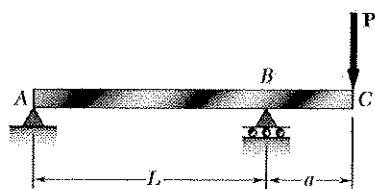
$$y_A = \frac{a}{L} t_{C/B} + t_{A/B}$$

$$= \frac{a}{L} \left(-\frac{M_o L^2}{3EI} \right) - \frac{M_o a^2}{2EI}$$

$$y_A = \frac{M_o a}{6EI} (2L + 3a) \downarrow$$

Problem 9.133

9.133 For the beam and loading shown, determine (a) the slope at point C, (b) the deflection at point C.



Deflection at point C.

$$A_1 = -\frac{PaL}{2EI}$$

$$A_2 = -\frac{Pa^2}{2EI}$$

$$t_{A/B} = A_1 \left(\frac{2}{3}L \right) = -\frac{PaL^2}{3EI}$$

$$\theta_B = \frac{t_{A/B}}{L} = -\frac{PaL}{3EI}$$

(a) Slope at C. $\theta_C = \theta_B + \theta_{C/B}$

$$\begin{aligned} \theta_C &= -\frac{PaL}{3EI} - \frac{Pa^2}{2EI} \\ &= -\frac{Pa(2L+3a)}{6EI} \end{aligned}$$

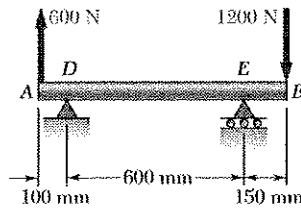
$$\theta_C = \frac{Pa(2L+3a)}{6EI} \quad \swarrow$$

$$y_C = a\theta_B + t_{C/B}$$

$$\begin{aligned} y_C &= -\frac{Pa^2L}{3EI} + \left(-\frac{Pa^2}{2EI} \right) \left(\frac{2}{3}a \right) \\ &= -\frac{Pa^2(L+a)}{3EI} \end{aligned}$$

$$y_C = \frac{Pa^2(L+a)}{3EI} \quad \downarrow \quad \swarrow$$

Problem 9.134



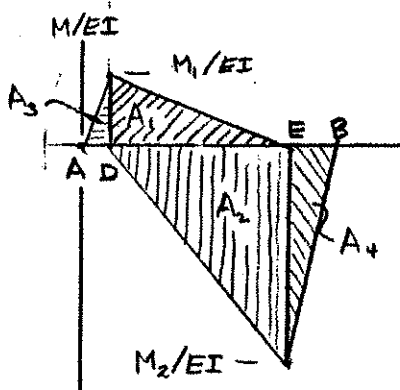
9.134 Knowing that the beam AB is made of a solid steel rod of diameter $d = 18$ mm, determine for the loading shown (a) the slope at point D , (b) the deflection at point A . Use $E = 200$ GPa.

Units: Forces in kN; lengths in m.

$$c = \frac{1}{2} = \frac{1}{2}(18) = 9 \text{ mm.}$$

$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (9)^4 = 5153 \text{ mm}^4.$$

$$EI = (200 \times 10^6)(5153 \times 10^{-12}) = 1.0306 \text{ kNm}^2.$$



Draw the $\frac{M}{EI}$ diagram by parts by considering the bending moment diagram due to each of the applied loads.

$$\frac{M_1}{EI} = \frac{(0.6)(0.1)}{1.0306} = 0.0582 \text{ m}^{-1}$$

$$\frac{M_2}{EI} = \frac{-(1.2)(0.15)}{1.0306} = -0.1747 \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(0.6)(0.0582) = 0.01746$$

$$A_2 = \frac{1}{2}(0.6)(-0.1747) = -0.05241$$

$$A_3 = \frac{1}{2}(0.1)(0.0582) = 0.00291$$

Place reference tangent at D .

(a) Slope at point D .

$$y_E = y_D + L\theta_D + t_{E/D}$$

$$\theta_D = -t_{E/D}/L$$

$$t_{E/D} = 0.4A_1 + 0.2A_2 = -3.498 \times 10^{-3}$$

$$\theta_D = \frac{-0.003498}{0.6} = -5.83 \times 10^{-3}$$

$$\theta_D = 5.83 \times 10^{-3} \text{ rad} \swarrow$$

(b) Deflection at A .

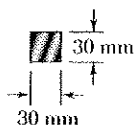
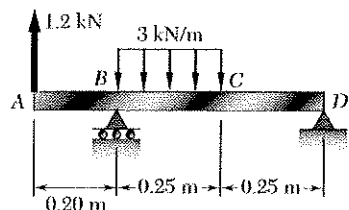
$$y_A = y_D - a\theta_D + t_{A/D} = t_{A/D} - a\theta_D$$

$$y_A = A_3\left(\frac{2}{3}\right)(0.1) - (0.1)(0.00583) = -0.389 \times 10^{-3} \text{ m}$$

$$y_A = 0.39 \text{ mm} \downarrow$$

Problem 9.135

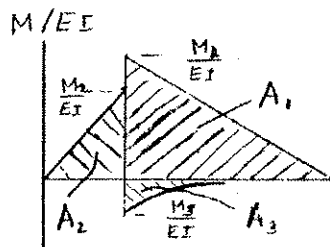
9.135 Knowing that the beam AD is made of a solid steel bar, determine the (a) slope at point B , (b) the deflection at point A . Use $E = 200$ GPa.



$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \frac{1}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4 = 67.5 \times 10^{-9} \text{ m}^4$$

$$EI = (200 \times 10^9)(67.5 \times 10^{-9}) = 13500 \text{ N} \cdot \text{m}^2 = 13.5 \text{ kN} \cdot \text{m}^2$$



$$\sum M_B = 0: -(0.2)(1.2) - (3)(0.25)(0.125) + 5R_D = 0$$

$$R_D = 0.6675 \text{ kN}$$

Draw $\frac{M}{EI}$ diagram by parts.

$$M_1 = (0.6675)(0.5) = 0.33375 \text{ kN} \cdot \text{m}$$

$$M_2 = (1.2)(0.2) = 0.240 \text{ kN} \cdot \text{m}$$

$$M_3 = -\frac{1}{2}(3)(0.25)^2 = -0.09375 \text{ kN} \cdot \text{m}$$

$$A_1 = \frac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$$

$$A_2 = \frac{1}{2}(0.240)(0.2)/EI = 0.024/EI$$

$$A_3 = \frac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$$

Place reference tangent at B .

$$t_{D/B} = A_1\left(\frac{2}{3} \cdot 0.5\right) + A_3\left(\frac{3}{4} \cdot 0.25 + 0.25\right) = 0.024395/EI$$

$$(a) \text{ Slope at } B, \quad \theta_B = -\frac{t_{D/B}}{L} = -\frac{0.024395}{0.5 EI} = -\frac{0.048789}{EI}$$

$$= -3.6140 \times 10^{-3} \quad \theta_B = 3.61 \times 10^{-3} \text{ rad} \swarrow$$

$$t_{A/B} = A_2\left(\frac{2}{3}(0.20)\right) = 0.0032/EI = 0.23704 \times 10^{-3} \text{ m}$$

(b) Deflection at A .

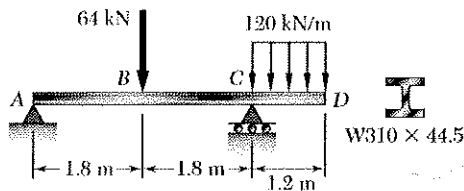
$$y_A = t_{A/B} - L_{AB} \theta_B$$

$$= 0.23704 \times 10^{-3} - (0.2)(-3.6140 \times 10^{-3}) = 0.960 \times 10^{-3} \text{ m}$$

$$y_A = 0.960 \text{ mm} \uparrow$$

Problem 9.136

9.136 For the beam and loading shown, determine (a) the slope at point C, (b) the deflection at point D. Use $E = 200 \text{ GPa}$.



FREE BODY AD:

$$+\circlearrowleft \sum M_C = 0: (64)(1.8) - (144)(0.6) - 3.6R_A = 0$$

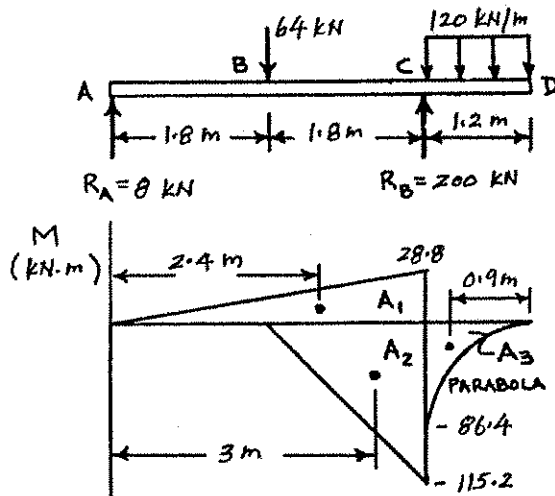
$$R_A = 8 \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0: 8 - 64 + R_B - 144 = 0$$

$$R_B = 200 \text{ kN} \uparrow$$

FOR W310x44.5 : $I = 99.2 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^6)(99.2 \times 10^{-6}) = 19840 \text{ kN}\cdot\text{m}^2$$



(a) SLOPE AT C:

$$A_1 = \frac{1}{2}(28.8)(3.6) = 51.84 \text{ kN}\cdot\text{m}^2$$

$$A_2 = \frac{1}{2}(-115.2)(1.8) = -103.68 \text{ kN}\cdot\text{m}^2$$

$$EI t_{A/C} = A_1(2.4) + A_2(3) = (51.84)(2.4) + (-103.68)(3) = -186.624 \text{ kN}\cdot\text{m}^3$$

$$t_{A/C} = \frac{-186.624}{19840} = -9.406 \times 10^{-3} \text{ m}$$

$$\theta_C = \frac{t_{A/C}}{L} = \frac{-9.406 \times 10^{-3} \text{ m}}{3.6 \text{ m}}$$

$$\theta_C = 2.61 \times 10^{-3} \text{ rad} \searrow$$

(b) DEFLECTION AT D:

$$EI t_{D/C} = A_1(0.9) = \frac{1}{3}(-86.4)(1.2)(0.9) = -31.1 \text{ kN}\cdot\text{m}$$

$$t_{D/C} = \frac{-31.1}{19840} = -1.5675 \times 10^{-3} \text{ m}$$

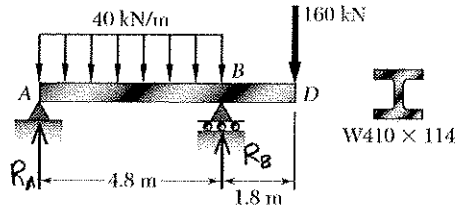
$$y_D = t_{D/C} + \frac{1.2}{3.6} t_{A/C} = -1.5675 \times 10^{-3} + \frac{1}{3}(-9.406 \times 10^{-3}) = -4.703 \times 10^{-3} \text{ m}$$

$$y_C = 4.7 \text{ mm} \downarrow$$

Proprietary Material. © 2009 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

Problem 9.137

9.137 For the beam and loading shown, determine (a) the slope at point B, (b) the deflection at point D. Use $E = 200 \text{ GPa}$.



Units: Forces in kN; lengths in meters.

$$I = 462 \times 10^6 \text{ mm}^4 = 462 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(462 \times 10^{-6}) = 92.4 \times 10^6 \text{ N}\cdot\text{m}^2 = 92400 \text{ kN}\cdot\text{m}^2$$

$$+\circlearrowleft \sum M_B = 0: -4.8 R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36 \text{ kN}$$

Draw bending moment diagram by parts.

$$A_1 = \frac{1}{2}(4.8)(172.8) = 414.72 \text{ kN}\cdot\text{m}^2$$

$$A_2 = \frac{1}{3}(4.8)(-460.8) = -737.28 \text{ kN}\cdot\text{m}^2$$

$$A_3 = \frac{1}{2}(1.8)(-288) = -259.2 \text{ kN}\cdot\text{m}^2$$

Place reference tangent at B.

$$(a) \text{ Slope at B. } y_A = y_B - L\theta_B + t_{A/B}$$

$$\theta_B = \frac{t_{B/A}}{L} = \frac{1}{EIL} \left\{ A_1 \left(\frac{2}{3} \right) (4.8) + A_2 \left(\frac{3}{4} \right) (4.8) \right\}$$

$$= \frac{-1327.04}{(92400)(4.8)} = -2.9922 \times 10^{-3}$$

$$\theta_B = 2.99 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

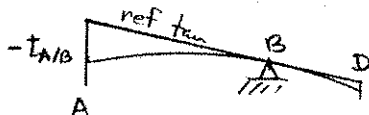
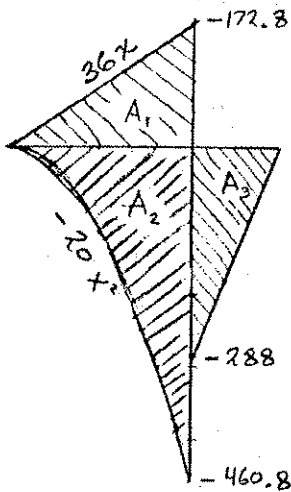
$$(b) \text{ Deflection at D. } y_D = y_B + a\theta_B + t_{D/B}$$

$$= 0 + (1.8)(-2.9922 \times 10^{-3})$$

$$- \frac{1}{EI} \left\{ A_3 \left(\frac{2}{3} \right) (1.8) \right\}$$

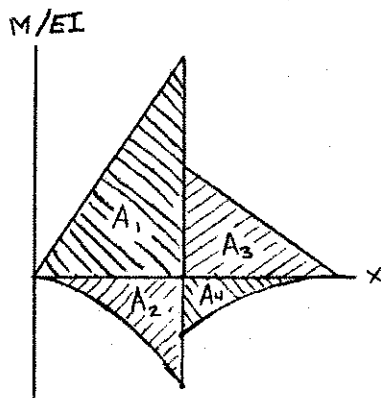
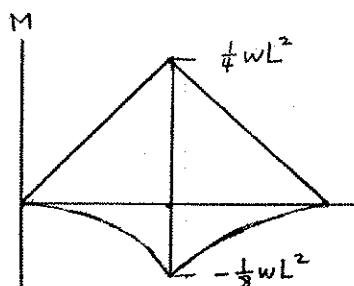
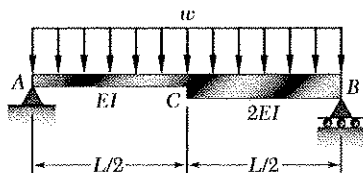
$$= -5.3860 \times 10^{-3} - \frac{311.04}{92400}$$

$$= -8.75 \times 10^{-3} \text{ m} \quad y_B = 8.75 \text{ mm} \quad \blacktriangleleft$$



Problem 9.138

9.138 For the beam and loading shown, determine (a) the slope at end A, (b) the slope at end B, (c) the deflection at the midpoint C.



Reactions: $R_A = R_B = \frac{1}{2} WL$

Draw bending moment and M/EI diagrams by parts as shown.

$$A_1 = \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{WL^2}{4EI} = \frac{WL^3}{16EI}$$

$$A_2 = -\frac{1}{3} \cdot \frac{L}{2} \cdot \frac{WL^2}{8EI} = -\frac{WL^3}{48EI}$$

$$A_3 = \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{WL^2}{8EI} = \frac{WL^3}{32EI}$$

$$A_4 = -\frac{1}{3} \cdot \frac{L}{2} \cdot \frac{WL^2}{16EI} = -\frac{WL^3}{96EI}$$

Place reference tangent at A.

(a) Slope at end A.

$$y_B = y_A + L\theta_A + t_{B/A}$$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \left(\frac{1}{2} + \frac{1}{6}\right)A_1 + \left(\frac{1}{2} + \frac{1}{8}\right)A_2 + \frac{1}{3}A_3 + \frac{3L}{8}A_4$$

$$= \frac{WL^4}{EI} \left(\frac{1}{24} - \frac{5}{384} + \frac{1}{96} - \frac{1}{256}\right) = \frac{9WL^4}{256EI}$$

$$\theta_A = -\frac{9WL^4}{256EI} \cdot \frac{1}{L} = -\frac{9WL^3}{256EI} \quad \theta_A = \frac{9WL^3}{256EI} \quad \leftarrow$$

(b) Slope at end B.

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{9WL^3}{256EI} + A_1 + A_2 + A_3 + A_4$$

$$\theta_B = \frac{7WL^3}{256EI}$$

$$\theta_B = \frac{7WL^3}{256EI} \quad \leftarrow$$

(c) Deflection at midpoint C.

$$y_C = y_C + \frac{L}{2}\theta_A + t_{C/A}$$

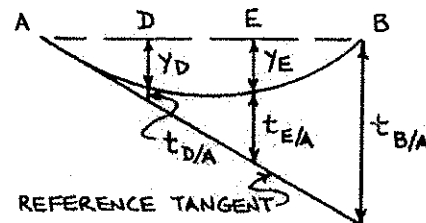
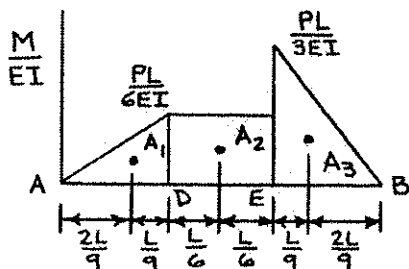
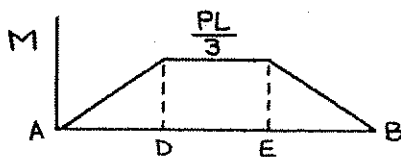
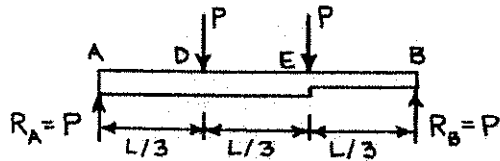
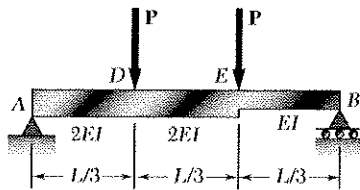
$$t_{C/A} = \left(\frac{1}{6}\right)A_1 + \left(\frac{1}{8}\right)A_2 = \frac{WL^4}{128EI}$$

$$y_C = 0 + \left(\frac{L}{2}\right)\left(-\frac{9WL^3}{256EI}\right) + \frac{WL^4}{128EI} = -\frac{5WL^4}{512EI}$$

$$y_C = \frac{5WL^4}{512EI} \quad \downarrow$$

Problem 9.139

9.139 For the beam and loading shown, determine the deflection (a) at point D, (b) at point E.



$$A_1 = \frac{1}{2} \left(\frac{PL}{6EI} \right) \left(\frac{L}{3} \right) = \frac{PL^2}{36EI}$$

$$A_2 = \left(\frac{PL}{6EI} \right) \left(\frac{L}{3} \right) = \frac{PL^2}{18EI}$$

$$A_3 = \frac{1}{2} \left(\frac{PL}{3EI} \right) \left(\frac{L}{3} \right) = \frac{PL^2}{18EI}$$

$$t_{D/A} = A_1 \left(\frac{L}{9} \right) = \left(\frac{PL^2}{36EI} \right) \left(\frac{L}{9} \right) = \frac{PL^3}{324EI}$$

$$t_{E/A} = A_1 \left(\frac{L}{9} + \frac{L}{3} \right) + A_2 \left(\frac{L}{6} \right) + A_3 \left(\frac{L}{6} \right) = \frac{7PL^3}{324EI}$$

$$t_{B/A} = A_1 \left(\frac{7L}{9} \right) + A_2 \left(\frac{L}{2} \right) + A_3 \left(\frac{2L}{9} \right) = \left(\frac{PL^3}{36EI} \right) \left(\frac{7L}{9} \right) + \left(\frac{PL^2}{18EI} \right) \left(\frac{L}{2} \right) + \left(\frac{PL^2}{18EI} \right) \left(\frac{2L}{9} \right) = \frac{5PL^3}{81EI}$$

(a) DEFLECTION AT D:

$$y_D = \frac{1}{3} t_{B/A} - t_{D/A} = \frac{1}{3} \left(\frac{5PL^3}{81EI} \right) - \frac{PL^3}{324EI} = \frac{17PL^3}{972EI}$$

$$y_D = \frac{17PL^3}{972EI} \downarrow$$

(b) DEFLECTION AT E:

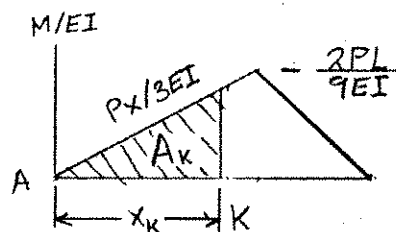
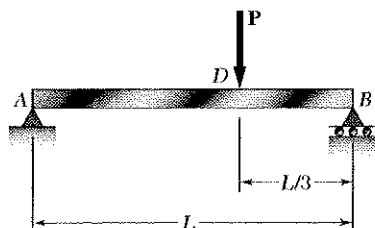
$$y_E = \frac{2}{3} t_{B/A} - t_{E/A} = \frac{2}{3} \left(\frac{5PL^3}{81EI} \right) - \frac{7PL^3}{324EI} = \frac{19PL^3}{972EI}$$

$$y_E = \frac{19PL^3}{972EI} \downarrow$$

Problem 9.140

9.140 through 9.143 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.140 Beam and loading of Prob. 9.126



From the solution to Problem 9.126,

$$R_A = \frac{1}{3}P \uparrow, \quad \theta_A = -\frac{4PL^2}{81EI}$$

Over portion AD, $\frac{M}{EI} = \frac{R_A x}{EI} = \frac{Px}{3EI}$

$$A_K = \frac{1}{2} x_K \left(\frac{Px_K}{3EI} \right) = \frac{Px_K^2}{6EI}$$

$$\theta_K = \theta_A + A_K = 0$$

$$-\frac{4PL^2}{81EI} + \frac{Px_K^2}{6EI} = 0$$

$$x_K^2 = \frac{8}{27} L^2$$

$$x_K = 0.54433 L$$

$$A_K = -\theta_A = \frac{4PL^2}{81EI}$$

$$t_{A/K} = \left(\frac{2}{3} x_K \right) A_K = 0.01792 \frac{PL^3}{EI}$$

$$y_A = y_K + t_{A/K} = 0$$

$$y_K = -t_{A/K} = -0.01792 \frac{PL^3}{EI}$$

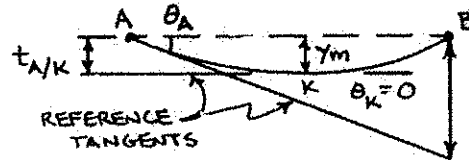
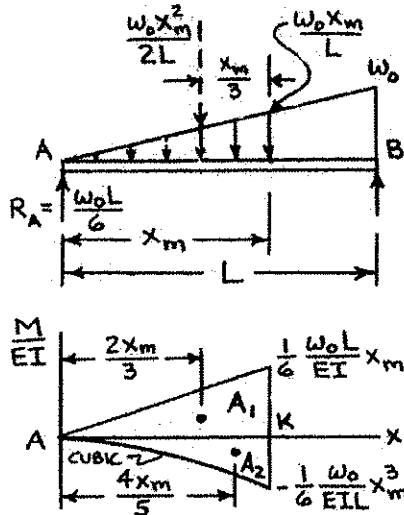
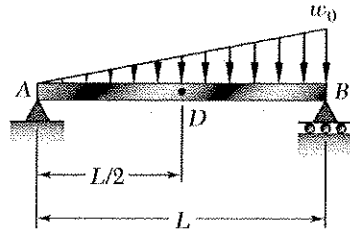
$$y_K = 0.01792 \frac{PL^3}{EI} \downarrow \blacktriangleleft$$

$$x_K = 0.544 L \quad \blacktriangleleft$$

Problem 9.141

9.140 through 9.143 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.141 Beam and loading of Prob. 9.128



$$\text{FROM PROB. 9.128: } \theta_A = -\frac{7w_0 L^3}{360EI}$$

$$A_1 = \frac{1}{2} \left(\frac{w_0 L}{6EI} x_m \right) (x_m) = \frac{w_0 L x_m^2}{12EI}$$

$$A_2 = \frac{1}{4} \left(-\frac{w_0 x_m^3}{6EIL} \right) (x_m) = -\frac{w_0 x_m^4}{24EIL}$$

MAX. DEFLECTION OCCURS AT K, WHERE $\theta_K = 0$.

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A_1 + A_2$$

$$0 = -\frac{7w_0 L^3}{360EI} + \frac{w_0 L x_m^2}{12EI} - \frac{w_0 x_m^4}{24EIL}$$

$$\text{REARRANGING: } 0 = \frac{w_0 L^3}{360EI} \left[-7 + 30 \left(\frac{x_m}{L} \right)^2 - 15 \left(\frac{x_m}{L} \right)^4 \right]$$

$$\text{SOLVING BIQUADRATIC: } \left(\frac{x_m}{L} \right)^2 = 0.26970 \quad x_m = 0.51933 L$$

y_m IS 0.519 L FROM A

$$\begin{aligned} t_{A/K} &= A_1 \frac{2x_m}{3} + A_2 \frac{4x_m}{5} = \left(\frac{w_0 L x_m^2}{12EI} \right) \frac{2x_m}{3} + \left(-\frac{w_0 x_m^4}{24EIL} \right) \frac{4x_m}{5} \\ &= \frac{w_0 L^4}{90EI} \left[5 \left(\frac{x_m}{L} \right)^3 - 3 \left(\frac{x_m}{L} \right)^5 \right] = \frac{w_0 L^4}{90EI} \left[5(0.51933)^3 - 3(0.51933)^5 \right] \\ &= 0.0065222 \frac{w_0 L^4}{EI} \end{aligned}$$

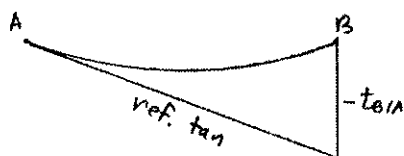
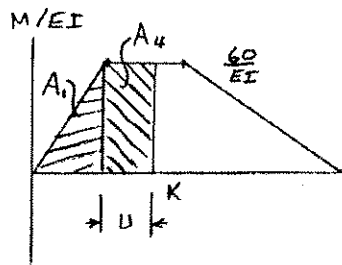
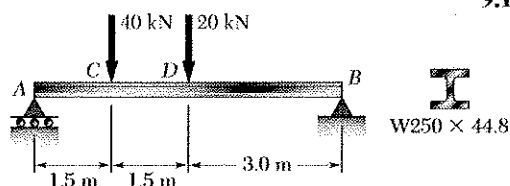
$$y_m = |t_{A/K}|$$

$$y_m = 6.52 \times 10^{-3} \frac{w_0 L^4}{EI}$$

Problem 9.142

9.140 through 9.143 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.142 Beam and loading of Prob. 9.129



Referring to the solution to Prob. 9.129,

$$EI = 14220 \text{ kN} \cdot \text{m}^2$$

$$R_A = 40 \text{ kN}, \quad A_1 = \frac{45}{EI}$$

$$t_{B/A} = \frac{742.5}{EI} \text{ m}$$

$$\theta_A = -\frac{123.75}{EI}$$

Let K be the location of the maximum deflection. Assume that K lies between C and D .

$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{123.75}{EI} + A_1 + A_4$$

$$= -\frac{123.75}{EI} + \frac{45}{EI} + \frac{60U}{EI} = 0$$

$$U = \frac{123.75 - 45}{60} = 1.3125 \text{ m}$$

$$x_K = 1.5 + U = 2.8125 \text{ m}$$

$$t_{K/A} = A_1(U + 0.5) + A_4\left(\frac{1}{2}U\right)$$

$$= \frac{45}{EI}(1.8125) + \frac{(60)(1.3125)(\frac{1}{2})(1.3125)}{EI} = \frac{133.242}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A}$$

$$= \frac{133.242}{EI} - \frac{2.8125}{6} \left(\frac{742.5}{EI} \right) = -\frac{214.80}{EI} = -\frac{214.80}{14220}$$

$$= -15.11 \times 10^{-3} \text{ m}$$

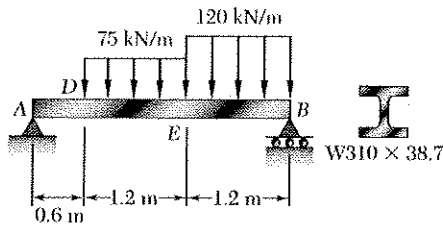
$$y_K = 15.11 \text{ mm} \downarrow$$

$$x_K = 2.81 \text{ m}$$

Problem 9.143

9.140 through 9.143 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.143 Beam and loading of Prob. 9.130.



From the solution to Problem 9.130

$$EI = 17020 \text{ kNm}^2.$$

$$R_A = 82.8 \text{ kN}$$

$$A_1 = 134 \text{ kNm}^2.$$

$$A_3 = -21.6 \text{ kNm}^2.$$

$$\theta_A = -0.005183$$

Slope at E. $\theta_E = \theta_A + \theta_{E/A}$

$$\theta_{E/A} = \frac{1}{EI} \{A_1 + A_3\} = \frac{278.767}{41083} = 6.604 \times 10^{-3}$$

$$\theta_E = 1.421 \times 10^{-3}$$

Since $\theta_E > 0$, the point K of zero slope lies to the left of point E. Let x_K be the coordinate of point K.

$$A_6 = \frac{1}{2} R_A x_K^2 = 41.4 x_K^2$$

$$A_7 = -\frac{1}{6} (75) (x_K - 0.6)^3$$

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + \frac{1}{EI} \{A_6 + A_7\} = 0$$

$$A_6 + A_7 + EI \theta_A = 0$$

$$f(x_K) = 41.4 x_K^2 - \frac{75}{6} (x_K - 0.6)^3 - 88.2 = 0$$

$$\frac{df}{dx_K} = 82.8 x_K - 37.5 (x_K - 0.6)^2$$

Solve for x_K by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx_K}$$

x_K	1.5	1.5442	1.54575	$x_K = 1.545 \text{ m}$
f	-4.1625	-0.00157	0.145	
df/dx_K	93.825	94.428		

$$A_6 = 98.823 \text{ kNm}^2,$$

$$A_7 = -10.55 \text{ kNm}^2.$$

Maximum deflection.

$$y_A = y_K + t_{A/K} = 0$$

$$y_K = -t_{A/K}$$

$$\bar{x}_6 = \frac{2}{3} x_K, \quad \bar{x}_7 = 0.6 + \frac{3}{4} (x_K - 0.6) = \frac{3x_K + 0.6}{4}$$

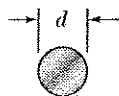
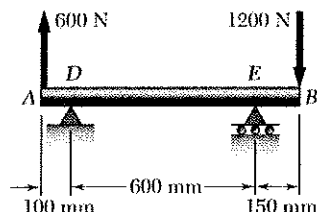
$$x_K = 1.545 \text{ m} \quad \blacktriangleleft$$

$$y_1 = -\frac{1}{EI} \{A_6 \bar{x}_6 + A_7 \bar{x}_7\} = -\frac{87.98}{17020} = -0.005169 \text{ m}$$

$$y_K = 5.2 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.144

9.144 For the beam and loading of Prob. 9.134, determine the largest upward deflection in span DE.



Units: Forces in kN; lengths in m.

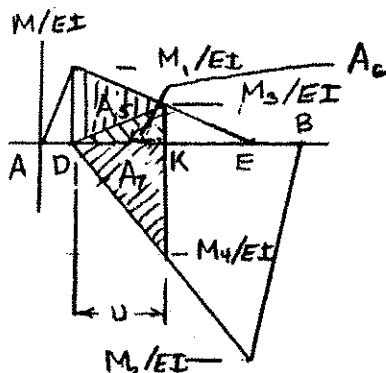
From the solution to Problem 9.134

$$EI = 1.0306 \text{ kN m}^2$$

$$\frac{M_1}{EI} = 0.0582 \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -0.1747 \text{ m}^{-1}$$

$$\theta_0 = 5.83 \times 10^{-3}$$



Location of maximum deflection.

$$\frac{M_3}{EI} = \frac{M_1}{EI} \left(1 - \frac{U}{0.6}\right)$$

$$\frac{M_4}{EI} = \frac{M_2}{EI} \frac{U}{0.6}$$

$$A_5 = \frac{1}{2} \frac{M_1}{EI} \cdot U = 0.0291 U$$

$$A_6 = \frac{1}{2} \frac{M_3}{EI} \left(\frac{U}{0.6}\right) = 0.0291 \left(1 - \frac{U}{0.6}\right) U$$

$$A_7 = \frac{1}{2} \frac{M_4}{EI} \frac{U}{0.6} = -0.0873 \left(\frac{U}{0.6}\right) U$$

$$\theta_K = \theta_0 + A_5 + A_6 + A_7 = 0 \quad \text{Multiply by } 10^3.$$

$$0.00583 + 0.0291 U + 0.0291 \left(1 - \frac{U}{0.6}\right) U - (0.0873) \frac{U}{0.6} U = 0$$

$$0.00583 + 0.0582 U - 0.097 U^2 = 0$$

$$U = 0.3897 \text{ m} = 381 \text{ mm}$$

$$A_5 = 0.0111, \quad A_6 = 0.00465, \quad A_7 = -0.0211$$

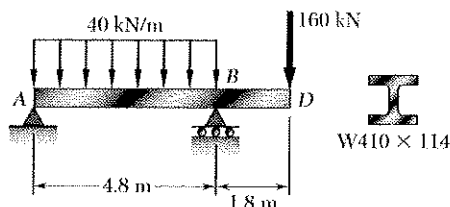
Maximum deflection in portion DE. $y_0 = y_K + t_{0/K} = 0$

$$y_K = -t_{0/K} = -\left\{ A_5 \left(\frac{U}{3}\right) + A_6 \left(\frac{2U}{3}\right) + A_7 \left(\frac{2U}{3}\right) \right\}$$

$$= -\{-0.00292\} = 2.9 \text{ mm} \uparrow$$

Problem 9.145

9.145 For the beam and loading of Prob. 9.137, determine the largest upward deflection in span AB.



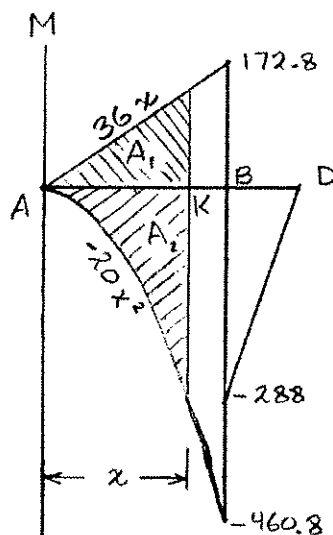
Units: Forces in kN. Lengths in meters.

$$I = 462 \times 10^6 \text{ mm}^4 = 462 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(462 \times 10^{-6}) = 92.4 \times 10^6 \text{ N}\cdot\text{m}^2 = 92400 \text{ kN}\cdot\text{m}$$

$$+\circlearrowleft M_B = 0: -4.8 R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36 \text{ kN}$$



$$A_1 = \frac{1}{2} x (36x) = 18x^2$$

$$A_2 = \frac{1}{3} x (-20x^2) = -\frac{20}{3} x^3$$

Place reference tangent at A.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

$$\theta_A = -\frac{t_{B/A}}{L}$$

$$(A_1)_B = (18)(4.8)^2 = 414.72 \text{ kN}\cdot\text{m}^2$$

$$(A_2)_B = \left(\frac{20}{3}\right)(4.8)^3 = -737.28 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{1}{EIL} \left\{ (A_1)_B \left(\frac{1}{3}\right)(4.8) + (A_2)_B \left(\frac{1}{4}\right)(4.8) \right\}$$

$$= -\frac{-221.184}{(92400)(4.8)} = 0.49870 \times 10^{-3}$$

Locate point K of maximum deflection. $\theta_K = \theta_A + \theta_{K/A} = 0$

$$EI\theta_A + A_1 + A_2 = 0$$

$$f = 46.08 + 18x_K^2 - \frac{20}{3}x_K^3 = 0 \quad \frac{df}{dx} = 36x_K - 20x_K^2$$

Solve by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx}$$

x_K	3	3.39	3.327	3.3251	3.32514 ←
df/dx	-72	-107.8	-101.6	-101.42	
f	28.08	-6.78	-0.188	0.005	

Place reference tangent at K. $y_A = y_K + t_{A/K}$

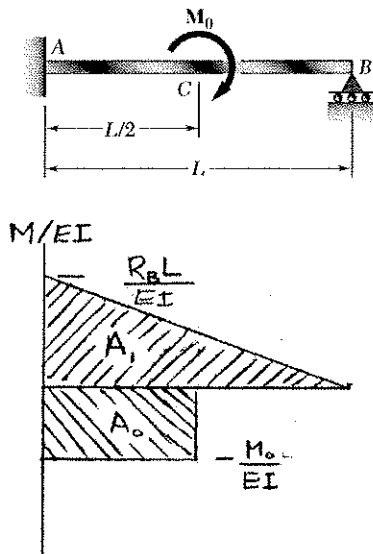
$$y_A - y_K = -t_{A/K} = -\frac{1}{EI} \left\{ (A_1) \left(\frac{2}{3}x_K\right) + A_2 \left(\frac{3}{4}x_K\right) \right\} = -\frac{1}{EI} \left\{ 12x_K^3 - 5x_K^4 \right\}$$

$$= -\frac{170.064}{92400} = -1.841 \times 10^{-3} \text{ m}$$

$$y_K = 1.841 \text{ mm}$$

Problem 9.146

9.146 through 1.149 For the beam and loading shown, determine the reaction at the roller support.



Remove support B and treat R_B as redundant.

Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2} L \frac{R_B L}{EI} = \frac{R_B L^2}{2EI}$$

$$A_2 = \frac{L}{2} \cdot \frac{M_0 L}{EI} = \frac{M_0 L^2}{2EI}$$

Place reference tangent at A.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

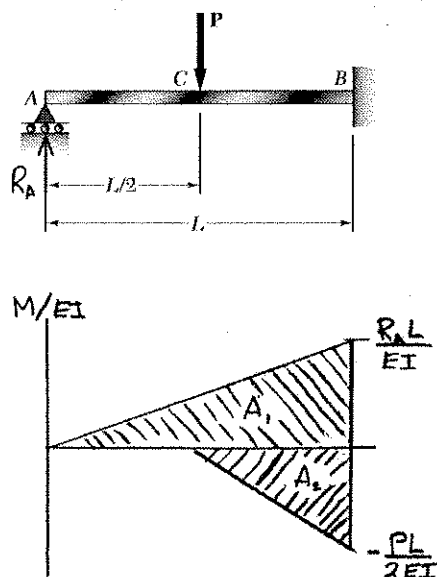
$$t_{B/A} = 0$$

$$A_1 \left(\frac{2L}{3} \right) + A_2 \left(\frac{L}{2} + \frac{L}{4} \right) = 0$$

$$\frac{R_B L^3}{3EI} - \frac{3M_0 L^2}{8EI} = 0 \quad R_B = \frac{9}{8} \frac{M_0}{L} \uparrow \blacktriangleleft$$

Problem 9.147

9.146 through 1.149 For the beam and loading shown, determine the reaction at the roller support.



Remove support A and treat R_A as redundant.

Draw the $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = -\frac{1}{2} \frac{L}{2} \frac{PL}{2} = -\frac{PL^2}{8EI}$$

Place reference tangent at B.

$$y_A = y_B - \theta_B L + t_{A/B} = 0$$

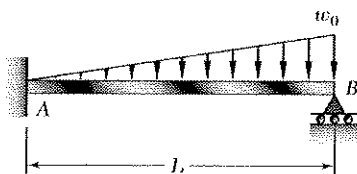
$$t_{A/B} = 0$$

$$A_1 \left(\frac{2L}{3} \right) + A_2 \left(\frac{L}{2} + \frac{L}{3} \right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{5PL^3}{48EI} = 0 \quad R_A = \frac{5}{16} P \uparrow \blacktriangleleft$$

Problem 9.148

9.146 through 1.149 For the beam and loading shown, determine the reaction at the roller support.



Remove support B and treat R_B as redundant.

Replace loading by equivalent shown at left.

Draw M/EI diagram for load w_0 and R_B .

Use parts as shown.

$$A_1 = \frac{1}{2} \left(\frac{R_B L}{EI} \right) (L) = \frac{1}{2} \frac{R_B L^2}{EI}$$

$$M_2 = -\frac{1}{2} w_0 L^2$$

$$A_2 = \frac{1}{3} \left(-\frac{1}{2} \frac{w_0 L^2}{EI} \right) L = -\frac{1}{6} \frac{w_0 L^3}{EI}$$

$$M_3 = \frac{1}{6} \frac{w_0}{L} L^3 = \frac{1}{6} w_0 L^2$$

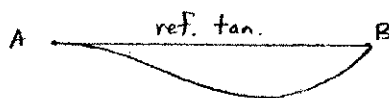
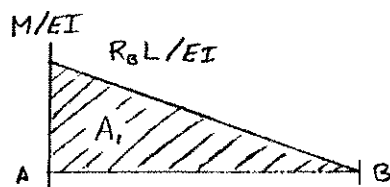
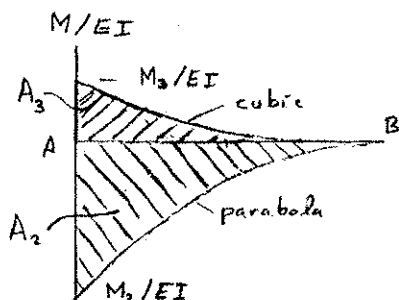
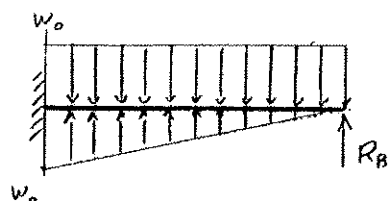
$$A_3 = \frac{1}{4} \left(\frac{1}{6} \frac{w_0 L^2}{EI} \right) L = \frac{1}{24} \frac{w_0 L^3}{EI}$$

Place reference tangent at A.

$$\Delta_{B/A} = A_1 \left(\frac{2}{3} L \right) + A_2 \left(\frac{3}{4} L \right) + A_3 \left(\frac{4}{5} L \right)$$

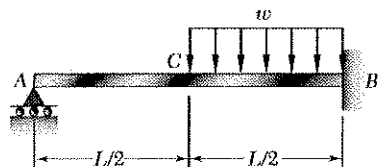
$$= \frac{1}{3} \frac{R_B L^3}{EI} - \frac{1}{8} \frac{w_0 L^4}{EI} + \frac{1}{30} \frac{w_0 L^4}{EI} = 0$$

$$R_B = \frac{11}{40} w_0 L \uparrow \quad R_B = 0.275 w_0 L \uparrow$$



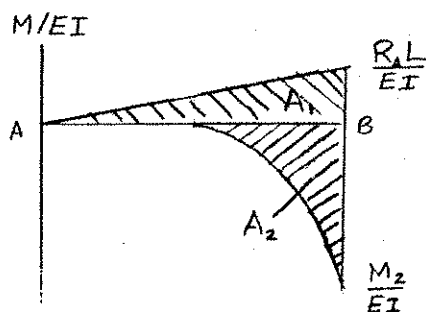
Problem 9.149

9.146 through 1.149 For the beam and loading shown, determine the reaction at the roller support.



Remove support A and treat R_A as redundant.

Draw M/EI diagram for loads R_A and w .



$$M_2 = -\frac{1}{2} w \left(\frac{L}{2}\right)^2 = -\frac{1}{8} w L^2$$

$$A_1 = \frac{1}{2} \left(\frac{R_A L}{EI}\right) L = \frac{1}{2} \frac{R_A L^2}{EI}$$

$$A_2 = \frac{1}{3} \left(-\frac{1}{8} \frac{w L^2}{EI}\right) \left(\frac{L}{2}\right) = -\frac{1}{48} \frac{w L^3}{EI}$$

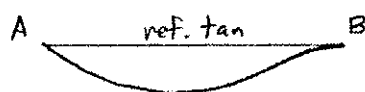
Place reference tangent at B.

$$t_{A/B} = A_1 \left(\frac{2}{3} L\right) + A_2 \left(\frac{L}{2} + \frac{3}{4} \frac{L}{2}\right)$$

$$= \frac{1}{3} \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} = 0$$

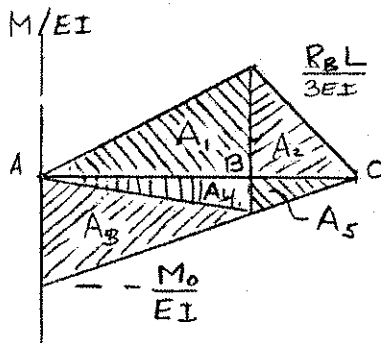
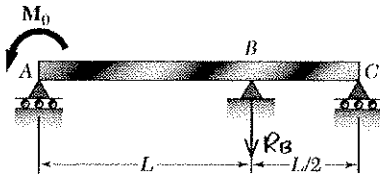
$$R_A = \frac{7}{128} w L$$

$$R_A = \frac{7}{128} w L \uparrow$$



Problem 9.150

9.150 and 9.151 For the beam and loading shown, determine the reaction at each support.



Choose R_B as the redundant reaction.

Draw $\frac{M}{EI}$ diagram for the loads R_B and M_0 .

$$A_1 = \frac{1}{2}(L)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{6EI}$$

$$A_2 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{12EI}$$

$$A_3 = \frac{1}{2}(L)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{2EI}$$

$$A_4 = \frac{1}{2}(L)\left(\frac{1}{3}\right)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{6EI}$$

$$A_3 + A_4 + A_5 = \frac{1}{2}\left(\frac{3L}{2}\right)\left(-\frac{M_0}{EI}\right) = -\frac{3M_0 L}{4EI}$$

$$y_B = y_A + L\theta_A + t_{B/A} \quad \theta_A = -t_{B/A}/L$$

$$y_C = y_A + \frac{3L}{2}\theta_A + t_{C/A} = 0 \quad -\frac{3}{2}t_{B/A} + t_{C/A} = 0$$

$$t_{B/A} = (A_1)\left(\frac{L}{3}\right) + A_3\left(\frac{2L}{3}\right) + A_4\left(\frac{L}{3}\right) = \frac{R_B L^3}{18EI} - \frac{7M_0 L^2}{18EI}$$

$$t_{C/A} = (A_1)\left(\frac{L}{2} + \frac{L}{3}\right) + A_2\left(\frac{L}{3}\right) + (A_3 + A_4 + A_5)(L) = \frac{R_B L^3}{6EI} - \frac{3M_0 L^2}{4EI}$$

$$-\frac{3}{2}t_{B/A} + t_{C/A} = \frac{R_B L^3}{12EI} - \frac{M_0 L^2}{6EI} = 0$$

$$R_B = \frac{2M_0}{L} \downarrow$$

$$+\circlearrowleft \sum M_C = 0: M_0 + \frac{1}{2}R_B - \frac{3L}{2}R_A = 0$$

$$R_A = \frac{2}{3L}[M_0 + M_0]$$

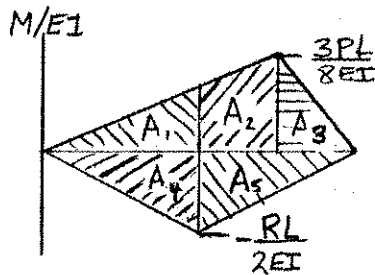
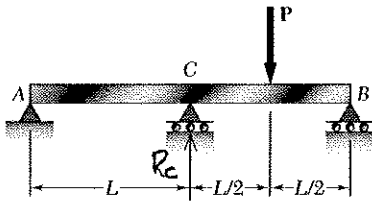
$$R_A = \frac{4M_0}{3L} \uparrow$$

$$+\uparrow \sum F_y = 0: R_A + R_B + R_C = 0 \quad \frac{4M_0}{3L} - \frac{2M_0}{L} + R_C = 0$$

$$R_C = \frac{2M_0}{3L} \uparrow$$

Problem 9.151

9.150 and 9.151 For the beam and loading shown, determine the reaction at each support.



Remove support C and add reaction R_C .

Draw $\frac{M}{EI}$ due to each of the loads (P and R_C).

$$A_1 = \frac{1}{2} \cdot L \cdot \frac{2}{3} \cdot \frac{3PL}{8EI} = \frac{PL^2}{8EI}$$

$$A_1 + A_2 = \frac{1}{2} \cdot \frac{3L}{2} \cdot \frac{3PL}{8EI} = \frac{9PL^2}{32EI}$$

$$A_3 = \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{3PL}{8EI} = \frac{3PL^2}{32EI}$$

$$A_4 = \frac{1}{2} \cdot L \cdot \left(-\frac{R_C L}{2EI}\right) = -\frac{R_C L^2}{4EI}$$

$$A_4 + A_5 = \frac{1}{2} (2L) \cdot \left(-\frac{R_C L}{2EI}\right) = -\frac{R_C L^2}{2EI}$$

Place reference tangent at A. $y_A = 0$

$$y_C = L\theta_A + t_{C/A} = 0 \quad \theta_A = -\frac{t_{C/A}}{L}$$

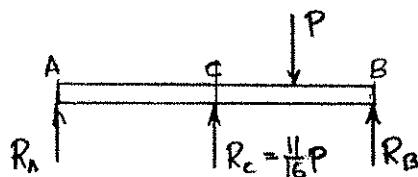
$$y_B = 2L\theta_A + t_{B/A} = 0 \quad -2t_{C/A} + t_{B/A} = 0$$

$$-2 \left[A_1 \frac{L}{3} + A_4 \cdot \frac{L}{3} \right] + [(A_1 + A_2) \left(\frac{L}{2} + \frac{2}{3} \cdot \frac{3L}{2} \right) + A_3 \cdot \frac{2}{3} \cdot \frac{L}{2} + (A_4 + A_5) \cdot L] = 0$$

$$-2 \left[\frac{PL^3}{24EI} - \frac{R_C L^3}{12EI} \right] + \left[\frac{9PL^3}{32EI} + \frac{PL^3}{32EI} - \frac{R_C L^3}{2EI} \right] = 0$$

$$-\frac{1R_C L^3}{3EI} + \frac{11PL^3}{48EI} = 0$$

$$R_C = \frac{11}{16} P \uparrow$$



$$+\circlearrowleft \Sigma M_B = 0:$$

$$-2LR_A - LR_C + \frac{1}{2}P = 0$$

$$R_A = \frac{P}{4} - \frac{1}{2}R_C = -\frac{3}{32}P$$

$$R_A = \frac{3}{32} P \downarrow$$

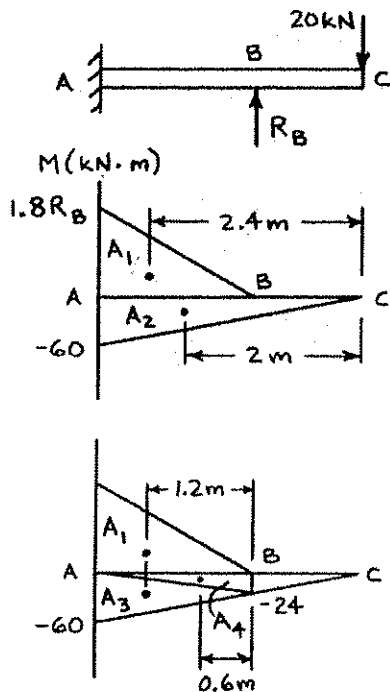
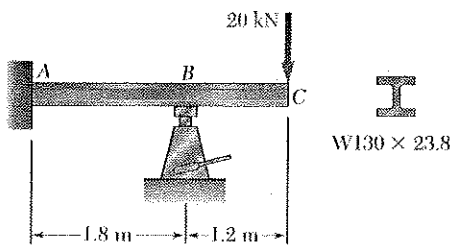
$$+\circlearrowleft \Sigma M_A = 0:$$

$$2LR_B + LR_C - \frac{3L}{2}P = 0$$

$$R_B = \frac{3P}{4} - \frac{1}{2}R_C = \frac{13}{32}P$$

$$R_B = \frac{13}{32} P \uparrow$$

Problem 9.152



9.152 A hydraulic jack can be used to raise point B of the cantilever beam ABC . The beam was originally straight, horizontal, and unloaded. A 20-kN load was then applied at point C , causing this point to move down. Determine (a) how much point B should be raised to return point C to its original position, (b) the final value of the reaction at B . Use $E = 200$ GPa.

FOR $W130 \times 23.8$ $I_x = 8.80 \times 10^6 \text{ mm}^4$

$EI = (200 \times 10^6 \text{ kPa})(8.80 \times 10^{-6} \text{ m}^4) = 1760 \text{ kN}\cdot\text{m}^2$

LET R_B BE THE JACK FORCE IN kN.

$A_1 = \frac{1}{2}(1.8R_B)(1.8) = 1.62R_B$

$A_2 = \frac{1}{2}(-60)(3) = -90 \text{ kN}\cdot\text{m}^2$

$EI t_{C/A} = (2.4)A_1 + (2)A_2$

$0 = (2.4)(1.62R_B) + (2)(-90)$

$R_B = 46.296 \text{ kN}$

$A_1 = 75 \text{ kN}\cdot\text{m}^2$

$A_3 = \frac{1}{2}(-60)(1.8) = -54 \text{ kN}\cdot\text{m}^2$

$A_4 = \frac{1}{2}(-24)(1.8) = -21.6 \text{ kN}\cdot\text{m}^2$

$EI t_{B/A} = (1.2)A_1 + (1.2)A_3 + (0.6)A_4$

$= (1.2)(75) + (1.2)(-54) + (0.6)(-21.6)$

$= 12.24 \text{ kN}\cdot\text{m}^2$

(a) $y_B = t_{B/A} = \frac{EI t_{B/A}}{EI} = \frac{12.24}{1760} = 6.9545 \times 10^{-3} \text{ m}$

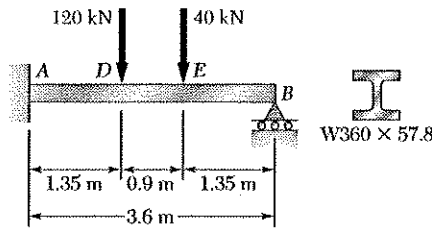
$y_B = 6.95 \text{ mm} \uparrow$

(b)

$R_B = 46.3 \text{ kN} \uparrow$

Problem 9.153

9.152 and 9.153 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Units. Forces in kN. Lengths in m.

Let R_B be the redundant reaction.
Remove support B and add load R_B .

Draw bending moment diagram by parts.

$$M_1 = 3.6 R_B \text{ kNm}$$

$$M_2 = -(1.35 + 0.9)(40) = -90 \text{ kNm}$$

$$M_3 = -(1.35)(120) = -162 \text{ kNm}$$

$$A_1 = \frac{1}{2}(3.6)(3.6 R_B) = R_B 6.48 \text{ kNm}^2$$

$$A_2 = \frac{1}{2}(2.25)(-90) = -101.25 \text{ kNm}^2$$

$$A_3 = \frac{1}{2}(1.35)(-162) = -109.35 \text{ kNm}^2$$

$$y_B = y_A + 3.6 \theta_A + t_{B/A} = 0$$

$$t_{B/A} = 0$$

$$t_{B/A} = \frac{1}{EI} \left\{ (6.48 R_B)(2.4) + (-101.25)(2.25) + (-109.35)(3.15) \right\} = 0$$

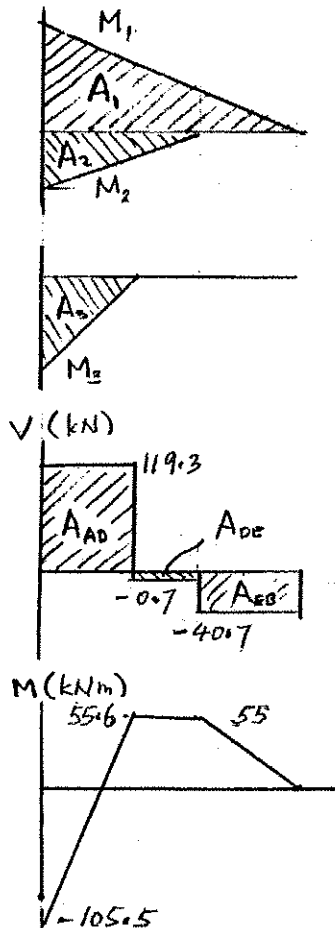
$$15.552 R_B - 633.015 = 0 \quad R_B = 40.7 \text{ kN} \uparrow$$

Draw shear diagram working from right to left.

$$B \text{ to } E \quad V = -R_B = -40.7 \text{ kN}$$

$$E \text{ to } D \quad V = -40.7 + 40 = -0.7 \text{ kN}$$

$$D \text{ to } A \quad V = -0.7 + 120 = 119.3 \text{ kN}$$



Areas of shear diagram.

$$A_{AD} = (1.35)(119.3) = 161.055 \text{ kNm}$$

$$A_{DE} = (0.9)(-0.7) = -0.63 \text{ kNm}$$

$$A_{EB} = (1.35)(-40.7) = -54.945 \text{ kNm}$$

Bending moments.

$$M_A = M_1 + M_2 + M_3 = -105.5 \text{ kNm}$$

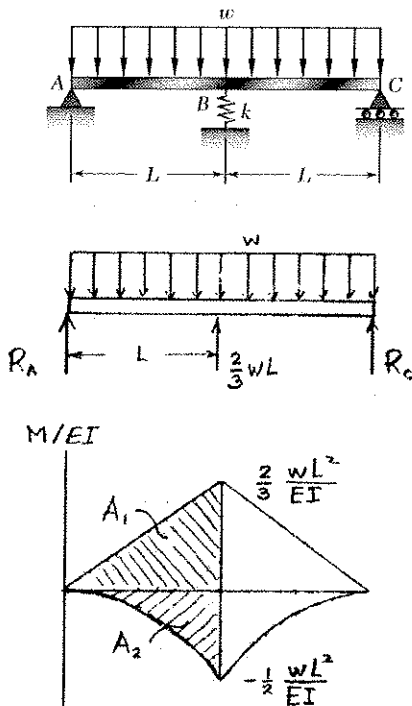
$$M_D = M_A + A_{AD} = 55.6 \text{ kNm}$$

$$M_E = M_D + A_{DE} = 55 \text{ kNm}$$

$$M_B = M_E + A_{EB} = 0$$

Problem 9.154

9.154 For the beam and loading shown, determine the spring constant k for which the force in the spring is equal to one-third of the total load on the beam.



Symmetric beam and loading. $R_C = R_A$

$$\text{Spring force: } F = \frac{1}{3}(2wL) = \frac{2}{3}wL$$

$$+\uparrow \Sigma F_y = 0: R_A + F - 2wL + R_C = 0$$

$$R_A = R_C = \frac{2}{3}wL$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{2}{3} \frac{wL^2}{EI} \right) L = \frac{1}{3} \frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3} \left(\frac{1}{2} \frac{wL^2}{EI} \right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$

$$y_B = -t_{A/B}$$

$$= - \left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L \right)$$

$$= -\frac{7}{72} \frac{wL^4}{EI}$$

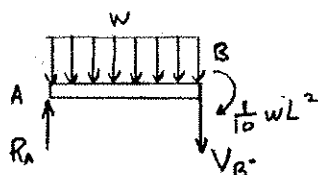
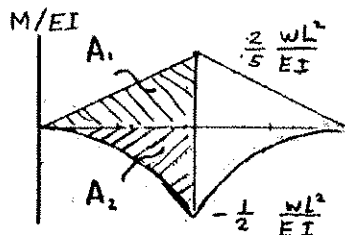
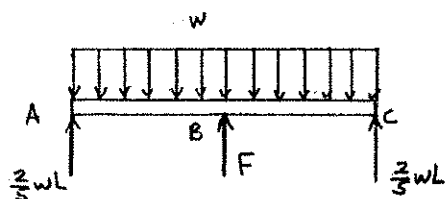
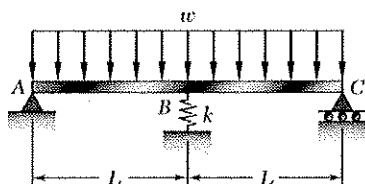
$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{2}{3}wL}{\frac{7}{72} \frac{wL^4}{EI}}$$

$$k = \frac{48}{7} \frac{EI}{L^3} \quad \blacktriangleleft$$

Problem 9.155

9.155 For the beam and loading shown, determine the spring constant k for which the bending moment at B is $M_B = -wL^2/10$.



Using free body AB,

$$+\circlearrowleft M_B = 0:$$

$$-R_A L + (wL)(\frac{L}{2}) - \frac{1}{10} wL^2 = 0$$

$$R_A = \frac{3}{5} wL \uparrow$$

Symmetric beam and loading. $R_C = R_A$

Using free body ABC, $+\uparrow \Sigma F_y = 0:$

$$\frac{2}{5} wL + F + \frac{2}{5} wL - 2wL = 0$$

$$F = \frac{6}{5} wL$$

Draw $\frac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{2}{5} \frac{wL^2}{EI} \right) L = \frac{1}{5} \frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3} \left(\frac{1}{2} \frac{wL^2}{EI} \right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$

$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3} L + A_2 \cdot \frac{3}{4} L \right)$$

$$= -\frac{1}{120} \frac{wL^4}{EI}$$

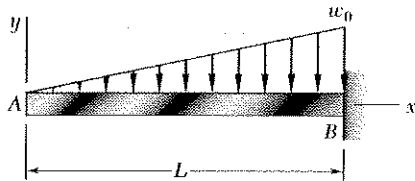
$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{6}{5} wL}{\frac{1}{120} \frac{wL^4}{EI}}$$

$$k = 144 \frac{EI}{L^3} \quad \blacktriangleleft$$

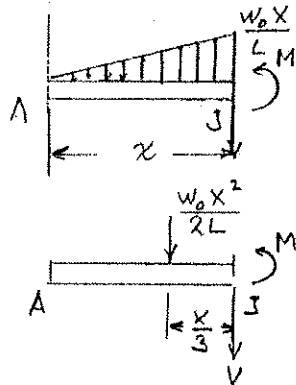
Problem 9.156

9.156 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.



$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



Use free body AJ.

$$+\circlearrowleft \sum M_J = 0: M + \frac{w_0 x^2}{2L} \cdot \frac{x}{3} = 0$$

$$M = -\frac{1}{6} \frac{w_0 x^3}{L}$$

$$\frac{d^2 y}{dx^2} = -\frac{1}{6} \frac{w_0 x^3}{L}$$

$$EI \frac{dy}{dx} = -\frac{1}{24} \frac{w_0 x^4}{L} + C_1$$

$$EI y = -\frac{1}{120} \frac{w_0 x^5}{L} + C_1 x + C_2$$

$$[x=L, \frac{dy}{dx}=0]: -\frac{1}{24} w_0 L^3 + C_1$$

$$C_1 = \frac{1}{24} w_0 L^3$$

$$[x=L, y=0]$$

$$EI y = -\frac{1}{120} w_0 L^4 + \frac{1}{24} w_0 L^4 + C_2 = 0$$

$$C_2 = \frac{1}{30} w_0 L^4$$

(a) Elastic curve.

$$y = -\frac{w_0}{120 EIL} (x^5 - 5L^4 x + 4L^5)$$

$$\frac{dy}{dx} = \frac{w_0}{24 EIL} (-x^4 + L^4)$$

(b) y @ x=0:

$$y_A = -\frac{w_0 L^4}{30 EI}$$

$$y_A = \frac{w_0 L^4}{30 EI} \downarrow$$

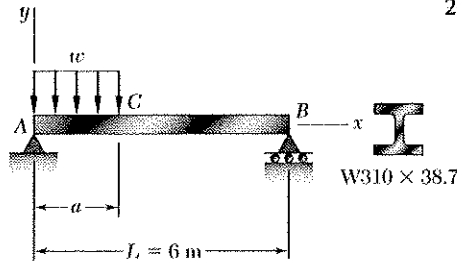
(c) \frac{dy}{dx} @ x=0:

$$\frac{dy}{dx} \Big|_A = \frac{w_0 L^3}{24 EI}$$

$$\theta_A = \frac{w_0 L^3}{24 EI} \nearrow$$

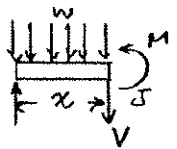
Problem 9.157

9.157 For the beam and loading shown, knowing that $a = 2 \text{ m}$, $w = 50 \text{ kN/m}$, and $E = 200 \text{ GPa}$, determine (a) the slope at support A, (b) the deflection at point C.



$$\begin{aligned} [x=0, y=0] & \quad [x=L, y=0] \\ [x=a, y=y] & \\ [x=a, \frac{dy}{dx} = \frac{dy}{dx}] & \end{aligned}$$

$$0 \leq x \leq a$$



$$+\circlearrowleft \sum M_J = 0:$$

$$M - R_A x + (wx)(\frac{x}{2}) = 0$$

$$M = R_A x - \frac{1}{2} wx^2$$

$$EI \frac{d^2 y}{dx^2} = R_A x - \frac{1}{2} wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} wx^3 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} wx^4 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2$$

$$C_2 = 0$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} wx^4 + C_1 x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} wx^3 + C_1$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2} R_A a^2 - \frac{1}{6} wa^3 + C_1 = -\frac{1}{2} R_B (2a)^2 + C_3$$

$$C_3 = C_1 + \frac{1}{2} R_A a^2 - \frac{1}{6} wa^3 + \frac{1}{2} R_B (2a)^2 = C_1 + \frac{7}{12} wa^3$$

$$[x=a, y=y] \quad \frac{1}{6} R_A a^3 - \frac{1}{24} wa^4 + C_1 a = \frac{1}{6} R_B (2a)^3 - (C_1 + \frac{7}{12} wa^3)(2a)$$

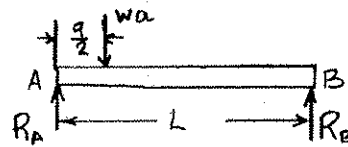
$$3C_1 a = -\frac{1}{6} R_A a^3 + \frac{1}{24} wa^4 + \frac{1}{6} R_B (2a)^3 - \frac{7}{12} wa^2 (2a) = -\frac{25}{24} wa^4$$

$$C_1 = -\frac{25}{72} wa^3$$

$$\text{For } 0 \leq x \leq a, \quad EI y = \frac{5}{36} wax^3 - \frac{1}{24} wx^4 - \frac{25}{72} wa^3 x$$

$$EI \frac{dy}{dx} = \frac{5}{12} wax^2 - \frac{1}{6} wx^3 - \frac{25}{72} wa^3$$

Using ACB as a free body and noting that $L = 3a$,



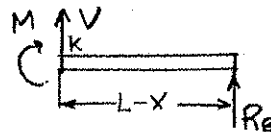
$$+\circlearrowleft \sum M_A = 0:$$

$$R_B L - (wa)(\frac{L}{2}) = 0$$

$$R_B = (wa) \frac{a}{2L} = \frac{1}{6} wa$$

$$+\uparrow \sum F_y = 0: \quad R_A + R_B - wa = 0 \quad R_A = \frac{5}{6} wa$$

$$a \leq x \leq L$$



$$+\circlearrowleft \sum M_k = 0$$

$$-M + R_B (L-x) = 0$$

$$M = R_B (L-x)$$

$$EI \frac{d^2 y}{dx^2} = R_B (L-x)$$

$$EI \frac{dy}{dx} = -\frac{1}{2} R_B (L-x)^2 + C_3$$

$$EI y = \frac{1}{6} R_B (L-x)^3 + C_3 x + C_4$$

$$[x=L, y=0] \quad 0 = 0 + C_3 L + C_4$$

$$C_4 = -C_3 L$$

$$EI y = \frac{1}{6} R_B (L-x)^3 - C_3 (L-x)$$

$$EI \frac{dy}{dx} = -\frac{1}{2} R_B (L-x)^2 + C_3$$

continued

Problem 9.157 continued

Data: $w = 50 \times 10^3 \text{ N}\cdot\text{m}$, $a = 2 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$
 $I = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4$, $EI = 17.02 \times 10^6 \text{ N}\cdot\text{m}^2$

(a) Slope at $x = 0$.

$$17.02 \times 10^6 \left. \frac{dy}{dx} \right|_A = 0 - 0 - \frac{25}{72} (50 \times 10^3) (2)^3$$

$$\left. \frac{dy}{dx} \right|_A = \theta_A = -8.16 \times 10^{-3}$$

$$\theta_A = 8.16 \times 10^{-3} \text{ rad} \quad \swarrow$$

(b) Deflection at $x = 2 \text{ m}$.

$$EI y_c = \frac{5}{36} wa^4 - \frac{1}{24} wa^4 - \frac{25}{72} wa^4 = -\frac{1}{4} wa^4$$

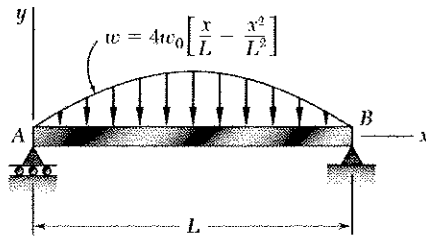
$$17.02 \times 10^6 y_c = -\frac{1}{4} (50 \times 10^3) (2)^4$$

$$y_c = -11.75 \times 10^{-3} \text{ m}$$

$$y_c = 11.75 \text{ mm} \quad \downarrow$$

Problem 9.158

9.158 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection at the midpoint of the span.



$$\begin{aligned} [x=0, M=0] & \quad [x=L, M=0] \\ [x=0, y=0] & \quad [x=L, y=0] \end{aligned}$$

Boundary conditions at A and B are noted.

$$w = \frac{w_0}{L^2} (4Lx - 4x^2)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} (4x^2 - 4Lx)$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2} \left(\frac{4}{3}x^3 - 2Lx^2 \right) + C_1$$

$$M = \frac{w_0}{L^2} \left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 \right) + C_1x + C_2$$

$$[x=0, M=0] \quad 0 = 0 + 0 + 0 + C_2$$

$$C_2 = 0$$

$$[x=L, M=0] \quad 0 = \frac{w_0}{L^2} \left(\frac{1}{3}L^4 - \frac{2}{3}L^4 \right) + C_1L + 0$$

$$C_1 = \frac{1}{3}w_0L^2$$

$$EI \frac{d^2y}{dx^2} = M = \frac{w_0}{L^2} \left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 + \frac{1}{3}L^3x \right)$$

$$EI \frac{dy}{dx} = \frac{w_0}{L^2} \left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2 \right) + C_3$$

$$EI y = \frac{w_0}{L^2} \left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3 \right) + C_3x + C_4$$

$$[x=0, y=0] \quad 0 = 0 + 0 + 0 + 0 + C_4$$

$$C_4 = 0$$

$$[x=L, y=0] \quad 0 = \frac{w_0}{L^2} \left(\frac{1}{90}L^6 - \frac{1}{30}L^6 + \frac{1}{18}L^6 \right) + C_3L + 0$$

$$C_3 = -\frac{1}{30}w_0L^3$$

(a) Elastic curve. $y = \frac{w_0}{EIL^2} \left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3 - \frac{1}{30}L^5x \right)$

$$\frac{dy}{dx} = \frac{w_0}{EIL^2} \left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2 - \frac{1}{30}L^5 \right)$$

(b) Slope at end A. Set $x=0$ in $\frac{dy}{dx}$. $\frac{dy}{dx} \Big|_A = -\frac{1}{30} \frac{w_0L^3}{EI}$

$$\theta_A = \frac{1}{30} \frac{w_0L^3}{EI}$$

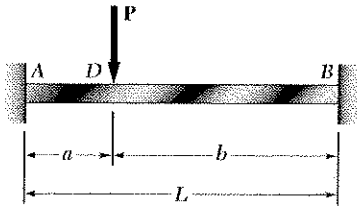
(c) Deflection at midpoint. Set $x = \frac{L}{2}$ in y .

$$\begin{aligned} y_c &= \frac{w_0L^4}{EI} \left\{ \left(\frac{1}{90} \right) \left(\frac{1}{2} \right)^6 - \left(\frac{1}{30} \right) \left(\frac{1}{2} \right)^5 + \frac{1}{18} \left(\frac{1}{2} \right)^3 - \frac{1}{30} \left(\frac{1}{2} \right) \right\} \\ &= \frac{w_0L^4}{EI} \left\{ \frac{1}{5760} - \frac{1}{960} + \frac{1}{144} - \frac{1}{60} \right\} = -\frac{61}{5760} \frac{w_0L^4}{EI} \end{aligned}$$

$$y_c = \frac{61}{5760} \frac{w_0L^4}{EI} \downarrow$$

Problem 9.159

9.159 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

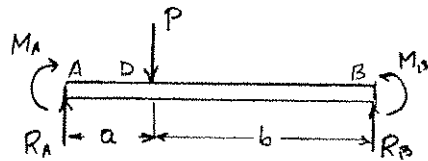


$$[x=0, y=0]$$

$$[x=0, \frac{dy}{dx}=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



Reactions are statically indeterminate.

$$0 \leq x \leq a$$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \rightarrow C_1 = 0$$

$$[x=0, y=0] \rightarrow C_2 = 0$$

$$a \leq x \leq L$$

$$EI \frac{d^2 y}{dx^2} = M_A + R_A x - P(x-a)$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P(x-a)^2 + C_3$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P(x-a)^3 + C_3 x + C_4$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad M_A a + \frac{1}{2} R_A a^2 = M_A a + \frac{1}{2} R_A a^2 - 0 + C_3 \quad C_3 = 0$$

$$[x=a, y=y] \quad \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 = \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 - 0 + 0 + C_4 \quad C_4 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad M_A L + \frac{1}{2} R_A L^2 - \frac{1}{2} P b^2 = 0 \quad (1)$$

$$[x=L, y=0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P b^3 = 0 \quad (2)$$

Solving (1) and (2) simultaneously, $M_A = -\frac{P b^2 (L-b)}{L^2} = -\frac{P a b^2}{L^2}$

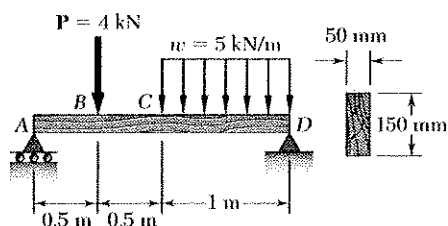
$$R_A = \frac{P b^2}{L^2} - \frac{2 M_A}{L} = \frac{P b^2}{L^2} (1 + 2 \frac{a}{L}) = \frac{P b^2 (3a+b)}{L^3}$$

$$M_B = M_A + R_A L - P b = -\frac{P a b^2}{L^2} + \frac{P b^2 (3a+b)(a+b)}{L^3} - P b = -\frac{P b a^2}{L^2}$$

$$M_D = M_A + R_A a = -\frac{P a b^2}{L^2} + \frac{P b^2 (3a+b) a}{L^3} = \frac{2 P a^2 b^2}{L^3}$$

Problem 9.160

9.160 For the timber beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use $E = 12 \text{ GPa}$.



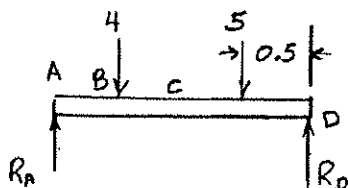
Units: Forces in kN, lengths in meters.

$$I = \frac{1}{12}(50)(150)^3 = 14.0625 \times 10^6 \text{ mm}^4 = 14.0625 \times 10^{-6} \text{ m}^4$$

$$EI = (12 \times 10^9)(14.0625 \times 10^{-6}) = 168.75 \times 10^3 \text{ N} \cdot \text{m}^2 = 168.75 \text{ kN} \cdot \text{m}^2$$

$$+\circlearrowleft M_D = 0: -2R_A + (1.5)(4) + (0.5)(5) = 0$$

$$R_A = 4.25 \text{ kN}$$



$$w(x) = 5\langle x-1 \rangle^0 \quad \text{kN} \cdot \text{m}$$

$$\frac{dV}{dx} = -w = -5\langle x-1 \rangle^0 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = -5\langle x-1 \rangle^1 + 4.25 - 4\langle x-0.5 \rangle^0 \quad \text{kN}$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{5}{2}\langle x-1 \rangle^2 + 4.25x - 4\langle x-0.5 \rangle^1 \quad \text{kN} \cdot \text{m}$$

$$EI \frac{dy}{dx} = -\frac{5}{6}\langle x-1 \rangle^3 + 2.125x^2 - 2\langle x-0.5 \rangle^2 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EI y = -\frac{5}{24}\langle x-1 \rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5 \rangle^3 + C_1x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x=0, y=0] \quad -0 + 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=2\text{m}, y=0] \quad -\left(\frac{5}{24}\right)(1)^4 + \left(\frac{2.125}{3}\right)(2)^3 - \left(\frac{2}{3}\right)(1.5)^3 + 2C_1 = 0$$

$$C_1 = -1.60417 \text{ kN} \cdot \text{m}^2$$

(a) Slope at end A. $\left(\frac{dy}{dx}\right)$ at $x=0$

$$EI \left(\frac{dy}{dx}\right)_A = -0 + 0 - 0 + C_1$$

$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-1.60417}{168.75} = -9.51 \times 10^{-3}$$

$$\theta_A = 9.51 \times 10^{-3} \text{ rad} \quad \swarrow$$

(b) Deflection at midpoint C. (y) at $x=1\text{m}$

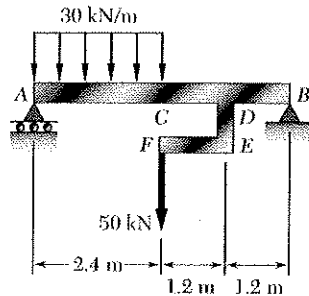
$$EI y_C = -0 + \left(\frac{2.125}{3}\right)(1)^3 - \left(\frac{2}{3}\right)(0.5)^3 + (-1.60417)(1) = -979.17 \times 10^{-3} \text{ kN} \cdot \text{m}^3$$

$$y_C = \frac{-979.17 \times 10^{-3}}{168.75} = -5.80 \times 10^{-3} \text{ m}$$

$$y_C = 5.80 \text{ mm} \downarrow$$

Problem 9.161

9.161 The rigid bar DEF is welded at point D to the rolled-steel beam AB. For the loading shown, determine (a) the slope at point A, (b) the deflection at midpoint C of the beam. Use $E = 200 \text{ GPa}$.



Units: Forces in kN; lengths in meters.

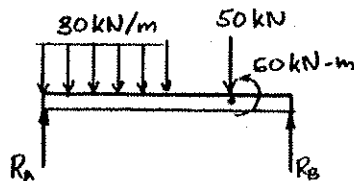
$$+\circlearrowleft \sum M_B = 0:$$

$$-4.8 R_A + (30)(2.4)(3.6) + (50)(2.4) = 0$$

$$R_A = 79 \text{ kN} \uparrow$$

$$I = 212 \times 10^6 \text{ mm}^4 = 212 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(212 \times 10^{-6}) = 42.4 \times 10^6 \text{ N} \cdot \text{m}^2 = 42400 \text{ kN} \cdot \text{m}^2$$



$$w(x) = 30 - 30\langle x - 2.4 \rangle^0$$

$$\frac{dV}{dx} = -w = -30 + 30\langle x - 2.4 \rangle^0 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = 79 - 30x + 30\langle x - 2.4 \rangle^1 - 50\langle x - 3.6 \rangle^0 \quad \text{kN}$$

$$EI \frac{d^2y}{dx^2} = M = 79x - 15x^2 + 15\langle x - 2.4 \rangle^2 - 50\langle x - 3.6 \rangle^1 - 60\langle x - 3.6 \rangle^0 \quad \text{kN} \cdot \text{m}$$

$$EI \frac{dy}{dx} = \frac{79}{2}x^2 - 5x^3 + 5\langle x - 2.4 \rangle^3 - 25\langle x - 3.6 \rangle^2 - 60\langle x - 3.6 \rangle^1 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EI y = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}\langle x - 2.4 \rangle^4 - \frac{25}{3}\langle x - 3.6 \rangle^3 - 30\langle x - 3.6 \rangle^2 + C_1x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x = 0, y = 0] \quad 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x = 4.8, y = 0] \quad \left(\frac{79}{6}\right)(4.8)^3 - \left(\frac{5}{4}\right)(4.8)^4 + \left(\frac{5}{4}\right)(2.4)^4 - \left(\frac{25}{3}\right)(1.2)^3 - (30)(1.2)^2 + 4.8C_1 = 0$$

$$C_1 = -161.76 \text{ kN} \cdot \text{m}^2$$

(a) Slope at point A. $\left(\frac{dy}{dx}\right)_A$ at $x = 0$

$$EI \left(\frac{dy}{dx}\right)_A = 0 - 0 + 0 - 0 - 0 - 161.76 = -161.76 \text{ kN} \cdot \text{m}^2$$

$$\left(\frac{dy}{dx}\right)_A = \frac{-161.76}{42400} = -3.82 \times 10^{-3}$$

$$\theta_A = 3.82 \times 10^{-3} \text{ rad} \quad \swarrow$$

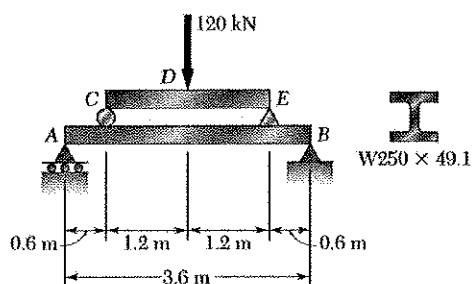
(b) Deflection at midpoint C. $(y \text{ at } x = 2.4)$

$$EI y_C = \left(\frac{79}{6}\right)(2.4)^3 - \left(\frac{5}{4}\right)(2.4)^4 + 0 - 0 - 0 - (161.76)(2.4) + 0 = -247.68 \text{ kN} \cdot \text{m}^3$$

$$y_C = \frac{-247.68}{42400} = -5.84 \times 10^{-3} \text{ m}$$

$$y_C = 5.84 \text{ mm} \downarrow \quad \blacktriangleleft$$

Problem 9.162



9.162 Beam CE rests on beam AB , as shown. Knowing that a $W250 \times 49.1$ rolled-steel shape is used for each beam, determine for the loading shown the deflection at point D . Use $E = 200$ GPa.

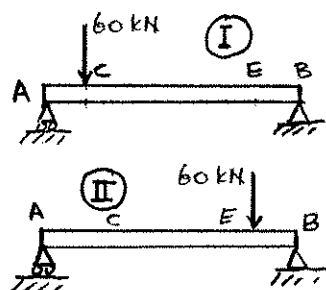
$$\text{For } W250 \times 49.1 \quad I = 70.6 \times 10^6 \text{ mm}^4$$

$$EI = (200 \times 10^9)(70.6 \times 10^{-6})$$

$$= 14120 \text{ kN mm}^2$$

Beam AB : 60 kN downward loads at C and E .

Refer to Case 5 of Appendix D.



$$\text{Loading I. } (y_c)_1 = -\frac{Pa^2b^2}{3EIL}$$

$$\text{with } a = 0.6 \text{ m, } b = 3 \text{ m, } L = 3.6 \text{ m}$$

$$(y_c)_1 = -\frac{(60)(0.6)^2(3)^2}{(3)(14120)(3.6)} = -1.275 \times 10^{-3} \text{ m}$$

$$\text{Loading II. } (y_c)_2 = \frac{Pb[x^3 - (L^2 - b^2)x]}{6EIL}$$

$$\text{with } b = 0.6 \text{ m, } x = 0.6 \text{ m, } L = 3.6 \text{ m}$$

$$(y_c)_2 = \frac{(60)(0.6)[0.6^3 - (3.6^2 - 0.6^2)(0.6)]}{(6)(14120)(3.6)} = -0.0867 \times 10^{-3} \text{ m}$$

$$y_c = (y_c)_1 + (y_c)_2 = -1.362 \times 10^{-3} \text{ m}$$

By symmetry $y_E = y_c$

Beam CDE . 120 kN downward load at D .

Refer to Case 4 of Appendix D.

$$y_{D/C} = -\frac{PL^3}{48EI}$$

$$\text{with } P = 120 \text{ kN and } L = 2.4 \text{ m}$$

$$y_{D/C} = -\frac{(120)(2.4)^3}{(48)(14120)} = -2.448 \times 10^{-3} \text{ m}$$

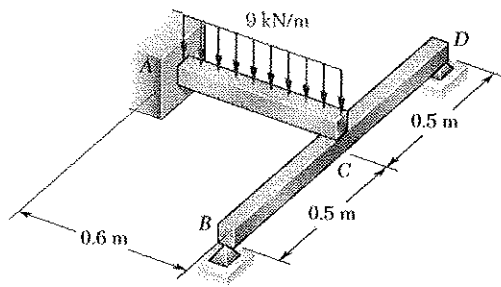
Total deflection at D .

$$y_D = y_c + y_{D/C} = -3.81 \times 10^{-3} \text{ m}$$

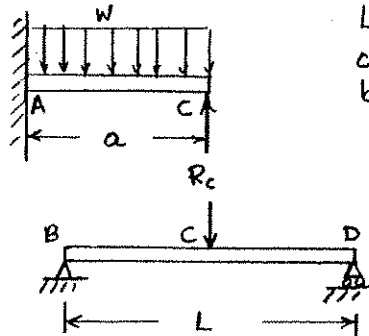
$$= 3.81 \text{ mm} \downarrow$$

Problem 9.163

9.163 For the loading shown, knowing that beams AC and BD have the same flexural rigidity, determine the reaction at B.



Consider the two beams shown below.



Let R_c be the contact force between beams AC and BCD.

Applying Cases 1 and 2 of Appendix D to cantilever beam AC,

$$y_c = \frac{R_c a^3}{3EI} - \frac{wa^4}{8EI}$$

Applying Case 4 of Appendix D to simply supported beam BCD,

$$y_c = -\frac{R_c L^3}{48EI}$$

Equating expressions for y_c ,

$$\frac{R_c a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{R_c L^3}{48EI}$$

$$(16a^3 + L^3)R_c = 6wa^4$$

$$R_c = \frac{6wa^4}{16 + L^3/a^3}$$

Data: $w = 9 \text{ kN/m}$, $a = 0.6 \text{ m}$, $L = 1 \text{ m}$

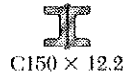
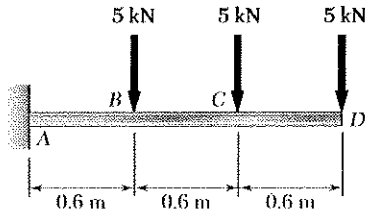
$$R_c = \frac{(6)(9)(0.6)^4}{16 + (1/0.6)^3} = 1.57 \text{ kN}$$

Using beam BCD as a free body,

$$\sum M_D = 0: -R_B L + R_c \frac{L}{2} = 0 \quad R_B = \frac{1}{2} R_c = 0.79 \text{ kN} \uparrow$$

Problem 9.164

9.164 Two C150 × 12.2 channels are welded back to back and loaded as shown. Knowing that $E = 200$ GPa, determine (a) the slope at point D, (b) the deflection at point D.



Units: Forces in kN; lengths in m

$$E = 200 \text{ GPa}$$

$$I = (2)(5.35 \times 10^{-6}) = 10.7 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^6)(10.7 \times 10^{-6}) \\ = 2140 \text{ kN m}^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{(5)(1.8)}{EI} = -\frac{9}{EI} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2} \left(\frac{9}{EI} \right) (1.8) = -\frac{8.1}{EI}$$

$$\bar{x}_1 = \frac{1}{3} (1.8) = 0.6 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(5)(1.2)}{EI} = -\frac{6}{EI} \text{ m}^{-1}$$

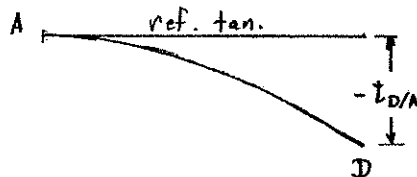
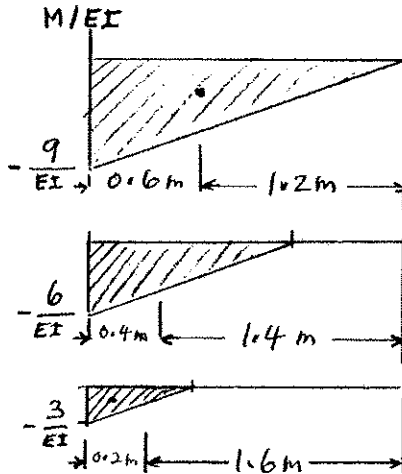
$$A_2 = \frac{1}{2} \left(-\frac{6}{EI} \right) (1.2) = -\frac{3.6}{EI}$$

$$\bar{x}_2 = \frac{1}{3} (1.2) = 0.4 \text{ m}$$

$$\frac{M_3}{EI} = -\frac{(5)(0.6)}{EI} = -\frac{3}{EI} \text{ m}^{-1}$$

$$A_3 = \frac{1}{2} \left(-\frac{3}{EI} \right) (0.6) = -\frac{0.9}{EI}$$

$$\bar{x}_3 = \frac{1}{3} (0.6) = 0.2 \text{ m}$$



Place reference tangent at A. $\theta_A = 0$

$$\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{12.6}{EI} = -\frac{12.6}{2140} = -5.89 \times 10^{-3} \text{ rad.}$$

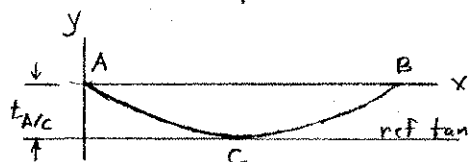
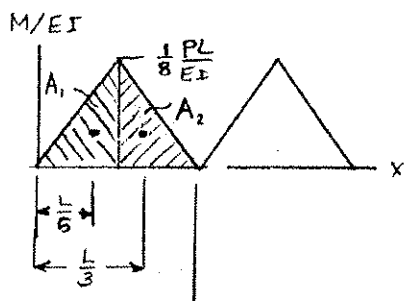
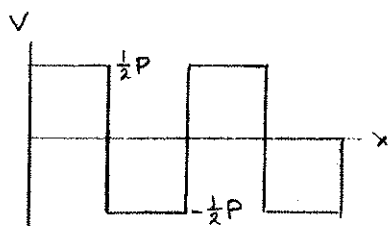
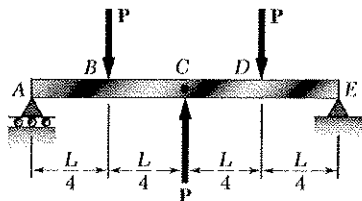
$$\theta_D = \theta_A + \theta_{D/A} = 5.89 \times 10^{-3} \text{ rad.}$$

$$t_{D/A} = \left(-\frac{8.1}{EI} \right) (1.2) + \left(-\frac{3.6}{EI} \right) (1.4) + \left(-\frac{3}{EI} \right) (1.6) = -\frac{19.56}{EI} = -9.14 \times 10^{-3} \text{ m}$$

$$y_D = t_{D/A} = 9.14 \text{ mm} \downarrow$$

Problem 9.165

9.165 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



Symmetric beam and loading.

Place reference tangent at C. $\theta_c = 0$

Reactions: $R_A = R_E = \frac{1}{2}P$

Draw V (shear) and M/EI diagrams.

$$A_1 = A_2 = \frac{1}{2} \left(\frac{1}{8} \frac{PL}{EI} \right) \frac{L}{4} = \frac{1}{64} \frac{PL^2}{EI}$$

(a) Slope at A.

$$\theta_A = \theta_c - \theta_{A/C} = 0 - A_1 - A_2$$

$$= -\frac{1}{32} \frac{PL^2}{EI} \quad \theta_A = \frac{PL^2}{32EI}$$

(b) Deflection at C.

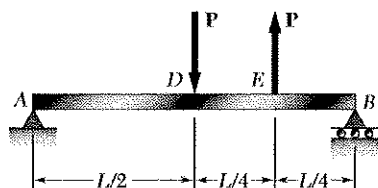
$$y_C = -t_{A/C} = -(A_1 \frac{L}{6} + A_2 \frac{L}{3})$$

$$= - \left(\frac{1}{64} \frac{PL^3}{EI} \cdot \frac{L}{6} + \frac{1}{64} \frac{PL^3}{EI} \cdot \frac{L}{3} \right)$$

$$= -\frac{1}{128} \frac{PL^3}{EI} \quad y_C = \frac{PL^3}{128EI}$$

Problem 9.166

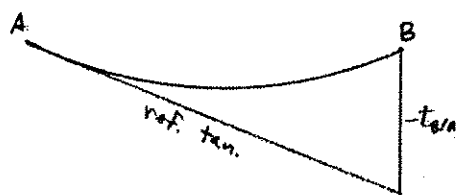
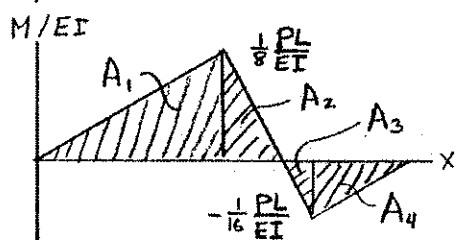
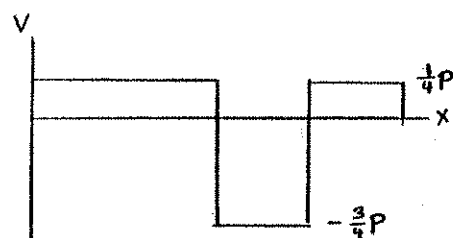
9.166 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.



$$+\circlearrowleft \sum M_B = 0: -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \quad R_A = \frac{1}{4}P \uparrow$$

$$+\circlearrowleft \sum M_A = 0: -\frac{PL}{2} + P\frac{3L}{4} + R_B L = 0 \quad R_B = \frac{1}{4}P \downarrow$$

Draw V (shear) diagram and $\frac{M}{EI}$ diagram.



$$A_1 = \frac{1}{2} \left(\frac{1}{8} \frac{PL}{EI} \right) \left(\frac{L}{2} \right) = \frac{1}{32} \frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2} \left(\frac{1}{96} \frac{PL}{EI} \right) \left(\frac{L}{2} \right) = \frac{1}{96} \frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2} \left(-\frac{1}{16} \frac{PL}{EI} \right) \left(\frac{L}{2} \right) = -\frac{1}{384} \frac{PL^2}{EI}$$

$$A_4 = \frac{1}{2} \left(-\frac{1}{128} \frac{PL}{EI} \right) \left(\frac{L}{4} \right) = -\frac{1}{128} \frac{PL^2}{EI}$$

Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= \left(\frac{1}{32} \frac{PL^2}{EI} \right) \left(\frac{2L}{3} \right) + \left(\frac{1}{96} \frac{PL^2}{EI} \right) \left(\frac{L}{2} - \frac{1}{3} \frac{L}{6} \right) \\ &\quad + \left(-\frac{1}{384} \frac{PL^2}{EI} \right) \left(\frac{L}{4} + \frac{1}{3} \frac{L}{12} \right) + \left(-\frac{1}{128} \frac{PL^2}{EI} \right) \left(\frac{2}{3} \cdot \frac{L}{4} \right) \\ &= \left(\frac{1}{48} + \frac{1}{216} - \frac{5}{6912} - \frac{1}{768} \right) \frac{PL^3}{EI} = \frac{3}{128} \frac{PL^3}{EI} \end{aligned}$$

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{128} \frac{PL^2}{EI}$$

Let point K be the location of $|y|_{\max}$.

$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{3}{128} \frac{PL^2}{EI} + A_K$$

$$= -\frac{3}{128} \frac{PL^2}{EI} + \frac{1}{2} \left(\frac{1}{4} \frac{Px_K}{EI} \right) x_K$$

$$= \frac{P}{EI} \left(-\frac{3}{128} L^2 + \frac{1}{8} x_K^2 \right) = 0$$

$$x_K = \sqrt{\frac{3}{16}} L = \frac{1}{4} \sqrt{3} L = 0.433 L$$

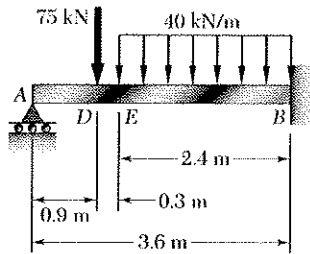
$$t_{K/A} = A_K \left(\frac{1}{3} x_K \right) = \frac{1}{2} \left(\frac{1}{4} \frac{Px_K}{EI} \right) \frac{x_K}{3} = \frac{1}{24} \frac{Px_K^3}{EI} = \frac{\sqrt{3}}{512} \frac{PL^3}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A} = \frac{\sqrt{3}}{512} \frac{PL^3}{EI} - \left(\frac{1}{4} \sqrt{3} \right) \frac{3}{128} \frac{PL^3}{EI} = -\frac{\sqrt{3}}{256} \frac{PL^3}{EI}$$

$$y_K = 0.00677 \frac{PL^3}{EI} \downarrow$$

Problem 9.167

9.167 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Units: Forces in kN. Lengths in meters.

Let R_A be the redundant reaction.

Remove support at A and add reaction R_A .

Draw bending-moment diagram by parts.

$$M_1 = 3.6 R_A \text{ kN}\cdot\text{m}$$

$$M_2 = -(75)(0.3 + 2.4) = -202.5 \text{ kN}\cdot\text{m}$$

$$M_3 = -\frac{1}{2}(40)(2.4)^2 = -115.2 \text{ kN}\cdot\text{m}$$

$$A_1 = \frac{1}{2}(3.6)(3.6 R_A) = 6.48 R_A \text{ kN}\cdot\text{m}^2$$

$$A_2 = \frac{1}{2}(2.7)(-202.5) = -273.375 \text{ kN}\cdot\text{m}^2$$

$$A_3 = \frac{1}{3}(2.4)(-115.2) = -92.16 \text{ kN}\cdot\text{m}^2$$

Place reference tangent at B, where

$$\theta_B = 0 \text{ and } y_B = 0$$

$$\text{Then, } y_A = t_{A/B} = 0$$

$$t_{A/B} = \frac{1}{EI} \left\{ \left(\frac{2}{3} \cdot 3.6 \right) A_1 + \left(0.9 + \frac{2}{3} \cdot 2.7 \right) A_2 + \left(0.9 + 0.3 + \frac{2}{3} \cdot 2.4 \right) A_3 \right\}$$

$$= \frac{1}{EI} \{ 15.552 R_A - 1014.5925 \} = 0$$

$$R_A = 65.24 \text{ kN} \uparrow$$

Draw shear diagram.

$$A \text{ to } D \quad V = R_A = 65.24 \text{ kN}$$

$$D \text{ to } E \quad V = 65.24 - 75 = -9.76 \text{ kN}$$

$$E \text{ to } B \quad V = -9.76 - 40(x - 1.2) \text{ kN}$$

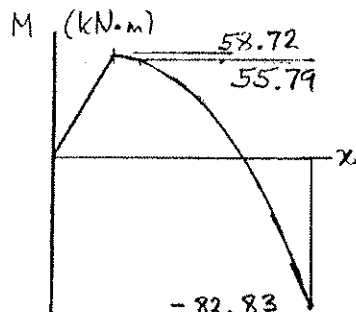
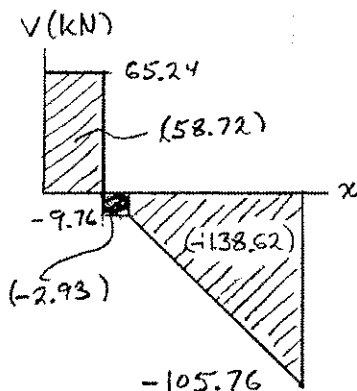
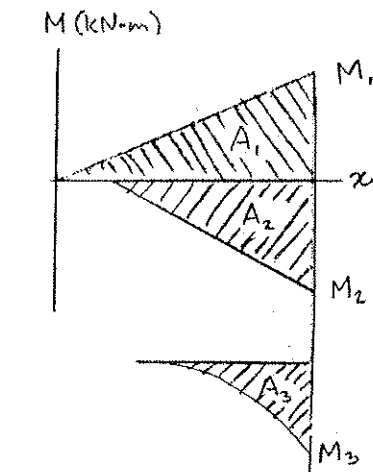
$$\text{At } B \quad V_B = -105.76 \text{ kN.}$$

Bending moment diagram. $M_A = 0$

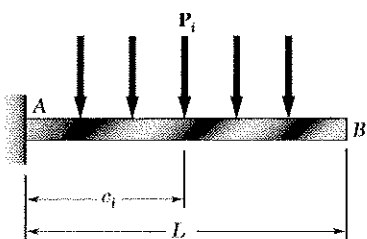
$$M_D = M_A + 58.72 = 58.72 \text{ kN}\cdot\text{m}$$

$$M_E = 58.72 - 2.93 = 55.79 \text{ kN}\cdot\text{m}$$

$$M_B = 55.79 - 138.62 = -82.83 \text{ kN}\cdot\text{m}$$



PROBLEM 9.C1



9.C1 Several concentrate loads can be applied to the cantilever beam AB . Write a computer program to calculate the slope and deflection of beam AB from $x = 0$ to $x = L$, using given increments Δx . Apply this program with increments $\Delta x = 50$ mm to the beam and loading of Probs. 9.73 and Prob. 9.74.

SOLUTION

FOR EACH LOAD, ENTER

$$P_i, c_i$$

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS

$$R_A = R_A + P_i$$

$$M_A = M_A - P_i c_i$$

COMPUTE SLOPE AND DEFLECTION

USE METHOD OF INTEGRATION

STARTING WITH $x=0$ AND UPDATING
THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A:

$$\theta = (1/EI)(R_A x^2/2.0 + M_A x)$$

$$y = (1/EI)(R_A x^3/6.0 + M_A x^2/2.0)$$

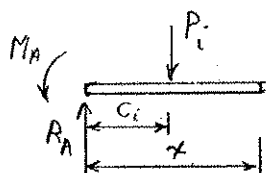
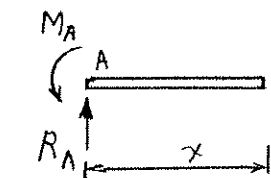
(2) DUE TO EACH LOAD WITH $c_i < x$:

$$\theta = -(1/EI)(P_i/2.0)(x - c_i)^2$$

$$y = -(1/EI)(P_i/6.0)(x - c_i)^3$$

AT $x=0$, $y = \frac{dy}{dx} = 0$

$\therefore$ THE CONSTANTS OF
INTEGRATION EQUAL ZERO



CONTINUED

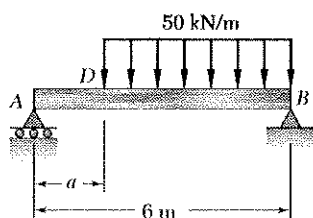
PROBLEM 9.C1 CONTINUED

PROGRAM OUTPUT

Problem 9.73 and 9.74

At A: Force = 6.0 kN Couple = -6.0 kN·m

x m	Slope radians	Deflection m
.00	.000000	.000000
.05	-.000578	-.000015
.10	-.001126	-.000057
.15	-.001645	-.000127
.20	-.002134	-.000221
.25	-.002594	-.000340
.30	-.003024	-.000480
.35	-.003424	-.000642
.40	-.003794	-.000822
.45	-.004135	-.001021
.50	-.004447	-.001235
.55	-.004728	-.001465
.60	-.004980	-.001708
.65	-.005203	-.001962
.70	-.005395	-.002227
.75	-.005558	-.002501 ◀
.80	-.005699	-.002783
.85	-.005825	-.003071
.90	-.005936	-.003365
.95	-.006033	-.003664
1.00	-.006114	-.003968
1.05	-.006181	-.004275
1.10	-.006233	-.004586
1.15	-.006270	-.004898
1.20	-.006292	-.005213
1.25	-.006299	-.005527 ◀

PROBLEM 9.C2


9.C2 The 6-m beam AB consists of a $W530 \times 92$ rolled-steel shape and supports a 50-kN/m distributed load as shown. Write a computer program and use it to calculate for values of a from 0 to 6 m, using 0.3-m increments, (a) the slope and deflection at D , (b) the location and magnitude of the maximum deflection. Use $E = 200$ GPa.

SOLUTION

ENTER LOAD w , LENGTH L , a
COMPUTE REACTION AT A

$$R_A = w(L-a)^2 / (2.0L)$$

COMPUTE SLOPE AND DEFLECTION AT D

USING SINGULARITY FUNCTIONS:

$$C_1 = -\frac{w}{24L} (L-a)^4 - \frac{1}{6} R_A L^2$$

$$\theta = (1/EI) (R_A a^2 / 2.0 + C_1)$$

$$y = (1/EI) (R_A a^3 / 6.0 + C_1 a)$$

$$EI \frac{d^2 y}{dx^2} = R_A x - \frac{w}{2} \langle x-a \rangle^2$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{w}{6} \langle x-a \rangle^3 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{w}{24} \langle x-a \rangle^4 + C_1 x + C_2$$

FROM BOUNDARY CONDITIONS:

$$C_2 = 0$$

$$C_1 = -\frac{w}{24L} (L-a)^4 - \frac{1}{6} R_A L^2$$

COMPUTE LOCATION AND MAGNITUDE OF MAXIMUM DEFLECTION

MAXIMUM y AT $\theta = 0$:

$$0 = \frac{1}{2} R_A x^2 - \frac{w}{6} \langle x-a \rangle^3 + C_1$$

IF $x_{\max} \leq a$

$$\frac{1}{2} R_A x^2 + C_1 = 0$$

$$x_{\max} = \sqrt{\frac{-2.0 C_1}{R_A}}$$

$$y_{\max} = \frac{1}{6} R_A x_{\max}^3 + C_1 x_{\max}$$

ASSUME $x < a$:

$$x_{\max} = (-2.0 C_1 / R_A)^{1/2}$$

IF $x_{\max} < a$, THEN

$$y_{\max} = (1/EI) \left(\frac{1}{6} R_A x_{\max}^3 + C_1 x_{\max} \right)$$

IF $x_{\max} > a$, THEN

BEGIN WITH $x = a$

$$\theta = (1/EI) \left(\frac{1}{2} R_A x - \frac{1}{6} \langle x-a \rangle^3 + C_1 \right)$$

INCREASE x BY SMALL AMOUNT
 UNTIL θ IS APPROXIMATELY 0

$$y_{\max} = (1/EI) \left(\frac{1}{6} R_A x^3 - \frac{w}{24} \langle x-a \rangle^4 + C_1 x \right)$$

CONTINUED

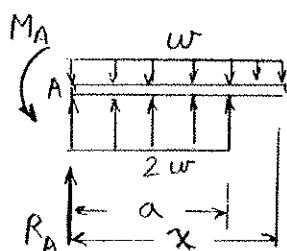
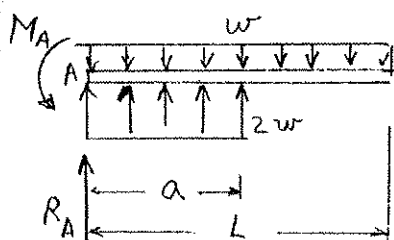
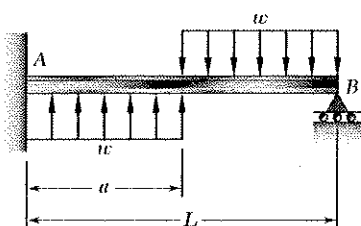
PROBLEM 9.C2 CONTINUED

PROGRAM OUTPUT

a m	theta D radians	yD mm	xm m	ym mm
0	-0.00554	0	3.300	9.588
1.5	-0.00361	5.427	3.357	8.443
3.0	-0.000764	5.350	3.523	5.520
4.5	-0.000497	2.0524	3.713	2.212
6.0	-0.000134	0.0781	3.803	0.194

Proprietary Material. © 2009 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.C3



AT $x=0$, $y = \frac{dy}{dx} = 0$

∴ THE CONSTANTS OF
INTEGRATION ARE
ZERO

9.C3 The cantilever beam AB carries the distributed loads shown. Write a computer program to calculate the slope and deflection of beam AB from $x=0$ to $x=L$ using given increments Δx . Apply this program with increments $\Delta x = 100$ mm, assuming that $L = 2.4$ m, $w = 36$ kN/m, and (a) $a = 0.6$ m, (b) $a = 1.2$ m, (c) $a = 1.8$ m. Use $E = 200$ GPa.

SOLUTION

ENTER w, a, L

COMPUTE REACTION AT A

$$R_A = wL - 2.0wa$$

$$M_A = \frac{1}{2} wL^2 - \frac{1}{2} wa^2$$

COMPUTE SLOPE AND DEFLECTION

USE EQUATION OF ELASTIC CURVE

STARTING WITH $x=0$ AND UPDATING THROUGH
INCREMENTS, SUPERPOSE:

(1) DUE TO REACTIONS AT A

$$\theta = (1/EI) \left(\frac{1}{2} R_A x^2 + M_A x \right)$$

$$y = (1/EI) \left(\frac{1}{6} R_A x^3 + \frac{1}{2} M_A x^2 \right)$$

(2) DUE TO LOAD w

$$\theta = -(1/EI) \left(\frac{1}{6} w x^3 \right)$$

$$y = -(1/EI) \left(\frac{1}{24} w x^4 \right)$$

(3) DUE TO LOAD $2w$

IF $x \leq a$

$$\theta = (1/EI) \left(\frac{1}{3} w x^3 \right)$$

$$y = (1/EI) \left(\frac{1}{12} w x^4 \right)$$

IF $x > a$

$$\theta = (1/EI) \left(\frac{1}{3} w x^3 - \frac{1}{3} w (x-a)^3 \right)$$

$$y = (1/EI) \left(\frac{1}{12} w x^4 - \frac{1}{12} w (x-a)^4 \right)$$

CONTINUED

PROBLEM 9.C3 CONTINUEDPROGRAM OUTPUTProblem 9.C3 (a) $a = 0.6 \text{ m}$

At A: Force = 43.2 kN Couple = -90.7 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000905	-.000046
.20	-.001762	-.000179
.30	-.002567	-.000396
.40	-.003318	-.000691
.50	-.004009	-.001058
.60	-.004638	-.001491
.70	-.005202	-.001983
.80	-.005703	-.002529
.90	-.006145	-.003122
1.00	-.006533	-.003756
1.10	-.006868	-.004427
1.20	-.007156	-.005128
1.30	-.007399	-.005856
1.40	-.007602	-.006607
1.50	-.007769	-.007376
1.60	-.007902	-.008160
1.70	-.008006	-.008955
1.80	-.008083	-.009760
1.90	-.008139	-.010571
2.00	-.008177	-.011387
2.10	-.008199	-.012206
2.20	-.008211	-.013027
2.30	-.008215	-.013848
2.40	-.008216	-.014669

Problem 9.C3 (b) $a = 1.2 \text{ m}$

At A: Force = 0.0 kN Couple = -51.8 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000529	-.000026
.20	-.001055	-.000106
.30	-.001574	-.000237
.40	-.002081	-.000420
.50	-.002574	-.000653
.60	-.003048	-.000934
.70	-.003500	-.001262
.80	-.003926	-.001633
.90	-.004323	-.002046
1.00	-.004687	-.002497
1.10	-.005014	-.002982
1.20	-.005301	-.003498
1.30	-.005544	-.004041
1.40	-.005747	-.004606
1.50	-.005913	-.005189
1.60	-.006047	-.005787
1.70	-.006150	-.006398
1.80	-.006228	-.007017
1.90	-.006284	-.007642
2.00	-.006321	-.008273
2.10	-.006344	-.008906
2.20	-.006356	-.009541
2.30	-.006360	-.010177
2.40	-.006361	-.010813

CONTINUED

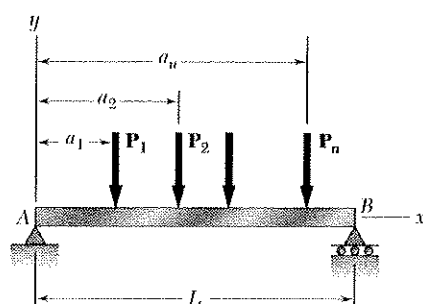
PROBLEM 9.C3 PROGRAM OUTPUTS CONTINUED

Problem 9.C3 (c) $a = 1.8$ m

At A: Force = -43.2 kN Couple = 13.0 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	.000111	.000006
.20	.000182	.000021
.30	.000215	.000041
.40	.000216	.000063
.50	.000187	.000083
.60	.000133	.000099
.70	.000056	.000109
.80	-.000039	.000110
.90	-.000149	.000101
1.00	-.000270	.000080
1.10	-.000398	.000046
1.20	-.000530	.000000
1.30	-.000662	-.000060
1.40	-.000790	-.000132
1.50	-.000911	-.000217
1.60	-.001021	-.000314
1.70	-.001116	-.000421
1.80	-.001193	-.000537
1.90	-.001248	-.000659
2.00	-.001286	-.000786
2.10	-.001309	-.000916
2.20	-.001320	-.001047
2.30	-.001325	-.001179
2.40	-.001325	-.001312

PROBLEM 9.C4



9.C4 The simple beam AB is of constant flexural rigidity EI and carries several concentrated loads as shown. Using the *Method of Integration*, write a computer program that can be used to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx . Apply this program to the beam and loading of (a) Prob. 9.13 with $\Delta x = 0.3$ m, (b) Prob. 9.16 with $\Delta x = 0.05$ m, (c) Prob. 9.129 $\Delta x = 0.25$ m.

SOLUTION

FOR EACH LOAD, ENTER P_i, a_i

COMPUTE REACTION AT A

FOR $i = 1$ TO NUMBER LOADS:

$$M_A = M_A + P_i a_i$$

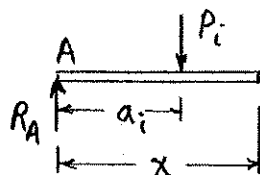
$$\text{LOAD} = \text{LOAD} + P_i$$

THEN:

$$R_B = M_A / L$$

$$R_A = \text{LOAD} - R_B$$

FOR LOAD P_i :



FOR $x < a_i$

$$EI \frac{d^2 y}{dx^2} = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

FOR $x > a_i$

$$EI \frac{d^2 y}{dx^2} = R_A x - P_i (x - a_i)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P_i (x - a_i)^2 + C_3$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{6} (x - a_i)^3 + C_3 x + C_4$$

FROM BOUNDARY CONDITIONS

$$C_2 = C_4 = 0$$

$$C_1 = C_3 = \frac{P_i}{6L} (L - a_i)^3 - \frac{1}{6} R_A L^2$$

NOTE: R_A FOR LOAD P_i

COMPUTE SLOPE AND DEFLECTION

STARTING WITH $x = 0$ AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A

$$\theta = (1/EI) \left(\frac{1}{2} R_A x^2 \right)$$

$$y = (1/EI) \left(\frac{1}{6} R_A x^3 \right)$$

(2) DUE TO LOADS - CONSTANT PART

$$\text{CONST}_1 = -\frac{1}{6} R_A L^2$$

FOR 1 TO NUMBER LOADS

$$\text{CONST}_2 = \frac{1}{6L} P_i (L - a_i)^3 + \text{CONST}_2$$

THEN, TOTAL CONTRIBUTION FOR CONSTANT

$$\text{CONST} = (1/EI) (\text{CONST}_1 + \text{CONST}_2)$$

(3) DUE TO LOADS - REMAINING PART

IF $x \leq a_i$

$$\theta = (1/EI) \left(\frac{1}{2.0} R_A x^2 \right)$$

$$y = (1/EI) \left(\frac{1}{6.0} R_A x^3 \right)$$

IF $x > a_i$

$$\theta = (1/EI) \left(\frac{1}{2.0} R_A x^2 - \frac{1}{2.0} P_i (x - a_i)^2 \right)$$

$$y = (1/EI) \left(\frac{1}{6.0} R_A x^3 - \frac{1}{6.0} P_i (x - a_i)^3 \right)$$

CONTINUED

PROBLEM 9.C4 CONTINUED

PROGRAM OUTPUT

Problem 9.13

x m	theta rad*10**3	y mm.
0	-7.6845	0
1.5	-3.0738	-9.221 ←
2.4	1.6291	-9.747
3.0	3.8423	-8.068
4.5	6.1477	0

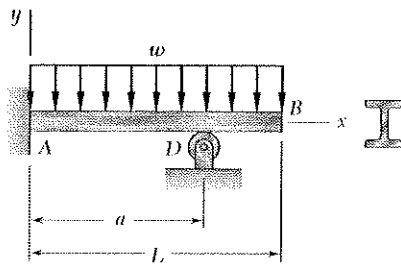
Problem 9.129

x m	theta rad*10**3	y mm
.000	-8.703	.000
.250	-8.615	-2.168
.500	-8.351	-4.293
.750	-7.911	-6.329
1.000	-7.296	-8.234
1.250	-6.505	-9.962
1.500	-5.538	-11.472
1.750	-4.483	-12.724
2.000	-3.428	-13.713
2.250	-2.373	-14.438
2.500	-1.319	-14.900
2.750	-.264	-15.098
3.000	.791	-15.032
3.250	1.802	-14.706
3.500	2.725	-14.138
3.750	3.560	-13.350
4.000	4.307	-12.365
4.250	4.967	-11.204
4.500	5.538	-9.889
4.750	6.021	-8.442
5.000	6.417	-6.886
5.250	6.725	-5.241
5.500	6.944	-3.531
5.750	7.076	-1.776
6.000	7.120	.000

Problem 9.16

x m	theta rad*10**3	y mm
.000	-2.490	.000
.050	-2.485	-.124
.100	-2.471	-.248
.150	-2.448	-.371
.200	-2.416	-.493
.250	-2.375	-.613
.300	-2.325	-.730
.350	-2.265	-.845
.400	-2.197	-.957
.450	-2.119	-1.065
.500	-2.032	-1.168
.550	-1.936	-1.268
.600	-1.831	-1.362
.650	-1.716	-1.451
.700	-1.593	-1.533
.750	-1.460	-1.610
.800	-1.318	-1.679
.850	-1.172	-1.741
.900	-1.025	-1.796
.950	-.879	-1.844
1.000	-.732	-1.884
1.050	-.586	-1.917
1.100	-.439	-1.943
1.150	-.293	-1.961
1.200	-.146	-1.972
1.250	.000	-1.976
1.300	.146	-1.972
1.350	.293	-1.961
1.400	.439	-1.943
1.450	.586	-1.917
1.500	.732	-1.884
1.550	.879	-1.844
1.600	1.025	-1.796
1.650	1.172	-1.741
1.700	1.318	-1.679
1.750	1.460	-1.610
1.800	1.593	-1.533
1.850	1.716	-1.451
1.900	1.831	-1.362
1.950	1.936	-1.268
2.000	2.032	-1.168
2.050	2.119	-1.065
2.100	2.197	-.957
2.150	2.265	-.845
2.200	2.325	-.730
2.250	2.375	-.613
2.300	2.416	-.493
2.350	2.448	-.371
2.400	2.471	-.248
2.450	2.485	-.124
2.500	2.490	.000

PROBLEM 9.C5



9.C5 The supports of beam AB consist of a fixed support at end A and a roller support located at point D . Write a computer program that can be used to calculate the slope and deflection at the free end of the beam for values of a from 0 to L using given increments Δa . Apply this program to calculate the slope and deflection at point B for each of the following cases:

	L	ΔL	w	E	Shape
(a)	3.6 m	0.15 m	24 k/Nm	200 GPa	W410 $\times$ 65
(b)	3 m	0.2m	18 kN/m	200 GPa	W460 $\times$ 113

SOLUTION

BEAM IS INDETERMINATE

USE APPENDIX D AND SUPERPOSITION

DETERMINE REACTION AT D

DUE TO DISTRIBUTED LOAD

$$(y_D)_w = -\frac{w}{24EI} (a^4 - 4La^3 + 6L^2a^2)$$

DUE TO REDUNDANT LOAD:

$$(y_D)_R = \frac{R_D L^3}{3EI}$$

REDUNDANT REACTION:

$$\text{SINCE } (y_D)_w + (y_D)_R = 0:$$

$$R_D = \frac{3EI}{L^3} (y_D)_w$$

COMPUTE SLOPE AND DEFLECTION AT B

SUPERPOSE:

DUE TO DISTRIBUTED LOAD:

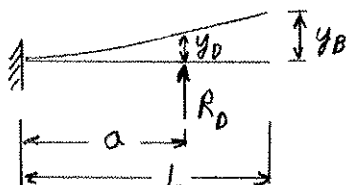
$$\theta_B = -\frac{wL^3}{6EI}$$

$$y_B = -\frac{wL^4}{8EI}$$

DUE TO R_D :

$$\theta_B = \frac{Pa^2}{2EI}$$

$$y_B = \frac{Pa^3}{3EI} + (L-a) \frac{Pa^2}{2EI}$$



$$\theta_B = \theta_D$$

$$y_B = y_D + (L-a)\theta_D$$

CONTINUED

PROBLEM 9.C5 CONTINUED

PROGRAM OUTPUT

Problem 9.C5 (a)

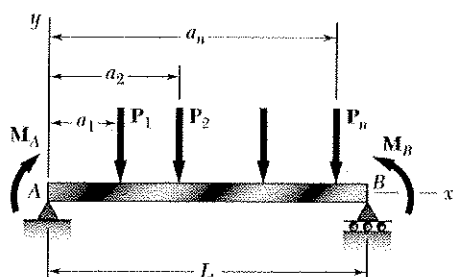
a m	theta B rad*10 ⁻³	y at B mm
0	-3.019	-8.15
0.6	-2.007	-4.7125
1.2	-1.216	-2.3825
1.8	-0.613	-0.935
2.4	-0.168	-0.1675
3.0	0.152	0.0925
3.6	0.377	0

Problem 9.C5 (b)

a m	theta B rad*10 ⁻³	y at B mm
.0	-.728	-1.6389
.2	-.624	-1.3324
.4	-.529	-1.0663
.6	-.442	-.8374
.8	-.364	-.6426
1.0	-.293	-.4789
1.2	-.230	-.3435
1.4	-.174	-.2338
1.6	-.124	-.1472
1.8	-.079	-.0813
2.0	-.040	-.0337
2.2	-.006	-.0024
2.4	.023	.0149
2.6	.049	.0198
2.8	.072	.0143

Proprietary Material. © 2009 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.C6



9.C6 For the beam and loading shown, use the *Moment-Area Method* to write a computer to calculate the slope and deflection at points along the beam from $x = 0$ to $x = L$ using given increments Δx . Apply this program to calculate the slope and deflection at each concentrated load for the beam of (a) Prob. 9.77 with $\Delta x = 0.5$ m, (b) Prob. 9.119 with $\Delta x = 0.5$ m.

SOLUTION

ENTER M_A AND M_B
 FOR EACH LOAD ENTER P_i AND a_i
 DETERMINE REACTION AT A

DUE TO MOMENTS AT ENDS:

$$(R_A)_1 = -(M_A - M_B)/L$$

DUE TO LOADS P_i :

FOR $i = 1$ TO NUMBER OF LOADS

$$R_B = R_B + P_i a_i / L$$

$$\text{LOAD} = \text{LOAD} + P_i$$

$$(R_A)_2 = \text{LOAD} - R_B$$

$$R_A = (R_A)_1 + (R_A)_2$$

DETERMINE SLOPE AT A

USE SECOND MOMENT-AREA THEOREM
 TO GET TANGENTIAL DEVIATION AT B

DUE TO M_A :

$$t_{B/A} = M_A L^2 / (2.0 EI)$$

DUE TO R_A :

$$t_{B/A} = R_A L^3 / (6.0 EI)$$

DUE TO LOADS P_i :

FOR $i = 1$ TO NUMBER OF LOADS

$$t_{B/A} = -P_i (L - a_i)^3 / (6.0 EI)$$

SUM $t_{B/A}$:

$$\theta_A = -t_{B/A} / L$$

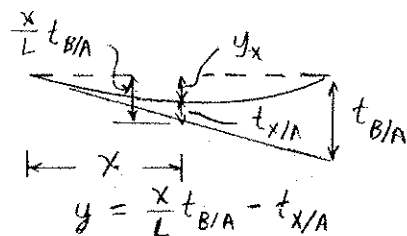
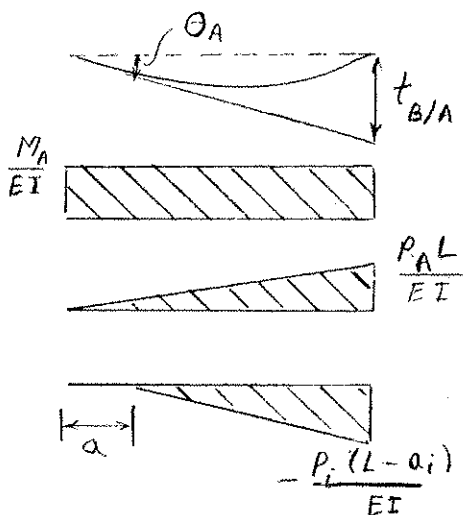
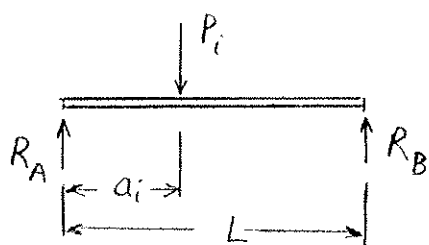
DETERMINE SLOPE AND DEFLECTIONS

FOR $x = 0$ TO L , SUPERPOSE:

DUE TO M_A AND R_A :

$$\theta_y = \theta_A + (M_A x + R_A x^2 / 2.0) / EI$$

CONTINUED



PROBLEM 9.C6 CONTINUED

$$y_x = \frac{x}{L} t_{B/A} - M_A x^2 / (2.0 EI) - R_A x^3 / (6.0 EI)$$

DUE TO LOADS P_i :

DO FOR ALL LOADS WITH $a_i < x$

$$\theta_x = P_i (x - a_i)^2 / (2.0 EI)$$

$$y_x = P_i (x - a_i)^3 / (6.0 EI)$$

PROGRAM OUTPUT

Problem 9.77

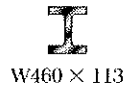
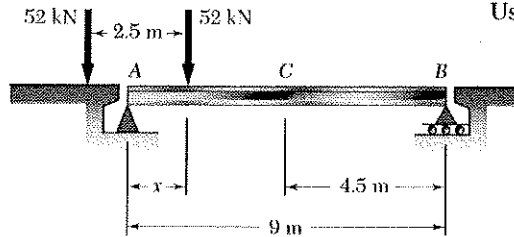
x m	theta rad*1000	y at x mm	
.000	-.600962	.000000	◀ (a)
.500	-1.602564	.574252	
1.000	-2.043269	1.509081	
1.500	-1.923077	2.524039	
2.000	-1.241987	3.338675	
2.500	.000000	3.672543	◀ (b)
3.000	1.241987	3.338676	
3.500	1.923077	2.524039	
4.000	2.043269	1.509082	
4.500	1.602564	.574253	
5.000	.600962	.000000	

Problem 9.119

x m	theta rad*1000	y at x mm	
.000	-4.504505	.000000	◀ (a)
.500	-4.673423	2.317943	
1.000	-4.279279	4.579579	
1.500	-3.322072	6.503378	
2.000	-1.801802	7.807808	
2.500	.000000	8.258258	◀ (b)
3.000	1.801802	7.807808	
3.500	3.322072	6.503378	
4.000	4.279279	4.579579	
4.500	4.673423	2.317943	
5.000	4.504505	.000000	

PROBLEM 9.C7

9.C7 Two 52 kN loads are maintained 2.5 m apart as they are moved slowly across beam AB. Write a computer program to calculate the deflection at the midpoint C of the beam for values of x from 0 to 9 m, using 0.5-m increments. Use $E = 200$ GPa.


SOLUTION

ENTER LOAD P , BEAM LENGTH L AND SPACE BETWEEN LOADS D

WILL SOLVE WITH MOMENT-AREA METHOD

DETERMINE DEFLECTION AT C

FOR $x = 0$ TO L

IF $0 \leq x \leq D$:

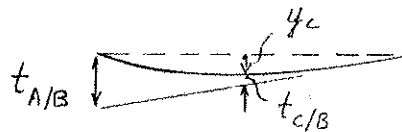
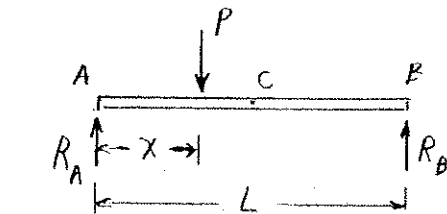
HAVE ONE LOAD TO LEFT OF C

$$R_B = Px/L$$

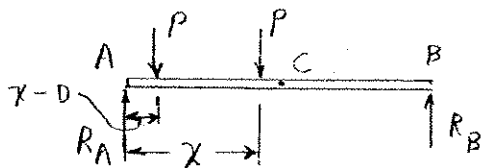
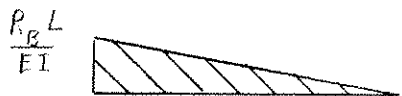
$$t_{A/B} = (R_B L^3 - Px^3)/(6.0EI)$$

$$t_{C/B} = R_B L^3/(48.0EI)$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



IF $D < x \leq L/2$

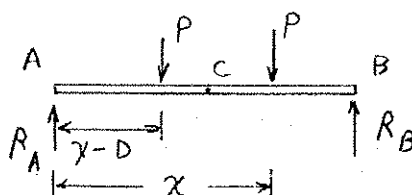
HAVE TWO LOADS TO LEFT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

$$t_{C/B} = R_B L^3/(48.0EI)$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

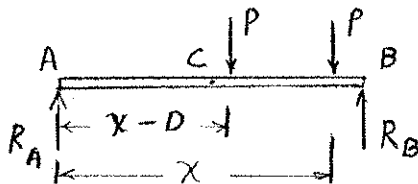


IF $L/2 < x \leq (L/2 + D)$

HAVE ONE LOAD TO LEFT OF C AND ONE TO RIGHT OF C OR AT C

CONTINUED

PROBLEM 9.C7 CONTINUED



$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

$$t_{C/B} = (R_B L^3/48.0 - P(x - \frac{L}{2})^3/6.0)/EI$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

IF $(L/2 + D) < x < L$

HAVE BOTH LOADS TO RIGHT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-b)^3)/(6.0EI)$$

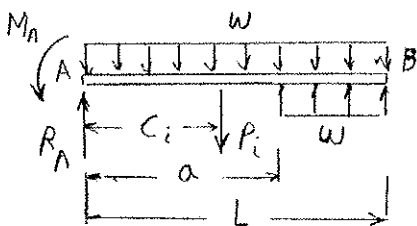
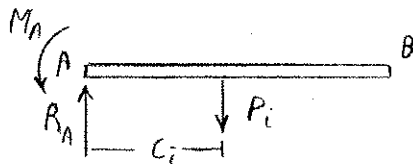
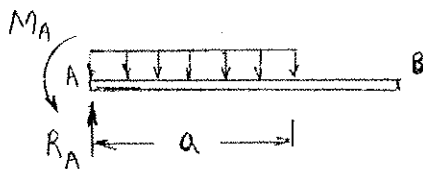
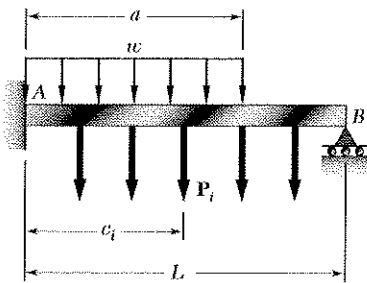
$$t_{C/B} = (R_B L^3/48.0 - P(x - \frac{L}{2})^3/6.0 - P(x-D - \frac{L}{2})^3/6.0)/EI$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

PROGRAM OUTPUT

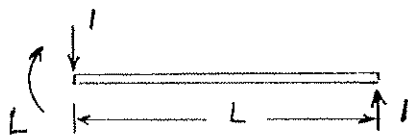
x m	RB kN	ThetaB rad	YC mm
.0	.000	.00000	.00000
.5	2.889	.00315	1.17881
1.0	5.778	.00624	2.32839
1.5	8.667	.00921	3.41951
2.0	11.556	.01200	4.42296
2.5	14.444	.01456	5.30950
3.0	20.222	.01998	7.22872
3.5	26.000	.02499	8.94335
4.0	31.778	.02947	10.39493
4.5	37.556	.03331	11.52503
5.0	43.333	.03639	12.28492
5.5	49.111	.03859	12.66487
6.0	54.889	.03980	12.66487
6.5	60.667	.03989	12.28492
7.0	66.444	.03876	11.52503
7.5	72.222	.03629	10.39493
8.0	78.000	.03235	8.94335
8.5	83.778	.02684	7.22872
9.0	89.556	.01963	5.30950

PROBLEM 9.C8



$$\text{AT } x=0, y = \frac{dy}{dx} = 0$$

∴ THE CONSTANTS OF INTEGRATION ARE ZERO



$$EI \frac{d^2y}{dx^2} = -x + L$$

$$EI \frac{dy}{dx} = -\frac{1}{2}x^2 + Lx + C_1$$

$$EI y = -\frac{1}{6}x^3 + \frac{1}{2}Lx^2 + C_1x + C_2$$

BOUNDARY CONDITIONS GIVE $C_1 = C_2 = 0$

9.C8 A uniformly distributed load w and several distributed loads P_i may be applied to the cantilever beam AB . Write a computer program to determine the reaction at the roller support and apply this program to the beam and loading of (a) Prob. 9.53a, (b) Prob. 9.154.

SOLUTION

THE BEAM IS INDETERMINATE

USE EQUATION OF ELASTIC CURVE

ENTER w AND FOR EACH LOAD P_i AND c_i

COMPUTE DISPLACEMENT AT B DUE TO LOADS

REACTION AT A:

DUE TO w

$$R_A = wa$$

$$M_A = \frac{1}{2}wa^2$$

FOR $i = 1$ TO NUMBER LOADS P_i

$$R_A = R_A - P_i$$

$$M_A = M_A - P_i c_i$$

FOR DISPLACEMENT AT B, SUPERPOSE:

DUE TO REACTION AT A

$$EI y_B = \frac{1}{6}R_A L^3 + \frac{1}{2}M_A L^2$$

DUE TO DISTRIBUTED LOADS

$$EI y_B = \frac{1}{24}(-wL^4 + w(L-a)^4)$$

DUE TO P_i

FOR $i = 1$ TO NUMBER LOADS

$$EI y_B = \frac{1}{6}P_i (L-c_i)^3$$

COMPUTE DISPLACEMENT AT B DUE TO UNIT R_B

$$EI(y_B)_{UNIT} = \frac{1}{3}L^3$$

COMPUTE REACTION AT B

$$\text{FROM } EI y_B + R_B EI(y_B)_{UNIT} = 0$$

$$R_B = -y_B / (y_B)_{UNIT}$$

CONTINUED

PROBLEM 9.C8 CONTINUED

PROGRAM OUTPUT

Problem 9.53 (a)

Reaction at Roller Support = 11.5356 kN

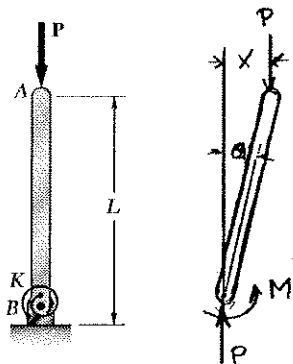
Problem 9.154

Reaction at Roller Support = 10.1758 kN

Chapter 10

Problem 10.1

10.1 Knowing that the torsional spring at B is of constant K and that the bar AB is rigid, determine the critical load P_{cr} .



Let θ be the angle change of bar AB .

$$M = K\theta, \quad x = L \sin \theta \approx L\theta$$

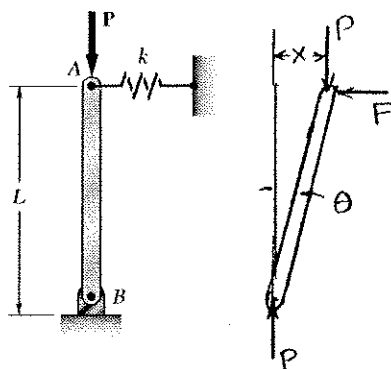
$$\sum M_B = 0: \quad M - Px = 0 \quad K\theta - PL\theta = 0$$

$$(K - PL)\theta = 0$$

$$P_{cr} = K/L \quad \rightarrow$$

Problem 10.2

10.2 Knowing that the spring at A is of constant k and that the bar AB is rigid, determine the critical load P_{cr} .



Let θ be the angle change of bar AB .

$$F = kx = kL \sin \theta$$

$$\sum M_B = 0: \quad Fh \cos \theta - Px = 0$$

$$kL^2 \sin \theta \cos \theta - PL \sin \theta = 0$$

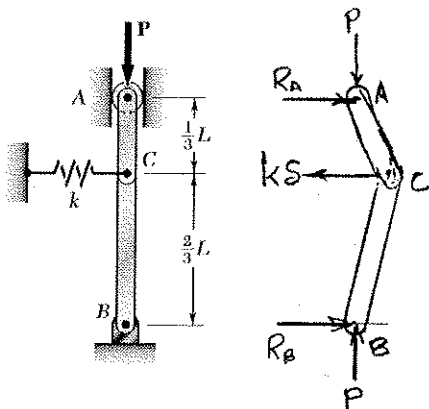
Using $\sin \theta \approx \theta$ and $\cos \theta \approx 1$, $kL^2 \theta - PL \theta = 0$

$$(kL^2 - PL)\theta = 0$$

$$P_{cr} = kL \quad \rightarrow$$

Problem 10.3

10.3 Two rigid bars AC and BC are connected as shown to a spring of constant k . Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.



Let S be the deflection of point C .

Using free body AC and $+\circlearrowleft \Sigma M_C = 0$:

$$-\frac{1}{3}LR_A + PS = 0 \quad R_A = \frac{3PS}{L}$$

Using free body BC and $+\circlearrowleft \Sigma M_C = 0$:

$$\frac{2}{3}LR_B - PS = 0 \quad R_B = \frac{3PS}{2L}$$

Using both free bodies together,

$$+\rightarrow \Sigma F_x = 0: R_A + R_B - kS = 0$$

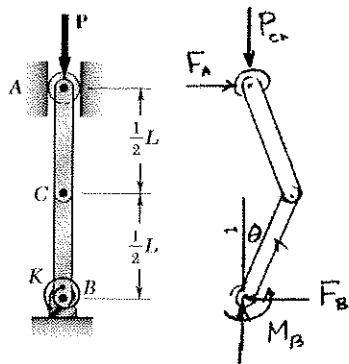
$$\frac{3PS}{L} + \frac{3PS}{2L} - kS = 0$$

$$\left(\frac{9}{2} \frac{P}{L} - k\right) S = 0$$

$$P_{cr} = \frac{2kL}{9}$$

Problem 10.4

10.4 Two rigid bars AC and BC are connected by a pin at C as shown. Knowing that the torsional spring at B is of constant K , determine the critical load P_{cr} for the system.



Let θ be the angle change of each bar.

$$M_B = K\theta$$

$$+\circlearrowleft \Sigma M_B = 0: K\theta - F_A L = 0 \quad F_A = \frac{K\theta}{L}$$

Bar AC $+\circlearrowleft \Sigma M_C = 0$:

$$P_{cr} \frac{1}{2} L \theta - \frac{1}{2} L F_A = 0$$

$$P_{cr} = \frac{F_A}{\theta} \quad P_{cr} = \frac{K}{L}$$

Problem 10.9

10.9 Determine the critical load of a wooden meter stick which has a 7×24 -mm rectangular cross section. Use $E = 12$ GPa.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (24)(7)^3 = 686 \text{ mm}^4 = 686 \times 10^{-12} \text{ m}^4$$

$$L = 1.00 \text{ m}$$

$$E = 12 \times 10^9 \text{ Pa}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (12 \times 10^9)(686 \times 10^{-12})}{(1.00)^2}$$

$$P_{cr} = 81.2 \text{ N} \quad \blacktriangleleft$$

Problem 10.10

10.10 Determine the critical load of a round wooden dowel that is 0.9 m long and has a diameter of (a) 10 mm, (b) 15 mm. Use $E = 12$ GPa.

$$(a) \quad c = \frac{1}{2} d = 5 \text{ mm} \quad I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (5)^4 = 490.87 \text{ mm}^4$$

$$I = 490.87 \times 10^{-12} \text{ m}^4 \quad E = 12 \times 10^9 \text{ Pa} \quad L = 0.9 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (12 \times 10^9)(490.87 \times 10^{-12})}{(0.9)^2}$$

$$P_{cr} = 71.8 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad c = \frac{1}{2} d = 7.5 \text{ mm} \quad I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (7.5)^4 = 2.485 \times 10^3 \text{ mm}^4$$

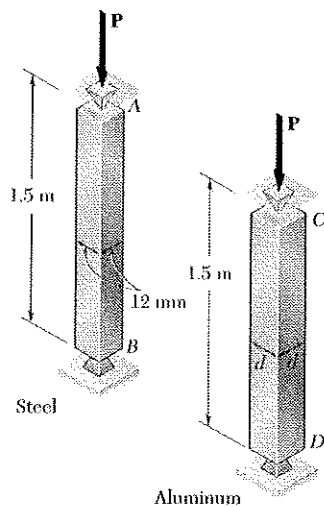
$$I = 2.485 \times 10^{-9} \text{ m}^4 \quad E = 12 \times 10^9 \text{ Pa} \quad L = 0.9 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (12 \times 10^9)(2.485 \times 10^{-9})}{(0.9)^2}$$

$$P_{cr} = 363 \text{ N} \quad \blacktriangleleft$$

Problem 10.11

10.11 Determine the dimension d so that the aluminum and steel struts will have the same weight, and compute the critical load for each strut.



$$\text{Steel: } E = 200 \text{ GPa} \\ \gamma = 7860 \text{ kg/m}^3$$

$$\text{Aluminum: } E = 70 \text{ GPa} \\ \gamma = 2800 \text{ kg/m}^3$$

$$\text{Length: } L = 1.5 \text{ m}$$

$$\text{Weight of steel strut: } W_s = \gamma L d_s^2$$

$$W_s = (7860)(9.81)(1.5)(0.012^2) = 16.655 \text{ N}$$

$$\text{Weight of aluminum strut: } W_a = \gamma L d^2 = (2800)(9.81)(1.5)d^2 = 41202 d^2$$

$$\text{For } W_a = W_s, \quad 41202 d^2 = 16.655 \quad d^2 = 0.404227 \times 10^{-3} \text{ m}^2$$

$$d = 20.1 \text{ mm}$$

$$\text{Steel strut: } I = \frac{1}{12} d_s^4 = \frac{1}{12} (0.012)^4 = 1728 \times 10^{-12} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (1728 \times 10^{-12})}{(1.5)^2}$$

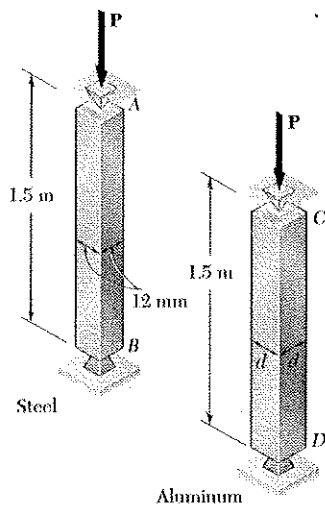
$$P_{cr} = 1516 \text{ N}$$

$$\text{Aluminum strut: } I = \frac{1}{12} d^4 = \frac{1}{12} (0.0201)^4 = 13602 \times 10^{-12} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (70 \times 10^9) (13602 \times 10^{-12})}{(1.5)^2}$$

$$P_{cr} = 4177 \text{ N}$$

Problem 10.12



10.12 Determine (a) the critical load for the steel strut, (b) the dimension d for which the aluminum strut will have the same critical load. (c) Express the weight of the aluminum strut as a percent of the weight of the steel strut.

Steel: $E = 200 \text{ GPa}$
 $\gamma = 7860 \text{ kg/m}^3$

Aluminum: $E = 70 \text{ GPa}$
 $\gamma = 2800 \text{ kg/m}^3$

Length: $L = 1.5 \text{ m}$

(a) Steel strut:

$$I = \frac{1}{12} d_s^4 = \frac{1}{12} (0.012)^4 = 1728 \times 10^{-12} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (1728 \times 10^{-12})}{(1.5)^2} \quad P_{cr} = 1516 \text{ N}$$

$$\text{Weight: } W_s = \gamma_s L d_s^2 = (7860) (9.81) (1.5) (0.012)^2 = 16.655 \text{ N}$$

(b) Aluminum strut: $P_{cr} = \frac{\pi^2 EI}{L^2}$

$$I = \frac{P_{cr} L^2}{\pi^2 E} = \frac{(1516) (1.5)^2}{\pi^2 (70 \times 10^9)} = 3290 \times 10^{-12} \text{ m}^4 = 3290 \text{ mm}^4$$

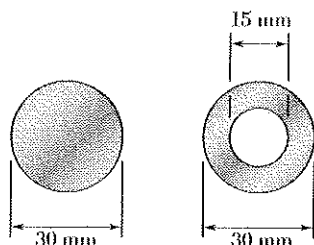
$$I = \frac{1}{12} d^4 \quad d = \sqrt[4]{12 I} = \sqrt[4]{(12) 3290} \quad d = 14.1 \text{ mm}$$

$$\text{Weight: } W_a = \gamma_a L d^2 = (2800) (9.81) (1.5) (0.0141)^2 = 8.191 \text{ N}$$

(c) Weight ratio as a percent: $\frac{W_a}{W_s} \times 100\%$

$$\frac{8.191}{16.655} \times 100\% \quad 49.2\%$$

Problem 10.13



10.13 A compression member of 1.5-m effective length consists of a solid 30-mm-diameter brass rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (a) the percent reduction in the critical load, (b) the value of the critical load for the hollow rod. Use $E = 105 \text{ GPa}$.

$$(a) \quad P_{cr} = \frac{\pi^2 EI}{L^2}$$

P_{cr} is proportional to I .

$$\text{For solid rod, } c = \frac{1}{2}d, \quad I_s = \frac{\pi}{4}c^4$$

$$c = \frac{1}{2}(30) = 15 \text{ mm} \quad I_s = \frac{\pi}{4}(15)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$$

$$\text{For hollow rod, } c_o = \frac{1}{2}d_o \quad I_h = \frac{\pi}{4}(c_o^4 - c_i^4)$$

$$\begin{aligned} \frac{(P_{cr})_h}{(P_{cr})_s} &= \frac{I_h}{I_s} = \frac{c_o^4 - c_i^4}{c^4} = 1 - \left(\frac{c_i}{c_o}\right)^4 = 1 - \left(\frac{d_i}{d_o}\right)^4 \\ &= 1 - \left(\frac{15}{30}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16} \end{aligned}$$

$$\text{Percent reduction in } P_{cr}: \frac{1}{16} \times 100\%$$

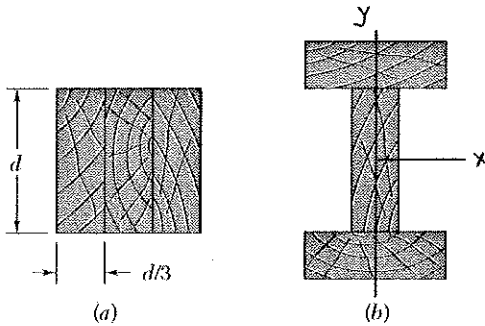
$$6.25\% \quad \blacktriangleleft$$

$$(b) \quad P_{cr} = \frac{15}{16} \frac{\pi^2 EI_s}{L^2} = \frac{15}{16} \frac{\pi^2 (105 \times 10^9)(39.761 \times 10^{-9})}{(1.5)^2} = 17.17 \times 10^3 \text{ N}$$

$$P_{cr} = 17.17 \text{ kN} \quad \blacktriangleleft$$

Problem 10.14

10.14 A column of effective length L can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement a to the critical load using the arrangement b .



Arrangement (a).

$$I_a = \frac{1}{12} d^4$$

$$P_{cr,a} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d^4}{12 L_e^2}$$

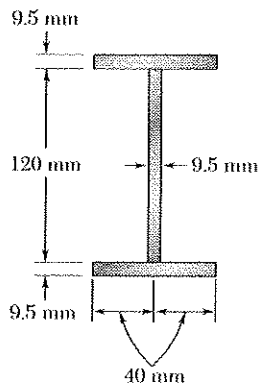
Arrangement (b). $I_{min} = I_y = \frac{1}{12} \left(\frac{d}{3} d^3 \right) + \frac{1}{12} \left(d \left(\frac{d}{3} \right)^3 \right) + \frac{1}{12} \left(\frac{d}{3} d^3 \right) = \frac{19}{324} d^4$

$$P_{cr,b} = \frac{\pi^2 EI}{L_e^2} = \frac{19 \pi^2 E d^4}{324 L_e^2}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = \frac{1}{12} \cdot \frac{324}{19} = \frac{27}{19}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = 1.421 \quad \blacktriangleleft$$

Problem 10.15



$$P = 16 \text{ kN}$$

10.15 A column of 6-m effective length is to be made from three plates as shown. Using $E = 200 \text{ GPa}$, determine the factor of safety with respect to buckling for a centric load of 16 kN.

Moment of inertia is smallest about the vertical centroidal axis.

$$I_{\min} = 2 \left[\frac{1}{12} (9.5) (80)^3 \right] + \frac{1}{12} (120) (9.5)^3$$

$$= 819.24 \times 10^3 \text{ mm}^4 = 819.24 \times 10^{-9} \text{ m}^4$$

$$E = 200 \times 10^9 \text{ Pa}$$

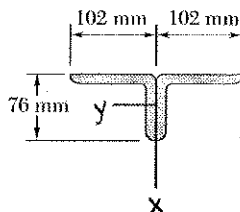
$$P_{cr} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9) (819.24 \times 10^{-9})}{(6)^2}$$

$$= 44.92 \times 10^3 \text{ N} = 44.92 \text{ kN}$$

$$F.S. = \frac{P_{cr}}{P} = \frac{44.92}{16}$$

$$F.S. = 2.81 \quad \blacktriangleleft$$

Problem 10.16



10.16 and 10.17 A compression member of 3.6 m effective length is made by welding together two $102 \times 76 \times 6.4 \text{ mm}$ steel angles as shown. Using $E = 200 \text{ GPa}$, determine the allowable centric load for the member if a factor of safety of 2.5 is required.

For $102 \times 76 \times 6.4 \text{ mm}$ steel angle,

$$A = 1100 \text{ mm}^2, \quad \bar{I}_x = 1.17 \times 10^6 \text{ mm}^4, \quad \bar{y} = 31.6 \text{ mm}$$

$$\bar{I}_y = 0.563 \times 10^6 \text{ mm}^4, \quad \bar{x} = 18.6 \text{ mm}$$

For welded member,

$$I_x = 2 \left[1.17 \times 10^6 + (1100) (31.6)^2 \right] = 4.5368 \times 10^6 \text{ mm}^4$$

$$I_y = 2 \left[0.563 \times 10^6 \right] = 1.126 \times 10^6 \text{ mm}^4 = I_{\min}$$

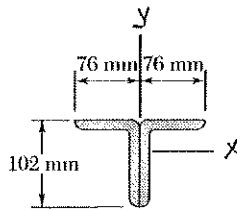
$$L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9) (1.126 \times 10^{-6})}{(3.6)^2} = 171500 \text{ N}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{171500}{2.5}$$

$$P_{all} = 68.6 \text{ kN} \quad \blacktriangleleft$$

Problem 10.17



10.16 and 10.17 A compression member of 3.6 m effective length is made by welding together two $102 \times 76 \times 6.4$ mm steel angles as shown. Using $E = 200$ GPa, determine the allowable centric load for the member if a factor of safety of 2.5 is required.

For $102 \times 76 \times 6.4$ mm steel angle,

$$A = 1100 \text{ mm}^2, \quad \bar{I}_x = 1.17 \times 10^6 \text{ mm}^4, \quad \bar{y} = 31.6 \text{ mm}$$

$$\bar{I}_y = 0.563 \times 10^6 \text{ mm}^4, \quad \bar{x} = 18.6 \text{ mm}$$

For welded member,

$$I_x = 2[1.17 \times 10^6] = 2.34 \times 10^6 \text{ mm}^4$$

$$I_y = 2[0.563 \times 10^6 + (1100)(18.6)^2] = 1.887 \times 10^6 \text{ mm}^4 = I_{\min}$$

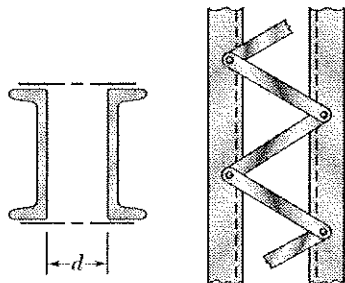
$$L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9) (1.887 \times 10^{-6})}{(3.6)^2} = 287407 \text{ N}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{287407}{2.5}$$

$$P_{all} = 115 \text{ kN}$$

Problem 10.18



10.18 A single compression member of 8.2-m effective length is obtained by connecting two C 200 $\times$ 17.1 steel channels with lacing bars as shown. Knowing that the factor of safety is 1.85, determine the allowable centric load for the member. Use $E = 200$ GPa and $d = 100$ mm.

For C 200 $\times$ 17.1 steel channel, $A = 2170 \text{ mm}^2$

$$\bar{I}_x = 13.4 \times 10^6 \text{ mm}^4, \quad \bar{I}_y = 0.538 \times 10^6 \text{ mm}^4$$

$$\bar{x} = 14.4 \text{ mm}$$

For the fabricated column,

$$I_x = 2\bar{I}_x = (2)(13.4 \times 10^6) = 26.8 \times 10^6 \text{ mm}^4$$

$$I_y = 2[\bar{I}_y + A(\frac{d}{2} + \bar{x})^2]$$

$$= 2[0.538 \times 10^6 + 2170(\frac{100}{2} + 14.4)^2]$$

$$= 19.0755 \times 10^6 \text{ mm}^4$$

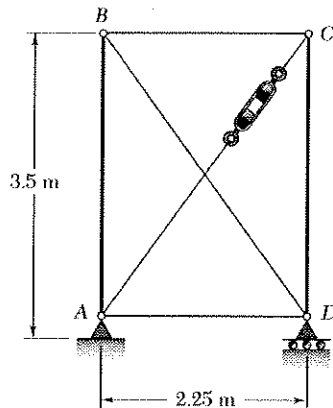
$$I_{\min} = I_y = 19.0755 \times 10^6 \text{ mm}^4 = 19.0755 \times 10^{-6} \text{ m}^4 \quad E = 200 \times 10^9 \text{ Pa}$$

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9) (19.0755 \times 10^{-6})}{(8.2)^2} = 559.99 \times 10^3 \text{ N}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{559.99 \times 10^3}{1.85} = 303 \times 10^3 \text{ N}$$

$$P_{all} = 303 \text{ kN}$$

Problem 10.19



10.19 Members AB and CD are 30-mm-diameter steel rods, and members BC and AD are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that a factor of safety with respect to buckling of 2.75 is required, determine the largest allowable tension in AC . Use $E = 200$ GPa and consider only buckling in the plane of the structure.

$$L_{AC} = \sqrt{(3.5)^2 + (2.25)^2} = 4.1608 \text{ m}$$

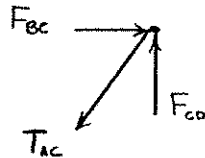
Joint C:

$$\pm \sum F_x = 0: F_{BC} - \frac{2.25}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.84926 F_{BC}$$

$$+\sum F_y = 0: F_{CD} - \frac{3.5}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.1888 F_{CD}$$



Members BC and AD: $I_{BC} = \frac{\pi}{4} \left(\frac{d_{BC}}{2} \right)^4 = \frac{\pi}{4} \left(\frac{22}{2} \right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$

$$L_{BC} = 2.25 \text{ m} \quad F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-9})}{(2.25)^2} = 4.4836 \times 10^3 \text{ N}$$

$$F_{BC,all} = \frac{F_{BC,cr}}{F.S.} = 1.6304 \times 10^3 \text{ N} \quad T_{AC,all} = 3.02 \times 10^3 \text{ N}$$

Members AB and CD: $I_{CD} = \frac{\pi}{4} \left(\frac{d_{CD}}{2} \right)^4 = \frac{\pi}{4} \left(\frac{30}{2} \right)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$

$$L_{CD} = 3.5 \text{ m} \quad F_{CD,cr} = \frac{\pi^2 E I_{CD}}{L_{CD}^2} = \frac{\pi^2 (200 \times 10^9) (39.761 \times 10^{-9})}{(3.5)^2} = 6.4069 \times 10^3 \text{ N}$$

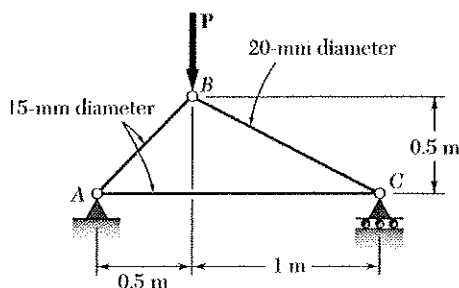
$$F_{CD,all} = \frac{F_{CD,cr}}{F.S.} = 2.3298 \times 10^3 \text{ N} \quad T_{AC,all} = 2.77 \times 10^3 \text{ N}$$

Smaller value for $T_{AC,all}$ governs.

$$T_{AC,all} = 2.77 \text{ kN} \quad \blacktriangleleft$$

Problem 10.20

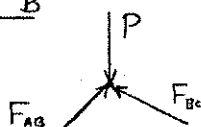
10.20 Knowing that a factor of safety of 2.6 is required, determine the largest load P that can be applied to the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



$$\begin{aligned} \text{BC: } L_{BC} &= \sqrt{1^2 + 0.5^2} = 1.1180 \text{ m} \\ I &= \frac{\pi}{64} d_{BC}^4 = \frac{\pi}{64} (20)^4 = 7.854 \times 10^3 \text{ mm}^4 \\ &= 7.854 \times 10^{-9} \text{ m}^4 \\ P_{cr} &= \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (7.854 \times 10^{-9})}{(1.1180)^2} \\ &= 12.403 \times 10^3 \text{ N} = 12.403 \text{ kN} \\ F_{BC,all} &= \frac{P_{cr}}{F.S.} = \frac{12.403}{2.6} = 4.770 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{AB: } L_{AB} &= \sqrt{0.5^2 + 0.5^2} = 0.70711 \text{ m} \\ I &= \frac{\pi}{64} d^4 = \frac{\pi}{64} (15)^4 = 2.485 \times 10^3 \text{ mm}^4 = 2.485 \times 10^{-9} \text{ m}^4 \\ P_{cr} &= \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (2.485 \times 10^{-9})}{(0.70711)^2} = 9.8106 \times 10^3 \text{ N} = 9.8106 \text{ kN} \\ F_{AB,all} &= \frac{P_{cr}}{F.S.} = \frac{9.8106}{2.6} = 3.773 \text{ kN} \end{aligned}$$

Joint B



$$\sum F_x = 0: \frac{0.5}{0.70711} F_{AB} - \frac{1.0}{1.1180} F_{BC} = 0$$

$$F_{BC} = 0.79057 F_{AB}$$

$$\sum F_y = 0: \frac{0.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} + P = 0$$

$$0.70711 F_{AB} + (0.44721)(0.79057 F_{AB}) - P = 0$$

$$P = 1.06066 F_{AB}$$

$$P = (1.06066) \frac{F_{BC}}{0.79057} = 1.3416 F_{BC}$$

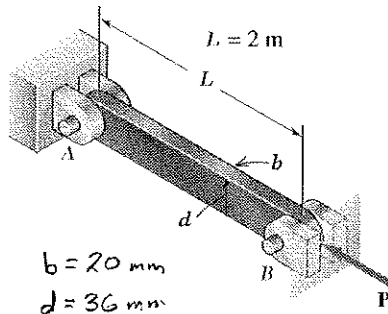
Allowable value for P.

$$P < 1.06066 F_{AB,all} = (1.06066)(3.773) = 4.00 \text{ kN}$$

$$P < 1.3416 F_{BC,all} = (1.3416)(4.770) = 6.40 \text{ kN}$$

$$P_{all} = 4.00 \text{ kN}$$

Problem 10.21



10.21 The uniform aluminum bar AB has a $20 \times 36\text{-mm}$ rectangular cross section and is supported by pins and brackets as shown. Each end of the bar may rotate freely about a horizontal axis through the pin, but rotation about a vertical axis is prevented by the brackets. Using $E = 70\text{ GPa}$, determine the allowable centric load P if a factor of safety of 2.5 is required.

Buckling in horizontal plane.

$$(L_e)_1 = \frac{L}{2} = \frac{2}{2} = 1.00\text{ m}$$

$$I_1 = \frac{1}{12} d b^3 = \frac{1}{12} (36)(20)^3 = 24 \times 10^3 \text{ mm}^4 = 24 \times 10^{-9} \text{ m}^4$$

$$(P_{cr})_1 = \frac{\pi^2 E I_1}{(L_e)_1^2} = \frac{\pi^2 (70 \times 10^9)(24 \times 10^{-9})}{(1.00)^2}$$

$$= 16.581 \times 10^3 \text{ N} = 16.581 \text{ kN}$$

Buckling in vertical plane. $(L_e)_2 = L = 2\text{ m}$

$$I_2 = \frac{1}{12} b d^3 = \frac{1}{12} (20)(36)^3 = 77.76 \times 10^3 \text{ mm}^4 = 77.76 \times 10^{-9} \text{ m}^4$$

$$(P_{cr})_2 = \frac{\pi^2 E I_2}{(L_e)_2^2} = \frac{\pi^2 (70 \times 10^9)(77.76 \times 10^{-9})}{(2)^2} = 13.4306 \times 10^3 \text{ N}$$

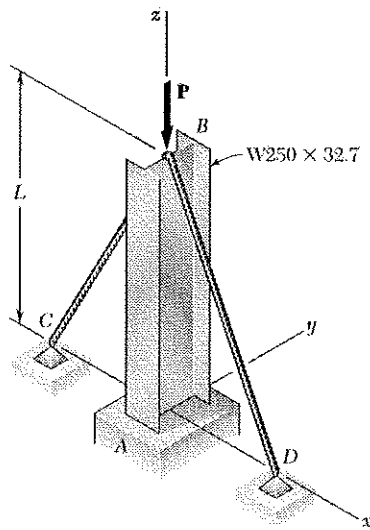
$$= 13.4306 \text{ kN}$$

$$P_{cr} = \min [(P_{cr})_1, (P_{cr})_2] = 13.4306 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{13.4306}{2.5}$$

$$P = 5.37 \text{ kN} \quad \blacktriangleleft$$

Problem 10.22



10.22 Column AB carries a centric load P of magnitude 60 kN. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L . Use $E = 200$ GPa.

$$W250 \times 32.7: I_x = 48.9 \times 10^6 \text{ mm}^4, I_y = 4.73 \times 10^6 \text{ mm}^4$$

$$P = 60 \text{ kN}$$

$$P_{cr} = (F.S.)P = (2.2)(60) = 132 \text{ kN}$$

Buckling in xz -plane: $L_e = 0.7L$

$$P_{cr} = \frac{\pi^2 EI_y}{(0.7L)^2} \quad L = \frac{\pi}{0.7} \sqrt{\frac{EI_y}{P_{cr}}}$$

$$L = \frac{\pi}{0.7} \sqrt{\frac{(200 \times 10^9)(4.73 \times 10^{-6})}{132000}} = 12.01 \text{ m}$$

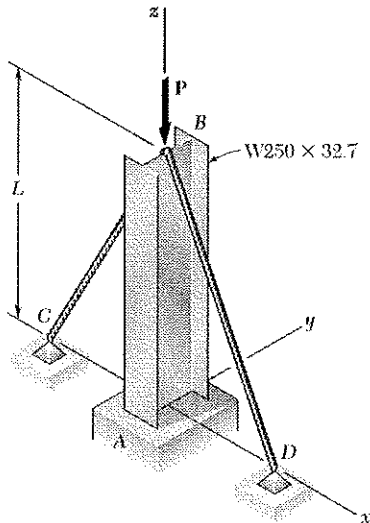
Buckling in yz -plane: $L_e = 2L$

$$P_{cr} = \frac{\pi^2 EI_x}{(2L)^2} \quad L = \frac{\pi}{2} \sqrt{\frac{EI_x}{P_{cr}}} = \frac{\pi}{2} \sqrt{\frac{(200 \times 10^9)(48.9 \times 10^{-6})}{132000}} = 13.52 \text{ m}$$

Smaller value for L governs.

$$L = 12.01 \text{ m}$$

Problem 10.23



10.23 A $W200 \times 31.3$ rolled-steel shape is used with the support and cable arrangement shown in Prob. 10.21. Knowing that $L = 7$ m, determine the allowable centric load P if a factor of safety of 2.2 is required. Use $E = 200$ GPa.

10.22 Column AB carries a centric load P of magnitude 60 kN. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L . Use $E = 200$ GPa.

$$W200 \times 31.3: I_x = 31.4 \times 10^6 \text{ mm}^4, I_y = 4.1 \times 10^6 \text{ mm}^4$$

$$L = 7 \text{ m}$$

Buckling in xz -plane: $L_e = 0.7L = 4.9 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(4.1 \times 10^{-6})}{(4.9)^2} = 337.07 \text{ kN}$$

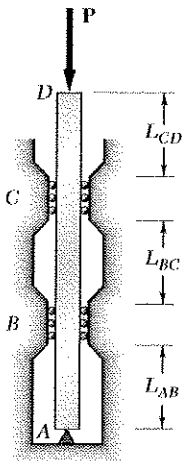
Buckling in yz -plane: $L_e = 2L = 14 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI_x}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(31.4 \times 10^{-6})}{(14)^2} = 316.23 \text{ kN}$$

The smaller value governs. $P_c = 316.23 \text{ kN}$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{316.23}{2.2} = 143.7 \text{ kN}$$

Problem 10.24



10.24 A 25-mm-square aluminum strut is maintained in the position shown by a pin support at A and by sets of rollers at B and C that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 0.9$ m, determine (a) the largest values of L_{BC} and L_{CD} that may be used if the allowable load P is to be as large as possible, (b) the magnitude of the corresponding allowable load. Consider only buckling in the plane of the figure and use $E = 72$ GPa.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (25)(25)^3 = 32.552 \times 10^3 \text{ mm}^4$$

(a) Equivalent lengths: AB $L_e = 0.7 L_{AB} = 0.63 \text{ m}$

BC $L_e = 0.5 L_{BC}$

$$L_{BC} = \frac{0.63}{0.5} = 1.26 \text{ m}$$

CD $L_e = 2 L_{CD}$

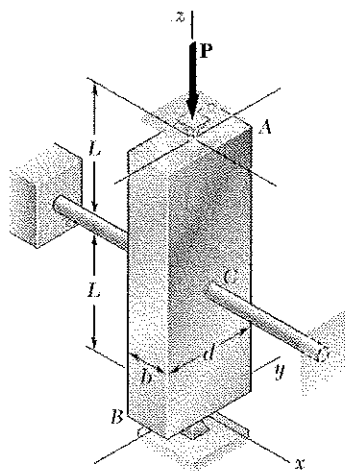
$$L_{CD} = \frac{0.63}{2} = 0.315 \text{ m}$$

(b) $P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{(F.S.) L_e^2} = \frac{\pi^2 (72 \times 10^9) (32.552 \times 10^{-9})}{(3.2)(0.63)^2} = 18213 \text{ N}$

$$P_{all} = 18.2 \text{ kN}$$

Problem 10.25

10.25 Column ABC has a uniform rectangular cross section with $b = 12$ mm and $d = 22$ mm. The column is braced in the xz plane at its midpoint C and carries a centric load P of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length L . Use $E = 200$ GPa.



$$P_{cr} = (F.S.)P = (3.2)(3.8 \times 10^3) = 12.16 \times 10^3 \text{ N}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad L_e = \pi \sqrt{\frac{EI}{P_{cr}}}$$

Buckling in xz -plane. $L = L_e = \pi \sqrt{\frac{EI}{P_{cr}}}$

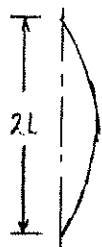


$$I = \frac{1}{12} db^3 = \frac{1}{12} (22)(12)^3 = 3.168 \times 10^3 \text{ mm}^4 = 3.168 \times 10^{-9} \text{ m}^4$$

$$L = \pi \sqrt{\frac{(200 \times 10^9)(3.168 \times 10^{-9})}{12.16 \times 10^3}} = 0.717 \text{ m}$$

Buckling in yz -plane.

$$L_e = 2L \quad L = \frac{L_e}{2} = \frac{\pi}{2} \sqrt{\frac{EI}{P_{cr}}}$$



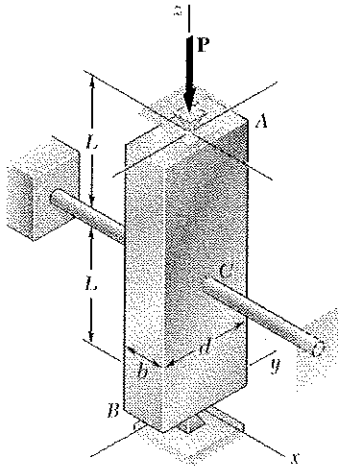
$$I = \frac{1}{12} bd^3 = \frac{1}{12} (12)(22)^3 = 10.648 \times 10^3 \text{ mm}^4 = 10.648 \times 10^{-9} \text{ m}^4$$

$$L = \frac{\pi}{2} \sqrt{\frac{(200 \times 10^9)(10.648 \times 10^{-9})}{12.16 \times 10^3}} = 0.657 \text{ m}$$

The smaller length governs. $L = 0.657 \text{ m} = 657 \text{ mm}$ ◀

Problem 10.26

10.26 Column ABC has a uniform rectangular cross section and is braced in the xz plane at its midpoint C . (a) Determine the ratio b/d for which the factor of safety is the same with respect to buckling in the xz and yz planes. (b) Using the ratio found in part a, design the cross section of the column so that the factor of safety will be 3.0 when $P = 4.4 \text{ kN}$, $L = 1 \text{ m}$, and $E = 200 \text{ GPa}$.



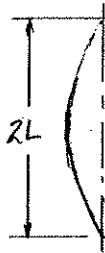
Buckling in xz -plane. $L_e = L$, $I = \frac{db^3}{12}$



$$(P_{cr})_1 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d b^3}{12 L^2}$$

$$(F.S.)_1 = \frac{(P_{cr})_1}{P} = \frac{\pi^2 E d b^3}{12 P L^2}$$

Buckling in yz -plane. $L_e = 2L$, $I = \frac{bd^3}{12}$



$$(P_{cr})_2 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E b d^3}{12 (2L)^2}$$

$$(F.S.)_2 = \frac{(P_{cr})_2}{P} = \frac{\pi^2 E b d^3}{48 P L^2}$$

(a) Equating the two factors of safety,

$$\frac{\pi^2 E d b^3}{12 P L^2} = \frac{\pi^2 E b d^3}{48 P L^2}$$

$$b^2 = \frac{1}{4} d^2 \quad b/d = 1/2 \quad \blacktriangleleft$$

$$\text{Then, } (F.S.) = \frac{\pi^2 E d^4}{96 P L^2}$$

$$d^4 = \frac{96 (F.S.) P L^2}{\pi^2 E} = \frac{(96)(3.0)(4.4 \times 10^3)(1)^2}{\pi^2 (200 \times 10^9)} = 641.97 \times 10^{-9} \text{ m}^4$$

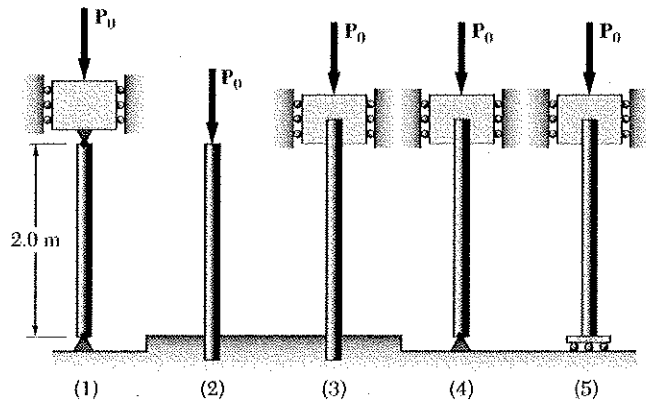
(b) $d = 28.3 \times 10^{-3} \text{ m}$

$$d = 28.3 \text{ mm} \quad \blacktriangleleft$$

$$b = 14.15 \text{ mm} \quad \blacktriangleleft$$

Problem 10.27

10.27 Each of the five struts consists of an aluminum tube that has a 32-mm outer diameter and a 4-mm wall thickness. Using $E = 70$ GPa and a factor of safety of 2.3, determine the allowable load P_0 for each support condition shown.



$$C_o = \frac{1}{2} d_o = \frac{1}{2}(32) = 16 \text{ mm}$$

$$C_i = C_o - t = 16 - 4 = 12 \text{ mm}$$

$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = 35.1858 \times 10^3 \text{ mm}^4$$

$$= 35.1858 \times 10^{-9} \text{ m}^4$$

$$\pi^2 EI = \pi^2 (70 \times 10^9)(35.1858 \times 10^{-9})$$

$$= 24309 \text{ N} \cdot \text{m}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{24309}{L_e^2}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{10569}{L_e^2}$$

$$(1) L_e = (1)(2.0) = 2.0 \text{ m}, \quad P_{all} = 2642 \text{ N}$$

$$(2) L_e = (2)(2.0) = 4.0 \text{ m}, \quad P_{all} = 661 \text{ N}$$

$$(3) L_e = \left(\frac{1}{2}\right)(2.0) = 1.0 \text{ m}, \quad P_{all} = 10569 \text{ N}$$

$$(4) L_e = (0.7)(2.0) = 1.4 \text{ m}, \quad P_{all} = 5392 \text{ N}$$

$$(5) L_e = (1.0)(2.0) = 2.0 \text{ m}, \quad P_{all} = 2642 \text{ N}$$

$$P_{all} = 2.64 \text{ kN} \rightarrow$$

$$P_{all} = 0.661 \text{ kN} \rightarrow$$

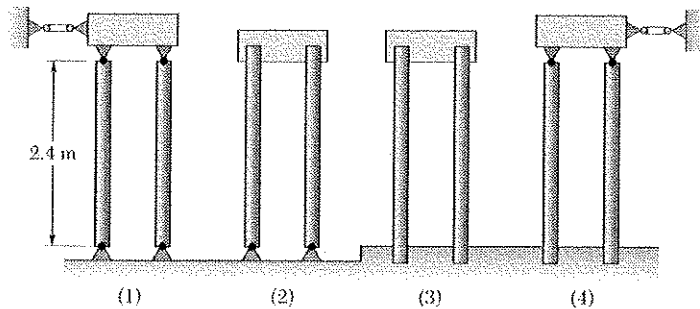
$$P_{all} = 10.57 \text{ kN} \rightarrow$$

$$P_{all} = 5.39 \text{ kN} \rightarrow$$

$$P_{all} = 2.64 \text{ kN} \rightarrow$$

Problem 10.28

10.28 Two columns are used to support a block weighing 14 kN in each of the four ways shown. (a) Knowing that the column of Fig. (1) is made of steel with a 30-mm diameter, determine the factor of safety with respect to buckling for the loading shown. (b) Determine the diameter of each of the other columns for which the factor of safety is the same as the factor of safety obtained in part a. Use $E = 200$ GPa.



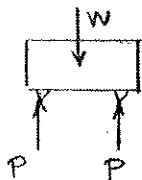
$$(a) \quad I = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} (30)^4$$

$$= 39761 \text{ mm}^4$$

$$L = 2.4 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 (200 \times 10^9) (39761 \times 10^{-12})}{(2.4)^2} = 13.626 \text{ kN for one column.}$$



$$P = \frac{1}{2} W = \frac{14}{2} = 7 \text{ kN.}$$

$$F.S. = \frac{P_{cr}}{P} = \frac{13.626}{7} = 1.95$$

$$P_{cr(1)} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr(\alpha)} = \frac{\pi^2 E I_{\alpha}}{(L_{e,\alpha})^2}$$

$$\frac{P_{cr(\alpha)}}{P_{cr(1)}} = 1$$

$$\frac{I_{\alpha}}{I_{(1)}} \cdot \frac{L^2}{L_{e^2}} = 1 \quad \left(\frac{d_{\alpha}}{d_{(1)}} \right)^4 \left(\frac{L_e}{L} \right)^2 = 1$$

$$d_{\alpha} = d_{(1)} \sqrt[4]{\frac{L_{e(1)}}{L}}$$

$$(b) \quad (2) \quad L_{e(2)} / L = 2.0$$

$$d_{(2)} = 30 \sqrt[4]{2.0} = 42.4 \text{ mm.}$$

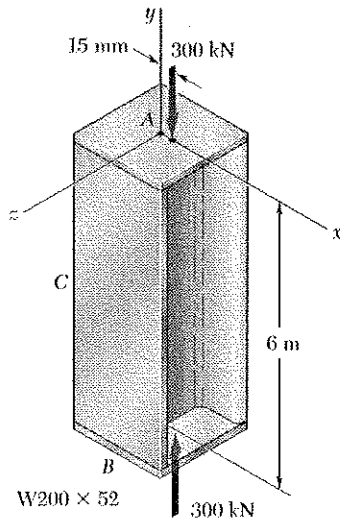
$$(3) \quad L_{e(3)} / L = 1.0$$

$$d_{(3)} = 30 \text{ mm}$$

$$(4) \quad L_{e(4)} / L = 0.7$$

$$d_{(4)} = 30 \sqrt[4]{0.7} = 25.1 \text{ mm}$$

Problem 10.29



10.29 The line of action of the 300-kN axial load is parallel to the geometric axis of the column AB and intersects the x axis at $x = 15$ mm. Using $E = 200$ GPa, determine (a) the horizontal deflection of the midpoint C of the column, (b) the maximum stress in the column.

$$L_e = L = 6 \text{ m}$$

$$e = 15 \text{ mm}$$

$$W_{200 \times 52}$$

$$A = 6660 \text{ mm}^2$$

$$I_y = 17.8 \times 10^6 \text{ mm}^4$$

$$S_y = 175 \times 10^3 \text{ mm}^3$$

$$E = 200 \text{ GPa}$$

$$P_{cr} = \frac{\pi^2 EI_y}{L^2} = \frac{\pi^2 (200 \times 10^9) (17.8 \times 10^{-6})}{(6)^2} = 976 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{300}{976} = 0.3074$$

$$\begin{aligned} (a) \quad y_m &= e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \\ &= (15) \left[\sec\left(\frac{\pi}{2} \sqrt{0.3074}\right) - 1 \right] \\ &= (15) \left[\sec(0.8709) - 1 \right] \\ &= 8.287 \text{ mm} \end{aligned}$$

$$y_m = 8.3 \text{ mm}$$

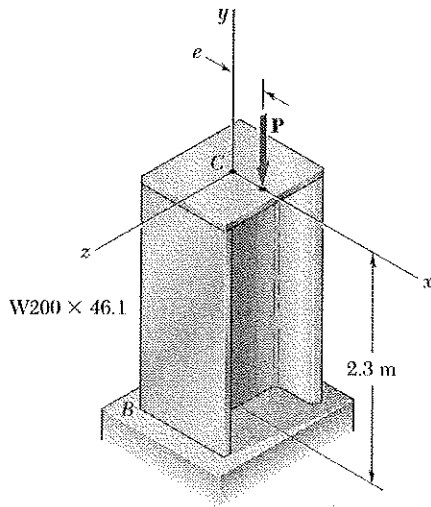
$$(b) \quad M_{max} = P(y_m + e) = (300)(0.015 + 0.008287) = 6.986 \text{ kN}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{300 \times 10^3}{6660 \times 10^{-6}} + \frac{6.986 \times 10^3}{175 \times 10^{-6}}$$

$$\sigma_{max} = 85 \text{ MPa}$$

Problem 10.30

10.30 An axial load P of magnitude 560 kN is applied at a point on the x axis at a distance $e = 6$ mm from the geometric axis of the W200 $\times$ 46.1 rolled-steel column BC . Using $E = 200$ GPa, determine (a) the horizontal deflection of end C , (b) the maximum stress in the column.



$$L_e = 2L = (2)(2.3) = 4.6 \text{ m} \quad e = 6 \times 10^{-3} \text{ m}$$

$$W 200 \times 46.1 \quad A = 5860 \text{ mm}^2 = 5860 \times 10^{-6} \text{ m}^2$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (15.3 \times 10^{-6})}{(4.6)^2}$$

$$= 1.42727 \times 10^6 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{560 \times 10^3}{1.42727 \times 10^6} = 0.39236$$

$$(a) \quad y_m = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = (6 \times 10^{-3}) \left[\sec\left(\frac{\pi}{2} \sqrt{0.39236}\right) - 1 \right]$$

$$= (6 \times 10^{-3}) [\sec(0.98393) - 1] = (6 \times 10^{-3}) (1.8058 - 1)$$

$$= 4.835 \times 10^{-3} \text{ m}$$

$$y_m = 4.84 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad M_{max} = P(y_m + e) = (560 \times 10^3)(4.835 \times 10^{-3} + 6 \times 10^{-3}) = 6.0676 \times 10^3 \text{ N}\cdot\text{m}$$

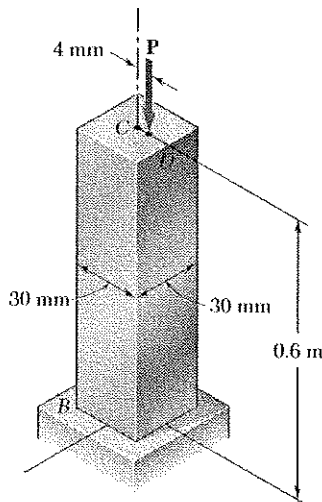
$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S_y} = \frac{560 \times 10^3}{5860 \times 10^{-6}} + \frac{6.0676 \times 10^3}{151 \times 10^{-6}} = 135.7 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 135.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.31

10.31 An axial load $P = 15 \text{ kN}$ is applied at point D that is 4 mm from the geometric axis of the square aluminum bar BC . Using $E = 70 \text{ GPa}$, determine (a) the horizontal deflection of end C , (b) the maximum stress in the column.



$$A = (30)^2 = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4 = 67.5 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} (30) = 15 \text{ mm} = 0.015 \text{ m} \quad e = 4 \times 10^{-3} \text{ m}$$

$$L_e = 2L = (2)(0.6) = 1.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (70 \times 10^9) (67.5 \times 10^{-9})}{(1.2)^2}$$

$$= 32.385 \times 10^3 \text{ N} = 32.385 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{15}{32.385} = 0.46318$$

$$(a) \quad y_m = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = (4 \times 10^{-3}) \left[\sec\left(\frac{\pi}{2} \sqrt{0.46318}\right) - 1 \right]$$

$$= (4 \times 10^{-3}) \left[\sec(1.06904) - 1 \right] = 4.3166 \times 10^{-3} \text{ m}$$

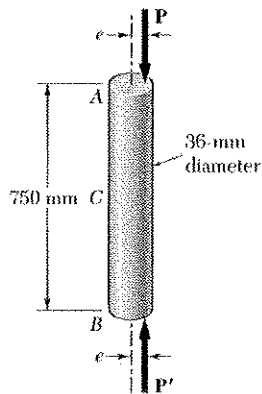
$$y_m = 4.32 \text{ mm}$$

$$(b) \quad M_{max} = P(e + y_m) = (15 \times 10^3) (4 \times 10^{-3} + 4.3166 \times 10^{-3}) = 124.75 \text{ N} \cdot \text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{15 \times 10^3}{900 \times 10^{-6}} + \frac{(124.75)(0.015)}{67.5 \times 10^{-9}} = 44.4 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 44.4 \text{ MPa}$$

Problem 10.32



10.32 An axial load P is applied to the 36-mm-diameter steel rod AB as shown. When $P = 90$ kN, it is observed that the horizontal deflection of the midpoint C is 0.8 mm. Using $E = 200$ GPa, determine (a) the eccentricity e of the load, (b) the maximum stress in the rod.

$$c = \frac{1}{2}d = 18 \text{ mm.} \quad A = \pi c^2 = 1017.9 \text{ mm}^2$$

$$I = \frac{\pi}{4} c^4 = 82448 \text{ mm}^4$$

$$L_e = L = 0.75 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (82448 \times 10^{-12})}{(0.75)^2} = 289.325 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{90}{289.325} = 0.31107$$

$$(a) \quad y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = e [\sec 0.8761 - 1] \\ = 0.5621 e$$

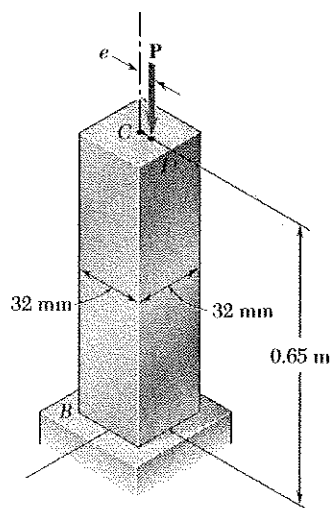
$$e = \frac{y_{max}}{0.5621} = \frac{0.8}{0.5621} = 1.42 \text{ mm}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (90)(1.42 + 0.8) = 199.8 \text{ Nm}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{90 \times 10^3}{1017.9} + \frac{(199.8)(18)(10^3)}{82448} = 132.03 \text{ MPa} \\ = 132 \text{ MPa}$$

Problem 10.33

10.33 An axial load P is applied to the 32-mm-square aluminum bar BC as shown. When $P = 24$ kN, the horizontal deflection at end C is 4 mm. Using $E = 70$ GPa, determine (a) the eccentricity e of the load, (b) the maximum stress in the rod.



$$I = \frac{1}{12} (32)^4 = 87.3813 \times 10^3 \text{ mm}^4 = 87.3813 \times 10^{-9} \text{ m}^4$$

$$A = (32)^2 = 1.024 \times 10^3 \text{ mm}^2 = 1.024 \times 10^{-3} \text{ m}^2$$

$$L_e = 2L = (2)(0.65) = 1.30 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (70 \times 10^9) (87.3813 \times 10^{-9})}{(1.30)^2} = 35.7215 \times 10^3 \text{ N} = 35.7215 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{24}{35.7215} = 0.67186$$

$$(a) \quad y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = e \left[\sec\left(\frac{\pi}{2} \sqrt{0.67186}\right) - 1 \right] = e [\sec 1.28754 - 1] = 2.5780 e$$

$$e = \frac{y_{max}}{2.5780} = \frac{4}{2.5780} \quad e = 1.552 \text{ mm} \blacktriangleleft$$

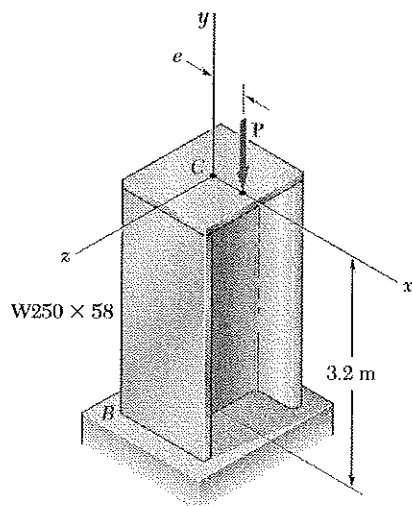
$$(b) \quad M_{max} = P(e + y_{max}) = (24 \times 10^3) [1.552 \times 10^{-3} + 4 \times 10^{-3}] = 133.24 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{24 \times 10^3}{1.024 \times 10^{-3}} + \frac{(133.24)(0.016)}{87.3813 \times 10^{-9}} = 47.8 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 47.8 \text{ MPa} \blacktriangleleft$$

Problem 10.34

10.34 The axial load P is applied at a point located on the x axis at a distance e from the geometric axis of the rolled-steel column BC . When $P = 350$ kN, the horizontal deflection of the top of the column is 5 mm. Using $E = 200$ GPa, determine (a) the eccentricity e of the load, (b) the maximum stress in the column.



$$W 250 \times 58 \quad A = 7420 \text{ mm}^2 = 7420 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.8 \times 10^6 \text{ mm}^4 = 18.8 \times 10^{-6} \text{ m}^4$$

$$S_y = 185 \times 10^3 \text{ mm}^3 = 185 \times 10^{-6} \text{ m}^3$$

$$L_e = 2L = (2)(3.2) = 6.4 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.8 \times 10^{-6})}{(6.4)^2}$$

$$= 906 \times 10^3 \text{ N} = 906 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{350}{906} = 0.38631$$

$$(a) \quad y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = e \left[\sec 0.97632 - 1 \right] = 0.78546 e$$

$$e = \frac{y_{max}}{0.78540} = \frac{5}{0.78540}$$

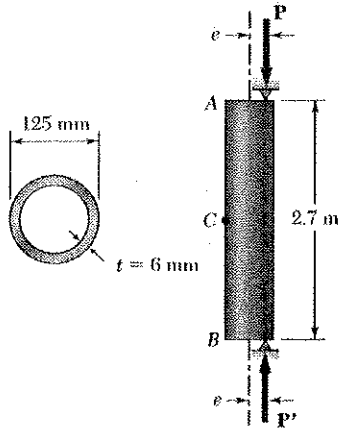
$$e = 6.37 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad M_{max} = P(e + y_{max}) = (350 \times 10^3)(6.37 \times 10^{-3} + 5 \times 10^{-3}) = 3.978 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{350 \times 10^3}{7420 \times 10^{-6}} + \frac{3.978 \times 10^3}{185 \times 10^{-6}} = 68.7 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 68.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.35



10.35 A brass pipe having the cross section shown has an axial load P applied 4 mm from its geometric axis. Using $E = 120 \text{ GPa}$, determine (a) the load P for which the horizontal deflection at the midpoint C is 5 mm, (b) the corresponding maximum stress in the column.

$$c_o = \frac{1}{2} d_o = \frac{1}{2} (125) = 62.5 \text{ mm} \quad c_i = c_o - t = 56.5 \text{ mm}$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = \frac{\pi}{4} (62.5^4 - 56.5^4) = 3.9807 \times 10^6 \text{ mm}^4$$

$$L_e = L = 2.7 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (120 \times 10^9) (3.9807 \times 10^{-6})}{(2.7)^2} = 646.71 \text{ kN}$$

$$(a) \quad y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{e + y_{max}} \quad \frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2$$

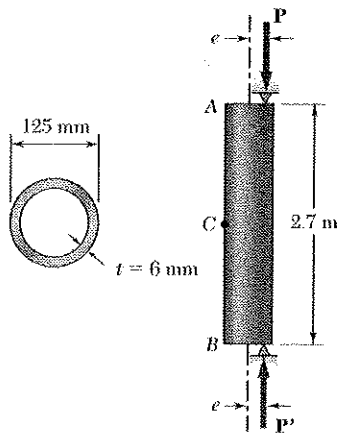
$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{4}{5 + 4} \right]^2 = 0.4996 \quad P = 0.4996 P_{cr} = 323 \text{ kN}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (323)(5 + 4) = 2907 \text{ kN}\cdot\text{mm}$$

$$A = \pi (c_o^2 - c_i^2) = \pi (62.5^2 - 56.5^2) = 2243 \text{ mm}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{323000}{2243} + \frac{(2907000)(62.5)}{3.9807 \times 10^6} = 189.7 \text{ MPa}$$

Problem 10.36



10.36 Solve Prob. 10.35, assuming that the axial load P is applied 8 mm from the geometric axis of the column.

10.35 A brass pipe having the cross section shown has an axial load P applied 4 mm from its geometric axis. Using $E = 120$ GPa, determine (a) the load P for which the horizontal deflection at the midpoint C is 5 mm, (b) the corresponding maximum stress in the column.

$$c_o = \frac{1}{2} d_o = \frac{1}{2}(125) = 62.5 \text{ mm} \quad c_i = c_o - t = 56.5 \text{ mm}$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = \frac{\pi}{4} (62.5^4 - 56.5^4) = 3.9807 \times 10^6 \text{ mm}^4$$

$$L_e = L = 2.7 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (120 \times 10^9) (3.9807 \times 10^{-6})}{(2.7)^2} = 646.71 \text{ kN}$$

$$(a) \quad y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] \quad \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{y_{max} + e}{e} \quad \cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2 = \left[\frac{2}{\pi} \arccos \frac{8}{5 + 8} \right]^2 = 0.3341$$

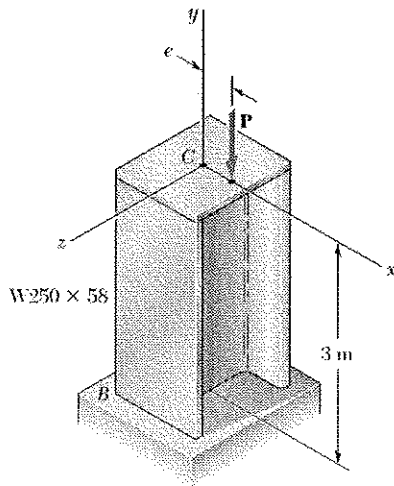
$$P = 0.3341 P_{cr} = 216 \text{ kN}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (216)(5 + 8) = 2808 \text{ kN} \cdot \text{mm}$$

$$A = \pi (c_o^2 - c_i^2) = \pi (62.5^2 - 56.5^2) = 2243 \text{ mm}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{216000}{2243} + \frac{(2808000)(62.5)}{3.9807 \times 10^6} = 140.4 \text{ MPa}$$

Problem 10.37



10.37 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W250 $\times$ 58 rolled-steel column BC . Using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection of the top of the column is 15 mm, (b) the corresponding maximum stress in the column.

$$W250 \times 58 \quad A = 7420 \text{ mm}^2, \quad I_y = 18.8 \times 10^6 \text{ mm}^4$$

$$S_y = 185 \times 10^3 \text{ mm}^3$$

$$L_e = 2L = (2)(3) = 6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.8 \times 10^{-6})}{(6)^2}$$

$$= 1030.8 \text{ kN}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos\left(\frac{e}{y_{max} + e}\right) \right]^2 = \left[\frac{2}{\pi} \arccos\left(\frac{12}{15 + 12}\right) \right]^2$$

$$= 0.49957$$

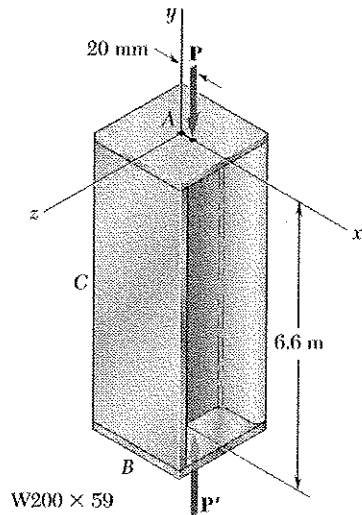
$$(a) \quad P = (0.49957)(1030.8) = 463.8 \text{ kN}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (463.8)(15 + 12) = 12.52 \text{ kNm}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{463800}{7420} + \frac{12520000}{185 \times 10^3} = 130.2 \text{ MPa}$$

Problem 10.38

10.38 The line of action of an axial load P is parallel to the geometric axis of the column AB and intersects the x axis at $x = 20$ mm. Using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection of the midpoint C of the column is 12 mm, (b) the corresponding maximum stress in the column.



$$L_e = L = 6.6 \text{ m}$$

$$e = 20 \text{ mm}$$

$$W200 \times 59 \quad A = 7560 \text{ mm}^2 \quad I_y = 20.4 \times 10^6 \text{ mm}^4 \quad S_y = 199 \times 10^3 \text{ mm}^3$$

$$E = 200 \text{ GPa}$$

$$y_m = 20 \text{ mm}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (20.4 \times 10^{-6})}{(6.6)^2} = 92442.5 \text{ N}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = \frac{y_m + e}{e}$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_m + e} = \frac{20}{12 + 20} = 0.625$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.89567$$

$$\frac{P}{P_{cr}} = \left[\frac{(2)(0.89567)}{\pi} \right]^2 = 0.32513$$

$$(a) \quad P = (0.32513)(92442.5) = 30055.8$$

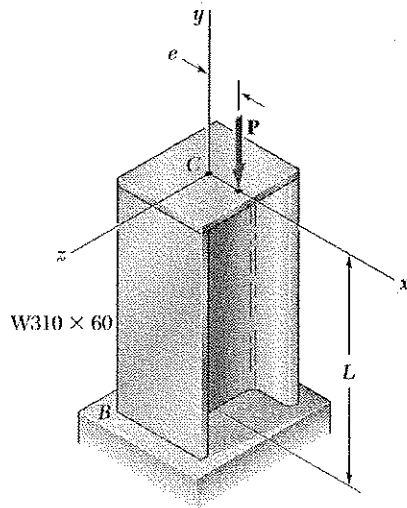
$$P = 300.6 \text{ kN}$$

$$(b) \quad M_{max} = P(e + y_m) = (30055.8)(0.02 + 0.012) = 9617.9 \text{ N}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{30055.8}{7560 \times 10^{-6}} + \frac{9617.9}{199 \times 10^{-6}} \quad \sigma_{max} = 52.3 \text{ MPa}$$

Problem 10.39

10.39 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 $\times$ 60 rolled-steel column BC . Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection at end C is 15 mm, (b) the corresponding maximum stress in the column.



W310 $\times$ 60

$$\begin{aligned} A &= 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2 \\ I_y &= 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4 \\ S_y &= 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$L = 3.5 \text{ m} \quad L_e = 2L = 7.0 \text{ m}$$

$$\begin{aligned} P_{cr} &= \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.3 \times 10^{-6})}{(7.0)^2} \\ &= 737.2 \times 10^3 \text{ N} = 737.2 \text{ kN} \end{aligned}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \quad \cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos\left(\frac{e}{y_{max} + e}\right) \right]^2 = \left[\frac{2}{\pi} \arccos\left(\frac{12}{15 + 12}\right) \right]^2 = 0.49957$$

$$(a) \quad P = 0.49957 P_{cr} = 368.28 \text{ kN} \quad P = 368 \text{ kN} \quad \blacktriangleleft$$

$$M_{max} = P(e + y_{max}) = (368.28 \times 10^3)(12 + 15)(10^{-3}) = 9944 \text{ N}\cdot\text{m}$$

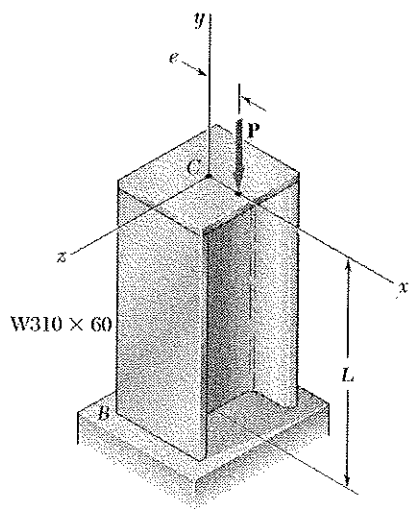
$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{368.28 \times 10^3}{7590 \times 10^{-6}} + \frac{9944}{180 \times 10^{-6}} = 103.8 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 103.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.40

10.40 Solve Prob. 10.39, assuming that L is 4.5 m.

10.39 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 $\times$ 60 rolled-steel column BC . Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection at end C is 15 mm, (b) the corresponding maximum stress in the column.



W 310 $\times$ 60

$$A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4$$

$$S_y = 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3$$

$$L = 4.5 \text{ m}$$

$$L_e = 2L = 9.0 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.3 \times 10^{-6})}{(9.0)^2}$$

$$= 445.96 \times 10^3 \text{ N} = 445.96 \text{ kN}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \quad \cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos\left(\frac{e}{y_{max} + e}\right) \right]^2 = \left[\frac{2}{\pi} \arccos\left(\frac{12}{15 + 12}\right) \right]^2 = 0.49957$$

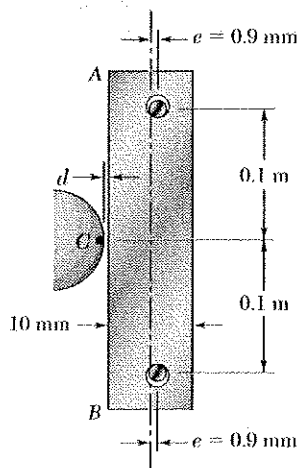
$$(a) \quad P = 0.49957 P_{cr} = 222.79 \text{ kN} \quad P = 223 \text{ kN} \quad \blacktriangleleft$$

$$M_{max} = P(e + y_{max}) = (222.79 \times 10^3)(12 + 15)(10^{-3}) = 6015 \text{ N}\cdot\text{m}$$

$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{222.79 \times 10^3}{7590 \times 10^{-6}} + \frac{6015}{180 \times 10^{-6}} = 62.8 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 62.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.41



10.41 The steel bar AB has a 10×10 -mm square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.9$ mm from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.3$ mm. Use $E = 200$ GPa and the coefficient of thermal expansion $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$.

$$A = (10)^2 = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (10)^4 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = (200 \times 10^9)(833.33 \times 10^{-12}) = 166.667 \text{ N}\cdot\text{m}^2$$

$$L_e = L = 0.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (166.667)}{(0.2)^2} = 41.1234 \times 10^3 \text{ N}$$

Calculate P using the secant formula.

$$y_{max} = d = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] \quad \text{or} \quad \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 1 + \frac{d}{e} =$$

$$\cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{e}{e+d} = \frac{0.9}{0.9+0.3} = 0.75$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = \cos^{-1}(0.75) = 0.72273$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.72273) \right]^2 = 0.21170$$

$$P = 0.21170 P_{cr} = 8.7058 \times 10^3 \text{ N}$$

If the effects of eccentricity and the shortening due to bending are neglected, the shortening of the bar, if not constrained by the pins, would be

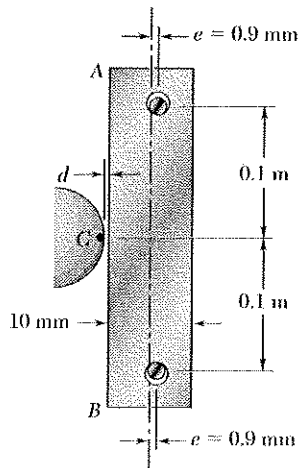
$$\delta = \frac{PL}{EA} = \frac{(8.7058 \times 10^3)(0.2)}{(200 \times 10^9)(100 \times 10^{-6})} = 87.058 \times 10^{-6} \text{ m}$$

This mechanical shortening is compensated by thermal expansion.

$$L\alpha(\Delta T) - \delta = 0$$

$$\Delta T = \frac{\delta}{L\alpha} = \frac{87.058 \times 10^{-6}}{(0.2)(11.7 \times 10^{-6})} = 37.2^\circ\text{C}$$

Problem 10.42



10.42 For the bar of Prob. 10.41, determine the required distance d for which the bar will just make contact with point C when the temperature increases by 60°C .

10.41 The steel bar AB has a $10 \times 10\text{-mm}$ square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.9\text{ mm}$ from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.3\text{ mm}$. Use $E = 200\text{ GPa}$ and the coefficient of thermal expansion $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$.

$$A = (10)^2 = 100\text{ mm}^2 = 100 \times 10^{-6}\text{ m}^2$$

$$I = \frac{1}{12}(10)^4 = 833.33\text{ mm}^4 = 833.33 \times 10^{-12}\text{ m}^4$$

$$EI = (200 \times 10^9)(833.33 \times 10^{-12}) = 166.667\text{ N}\cdot\text{m}^2$$

$$L_e = L = 0.2\text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (166.667)}{(0.2)^2} = 41.1234 \times 10^3\text{ N}$$

Free thermal expansion of the bar:

$$\delta = L\alpha(\Delta T) = (0.2)(11.7 \times 10^{-6})(60) = 140.4 \times 10^{-6}\text{ m}$$

The free expansion is prevented by constraint of the pins.

If eccentricity and shortening due to bending are neglected, the constraining force P is given by

$$P = \frac{EAS}{L} = \frac{(200 \times 10^9)(100 \times 10^{-6})(140.4 \times 10^{-6})}{0.2} = 14.04 \times 10^3\text{ N}$$

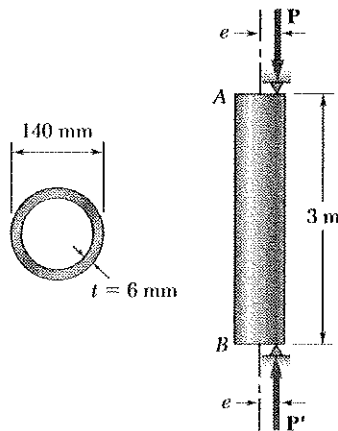
Using the secant formula,

$$y_{max} = d = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right]$$

$$= (9 \times 10^{-3}) \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{14.04 \times 10^3}{41.1234 \times 10^3}}\right) - 1 \right] = 5.81 \times 10^{-3}\text{ m}$$

$$d = 5.81\text{ mm} \quad \blacktriangleleft$$

Problem 10.43



10.43 A pipe having the cross section shown is used as a 3-m column. For the grade of steel used $\sigma_Y = 252 \text{ MPa}$ and $E = 200 \text{ GPa}$. Knowing that a factor of safety of 2.8 with respect to permanent deformation is required, determine the allowable load P when the eccentricity e is (a) 8 mm, (b) 4 mm (Hint: Since the factor of safety must be applied to the load P , not to the stress, use Fig. 10.24 to determine P_Y).

$$c = \frac{1}{2}d = \frac{140}{2} = 70 \text{ mm}$$

$$c_1 = c - t = 70 - 6 = 64 \text{ mm}$$

$$A = \pi(c^2 - c_1^2) = 2525.8 \text{ mm}^2$$

$$I = \frac{\pi}{4}(c^4 - c_1^4) = 5680615 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 47.4 \text{ mm}$$

$$L_e = L = 3 \text{ m}$$

$$\frac{L}{r} = \frac{3000}{47.4} = 63.32$$

$$(a) \quad e = 8 \text{ mm} \quad \frac{ec}{r^2} = \frac{(8)(70)}{(47.4)^2} = 0.2492$$

$$\text{Fig 10.24} \quad \frac{P_Y}{A} = 160 \text{ MPa}$$

$$P_Y = (2525.8 \times 10^{-6})(160 \times 10^6) = 404128 \text{ N}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{404128}{2.8}$$

$$P_{all} = 144.3 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 4 \text{ mm} \quad \frac{ec}{r^2} = \frac{(4)(70)}{(47.4)^2} = 0.1246$$

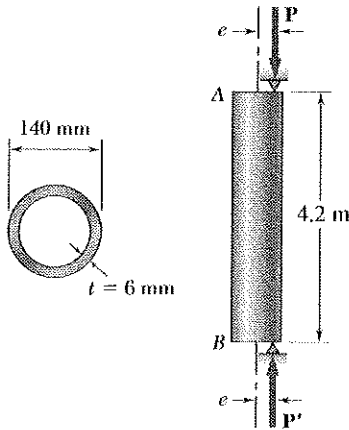
$$\text{Fig 10.24} \quad \frac{P_Y}{A} = 196 \text{ MPa}$$

$$P_Y = (2525.8 \times 10^{-6})(196 \times 10^6) = 495056.8$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{495056.8}{2.8}$$

$$P_{all} = 176.8 \text{ kN} \quad \blacktriangleleft$$

Problem 10.44



10.44 Solve Prob. 10.43, assuming that the length of the column is increased to 4.2 m.

10.43 A pipe having the cross section shown is used as a 4.2 m column. For the grade of steel used $\sigma_Y = 252 \text{ MPa}$ and $E = 200 \text{ GPa}$. Knowing that a factor of safety of 2.8 with respect to permanent deformation is required, determine the allowable load P when the eccentricity e is (a) 8 mm, (b) 4 mm (Hint: Since the factor of safety must be applied to the load P , not to the stress, use Fig. 10.24 to determine P_Y).

$$c = \frac{1}{2}d = \frac{140}{2} = 70 \text{ mm}$$

$$c_1 = c - t = 70 - 6 = 64 \text{ mm}$$

$$A = \pi(c^2 - c_1^2) = 2525.8 \text{ mm}^2$$

$$I = \frac{\pi}{4}(c^4 - c_1^4) = 5680615 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 47.4 \text{ mm}$$

$$L_e = L = 4.2 \text{ m}$$

$$\frac{L_e}{r} = \frac{4200}{47.4} = 88.608$$

$$(a) \quad e = 8 \text{ mm} \quad \frac{ec}{r^2} = \frac{(8)(70)}{(47.4)^2} = 0.2492$$

$$\text{Fig 10.24} \quad \frac{P_Y}{A} = 135 \text{ MPa}$$

$$P_Y = (2525.8 \times 10^{-6})(135 \times 10^6) = 340983 \text{ N}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{340983}{2.8}$$

$$P_{all} = 121.8 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 4 \text{ mm} \quad \frac{ec}{r^2} = \frac{(4)(70)}{(47.4)^2} = 0.1246$$

$$\text{Fig 10.24} \quad \frac{P_Y}{A} = 165 \text{ MPa}$$

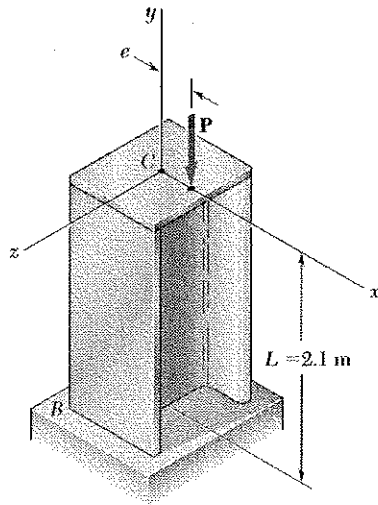
$$P_Y = (2525.8 \times 10^{-6})(165 \times 10^6) = 416757 \text{ N}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{416757}{2.8}$$

$$P_{all} = 148.8 \text{ kN} \quad \blacktriangleleft$$

Problem 10.45

10.45 An axial load P is applied to the W250 $\times$ 44.8 rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 12$ mm and that for the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine (a) the magnitude of P of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (b) the ratio of the load found in part a to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43).



$$\text{W250} \times 44.8 \quad c = \frac{bf}{2} = 74 \text{ mm}$$

$$A = 5720 \text{ mm}^2 = 5.720 \times 10^{-3} \text{ m}^2$$

$$I_y = 7.03 \times 10^6 \text{ mm}^4 = 7.03 \times 10^{-6} \text{ m}^4$$

$$r_y = 35.1 \text{ mm} = 35.1 \times 10^{-3} \text{ m} \quad e = 12 \text{ mm}$$

$$L_c = 2L = (2)(2.1) = 4.2 \text{ m}$$

$$\frac{L_c}{r} = \frac{4.2}{35.1 \times 10^{-3}} = 119.658$$

$$\frac{ec}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$$

(a) Fig. 10.24 (b) $\frac{P_Y}{A} = 82.0 \text{ MPa}$

$$P_Y = (5.720 \times 10^{-3})(82.0 \times 10^6) = 469 \times 10^3 \text{ N} = 469 \text{ kN}$$

$$P_{all} = \frac{P_Y}{\text{F.S.}} = \frac{469}{2.4}$$

$$P_{all} = 195.4 \text{ kN} \quad \blacktriangleleft$$

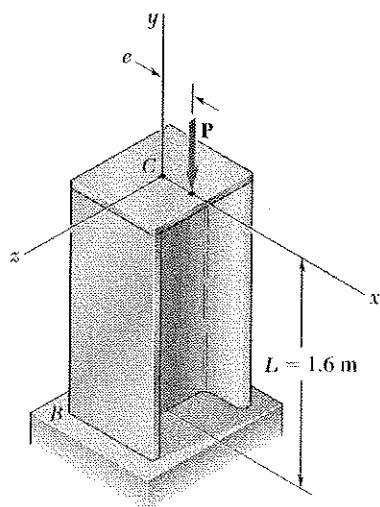
(b) $P_{cr} = \frac{\pi^2 EI}{L_c^2} = \frac{\pi^2 (200 \times 10^9)(7.03 \times 10^{-6})}{(4.2)^2} = 787 \times 10^3 \text{ N} = 787 \text{ kN}$

$$P_{all} = \frac{P_{cr}}{\text{F.S.}} = \frac{787}{2.4} = 328 \text{ kN}$$

$$\text{ratio} = \frac{195.4 \text{ kN}}{328 \text{ kN}}$$

$$\text{ratio} = 0.596 \quad \blacktriangleleft$$

Problem 10.46



10.46 Solve Prob. 10.45, assuming that the length of the column is reduced to 1.6 m.

10.45 An axial load P is applied to the $W250 \times 44.8$ rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 12$ mm and that for the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine (a) the magnitude of P of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (b) the ratio of the load found in part a to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43).

$$W250 \times 44.8 \quad c = \frac{bf}{2} = 74 \text{ mm}$$

$$A = 5720 \text{ mm}^2 = 5.720 \times 10^{-3} \text{ m}^2$$

$$I_y = 7.03 \times 10^6 \text{ mm}^4 = 7.03 \times 10^{-6} \text{ m}^4$$

$$r_y = 35.1 \text{ mm} = 35.1 \times 10^{-3} \text{ m} \quad e = 12 \text{ mm}$$

$$L_e = 2L = (2)(1.6) = 3.2 \text{ m}$$

$$\frac{L_e}{r} = \frac{3.2}{35.1 \times 10^{-3}} = 91.168$$

$$\frac{ec}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$$

(a) Fig. 10.24(b) $\frac{P_Y}{A} = 103.5 \text{ MPa}$

$$P_Y = (5.72 \times 10^{-3})(103.5 \times 10^6) = 592 \times 10^3 = 592 \text{ kN}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{592}{2.4}$$

$$P_{all} = 247 \text{ kN}$$

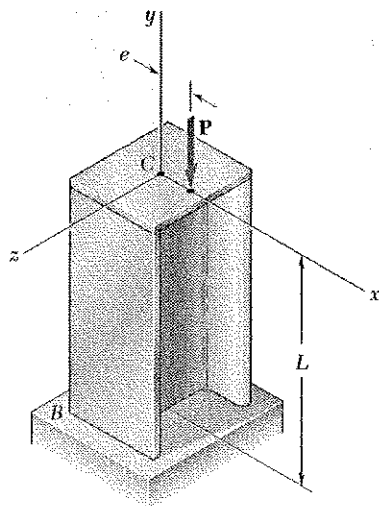
(b) $P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(7.03 \times 10^{-6})}{(3.2)^2} = 1.355 \times 10^6 \text{ N} = 1355 \text{ kN}$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{1355}{2.4} = 565 \text{ kN}$$

$$\text{ratio} = \frac{247 \text{ kN}}{565 \text{ kN}}$$

$$\text{ratio} = 0.437$$

Problem 10.47



10.47 A 100-kN axial load P is applied to the W150 $\times$ 18 rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 6$ mm, determine the largest permissible length L if the allowable stress in the column is 80 MPa. Use $E = 200$ GPa.

$$W150 \times 18 \quad A = 2290 \text{ mm}^2 = 2290 \times 10^{-6} \text{ m}^2$$

$$b_p = 102 \text{ mm} \quad c = \frac{b_p}{2} = 51 \text{ mm}$$

$$I_y = 1.26 \times 10^6 \text{ mm}^4 = 1.26 \times 10^{-6} \text{ m}^4 \quad r_y = 23.5 \text{ mm}$$

$$\sigma_{max} = 80 \times 10^6 \text{ Pa} \quad P = 100 \times 10^3 \text{ N}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

$$\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{r_y^2}{ec} \left(\frac{A\sigma_{max}}{P} - 1 \right) = \frac{(23.5)^2}{(6)(51)} \left[\frac{(2290 \times 10^{-6})(80 \times 10^6)}{100 \times 10^3} - 1 \right]$$

$$= 1.50154$$

$$\cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 0.66598 \quad \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.84199$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.84199) \right]^2 = 0.28732$$

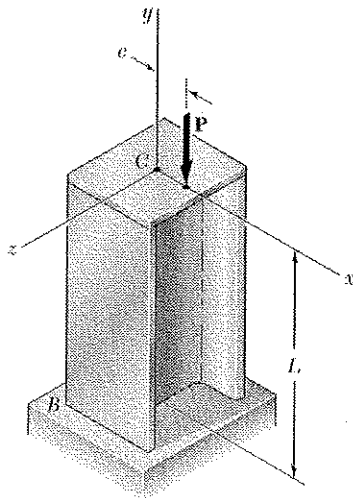
$$P_{cr} = \frac{P}{0.28732} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.28732 \pi^2 EI}{P} = \frac{0.28732 \pi^2 (200 \times 10^9)(1.26 \times 10^{-6})}{100 \times 10^3} = 7.1461 \text{ m}^2$$

$$L_e = 2.6732 = 2L$$

$$L = 1.337 \text{ m} \quad \blacktriangleleft$$

Problem 10.48



10.48 A 250-kN axial load P is applied to the W200 $\times$ 35.9 rolled-steel column BC which is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 6$ mm, determine the largest permissible length L if the allowable stress in the column is 80 MPa. Use $E = 200$ GPa.

$$W200 \times 35.9 \quad A = 4580 \text{ mm}^2 = 4580 \times 10^{-6} \text{ m}^2$$

$$b_f = 165 \text{ mm} \quad c = \frac{b_f}{2} = 82.5 \text{ mm}$$

$$I_y = 7.64 \times 10^6 \text{ mm}^4 = 7.64 \times 10^{-6} \text{ m}^4 \quad r_y = 40.8 \text{ mm}$$

$$\sigma_{\max} = 80 \times 10^6 \text{ Pa} \quad P = 250 \times 10^3 \text{ N}$$

$$\sigma_{\max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{\max}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec} \left(\frac{A\sigma_{\max}}{P} - 1 \right) = \frac{(40.8)^2}{(6)(82.5)} \left[\frac{(4580 \times 10^{-6})(80 \times 10^6)}{250 \times 10^3} - 1 \right]$$

$$= 1.56657$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = 0.63866 \quad \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.87804$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.87804) \right]^2 = 0.31245$$

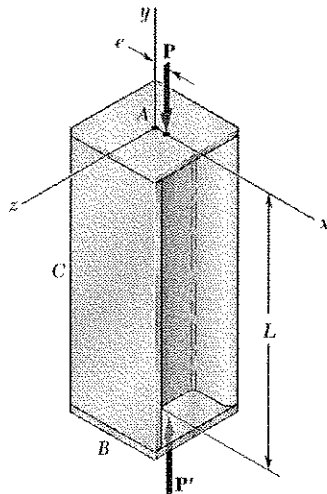
$$P_{cr} = \frac{P}{0.31245} = \frac{\pi^2 EI}{Le^2}$$

$$Le^2 = \frac{0.31245 \pi^2 EI}{P} = \frac{0.31245 \pi^2 (200 \times 10^9)(7.64 \times 10^{-6})}{250 \times 10^3} = 18.848 \text{ m}^2$$

$$Le = 4.34 \text{ m} = 2L$$

$$L = 2.17 \text{ m}$$

Problem 10.49



10.49 Axial loads of magnitude $P = 600 \text{ kN}$ are applied parallel to the geometric axis of the $W250 \times 80$ rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{all} = 84 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the largest permissible length L when (a) $e = 6 \text{ mm}$, (b) $e = 12 \text{ mm}$.

Data: $P = 600 \text{ kN}$ $E = 200 \text{ GPa}$

$W250 \times 80$ $A = 10200 \text{ mm}^2$ $c = \frac{b_f}{2} = 128 \text{ mm}$

$I_y = 43.1 \times 10^6 \text{ mm}^4$ $r_y = 65 \text{ mm}$

$\sigma_{all} = \sigma_{max} = 84 \text{ MPa}$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec} \left[\frac{A\sigma_{max}}{P} - 1 \right]$$

(a) $e = 6 \text{ mm}$ $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{(65)^2}{(6)(128)} \left[\frac{(10200)(84)}{600000} - 1 \right] = 2.3546$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = 0.4247$ $\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 1.132$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (1.132) \right]^2 = 0.519$

$P_{cr} = \frac{P}{0.519} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.519 \pi^2 EI}{P} = \frac{0.519 \pi^2 (200 \times 10^9) (43.1 \times 10^{-6})}{600000} = 73.59 \text{ m}^2$

$L_e = 8.579$ $L = L_e = 8.6 \text{ m}$

(b) $e = 12 \text{ mm}$ $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{(65)^2}{(12)(128)} \left[\frac{(10200)(84)}{600000} - 1 \right] = 1.1773$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = 0.8494$ $\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.5559$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.5559) \right]^2 = 0.1252$

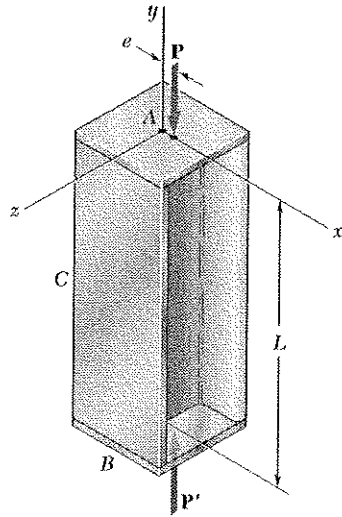
$P_{cr} = \frac{P}{0.1252} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.1252 \pi^2 EI}{P} = \frac{0.1252 \pi^2 (200 \times 10^9) (43.1 \times 10^{-6})}{600000} = 17.75 \text{ m}^2$

$L_e = 4.21 \text{ m}$ $L = L_e = 4.2 \text{ m}$

Problem 10.50

10.50 Axial loads of magnitude $P = 580 \text{ kN}$ are applied parallel to the geometric axis of the $W250 \times 80$ rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{\text{all}} = 75 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the largest permissible length L when (a) $e = 5 \text{ mm}$ (b) $e = 10 \text{ mm}$.



Data: $P = 580 \times 10^3 \text{ N}$ $E = 200 \times 10^9 \text{ Pa}$

$W 250 \times 80$ $A = 10200 \text{ mm}^2 = 10200 \times 10^{-6} \text{ m}^2$

$b_f = 255 \text{ mm}$ $c = \frac{b_f}{2} = 127.5 \text{ mm}$ $r_y = 65.0 \text{ mm}$

$I_y = 43.1 \times 10^6 \text{ mm}^4 = 43.1 \times 10^{-6} \text{ m}^4$

$\sigma_{\text{all}} = \sigma_{\text{max}} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$

$$\sigma_{\text{max}} = \frac{P}{A} \left[1 + \frac{ec}{r_y^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) \right]$$

$$\frac{A\sigma_{\text{max}}}{P} - 1 = \frac{ec}{r_y^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{r_y^2}{ec} \left(\frac{A\sigma_{\text{max}}}{P} - 1 \right)$$

(a) $e = 5 \text{ mm}$, $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{(65.0)^2}{(5)(127.5)} \left[\frac{(10200 \times 10^{-6})(75 \times 10^6)}{580 \times 10^3} - 1 \right] = 2.1139$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.47305$ $\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}} = 1.07804$

$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi} (1.07804) \right]^2 = 0.47101$

$P_{\text{cr}} = \frac{P}{0.47101} = \frac{\pi^2 EI_y}{L_e^2}$

$L_e^2 = \frac{0.47101 \pi^2 EI_y}{P} = \frac{0.47101 \pi^2 (200 \times 10^9)(43.1 \times 10^{-6})}{580 \times 10^3} = 69.09 \text{ m}^2$

$L_e = 8.31 \text{ m}$

$L = L_e$

$L = 8.31 \text{ m}$ $\blacktriangleleft$

(b) $e = 10 \text{ mm}$, $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{(65)^2}{(10)(127.5)} \left[\frac{(10200 \times 10^{-6})(75 \times 10^6)}{580 \times 10^3} - 1 \right] = 1.05696$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.94611$ $\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}} = 0.32980$

$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi} (0.32980) \right]^2 = 0.044083$

$P_{\text{cr}} = \frac{P}{0.044083} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.044083 \pi^2 EI}{P} = \frac{0.044083 \pi^2 (200 \times 10^9)(43.1 \times 10^{-6})}{580 \times 10^3} = 6.466 \text{ m}^2$

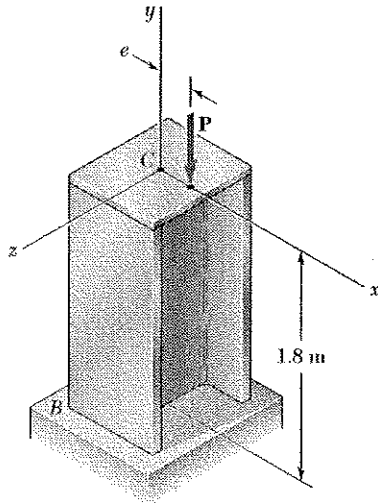
$L_e = 2.54 \text{ m}$

$L = L_e$

$L = 2.54 \text{ m}$ $\blacktriangleleft$

Problem 10.51

10.51 An axial load of magnitude $P = 220 \text{ kN}$ is applied at a point located on the x axis at a distance $e = 6 \text{ mm}$ from the geometric axis of the wide-flange column BC . Knowing that $E = 200 \text{ GPa}$, choose the lightest W200 shape that can be used if $\sigma_{\text{all}} = 120 \text{ MPa}$.



$$P = 220 \times 10^3 \text{ N} \quad L = 1.8 \text{ m} \quad L_e = 2L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{3.6^2} = 152.3 \times 10^3 I_y \text{ N}$$

$$e = 6 \text{ mm} \quad c = \frac{b_f}{2} \quad \frac{ec}{r^2} = \frac{eb_f}{2r_y^2} =$$

$$\sigma_{\max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

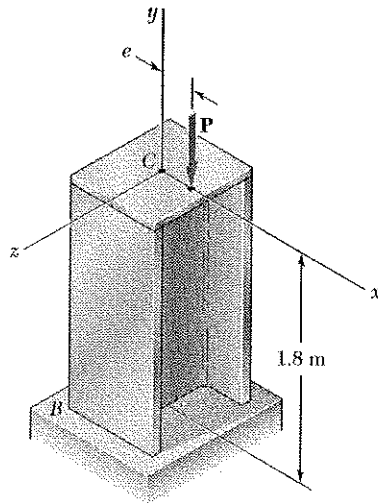
Shape	$A (10^6 \text{ m}^2)$	$b_f (\text{mm})$	$I_y (10^6 \text{ m}^4)$	$r_y (\text{mm})$	$P_{cr} (\text{kN})$	$\frac{ec}{r^2}$	$\sigma_{\max} (\text{MPa})$
W200 $\times$ 41.7	5310	166	9.01	41.2	1372	0.2934	56.5
W200 $\times$ 26.6	3390	133	3.30	31.2	502.6	0.4099	117.4
W200 $\times$ 22.5	2860	102	1.42	22.3	*216.3		

* < P

$$\sigma_{\max} = 117.4 \text{ MPa} < 120 \text{ MPa}$$

Use W200 $\times$ 26.6. $\blacktriangleleft$

Problem 10.52



10.52 Solve Prob. 10.51, assuming that the magnitude of the axial load is $P = 345$ kN.

10.51 An axial load of magnitude $P = 220$ kN is applied at a point located on the x axis at a distance $e = 6$ mm from the geometric axis of the wide-flange column BC . Knowing that $E = 200$ GPa, choose the lightest W200 shape that can be used if $\sigma_{all} = 120$ MPa.

$$P = 345 \times 10^3 \text{ N} \quad L = 1.8 \text{ m} \quad L_e = 2L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 E I_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{(3.6)^2} = 152.3 \times 10^9 I_y \text{ N}$$

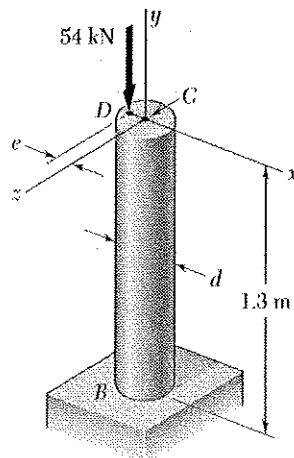
$$e = 6 \text{ mm} \quad c = \frac{b_f}{2} \quad \frac{ec}{r^2} = \frac{e b_f}{2 r_y^2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

Shape	$A (10^6 \text{ m}^2)$	$b_f (\text{mm})$	$I_y (10^6 \text{ m}^4)$	$r_y (\text{mm})$	$P_{cr} (\text{kN})$	$\frac{ec}{r^2}$	$\sigma_{max} (\text{MPa})$
W 200 $\times$ 41.7	5310	166	9.01	41.2	1372	0.2934	92.0
W 200 $\times$ 26.6	3390	133	3.30	31.2	502.6	0.4099	258
W 200 $\times$ 35.9	4580	165	7.64	40.8	1164	0.2974	109.5
W 200 $\times$ 31.3	4000	134	4.10	32.0	624.4	0.3926	172.6

$$\sigma_{max} = 109.5 \text{ MPa} < 120 \text{ MPa} \quad \text{Use W200} \times 35.9$$

Problem 10.53



10.53 A 54 kN axial load is applied with an eccentricity $e = 10$ mm to the circular steel rod BC that is free at its top C and fixed at its base B . Knowing that the stock of rods available for use have diameters in increments of 4 mm from 44 mm to 72 mm, determine the lightest rod that may be used if $\sigma_{all} = 110$ MPa. Use $E = 200$ GPa.

For a solid circular section $c = \frac{1}{2}d$

$$A = \pi c^2 = \frac{\pi}{4} d^2 \quad I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$$

$$r^2 = \frac{I}{A} = \frac{1}{16} d^2 \quad \frac{ec}{r^2} = \frac{8e}{d}$$

$$P = 54 \times 10^3 \text{ N} \quad e = 10 \text{ mm} = 0.010 \text{ m}$$

$$L_e = 2L = (2)(1.3) = 2.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P}{P_{cr}} = \frac{PL_e^2}{\pi^2 EI} = \frac{(54 \times 10^3)(2.6)^2(64)}{\pi^2(200 \times 10^9)\pi d^4} = 3.7674 \times 10^{-6} d^{-4}$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 3.0489 \times 10^{-3} d^{-2}$$

$$\frac{P}{A} = \frac{(54 \times 10^3)(4)}{\pi d^2} = 68.755 \times 10^3 d^{-2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right] \quad \frac{ec}{r^2} = \frac{0.08}{d}$$

$$\sigma_{max} = 68.755 \times 10^3 d^{-2} \left[1 + 0.08 d^{-1} \sec(3.0489 \times 10^{-3} d^{-2}) \right]$$

d (mm)	σ_{max} (MPa)
44	-1.5919 ($P/P_{cr} > 1$)
48	232
52	116.6
56	77.5 ←
60	57.5
64	45.3
68	37.0
72	31.0

The lightest section with $\sigma_{max} < 110$ MPa is $d = 56$ mm.

Checking: $d = 56 \text{ mm} = 0.056 \text{ m} \quad c = 0.028 \text{ m}$

$$A = \frac{\pi}{4} d^2 = 2.4630 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{64} d^4 = 482.75 \times 10^{-9} \text{ m}^4$$

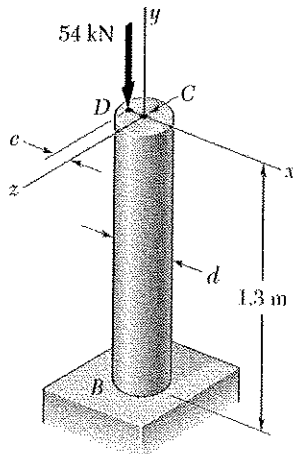
$$r^2 = \frac{I}{A} = 196 \times 10^{-6} \quad \frac{ec}{r^2} = \frac{(0.010)(0.028)}{196 \times 10^{-6}} = 1.4286$$

$$P_{cr} = \frac{\pi^2 (200 \times 10^9)(482.75 \times 10^{-9})}{(2.6)^2} = 140.96 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{54 \times 10^3}{140.96 \times 10^3} = 0.38308$$

$$\sigma_{max} = \frac{54 \times 10^3}{2.4630 \times 10^{-3}} \left[1 + 1.4286 \sec\left(\frac{\pi}{2} \sqrt{0.38308}\right) \right] = 77.5 \times 10^6 \text{ Pa}$$

Problem 10.54



10.54 Solve Prob. 10.53, assuming that the 54 kN axial load will be applied to the rod with an eccentricity $e = \frac{1}{2} d$.

10.53 A 54 kN axial load is applied with an eccentricity $e = 10$ mm to the circular steel rod BC that is free at its top C and fixed at its base B. Knowing that the stock of rods available for use have diameters in increments of 4 mm from 44 mm to 72 mm, determine the lightest rod that may be used if $\sigma_{all} = 110$ MPa. Use $E = 200$ GPa.

For a solid circular section $c = \frac{1}{2} d$
 $A = \pi c^2 = \frac{\pi}{4} d^2$ $I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$

$$r^2 = \frac{I}{A} = \frac{1}{16} d^2$$

Use $e = \frac{1}{2} d$ $\frac{ec}{r^2} = (\frac{1}{2})(\frac{1}{2})(16) = 4$

$$P = 54 \times 10^3 \text{ N}$$

$$L_e = 2L = (2)(1.3) = 2.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P}{P_{cr}} = \frac{PL_e^2}{\pi^2 EI} = \frac{(54 \times 10^3)(2.6)^2(64)}{\pi^2(200 \times 10^9) \pi d^4} = 3.7674 \times 10^{-6} d^{-4}$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 3.0489 \times 10^{-3} d^{-2}$$

$$\frac{P}{A} = \frac{(54 \times 10^3)(4)}{\pi d^2} = 68.755 \times 10^3 d^{-2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\sigma_{max} = 68.755 \times 10^3 d^{-2} \left[1 + 4 \sec(3.0489 \times 10^{-3} d^{-2}) \right]$$

$d(\text{mm})$	$\sigma_{max}(\text{MPa})$
44	-35.1
48	517
52	263
56	177.6
60	134.4
64	108.1 ←
68	90.1
72	77.0

The lightest section
 with $\sigma_{max} < 110$ MPa
 is $d = 64$ mm

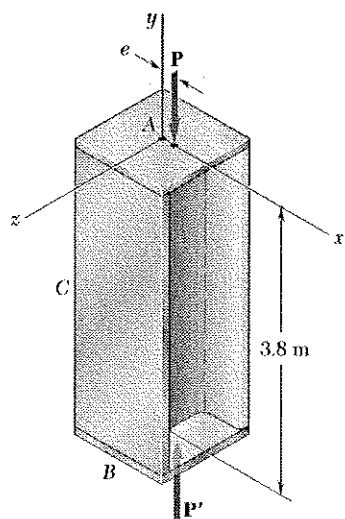
Checking $A = \frac{\pi}{4} (0.064)^2 = 3.217 \times 10^{-3} \text{ m}^2$, $I = \frac{\pi}{64} (0.064)^4 = 823.55 \times 10^{-9} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 (200 \times 10^9) (823.55 \times 10^{-9})}{(2.6)^2} = 240.48 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{54 \times 10^3}{240.48 \times 10^3} = 0.22455$$

$$\sigma_{max} = \frac{54 \times 10^3}{3.217 \times 10^{-3}} \left[1 + 4 \sec\left(\frac{\pi}{2} \sqrt{0.22455}\right) \right] = 108.1 \times 10^6 \text{ Pa}$$

Problem 10.55



10.55 Axial loads of magnitude $P = 175$ kN are applied parallel to the geometric axis of a $W250 \times 44.8$ rolled-steel column AB and intersect the axis at a distance $e = 12$ mm from its geometric axis. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (Hint: Since the factor of safety must be applied to the load P , not to the stresses, use Fig. 10.24 to determine P_Y .)

For W 250 x 44.8 $A = 5720 \text{ mm}^2$, $r_y = 35.1 \text{ mm}$

$L_e = 3800 \text{ mm}$ $L_e/r = 108.26$

$C = \frac{b_F}{2} = \frac{148}{2} = 74 \text{ mm}$ $e = 12 \text{ mm}$

$\frac{eC}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$

Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{eC}{r^2} = 0.72077$,

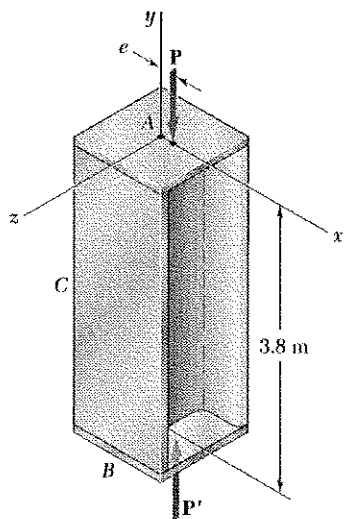
$P_Y/A = 90.37 \text{ MPa} = 90.37 \times 10^6 \text{ N/m}^2$

$P_Y = A(P_Y/A) = (5720 \times 10^{-6})(90.37 \times 10^6) = 517 \times 10^3 \text{ N} = 517 \text{ kN}$

F.S. = $\frac{P_Y}{P} = \frac{517}{175}$

F.S. = 2.95

Problem 10.56



10.56 Solve Prob. 10.55, assuming that $e = 0.16$ mm and $P = 155$ kN.

10.55 Axial loads of magnitude $P = 175$ kN are applied parallel to the geometric axis of a $W250 \times 44.8$ rolled-steel column AB and intersect the axis at a distance $e = 12$ mm from its geometric axis. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (Hint: Since the factor of safety must be applied to the load P , not to the stresses, use Fig. 10.24 to determine P_Y .)

For W 250 x 44.8 $A = 5720 \text{ mm}^2$, $r_y = 35.1 \text{ mm}$

$L_e = 3800 \text{ mm}$ $L_e/r = 108.26$

$C = \frac{b_F}{2} = \frac{148}{2} = 74 \text{ mm}$ $e = 16 \text{ mm}$

$\frac{eC}{r^2} = \frac{(16)(74)}{(35.1)^2} = 0.96103$

Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{eC}{r^2} = 0.96103$,

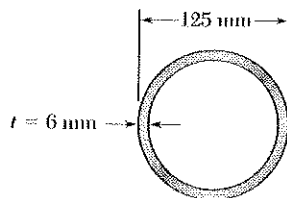
$P_Y/A = 81.17 \text{ MPa} = 81.17 \times 10^6 \text{ N/m}^2$

$P_Y = A(P_Y/A) = (5720 \times 10^{-6})(81.17 \times 10^6) = 464 \times 10^3 \text{ N} = 464 \text{ kN}$

F.S. = $\frac{P_Y}{P} = \frac{464}{155}$

F.S. = 3.00

Problem 10.57



10.57 A steel pipe having the cross section shown is used as a column. Using the AISC allowable stress design formulas, determine the allowable centric load if the effective length of the column is (a) 6 m, (b) 4 m. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$c = \frac{1}{2}d = 62.5 \text{ mm} \quad c_1 = c - t = 56.5 \text{ mm}$$

$$A = \pi(c^2 - c_1^2) = 2.2431 \times 10^3 \text{ mm}^2 = 2.2431 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{4}(c^4 - c_1^4) = 3.9807 \times 10^6 \text{ mm}^4 = 3.9807 \times 10^{-6} \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = 42.149 \text{ mm} = 42.149 \times 10^{-3} \text{ m}$$

Steel: $\sigma_Y = 250$ MPa $E = 200\,000$ MPa

transition $\frac{L_e}{r}$: $4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200\,000}{250}} = 133.22$

(a) $L_e = 6 \text{ m}$. $\frac{L_e}{r} = \frac{6}{42.149 \times 10^{-3}} = 142.352 > 133.22$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(142.352)^2} = 97.41 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_{cr}}{F.S.} = \frac{0.877 \sigma_e}{1.67} = 51.155 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (51.155 \times 10^6)(2.2431 \times 10^{-3}) \quad P_{all} = 114.7 \text{ kN} \quad \blacktriangleleft$$

(b) $L = 4 \text{ m}$. $\frac{L_e}{r} = \frac{4}{42.149 \times 10^{-3}} = 94.901 < 133.22$

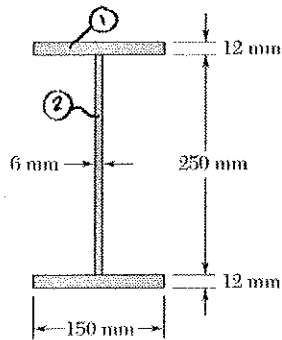
$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(94.901)^2} = 219.17 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{\sigma_Y/\sigma_e}] \sigma_Y = [0.658^{250/219.17}] (250) = 155.09 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_{cr}}{F.S.} = \frac{155.09}{1.67} = 92.871 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (92.871 \times 10^6)(2.2431 \times 10^{-3}) \quad P_{all} = 208 \text{ kN} \quad \blacktriangleleft$$

Problem 10.58



10.58 A column with the cross section shown has a 4-m effective length. Using allowable stress design, determine the largest centric load that can be applied to the column. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$A = 2A_1 + A_2 = (2)(12)(150) + (250)(6) = 5100 \text{ mm}^2$$

$$I_y = 2I_1 + I_2 = (2)\left(\frac{1}{12}\right)(12)(150)^3 + \left(\frac{1}{12}\right)(250)(6)^3 = 6.7545 \times 10^6 \text{ mm}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{6.7545 \times 10^6}{5100}} = 36.4 \text{ mm}$$

$$L_e = 4 \text{ m} \quad \frac{L_e}{r} = 109.9 < C_c$$

$$\text{Steel: } E = 200 \text{ GPa}, \sigma_Y = 250 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$\frac{L_e/r}{C_c} = 0.8743 \quad \text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.8743) - \frac{1}{8}(0.8743)^3 = 1.911$$

$$\sigma_{all} = \frac{\sigma_Y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.911} \left[1 - \frac{1}{2} (0.8743)^2 \right] = 80.82 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (80.82)(5100) = 412.2 \text{ kN}$$

Problem 10.59

10.59 Using allowable stress design, determine the allowable centric load for a column of 6-m effective length that is made from the following rolled-steel shape: (a) W200 × 35.9, (b) W200 × 86. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

Steel: $\sigma_Y = 250$ MPa $E = 200 \times 10^3$ MPa

transition L/r : $4.71 \sqrt{\frac{E}{\sigma_Y}} = 133.22$

(a) W 200 × 35.9 $A = 4580 \times 10^{-6} \text{ m}^2$ $r_y = 40.8 \times 10^{-3} \text{ m}$

$$\frac{L_e}{r_y} = \frac{6}{40.8 \times 10^{-3}} = 147.06 > 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^3)}{(147.06)^2} = 91.27 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_e}{F.S.} = \frac{(0.877)(91.27)}{1.67} = 47.932 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (47.932 \times 10^6)(4580 \times 10^{-6}) \quad P_{all} = 220 \text{ kN} \quad \blacktriangleleft$$

(b) W 200 × 86 $A = 11000 \times 10^{-6} \text{ m}^2$ $r_y = 53.2 \times 10^{-3} \text{ m}$

$$\frac{L_e}{r_y} = \frac{6}{53.2 \times 10^{-3}} = 112.782 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^3)}{(112.782)^2} = 155.185 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_e}{F.S.} = \frac{1}{1.67} [0.658^{250/155.185}] (250) = 76.276 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (76.276 \times 10^6)(11000 \times 10^{-6}) \quad P_{all} = 839 \text{ kN} \quad \blacktriangleleft$$

Problem 10.60

10.60 A W200 × 46.1 rolled-steel shape is used to form a column of 6-m effective length. Using allowable stress design, determine the allowable centric load if the yield strength of the grade of steel used is (a) $\sigma_Y = 250$ MPa, (b) $\sigma_Y = 345$ MPa. Use $E = 200$ GPa.

Steel: $E = 200 \text{ GPa}$

W200X46.1 $A = 5860 \text{ mm}^2$

$r_{\min} = 51.1 \text{ mm}$

$L_e = 6 \text{ m}$

$L_e/r = 117.4$

$$(a) \sigma_Y = 250 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{250}} = 125.7$$

$$L_e/r < C_c \quad \frac{L/r}{C_c} = 0.934$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.934) - \frac{1}{8}(0.934)^3 = 1.915$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L/r}{C_c} \right)^2 \right] = \frac{250}{1.915} \left[1 - \frac{1}{2} (0.934)^2 \right] = 73.6 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (73.6)(5860) = 431.3 \text{ kN}$$

$$(b) \sigma_Y = 345 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{345}} = 106.97$$

$$L_e/r > C_c \quad \sigma_{all} = \frac{\pi^2 E}{\frac{23}{12} (L/r)^2} = 74.72 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (74.72)(5860) = 437.9 \text{ kN}$$

Problem 10.61

10.61 A column having a 3.5-m effective length is made of sawn lumber with a 114×140 -mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 7.6$ MPa and the adjusted modulus $E = 2.8$ GPa, determine the maximum allowable centric load for the column.

Sawn lumber: $c = 0.8$ $\sigma_c = 7.6$ MPa $E = 2800$ MPa

$$A = (114)(140) = 15960 \text{ mm}^2 = 15960 \times 10^{-6} \text{ m}^2$$

$$d = 114 \text{ mm} = 114 \times 10^{-3} \text{ m}$$

$$L/d = 3.5 / 114 \times 10^{-3} = 30.70$$

$$\sigma_{ce} = \frac{0.822 E}{(L/d)^2} = \frac{(0.822)(2800)}{(30.70)^2} = 2.442 \quad \frac{\sigma_{ce}}{\sigma_c} = 0.32132$$

$$U = \frac{1 + \sigma_{ce}/\sigma_c}{2c} = \frac{1 + 0.32132}{(2)(0.8)} = 0.82583 \quad V = \frac{\sigma_{ce}/\sigma_c}{c} = 0.40165$$

$$C_p = U - \sqrt{U^2 - V} = 0.29635$$

$$\sigma_{all} = C_p \sigma_c = (0.29635)(7.6) = 2.252 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (2.252 \times 10^6)(15960 \times 10^{-6}) \quad P_{all} = 35.9 \text{ kN}$$

Problem 10.62

10.62 A sawn lumber column with a 190×140 -mm cross section has a 5.5-m effective length. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 8.5$ MPa and that $E = 9$ GPa, determine the maximum allowable centric load for the column.

Sawn lumber: $c = 0.8$, $\sigma_c = 8.5$ MPa $E = 9600$ MPa $K_{ce} = 0.3$

$$A = (190)(140) = 26600 \text{ mm}^2 \quad d = 140 \text{ mm} \quad L = 5.5 \text{ m}$$

$$L/d = 5500 / 140 = 39.286$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{(0.3)(9 \times 10^9)}{(39.286)^2} = 1.75 \text{ MPa} \quad \frac{\sigma_{ce}}{\sigma_c} = 0.206$$

$$U = \frac{1 + \sigma_{ce}/\sigma_c}{2c} = \frac{1.206}{(2)(0.8)} = 0.75375 \quad V = \frac{\sigma_{ce}/\sigma_c}{c} = 0.2575$$

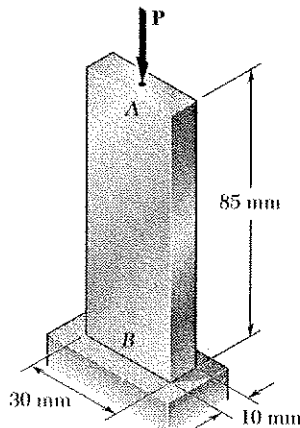
$$C_p = U - \sqrt{U^2 - V} = 0.1964$$

$$\sigma_{all} = \sigma_c C_p = (8.5)(0.1964) = 1.67 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (1.67)(26600) = 44.4 \text{ kN}$$

Problem 10.63

10.63 Bar AB is free at its end A and fixed at its base B. Determine the allowable centric load **P** if the aluminum alloy is (a) 6061-T6, (b) 2014-T6.



$$A = (30)(10) = 300 \text{ mm}^2 = 300 \times 10^{-6} \text{ m}^2$$

$$I_{\min} = \frac{1}{12}(30)(10)^3 = 2.50 \times 10^3 \text{ mm}^4$$

$$r_{\min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.50 \times 10^3}{300}} = 2.887 \text{ mm}$$

$$L_e = 2L = (2)(85) = 170 \text{ mm} \quad \frac{L_e}{r_{\min}} = 58.88$$

(a) 6061-T6 $L/r < 66$

$$\sigma_{\text{all}} = 139 - 0.868(L/r) = 139 - (0.868)(58.88) = 87.9 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (87.9 \times 10^6)(300 \times 10^{-6}) = 26.4 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 26.4 \text{ kN} \quad \blacktriangleleft$$

(b) 2014-T6 $L/r > 55$

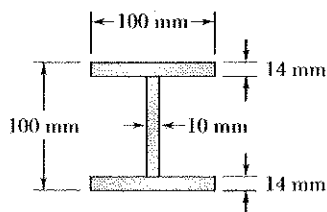
$$\sigma_{\text{all}} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(58.88)^2} = 107.3 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (107.3 \times 10^6)(300 \times 10^{-6}) = 32.2 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 32.2 \text{ kN} \quad \blacktriangleleft$$

Problem 10.64

10.64 A compression member has the cross section shown and an effective length of 1.5 m. Knowing that the aluminum alloy used is 6061-T6, determine the allowable centric load.



$$I_x = \frac{1}{12}(10)(72)^3 + 2\left[\frac{1}{12}(100)(14)^3 + (100)(14)(43)^2\right] = 5533973 \text{ mm}^4$$

$$I_y = \frac{1}{12}(72)(10)^3 + 2\left[\frac{1}{12}(14)(100)^3\right] = 2339333 \text{ mm}^4 = I_{\min}$$

$$A = (10)(72) + (2)(100)(14) = 3520 \text{ mm}^2$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{2339333}{3520}} = 25.78 \text{ mm}$$

$$L = 1.5 \text{ m} \quad \frac{L}{r} = \frac{1500}{25.78} = 58.185$$

6061 T6 aluminum alloy with $L/r < 66$.

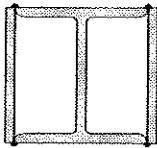
$$\sigma_{\text{all}} = 139 - (0.868)(58.185) = 88.49 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (88.49 \times 10^6)(3520 \times 10^{-6})$$

$$P_{\text{all}} = 311.2 \text{ kN} \quad \blacktriangleleft$$

Problem 10.65

10.65 and 10.66 A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a W250 × 80 rolled-steel shape as shown. Knowing that $\sigma_y = 345$ MPa, and $E = 200$ GPa and using allowable stress design, determine the allowable centric load for the compression member.



$$\text{For } W250 \times 80, \quad A = 10200 \text{ mm}^2, \quad d = 256 \text{ mm}, \quad b_f = 255 \text{ mm} \\ I_x = 126 \times 10^6 \text{ mm}^4, \quad I_y = 43.1 \times 10^6 \text{ mm}^4$$

$$\text{For one plate: } A = (256)(10) = 2560 \text{ mm}^2$$

$$I_x = \frac{1}{12}(10)(256)^3 = 13.981 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12}(256)(10)^3 + (2560)\left(\frac{255}{2} + \frac{10}{2}\right)^2 = 44.965 \times 10^6 \text{ mm}^4$$

$$\text{For column: } A = 10200 + (2)(2560) = 15.32 \times 10^3 \text{ mm}^2 = 15.32 \times 10^{-3} \text{ m}^2$$

$$I_x = 126 \times 10^6 + (2)(13.981 \times 10^6) = 153.962 \times 10^6 \text{ mm}^4$$

$$I_y = 43.1 \times 10^6 + (2)(44.965 \times 10^6) = 133.03 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{133.03 \times 10^6}{15.32 \times 10^3}} = 93.185 \text{ mm} = 93.185 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{9}{93.185 \times 10^{-3}} = 96.582$$

$$\text{Steel: transition } L/r = 4.71 \sqrt{\frac{E}{\sigma_y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40 > 96.582$$

$$\sigma_c = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(96.582)^2} = 211.61 \text{ MPa}$$

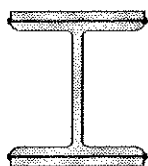
$$\sigma_{\text{all}} = \frac{\sigma_c}{\text{F.S.}} = \frac{1}{1.67} \left[0.658^{345/211.61} \right] (345) = 104.41 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (104.41 \times 10^6)(15.32 \times 10^{-3})$$

$$P_{\text{all}} = 1600 \text{ kN} \quad \blacktriangleleft$$

Problem 10.66

10.65 and 10.66 A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a W250 × 80 rolled-steel shape as shown. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa and using allowable stress design, determine the allowable centric load for the compression member.



For W 250 × 80 $A = 10200 \text{ mm}^2$, $d = 256 \text{ mm}$, $b_f = 255 \text{ mm}$
 $I_x = 126 \times 10^6 \text{ mm}^4$, $I_y = 43.1 \times 10^6 \text{ mm}^4$

For one plate: $A = (255)(10) = 2550 \text{ mm}^2$
 $I_x = \frac{1}{12}(255)(10)^3 + (2550)\left(\frac{256}{2} + \frac{10}{2}\right)^2 = 45.128 \times 10^6 \text{ mm}^4$
 $I_y = \frac{1}{12}(10)(255)^3 = 13.818 \times 10^6 \text{ mm}^4$

For column: $A = 10200 + (2)(2550) = 15300 \text{ mm}^2 = 15.3 \times 10^{-3} \text{ m}^2$

$I_x = 126 \times 10^6 + (2)(45.128 \times 10^6) = 216.256 \times 10^6 \text{ mm}^4$

$I_y = 43.1 \times 10^6 + (2)(13.818 \times 10^6) = 70.736 \times 10^6 \text{ mm}^4 = I_{\min}$

$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{70.736 \times 10^6}{15300}} = 67.995 \text{ mm} = 67.995 \times 10^{-3} \text{ m}$

$\frac{L_e}{r} = \frac{9}{67.995 \times 10^{-3}} = 132.363$

Steel: transition $\frac{L}{r}: 4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40 < 132.363$

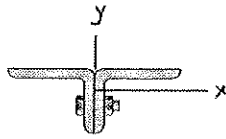
$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(132.363)^2} = 112.67 \text{ MPa}$

$\sigma_{\text{all}} = \frac{\sigma_e}{\text{F.S.}} = \frac{0.877 \sigma_e}{1.67} = 59.167 \text{ MPa}$

$P_{\text{all}} = \sigma_{\text{all}} A = (59.167 \times 10^6)(15.3 \times 10^{-3})$

$P_{\text{all}} = 905 \text{ kN}$

Problem 10.67



10.67 A compression member of 2.3-m effective length is obtained by bolting together two $127 \times 76 \times 12.7$ -mm steel angles as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 250$ MPa, and $E = 200$ GPa.

$$4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

L $127 \times 76 \times 12.7$ mm Table gives $A_L = 2420 \text{ mm}^2$, $I_x = 3.93 \times 10^6 \text{ mm}^4$

$$y = 44.4 \text{ mm}, \quad I_y = 1.06 \times 10^6 \text{ mm}^4, \quad x = 19.0 \text{ mm}, \quad r_y =$$

For column,

$$I_x = 2(I_y)_{\text{angle}} = (2)(1.06 \times 10^6) = 2.12 \times 10^6 \text{ mm}^4$$

$$I_y > I_x \quad I_{\min} = I_x = 2.12 \times 10^6 \text{ mm}^4 = 2.12 \times 10^{-6} \text{ m}^4$$

$$A = 2A_L = 4840 \text{ mm}^2 = 4840 \times 10^{-6} \text{ m}^2$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{2.12 \times 10^{-6}}{4840 \times 10^{-6}}} = 20.93 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.3}{20.93 \times 10^{-3}} = 109.90 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(109.90)^2} = 163.43 \text{ MPa}$$

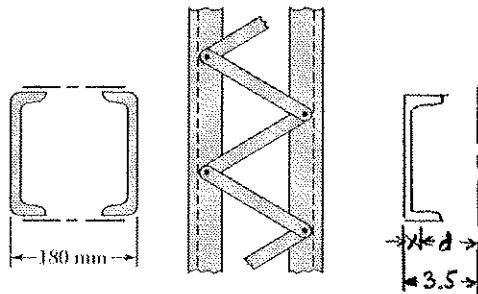
$$\sigma_{\text{all}} = \frac{\sigma_c}{F.S.} = \frac{1}{1.67} \left[0.658^{250/163.43} \right] (250) = 78.916 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (78.916 \times 10^6) (4840 \times 10^{-6})$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (78.916 \times 10^6) (4840 \times 10^{-6})$$

$$P_{\text{all}} = 382 \text{ kN}$$

Problem 10.68



10.68 A column of 6-m effective length is obtained by connecting two C250 × 30 steel channels with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

C250 × 30 $A = 3780 \text{ mm}^2$ $x = 15.3 \text{ mm}$
 $I_x = 32.6 \times 10^6 \text{ mm}^4$ $I_y = 1.14 \times 10^6 \text{ mm}^4$
 $d = 90 - x = 74.7 \text{ mm}$

For the column: $A = 2(3780) = 7560 \text{ mm}^2$

$I_x = (2)(32.6 \times 10^6) = 65.2 \times 10^6 \text{ mm}^4$

$I_y = 2[1.14 \times 10^6 + (3780)(74.7)^2] = 44.47 \times 10^6 \text{ mm}^4$

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{44.47 \times 10^6}{7560}} = 76.7 \text{ mm}$ $L_e = 6 \text{ m}$

$\frac{L_e}{r} = 78.23$ $C_c = \sqrt{\frac{2\pi^2 E'}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{250}} = 125.7$

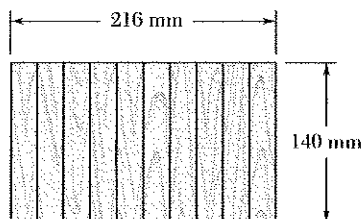
$\frac{L_e}{r} < C_c$ $\frac{L_e/r}{C_c} = 0.6224$

F.S. = $\frac{5}{3} + \frac{3}{8}(0.6224) - \frac{1}{8}(0.6224)^3 = 1.87$

$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.87} \left[1 - \frac{1}{2} (0.6224)^2 \right] = 107.8 \text{ MPa}$

$P_{all} = \sigma_{all} A = (107.8)(7560) = 815 \text{ kN}$

Problem 10.69



10.69 A rectangular column with a 4.4-m effective length is made of glued laminated wood. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 8.3$ MPa and the adjusted modulus $E = 4.6$ GPa, determine the maximum allowable centric load for the column.

Glued laminated column: $c = 0.9$ $E = 4600 \text{ MPa}$

$A = (216)(140) = 30240 \text{ mm}^2 = 30240 \times 10^{-6} \text{ m}^2$

$d = 140 \text{ mm} = 140 \times 10^{-3} \text{ m}$ $L = 4.4 \text{ m}$

$\frac{L}{d} = \frac{4.4}{140 \times 10^{-3}} = 31.429$

$\sigma_{ce} = \frac{0.822 E}{(L/d)^2} = \frac{(0.822)(4600)}{(31.429)^2} = 3.8281 \text{ MPa}$

$\frac{\sigma_{ce}}{\sigma_c} = \frac{3.8281}{8.3} = 0.46121$

$U = \frac{1 + \sigma_{ce}/\sigma_c}{2c} = \frac{1.46121}{2(0.9)} = 0.81178$

$v = \frac{\sigma_{ce}/\sigma_c}{c} = 0.51246$

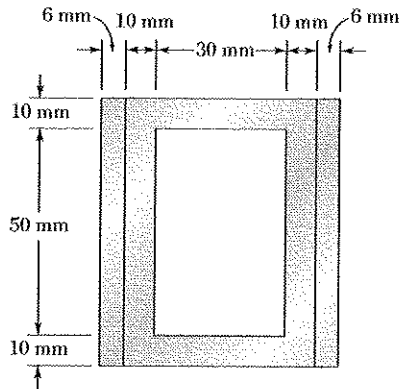
$C_p = U - \sqrt{U^2 - v} = 0.42908$

$\sigma_{all} = C_p \sigma_c = (0.42908)(8.3) = 3.5614 \text{ MPa}$

$P_{all} = \sigma_{all} A = (3.5614 \times 10^6)(30240 \times 10^{-6})$

$P_{all} = 107.7 \text{ kN}$

Problem 10.70



10.70 An aluminum structural tube is reinforced by riveting two plates to it as shown for use as a column of 1.7-m effective length. Knowing that all material is aluminum alloy 2014-T6, determine the maximum allowable centric load.

$$b_o = 2(6) + 2(10) + 30 = 62 \text{ mm}$$

$$b_i = 30 \text{ mm}$$

$$h_o = 2(10) + 50 = 70 \text{ mm}$$

$$h_i = 50 \text{ mm}$$

$$A = b_o h_o - b_i h_i = (62)(70) - (30)(50) = 2840 \text{ mm}^2$$

$$I_x = \frac{1}{12} [b_o h_o^3 - b_i h_i^3] = \frac{1}{12} [(62)(70)^3 - (30)(50)^3] = 1.46 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12} [h_o b_o^3 - h_i b_i^3] = \frac{1}{12} [(70)(62)^3 - (50)(30)^3] = 1.278 \times 10^6 \text{ mm}^4 = I_{\min}$$

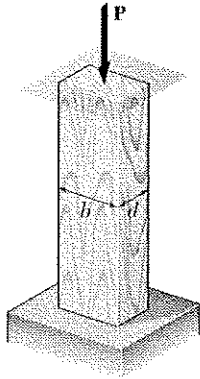
$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1.278 \times 10^6}{2840}} = 21.2 \text{ mm} \quad L_e = 1700 \text{ mm}$$

$$\frac{L_e}{r} = \frac{1700}{21.2} = 80.2 \geq 55$$

$$\sigma_{\text{all}} = \frac{372000}{(L_e/r)^2} = 57.84 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (57.84)(2840) = 164.3 \text{ kN}$$

Problem 10.71



10.71 An 75-kN centric load is applied to a rectangular sawn lumber column of 6.5-m effective length. Using sawn lumber for which the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 7.3$ MPa and knowing that $E = 10$ GPa, determine the smallest square cross section that may be used. Use $b = 2d$.

Sawn lumber: $c = 0.8$ $K_{CE} = 0.3$

$\sigma_c = 7.3$ MPa $E = 10$ GPa

$A = 2d^2$ $L = 6.5$ m

$L/d = \frac{6000}{d}$

Assumed $C_p = 0.5$

$\sigma_{all} = \sigma_c C_p = (7.3)(0.5) = 3.65$ MPa.

$P_{all} = \sigma_{all} A = 2\sigma_{all} d^2$

$d = \sqrt{\frac{P_{all}}{2\sigma_{all}}} = \sqrt{\frac{75000}{2(3.65)}} = \frac{193.65}{\sqrt{7.3}} = 101.4$ mm

$L/d = 59.17$

$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.3)(10 \times 10^3)}{(L/d)^2} = \frac{3 \times 10^3}{(L/d)^2} = 0.86$ MPa.

$\sigma_{CE} / \sigma_c = 0.1178$

Checked: $C_p = \frac{1 + \sigma_{CE} / \sigma_c}{2C} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2C}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{C}} = 0.1148$

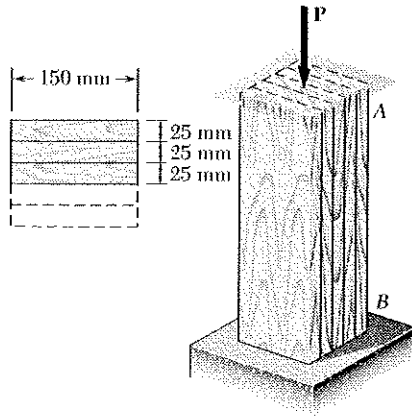
Results of similar trials are summarized in the table below:

Assumed C_p	σ_{all} (MPa)	d (mm)	L/d	σ_{CE} (MPa)	σ_{CE} / σ_c	Checked C_p	ΔC_p
0.5	3.65	101.4	59.17	0.86	0.1178	0.1148	-0.3852
0.2983	2.18	131.3	45.70	1.44	0.1975	0.2017	-0.0966
0.2340	1.708	148.2	40.49	1.84	0.2517	0.2538	0.0198
0.25	1.825	143.3	41.87	1.71	0.2342	0.2517	0.0017

$d = 143$ mm.

Problem 10.72

10.72 A column of 2.1-m effective length is to be made by gluing together laminated wood pieces of 25×150 -mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 7.7$ MPa and the adjusted modulus is $E = 5.4$ GPa, determine the number of wood pieces that must be used to support the concentric load shown when (a) $P = 52$ kN, (b) $P = 108$ kN.



Glued laminated column $c = 0.90$

Let n = number of pieces

$$A = (150)(25n) = 3750n \text{ mm}^2 = 3750 \times 10^{-6} n \text{ m}^2$$

$$d = 25 \times 10^{-3} n \text{ m if } n \leq 6$$

$$d = 150 \times 10^{-3} \text{ m if } n > 6$$

$$L_e = 2.1 \text{ m}$$

$$\frac{L_e}{d} = \frac{2.1}{25 \times 10^{-3} n} = \frac{84}{n} \text{ if } n \leq 6$$

$$= \frac{2.1}{0.15} = 14 \text{ if } n \geq 6$$

$$\sigma_c = 7.7 \text{ MPa} \quad E = 5400 \text{ MPa} \quad \sigma_{ce} = \frac{(0.822)(5400)}{(L/d)^2} = \frac{4439}{(L/d)^2} \text{ MPa}$$

$$C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \left(\frac{1 + \sigma_{ce}/\sigma_{ce}}{2c} \right)^2 - \frac{\sigma_{ce}/\sigma_c}{c}$$

$$\sigma_{all} = C_p \sigma_c$$

$$P_{all} = \sigma_{all} A$$

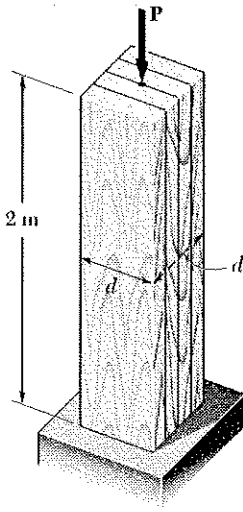
Calculations are carried out in the table below.

n	d (m)	A (10^{-6} m^2)	L/d	σ_{ce} (MPa)	σ_{ce}/σ_c	C_p	σ_{all} (MPa)	P_{all} kN
3	0.075	11250	28	5.662	0.7353	0.6288	4.842	54.5
4	0.100	15000	21	10.066	1.3073	0.8453	6.509	97.6
5	0.125	18750	16.8	15.727	2.0421	0.9237	7.412	133.6

(a) For $P = 52$ kN use $n = 3$ ▶

(b) For $P = 108$ kN use $n = 5$ ▶

Problem 10.73



10.73 The glued laminated column shown is free at its top *A* and fixed at its base *B*. Using wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 9.2$ MPa and an adjusted modulus of elasticity $E = 5.7$ GPa, determine the smallest cross section that can support a centric load of 62 kN.

Glued laminated column $c = 0.9$ $E = 5700$ MPa

$$L_e = 2L = (2)(2) = 4 \text{ m} = 4000 \text{ mm}$$

$$\sigma_c = 9.2 \text{ MPa}$$

$$A = d^2 \quad P_{all} = 62000 \text{ N}$$

$$L_e/d = 4000/d \quad \text{with } d \text{ in mm.}$$

$$\sigma_{all} = \sigma_c C_p$$

$$\text{Assume } C_p, \quad \sigma_{all} = 9.2 C_p \text{ (MPa)}$$

$$d = \sqrt{\frac{P_{all}}{\sigma_{all}}} = \sqrt{\frac{62000}{\sigma_{all}}} = \frac{249}{\sqrt{\sigma_{all}}}$$

$$\sigma_{ce} = \frac{0.822 E}{(L/d)^2} = \frac{(0.822)(5700)}{(L/d)^2} = \frac{4685}{(L/d)^2} \text{ (MPa)}$$

$$\text{Checking: } C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{ce}/\sigma_c}{c}}$$

Calculations are carried out in the following table:

C_p (assumed)	σ_{all} (MPa)	d mm	L/d	σ_{ce} (MPa)	σ_{ce}/σ_c	C_p (calc)	ΔC_p
0.5	4.6	116.1	34.45	3.948	0.4291	0.4021	-0.0979
0.4	3.68	129.8	30.82	4.932	0.5361	0.4892	+0.0892
0.448	4.122	121.7	32.87	4.336	0.4713	0.4373	-0.0107
0.443	4.076	123.3	32.44	4.452	0.4839	0.4476	+0.0046

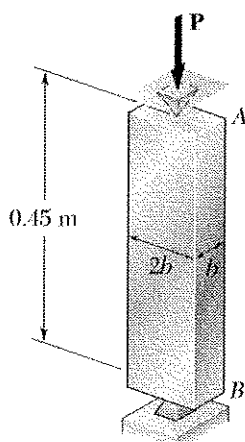
$$C_p = 0.443 + \frac{0.0046}{0.0153}(0.005) = 0.4445$$

$$\sigma_{all} = (0.4445)(9.2) = 4.0894 \text{ MPa}$$

$$d = \frac{249}{\sqrt{4.0894}} = 123.1 \text{ mm}$$

$$d = 123.1 \text{ mm} \quad \blacktriangleleft$$

Problem 10.74



10.74 A 72-kN centric load must be supported by an aluminum column as shown. Using the aluminum alloy 6061-T6, determine the minimum dimension b that can be used.

$$L_e = L = 0.45 \text{ m}, \quad A = 2b^2 \quad I_{min} = \frac{1}{12} (2b)(b)^3 = \frac{1}{6} b^4$$

$$r = \sqrt{\frac{I_{min}}{A}} = \frac{b}{\sqrt{12}} \quad \frac{L_e}{r} = \frac{0.45\sqrt{12}}{b} = \frac{1.55885}{b}$$

6061-T6 aluminum alloy. Assume $\frac{L_e}{r} < 66$.

$$\begin{aligned} \sigma_{all} &= 139 - 0.868 \left(\frac{L_e}{r} \right) = 139 - \frac{(0.868)(1.55885)}{b} \\ &= 139 - \frac{1.35308}{b} \text{ MPa} \end{aligned}$$

$$P_{all} = \sigma_{all} A = \left(139 - \frac{1.35308}{b} \right) (10^6) (2b^2)$$

$$72 \times 10^3 = 278 \times 10^6 b^2 - 2.70616 \times 10^6 b$$

$$278 b^2 - 2.70616 b - 0.072 = 0$$

$$b = \frac{2.70616 + \sqrt{(2.70616)^2 - (4)(278)(-0.072)}}{(2)(278)} = 21.680 \times 10^{-3} \text{ m}$$

$$r = 6.2586 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{0.45}{6.2586 \times 10^{-3}} = 71.9 \quad \text{Assumption is not verified.}$$

$$\text{Assume } \frac{L_e}{r} > 66. \quad \sigma_{all} = \frac{351 \times 10^3}{\left(\frac{L_e}{r} \right)^2} = \frac{351 \times 10^3 b^2}{(1.55885)^2} = 144.444 \times 10^3 b^2 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (144.444 \times 10^3 b^2) (10^6) (2b^2) = 288.89 \times 10^9 b^4 \text{ N}$$

$$72 \times 10^3 = 288.89 \times 10^9 b^4 \quad b^4 = 249.23 \times 10^{-9} \text{ m}^4$$

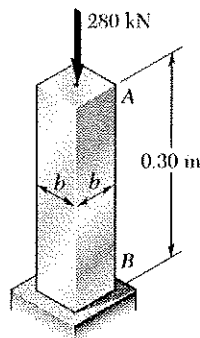
$$b = 22.3 \times 10^{-3} \text{ m} \quad r = 6.45 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = 69.77 > 66$$

Assumption is verified.

$$b = 22.3 \times 10^{-3} \text{ m} = 22.3 \text{ mm} \quad \blacktriangleleft$$

Problem 10.75

10.75 A 280-kN centric load is applied to the column shown, which is free at its top *A* and fixed at its base *B*. Using aluminum alloy 2014-T6, select the smallest square section that can be used.



$$L_e = 2L = (2)(0.30) = 0.60 \text{ m}$$

$$A = b^2 \quad I = \frac{1}{12} b^4 \quad r = \sqrt{\frac{I}{A}} = \frac{b}{\sqrt{12}}$$

$$\frac{L_e}{r} = \frac{0.60 \sqrt{12}}{b} = \frac{2.0785}{b}$$

2014-T6 aluminum alloy

$$\text{Assume } \frac{L_e}{r} < 55. \quad \sigma_{all} = 212 - 1.585(L/r) = 212 - (1.585)(2.0785/b) \\ = \left(212 - \frac{3.294}{b}\right) \text{ MPa} = \left[212 - \frac{3.294}{b}\right](10^6) \text{ Pa}$$

$$P_{all} = \sigma_{all} A = \left[212 b^2 - 3.294 b\right](10^6) = 280 \times 10^3$$

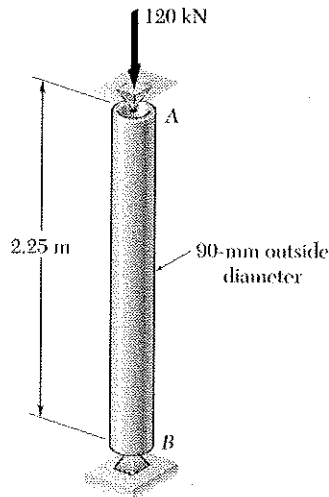
$$212 b^2 - 3.294 b - 280 \times 10^{-3} = 0$$

$$b = \frac{3.294 + \sqrt{(3.294)^2 + (4)(212)(280 \times 10^{-3})}}{(2)(212)} = 44.9 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.0785}{b} = \frac{2.0785}{44.9 \times 10^{-3}} = 46.26 < 55 \quad \text{Assumption is verified.}$$

$$b = 44.9 \text{ mm} \quad \blacktriangleleft$$

Problem 10.76



10.76 An aluminum tube of 90-mm outer diameter is to carry a centric load of 120 kN. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and with wall thicknesses in increments of 3 mm from 6 mm to 15 mm, determine the lightest tube that can be used.

$$L = 2250 \text{ mm}, P = 120 \times 10^3 \text{ N} \quad r_o = 45 \text{ mm}$$

$$r_i = r_o - t \quad A = \pi(r_o^2 - r_i^2) \quad I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$r = \sqrt{I/A}$$

For 2014-T6 aluminum alloy,

$$\sigma_{all} = 212 - 1.585(L/r) \text{ MPa} \quad \text{if } L/r < 55$$

$$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} \text{ MPa} \quad \text{if } L/r > 55$$

$$P_{all} = \sigma_{all} A$$

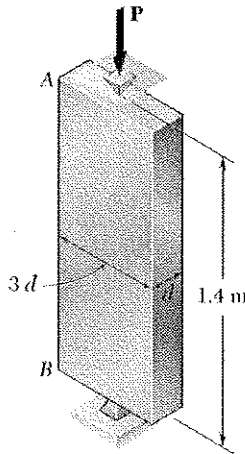
Calculate P_{all} for each thickness.

t mm	r_i mm	A mm^2	I 10^6 mm^4	r mm	L/r	σ_{all} MPa	P_{all} kN
6	39	1583	1.404	29.78	75.56	65.16	103.1
9	36	2290	1.901	28.82	78.08	61.01	139.7
12	33	2941	2.289	27.90	80.65	57.20	168.2
15	30	3534	2.584	27.04	83.20	53.74	189.9

Since P_{all} must be greater than 120 kN, use $t = 9 \text{ mm}$.

Problem 10.77

10.77 A centric load P must be supported by the steel bar AB . Using allowable stress design, determine the smallest dimension d of the cross section that can be used when (a) $P = 108 \text{ kN}$, (b) $P = 166 \text{ kN}$. Use $\sigma_r = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$.



$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250}} = 125.66$$

$$L_e = L = 1.4 \text{ m}$$

$$A = (3d)(d) = 3d^2$$

$$I = \frac{1}{12}(3d)(d)^3 = \frac{1}{4}d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} = 0.288675 d$$

(a) $P = 108 \times 10^3 \text{ N}$. Assume $\frac{L_e}{r} > C_c$.

$$P_{cr} = \frac{\pi^2 EI}{1.92 L_e^2} \quad I = \frac{(1.92)(P_{all} L_e^2)}{\pi^2 E} = \frac{1}{4}d^4$$

$$d^4 = \frac{(4)(1.92) P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(108 \times 10^3)(1.4)^2}{\pi^2 (200 \times 10^9)} = 823.59 \times 10^{-9} \text{ m}^4$$

$$d = 30.125 \times 10^{-3} \text{ m} \quad r = 8.696 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{8.696 \times 10^{-3}} = 160.99 > 125.66 \quad d = 30.1 \text{ mm}$$

(b) $P = 166 \times 10^3 \text{ N}$. Assume $\frac{L_e}{r} > C_c$

$$d^4 = \frac{(4)(1.92) P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(166 \times 10^3)(1.4)^2}{\pi^2 (200 \times 10^9)} = 1.26588 \times 10^{-6} \text{ m}^4$$

$$d = 33.543 \times 10^{-3} \text{ m} \quad r = 9.68295 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{9.68295 \times 10^{-3}} = 144.58 > 125.66 \quad d = 33.5 \text{ mm}$$

Problem 10.78

10.78 A column of 4.5-m effective length must carry a centric load of 900 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use allowable stress design to select the wide-flange shape of 250-mm nominal depth that should be used.

$$\text{transition } \frac{L}{r}: \quad 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40$$

$$P < \frac{\sigma_Y A}{1.67} \quad A > \frac{1.67 P}{\sigma_Y} = \frac{(1.67)(900 \times 10^3)}{345 \times 10^6} = 4357 \times 10^{-6} \text{ m}^2 = 4357 \text{ mm}^2$$

$$P < \frac{\sigma_{cr} A}{1.67} = \frac{0.877 \pi^2 E I}{1.67 L_e^2}$$

$$I > \frac{1.67 P L_e^2}{0.877 \pi^2 E} = \frac{(1.67)(900 \times 10^3)(4.5)^2}{0.877 \pi^2 (200 \times 10^9)} = 17.58 \times 10^{-6} \text{ m}^4 = 17.58 \times 10^6 \text{ mm}^4$$

Try W 250 × 58. $A = 7420 \text{ mm}^2$ $r_y = 50.3 \text{ mm}$

$$\frac{L_e}{r} = \frac{4.5}{50.3 \times 10^{-3}} = 89.46 < 113.40$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(89.46)^2} = 246.64 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_e}) \sigma_Y = [0.658^{345/246.64}] (345) = 192.11 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(192.11 \times 10^6)(7420 \times 10^{-6})}{1.67} = 854 \text{ kN} < 900 \text{ kN}$$

(not acceptable)

Try W 250 × 67 $A = 8580 \text{ mm}^2$ $r_y = 51.0 \text{ mm}$

$$\frac{L_e}{r} = \frac{4.5}{51.0 \times 10^{-3}} = 88.235 < 113.40$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(88.235)^2} = 253.54 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_e}) \sigma_Y = [0.658^{345/253.54}] (345) = 195.20 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(195.20 \times 10^6)(8580 \times 10^{-6})}{1.67} = 1003 \text{ kN} > 900 \text{ kN}$$

(acceptable)

Use W 250 × 67. 

Problem 10.79

10.79 A column of 6.8-m effective length must carry a centric load of 1200 kN. Using allowable stress design, select the wide-flange shape of 350-mm nominal depth that should be used. Use $\sigma_y = 350$ MPa and $E = 200$ GPa.

$$P < \frac{\sigma_y A}{F.S.} \quad A > \frac{(F.S.)P}{\sigma_y} = \frac{(5/3)(1200 \times 10^3)}{350 \times 10^6} = 5.714 \times 10^{-6} \text{ m}^2$$

$$L_e = 6.8 \text{ m}$$

$$E = 200 \text{ GPa}$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2} \quad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(1.2 \times 10^6)(6.8)^2}{\pi^2 (200 \times 10^9)} = 53.97 \times 10^{-6} \text{ m}^4$$

$$\text{Try } W360 \times 122 \quad A = 15500 \text{ mm}^2, \quad I_{\min} = 61.5 \times 10^6 \text{ mm}^4, \quad r = 63 \text{ mm}$$

$$C_e = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{350}} = 106.2$$

$$\frac{L_e}{r} = \frac{6800}{63} = 107.9 > 106.2$$

$$\sigma_{\text{all}} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = \frac{\pi^2 (200000)}{(1.92)(107.9)^2} = 88.3 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (88.3)(15500) = 1368.7 \text{ kN} > 1200 \text{ kN}$$

Use W360 x 122

Problem 10.80

10.80 A column of 4.6-m effective length must carry a centric load of 525 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use allowable stress design to select the wide-flange shape of 200-mm nominal depth that should be used.

$$\text{transition } \frac{L}{r}: \quad 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40$$

$$P < \frac{\sigma_Y A}{1.67} \quad A > \frac{1.67 P}{\sigma_Y} = \frac{(1.67)(525 \times 10^3)}{345 \times 10^6} = 2.541 \times 10^{-3} \text{ m}^2 = 2541 \text{ mm}^2$$

$$P < \frac{0.877 \pi^2 E I_{\min}}{1.67 L^2}$$

$$I_{\min} > \frac{1.67 P L^2}{0.877 \pi^2 E} = \frac{(1.67)(525 \times 10^3)(4.6)^2}{0.877 \pi^2 (200 \times 10^9)} = 10.72 \times 10^{-6} \text{ m}^4 = 10.72 \times 10^{-6} \text{ mm}^4$$

$$\text{Try } W 200 \times 46.1 \quad A = 5860 \text{ mm}^2 \quad I_{\min} = 15.3 \times 10^6 \text{ mm}^4 \quad r = 51.1 \text{ mm}$$

$$\frac{L}{r} = \frac{4.6}{51.1 \times 10^{-3}} = 90.02 < 113.40$$

$$\sigma_c = \frac{\pi^2 E}{(L/r)^2} = 243.59 \text{ MPa}$$

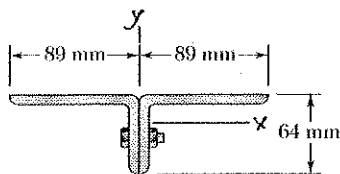
$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_c}) \sigma_Y = [0.658^{345/243.59}] (345) = 190.708 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(190.708 \times 10^6)(5860 \times 10^{-6})}{1.67} = 669 \text{ kN} > 525 \text{ kN}$$

All lighter sections fail the minimum moment of inertia criterion.

Use $W 200 \times 46.1$. 

Problem 10.81



10.81 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 180 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$\text{Transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_Y}}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Use x and y axes as added above.

Try L 89 × 64 × 9.5 $A = (2)(1360) = 2720 \text{ mm}^2 = 2720 \times 10^{-6} \text{ m}^2$

$$r_x = 18.6 \text{ mm} = 18.6 \times 10^{-3} \text{ m} \quad (r_y \text{ in Appendix C})$$

$$L_e/r = 2.4 / 18.6 \times 10^{-3} = 129.032 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(129.032)^2} = 118.559 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_e}) \sigma_Y = [0.658^{250/118.559}] (250) = 103.43 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(103.43 \times 10^6)(2720 \times 10^{-6})}{1.67} = 168.5 \text{ kN} < 180 \text{ kN}$$

Do not use.

Try L 89 × 64 × 12.7 $A = (2)(1780) = 3560 \text{ mm}^2 = 3560 \times 10^{-6} \text{ m}^2$

$$r_x = 18.1 \text{ mm} = 18.1 \times 10^{-3} \text{ m}$$

$$L_e/r = 2.4 / 18.1 \times 10^{-3} = 132.597 < 133.22$$

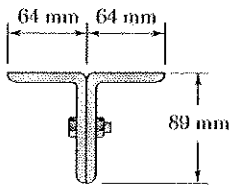
$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(132.597)^2} = 112.27 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_e}) \sigma_Y = [0.658^{250/112.27}] (250) = 98.44 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(98.44 \times 10^6)(3560 \times 10^{-6})}{1.67} = 233 \text{ kN} > 180 \text{ kN}$$

Use L 89 × 64 × 12.7 mm. ◀

Problem 10.82



10.82 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 325 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

$$\text{transition } \frac{L_e}{r} : 4.71 \sqrt{\frac{E}{\sigma_y}}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Try L 89 × 64 × 9.5. $A = (2)(1360) = 2720 \text{ mm}^2 = 2720 \times 10^{-6} \text{ m}^2$

$$I_x = (2)(1.07 \times 10^6) = 2.14 \times 10^6 \text{ mm}^4$$

$$I_y = (2)[0.463 \times 10^6 + (1360)(16.9)^2] = 1.70285 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1.70285 \times 10^6}{2720}} = 25.021 \text{ mm} = 25.021 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.021 \times 10^{-3}} = 95.919 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(95.919)^2} = 214.55 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_y/\sigma_c}) \sigma_y = [0.658^{250/214.55}] (250) = 153.51 \text{ MPa}$$

$$P_{\text{all}} = \frac{\sigma_{cr} A}{1.67} = \frac{(153.51 \times 10^6)(2720 \times 10^{-6})}{1.67} = 250 \text{ kN} < 325 \text{ kN}$$

Do not use.

Try L 89 × 64 × 12.7. $A = (2)(1780) = 3560 \text{ mm}^2 = 3560 \times 10^{-6} \text{ m}^2$

$$I_x = (2)(1.36 \times 10^6) = 2.72 \times 10^6 \text{ mm}^4$$

$$I_y = (2)[0.581 \times 10^6 + (1780)(18.1)^2] = 2.3283 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{2.3283 \times 10^6}{3560}} = 25.574 \text{ mm} = 25.574 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.574 \times 10^{-3}} = 93.846 < 133.22$$

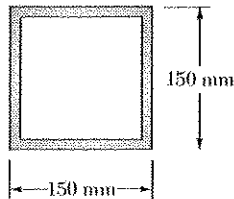
$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(93.846)^2} = 224.13 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{250/224.13}] (250) = 156.74 \text{ MPa}$$

$$P_{\text{all}} = \frac{\sigma_{cr} A}{1.67} = \frac{(156.74 \times 10^6)(3560 \times 10^{-6})}{1.67} = 334 \text{ kN} > 325 \text{ kN}$$

Use L 89 × 64 × 12.7 mm. ◀

Problem 10.83



10.83 A square steel tube having the cross section shown is used as a column of 7.8-m effective length to carry a centric load of 260 kN. Knowing that the tubes available for use are made with wall thicknesses ranging from 6 mm to 18 mm in increments of 1.5 mm, use allowable stress design to determine the lightest tube that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$b_o = 150 \text{ mm} \quad b_i = b_o - 2t \quad A = b_o^2 - b_i^2$$

$$I = \frac{1}{12} (b_o^4 - b_i^4)$$

$$L_e = 7.8 \text{ m}$$

$$P = 260 \text{ kN}$$

$$\sigma_Y = 250 \text{ MPa}$$

$$E = 200 \text{ GPa}$$

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$\text{Try } t = 12 \text{ mm} \quad b_i = 126 \text{ mm} \quad A = 150^2 - 126^2 = 6624 \text{ mm}^2$$

$$I = \frac{1}{12} [(150)^4 - (126)^4] = 21.18 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 55.6 \text{ mm}$$

$$\frac{L_e}{r} = \frac{7800}{55.6} = 140.3 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = 52.23 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (52.23)(6624) = 346 \text{ kN}$$

P_{all} is approximately proportional to t .

$$\frac{t}{12} = \frac{260}{346} \quad t \approx 9.02 \text{ mm}$$

$$\text{Try } t = 9 \text{ mm} \quad b_i = 132 \text{ mm} \quad A = 5076 \text{ mm}^2$$

$$I = \frac{1}{12} [(150)^4 - (132)^4] = 16.888 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 57.68 \text{ mm}$$

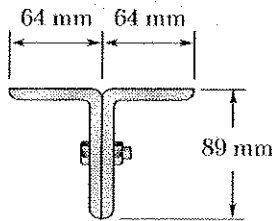
$$\frac{L_e}{r} = \frac{7800}{57.68} = 135.23 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = 56.2 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (56.2)(5076) = 285.3 \text{ kN} > 260 \text{ kN}$$

Use $t = 9 \text{ mm}$

Problem 10.84



10.84 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 325 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

Steel: $\sigma_y = 250 \times 10^6 \text{ Pa}$ $E = 200 \times 10^9 \text{ Pa}$
 $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.664$

Try L 89 × 64 × 9.5. $A = (2)(1360) = 2720 \text{ mm}^2 = 2720 \times 10^{-6} \text{ m}^2$
 $I_x = (2)(1.07 \times 10^6) = 2.14 \times 10^6 \text{ mm}^4$
 $I_y = (2)[0.463 \times 10^6 + (1360)(16.9)^2] = 1.70285 \times 10^6 \text{ mm}^4 = I_{min}$
 $r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{1.70285 \times 10^6}{2720}} = 25.021 \text{ mm} = 25.021 \times 10^{-3} \text{ m}$

$\frac{L_e}{r} = \frac{2.4}{25.021 \times 10^{-3}} = 95.919 < C_c$ $\frac{L_e/r}{C_c} = 0.76330$

F.S. = $\frac{5}{3} + \frac{3}{8}(0.76330) - \frac{1}{8}(0.76330)^3 = 1.8973$

$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.8973} \left[1 - \frac{1}{2} (0.76330)^2 \right] = 93.38 \times 10^6 \text{ Pa}$

$P_{all} = \sigma_{all} A = (93.38 \times 10^6)(2720 \times 10^{-6}) = 254 \times 10^3 \text{ N} = 254 \text{ kN} < 325 \text{ kN}$

Do not use.

Try L 89 × 64 × 12.7. $A = (2)(1780) = 3560 \text{ mm}^2 = 3560 \times 10^{-6} \text{ m}^2$
 $I_x = (2)(1.36 \times 10^6) = 2.72 \times 10^6 \text{ mm}^4$
 $I_y = (2)[0.581 \times 10^6 + (1780)(18.1)^2] = 2.3283 \times 10^6 \text{ mm}^4 = I_{min}$
 $r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.3283 \times 10^6}{3560}} = 25.574 \text{ mm} = 25.574 \times 10^{-3} \text{ m}$

$\frac{L_e}{r} = \frac{2.4}{25.574 \times 10^{-3}} = 93.846 < C_c$ $\frac{L_e/r}{C_c} = 0.7468$

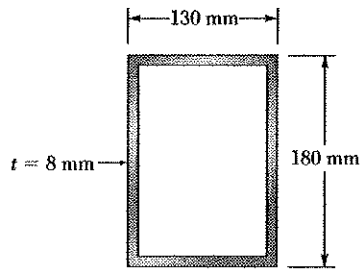
F.S. = $\frac{5}{3} + \frac{3}{8}(0.7468) - \frac{1}{8}(0.7468)^3 = 1.89465$

$\sigma_{all} = \frac{250 \times 10^6}{1.89465} \left[1 - \left(\frac{1}{2} \right) (0.7468)^2 \right] = 95.155 \times 10^6 \text{ Pa}$

$P_{all} = \sigma_{all} A = (95.155 \times 10^6)(3560 \times 10^{-6}) = 339 \times 10^3 \text{ N} = 339 \text{ kN} > 325 \text{ kN}$

Use L 89 × 64 × 12.7.

Problem 10.85



***10.85** A rectangular tube having the cross section shown is used as a column of 4.4-m effective length. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use load and resistance factor design to determine the largest centric live load that can be applied if the centric dead load is 220 kN. Use a dead load factor $\gamma_D = 1.2$, a live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.

$$L = 4.4 \text{ m}$$

$$b_o = 180 \text{ mm} \quad b_i = 180 - 2(8) = 164 \text{ mm}$$

$$h_o = 130 \text{ mm} \quad h_i = 114 \text{ mm}$$

$$A = (180)(130) - (164)(114) = 4704 \text{ mm}^2$$

$$I = \frac{1}{12} [(180)(130)^3 - (164)(114)^3] = 12707232 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 51.98 \text{ mm}$$

$$\frac{L_e}{r} = 84.65$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_Y}{E}} = \frac{84.65}{\pi} \sqrt{\frac{250}{200000}} = 0.9527 < 1.5$$

$$\lambda_c^2 = 0.9076$$

$$P_v = A (0.658)^{\lambda_c^2} \sigma_Y = (4704) (0.658)^{0.9076} (250) = 804.3 \text{ kN}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_v$$

$$(1.2)(220) + (1.6) P_L = (0.85)(804.3)$$

$$P_L = 262.3 \text{ kN}$$

Problem 10.86

10.86 A column with a 5.8-m effective length supports a centric load, with ratio of dead to live load equal to 1.35. The dead load factor is $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$, and the resistance factor $\phi = 0.90$. Use load and resistance factor design to determine the allowable centric dead and live loads if the column is made of the following rolled-steel shape: (a) W250 $\times$ 67, (b) W 360 $\times$ 101. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40$$

(a) W 250 $\times$ 67. $A = 8580 \text{ mm}^2 = 8580 \times 10^{-6} \text{ m}^2$, $r_y = 51.0 \text{ mm} = 51.0 \times 10^{-3} \text{ m}$

$$L_e/r_y = \frac{5.8}{51.0 \times 10^{-3}} = 113.725 > 113.40$$

$$\sigma_{cr} = \frac{0.877 \pi^2 E}{(L_e/r_y)^2} = \frac{0.877 \pi^2 (200 \times 10^9)}{(113.725)^2} = 133.85 \text{ MPa}$$

$$P_u = A \sigma_{cr} = (8580 \times 10^{-6})(133.85 \times 10^6) = 1.1484 \times 10^6 \text{ N}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u \quad \frac{P_D}{P_L} = 1.35 \quad P_L = \frac{P_D}{1.35}$$

$$1.2 P_D + 1.6 \frac{P_D}{1.35} = (0.9)(1.1484 \times 10^6) \quad P_D = 433 \text{ kN} \quad \blacktriangleleft$$

$$P_L = \quad P_L = 321 \text{ kN} \quad \blacktriangleleft$$

(b) W 360 $\times$ 101. $A = 12900 \text{ mm}^2 = 12900 \times 10^{-6} \text{ m}^2$, $r_y = 62.6 \text{ mm} = 62.6 \times 10^{-3} \text{ m}$

$$L_e/r_y = \frac{5.8}{62.6 \times 10^{-3}} = 92.652 < 113.40$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_y)^2} = \frac{\pi^2 (200 \times 10^9)}{(92.652)^2} = 229.94 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{\sigma_Y/\sigma_c}] \sigma_Y = [0.658^{345/229.94}] (345) = 184.12 \text{ MPa}$$

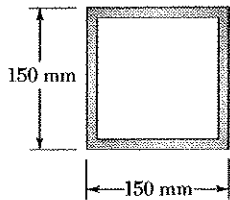
$$P_u = A \sigma_{cr} = (12900 \times 10^{-6})(184.12 \times 10^6) = 2.3751 \times 10^6 \text{ N}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$1.2 P_D + 1.6 \frac{P_D}{1.35} = (0.90)(2.3751 \times 10^6) \quad P_D = 896 \text{ kN} \quad \blacktriangleleft$$

$$P_L = 664 \text{ kN} \quad \blacktriangleleft$$

Problem 10.87



***10.87** The steel tube having the cross section shown is used as a column of 4.5-m effective length to carry a centric dead load of 210 kN and a centric live load of 240 kN. Knowing that the tubes available for use are made with wall thicknesses in increments of 2 mm from 4 mm to 10 mm, use load and resistance factor design to determine the lightest tube that can be used. Use $\sigma_y = 250$ MPa and $E = 200$ GPa. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$, and the resistance factor $\phi = 0.85$.

$$L_e = 4.5 \text{ m}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$\text{Required } P_u = \frac{\gamma_D P_D + \gamma_L P_L}{\phi} = \frac{(1.2)(210) + (1.6)(240)}{0.85} = 748.2 \text{ kN}$$

$$\text{Try } t = 6 \text{ mm}$$

$$b_o = 150 \text{ mm} \quad b_i = b_o - 2t = 138 \text{ mm}$$

$$A = b_o^2 - b_i^2 = (150)^2 - (138)^2 = 3456 \text{ mm}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = \frac{1}{12}[(150)^4 - (138)^4] = 11964672 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 58.84 \text{ mm}$$

$$\frac{L_e}{r} = \frac{4500}{58.84} = 76.48$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_y}{E}} = \frac{76.48}{\pi} \sqrt{\frac{250}{200000}} = 0.8607 < 1.5 \quad \lambda_c^2 = 0.7408$$

$$P_u = A (0.658)^{\lambda_c^2} \sigma_y = (3456)(0.658)^{0.7408} (250) = 633.66 < 748.2 \text{ kN}$$

Thickness is too small.

Since P_u is approximately proportional to thickness, the required thickness is approximately

$$\frac{t_{\text{req}}}{6} \approx \frac{P_{u(\text{req})}}{633.66} = \frac{748.2}{633.66} \quad t_{\text{req}} \approx 7.1 \text{ mm}$$

$$\text{Try } t = 8 \text{ mm}$$

$$b_i = 134 \text{ mm}$$

$$A = 4544 \text{ mm}^2, \quad I = 15319338 \text{ mm}^4, \quad r = 58.06 \text{ mm} \quad \frac{L_e}{r} = 77.5$$

$$\lambda_c = \frac{77.5}{\pi} \sqrt{\frac{250}{200000}} = 0.8722 < 1.5 \quad \lambda_c^2 = 0.7607$$

$$P_u = (4544)(0.658)^{0.7607} (250) = 826.2 \text{ kN} > 748.2 \text{ kN}$$

$$\text{Use } t = 8 \text{ mm}$$

Problem 10.88

*10.88 A steel column of 5.5-m effective length must carry a centric dead load of 310 kN and a centric live load of 375 kN. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use load and resistance factor design to select the wide-flange shape of 310-mm nominal depth that should be used. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.90$.

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$\text{Required: } P_u = \frac{\gamma_D P_D + \gamma_L P_L}{\phi} = \frac{(1.2)(310) + (1.6)(375)}{0.90} = 1080 \text{ kN}$$

Preliminary calculations.

$$P_u < \sigma_Y A \quad A > \frac{P_u}{\sigma_Y} = \frac{1080 \times 10^3}{250 \times 10^6} = 4.32 \times 10^{-3} \text{ m}^2 = 4320 \text{ mm}^2$$

$$P_u < \frac{0.877 \pi^2 E I_y}{L^2}$$

$$I_y > \frac{P_u L^2}{0.877 \pi^2 E} = \frac{(1080 \times 10^3)(5.5)^2}{0.877 \pi^2 (200 \times 10^9)} = 18.87 \times 10^{-6} \text{ m}^4 = 18.87 \times 10^6 \text{ mm}^4$$

$$\text{Try } W 310 \times 74. \quad A = 9480 \text{ mm}^2 \quad I_y = 23.4 \times 10^6 \text{ mm}^4 \quad r_y = 49.7 \text{ mm}$$

$$\frac{L_e}{r_y} = \frac{5.5}{49.7 \times 10^{-3}} = 110.66 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r_y)^2} = \frac{\pi^2 (200 \times 10^9)}{(110.66)^2} = 161.19 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{\sigma_Y/\sigma_e}] \sigma_Y = [0.658^{250/161.19}] (250) = 130.62 \text{ MPa}$$

$$P_u = A \sigma_{cr} = (9480 \times 10^{-6} \text{ m}^2)(130.62 \times 10^6 \text{ Pa}) = 1238 \text{ kN} > 1080 \text{ kN}$$

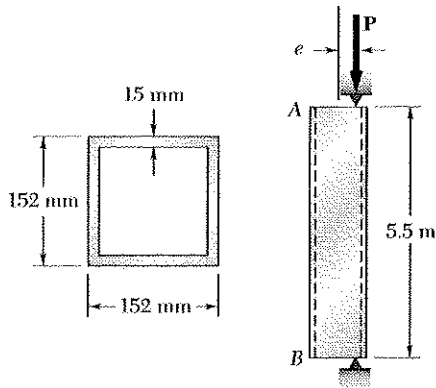
(Acceptable)

The next lighter shape, $W 310 \times 60$, with $I_y = 18.3 \times 10^6 \text{ mm}^4$ fails the moment of inertia criterion given above.

Use $W 310 \times 74$ 

Problem 10.89

10.89 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load P , knowing that the eccentricity is (a) $e = 0$, (b) $e = 40$ mm.



$$b_o = 152 \text{ mm} \quad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{5.5}{56.26 \times 10^{-3}} = 97.76 > 55$$

$$\sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(97.76)^2} = 38.92 \text{ MPa for centric loading}$$

$$\frac{P}{A \sigma_{all,c}} + \frac{P e c}{I \sigma_{all,b}} = 1$$

$$(a) \quad e = 0 \quad P = A \sigma_{all,c} = (8220 \times 10^{-6})(38.92 \times 10^6) = 320 \times 10^3 \text{ N} = 320 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 40 \times 10^{-3} \text{ m} \quad c = \frac{1}{2}(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$$

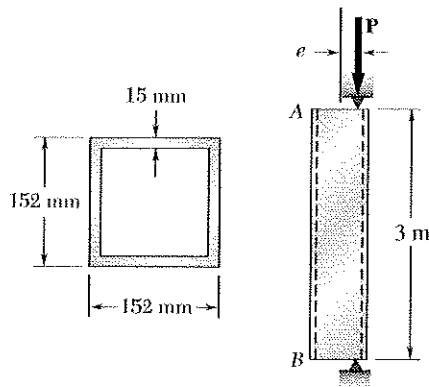
$$\frac{P}{(8220 \times 10^{-6})(38.92 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^6)(220 \times 10^6)} = 3.6568 \times 10^{-6} P = 1$$

$$P = 273 \times 10^3 \text{ N}$$

$$P = 273 \text{ kN} \quad \blacktriangleleft$$

Problem 10.90

10.90 Solve Prob. 10.89, assuming that the effective length of the column is 3.0 m.



10.89 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load P , knowing that the eccentricity is (a) $e = 0$, (b) $e = 40$ mm.

$$b_o = 152 \text{ mm} \quad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{3.0}{56.26 \times 10^{-3}} = 53.32 < 55$$

$$\sigma_{all,c} = 212 - 1.585 (L/r) = 212 - (1.585)(53.32) = 127.5 \text{ MPa}$$

$$\frac{P}{A \sigma_{all,c}} + \frac{P e c}{I \sigma_{all,b}} = 1$$

$$(a) \quad e = 0 \quad P = A \sigma_{all} = (8220 \times 10^6)(127.5 \times 10^6) = 1048 \times 10^3 \text{ N} = 1048 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} \quad c = \left(\frac{1}{2}\right)(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$$

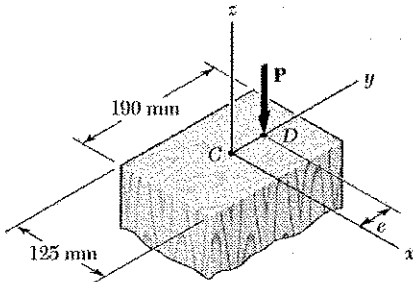
$$\frac{P}{(8220 \times 10^6)(127.5 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^6)(220 \times 10^6)} = 1.4852 \times 10^{-6} P = 1$$

$$P = 673 \times 10^3 \text{ N}$$

$$P = 673 \text{ kN} \quad \blacktriangleleft$$

Problem 10.91

10.91 A sawn lumber column of 125×190 -mm cross section has an effective length of 2.5 m. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8$ MPa and a modulus of elasticity $E = 8.3$ GPa. Using the allowable-stress method, determine the largest eccentric load P that can be applied when (a) $e = 12$ mm, (b) $e = 24$ mm.



Sawn lumber: $\sigma_c = 8 \text{ MPa}$ $E = 8.3 \text{ GPa}$
 $C = 0.8$ $K_{CE} = 0.300$

$L_e = 2.5 \text{ m}$

$b = 190 \text{ mm}$, $d = 125 \text{ mm}$, $c = \frac{b}{2} = 95 \text{ mm}$

$A = bd = (190)(125) = 23750 \text{ mm}^2$ $I_x = \frac{1}{12}(125)(190)^3 = 71.448 \times 10^6 \text{ mm}^4$

$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{K_{CE} E d^2}{L^2} = \frac{(0.300)(8.3 \times 10^3)(125)^2}{(2500)^2} = 6.225 \text{ MPa}$

$\sigma_{CE} / \sigma_c = 6.225 / 8 = 0.7781$

$C_P = \frac{1 + \sigma_{CE} / \sigma_c}{2C} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2C}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{C}} = 0.5991$

$\sigma_{all} = \sigma_c C_P = (8)(0.5991) = 4.793 \text{ MPa}$

$\frac{P_{all}}{A} + \frac{P_{all} e c}{I_x} = \sigma_{all}$

$P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{e c}{I_x}}$

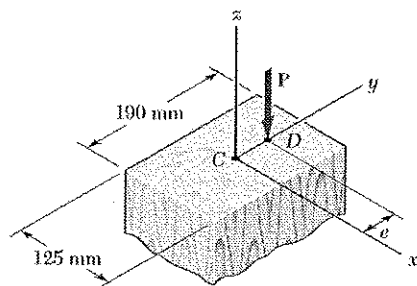
(a) $e = 12 \text{ mm}$

$P_{all} = \frac{4.793}{\frac{1}{23750} + \frac{(12)(95)}{71.448 \times 10^6}} = 82.55 \text{ kN} = 82.6 \text{ kN}$

(b) $e = 24 \text{ mm}$

$P_{all} = \frac{4.793}{\frac{1}{23750} + \frac{(24)(95)}{71.448 \times 10^6}} = 64.76 \text{ kN} = 64.8 \text{ kN}$

Problem 10.92



10.92 Solve Prob. 10.91 using the interaction method and an allowable stress in bending of 9 MPa.

10.91 A sawn lumber column of 125×190 -mm cross section has an effective length of 2.5 m. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8$ MPa and a modulus of elasticity $E = 8.3$ GPa. Using the allowable-stress method, determine the largest eccentric load P that can be applied when (a) $e = 12$ mm, (b) $e = 24$ mm.

$$\begin{aligned} \text{Sawn lumber: } \sigma_c &= 8 \text{ MPa} & E &= 8.3 \text{ MPa} \\ c &= 0.8 & K_{ce} &= 0.300 \\ L_e &= 2.5 \text{ m} \end{aligned}$$

$$b = 190 \text{ mm}, \quad d = 125 \text{ mm}, \quad c = \frac{b}{2} = 95 \text{ mm}$$

$$A = bd = (190)(125) = 23750 \text{ mm}^2 \quad I_x = \frac{1}{12} (125)(190)^3 = 71.448 \times 10^6 \text{ mm}^4$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{K_{ce} E d^2}{L^2} = \frac{(0.300)(8.3 \times 10^3)(125)^2}{(2500)^2} = 6.225 \text{ MPa}$$

$$\sigma_{ce} / \sigma_c = 6.225 / 8 = 0.7781$$

$$C_p = \frac{1 + \sigma_{ce} / \sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{ce} / \sigma_c}{2c} \right)^2 - \frac{\sigma_{ce} / \sigma_c}{c}} = 0.5991$$

$$\sigma_{all} = \sigma_c C_p = (8)(0.5991) = 4.793 \text{ MPa}$$

$$\frac{P_{all}}{A \sigma_{all}} + \frac{P e c}{I_x (\sigma_{all})_b} = B P \leq 1 \quad P \leq \frac{1}{B}$$

(a) $e = 12 \text{ mm}$

$$B = \frac{1}{(23750)(4.793)} + \frac{(12)(95)}{(71.448 \times 10^6)(9)} = 10.5576 \times 10^{-6} \text{ N}^{-1}$$

$$P_{all} = \frac{1}{10.5576 \times 10^{-6}} = 94.718 \text{ kN} = 94.7 \text{ kN}$$

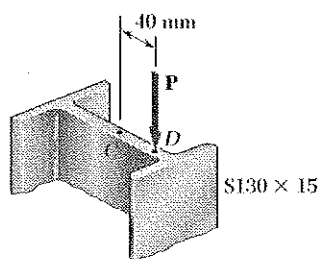
(b) $e = 24 \text{ mm}$

$$B = \frac{1}{(23750)(4.793)} + \frac{(24)(95)}{(71.448 \times 10^6)(9)} = 12.33 \times 10^{-6} \text{ N}^{-1}$$

$$P_{all} = \frac{1}{12.33 \times 10^{-6}} = 81.1 \text{ kN}$$

Problem 10.93

10.93 A steel compression member of 2.75-m effective length supports an eccentric load as shown. Using the allowable-stress method and assuming $e = 40$ mm, determine the maximum allowable load P . Use $\sigma_y = 250$ MPa and $E = 200$ GPa.



$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_y}}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$S130 \times 15: A = 1890 \text{ mm}^2 = 1890 \times 10^{-6} \text{ m}^2$$

$$r_{\min} = r_y = 16.3 \text{ mm} = 16.3 \times 10^{-3} \text{ m}$$

$$S_x = 79.8 \times 10^3 \text{ mm}^3 = 79.8 \times 10^{-6} \text{ m}^3$$

$$e_x = 0$$

$$e_y = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r_{\min}} = \frac{2.75}{16.3 \times 10^{-3}} = 168.712 > 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_{\min})^2} = \frac{\pi^2 (200 \times 10^9)}{(168.712)^2} = 69.35 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{0.877 \sigma_c}{1.67} = \frac{(0.877)(69.35)}{1.67} = 36.418 \text{ MPa}$$

$$\frac{P}{A} + \left| \frac{M_x}{S_x} \right| + \left| \frac{M_y}{S_y} \right| = \frac{P}{A} + \frac{P e_y}{S_x} + \frac{P e_x}{S_y} = P \left(\frac{1}{A} + \frac{e_y}{S_x} + \frac{e_x}{S_y} \right) = KP$$

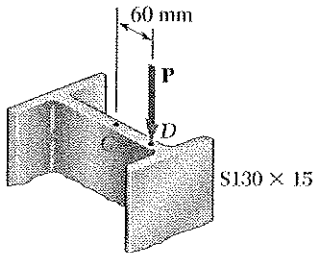
$$KP \leq \sigma_{\text{all}} \quad P \leq \frac{\sigma_{\text{all}}}{K}$$

$$K = \frac{1}{1890 \times 10^{-6}} + \frac{40 \times 10^{-3}}{79.8 \times 10^{-6}} + 0 = 1.03035 \times 10^3 \text{ m}^{-2}$$

$$P \leq \frac{36.418 \times 10^6}{1.03035 \times 10^3} = 35.3 \times 10^3 \text{ N}$$

$$P = 35.3 \text{ kN}$$

Problem 10.94



10.94 Solve Prob. 10.93, using $e = 60$ mm.

10.93 A steel compression member of 2.75-m effective length supports an eccentric load as shown. Using the allowable-stress method and assuming $e = 40$ mm, determine the maximum allowable load P. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_y}}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$S_{130 \times 15} \quad A = 1890 \text{ mm}^2 = 1890 \times 10^{-6} \text{ m}^2$$

$$r_{\min} = r_y = 16.3 \text{ mm} = 16.3 \times 10^{-3} \text{ m}$$

$$S_x = 79.8 \times 10^3 \text{ mm}^3 = 79.8 \times 10^{-6} \text{ m}^3$$

$$e_x = 0$$

$$e_y = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r_{\min}} = \frac{2.75}{16.3 \times 10^{-3}} = 168.712 > 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_{\min})^2} = \frac{\pi^2 (200 \times 10^9)}{(168.712)^2} = 69.35 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{0.877 \sigma_c}{1.67} = \frac{(0.877)(69.35)}{1.67} = 36.418 \text{ MPa}$$

$$\frac{P}{A} + \left| \frac{M_x}{S_x} \right| + \left| \frac{M_y}{S_y} \right| = \frac{P}{A} + \frac{P e_y}{S_x} + \frac{P e_x}{S_y} = P \left(\frac{1}{A} + \frac{e_y}{S_x} + \frac{e_x}{S_y} \right) = KP$$

$$KP \leq \sigma_{\text{all}}$$

$$P \leq \frac{\sigma_{\text{all}}}{K}$$

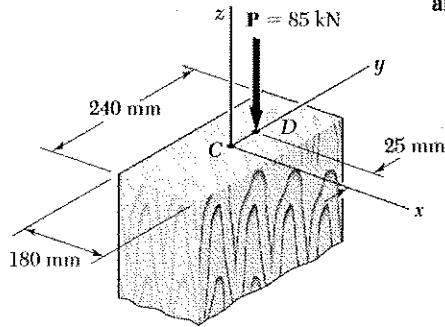
$$K = \frac{1}{1890 \times 10^{-6}} + \frac{60 \times 10^{-3}}{79.8 \times 10^{-6}} + 0 = 1.28098 \times 10^3 \text{ m}^{-2}$$

$$P \leq \frac{36.418 \times 10^6}{1.28098 \times 10^3} = 28.4 \times 10^3 \text{ N}$$

$$P = 28.4 \text{ kN}$$

Problem 10.95

10.95 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $F_c = 8.3 \text{ MPa}$ and a modulus of elasticity $E = 11.1 \text{ GPa}$. Using the allowable-stress method, determine the largest allowable effective length L that can be used.



$$d = 180 \text{ mm} = 0.180 \text{ m} \quad b = 240 \text{ mm} = 0.240 \text{ m}$$

$$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \quad E = 11100 \text{ MPa}$$

$$I_x = \frac{1}{12} db^3 = \frac{1}{12} (0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$$

$$e = 25 \text{ mm} = 0.025 \text{ m} \quad c = \frac{b}{2} = 0.120 \text{ m}$$

$$\frac{P}{A} + \frac{Pec}{I} \leq \sigma_{all} \quad \sigma_{all} \geq \frac{85 \times 10^3}{43.2 \times 10^{-3}} + \frac{(85 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa}$$

$$= \text{MPa}$$

$$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.9496}{8.3} = 0.38522 = y \quad \text{Let } x = \sigma_{ce}/\sigma_c$$

$$y = \frac{1+x}{2c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \quad \text{where } c = 0.8 \text{ for sawn lumber}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c}\right)^2 - y\left(\frac{1+x}{c}\right) + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$x = y \frac{(1-cy)}{1-y} = (0.38522) \frac{1 - (0.8)(0.38522)}{1 - 0.38522} = 0.43350$$

$$\sigma_{ce} = \sigma_c (0.43350) = (8.3)(0.43350) = 3.598 \text{ MPa}$$

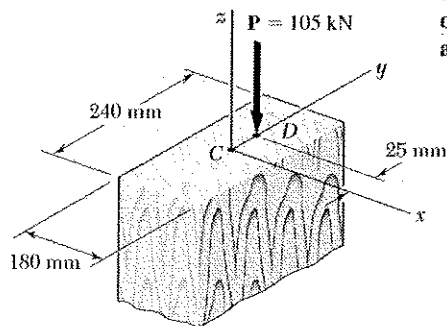
$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} \quad L^2 = \frac{K_{ce} E d^2}{\sigma_{ce}} \quad \text{where } K_{ce} = 0.300$$

$$L = d \sqrt{\frac{K_{ce} E}{\sigma_{ce}}} = (0.180) \sqrt{\frac{(0.300)(11100)}{3.598}} = 5.48 \text{ m}$$

Problem 10.96

10.96 Solve Prob. 10.95, assuming that $P = 105 \text{ kN}$.

10.95 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $F_c = 8.3 \text{ MPa}$ and a modulus of elasticity $E = 11.1 \text{ GPa}$. Using the allowable-stress method, determine the largest allowable effective length L that can be used.



$$d = 180 \text{ mm} = 0.180 \text{ m} \quad b = 240 \text{ mm} = 0.240 \text{ m}$$

$$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \quad E = 11100 \text{ MPa}$$

$$I_x = \frac{1}{12} db^3 = \frac{1}{12} (0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$$

$$e = 25 \text{ mm} = 0.025 \text{ m} \quad c = \frac{b}{2} = 0.120 \text{ m}$$

$$\frac{P}{A} + \frac{Pec}{I_x} \leq \sigma_{all} \quad \sigma_{all} \geq \frac{105 \times 10^3}{43.2 \times 10^{-3}} + \frac{(105 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa} = 3.9496 \text{ MPa}$$

$$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.9496}{8.3} = 0.47586 = y \quad \text{Let } x = \sigma_{ce}/\sigma_c$$

$$y = \frac{1+x}{c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \quad \text{where } c = 0.8 \text{ for sawn lumber}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c} - y\right)^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$\left(\frac{1+x}{2c}\right)^2 - \frac{1+x}{c}y + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

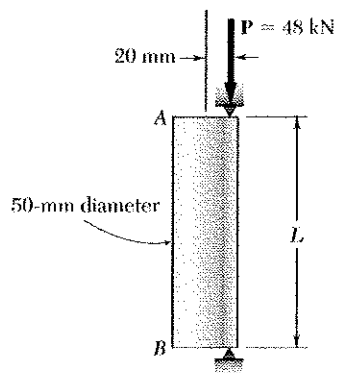
$$x = y \frac{1-cy}{1-y} = (0.47586) \frac{1 - (0.8)(0.47586)}{1 - 0.47586} = 0.56227$$

$$\sigma_{ce} = \sigma_c (0.56227) = (8.3)(0.56227) = 4.6668 \text{ MPa}$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} \quad L^2 = \frac{K_{ce} E d^2}{\sigma_{ce}} \quad \text{where } K_{ce} = 0.300$$

$$L = d \sqrt{\frac{K_{ce} E}{\sigma_{ce}}} = 0.180 \sqrt{\frac{(0.300)(11100)}{4.6668}} = 4.81 \text{ m}$$

Problem 10.97



10.97 An eccentric load $P = 48 \text{ kN}$ is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.

$$c = \frac{1}{2}d = 25 \text{ mm} \quad A = \pi c^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 306.8 \times 10^3 \text{ mm}^4 \quad r = \sqrt{\frac{I}{A}} = 12.5 \text{ mm}$$

$$e = 20 \text{ mm} \quad \sigma_{all,b} = 145 \times 10^6 \text{ Pa}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

$$\frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

$$\frac{1}{\sigma_{all,c}} = \frac{A}{P} \left[1 - \frac{Pec}{I\sigma_{all,b}} \right] = \frac{1.9635 \times 10^{-3}}{48 \times 10^3} \left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^{-9})(145 \times 10^6)} \right]$$

$$= 18.838 \times 10^{-9} \text{ Pa}^{-1} \quad \sigma_{all,c} = 53.086 \times 10^6 \text{ Pa} = 53.086 \text{ MPa}$$

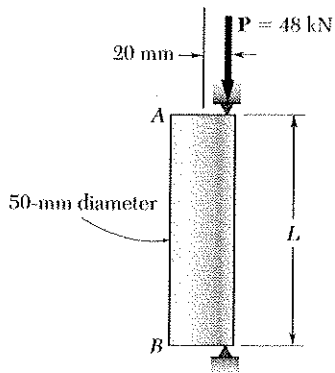
Assume $\frac{L}{r} > 66$. $\sigma_{all,c} = \frac{351 \times 10^3}{(L/r)^2}$

$$\frac{L}{r} = \sqrt{\frac{351 \times 10^3}{\sigma_{all,c}}} = \sqrt{\frac{351 \times 10^3}{53.086}} = 81.314 > 66$$

$$L = 81.314 r = (81.314)(12.5 \times 10^{-3})$$

$$L = 1.016 \text{ m} \quad \blacktriangleleft$$

Problem 10.98



10.98 Solve Prob. 10.97, assuming that the aluminum alloy used is 2014-T6 and that the allowable stress in bending is 180 MPa.

10.97 An eccentric load $P = 48 \text{ kN}$ is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.

$$c = \frac{1}{2}d = 25 \text{ mm} \quad A = \pi c^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4} c^4 = 306.8 \times 10^3 \text{ mm}^4 \quad r = \sqrt{\frac{I}{A}} = 12.5 \text{ mm}$$

$$e = 20 \text{ mm} \quad \sigma_{all,b} = 180 \times 10^6 \text{ Pa}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

$$\frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

$$\frac{1}{\sigma_{all,c}} = \frac{A}{P} \left[1 - \frac{Pec}{I\sigma_{all,b}} \right] = \frac{1.9635 \times 10^{-3}}{48 \times 10^3} \left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^{-9})(180 \times 10^6)} \right]$$

$$= 23.129 \times 10^{-9} \text{ Pa}^{-1} \quad \sigma_{all,c} = 43.236 \times 10^6 \text{ Pa} = 43.236 \text{ MPa}$$

Assume $\frac{L}{r} > 55$.

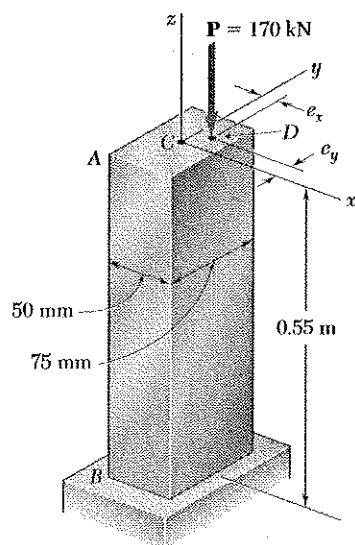
$$\sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2}$$

$$\frac{L}{r} = \sqrt{\frac{372 \times 10^3}{\sigma_{all,c}}} = \sqrt{\frac{372 \times 10^3}{43.236}} = 92.757 > 55$$

$$L = 92.757 r = (92.757)(12.5 \times 10^{-3})$$

$$L = 1.159 \text{ m} \quad \blacktriangleleft$$

Problem 10.99



10.99 The compression member AB is made of a steel for which $\sigma_y = 250$ MPa and $E = 200$ GPa. It is free at its top A and fixed at its base B . Using the allowable-stress method, determine the largest allowable eccentricity e_x , knowing that (a) $e_y = 0$, (b) $e_y = 8$ mm.

Steel: $\sigma_y = 250$ MPa $E = 200000$ MPa
 transition $\frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_y}} = 133.22$

$$A = (75 \times 10^{-3})(50 \times 10^{-3}) = 3750 \times 10^{-6} \text{ m}^2$$

$$I_y = \frac{1}{12} (75 \times 10^{-3})(50 \times 10^{-3})^3 = 781.25 \times 10^{-9} \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = 14.434 \times 10^{-3} \text{ m} = r_{\min}$$

$$I_x = \frac{1}{12} (50 \times 10^{-3})(75 \times 10^{-3})^3 = 1.7578 \times 10^{-6} \text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = 21.651 \times 10^{-3} \text{ m}$$

$$L_e = 2L = (2)(0.55) = 1.10 \text{ m} \quad L_e/r_{\min} = 1.10/14.434 \times 10^{-3} = 76.21 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_{\min})^2} = \frac{\pi^2 (200 \times 10^9)}{(76.21)^2} = 339.86 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_y}{1.67} = \frac{1}{1.67} [0.658^{250/339.86}] (250) = 110.03 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe_x}{S_y} + \frac{Pe_y}{S_x} = \sigma_{all} \quad \frac{Pe_y}{S_x} = \sigma_{all} - \frac{P}{A} - \frac{Pe_x}{S_y}$$

$$e_x = \frac{S_y}{P} \left[\sigma_{all} - \frac{P}{A} - \frac{Pe_x}{S_x} \right] = S_y \left[\frac{\sigma_{all}}{P} - \frac{1}{A} - \frac{e_y}{S_x} \right]$$

$$S_y = \frac{I_x}{x} = \frac{781.25 \times 10^{-9}}{25 \times 10^{-3}} = 31.25 \times 10^{-6} \text{ m}^3$$

$$S_x = \frac{I_y}{y} = \frac{1.7578 \times 10^{-6}}{37.5 \times 10^{-3}} = 46.875 \times 10^{-6} \text{ m}^3$$

$$P = 170 \times 10^3 \text{ N}$$

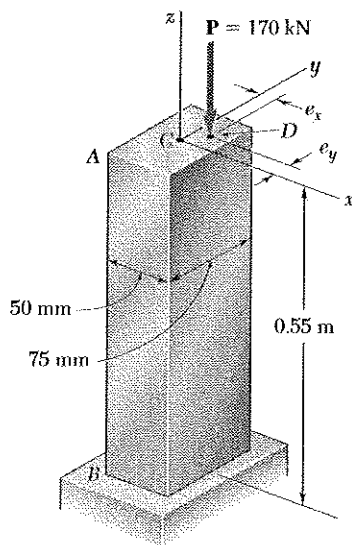
(a) $e_y = 0$
$$e_x = 31.25 \times 10^{-6} \left[\frac{110.03 \times 10^6}{170 \times 10^3} - \frac{1}{3750 \times 10^{-6}} - 0 \right]$$

$$= 11.89 \times 10^{-3} \text{ m} \quad e_x = 11.89 \text{ mm} \quad \blacktriangleleft$$

(b) $e_y = 8 \times 10^{-3} \text{ m}$
$$e_x = 31.25 \times 10^{-6} \left[\frac{110.03 \times 10^6}{170 \times 10^3} - \frac{1}{3750 \times 10^{-6}} - \frac{8 \times 10^{-3}}{46.875 \times 10^{-6}} \right]$$

$$= 6.56 \times 10^{-3} \text{ m} \quad e_x = 6.56 \text{ mm} \quad \blacktriangleleft$$

Problem 10.100



10.100 The compression member AB is made of a steel for which $\sigma_y = 250$ MPa and $E = 200$ GPa. It is free at its top A and fixed at its base B . Using the interaction method with an allowable bending stress equal to 120 MPa and knowing that the eccentricities e_x and e_y are equal, determine their largest allowable common value.

Steel: $\sigma_y = 250$ MPa $E = 200000$ MPa
 transition $\frac{L}{r} : 4.71\sqrt{\frac{E}{\sigma_y}} = 133.22$

$$A = (75 \times 10^{-3})(50 \times 10^{-3}) = 3750 \times 10^{-6} \text{ m}^2$$

$$I_y = \frac{1}{12}(75 \times 10^{-3})(50 \times 10^{-3})^3 = 781.25 \times 10^{-9} \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = 14.434 \times 10^{-3} \text{ m}$$

$$I_x = \frac{1}{12}(50 \times 10^{-3})(75 \times 10^{-3})^3 = 1.7578 \times 10^{-6} \text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = 21.651 \times 10^{-3} \text{ m}$$

$$L_e = 2L = (2)(0.55) = 1.10 \text{ m} \quad L_e/r_{min} = 1.10 / 14.434 \times 10^{-3} = 76.21 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_{min})^2} = \frac{\pi^2 (200 \times 10^9)}{(76.21)^2} = 339.86 \text{ MPa}$$

$$\sigma_{all(centric)} = \frac{\sigma_y}{1.67} = \frac{1}{1.67} [0.658^{250/339.86}] (250) = 110.03 \text{ MPa}$$

$$\sigma_{all(bending)} = 120 \text{ MPa}$$

$$\frac{P}{A\sigma_{all(centric)}} + \frac{Pe_y y}{I_x \sigma_{all(bending)}} + \frac{Pe_x x}{I_y \sigma_{all(bending)}} = 1 \quad \text{with } e_x = e_y$$

$$\frac{P}{\sigma_{all(bending)}} \left(\frac{y}{I_x} + \frac{x}{I_y} \right) e = 1 - \frac{P}{A\sigma_{all(centric)}}$$

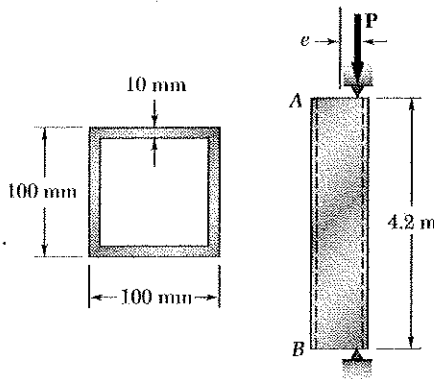
$$\frac{170 \times 10^3}{120 \times 10^6} \left(\frac{37.5 \times 10^{-3}}{1.7578 \times 10^{-6}} + \frac{25 \times 10^{-3}}{781.25 \times 10^{-9}} \right) e = 1 - \frac{170 \times 10^3}{(3750 \times 10^{-6})(110.03 \times 10^6)}$$

$$75.556 e = 1 - 0.41201$$

$$e = 7.78 \times 10^{-3} \text{ m}$$

$$e = 7.78 \text{ mm} \quad \blacktriangleleft$$

Problem 10.101



10.101 A column of 4.2-m effective length consists of a section of steel tubing having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 220$ kN, (b) $P = 140$ kN. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$E = 200 \text{ GPa}$$

$$\text{transition } \frac{L_e}{r} = 4.71 \sqrt{\frac{E}{\sigma_Y}} = 133.22$$

$$b_o = 100 \text{ mm} \quad b_i = b_o - 2t = 80 \text{ mm} \quad c = 50 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 3600 \text{ mm}^2 \quad I_y = \frac{1}{12}(b_o^4 - b_i^4) = 4.92 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I_y}{A}} = 36.97 \text{ mm} \quad L_e = 4.2 \text{ m}$$

$$\frac{L_e}{r} = \frac{4200}{36.97} = 113.61 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(113.61)^2} = 125.93 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_Y}{1.67} = \frac{1}{1.67} [0.658^{250/125.93}] (250) = 65.22 \text{ MPa}$$

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I} = \sigma_{all}$$

$$\frac{P_{all} e c}{I} = \sigma_{all} - \frac{P_{all}}{A}$$

$$e = \frac{I}{c P_{all}} \left(\sigma_{all} - \frac{P_{all}}{A} \right)$$

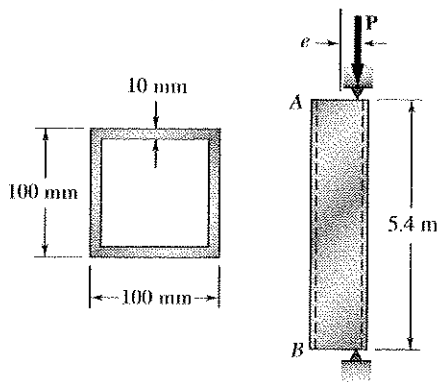
(a) $P_{all} = 220 \text{ kN}$

$$e = \frac{4.92 \times 10^{-6}}{(0.05)(220000)} \left[65.22 \times 10^6 - \frac{220000}{3600 \times 10^{-6}} \right] = 0.001838 \text{ m} \quad e = 1.84 \text{ mm}$$

(b) $P_{all} = 140 \text{ kN}$

$$e = \frac{4.92 \times 10^{-6}}{(0.05)(140000)} \left[65.22 \times 10^6 - \frac{140000}{3600 \times 10^{-6}} \right] = 0.01851 \text{ m} \quad e = 18.5 \text{ mm}$$

Problem 10.102



10.102 Solve Prob. 10.101, assuming that the effective length of the column is increased to 5.4 m and that (a) $P = 110$ kN, (b) $P = 70$ kN.

10.101 A column of 4.2-m effective length consists of a section of steel tubing having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 220$ kN, (b) $P = 140$ kN. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$E = 200 \text{ GPa}$$

$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_Y}} = 133.22$$

$$b_o = 100 \text{ mm} \quad b_i = b_o - 2t = 80 \text{ mm} \quad c = 50 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 3600 \text{ mm}^2 \quad I = \frac{1}{12}(b_o^4 - b_i^4) = 4.92 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 36.97$$

$$L_e = 5.4 \text{ m}$$

$$L_e/r = 146.06 > 133.22$$

$$\sigma_{cr} = \frac{0.877 \pi^2 E}{(L_e/r)^2} = 81.146 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_{cr}}{1.67} = 48.59 \text{ MPa}$$

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I} = \sigma_{all}$$

$$\frac{P_{all} e c}{I} = \sigma_{all} - \frac{P_{all}}{A}$$

$$e = \frac{I}{c P_{all}} \left(\sigma_{all} - \frac{P_{all}}{A} \right)$$

(a) $P_{all} = 110 \text{ kN}$

$$e = \frac{4.92 \times 10^{-6}}{(0.05)(110000)} \left[48.59 \times 10^6 - \frac{110000}{3600 \times 10^{-6}} \right] = 0.01613 \text{ m}$$

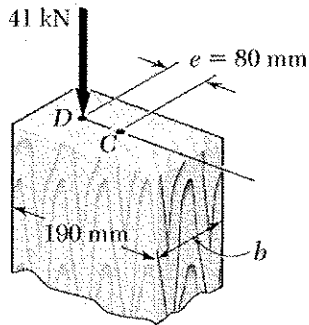
$$e = 16.1 \text{ mm}$$

(b) $P_{all} = 70 \text{ kN}$

$$e = \frac{4.92 \times 10^{-6}}{(0.05)(70000)} \left[48.59 \times 10^6 - \frac{70000}{3600 \times 10^{-6}} \right] = 0.04097 \text{ m}$$

$$e = 41 \text{ mm}$$

Problem 10.103



10.103 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41 kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8.1$ MPa and $E = 8.3$ GPa. Use the allowable-stress method to determine the lightest section that can be used.

Sawn lumber: $\sigma_c = 8.1$ MPa $E = 8.3$ GPa
 $c = 0.8$ $K_{CE} = 0.300$
 $L_e = 2.2$ m

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I_x} = \sigma_{all}$$

$$e = 80 \times 10^{-3} \text{ m}, \quad c = \frac{1}{2}(190) = 95 \text{ mm} = 95 \times 10^{-3} \text{ m}$$

$$A = 0.190 b \text{ m}^2 \quad I_x = \frac{1}{12} b (0.190)^3 = 571.58 \times 10^{-6} b \text{ m}^4$$

$$\frac{P_{all}}{0.190 b} + \frac{P_{all} (80 \times 10^{-3}) (95 \times 10^{-3})}{571.58 \times 10^{-6} b} = \frac{18.56 P_{all}}{b} = \sigma_{all}$$

$$P_{all} = 0.05388 \sigma_{all} b$$

$d = 0.190$ m or b , whichever is smaller.

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5 d^2 \text{ MPa}$$

$$\sigma_{CE} / \sigma_c = \frac{514.5 d^2}{8.1} = 63.51 d^2$$

$$C_P = \frac{1 + \sigma_{CE} / \sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2c} \right)^2 - \frac{\sigma_{CE} / \sigma_c}{c}}$$

$$= \frac{1 + \sigma_{CE} / \sigma_c}{1.6} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{1.6} \right)^2 - \frac{\sigma_{CE} / \sigma_c}{0.8}}$$

$$\sigma_{all} = \sigma_c C_P = (8.1 \times 10^6) C_P$$

$$P_{all} = (0.05388 b)(8.1 \times 10^6) C_P = 472.47 \times 10^3 b C_P$$

Calculate P_{all} for all four values of b . See table below.

b (m)	d (m)	σ_{CE} / σ_c	C_P	P_{all} (kN)
0.090	0.090	0.51443	0.44367	18.87
0.140	0.140	1.24480	0.76081	50.3
0.190	0.190	2.2927	0.8878	79.7
0.240	0.190	2.2927	0.8878	100.7

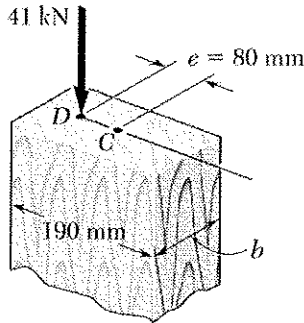
$\leftarrow P = 41 \text{ kN}$

Use $b = 0.140$ m

$= 140 \text{ mm}$

Problem 10.104

10.104 Solve Prob. 10.103, assuming that $e = 40$ mm.



10.103 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41 kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8.1$ MPa and $E = 8.3$ GPa. Use the allowable-stress method to determine the lightest section that can be used.

Sawn lumber: $\sigma_c = 8.1$ MPa $E = 8.3$ GPa
 $c = 0.8$ $K_{CE} = 0.300$
 $L_e = 2.2$ m

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I_x} = \sigma_{all}$$

$$e = 40 \times 10^{-3} \text{ m}$$

$$c = \frac{1}{2}(190) = 95 \text{ mm} = 95 \times 10^{-3} \text{ m}$$

$$A = 0.190 b \text{ m}^2$$

$$I_x = \frac{1}{12} b (0.190)^3 = 571.58 \times 10^{-6} b \text{ m}^4$$

$$\frac{P_{all}}{0.190 b} + \frac{P_{all} (40 \times 10^{-3})(95 \times 10^{-3})}{571.58 \times 10^{-6} b} = \frac{11.911 P_{all}}{b} = \sigma_{all}$$

$$P_{all} = 0.083953 \sigma_{all} b$$

$d = 0.190$ m or b , whichever is smaller.

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5 d^2 \text{ MPa}$$

$$\sigma_{CE}/\sigma_c = \frac{514.5 d^2}{8.1} = 63.51 d^2$$

$$C_p = \frac{1 + \sigma_{CE}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_c}{c}}$$

$$= \frac{1 + \sigma_{CE}/\sigma_c}{1.6} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_c}{1.6}\right)^2 - \frac{\sigma_{CE}/\sigma_c}{0.8}}$$

$$\sigma_{all} = \sigma_c C_p = (8.1 \times 10^6) C_p$$

$$P_{all} = (0.083953 k)(8.1 \times 10^6) C_p = 680.02 \times 10^3 b C_p$$

Calculate P_{all} for all four values of b . See table below.

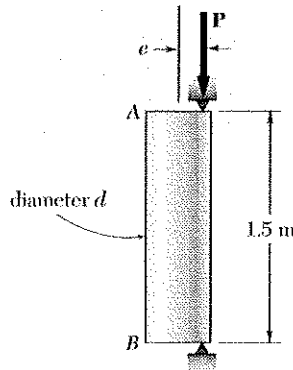
b (m)	d (m)	σ_{CE}/σ_c	C_p	P_{all} (kN)
0.090	0.090	0.51443	0.44367	27.2
0.140	0.140	1.24480	0.76081	72.4
0.190	0.190	2.2927	0.8878	114.7
0.240	0.190	2.2927	0.8878	144.9

→ $P = 41$ kN

Use $b = 0.140$ m
 $= 140$ mm

Problem 10.105

10.105 The eccentric load P has a magnitude of 85 kN and is applied at a point located at a distance $e = 30$ mm from the geometric axis of a rod made of the aluminum alloy 6016-T6. Using the interaction method with a 140-MPa allowable stress in bending, determine the smallest diameter d that can be used.



Assume $L/r > 66$. $\sigma_{all} = \frac{B}{(L/r)^2}$ $B = 351 \times 10^9 \text{ Pa}$

$$C = \frac{d}{2} \quad A = \pi C^2 = \frac{\pi}{4} d^2 \quad I = \frac{\pi}{4} C^4 = \frac{\pi}{64} d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{1}{4} d$$

$$\frac{P}{A \sigma_{all, \text{centric}}} + \frac{P e c}{I \sigma_{all, \text{bending}}} = 1$$

$$\frac{P L^2}{A B r^2} + \frac{P e d}{2 I \sigma_{all, \text{bending}}} = 1$$

$$\frac{64 P L^2}{\pi B d^4} + \frac{32 P e d}{\pi \sigma_{all, \text{bending}}} = 1$$

$$\frac{(64)(85 \times 10^3)(1.5)^2}{\pi (351 \times 10^9) d^4} + \frac{(32)(85 \times 10^3)(30 \times 10^{-3})}{\pi (140 \times 10^6) d^3} = 1 \quad \text{let } x = \frac{1}{d}$$

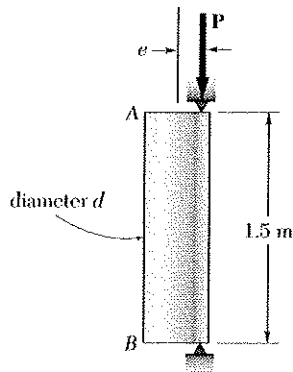
$$11.1 \times 10^{-6} x^4 + 185.53 \times 10^{-6} x^3 = 1 \quad \text{Solving, } x = 14.2725 \text{ m}^{-1}$$

$$d = \frac{1}{x} = 70.0 \times 10^{-3} \text{ m} \quad d = 70.0 \text{ mm} \quad \blacktriangleleft$$

$$r = \frac{d}{4} = 17.50 \times 10^{-3} \text{ m} \quad \frac{L}{r} = \frac{1.5}{17.5 \times 10^{-3}} = 85.7 > 66$$

Assumption is verified.

Problem 10.106



10.106 Solve Prob. 10.105, using the allowable-stress method and assuming that the aluminum alloy used is 2014-T6.

10.105 The eccentric load P has a magnitude of 85 kN and is applied at a point located at a distance $e = 30$ mm from the geometric axis of a rod made of the aluminum alloy 6016-T6. Using the interaction method with a 140-MPa allowable stress in bending, determine the smallest diameter d that can be used.

Assume $L/r > 55$. $\sigma_{all} = \frac{B}{(L/r)^2}$ $B = 372 \times 10^9 \text{ Pa}$

$c = \frac{d}{2}$ $A = \pi c^2 = \frac{\pi}{4} d^2$ $I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$

$r = \sqrt{\frac{I}{A}} = \frac{1}{4} d$

$\frac{P}{A} + \frac{Pec}{I} = \sigma_{all} = \frac{Br^2}{L^2}$

$\frac{PL^2}{ABr^2} + \frac{PL^2 e \frac{1}{4} d}{IBr^2} = 1$ $\frac{64 PL^2}{\pi d^4 B} + \frac{32 PL^2}{\pi d^3 B} = 1$ Let $x = \frac{1}{d}$

$\frac{64 PL^2}{\pi B} x^4 + \frac{(16)(64) PL^2 e}{2\pi B} x^5 = 1$

$\frac{(64)(85 \times 10^3)(1.5)^2}{\pi (372 \times 10^9)} x^4 + \frac{(16)(64)(85 \times 10^3)(1.5)^2 (30 \times 10^{-3})}{2\pi (372 \times 10^9)} x^5 = 1$

$10.473 \times 10^{-6} x^4 + 2.5136 \times 10^{-6} x^5 = 1$ $x = 12.441 \text{ m}^{-1}$

$d = \frac{1}{x} = 80.4 \times 10^{-3} \text{ m}$

$d = 80.4 \text{ mm}$ $\blacktriangleleft$

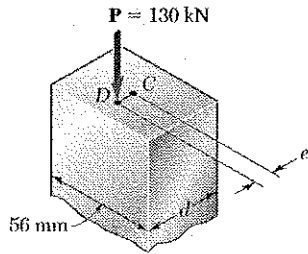
$r = \frac{d}{4} = 20.1 \times 10^{-3} \text{ m}$

$\frac{L}{r} = \frac{1.5}{20.1 \times 10^{-3}} = 74.5 > 55$

Assumption is verified.

Problem 10.107

10.107 A compression member of rectangular cross section has an effective length of 0.9 m and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 165 MPa. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 10$ mm.



$$A = 56d \quad c = \frac{1}{2}d \quad e = 10 \text{ mm} \quad L_e = 900 \text{ mm}$$

$$\sigma_{all,b} = 165 \text{ MPa} \quad P = 130 \text{ kN}$$

$$I_x = \frac{1}{12}(56)d^3 \quad r_x = \frac{d}{\sqrt{12}}$$

Assume $r_x = r_{min}$, i.e. $d < 56$ mm

$$L_e/r_{min} = \sqrt{12} L_e/d$$

Assume $L_e/r_{min} > 55$. $\sigma_{all,c} = \frac{372 \times 10^3}{(L_e/r_x)^2} = \frac{372000 d^2}{12 L_e^2} = \frac{372000}{(12)(900)^2} d^2 = 0.03827 d^2$

$$\frac{P}{A \sigma_{all,c}} + \frac{Pec}{I \sigma_{all,b}} = \frac{130000}{(56d)(0.03827 d^2)} + \frac{(12)(130000)(10)(\frac{1}{2}d)}{(56 d^3)(165)} = 1$$

$$\frac{60659}{d^3} + \frac{844.2}{d^2} = 1 \quad \text{Let } x = \frac{1}{d} \quad 60659x^3 + 844.2x^2 = 1$$

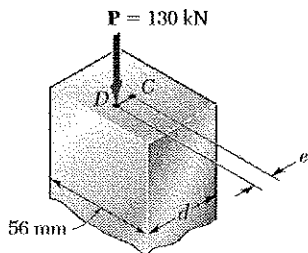
Solving for x , $x = 0.02107$, $d = \frac{1}{x} = 47.46 < 56$ mm

$$L/r_x = (\sqrt{12})(900)/(47.46) = 65.7 > 55 \quad d = 47.5 \text{ mm} \quad \blacktriangleleft$$

Problem 10.108

10.108 Solve Prob. 10.107, assuming that $e = 5$ mm.

10.107 A compression member of rectangular cross section has an effective length of 0.9 m and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 165 MPa. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 10$ mm.



$$A = 56d \quad c = \frac{1}{2}d \quad e = 5 \text{ mm} \quad L_e = 900 \text{ mm}$$

$$\sigma_{all,b} = 165 \text{ MPa} \quad P = 130 \text{ kN}$$

$$I_x = \frac{1}{12}(56)d^3 \quad r_x = \frac{d}{\sqrt{12}}$$

Assume $r_x = r_{min}$, i.e. $d < 56$ mm

$$L_e/r_{min} = \sqrt{12} L_e/d$$

Assume $L_e/r_{min} > 55$. $\sigma_{all,c} = \frac{372000}{(L_e/r_x)^2} = \frac{372000 d^2}{12 L_e^2} = \frac{372000 d^2}{(12)(900)^2} = 0.03827 d^2$

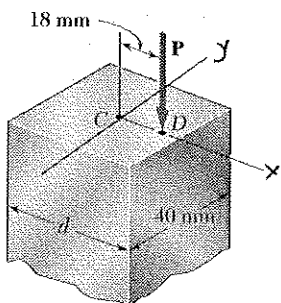
$$\frac{P}{A \sigma_{all,c}} + \frac{Pec}{I \sigma_{all,b}} = \frac{130000}{(56d)(0.03827 d^2)} + \frac{(12)(130000)(5)(\frac{1}{2}d)}{(56 d^3)(165)} = 1$$

$$\frac{60659}{d^3} + \frac{422.1}{d^2} = 1 \quad \text{Let } x = \frac{1}{d} \quad 60659x^3 + 422.1x^2 = 1$$

Solving for x , $x = 0.02267 \text{ m}^{-1}$ $d = \frac{1}{x} = 44.1 \text{ mm} < 56 \text{ mm}$

$$L_e/r_x = (\sqrt{12})(900)/44.1 = 70.7 > 55 \quad d = 44.1 \text{ mm} \quad \blacktriangleleft$$

Problem 10.109



10.109 A compression member made of steel has a 720-mm effective length and must support the 198-kN load P as shown. For the material used $\sigma_y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

Using dimensions in meters,

$$A = 40 \times 10^{-3} d \quad L_e = 720 \text{ mm} = 0.720 \text{ m}$$

$$I_x = \frac{1}{12} (40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12} (40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-3} d^3$$

$$|x| = \frac{d}{2}, \quad |y| = 20 \text{ mm} = 0.020 \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$\text{transition } \frac{L_e}{r}: \quad 4.71 \sqrt{\frac{E}{\sigma_y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Assume $d > 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$. Then $I_{\min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{40 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m}, \quad \frac{L_e}{r} = 62.35 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(62.35)^2} = 507.76 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = \frac{\sigma_c}{1.67} = \frac{1}{1.67} (0.658^{250/507.76}) (250) = 121.82 \text{ MPa}$$

$$\sigma_{\text{all, bending}} = 150 \text{ MPa}$$

$$\frac{P}{A \sigma_{\text{all, centric}}} + \frac{P e_x x}{I_y \sigma_{\text{all, bending}}} = 1$$

$$\frac{198 \times 10^3}{(40 \times 10^{-3} d)(121.82 \times 10^6)} + \frac{(198 \times 10^3)(18 \times 10^{-3})(\frac{1}{2} d)}{(3.3333 \times 10^{-3} d^3)(150 \times 10^6)} = 1$$

$$\frac{40.634 \times 10^{-3}}{d} + \frac{3.5640 \times 10^{-3}}{d^2} = 1$$

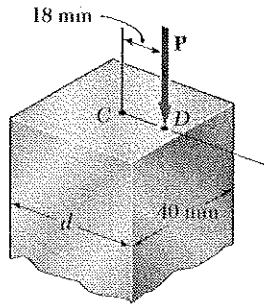
$$d^2 - 40.634 \times 10^{-3} d - 3.5640 \times 10^{-3} = 0$$

$$d = \frac{1}{2} \left\{ 40.634 \times 10^{-3} + \sqrt{(40.634 \times 10^{-3})^2 + (4)(3.5640 \times 10^{-3})} \right\}$$

$$d = 83.4 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m}$$

$$d = 83.4 \text{ mm} \quad \blacktriangleleft$$

Problem 10.110



10.110 Solve Prob. 10.109, assuming that the effective length is 1.62 m and that the magnitude P of the eccentric load is 128 kN.

10.109 A compression member made of steel has a 720-mm effective length and must support the 198-kN load P as shown. For the material used $\sigma_y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

Using dimensions in meters,

$$A = 40 \times 10^{-3} d \quad L_e = 1.62 \text{ m}$$

$$I_x = \frac{1}{12} (40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12} (40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-5} d^3$$

$$|x| = \frac{1}{2} d, \quad |y| = 20 \text{ mm} = 20 \times 10^{-3} \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Assume $d > 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$. Then $I_{\min} = I_x$.

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{3.3333 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = 140.29 > 133.22$$

$$\sigma_{cr} = \frac{0.877 \pi^2 E}{(L_e/r)^2} = \frac{0.877 \pi^2 (200 \times 10^9)}{(140.29)^2} = 87.958 \text{ MPa}$$

$$\sigma_{all, \text{centric}} = \frac{\sigma_{cr}}{1.67} = 52.67 \text{ MPa}$$

$$\sigma_{all, \text{bending}} = 150 \text{ MPa}$$

$$\frac{P}{A \sigma_{all, \text{centric}}} + \frac{P e_x}{I_y \sigma_{all, \text{bending}}} = 1$$

$$\frac{128 \times 10^3}{(40 \times 10^{-3} d)(52.67 \times 10^6)} + \frac{(128 \times 10^3)(18 \times 10^{-3})(\frac{1}{2} d)}{(3.3333 \times 10^{-5} d^3)(150 \times 10^6)} = 1$$

$$\frac{60.756 \times 10^{-3}}{d} + \frac{2.304 \times 10^{-3}}{d^2} = 1$$

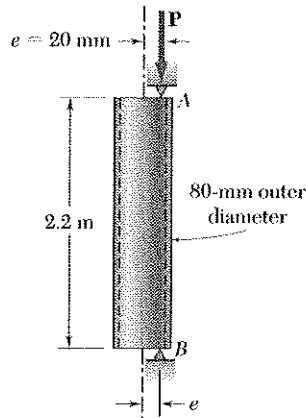
$$d^2 - 60.756 \times 10^{-3} d - 2.304 \times 10^{-3} = 0$$

$$d = \frac{1}{2} \left\{ 60.756 \times 10^{-3} + \sqrt{(60.756 \times 10^{-3})^2 + (4)(2.304 \times 10^{-3})} \right\}$$

$$d = 87.2 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m}$$

$$d = 87.2 \text{ mm} \quad \blacktriangleleft$$

Problem 10.111



10.111 A steel tube of 80-mm outer diameter is to carry a 93-kN load P with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable-stress method, determine the lightest tube that can be used. Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.

$$r_o = \frac{1}{2} d_o = 40 \text{ mm}, \quad r_i = r_o - t$$

$$A = \pi (r_o^2 - r_i^2), \quad I = \frac{\pi}{4} (r_o^4 - r_i^4) \quad r = \sqrt{\frac{I}{A}}$$

t (mm)	r_i (mm)	A (mm ²)	I (10 ⁶ mm ⁴)	r (mm)
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$$L_e = 2.2 \text{ m}$$

$$P = 93 \times 10^3 \text{ N}$$

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$\text{Try } t = 9 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(86.92)^2} = 261.27 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_e}{1.67} = \frac{1}{1.67} [0.658^{250/261.27}] (250) = 100.30 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2007 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.285 \times 10^{-6}} = 104.2 \text{ MPa} > 100.3 \text{ MPa}$$

(not allowed)

$$\text{Approximate required area: } \left(\frac{104.2}{100.3} \right) (2007) = 2085 \text{ mm}^2$$

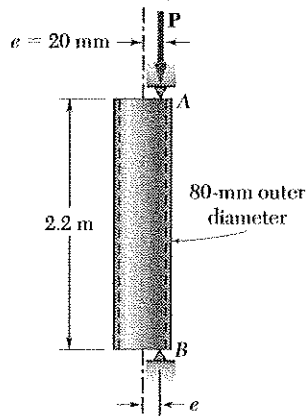
$$\text{Try } t = 12 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < 133.22$$

$$\sigma_e = 243.05 \text{ MPa} \quad \sigma_{all} = 97.33 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2564 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.528 \times 10^{-6}} = 85.0 \text{ MPa} < 97.3 \text{ MPa}$$

$$\text{Use } t = 12 \text{ mm.} \quad \blacktriangleleft$$

Problem 10.112



10.112 Solve Prob. 10.111, using the interaction method with $P = 165 \text{ kN}$, $e = 15 \text{ mm}$, and an allowable stress in bending of 150 MPa .

10.111 A steel tube of 80-mm outer diameter is to carry a 93-kN load P with an eccentricity of 20 mm . The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm . Using the allowable-stress method, determine the lightest tube that can be used. Assume $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$.

$$r_o = \frac{1}{2} d_o = 40 \text{ mm} \quad r_i = r_o - t$$

$$A = \pi (r_o^2 - r_i^2) \quad I = \frac{\pi}{4} (r_o^4 - r_i^4) \quad r = \sqrt{\frac{I}{A}}$$

t (mm)	r_i (mm)	A (mm ²)	I (10 ⁶ mm ⁴)	r (mm)
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$$L_e = 2.2 \text{ m}$$

$$P = 165 \times 10^3 \text{ N}$$

$$\sigma_{\text{all, bending}} = 150 \text{ MPa}$$

$$\text{transition } \frac{L_e}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$\text{Try } t = 9 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(86.92)^2} = 261.27 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = \frac{\sigma_Y}{1.67} = \frac{1}{1.67} [0.658^{250/261.27}] (250) = 100.30 \text{ MPa}$$

$$\frac{P}{A \sigma_{\text{all, centric}}} + \frac{P e c}{I \sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(2007 \times 10^{-6})(100.3 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.285 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.820 + 0.514 = 1.334 > 1 \quad (\text{not allowed})$$

$$\text{Approximate required area: } A = (1.334)(2007) = 2677 \text{ mm}^2$$

$$\text{Try } t = 12 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < 133.22$$

$$\sigma_c = 243.05 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = 97.33 \text{ MPa}$$

$$\frac{P}{A \sigma_{\text{all, centric}}} + \frac{P e c}{I \sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(2564 \times 10^{-6})(97.33 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.528 \times 10^{-6})(150 \times 10^6)}$$

$$0.661 + 0.432 = 1.093 > 1 \quad (\text{not allowed})$$

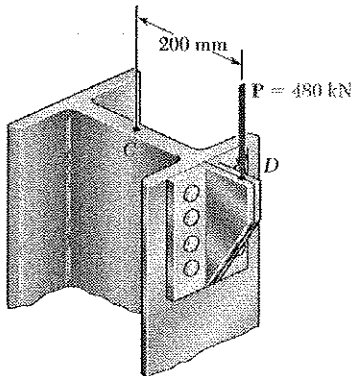
$$\text{Try } t = 15 \text{ mm. } \frac{L_e}{r} = 93.26 \quad \sigma_c = 226.95 \text{ MPa} \quad \sigma_{\text{all, centric}} = 94.40 \text{ MPa}$$

$$\frac{165 \times 10^3}{(3063 \times 10^{-6})(94.40 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.704 \times 10^{-6})(150 \times 10^6)} = 0.958 < 1$$

$$\text{Use } t = 15 \text{ mm.}$$

Problem 10.113

10.113 A steel column having a 7.2-m effective length is loaded eccentrically as shown. Using the allowable stress method, select the wide-flange shape of 350-mm nominal depth that should be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.



$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$L = 7.2 \text{ m}$$

If column is short and loading is centric

$$F.S. = \frac{5}{3} \quad \sigma_{all} = \frac{\sigma_Y}{F.S.} = \frac{250}{5/3} = 150 \text{ MPa}$$

$$A > \frac{P}{\sigma_{all}} = \frac{480000}{150} = 3200 \text{ mm}^2$$

If column is long and loading is centric, $F.S. = 1.92$

$$P < \frac{P_{cr}}{1.92} = \frac{\pi^2 E I_y}{1.92 L^2}, \quad I_y > \frac{1.92 P L^2}{\pi^2 E} = \frac{(1.92)(480000)(7200)^2}{\pi^2 (200000)} = 24.2 \times 10^6 \text{ mm}^4$$

$$\frac{P}{A} + \frac{P e c}{I_x} = \frac{P}{A} + \frac{P e}{S_x} < \sigma_{all}$$

Try W 360 X 101 $A = 12900 \text{ mm}^2$, $I_y = 50.6 \times 10^6 \text{ mm}^4$, $r_y = 62.6 \text{ mm}$, $S_x = 1690000 \text{ mm}^3$

$$\frac{L}{r_y} = \frac{7200}{62.6} = 115 < C_c \quad \frac{L/r_y}{C_c} = 0.9149$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.9149) - \frac{1}{8}(0.9149)^2 = 1.9051$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L/r_y}{C_c} \right)^2 \right] = \frac{250}{1.9051} \left[1 - \frac{1}{2} (0.9149)^2 \right] = 76.3 \text{ MPa}$$

$$\frac{P}{A} + \frac{P e}{S_x} = \frac{480000}{12900} + \frac{(480000)(200)}{1690000} = 90 \text{ MPa} > \sigma_{all} \quad \text{unsafe}$$

Assuming that σ_{max} is approximately proportional to $\frac{1}{A}$, the required area is

$$A = (12900) \frac{90}{76.3} = 15216.3 \text{ mm}^2$$

Try W 360 X 216 $A = 27600 \text{ mm}^2$, $S_x = 3860000 \text{ mm}^3$, $r_y = 101 \text{ mm}$

$$\frac{L}{r_y} = 71.29 \quad \frac{L/r_y}{C_c} = 0.5671 \quad F.S. = 1.8391 \quad \sigma_{all} = 114.08 \text{ MPa}$$

$$\frac{P}{A} + \frac{P e}{S_x} = 56.8 \text{ MPa} < 114.08 \text{ MPa} \quad \text{safe}$$

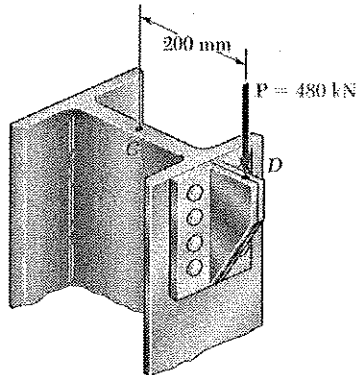
Try W 360 X 122 $A = 15500 \text{ mm}^2$, $S_x = 2010000 \text{ mm}^3$, $r_y = 63 \text{ mm}$

$$\frac{L}{r_y} = 114.3 \quad \frac{L/r_y}{C_c} = 0.9093 \quad F.S. = 1.9043 \quad \sigma_{all} = 77 \text{ MPa}$$

$$\frac{P}{A} + \frac{P e}{S_x} = 78.7 \text{ MPa} > 77 \text{ MPa} \quad \text{unsafe}$$

Use W 360 X 216

Problem 10.114



10.114 Solve Prob. 10.113 using the interaction method, assuming that $\sigma_Y = 350$ MPa and an allowable stress in bending of 200 MPa.

10.113 A steel column having a 7.2-m effective length is loaded eccentrically as shown. Using the allowable stress method, select the wide-flange shape of 350-mm nominal depth that should be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{350}} = 106.2$$

$$L_e = 7.2 \text{ m}$$

$$\text{Assume } \frac{L}{r_y} > 106.2$$

$$r_y < \frac{L}{106.2} = \frac{7200}{106.2} = 67.8$$

$$\sigma_{all,c} = \frac{\pi^2 E}{1.92 (L/r_y)^2} = \frac{\pi^2 E r_y^2}{1.92 L^2}$$

$$\text{Interaction formula: } \frac{P}{A \sigma_{all,c}} + \frac{P e c}{I_x \sigma_{all,b}} < 1 \quad (1)$$

The first term of inequality (1) becomes

$$\frac{1.92 P L^2}{\pi^2 E A r_y^2} = \frac{1.92 P L^2}{\pi^2 E I_y} = \frac{(1.92)(480000)(7200)^2}{\pi^2 (200000) I_y} = \frac{24.2 \times 10^6}{I_y}$$

The second term of inequality (1) becomes

$$\frac{P e}{S_x \sigma_{all,b}} = \frac{(480000)(200)}{(S_x)(200)} = \frac{0.457 \times 10^6}{S_x}$$

$$\text{Then inequality (1) becomes } \frac{24.2 \times 10^6}{I_y} + \frac{0.457 \times 10^6}{S_x} = \beta < 1$$

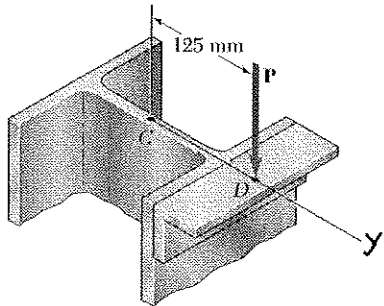
$$\text{Obviously } I_y > 24.2 \times 10^6 \text{ mm}^4 \text{ and } S_x > 457 \times 10^3 \text{ mm}^3$$

Shape	I_y (mm ⁴)	S_x (x10 ³ mm ³)	β
W200 x 71	25.4 x 10 ⁶	709	> 1
W360 x 101	50.6 x 10 ⁶	1690	0.75

W360 x 101 is the lightest shape with $\beta < 1$

Use W360 x 101

Problem 10.115



10.115 A steel compression member of 5.8-m effective length is to support a 296-kN eccentric load P . Using the interaction method, select the wide-flange shape of 200-mm nominal depth that should be used. Use $E = 200$ GPa, $\sigma_Y = 250$ MPa, and $\sigma_{all} = 150$ MPa in bending.

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} \quad L_e = 5.8 \text{ m}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$\text{At transition: } \frac{5.8}{r_y} = 133.22$$

$$r_y = 45.4 \times 10^{-3} \text{ m} = 45.4 \text{ mm}$$

For 200 mm nominal depth wide flange sections,

$$c = \frac{1}{2}d \approx 104 \text{ mm} \quad r_x \approx 88 \text{ mm} \quad \frac{ec}{r_x^2} \approx \frac{(125)(104)}{(88)^2} = 1.68$$

$$\text{For } r_y = 45.4 \text{ mm}, \quad \sigma_{all, \text{centric}} = \frac{0.877 \pi^2 E}{(1.67)(133.22)^2} = 58.4 \times 10^6 \text{ Pa}$$

$$\text{Interaction formula: } \frac{P}{A \sigma_{all, \text{centric}}} + \frac{Pec}{A r_x^2 \sigma_{all, \text{bending}}} < 1$$

$$A > P \left[\frac{1}{\sigma_{all, \text{centric}}} + \frac{ec/r_x^2}{\sigma_{all, \text{bending}}} \right] = 296 \times 10^3 \left(\frac{1}{58.4 \times 10^6} + \frac{1.68}{150 \times 10^6} \right) \\ = 8.38 \times 10^{-3} \text{ m}^2 = 8380 \text{ mm}^2$$

$$\text{Try W200} \times 59. \quad A = 7560 \text{ mm}^2, \quad S_x = 582 \times 10^3 \text{ mm}^3 \quad r_y = 51.9 \text{ mm}$$

$$\frac{L_e}{r_y} = \frac{5.8}{51.9 \times 10^{-3}} = 111.75 < 133.22 \quad \sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = 158.06 \text{ MPa}$$

$$\sigma_{all, \text{centric}} = \frac{\sigma_e}{1.67} = \frac{1}{1.67} [0.658^{250/158.06}] (250) = 77.217 \text{ MPa}$$

$$\frac{P}{A \sigma_{all, \text{centric}}} + \frac{Pe}{S_x \sigma_{all, \text{bending}}} = \frac{296 \times 10^3}{(7560 \times 10^{-6})(77.217 \times 10^6)} + \frac{(296 \times 10^3)(125 \times 10^{-3})}{(582 \times 10^{-6})(150 \times 10^6)} \\ = 0.5075 + 0.4238 = 0.9313 < 1 \quad (\text{allowed})$$

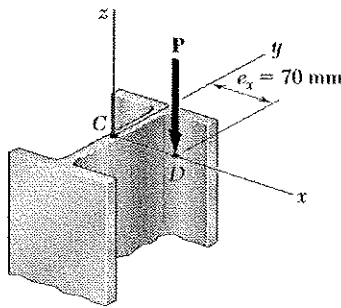
$$\text{Try W200} \times 52. \quad A = 6660 \text{ mm}^2 \quad S_x = 512 \times 10^3 \text{ mm}^3 \quad r_y = 51.7$$

$$\frac{L_e}{r_y} = 112.186 \quad \sigma_e = 156.84 \text{ MPa} \quad \sigma_{all, \text{centric}} = 76.821 \text{ MPa}$$

$$\frac{296 \times 10^3}{(6660 \times 10^{-6})(76.821 \times 10^6)} + \frac{(296 \times 10^3)(125 \times 10^{-3})}{(512 \times 10^{-6})(150 \times 10^6)} = 1.060 > 1 \quad (\text{not allowed})$$

Use W200 $\times$ 59 $\blacktriangleleft$

Problem 10.116



10.116 A steel column of 7.2-m effective length is to support an 83-kN eccentric load P at a point D , located on the x axis as shown. Using the allowable-stress method, select the wide-flange shape of 250-mm nominal depth that should be used. Use $E = 200$ GPa and $\sigma_y = 250$ MPa.

$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_y}} = 133.22$$

$$\text{At the transition: } \frac{7.2}{r_y} = 133.22$$

$$r_y = 54.05 \times 10^{-3} \text{ m} = 54.05 \text{ mm}$$

$$\text{All sections meet } \frac{L}{r_y} > 133.22.$$

$$\sigma_{all} = \frac{\Phi_{cr}}{1.67} = \frac{0.877 \pi^2 E}{1.67 (L/r_y)^2} = \frac{0.877 \pi^2 (200 \times 10^9)}{(1.67)(7.2/r_y)^2} = 19.9962 \times 10^9 r_y^2$$

$$\text{Try } W 250 \times 49.1 \quad A = 6250 \text{ mm}^2, \quad I_y = 15.1 \times 10^6 \text{ mm}^4$$

$$S_y = 150 \times 10^3 \text{ mm}^3 \quad r_y = 49.2 \text{ mm}$$

$$\sigma_{all} = (19.9962 \times 10^9)(49.2 \times 10^{-3})^2 = 48.404 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe}{S_y} = \frac{83 \times 10^3}{6250 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})}{150 \times 10^{-6}} = 52.013 \text{ MPa} > \sigma_{all}$$

(not allowed)

$$\text{Approximate required area: } A \approx \frac{52.013}{48.404} (6250) = 6716 \text{ mm}^2$$

$$\text{Try } W 250 \times 58 \quad A = 7420 \text{ mm}^2 \quad S_y = 185 \times 10^3 \text{ mm}^3 \quad r_y = 50.3 \text{ mm}$$

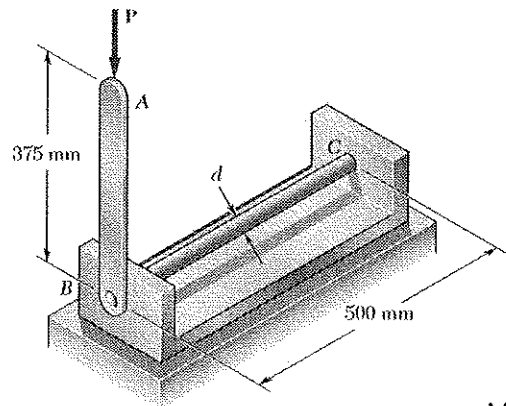
$$\sigma_{all} = (19.9962 \times 10^9)(50.3 \times 10^{-3})^2 = 50.592 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe}{S_y} = \frac{83 \times 10^3}{7420 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})}{185 \times 10^{-6}} = 42.591 \text{ MPa} < \sigma_{all}$$

Use $W 250 \times 58$ ◀

Problem 10.117

10.117 The steel rod BC is attached to the rigid bar AB and to the fixed support at C . Knowing that $G = 77 \text{ GPa}$, determine the critical load P_{cr} of the system when $d = 12 \text{ mm}$.



Look at torsion spring BC .

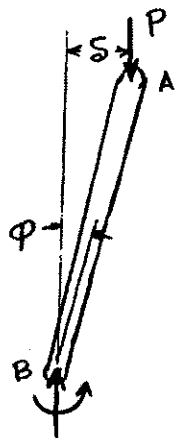
$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ}{L} \phi = K\phi$$

$$G = 77 \text{ GPa}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{2} (6)^4 = 2035.7 \text{ mm}^4$$

$$L = 0.5 \text{ m}$$

$$K = \frac{GJ}{L} = \frac{(77 \times 10^9)(2035.7 \times 10^{-12})}{0.5} = 313.5 \text{ Nm/rad.}$$



$$s = l \sin \phi = 0.375 \sin \phi$$

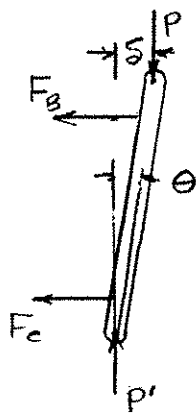
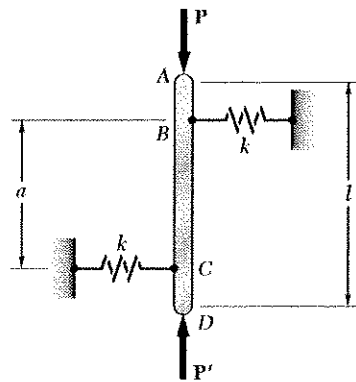
$$+\circlearrowleft \sum M_B = 0: -Ps + T = 0$$

$$P = \frac{T}{s} = \frac{K\phi}{l \sin \phi} = \frac{313.5 \phi}{0.375 \sin \phi} = 836 \frac{\phi}{\sin \phi} \text{ N}$$

$$\text{Let } \phi \rightarrow 0.$$

$$P_{cr} = 836 \text{ N} \quad \blacktriangleleft$$

Problem 10.118



10.118 The rigid bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Knowing that the equal and opposite loads P and P' remain vertical, determine the magnitude P_{cr} of the critical load for the system. Each spring can act in either tension or compression.

Let x_B and x_C be the deflections of points B and C , positive $\rightarrow$.

Then, $F_B = -kx_B$ and $F_C = -kx_C$

$$\pm \sum F_x = 0: -F_B - F_C = 0 \quad F_C = -F_B$$

$$x_C = -x_B \quad F_B \text{ and } F_C \text{ form a couple } \curvearrowright.$$

Let θ be the angle change. $y_B = -y_C = \frac{1}{2}a \sin \theta$

$$s = l \sin \theta$$

P and P' form a couple Ps

$$+\circlearrowleft \sum M = 0: k\left(\frac{1}{2}a \sin \theta\right)a \cos \theta - Pl \sin \theta = 0$$

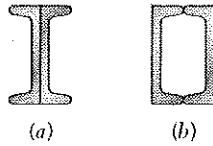
$$P = \frac{ka^2 \cos \theta}{2l}$$

Let $\theta \rightarrow 0$

$$P_{cr} = \frac{ka^2}{2l} \quad \blacktriangle$$

Problem 10.119

10.119 A column of 3-m effective length is to be made by welding together two C130 × 13 rolled-steel channels. Using $E = 200$ GPa, determine for each arrangement shown the allowable centric load if a factor of safety of 2.4 is required.



For channel C 130 × 13, $A = 1710 \text{ mm}^2$ $b_x = 48 \text{ mm}$
 $I_x = 3.70 \times 10^6 \text{ mm}^4$ $I_y = 0.264 \times 10^6 \text{ mm}^4$ $\bar{x} = 12.2 \text{ mm}$

Arrangement (a). $I_x = (2)(3.70 \times 10^6) = 7.40 \times 10^6 \text{ mm}^4$

$$I_y = 2[0.264 \times 10^6 + (1710)(12.2)^2] = 1.0370 \times 10^6 \text{ mm}^4$$

$$I_{\min} = I_y = 1.0370 \times 10^6 \text{ mm}^4 = 1.0370 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI_{\min}}{L_c^2} = \frac{\pi^2 (200 \times 10^9)(1.0370 \times 10^{-6})}{(3.0)^2} = 227 \times 10^3 \text{ N} = 227 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{227}{2.4}$$

$$P_{all} = 94.8 \text{ kN} \quad \blacktriangleleft$$

Arrangement (b). $I_x = (2)(3.70 \times 10^6) \text{ mm}^4$

$$I_y = 2[0.264 \times 10^6 + (1710)(48 - 12.2)^2] = 4.911 \times 10^6 \text{ mm}^4$$

$$I_{\min} = I_y = 4.911 \times 10^6 \text{ mm}^4 = 4.911 \times 10^{-6} \text{ m}^4$$

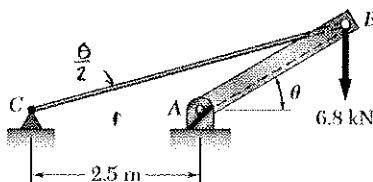
$$P_{cr} = \frac{\pi^2 EI_{\min}}{L_c^2} = \frac{\pi^2 (200 \times 10^9)(4.911 \times 10^{-6})}{(3.0)^2} = 1077 \times 10^3 \text{ N} = 1077 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{1077}{2.4}$$

$$P_{all} = 449 \text{ kN} \quad \blacktriangleleft$$

Problem 10.120

10.120 Member AB consists of a single C130 × 10.4 steel channel of length 2.5 m. Knowing that the pins A and B pass through the centroid of the cross section of the channel, determine the factor of safety for the load shown with respect to buckling in the plane of the figure when $\theta = 30^\circ$. Use $E = 200$ GPa.



Since $AB = 2.5 \text{ m}$, triangle ABC is isosceles.

$$+\uparrow \sum F_y = 0:$$

$$F_{AB} \sin 30^\circ - F_{AC} \sin 15^\circ - 6.8 = 0$$

$$F_{AB} \left(\sin 30^\circ - \frac{\sin 15^\circ \cos 30^\circ}{\cos 15^\circ} \right) = 0.26795 F_{AB} = 6.8$$

$$F_{AB} = 25.378 \text{ kN}$$

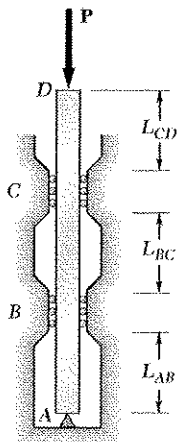
$$C 130 \times 10.4 \quad I_{\min} = 0.229 \times 10^6 \text{ mm}^4 = 0.229 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI_{\min}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9)(0.229 \times 10^{-6})}{(2.5)^2} = 72.324 \times 10^3 \text{ N} = 72.324 \text{ kN}$$

$$F.S. = \frac{P_{cr}}{F_{AB}} = \frac{72.324}{25.378}$$

$$F.S. = 2.85 \quad \blacktriangleleft$$

Problem 10.121



10.121 A 25-mm-square aluminum strut is maintained in the position shown by a pin support at *A* and by sets of rollers at *B* and *C* that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 0.9$ m, $L_{BC} = 1.2$ m, and $L_{CD} = 0.3$ m, determine the allowable load **P** using a factor of safety with respect to buckling of 3.2. Consider only buckling in the plane of the figure and use $E = 72$ GPa.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(25)^3 = 32552 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad P_{all} = \frac{(P_{cr})_{min}}{F.S.} = \frac{\pi^2 EI}{(F.S.)(L_e)_{max}^2}$$

$$\text{Portion AB: } L_e = 0.7L_{AB} = (0.7)(0.9) = 0.63 \text{ m}$$

$$\text{Portion BC: } L_e = 0.5L_{BC} = (0.5)(1.2) = 0.6 \text{ m}$$

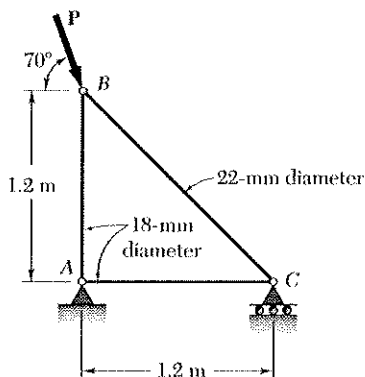
$$\text{Portion CD } L_e = 2L_{CD} = (2)(0.3) = 0.6 \text{ m}$$

$$(L_e)_{max} = 0.63 \text{ m}$$

$$P_{all} = \frac{\pi^2 (72 \times 10^9) (32552 \times 10^{-12})}{(3.2)(0.63)^2} = 18212.9 \text{ N} = 18.2 \text{ kN}$$

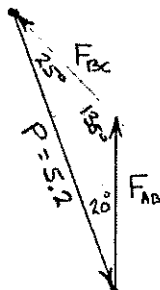
Problem 10.122

10.122 Knowing that $P = 5.2$ kN, determine the factor of safety for the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



Joint B :

From force triangle,



$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$F_{AB} = 3.1079 \text{ kN (comp)}$$

$$F_{BC} = 2.5152 \text{ kN (comp)}$$

$$\text{Member AB: } I_{AB} = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} \left(\frac{18}{2} \right)^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$$

$$F_{AB,cr} = \frac{\pi^2 E I_{AB}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9) (5.153 \times 10^{-9})}{(1.2)^2} = 7.0636 \times 10^3 \text{ N} = 7.0636 \text{ kN}$$

$$F.S. = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

$$\text{Member BC: } I_{BC} = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} \left(\frac{22}{2} \right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$$

$$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$$

$$F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-9})}{2.88} = 7.8813 \times 10^3 \text{ N} = 7.8813 \text{ kN}$$

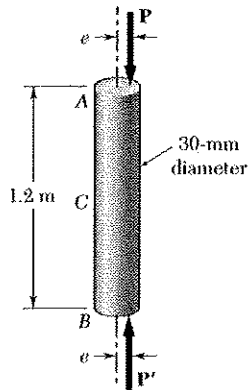
$$F.S. = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest F.S. governs.

$$F.S. = 2.27$$

Problem 10.123

10.123 An axial load P is applied to the 30-mm-diameter steel rod AB as shown. For $P = 35$ kN and $e = 1.5$ mm, determine (a) the deflection at the midpoint C of the rod, (b) the maximum stress in the rod. Use $E = 200$ GPa.



$$c = \frac{1}{2}d = \frac{30}{2} = 15 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(15)^4 = 39761 \text{ mm}^4$$

$$L_e = L = 1.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (39761 \times 10^{-12})}{(1.2)^2} = 54.5 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{35}{54.5} = 0.6422$$

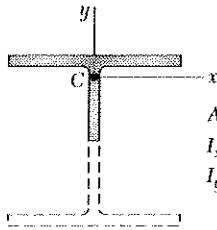
$$(a) \quad y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = 1.5 [2.2577] = 3.4 \text{ mm}$$

$$M_{max} = P(e + y_{max}) = (35)(1.5 + 3.4) = 171.5 \text{ N}\cdot\text{m}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(30)^2 = 706.9 \text{ mm}^2$$

$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{M_c}{I} = \frac{35000}{706.9} + \frac{(171.5 \times 10^3)(15)}{39761} = 114.21 \text{ MPa} = 114.2 \text{ MPa}$$

Problem 10.124



$$\begin{aligned} A &= 13.8 \times 10^3 \text{ mm}^2 \\ I_x &= 26.0 \times 10^6 \text{ mm}^4 \\ I_y &= 142.0 \times 10^6 \text{ mm}^4 \end{aligned}$$

10.124 A column is made from half of a W360 × 216 rolled-steel shape, with the geometric properties as shown. Using allowable stress design, determine the allowable centric load if the effective length of the column is (a) 4.0 m, (b) 6.5 m. Use $\sigma_Y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$.

$$\begin{aligned} r &= \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{26.0 \times 10^6}{13.8 \times 10^3}} = 43.406 \text{ mm} \\ &= 43.406 \times 10^{-3} \text{ m} \\ A &= 13.8 \times 10^{-3} \text{ m}^2 \end{aligned}$$

$$\text{transition } L/r: 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.4$$

$$(a) \quad L_e = 4.0 \text{ m}, \quad \frac{L_e}{r} = \frac{4.0}{43.406 \times 10^{-3}} = 92.153 < 113.4$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(92.153)^2} = 232.44 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{1}{F.S.} \sigma_{cr} = \frac{1}{1.67} \left[0.658^{345/232.44} \right] (345) = 111.0 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (111.0 \times 10^6) (13.8 \times 10^{-3}) \quad P_{\text{all}} = 1532 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad L_e = 6.5 \text{ m}, \quad \frac{L_e}{r} = \frac{6.5}{43.4 \times 10^{-3}} = 149.77 > 113.4$$

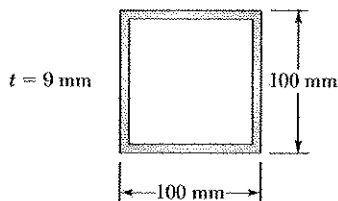
$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(149.77)^2} = 88.0 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{1}{F.S.} \sigma_{cr} = \frac{1}{1.67} \left[(0.877)(88.0) \right] = 46.21 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (46.21 \times 10^6) (13.8 \times 10^{-3}) \quad P_{\text{all}} = 638 \text{ kN} \quad \blacktriangleleft$$

Problem 10.125

10.125 A compression member has the cross section shown and an effective length of 1.5 m. Knowing that the aluminum alloy used is 2014-T6, determine the allowable centric load.



$$b_o = 100 \text{ mm} \quad b_i = b_o - 2t = 82 \text{ mm}$$

$$A = (100)^2 - (82)^2 = 3276 \text{ mm}^2$$

$$I = \frac{1}{12} [(100)^4 - (82)^4] = 4565652 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 37.3 \text{ mm}$$

$$L_e = 1.5 \text{ m}$$

$$\frac{L}{r} = \frac{1500}{37.3} = 40.21 < 55 \quad \text{for 2014-T6 aluminum alloy.}$$

$$\sigma_{\text{all}} = 212 - 1.585(L/r) = 212 - (1.585)(40.21) = 148.3 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (148.3) (3276) = 485.8 \text{ kN} \quad \blacktriangleleft$$

Problem 10.126

10.126 A column of 5-m effective length must carry a centric load of 950 kN. Using allowable stress design, select the wide-flange shape of 250-mm nominal depth that should be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$P < \frac{\sigma_Y A}{F.S.} \quad A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(950000)}{250 \times 10^6} = 6.33 \times 10^{-3} \text{ m}^2$$

$$L_e = 5 \text{ m}$$

$$E = 200 \text{ GPa}$$

$$P < \frac{\pi^2 EI}{1.92 L^2} \quad I > \frac{1.92 P L_e^3}{\pi^2 E} = \frac{(1.92)(950 \times 10^3)(5)^3}{\pi^2 (200 \times 10^9)} = 23.1 \times 10^{-6} \text{ m}^4$$

Try W250x80. $A = 10200 \text{ mm}^2$ $I_y = 43.1 \times 10^6 \text{ mm}^4$ $r = 65 \text{ mm}$.

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$\frac{L_e}{r} = \frac{5000}{65} = 76.92 < C_c \quad \frac{L_e/r}{C_c} = \frac{76.92}{125.7} = 0.6119$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.6119) - \frac{1}{8}(0.6119)^3 = 1.8675$$

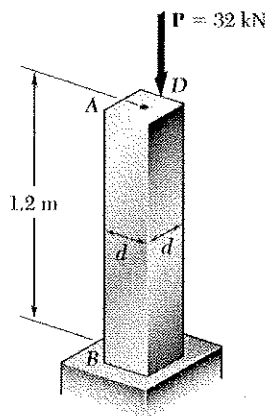
$$\sigma_{all} = \frac{\sigma_Y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.8675} \left[1 - \frac{1}{2} (0.6119)^2 \right] = 108.8 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (108.8)(10200) = 1109.7 \text{ kN} > 950 \text{ kN}$$

Use W250x80

Problem 10.127

10.127 A 32-kN vertical load P is applied at the midpoint of one edge of the square cross section of the aluminum compression member AB that is free at its top A and fixed at its base B . Knowing that the alloy used is 6061-T6 and using the allowable-stress method, determine the smallest allowable dimension d .



$$L_e = 2L = 2.4 \text{ m}$$

$$A = d^2 \quad I = \frac{1}{12} d^4 \quad r = \sqrt{\frac{I}{A}} = \frac{1}{\sqrt{12}} d \quad c = \frac{1}{2} d \quad e = \frac{1}{2} d$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{P}{d^2} + \frac{P(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{12}d^4} = \frac{4P}{d^2} = \sigma_{all}$$

Assume $L/r > 66$. $\sigma_{all} = \frac{B}{(L/r)^2}$ $B = 351 \times 10^9 \text{ Pa}$

$$\frac{Br^2}{L^2} = \frac{Bd^2}{12L^2} = \frac{4P}{d^2} \quad d^4 = \frac{48PL^2}{B}$$

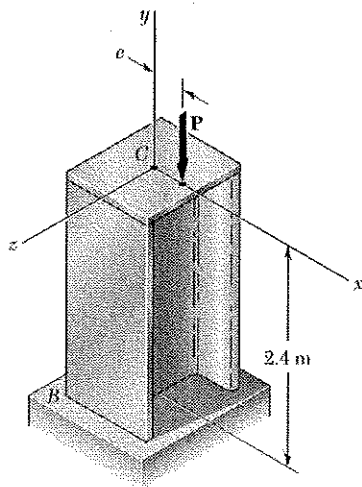
$$d = \sqrt[4]{\frac{48PL^2}{B}} = \sqrt[4]{\frac{(48)(32 \times 10^3)(2.4)^2}{351 \times 10^9}} = 70.9 \times 10^{-3} \text{ m}$$

$$r = \frac{70.9 \times 10^{-3}}{\sqrt{12}} = 20.45 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{20.45 \times 10^{-3}} = 117.3 > 66$$

$$d = 70.9 \text{ mm}$$

Problem 10.128



10.128 A 180-kN axial load P is applied to the rolled-steel column BC at a point on the x axis at a distance $e = 62$ mm from the geometric axis of the column. Using the allowable-stress method, select the wide-flange shape of 200-mm nominal depth that should be used. Use $E = 200$ GPa and $\sigma_Y = 250$ MPa.

Steel: $E = 200 \text{ GPa}$ $\sigma_Y = 250 \text{ MPa}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$L = 2.4 \text{ m}$$

$$L_e = 2L = 4.8 \text{ m}$$

Try W200x46.1 $r_y = 51.1 \text{ mm}$, $\frac{L_e}{r_y} = 93.93 < C_c$

$$\frac{L_e/r_y}{C_c} = 0.747$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.747) - \frac{1}{8}(0.747)^3 = 1.894$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r_y}{C_c} \right)^2 \right] = \frac{250}{1.894} \left[1 - \frac{1}{2} (0.747)^2 \right] = 95.1 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{180000}{5860} + \frac{(180000)(62)(204/2)}{15.3 \times 10^6} = 104.75 \text{ MPa} > 95.1 \text{ MPa}$$

(not allowed)

Approximate required area = $\left(\frac{104.75}{95.1} \right) (5860) = 6454.6 \text{ mm}^2$

Try W200x52. $r_y = 51.7 \text{ mm}$ $\frac{L_e}{r_y} = 92.84 < C_c$ $\frac{L_e/r_y}{C_c} = 0.7386$

$$F.S. = 1.893 \quad \sigma_{all} = 96 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{180000}{6660} + \frac{(180000)(62)(204/2)}{17.8 \times 10^6} = 90.97 < 96 \text{ MPa} \quad (\text{allowed})$$

Use W200x52

PROBLEM 10.C1

10.C1 A solid steel rod having an effective length of 500 mm is to be used as a compression strut to carry a centric load P . For the grade of steel used $E = 200$ GPa and $\sigma_y = 245$ MPa. Knowing that a factor of safety of 2.8 is required and using Euler's formula, write a computer program and use it to calculate the allowable centric load P_{all} for values of the radius of the rod from 6 mm to 24 mm, using 2-mm increments.

SOLUTION

ENTER RADIUS RAD, EFFECTIVE LENGTH L_e
AND FACTOR OF SAFETY FS

COMPUTE RADIUS OF GYRATION

$$A = \pi \text{ RAD}^2$$

$$I = \frac{1}{4} \pi \text{ RAD}^4$$

$$r = \sqrt{\frac{I}{A}}$$

DETERMINE ALLOWABLE CENTRIC LOAD

CRITICAL STRESS :

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

LET σ EQUAL SMALLER OF σ_{cr} AND σ_y

$$P_{all} = \frac{\sigma A}{FS}$$

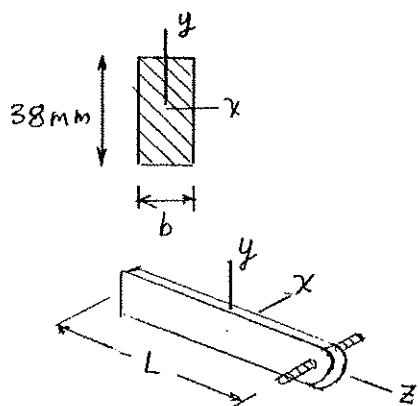
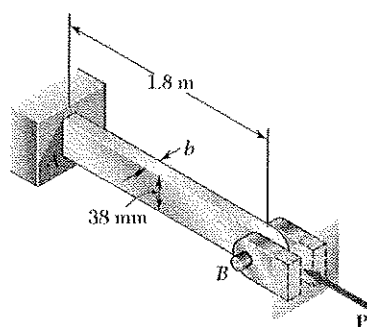
PROGRAM OUTPUT

Radius of rod m	Critical stress MPa	Allowable load kN
.006	71.1	2.87
.008	126.3	9.07
.010	197.4	22.15

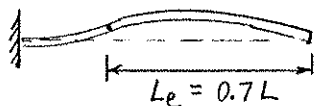
.012	284.2	39.58
.014	386.9	53.88
.016	505.3	70.37
.018	639.6	89.06
.020	789.6	109.96
.022	955.4	133.05
.024	1137.0	158.34

Below the dashed line we have:
critical stress > yield strength

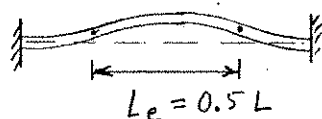
PROBLEM 10.C2



BUCKLING IN yz PLANE



BUCKLING IN xz PLANE



10.C2 An aluminum bar is fixed at end A and supported at end B so that it is free to rotate about a horizontal axis through the pin. Rotation about a vertical axis at end B is prevented by the brackets. Knowing that $E = 70$ GPa, use Euler's formula with a factor of safety of 2.5 to determine the allowable centric load P for values of b from 20 mm to 38 mm, using 3-mm increments.

SOLUTION

ENTER E , LENGTH L AND FACTOR OF SAFETY FS
FOR $b = 20$ mm TO 38 mm WITH 3 mm INCREMENTS

COMPUTE RADII OF GYRATION

$$A = 38 \quad b$$

$$I_x = \frac{1}{12} b \quad 38^3 \quad r_x = \sqrt{\frac{I_x}{A}}$$

$$I_y = \frac{1}{12} 38^3 \quad b \quad r_y = \sqrt{\frac{I_y}{A}}$$

COMPUTE CRITICAL STRESSES

$$(\sigma_{cr})_x = \frac{\pi^2 E}{(0.7L/r_x)^2}$$

$$(\sigma_{cr})_y = \frac{\pi^2 E}{(0.5L/r_y)^2}$$

LET σ_{cr} EQUAL SMALLER STRESS

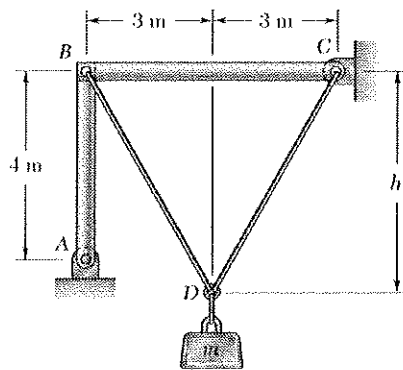
COMPUTE ALLOWABLE CENTRIC LOAD

$$P_{all} = \frac{\sigma_{cr} A}{FS}$$

PROGRAM OUTPUT

b mm	Critical stress x axis MPa	Critical stress y axis MPa	Allowable load kN
20	52.365	28.431	8.643
23	52.365	37.600	13.145
26	52.365	48.048	18.989
29	52.365	59.776	23.082
32	52.365	72.783	25.470
35	52.365	87.070	27.858
38	52.365	102.636	30.246

PROBLEM 10.C3



JOINT D:

$$\begin{aligned} & \text{Free body diagram of joint D: } T_x = \frac{3}{h} W, \quad T_y = \frac{1}{2} W \\ & \sum F_y = 0 \text{ YIELDS} \\ & T_y = \frac{1}{2} W \\ & \frac{T_x}{T_y} = \frac{3}{h} \text{ YIELDS} \\ & T_x = \frac{1.5 W}{h} \end{aligned}$$

JOINT B:

$$\begin{aligned} & \text{Free body diagram of joint B: } F_{AB} = \frac{1}{2} W, \quad F_{BC} = \frac{1.5 W}{h}, \quad T_x = \frac{1.5 W}{h}, \quad T_y = \frac{1}{2} W \end{aligned}$$

10.C3 The pin-ended members AB and BC consist of sections of aluminum pipe of 120-mm outer diameter and 10-mm wall thickness. Knowing that a factor of safety of 3.5 is required, determine the mass m of the largest block that can be supported by the cable arrangement shown for values of h from 4 m to 8 m, using 0.25-m increments. Use $E = 70$ GPa and consider only buckling in the plane of the structure.

SOLUTION

COMPUTE MOMENT OF INERTIA

$$I = \frac{\pi}{4} (0.06^4 - 0.05^4)$$

FOR $h = 4$ TO 8 USING 0.25 INCREMENTS

COMPUTE ALLOWABLE LOADS FOR MEMBERS

$$(F_{AB})_{cr} = \frac{\pi^2 EI}{3.5(4)^2}; \quad (F_{BC})_{cr} = \frac{\pi^2 EI}{3.5(6)^2}$$

DETERMINE ALLOWABLE W

$$(W_{all})_1 = 2(F_{AB})_{cr}; \quad (W_{all})_2 = \frac{h}{1.5}(F_{BC})_{cr}$$

W_{all} EQUALS SMALLER VALUE

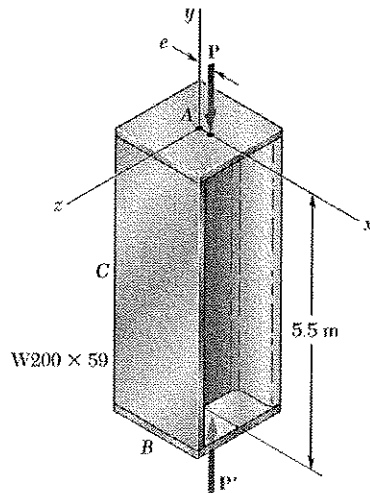
COMPUTE MASS m

$$m = \frac{W_{all}}{9.81}$$

PROGRAM OUTPUT

h	Weight critical stress AB	Weight critical stress BC	mass
m	kN	kN	kg
4.00	455.11	269.7	7854.88
4.25	455.11	286.6	8345.80
4.50	455.11	303.4	8836.74
4.75	455.11	320.3	9327.66
5.00	455.11	337.1	9818.59
5.25	455.11	354.0	10309.52
5.50	455.11	370.8	10800.45
5.75	455.11	387.7	11291.38
6.00	455.11	404.5	11782.31
6.25	455.11	421.4	12273.24
6.50	455.11	438.3	12764.17
6.75	455.11	455.1	13255.10
7.00	455.11	472.0	13255.10
7.25	455.11	488.8	13255.10
7.50	455.11	505.7	13255.10
7.75	455.11	522.5	13255.10
8.00	455.11	539.4	13255.10

PROBLEM 10.C4



10.C4 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W200 $\times$ 59 rolled-steel column AB . Using $E = 200$ GPa, write a computer program and use it to calculate for values of P from 100 to 300 kN, using 20-kN increments, (a) the horizontal deflection at the midpoint C , (b) the maximum stress in the column.

SOLUTION

ENTER LENGTH L , ECCENTRICITY e

ENTER PROPERTIES A, I_y, r_y, b_f

COMPUTE CRITICAL LOAD

$$P_{cr} = \frac{\pi^2 E I_y}{L^2}$$

FOR $P = 100$ kN TO 300 kN IN INCREMENTS OF 20 kN

COMPUTE HORIZONTAL DEFLECTION AT C

$$y_c = e \left(\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1.0 \right)$$

COMPUTE MAXIMUM STRESS

$$\sigma_{max} = \frac{P}{A} \left(1 + \frac{e b_f}{2 r_y^2} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

PROGRAM OUTPUT

Load kN	maximum deflection mm	maximum stress MPa
100	1.205	19.874
160	2.042	32.680
200	2.655	41.560
240	3.329	50.923
300	4.473	65.723

PROBLEM 10.C5

10.C5 A column of effective length L is made from a rolled-steel shape and carries a centric axial load P . The yield strength for the grade of steel used is denoted by σ_y , the modulus of elasticity by E , the cross-sectional area of the selected shape by A , and its smallest radius of gyration by r . Using the AISC design formulas for allowable stress design, write a computer program to determine the allowable load P . Use this program to solve (a) Prob. 10.59, (b) Prob. 10.60, (c) Prob. 10.124.

SOLUTION

ENTER L, E, σ_y

ENTER PROPERTIES A, r_y

DETERMINE ALLOWABLE STRESS

$$C = 4.71 \sqrt{\frac{E}{\sigma_y}}$$

$$\text{IF } L/r_y \geq C$$

$$\sigma_{all} = \frac{0.877 \pi^2 E}{1.67 (L/r_y)^2}$$

$$\text{IF } L/r_y < C$$

$$\sigma_e = \frac{\pi^2 E}{(L/r_y)^2}$$

$$\sigma_{all} = \frac{1}{1.67} \left[0.658^{\sigma_y / \sigma_e} \right] \sigma_y$$

CALCULATE ALLOWABLE LOAD:

$$P_{all} = \sigma_{all} A$$

CONTINUED

PROBLEM 10.C5 CONTINUED

PROGRAM OUTPUT

Problem 10.60 a

Effective Length = 6m
A = 5860 mm²
ry = 51.5 mm
Yield strength = 250 MPa
E = 200000 MPa

Allowable centroid load: P = 431.296 kN

Problem 10.60 b

Effective Length = 6m
A = 5860 mm²
ry = 51.5 mm
Yield strength = 345 MPa
E = 200000 MPa

Allowable centroid load: P = 437.859 kN

Problem 10.60 a

Effective Length = 6.00 m
A = 4580.0 mm²
ry = 40.8 mm
Yield strength = 250.0 MPa
E = 200 GPa

Allowable centroid load: P = 217.727 kN

Problem 10.60 b

Effective Length = 6.00 m
A = 11000.0 mm²
ry = 53.2 mm
Yield strength = 250.0 MPa
E = 200 GPa

Allowable centroid load: P = 858.637 kN

Problem 10.124 a

Effective Length = 4.00 m
A = 13.8 mm²
ry = 43.4 mm
Yield strength = 345.0 MPa
E = 200 GPa

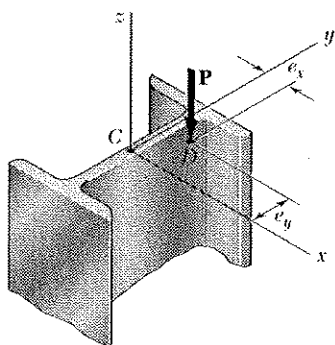
Allowable centroid load: P = 1532 kN

Problem 10.124 b

Effective Length = 6.50 m
A = 13800.0 mm²
ry = 43.4 mm
Yield strength = 345.0 MPa
E = 200 GPa

Allowable centroid load: P = 638.0 kN

PROBLEM 10.C6



10.C6 A column of effective length L is made from a rolled-steel shape and is loaded eccentrically as shown. The yield strength of the grade of steel used is denoted by σ_Y , the allowable stress in bending by σ_{all} , the modulus of elasticity by E , the cross-sectional area of the selected shape by A , and its smallest radius of gyration by r . Write a computer program to determine the allowable load P , using either the allowable-stress method or the interaction method. Use this program to check the given answer for (a) Prob. 10.113, (b) Prob. 10.114.

SOLUTION

ENTER $L, E, \sigma_Y, (\sigma_{all})_{bending}, e_x, e_y$

ENTER PROPERTIES A, s_x, s_y, r_y

DETERMINE ALLOWABLE STRESS

$$C = 4.71 \sqrt{\frac{E}{\sigma_Y}}$$

$$\text{IF } L/r_y \geq C$$

$$\sigma_{all} = \frac{0.877 \pi^2 E}{1.67 (L/r_y)^2}$$

$$\text{IF } L/r_y < C$$

$$\sigma_e = \frac{\pi^2 E}{(L/r_y)^2}$$

$$\sigma_{all} = \frac{1}{1.67} \left[0.658^{\sigma_Y/\sigma_e} \right] \sigma_Y$$

FOR ALLOWABLE-STRESS METHOD

$$COEF = \frac{1}{A} + \frac{e_x}{s_x} + \frac{e_y}{s_y}$$

$$P_{all} = \frac{\sigma_{all}}{COEF}$$

FOR INTERACTION METHOD

$$COFF = \frac{1}{A \sigma_{all}} + \frac{(e_x/s_x) + (e_y/s_y)}{(\sigma_{all})_{bending}}$$

$$P_{all} = \frac{1.0}{COFF}$$

CONTINUED

PROBLEM 10.C6 CONTINUED

PROGRAM OUTPUT

Problem 10.113

Effective Length = 7.2 m
A = 27600 mm²
R_Y = 101 mm
S_X = 3.8 x 10⁶ mm³
Yield strength = 250 MPa
E = 200000 MPa

Using Allowable-Stress Method
Allowable load: P = 1567.68 kN

Problem 10.114

Effective Length = 7.2 m
A = 12900 mm²
R_Y = 62.6 mm
S_X = 1690 x 10³ mm³
Yield strength = 350 MPa
E = 200000 MPa

Using Interaction Method
Allowable load: P = 1212.781 kN

Chapter 11

Problem 11.1**11.1** Determine the modulus of resilience for each of the following aluminum alloys:

- | | | |
|---------------|----------------------|------------------------------|
| (a) 1100-H14: | $E = 70 \text{ GPa}$ | $\sigma_Y = 55 \text{ MPa}$ |
| (b) 2014-T6 | $E = 72 \text{ GPa}$ | $\sigma_Y = 220 \text{ MPa}$ |
| (c) 6061-T6 | $E = 69 \text{ GPa}$ | $\sigma_Y = 150 \text{ MPa}$ |

Aluminum alloys

$$(a) \quad E = 70 \times 10^9 \text{ Pa} \quad \sigma_Y = 55 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(55 \times 10^6)^2}{(2)(70 \times 10^9)} = 21.6 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 21.6 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(b) \quad E = 72 \times 10^9 \text{ Pa} \quad \sigma_Y = 220 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(220 \times 10^6)^2}{(2)(72 \times 10^9)} = 336 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 336 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(c) \quad E = 69 \times 10^9 \text{ Pa} \quad \sigma_Y = 150 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(150 \times 10^6)^2}{(2)(69 \times 10^9)} = 163.0 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 163.0 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

Problem 11.2**11.2** Determine the modulus of resilience for each of the following grades of structural steel:

- | | |
|--------------------------|------------------------------|
| (a) ASTM A709 Grade 50: | $\sigma_Y = 350 \text{ MPa}$ |
| (b) ASTM A913 Grade 65: | $\sigma_Y = 450 \text{ MPa}$ |
| (c) ASTM A709 Grade 100: | $\sigma_Y = 700 \text{ MPa}$ |

Structural steel $E = 200 \text{ GPa}$ for all three steels given.

$$(a) \quad \sigma_Y = 350 \text{ MPa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(350 \times 10^6)^2}{(2)(200 \times 10^9)} = 306.25 \text{ N}\cdot\text{m}/\text{m}^3 = 306.3 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(b) \quad \sigma_Y = 450 \text{ MPa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(450 \times 10^6)^2}{(2)(200 \times 10^9)} = 506.25 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 506.3 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(c) \quad \sigma_Y = 700 \text{ MPa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(700 \times 10^6)^2}{(2)(200 \times 10^9)} = 1225 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 1225 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$= 1.225 \text{ MJ}/\text{m}^3$$

Proprietary Material. © 2009 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

Problem 11.3

11.3 Determine the modulus of resilience for each of the following alloys:

- | | |
|-----------------------------|--|
| (a) Titanium: | $E = 115 \text{ GPa}$, $\sigma_Y = 875 \text{ MPa}$ |
| (b) Magnesium: | $E = 45 \text{ GPa}$, $\sigma_Y = 200 \text{ MPa}$ |
| (c) Cupronickel (annealed): | $E = 140 \text{ GPa}$, $\sigma_Y = 125 \text{ MPa}$ |

(a) $E = 115 \text{ GPa}$, $\sigma_Y = 875 \text{ MPa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(875 \times 10^6)^2}{(2)(115 \times 10^9)} = 3328.8 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 3328.8 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$= 3.33 \text{ MJ}/\text{m}^3$$

(b) $E = 45 \text{ GPa}$, $\sigma_Y = 200 \text{ MPa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(200 \times 10^6)^2}{(2)(45 \times 10^9)} = 444.4 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 444.4 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

(c) $E = 140 \text{ GPa}$, $\sigma_Y = 125 \text{ MPa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(125 \times 10^6)^2}{(2)(140 \times 10^9)} = 55.8 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 55.8 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

Problem 11.4

11.4 Determine the modulus of resilience for each of the following metals:

- | | | |
|-----------------------------|-----------------------|------------------------------|
| (a) Stainless steel | | |
| AISI 302 (annealed): | $E = 190 \text{ GPa}$ | $\sigma_Y = 260 \text{ MPa}$ |
| (b) Stainless steel 2014-T6 | | |
| AISI 302 (cold-rolled): | $E = 190 \text{ GPa}$ | $\sigma_Y = 520 \text{ MPa}$ |
| (c) Malleable cast iron: | $E = 165 \text{ GPa}$ | $\sigma_Y = 230 \text{ MPa}$ |

(a) $E = 190 \times 10^9 \text{ Pa}$, $\sigma_Y = 260 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(260 \times 10^6)^2}{(2)(190 \times 10^9)} = 177.9 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 177.9 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

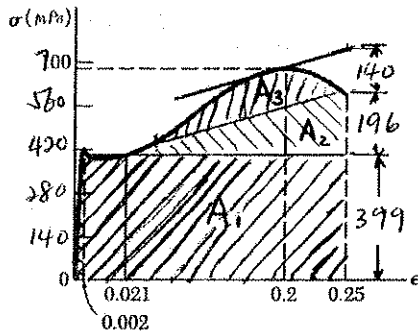
(b) $E = 190 \times 10^9 \text{ Pa}$, $\sigma_Y = 520 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(520 \times 10^6)^2}{(2)(190 \times 10^9)} = 712 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 712 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

(c) $E = 165 \times 10^9 \text{ Pa}$, $\sigma_Y = 230 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(230 \times 10^6)^2}{(2)(165 \times 10^9)} = 160.3 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 160.3 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

Problem 11.5



11.5 The stress-strain diagram shown has been drawn from data obtained during a tensile test of a specimen of structural steel. Using $E = 200$ GPa, determine (a) the modulus of resilience of the steel, (b) the modulus of toughness of the steel.

(a) $\sigma_Y = E \epsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2} E \epsilon_Y^2 = \frac{1}{2} (200 \times 10^9) (0.002)^2$$

$$= 400 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 400 \text{ kJ}/\text{m}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

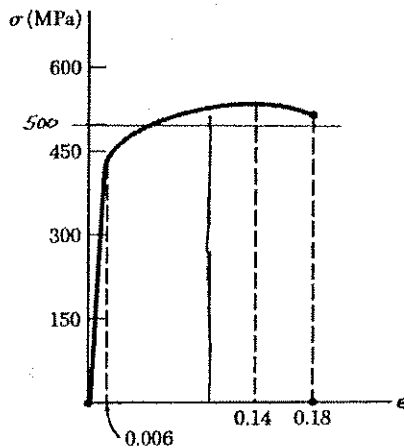
$$A_1 = (399) (0.25 - 0.002) = 98.95 \times 10^6 \text{ N}/\text{m}^2$$

$$A_2 = \frac{1}{2} (196) (0.25 - 0.021) = 22.44 \times 10^6 \text{ N}/\text{m}^2$$

$$A_3 = \frac{2}{3} (140) (0.25 - 0.075) = 16.33 \times 10^6 \text{ N}/\text{m}^2$$

$$\text{modulus of toughness} = U_Y + A_1 + A_2 + A_3 \approx 137.7 \text{ MJ}/\text{m}^3$$

Problem 11.6



11.6 The stress-strain diagram shown has been drawn from data obtained during a tensile test of an aluminum alloy. Using $E = 72$ GPa, determine (a) the modulus of resilience of the alloy, (b) the modulus of toughness of the alloy.

(a) $\sigma_Y = E \epsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2} E \epsilon_Y^2 = \frac{1}{2} (72 \times 10^9) (0.006)^2$$

$$= 1296 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 1296 \text{ kJ}/\text{m}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

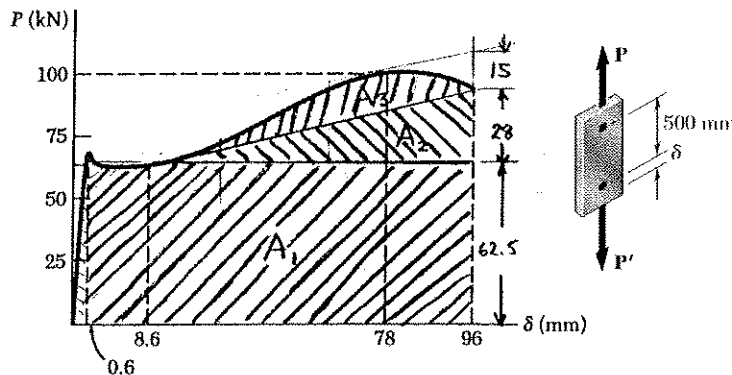
The average ordinate of the stress-strain curve is $500 \text{ MPa} = 500 \times 10^6 \text{ N}/\text{m}^2$

$$\text{The area under the curve is } A = (500 \times 10^6) (0.18) = 90 \times 10^6 \text{ N}/\text{m}^2$$

$$\text{modulus of toughness} = 90 \times 10^6 \text{ J}/\text{m}^3 = 90 \text{ MJ}/\text{m}^3$$

Problem 11.7

11.7 The load-deformation diagram shown has been drawn from data obtained during a tensile test of structural steel. Knowing that the cross-sectional area of the specimen is 250 mm^2 and that the deformation was measured using a 500-mm gage length, determine (a) the modulus of resilience of the steel, (b) the modulus of toughness of the steel.



Assuming that yielding occurs at $P = 62.5 \text{ kN}$ and $S = 0.6 \text{ mm}$

$$\begin{aligned} U_Y &= \frac{1}{2} (62.5 \times 10^3) (0.6 \times 10^{-3}) \\ &= 18.75 \text{ N}\cdot\text{m} \\ &= 18.75 \text{ J} \end{aligned}$$

Volume of stressed material: $V = AL = (250)(500) = 125 \times 10^3 \text{ mm}^3$
 $= 125 \times 10^{-6} \text{ m}^3$

$$U_Y = \frac{U_Y}{V} = \frac{18.75}{125 \times 10^{-6}} = 150 \times 10^3 = 150 \text{ kJ/m}^3$$

$$A_1 = (62.5 \times 10^3) (96 \times 10^{-3}) = 6 \times 10^3 \text{ N}\cdot\text{m} = 6 \times 10^3 \text{ J}$$

$$A_2 = \frac{1}{2} (28 \times 10^3) (96 - 8.6) \times 10^{-3} = 1.22 \times 10^3 \text{ N}\cdot\text{m} = 1.22 \times 10^3 \text{ J}$$

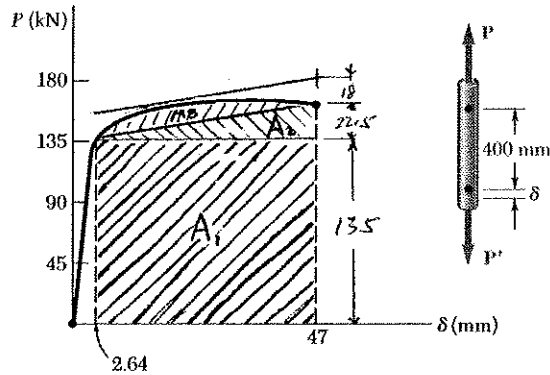
$$A_3 = \frac{2}{3} (15 \times 10^3) (61 \times 10^{-3}) = 0.61 \times 10^3 \text{ N}\cdot\text{m} = 0.61 \times 10^3 \text{ J}$$

Total energy: $U = U_Y + A_1 + A_2 + A_3 = 7.85 \times 10^3 \text{ J}$

$$\begin{aligned} \text{modulus of toughness} &= \frac{U}{V} = \frac{7.85 \times 10^3}{125 \times 10^{-6}} = 63 \times 10^6 \text{ J/m}^3 \\ &= 63 \text{ MJ/m}^3 \end{aligned}$$

Problem 11.8

11.8 The load-deformation diagram shown has been drawn from data obtained during a tensile test of a 18-mm-diameter rod of an aluminum alloy. Knowing that the deformation was measured using a 400-mm gage length, determine (a) the modulus of resilience of the alloy, (b) the modulus of toughness of the alloy.



Volume of stressed material involved in the measurement:

$$V = \frac{\pi}{4} d^2 L = \frac{\pi}{4} (0.018)^2 (0.4) = 101.79 \times 10^{-6} \text{ m}^3$$

(a) Modulus of resilience.

$$P_Y = 135 \text{ kN}, \quad \delta_Y = 0.00264 \text{ m}$$

$$U_Y = \frac{1}{2} P_Y \delta_Y = \frac{1}{2} (135000)(0.00264) = 178.2 \text{ N}\cdot\text{m}$$

$$\text{modulus of resilience} \quad u_Y = \frac{U_Y}{V} = \frac{178.2}{101.79} = 1.75 \text{ N}\cdot\text{m}/\text{m}^3 = 1.75 \text{ J}/\text{m}^3$$

(b) Modulus of toughness.

$$A_1 = (135000)(0.047 - 0.00264) = 5.9886 \text{ kN}\cdot\text{m} = 5.9886 \text{ kJ}$$

$$A_2 = \frac{1}{2} (22500)(0.047 - 0.00264) = 0.9981 \text{ kN}\cdot\text{m} = 0.9981 \text{ kJ}$$

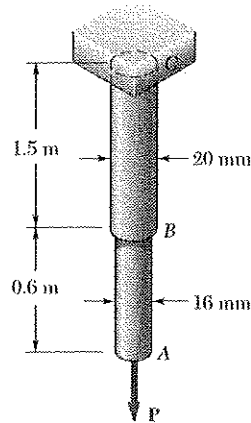
$$A_3 = \frac{2}{3} (18000)(0.047 - 0.00264) = 0.79848 \text{ kN}\cdot\text{m} = 0.79848 \text{ kJ}$$

$$U = U_Y + A_1 + A_2 + A_3 = 7.963 \text{ kJ}$$

$$\text{modulus of toughness} \quad \frac{U}{V} = \frac{7.963}{101.79 \times 10^{-6}} = 78.2 \text{ MJ}/\text{m}^3$$

Problem 11.9

11.9 Using $E = 200 \text{ GPa}$, determine (a) the strain energy of the steel rod ABC when $P = 35 \text{ kN}$, (b) the corresponding strain energy density in portions AB and BC of the rod.



$$P = 35 \text{ kN}, \quad E = 200 \text{ GPa}$$

$$A = \frac{\pi}{4} d^2, \quad V = AL, \quad \sigma = \frac{P}{A}, \quad u = \frac{\sigma^2}{2E}$$

$$U = uV$$

Portion	d m	L m	A m^2	V m^3	σ MPa	u kJ/m^3	U J
AB	0.020	1.5	201×10^{-6}	120.6×10^{-6}	174.1	75.777	9.093
BC	0.016	0.6	314.2×10^{-6}	471.3×10^{-6}	111.4	31.025	14.622
Σ							23.715

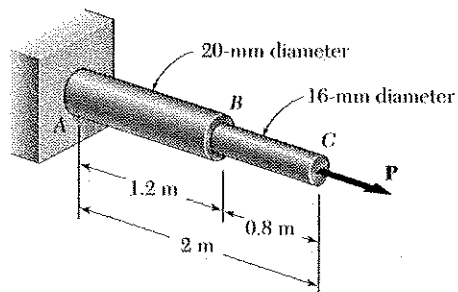
(a) $U = 23.7 \text{ J}$

(b) In AB, $u = 75.8 \text{ kJ/m}^3$

In BC, $u = 31.0 \text{ kJ/m}^3$

Problem 11.10

11.10 Using $E = 200 \text{ GPa}$, determine (a) the strain energy of the steel rod ABC when $P = 25 \text{ kN}$, (b) the corresponding strain-energy density in portions AB and BC of the rod.



$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$P = 25 \times 10^3 \text{ N}$$

$$\begin{aligned} U &= \sum \frac{P^2 L}{2EA} \\ &= \frac{(25 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(314.16 \times 10^{-6})} \\ &\quad + \frac{(25 \times 10^3)^2 (0.8)}{(2)(200 \times 10^9)(201.06 \times 10^{-6})} \end{aligned}$$

$$(a) \quad U = 5.968 + 6.213 = 12.18 \text{ N}\cdot\text{m} = 12.18 \text{ J}$$

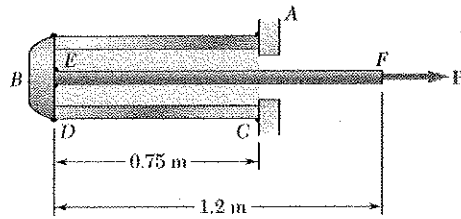
$$(b) \quad \sigma_{AB} = \frac{P}{A_{AB}} = \frac{25 \times 10^3}{314.16 \times 10^{-6}} = 79.58 \times 10^6 \text{ Pa}$$

$$U_{AB} = \frac{\sigma_{AB}^2}{2E} = \frac{(79.58 \times 10^6)^2}{(2)(200 \times 10^9)} = 15.83 \times 10^3 = 15.83 \text{ kJ/m}^3$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{25 \times 10^3}{201.06 \times 10^{-6}} = 124.28 \times 10^6 \text{ Pa}$$

$$U_{BC} = \frac{\sigma_{BC}^2}{2E} = \frac{(124.28 \times 10^6)^2}{(2)(200 \times 10^9)} = 38.6 \times 10^3 = 38.6 \text{ kJ/m}^3$$

Problem 11.11



11.11 A 0.75-m length of aluminum pipe of cross-sectional area 1190 mm^2 is welded to a fixed support A and to a rigid cap B . The steel rod EF , of 18-mm diameter, is welded to cap B . Knowing that the modulus of elasticity is 200 GPa for the steel and 73 GPa for the aluminum, determine (a) the total strain of the system when $P = 40 \text{ kN}$, (b) the corresponding strain-energy density of the pipe CD and in the rod EF .

For EF : $A = \frac{\pi}{4} d^2 = 254.47 \text{ mm}^2$

$$CD: U_{CD} = \frac{P^2 L}{2EA} = \frac{(40 \times 10^3)^2 (0.75)}{2(73 \times 10^9)(1190 \times 10^{-6})} = 6.907 \text{ J}$$

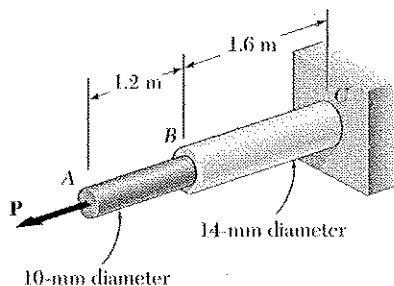
$$EF: U_{EF} = \frac{P^2 L}{2EA} = \frac{(40 \times 10^3)^2 (1.2)}{2(200 \times 10^9)(254.47 \times 10^{-6})} = 18.863 \text{ J}$$

(a) Total: $U = U_{CD} + U_{EF} = 25.77 \text{ J}$

(b) $CD: \epsilon = -\frac{40000}{1190} = -33.61 \text{ MPa}$, $U = \frac{\epsilon^2}{2E} = \frac{(-33.61)^2}{(2)(73 \times 10^3)} = 0.0077 \text{ J/m}^3$

$EF: \epsilon = \frac{40000}{254.47} = 157.19 \text{ MPa}$, $U = \frac{\epsilon^2}{2E} = \frac{157.19^2}{(2)(200 \times 10^3)} = 0.062 \text{ J/m}^3$

Problem 11.12



11.12 Rod AB is made of a steel for which the yield strength is $\sigma_Y = 450 \text{ MPa}$ and $E = 200 \text{ GPa}$; rod BC is made of an aluminum alloy for which $\sigma_Y = 280 \text{ MPa}$ and $E = 73 \text{ GPa}$. Determine the maximum strain energy that can be acquired by the composite rod ABC without causing any permanent deformations.

$$A_{AB} = \frac{\pi}{4} (10)^2 = 78.54 \text{ mm}^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (14)^2 = 153.94 \text{ mm}^2 = 153.94 \times 10^{-6} \text{ m}^2$$

$$P_{all} = \sigma_Y A \text{ for each portion.}$$

$$AB: P_{all} = (450 \times 10^6)(78.54 \times 10^{-6}) = 35.343 \times 10^3 \text{ N}$$

$$BC: P_{all} = (280 \times 10^6)(153.94 \times 10^{-6}) = 43.103 \times 10^3 \text{ N}$$

Use the smaller value.

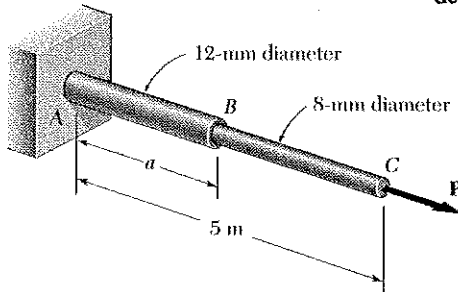
$$P = 35.343 \times 10^3 \text{ N}$$

$$U = \frac{P^2 L_{AB}}{2E_{AB} A_{AB}} + \frac{P^2 L_{BC}}{2E_{BC} A_{BC}} = \frac{(35.343 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})} + \frac{(35.343 \times 10^3)^2 (1.6)}{(2)(73 \times 10^9)(153.94 \times 10^{-6})}$$

$$U = 136.6 \text{ J}$$

Problem 11.13

11.13 Rods AB and BC are made of a steel for which the yield strength is $\sigma_Y = 300$ MPa and the modulus of elasticity is $E = 200$ GPa. Determine the maximum strain energy that can be acquired by the assembly without causing permanent deformation when the length a of rod AB is (a) 2 m, (b) 4 m.



$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(8)^2 = 50.265 \text{ mm}^2 = 50.265 \times 10^{-6} \text{ m}^2$$

$$P = \sigma_Y A_{\min} = (300 \times 10^6)(50.265 \times 10^{-6}) \\ = 15.08 \times 10^3 \text{ N}$$

$$U = \sum \frac{P^2 l}{2EA} = \frac{P^2}{2E} \sum \frac{l}{A}$$

(a) $a = 2 \text{ m}$, $L - a = 5 - 2 = 3 \text{ m}$

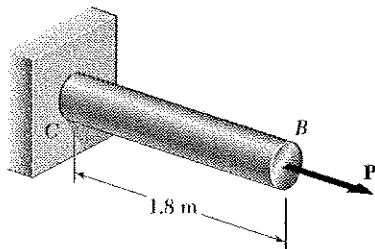
$$U = \frac{(15.08 \times 10^3)^2}{(2)(200 \times 10^9)} \left[\frac{2}{113.097 \times 10^{-6}} + \frac{3}{50.265 \times 10^{-6}} \right] = 44.0 \text{ N}\cdot\text{m} = 44.0 \text{ J}$$

(b) $a = 4 \text{ m}$, $L - a = 1 \text{ m}$

$$U = \frac{(15.08 \times 10^3)^2}{(2)(200 \times 10^9)} \left[\frac{4}{113.097 \times 10^{-6}} + \frac{1}{50.265 \times 10^{-6}} \right] = 31.4 \text{ N}\cdot\text{m} = 31.4 \text{ J}$$

Problem 11.14

11.14 Rod BC is made of a steel for which the yield strength is $\sigma_Y = 300$ MPa and the modulus of elasticity is $E = 200$ GPa. Knowing that a strain energy of 10 J must be acquired by the rod when the axial load P is applied, determine the diameter of the rod for which the factor of safety with respect to permanent deformation is six.



For factor of safety of six on the energy,

$$U_Y = (6)(10) = 60 \text{ J}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(300 \times 10^6)^2}{(2)(200 \times 10^9)} = 225 \times 10^3 \text{ J/m}^3$$

$$A = \frac{U_Y}{L U_Y} = \frac{60}{(1.8)(225 \times 10^3)} = 148.148 \times 10^{-6} \text{ m}^2$$

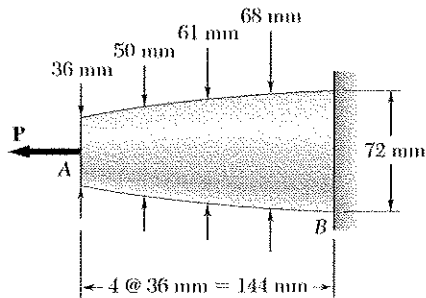
$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(148.148 \times 10^{-6})}{\pi}} = 13.73 \times 10^{-3} \text{ m} \\ d = 13.73 \text{ mm}$$

$$U_Y = A L U_Y$$

$$A = \frac{\pi d^2}{4}$$

Problem 11.15

11.15 Using $E = 74 \text{ GPa}$, determine by approximate means the maximum strain energy that can be acquired by the aluminum rod shown if the allowable normal stress is $\sigma_{all} = 154 \text{ MPa}$.



$$A_{min} = \frac{\pi}{4} (36)^2 = 1017.9 \text{ mm}^2$$

$$\sigma_{all} = 154 \text{ MPa}$$

$$P_{all} = \sigma_{all} A_{min} = 156756.6 \text{ N}$$

$$U = \int \frac{P^2 dx}{2EA} = \frac{P^2}{2E} \int \frac{dx}{\frac{\pi}{4} d^2} = \frac{2P^2}{\pi E} \int \frac{dx}{d^2}$$

Use Simpson's rule to compute the integral,

$$h = 36 \text{ mm}$$

Section	d(m)	1/d ² (m ⁻²)	multiplier	m(1/d ²) (m ⁻²)
1	0.036	771.6	1	771.6
2	0.05	400	4	1600
3	0.061	268.7	2	537.4
4	0.068	216.3	4	865.2
5	0.072	192.9	1	192.9
Σ				3967.1

$$\int_A^B \frac{dx}{d^2} = \frac{h}{3} \Sigma m\left(\frac{1}{d^2}\right) = \frac{0.036}{3} (3967.1) = 47.6052 \text{ m}^{-1}$$

$$U = \frac{(2)(156756.6)^2 (47.6052)}{\pi (74 \times 10^9)}$$

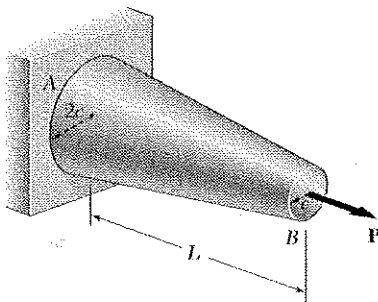
$$U = 10.06 \text{ J}$$

Problem 11.16

11.16 Show by integration that the strain energy of the tapered rod AB is

$$U = \frac{1}{4} \frac{P^2 L}{EA_{min}}$$

where A_{min} is the cross-sectional area at end B.



$$\text{radius: } r = \frac{cx}{L}$$

$$A_{min} = \pi c^2$$

$$A = \pi r^2 = \frac{\pi c^2}{L^2} x^2$$

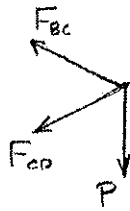
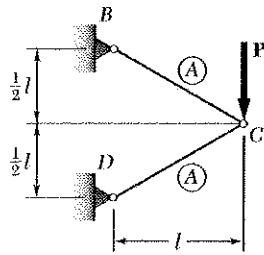
$$U = \int_L^{2L} \frac{P^2 dx}{2EA} = \frac{P^2}{2E} \int_L^{2L} \frac{L^2}{\pi c^2} \frac{dx}{x^2}$$

$$= \frac{P^2 L^2}{2E \pi c^2} \left(-\frac{1}{x} \right) \Big|_L^{2L}$$

$$= \frac{P^2 L^2}{2EA_{min}} \left(-\frac{1}{2L} + \frac{1}{L} \right)$$

$$U = \frac{1}{4} \frac{P^2 L}{EA_{min}}$$

Problem 11.17



11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.

$$L_{BC} = L_{CD} = \sqrt{l^2 + (\frac{1}{2}l)^2} = \frac{\sqrt{5}}{2}l$$

Joint C

$$+\rightarrow \Sigma F_x = 0: -\frac{2}{\sqrt{5}}F_{BC} - \frac{2}{\sqrt{5}}F_{CD} = 0$$

$$F_{CD} = -F_{BC}$$

$$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}}F_{BC} - \frac{1}{\sqrt{5}}F_{CD} - P = 0$$

$$F_{BC} = \frac{\sqrt{5}}{2}P$$

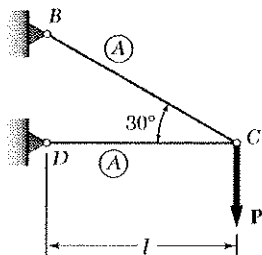
$$F_{CD} = -\frac{\sqrt{5}}{2}P$$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA} [F_{BC}^2 L_{BC} + F_{CD}^2 L_{CD}]$$

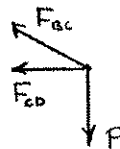
$$= \frac{1}{2EA} [(\frac{\sqrt{5}}{2}P)^2 (\frac{\sqrt{5}}{2}l) + (\frac{\sqrt{5}}{2}P)^2 (\frac{\sqrt{5}}{2}l)]$$

$$U = 1.398 \frac{P^2 l}{EA}$$

Problem 11.18



Joint C



$$+\uparrow \Sigma F_y = 0:$$

$$\frac{1}{2}F_{BC} - P = 0 \quad F_{BC} = 2P$$

$$+\rightarrow \Sigma F_x = 0:$$

$$-F_{CD} - \frac{\sqrt{3}}{2}F_{BC} = 0 \quad F_{CD} = -\sqrt{3}P$$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

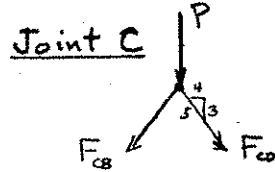
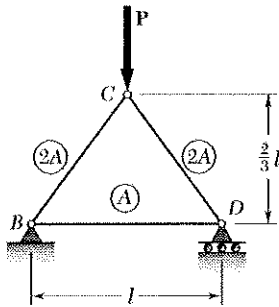
Member	F	L	A	$F^2 L / A$
BC	$2P$	$\frac{2}{\sqrt{3}}l$	A	$\frac{8}{\sqrt{3}}P^2 l / A$
CD	$-\sqrt{3}P$	l	A	$3P^2 l / A$
Σ				$7.62P^2 l / A$

$$U = \frac{1}{2E} (7.62 \frac{P^2 l}{A})$$

$$U = 3.81 \frac{P^2 l}{EA}$$

Problem 11.19

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.



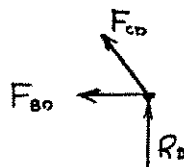
$$+\rightarrow \sum F_x = 0; \frac{3}{5} F_{CB} - \frac{3}{5} F_{CD} = 0$$

$$F_{CB} = F_{CD}$$

$$+\uparrow \sum F_y = 0; -P - 2 \cdot \frac{4}{5} F_{CD} = 0$$

$$F_{CB} = F_{CD} = -\frac{5}{8} P$$

Joint D



$$+\rightarrow \sum F_x = 0;$$

$$-F_{BD} - \frac{3}{5} F_{CD} = 0$$

$$F_{BD} = -\frac{3}{5} \cdot \frac{5}{8} P = \frac{3}{8} P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

$$= \frac{1}{2E} \left(\frac{179}{384} \frac{P^2 l}{A} \right)$$

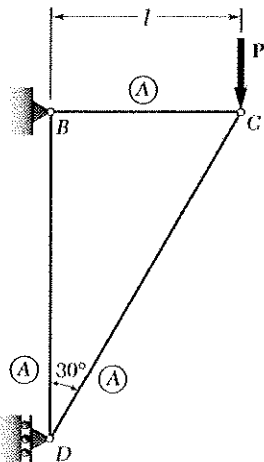
$$= \frac{179}{768} \frac{P^2 l}{EA}$$

$$U = 0.233 \frac{P^2 l}{EA}$$

Member	F	L	A	$F^2 L / A$
CB	$-\frac{5}{8} P$	$\frac{5}{6} l$	$2A$	$\frac{125}{768} \frac{P^2 l}{A}$
CD	$-\frac{5}{8} P$	$\frac{5}{6} l$	$2A$	$\frac{125}{768} \frac{P^2 l}{A}$
BD	$\frac{3}{8} P$	l	A	$\frac{9}{256} \frac{P^2 l}{A}$
Σ				$\frac{179}{384} \frac{P^2 l}{A}$

Problem 11.20

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.



Joint C

$$\uparrow \sum F_y = 0: -\frac{\sqrt{3}}{2} F_{CD} - P = 0$$

$$F_{CD} = -\frac{2}{\sqrt{3}} P$$

$$+\rightarrow \sum F_x = 0: -F_{BC} - \frac{1}{2} F_{CD} = 0$$

$$F_{BC} = \frac{1}{\sqrt{3}} P$$

Joint D

$$+\uparrow \sum F_y = 0:$$

$$F_{BD} + \frac{\sqrt{3}}{2} F_{CD} = 0$$

$$F_{BD} = P$$

Member	F	L	A	$F^2 L / A$
BC	$\frac{1}{\sqrt{3}} P$	l	A	$\frac{1}{3} P^2 l / A$
CD	$-\frac{2}{\sqrt{3}} P$	$2l$	A	$\frac{8}{3} P^2 l / A$
BD	P	$\sqrt{3} l$	A	$\sqrt{3} P^2 l / A$
Σ				$4.732 P^2 l / A$

$$U = \sum \frac{1}{2} \frac{F^2 L}{EA}$$

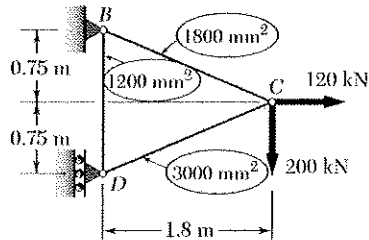
$$= \frac{1}{2E} \sum \frac{F^2 L}{A}$$

$$= \frac{1}{2E} (4.732 \frac{P^2 l}{A})$$

$$U = 2.37 \frac{P^2 l}{EA}$$

Problem 11.21

11.21 In the truss shown, all members are made of aluminum and have the uniform cross-sectional area shown. Using $E = 72 \text{ GPa}$, determine the strain energy of the truss for the loading shown.



$$l_{BC} = l_{CD} = \sqrt{1.8^2 + 0.75^2} = 1.95 \text{ m}$$

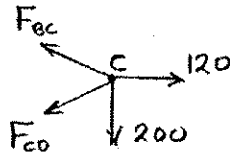
Joint C

$$+\rightarrow \Sigma F_x = 0: -\frac{1.8}{1.95} F_{BC} - \frac{1.8}{1.95} F_{CD} + 120 = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \frac{0.75}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} - 200 = 0 \quad (2)$$

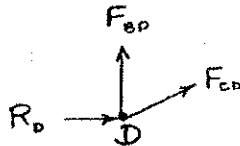
Solving (1) and (2) simultaneously,

$$F_{BC} = 325 \text{ kN} \quad F_{CD} = -195 \text{ kN}$$



Joint D

$$+\uparrow \Sigma F_y = 0: F_{BD} + \frac{0.75}{1.95} F_{CD} = 0 \quad F_{BD} = 75 \text{ kN}$$

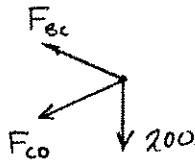
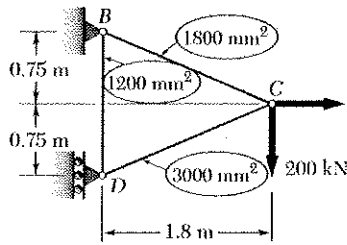


$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10^{-6} m^2)	$F^2 L / A$ (N^2 / m)
BC	325	1.95	1800	114.43×10^{12}
BD	75	1.5	1200	7.03×10^{12}
CD	-195	1.95	3000	24.72×10^{12}
Σ				146.18×10^{12}

$$U = \frac{146.18 \times 10^{12}}{(2)(72 \times 10^9)} = 1.015 \times 10^3 \text{ N-m} \quad U = 1015 \text{ J}$$

Problem 11.22



11.22 Solve Prob. 11.21, assuming that the 120-kN load is removed.

11.21 In the truss shown, all members are made of aluminum and have the uniform cross-sectional area shown. Using $E = 72 \text{ GPa}$, determine the strain energy of the truss for the loading shown.

$$l_{BC} = l_{CD} = \sqrt{1.8^2 + 0.75^2} = 1.95 \text{ m}$$

Joint C

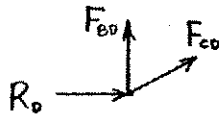
$$+\rightarrow \sum F_x = 0: -\frac{1.8}{1.95} F_{BC} - \frac{1.8}{1.95} F_{CD} = 0$$

$$+\uparrow \sum F_y = 0: \frac{0.75}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} - 200 = 0$$

Solving (1) and (2) simultaneously,

$$F_{BC} = 260 \text{ kN} \quad F_{CD} = -260 \text{ kN}$$

Joint D



$$+\uparrow \sum F_y = 0: F_{BD} + \frac{0.75}{1.95} F_{CD} = 0 \quad F_{BD} = 100 \text{ kN}$$

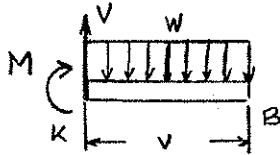
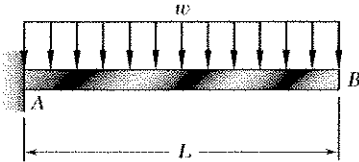
$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10^6 m^2)	$F^2 L / A \text{ (N}^2/\text{m)}$
BC	260	1.95	1800	73.23×10^{12}
BD	100	1.5	1200	12.50×10^{12}
CD	-260	1.95	3000	43.94×10^{12}
Σ				129.67×10^{12}

$$U = \frac{129.67 \times 10^{12}}{(2)(72 \times 10^9)} = 900 \text{ N}\cdot\text{m} \quad U = 900 \text{ J}$$

Problem 11.23

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\oplus \sum M_K = 0: -M - (wv)\left(\frac{v}{2}\right) = 0$$

$$M = -\frac{1}{2} wv^2$$

$$U = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \left(\frac{1}{2} wv^2\right)^2 dv$$

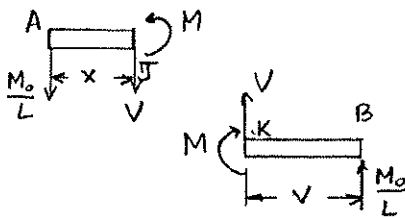
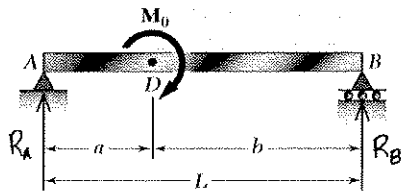
$$= \frac{w^2}{8EI} \int_0^L v^4 dv = \frac{w^2}{8EI} \left[\frac{v^5}{5} \right]_0^L$$

$$= \frac{w^2 L^5}{40EI}$$

$$U = \frac{w^2 L^5}{40EI}$$

Problem 11.24

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\oplus \sum M_B = 0: -R_A L - M_0 = 0 \quad R_A = \frac{M_0}{L} \downarrow$$

$$\oplus \sum M_A = 0: R_B L - M_0 = 0 \quad R_B = \frac{M_0}{L} \uparrow$$

$$\text{A to D: } \oplus \sum M_J = 0: \frac{M_0 x}{L} + M = 0$$

$$M = -\frac{M_0 x}{L}$$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_0^2}{2EIL^2} \int_0^a x^2 dx = \frac{M_0^2 a^3}{6EIL^2}$$

$$\text{D to B: } \oplus \sum M_K = 0: -M + \frac{M_0 v}{L}$$

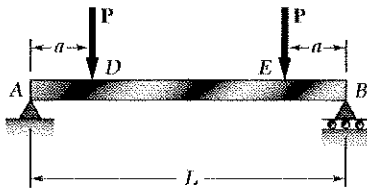
$$M = \frac{M_0 v}{L}$$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{M_0^2}{2EIL^2} \int_0^b v^2 dv = \frac{M_0^2 b^3}{6EIL^2}$$

$$\text{Total: } U = U_{AD} + U_{DB} = \frac{M_0^2 (a^3 + b^3)}{6EIL^2}$$

Problem 11.25

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



Symmetric beam and loading: $R_A = R_B$

$$+\uparrow \Sigma F_y = 0: R_A + R_B - 2P = 0 \quad R_A = R_B = P$$

Over portion AD: $M = R_A x = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^a = \frac{P^2 a^3}{6EI}$$

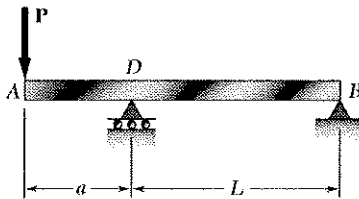
Over portion DE: $M = Pa$ $U_{DE} = \frac{P^2 a^2 (L - 2a)}{2EI}$

Over portion EB: By symmetry $U_{EB} = U_{AD} = \frac{P^2 a^3}{6EI}$

Total: $U = U_{AD} + U_{DE} + U_{EB} = \frac{P^2 a^2}{6EI} (3L - 4a)$ $\blacktriangleleft$

Problem 11.26

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\Sigma M_D = 0:$$

$$aP + LR_B = 0 \quad R_B = -\frac{aP}{L} = \frac{aP}{L} \downarrow$$

Over portion AD:

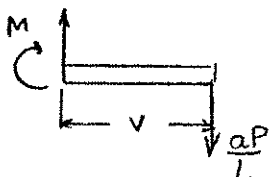
$$M = -Px$$



$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a P^2 x^2 dx = \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^a = \frac{P^2 a^3}{6EI}$$

Over portion DB:

$$M = -\frac{aP}{L} v$$



$$U_{DB} = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \frac{a^2 P^2}{L^2} v^2 dv = \frac{P^2 a^2}{2EIL^2} \int_0^L v^2 dv = \frac{P^2 a^2}{2EIL^2} \left[\frac{v^3}{3} \right]_0^L = \frac{P^2 a^2 L}{6EI}$$

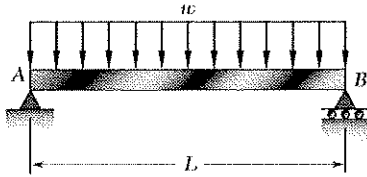
Total: $U = U_{AD} + U_{DB} = \frac{P^2 a^2}{6EI} (a + L)$ $\blacktriangleleft$

Problem 11.27

11.27 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is

$$u_{\max} = \frac{45 U}{8 V}$$

where U is the strain energy of the beam and V is its volume.



$$+\circlearrowleft M_B = 0: -R_A L + (wL) \frac{L}{2} = 0 \quad R_A = \frac{1}{2} wL$$

$$M = R_A x - \frac{1}{2} wL^2 = \frac{1}{2} w(Lx - x^2)$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{w^2}{8EI} \int_0^L (L^2 x^2 - 2Lx^3 + x^4) dx = \frac{w^2}{8EI} \left[\frac{L^2 x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right]_0^L$$

$$= \frac{w^2 L^5}{8EI} \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right] = \frac{w^2 L^5}{240EI}$$

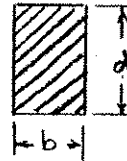
$$M_{\max} = \frac{1}{2} w \left[L \cdot \frac{L}{2} - \left(\frac{L}{2} \right)^2 \right] = \frac{1}{8} wL^2$$

$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{wL^2 c}{8I}$$

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{w^2 L^4 c^2}{128 EI^2}$$

$$\frac{U}{U_{\max}} = \frac{8LI}{15c^2} = \frac{8L(\frac{1}{2}bd^3)}{15(\frac{d}{2})^2} = \frac{8}{45} Lbd = \frac{8}{45} V$$

$$U_{\max} = \frac{45}{8} \frac{U}{V}$$

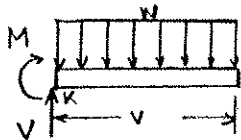
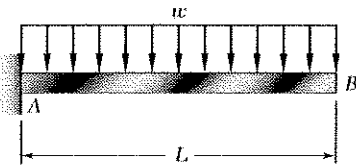


Problem 11.28

11.28 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is

$$u_{\max} = 15 \frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.



$$+\circlearrowleft \sum M_k = 0: \quad -M - (wv) \frac{v}{2} = 0$$

$$M = -\frac{1}{2} w v^2$$

$$U = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \left(\frac{1}{2} w v^2 \right)^2 dv = \frac{w^2}{8EI} \int_0^L v^4 dv = \frac{w^2}{8EI} \frac{v^5}{5} \Big|_0^L = \frac{w^2 L^5}{40EI}$$

$$M_{\max} = \frac{1}{2} w L^2$$

$$\sigma_{\max} = \frac{M_{\max} c}{I}$$

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{M_{\max}^2 c^2}{2EI^2} = \frac{\frac{1}{4} w^2 L^4 c^2}{2EI^2} = \frac{w^2 L^4 c^2}{8EI^2}$$

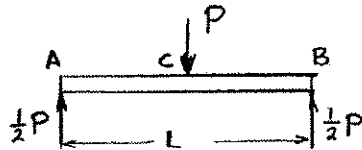
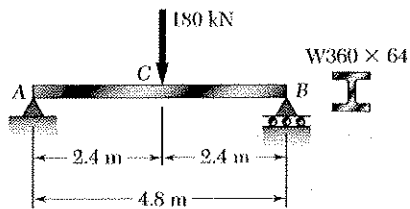
$$\frac{U}{U_{\max}} = \frac{LI}{5c^2} = \frac{L \left(\frac{1}{12} b d^3 \right)}{5 \left(\frac{d}{2} \right)^2} = \frac{1}{15} L b d = \frac{1}{15} V$$

$$U_{\max} = 15 \frac{U}{V}$$



Problem 11.29

11.29 and 11.31 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



Over portion AC: $M = \frac{1}{2}Px$

$$U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx = \frac{P^2}{8EI} \int_0^{\frac{L}{2}} x^2 dx$$

$$= \frac{P^2}{8EI} \left[\frac{x^3}{3} \right]_0^{\frac{L}{2}} = \frac{P^2 L^3}{192EI}$$

By symmetry, $U_{CB} = U_{AC} = \frac{P^2 L^3}{192EI}$

Total: $U = U_{AC} + U_{CB} = \frac{P^2 L^3}{96EI}$

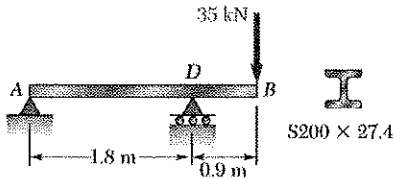
Data: $P = 180 \times 10^3 \text{ N}$, $L = 4.8 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$
 $I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6} \text{ m}^4$

$$U = \frac{(180 \times 10^3)^2 (4.8)^3}{(96)(200 \times 10^9)(178 \times 10^{-6})} = 1048 \text{ N} \cdot \text{m}$$

$$U = 1048 \text{ J}$$

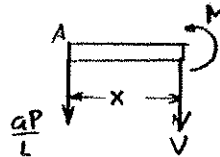
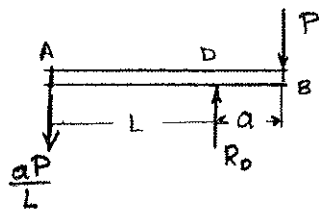
Problem 11.30

11.30 and 11.32 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



$$\sum M_D = 0 \quad -R_A L - aP = 0 \quad R_A = \frac{aP}{L} \downarrow$$

Over portion AD: $M = -\frac{aP}{L}x$



$$\begin{aligned} U_{AD} &= \int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^L \left(\frac{aP}{L} x \right)^2 dx \\ &= \frac{P^2 a^2}{2EI L^2} \int_0^L x^2 dx \\ &= \frac{P^2 a^2 L}{6EI} \end{aligned}$$

Over portion DB: $M = -Px$



$$U_{DB} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a P^2 x^2 dx = \frac{P^2 a^3}{6EI}$$

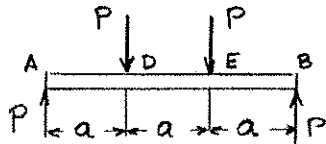
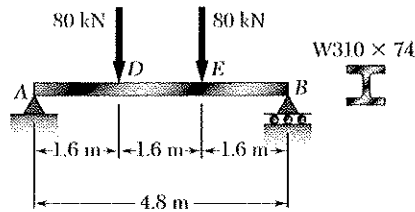
$$\text{Total: } U = U_{AD} + U_{DB} = \frac{P^2 a^2}{6EI} (a + L)$$

Data: $P = 35000$, $L = 1.8 \text{ m}$, $a = 0.9 \text{ m}$, $E = 200600 \text{ Pa}$
 $I = 23.9 \times 10^{-6} \text{ m}^4$

$$U = \frac{(35000)^2 (0.9)^2 (1.8 + 0.9)}{(6)(200 \times 10^9)(23.9 \times 10^{-6})} = 93.4 \text{ J}$$

Problem 11.31

11.29 and 11.31 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



Over portion AD: $M = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a (Px)^2 dx$$

$$= \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^a = \frac{P^2 a^3}{6EI}$$

Over portion DE: $M = Pa$

$$U_{DE} = \frac{(Pa)^2 a}{2EI} = \frac{P^2 a^3}{2EI}$$

By symmetry, $U_{EB} = U_{AD} = \frac{P^2 a^3}{6EI}$

$$U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{6} \frac{P^2 a^3}{EI}$$

Data: $P = 80 \times 10^3 \text{ N}$, $a = 1.6 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$

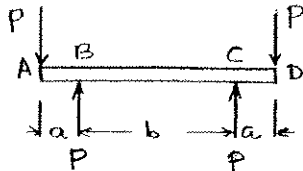
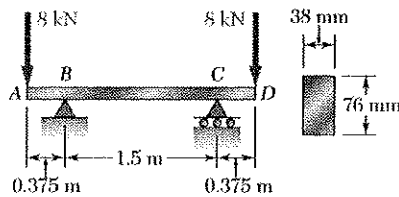
$$I = 165 \times 10^6 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$$

$$U = \frac{5}{6} \frac{(80 \times 10^3)^2 (1.6)^3}{(200 \times 10^9)(165 \times 10^{-6})} = 662 \text{ N}\cdot\text{m}$$

$$U = 662 \text{ J}$$

Problem 11.32

11.31 and 11.32 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



Over A to B: $M = -Px$

$$U_{AB} = \int_0^a \frac{M^2 dx}{2EI} = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over B to C: $-M = Pa = \text{constant}$

$$U_{BC} = \frac{M^2 b}{2EI} = \frac{P^2 a^2 b}{2EI}$$

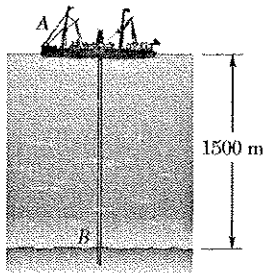
By symmetry, $U_{CD} = U_{AB} = \frac{P^2 a^3}{6EI}$

$$\text{Total } U = U_{AB} + U_{BC} + U_{CD} = \frac{P^2 a^2 (2a + 3b)}{6EI}$$

Data: $P = 8 \text{ kN}$ $a = 0.375 \text{ m}$ $b = 1.5 \text{ m}$ $I = \frac{1}{12} (76)^3 (38) = 1.39 \times 10^6 \text{ mm}^4$

$$U = \frac{(8 \times 10^3)^2 (0.375)^2 [(2)(0.375) + (3)(1.5)]}{(6)(200 \times 10^9)(1.39 \times 10^{-6})} = 28.3 \text{ J}$$

Problem 11.33



11.33 The ship at A has just started to drill for oil on the ocean floor at a depth of 1500 m. The steel drill pipe has an outer diameter of 200 mm and a uniform wall thickness of 12 mm. Knowing that the top of the drill pipe rotates through two complete revolutions before the drill bit at B starts to operate and using $G = 77 \text{ GPa}$, determine the maximum strain energy acquired by the drill pipe.

$$\phi = 2 \text{ rev} = (2)(2\pi) = 4\pi \text{ radians}$$

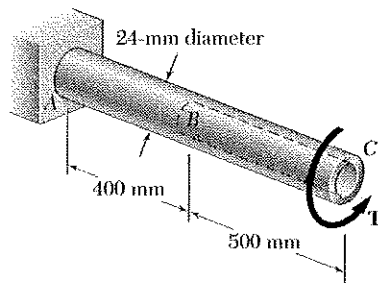
$$L = 1500 \text{ m}, \quad c_o = \frac{1}{2} d_o = 100 \text{ mm}, \quad c_i = c_o - t = 88 \text{ mm}$$

$$J = \frac{\pi}{2} (c_o^4 - c_i^4) = \frac{\pi}{2} (100^4 - 88^4) = 62.88 \times 10^6 \text{ mm}^4 = 62.88 \times 10^{-6} \text{ m}^4$$

$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ\phi}{L}$$

$$U = \frac{T^2 L}{2GJ} = \frac{\left(\frac{GJ\phi}{L}\right)^2 L}{2GJ} = \frac{GJ\phi^2}{2L} = \frac{(77 \times 10^9)(62.88 \times 10^{-6})(4\pi)^2}{(2)(1500)} = 254.86 \times 10^3 \text{ J} = 254.9 \text{ kJ}$$

Problem 11.34



11.34 Rod AC is made of aluminum and is subjected to a torque T applied at C. Knowing that $G = 73 \text{ GPa}$ and that portion BC of the rod is hollow and has an inner diameter of 16 mm, determine the strain energy of the rod for a maximum shearing stress of 120 MPa.

$$C_o = \frac{d_o}{2} = 12 \text{ mm}, \quad C_i = \frac{d_i}{2} = 8 \text{ mm}$$

$$J_{AB} = \frac{\pi}{2} C_o^4 = \frac{\pi}{2} (12)^4 = 32.572 \times 10^3 \text{ mm}^4 = 32.572 \times 10^{-9} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2} (C_o^4 - C_i^4) = \frac{\pi}{2} (12^4 - 8^4) = 26.138 \times 10^3 \text{ mm}^4 = 26.138 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{T C}{J_{\min}} \quad T = \frac{J_{\min} \tau_{\max}}{C} = \frac{(26.138 \times 10^{-9})(120 \times 10^6)}{12 \times 10^{-3}} = 261.38 \text{ N}\cdot\text{m}$$

$$U_{AB} = \frac{T^2 L_{AB}}{2 G J_{AB}} = \frac{(261.38)^2 (400 \times 10^{-3})}{(2)(73 \times 10^9)(32.572 \times 10^{-9})} = 5.747 \text{ J}$$

$$U_{BC} = \frac{T^2 L_{BC}}{2 G J_{BC}} = \frac{(261.38)^2 (500 \times 10^{-3})}{(2)(73 \times 10^9)(26.138 \times 10^{-9})} = 8.951 \text{ J}$$

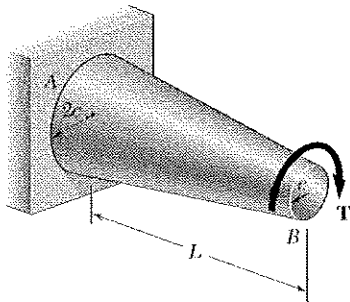
$$\text{Total: } U = U_{AB} + U_{BC} \quad U = 14.70 \text{ J} \quad \blacktriangleleft$$

Problem 11.35

11.35 Show by integration that the strain energy in the tapered rod AB is

$$U = \frac{7}{48} \frac{T^2 L}{G J_{\min}}$$

where $J_{\min}$ is the polar moment of inertia of the rod at end B.



$$r = \frac{C x}{L}$$

$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2} \frac{C^4}{L^4} x^4, \quad J_{\min} = \frac{\pi}{2} C^4$$

$$U = \int_L^{2L} \frac{T^2 dx}{2 G J} = \int_L^{2L} \frac{T^2}{2 G \left(\frac{\pi}{2} \frac{C^4}{L^4} x^4 \right)} dx$$

$$= \frac{T^2 L^4}{2 G J_{\min}} \int_L^{2L} \frac{dx}{x^4}$$

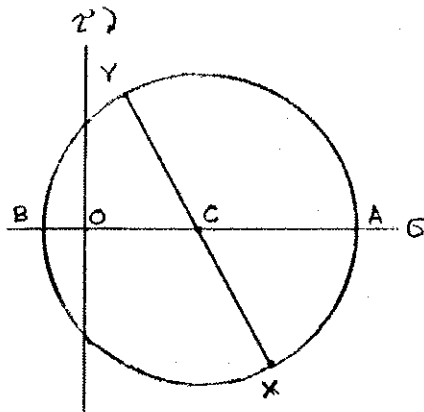
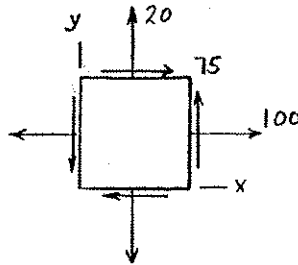
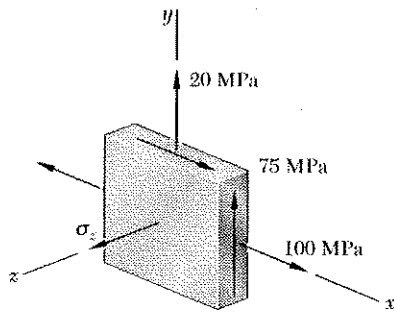
$$= \frac{T^2 L^4}{2 G J_{\min}} \left(-\frac{1}{3x^3} \right) \Big|_L^{2L}$$

$$U = \frac{T^2 L^4}{2 G J_{\min}} \left(-\frac{1}{3(2L)^3} + \frac{1}{3L^3} \right)$$

$$U = \frac{7}{48} \frac{T^2 L}{G J_{\min}} \quad \blacktriangleleft$$

Problem 11.36

11.36 The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a) $\sigma_z = +45$ MPa, (b) $\sigma_z = -45$ MPa.



$$\sigma_{ave} = \frac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\tau_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 \stackrel{?}{<} 2\sigma_Y^2$$

(a) $\sigma_c = \sigma_z = +45 \text{ MPa}$

$$(145 + 25)^2 + (-25 - 45)^2 + (45 - 145)^2 \stackrel{?}{<} 2(160)^2 = 51200$$

$$28900 + 4900 + 10000 = 43800 < 51200 \quad (\text{No yield})$$

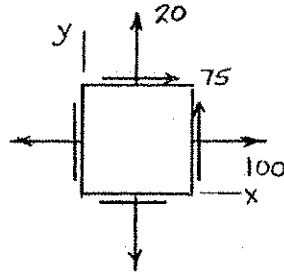
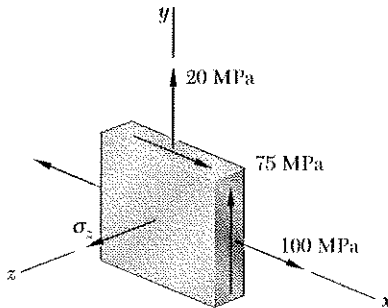
(b) $\sigma_c = \sigma_z = -45 \text{ MPa}$

$$(145 + 25)^2 + (-25 + 45)^2 + (-45 - 145)^2 \stackrel{?}{<} 51200$$

$$28900 + 400 + 36100 = 65400 > 51200 \quad (\text{Yield occurs})$$

Problem 11.37

11.37 The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine the range of values of σ_z for which yield does not occur.



$$\sigma_{ave} = \frac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\tau_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\sigma_Y^2$$

$$(145 + 25)^2 + (-25 - \sigma_z)^2 + (\sigma_z - 145)^2 = (2)(160)^2$$

$$28900 + (625 + 50\sigma_z + \sigma_z^2) + (\sigma_z^2 - 290\sigma_z + 21025) = 51200$$

$$2\sigma_z^2 - 240\sigma_z - 650 = 0$$

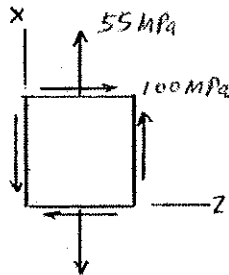
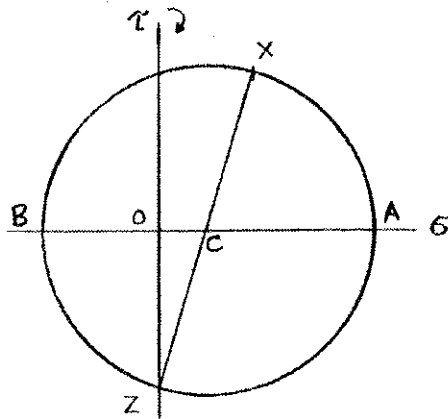
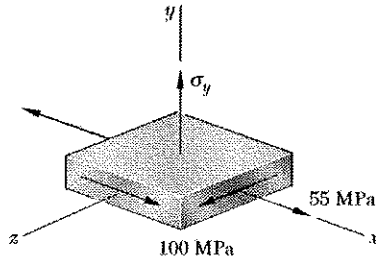
$$\sigma_z = \frac{240 \pm \sqrt{240^2 + (4)(2)(650)}}{(2)(2)} = 60 \pm 62.65$$

$$\sigma_z = 122.65 \text{ MPa}, -2.65 \text{ MPa}$$

$$-2.65 \text{ MPa} < \sigma_z < 122.65 \text{ MPa}$$

Problem 11.38

11.38 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_Y = 450$ MPa. Using the maximum-distortion-energy criterion, determine the factor of safety associated with the yield strength when (a) $\sigma_Y = +110$ MPa, (b) $\sigma_Y = -110$ MPa.



$$\sigma_{ave} = \frac{1}{2}(0 + 55) = 27.5 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{55 - 0}{2} = 27.5 \text{ MPa}$$

$$\tau_{xz} = 100 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{27.5^2 + 100^2} = 103.7 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 131.2 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -76.2 \text{ MPa}$$

$$\sigma_c = \sigma_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

(a) $\sigma_c = \sigma_y = 110 \text{ MPa}$

$$(131.2 + 76.2)^2 + (-76.2 - 110)^2 + (110 - 131.2)^2 = 2\left(\frac{450}{F.S.}\right)^2$$

$$43014.8 + 34670.4 + 449.4 = \frac{405000}{(F.S.)^2} \quad F.S. = 2.28$$

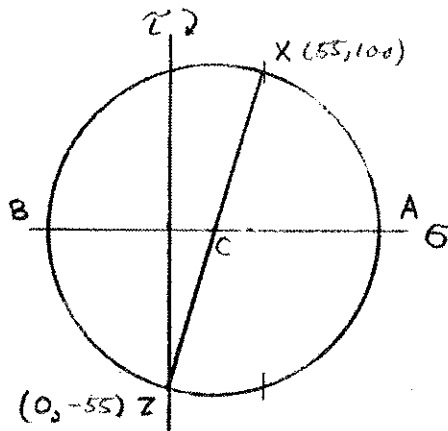
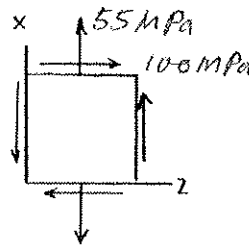
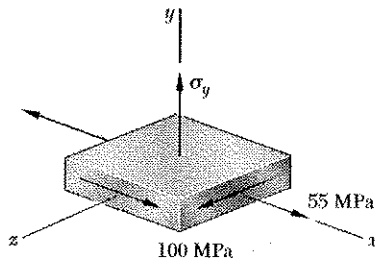
(b) $\sigma_c = \sigma_y = -110 \text{ MPa}$

$$(131.2 + 76.2)^2 + (-76.2 + 110)^2 + (-110 - 131.2)^2 = 2\left(\frac{450}{F.S.}\right)^2$$

$$43014.8 + 1142.4 + 58177.4 = \frac{405000}{(F.S.)^2} \quad F.S. = 1.99$$

Problem 11.39

11.39 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_y = 450$ MPa. Using the maximum-distortion-energy criterion, determine the range of values of σ_x for which the factor of safety associated with the yield strength is equal to or larger than 2.2.



$$\sigma_{ave} = \frac{1}{2} (0 + 100) = 50 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{55 - 0}{2} = 27.5 \text{ MPa}$$

$$\tau_{xz} = 100 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \sqrt{27.5^2 + 100^2} = 103.7 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 153.7 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -53.7 \text{ MPa}$$

$$\sigma_c = \sigma_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_y}{F.S.}\right)^2$$

$$(153.7 + 53.7)^2 + (-53.7 - \sigma_y)^2 + (\sigma_y - 153.7)^2 = 2\left(\frac{450}{2.2}\right)^2$$

$$43014.8 + (5806.4 + 152.4 \sigma_y + \sigma_y^2) + (\sigma_y^2 - 262.4 \sigma_y + 17213.4) = 83677.7$$

$$2\sigma_y^2 - 110\sigma_y - 17643 = 0$$

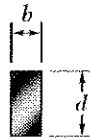
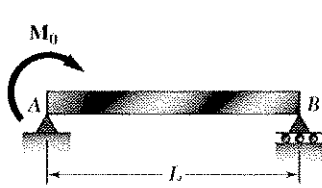
$$\sigma_y = \frac{110 \pm \sqrt{110^2 + (4)(2)(17643)}}{(2)(2)} = 27.5 \pm 97.9$$

$$\sigma_y = 125.4 \text{ MPa}, -70.4 \text{ MPa}$$

$$-70.4 \leq \sigma_y \leq 125.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 11.40

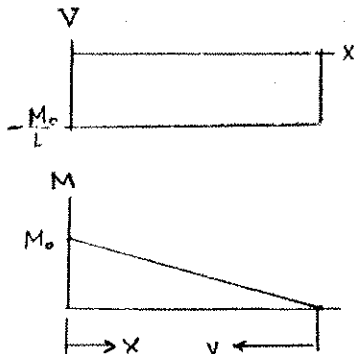
11.40 Determine the strain energy of the prismatic beam AB , taking into account the effect of both normal and shearing stresses.



Reactions $R_A = \frac{M_0}{L} \downarrow$, $R_B = \frac{M_0}{L} \uparrow$

Shear: $V = -\frac{M_0}{L}$

Bending moment: $M = \frac{M_0}{L} x$



For bending

$$U_1 = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^L x^2 dx = \frac{M_0^2 L^3}{6EI L^2} = \frac{M_0^2 L}{6EI}$$

For shear

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} \left(1 - \frac{y^2}{c^2}\right) \quad c = \frac{1}{2}d$$

$$u = \frac{\tau_{xy}^2}{2G} = \frac{9V^2}{8GA^2} \left(1 - \frac{y^2}{c^2}\right)^2 = \frac{9M_0^2}{8G(bd)^2 L^2} \left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right)$$

$$\begin{aligned} U_2 &= \int u dV = \int_0^L \int_{-c}^c u b dy dx = \frac{9M_0^2 b}{8G b^2 d^2 L^2} \int_0^L \int_{-c}^c \left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right) dy dx \\ &= \frac{9M_0^2}{8G b d^2 L^2} \int_0^L \left(y - \frac{2}{3} \frac{y^3}{c^2} + \frac{1}{5} \frac{y^5}{c^4}\right) \Big|_{-c}^c dx = \frac{9M_0^2}{8G b d^2 L^2} \int_0^L \left(2c - \frac{4}{3}c + \frac{2}{5}c\right) dx \\ &= \frac{9M_0^2}{8G b d^2 L^2} \left(\frac{16}{15}c\right) L = \frac{6}{5} \frac{M_0^2 c}{G b d^2 L} = \frac{3}{5} \frac{M_0^2}{G b d L} \end{aligned}$$

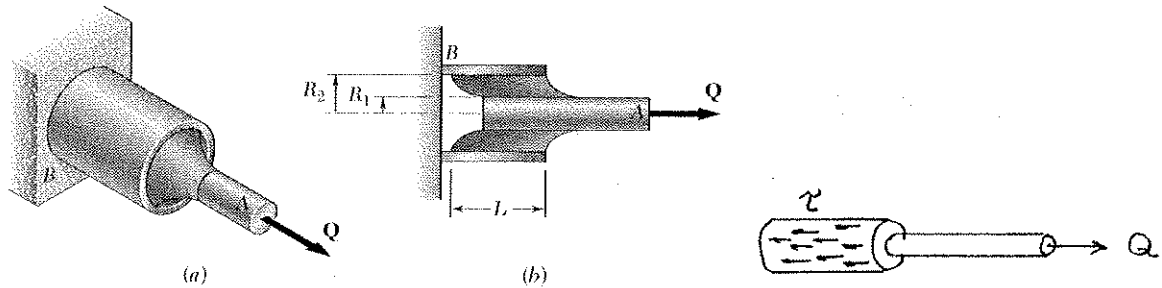
Total: $U = U_1 + U_2 = \frac{M_0^2 L}{6EI} + \frac{3}{5} \frac{M_0^2}{G b d L}$

With $I = \frac{1}{12} b d^3$,

$$U = \frac{2M_0^2 L}{E b d^3} + \frac{3}{5} \frac{M_0^2}{G b d L} = \frac{2M_0^2 L}{E b d^3} \left\{ 1 + \frac{3}{10} \cdot \frac{E}{G} \cdot \frac{d^2}{L^2} \right\}$$

Problem 11.41

*11.41 A vibration isolation support is made by bonding a rod A , of radius R_1 , and a tube B , of inner radius R_2 to a hollow rubber cylinder. Denoting by G the modulus of rigidity of the rubber, determine the strain energy of the hollow rubber cylinder for the loading shown.



$$+\rightarrow \sum F_x = 0: -\tau(2\pi r L) + Q = 0$$

$$\tau = \frac{Q}{2\pi r L}$$

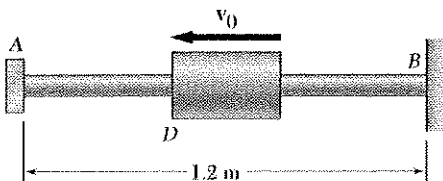
$$u = \frac{\tau^2}{2G} = \frac{Q^2}{8\pi^2 r^2 L^2 G}$$

$$U = \int u dV = \frac{Q^2}{8\pi^2 G L^2} \int \frac{dV}{r^2} = \frac{Q^2}{8\pi^2 G L^2} \int_0^L \int_{R_1}^{R_2} \frac{2\pi r dr}{r^2} dx$$

$$= \frac{Q^2}{4\pi G L^2} \int_0^L \int_{R_1}^{R_2} \frac{dr}{r} dx = \frac{Q^2}{4\pi G L^2} \int_0^L (\ln r \Big|_{R_1}^{R_2}) dx = \frac{Q^2}{4\pi G L} \ln \frac{R_2}{R_1}$$

Problem 11.42

11.42 A 6-kg collar has a speed $v_0 = 4.5$ m/s when it strikes a small plate attached to end A of the 20-mm-diameter rod AB . Using $E = 200$ GPa, determine the equivalent static load, (b) the maximum stress in the rod, (c) the maximum deflection of end A .



$$U_m = \frac{1}{2} m v_0^2 = \frac{1}{2} (6) (4.5)^2 = 60.75 \text{ J}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (20 \times 10^{-3})^2 = 314.159 \times 10^{-6} \text{ m}^2$$

$$U_m = \frac{P^2 L}{2EA}$$

$$(a) \quad P = \sqrt{\frac{2EAU_m}{L}} = \sqrt{\frac{(2)(200 \times 10^9)(314.159 \times 10^{-6})(60.75)}{1.2}}$$

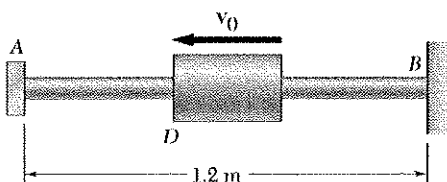
$$= 79.76 \times 10^3 \text{ N} \quad P = 79.8 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad \sigma_{max} = \frac{P}{A} = \frac{79.76 \times 10^3}{314.159 \times 10^{-6}} = 254 \times 10^6 \text{ Pa} \quad \sigma_{max} = 254 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \delta = \frac{PL}{EA} = \frac{(79.76 \times 10^3)(1.2)}{(200 \times 10^9)(314.159 \times 10^{-6})} = 1.523 \times 10^{-3} \text{ m} \quad \delta = 1.523 \text{ mm} \quad \blacktriangleleft$$

Problem 11.43

11.43 A 5-kg collar D moves along the uniform rod AB and has a speed $v_0 = 6$ m/s when it strikes a small plate attached to end A of the rod. Using $E = 200$ GPa and knowing that the allowable stress in the rod is 250 MPa, determine the smallest diameter that can be used for the rod.



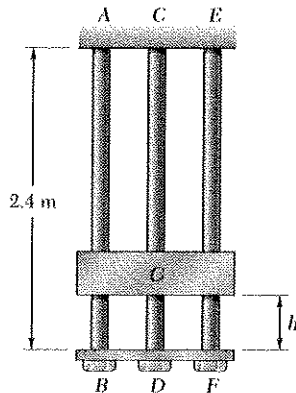
$$U_m = \frac{1}{2} m v_0^2 = \frac{1}{2} (5) (6)^2 = 90 \text{ J}$$

$$U_m = \frac{P_m^2 L}{2EA} = \frac{(A \sigma_{max})^2 L}{2EA}$$

$$A = \frac{2EU_m}{\sigma_{max}^2 L} = \frac{(2)(200 \times 10^9)(90)}{(250 \times 10^6)^2 (1.2)} = 480 \times 10^{-6} \text{ m}^2$$

$$\frac{\pi}{4} d^2 = A \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(480 \times 10^{-6})}{\pi}} = 24.7 \times 10^{-3} \text{ m} \quad d = 24.7 \text{ mm} \quad \blacktriangleleft$$

Problem 11.44



11.44 The 45-kg collar C is released from rest in the position shown and is stopped by Plate BDF that is attached the 22-mm-diameter steel rod CD and to the 16-mm-diameter steel rods AB and EF . Knowing that for the grade of steel used $\sigma_{all} = 165 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the largest allowable distance h .

Let Δ_m be the elongation.

$$\Delta_m = \frac{\sigma_{AB} L}{E} = \frac{\sigma_{CD} L}{E} = \frac{\sigma_{EF} L}{E}$$

$$\sigma_{AB} = \sigma_{CD} = \sigma_{EF} = 165 \text{ MPa}$$

$$L = 2.4 \text{ m}$$

$$\Delta_m = \frac{(165 \times 10^6)(2.4)}{200 \times 10^9} = 1.98 \times 10^{-3} \text{ m}$$

$$\text{For each rod } U = \frac{F_m^2 L}{2EA} = \frac{(EA \Delta_m / L)^2 L}{2EA} = \frac{EA \Delta_m^2}{2L}$$

$$\text{Rod } CD: A_{CD} = \frac{\pi}{4} (22)^2 = 380 \text{ mm}^2$$

$$U_{CD} = \frac{(200 \times 10^9)(380 \times 10^{-6})(1.98 \times 10^{-3})^2}{(2)(2.4)} = 62.07 \text{ J}$$

$$\text{Rods } AB \text{ and } EF: A_{AB} = A_{EF} = \frac{\pi}{4} (16)^2 = 201 \text{ mm}^2$$

$$U_{AB} = U_{EF} = \frac{(200 \times 10^9)(201 \times 10^{-6})(1.98 \times 10^{-3})^2}{(2)(2.4)} = 32.83 \text{ J}$$

$$\text{Total } U_m = U_{AB} + U_{CD} + U_{EF} = 127.73 \text{ J}$$

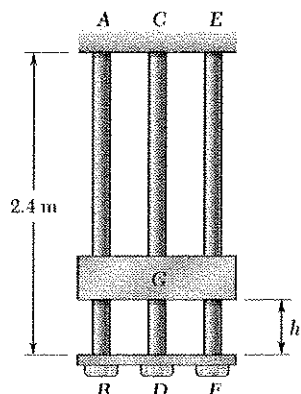
$$\text{Falling distance is } h + \Delta_m. \quad W = 45 \times 9.81 = 441.45 \text{ N}$$

$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{127.73}{441.45} = 0.28934 \text{ m}$$

$$h = 0.28934 - 0.00198 = 0.28736 \text{ m} = 287 \text{ mm} \quad \blacktriangleleft$$

Problem 11.45



11.45 Solve Prob. 11.44, assuming that the 22-mm-diameter steel rod CD is replaced by a 22-mm-diameter rod made of an aluminum alloy for which $\sigma_{all} = 140 \text{ MPa}$ and $E = 73 \text{ GPa}$.

11.44 The 45-kg collar C is released from rest in the position shown and is stopped by Plate BDF that is attached the 22-mm-diameter steel rod CD and to the 16-mm-diameter steel rods AB and EF. Knowing that for the grade of steel used $\sigma_{all} = 165 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the largest allowable distance h .

Let Δ_m be the elongation.

$$L = 2.4 \text{ m}$$

$$\Delta_m = \frac{\sigma_{AB} L}{E_{AB}} = \frac{\sigma_{CD} L}{E_{CD}} = \frac{\sigma_{EF} L}{E_{EF}}$$

$$\text{If } \sigma_{AB} = 165 \text{ MPa}, \quad \Delta_m = \frac{(165 \times 10^6)(2.4)}{200 \times 10^9} = 1.98 \times 10^{-3} \text{ m}$$

$$\text{If } \sigma_{CD} = 140 \text{ MPa}, \quad \Delta_m = \frac{(140 \times 10^6)(2.4)}{73 \times 10^9} = 4.603 \times 10^{-3} \text{ m}$$

$$\text{Smaller value governs} \quad \Delta_m = 1.98 \times 10^{-3} \text{ m}$$

$$\text{For each rod} \quad U = \frac{F^2 L}{2EA} = \frac{(EA \Delta_m / L)^2 L}{2EA} = \frac{EA \Delta_m^2}{2L}$$

$$\text{Rod CD:} \quad A_{CD} = \frac{\pi}{4} (22)^2 = 380 \text{ mm}^2, \quad E_{CD} = 73 \times 10^9 \text{ Pa}$$

$$U_{CD} = \frac{(73 \times 10^9)(380 \times 10^{-6})(1.98 \times 10^{-3})^2}{(2)(2.4)} = 22.66 \text{ J}$$

$$\text{Rods AB and EF:} \quad A_{AB} = A_{EF} = \frac{\pi}{4} (16)^2 = 201 \text{ mm}^2$$

$$U_{AB} = U_{EF} = \frac{(200 \times 10^9)(201 \times 10^{-6})(1.98 \times 10^{-3})^2}{(2)(2.4)} = 32.83 \text{ J}$$

$$\text{Total} \quad U_m = U_{AB} + U_{CD} + U_{EF} = 88.32 \text{ J}$$

$$\text{Falling distance is } h + \Delta_m \quad W = 45 \times 9.81 = 441.45 \text{ N}$$

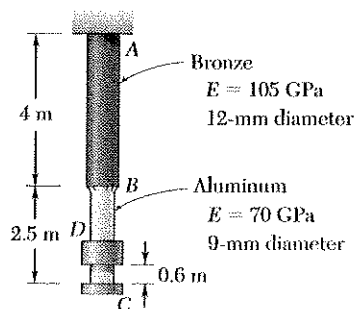
$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{88.32}{441.45} = 0.2 \text{ m}$$

$$h = 0.2 - 0.00198 = 0.198 \text{ m} = 198 \text{ mm}$$

Problem 11.46

11.46 Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC . Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.



Portion BC: $\sigma_m = 125 \times 10^6 \text{ Pa}$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952 \text{ N}$$

Corresponding strain energy.

$$U_{BC} = \frac{P_m^2 L_{BC}}{2 E_{BC} A_{BC}} = \frac{(7952)^2 (2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2 E_{AB} A_{AB}} = \frac{(7952)^2 (4)}{(2)(105 \times 10^9)(113.907 \times 10^{-6})} = 10.574 \text{ J}$$

$$U_m = U_{BC} + U_{AB} = 28.324 \text{ J}$$

Corresponding elongation Δ_m . $\frac{1}{2} P_m \Delta_m = U_m$

$$\Delta_m = \frac{2 U_m}{P_m} = \frac{(2)(28.324)}{7952} = 7.12 \times 10^{-3} \text{ m}$$

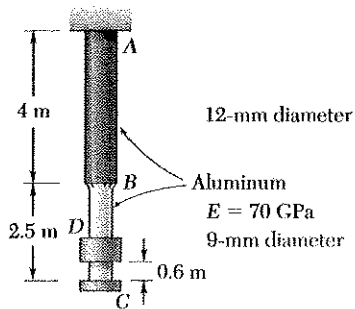
Falling distance: $h = 0.6 + 7.12 \times 10^{-3} = 0.60712 \text{ m}$

Work of weight = U_m $Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{28.324}{(9.81)(0.60712)}$$

$$m = 4.76 \text{ kg}$$

Problem 11.47



11.47 Solve Prob. 11.46, assuming that both portions of rod ABC are made of aluminum.

11.46 Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC. Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.

Portion BC: $\sigma_m = 125 \times 10^6 \text{ Pa}$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952 \text{ N}$$

Corresponding strain energy.

$$U_{BC} = \frac{P_m^2 L_{BC}}{2EA_{BC}} = \frac{(7952)^2 (2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2EA_{AB}} = \frac{(7952)^2 (4)}{(2)(70 \times 10^9)(113.907 \times 10^{-6})} = 15.861 \text{ J}$$

Total: $U_m = U_{BC} + U_{AB} = 33.611 \text{ J}$

Corresponding elongation Δ_m . $\frac{1}{2} P_m \Delta_m = U_m$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(33.611)}{7952} = 8.45 \times 10^{-3} \text{ m}$$

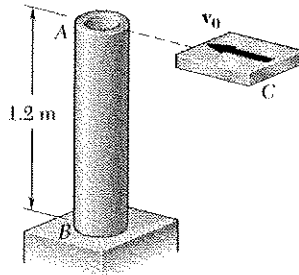
Falling distance. $h = 0.6 + \Delta_m = 0.60845 \text{ m}$

Work of weight = U_m . $Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{33.611}{(9.81)(0.60845)} =$$

$$m = 5.63 \text{ kg} \rightarrow$$

Problem 11.48



11.48 The post AB consists of a steel pipe of 80-mm-diameter and 6-mm wall thickness. A 6-kg block C moving horizontally with at velocity v_0 hits the post squarely at A . Using $E = 200$ GPa, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 180 MPa.

$$C_o = \frac{1}{2}d_o = \frac{1}{2}(80) = 40 \text{ mm} \quad C_i = C_o - t = 20 - 6 = 34 \text{ mm}$$

$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = \frac{\pi}{4}(40^4 - 34^4) = 961.06 \times 10^3 \text{ mm}^4 \\ = 961.06 \times 10^{-9} \text{ m}^4$$

$$\sigma_m = 180 \times 10^6 \text{ Pa} = \frac{M_m C}{I}$$

$$M_m = \frac{I \sigma_m}{C} = \frac{(961.06 \times 10^{-9})(180 \times 10^6)}{40 \times 10^{-3}} = 4.325 \times 10^3 \text{ N}\cdot\text{m}$$

$$P_m = \frac{M_m}{L} = \frac{4.325 \times 10^3}{1.2} = 3.604 \times 10^3 \text{ N}$$

By Appendix D, Case 1

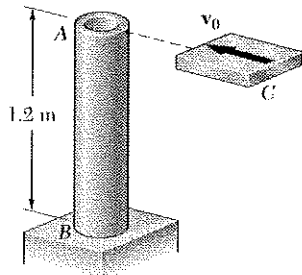
$$y_m = \frac{P_m L^3}{3EI} = \frac{(3.604 \times 10^3)(1.2)^3}{(3)(200 \times 10^9)(961.06 \times 10^{-9})} = 10.8 \times 10^{-3} \text{ m}$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2} (3.604 \times 10^3)(10.8 \times 10^{-3}) = 19.4616 \text{ J}$$

$$\frac{1}{2} m v_0^2 = U_m$$

$$v_0 = \sqrt{\frac{2U_m}{m}} = \sqrt{\frac{(2)(19.4616)}{6}} = 2.55 \text{ m/s}$$

Problem 11.49



11.49 Solve Prob. 11.48, assuming that the post AB consists of a solid steel rod of 80-mm diameter.

11.48 The post AB consists of a steel pipe of 80-mm-diameter and 6-mm wall thickness. A 6-kg block C moving horizontally with at velocity v_0 hits the post squarely at A . Using $E = 200$ GPa, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 180 MPa.

$$C = \frac{1}{2}d = \frac{1}{2}(80) = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$I = \frac{\pi}{4} C^4 = \frac{\pi}{4} (40 \times 10^{-3})^4 = 2.01062 \times 10^{-6} \text{ m}^4$$

$$\sigma_m = 180 \times 10^6 \text{ Pa} = \frac{M_m C}{I}$$

$$M_m = \frac{I \sigma_m}{C} = \frac{(2.01062 \times 10^{-6})(180 \times 10^6)}{40 \times 10^{-3}} = 9.048 \times 10^3 \text{ N}\cdot\text{m}$$

$$P_m = \frac{M_m}{L} = \frac{9.048 \times 10^3}{1.2} = 7.5398 \times 10^3 \text{ N}$$

By Appendix D, Case 1 $y_m = \frac{P_m L^3}{3EI}$

$$y_m = \frac{(7.5398 \times 10^3)(1.2)^3}{(3)(200 \times 10^9)(2.01062 \times 10^{-6})} = 10.8 \times 10^{-3} \text{ m}$$

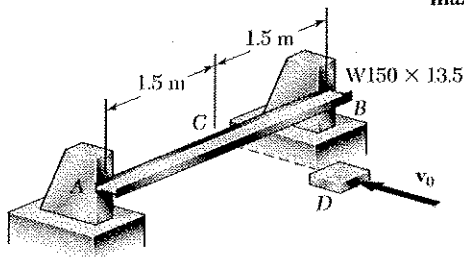
$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2} (7.5398 \times 10^3)(10.8 \times 10^{-3}) = 40.715 \text{ J}$$

$$\frac{1}{2} m v_0^2 = U_m$$

$$v_0 = \sqrt{\frac{2U_m}{m}} = \sqrt{\frac{(2)(40.715)}{6}} = 3.68 \text{ m/s}$$

Problem 11.50

11.50 The steel beam AB is struck squarely at its midpoint C by a 45-kg block moving horizontally with a speed $v_0 = 2$ m/s. Using $E = 200$ GPa, determine the equivalent static load, (b) the maximum normal stress in the beam, (c) the maximum deflection of the midpoint C of the beam.



From Appendix C, for $W150 \times 13.5$

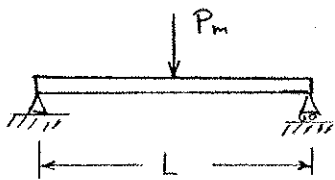
$$I_x = 6.87 \times 10^6 \text{ mm}^4 = 6.87 \times 10^{-6} \text{ m}^4$$

$$S_x = 91.6 \times 10^3 \text{ mm}^3 = 91.6 \times 10^{-6} \text{ m}^3$$

$$\text{Kinetic energy: } T = \frac{1}{2} m v_0^2 = \frac{1}{2} (45) (2)^2 = 90 \text{ J}$$

From Appendix D, Case 4

$$|y_m| = \frac{P L^3}{48 E I}, \quad M_{\max} = \frac{P L}{4}$$



$$U = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96 E I} = T$$

$$(a) \quad P_m = \sqrt{\frac{96 E I T}{L^3}} = \sqrt{\frac{(96)(200 \times 10^9)(6.87 \times 10^{-6})(90)}{(3.0)^3}} = 20.968 \times 10^3 \text{ N} = 21.0 \text{ kN}$$

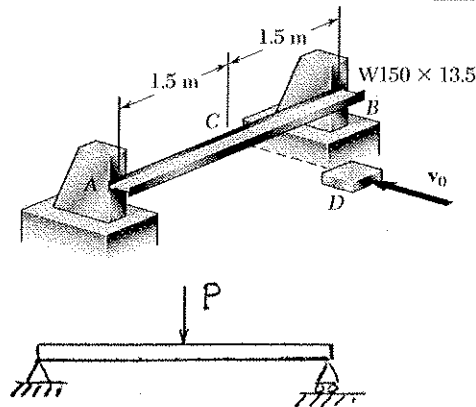
$$(b) \quad \sigma_m = \frac{M_{\max}}{S} = \frac{P_m L}{4 S} = \frac{(20.968 \times 10^3)(3.0)}{(4)(91.6 \times 10^{-6})} = 171.7 \times 10^6 \text{ Pa} = 171.7 \text{ MPa}$$

$$(c) \quad |y_m| = \frac{2U}{P_m} = \frac{(2)(90)}{20.968 \times 10^3} = 8.58 \times 10^{-3} \text{ m} = 8.58 \text{ mm}$$

Problem 11.51

11.51 Solve Prob. 11.50, assuming that the W150 × 13.5 rolled-steel beam is rotated by 90° about its longitudinal axis so that its web is vertical.

11.50 The steel beam AB is struck squarely at its midpoint C by a 45-kg block moving horizontally with a speed $v_0 = 2$ m/s. Using $E = 200$ GPa, determine the equivalent static load, (b) the maximum normal stress in the beam, (c) the maximum deflection of the midpoint C of the beam.



From Appendix C, for W 150 × 13.5

$$I_y = 0.918 \times 10^6 \text{ mm}^4 = 0.918 \times 10^{-6} \text{ m}^4$$

$$S_y = 18.4 \times 10^3 \text{ mm}^3 = 18.4 \times 10^{-6} \text{ m}^3$$

$$\text{Kinetic energy: } T = \frac{1}{2} m v_0^2 = \frac{1}{2} (45)(2)^2 = 90 \text{ J}$$

From Appendix D, Case 4

$$|y_m| = \frac{P L^3}{48 E I} \quad M_{\max} = \frac{P L}{4}$$

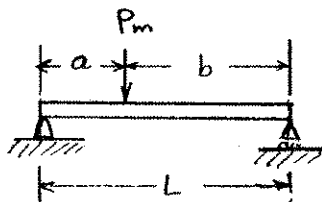
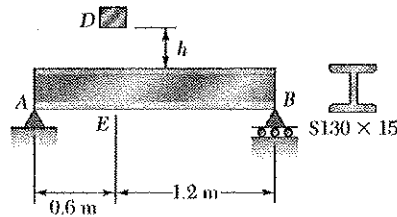
$$U = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96 E I} = T$$

$$(a) \quad P_m = \sqrt{\frac{96 E I T}{L^3}} = \sqrt{\frac{(96)(200 \times 10^9)(0.918 \times 10^{-6})(90)}{(3.0)^3}} = 7.665 \times 10^3 \text{ N} = 7.67 \text{ kN}$$

$$(b) \quad \sigma_m = \frac{M_{\max}}{S} = \frac{P_m L}{4 S} = \frac{(7.665 \times 10^3)(3.0)}{(4)(18.4 \times 10^{-6})} = 312 \times 10^6 \text{ Pa} = 312 \text{ MPa}$$

$$(c) \quad |y_m| = \frac{2U}{P_m} = \frac{(2)(90)}{7.665 \times 10^3} = 23.5 \times 10^{-3} \text{ m} = 23.5 \text{ mm}$$

Problem 11.52



11.52 The 20-kg block D is dropped from a height $h = 0.18$ m onto the steel beam AB . Knowing that $E = 200$ GPa, determine (a) the maximum deflection at point E , (b) the maximum normal stress in the beam.

$$I_x = 5107 \times 10^{-6} \text{ m}^4 \quad S_x = 79.8 \times 10^{-6} \text{ m}^3$$

$$a = 0.6 \text{ m} \quad b = 1.2 \text{ m}$$

$$L = 1.8 \text{ m} \quad h = 0.18 \text{ m}$$

$$R_A = \frac{P_m b}{L}, \quad R_B = \frac{P_m a}{L}, \quad M_m = M_E = \frac{P_m a b}{L}$$

From Appendix D Case 5

$$y_E = \frac{P_m a^2 b^2}{3 E I L} = \frac{P_m (0.6)^2 (1.2)^2}{(3)(200 \times 10^9)(5107 \times 10^{-6})(1.8)} = 94.67 \times 10^{-9} P_m$$

$$P_m = 10.563 \times 10^6 y_E$$

$$U_m = \frac{1}{2} P_m y_m = 5.2815 \times 10^6 y_E^2$$

Work of falling weight: $W(h + y_E) = (20 \times 9.81)(0.18 + y_E) = 35.3 + 196 y_E$

Equating work and energy, $35.3 + 196 y_E = 5.2815 \times 10^6 y_E^2$

$$y_E^2 - 37.1 \times 10^{-6} y_E - 6.68 \times 10^{-6} = 0$$

$$(a) \quad y_E = \frac{1}{2} \left[37.1 \times 10^{-6} + \sqrt{(37.1 \times 10^{-6})^2 - (4)(-6.68 \times 10^{-6})} \right] = 4.01 \times 10^{-3} \text{ m}$$

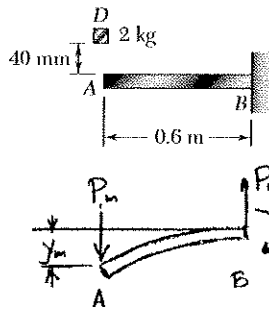
$$(b) \quad P_m = (10.563 \times 10^6)(0.00401) = 42357 \text{ N} \quad = 4 \text{ mm}$$

$$M_m = \frac{(42357)(0.6)(1.2)}{1.8} = 16942.8 \text{ Nm}$$

$$\sigma_m = \frac{M_m}{S_x} = \frac{16942.8}{79.8 \times 10^{-6}} = 212.3 \text{ MPa}$$

Problem 11.53

11.53 and 11.54 The 2-kg block *D* is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end *A*, (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



$$I = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} \left(\frac{16}{2} \right)^4 = 3.2170 \times 10^3 \text{ mm}^4 = 3.2170 \times 10^{-9} \text{ m}^4$$

$$c = \frac{d}{2} = 8 \text{ mm} = 8 \times 10^{-3} \text{ m} \quad L_{AB} = 0.6 \text{ m}$$

Appendix D, Case 1

$$y_m = \frac{P_m L_{AB}^3}{3EI} \quad M_m = P_m L_{AB}$$

$$P_m = \frac{3EI}{L_{AB}^3} y_m = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3} = 8.9361 \times 10^3 y_m$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2} (8.9361 \times 10^3) y_m^2 = 4.4681 \times 10^3 y_m^2$$

$$\begin{aligned} \text{Work of dropped weight: } mg(h + y_m) &= (2)(9.81)(0.040 + y_m) \\ &= 0.7848 + 19.62 y_m \end{aligned}$$

Equating work and energy,

$$0.7848 + 19.62 y_m = 4.4681 \times 10^3 y_m^2$$

$$y_m^2 - 4.3911 \times 10^{-3} y_m - 175.645 \times 10^{-6} = 0$$

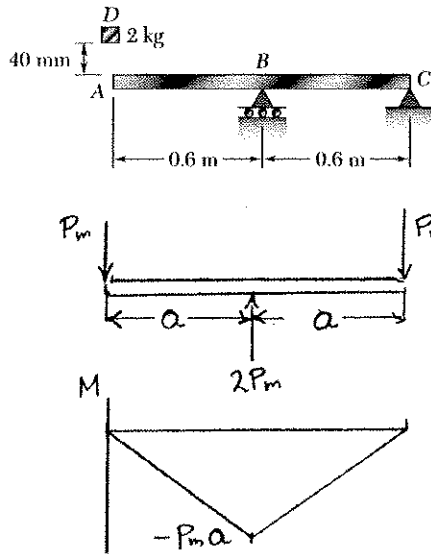
$$\begin{aligned} (a) \quad y_m &= \frac{1}{2} \left\{ 4.3911 \times 10^{-3} + \sqrt{(4.3911 \times 10^{-3})^2 + (4)(175.645 \times 10^{-6})} \right\} \\ &= 15.629 \times 10^{-3} \text{ m} \quad y_m = 15.63 \text{ mm} \quad \blacktriangleleft \end{aligned}$$

$$P_m = (8.9361 \times 10^3)(15.629 \times 10^{-3}) = 139.66 \text{ N}$$

$$(b) \quad M_m = -P_m L_{AB} = -(139.66)(0.6) \quad |M_m| = 83.8 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$\begin{aligned} (c) \quad \sigma_m &= \frac{|M_m|c}{I} = \frac{(83.8)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 208 \times 10^6 \text{ Pa} \\ \sigma_m &= 208 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$

Problem 11.54



11.53 and 11.54 The 2-kg block *D* is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end *A*, (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.

$$I = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} \left(\frac{16}{2} \right)^4 = 3.2170 \times 10^3 \text{ mm}^4 = 3.2170 \times 10^{-9} \text{ m}^4$$

$$c = \frac{d}{2} = 8 \text{ mm} = 8 \times 10^{-3} \text{ m} \quad a = 0.6 \text{ m}$$

Over AB: $M = -P_m x$ $M_m = -P_m a$

$$U_{AB} = \int_0^a \frac{P_m^2 x^2}{2EI} dx = \frac{P_m^2 a^3}{6EI}$$

$$= \frac{(0.6)^3}{(6)(200 \times 10^9)(3.2170 \times 10^{-9})} P_m^2$$

$$= 55.953 \times 10^{-6} P_m^2$$

By symmetry of bending moment diagram,

$$U_{BC} = U_{AB} = 55.953 \times 10^{-6} P_m^2$$

$$U_m = U_{AB} + U_{BC} = 111.906 \times 10^{-6} P_m^2$$

$$\frac{1}{2} P_m y_m = U_m = 111.906 \times 10^{-6} P_m^2 \quad P_m = 4.4681 \times 10^3 y_m$$

$$U_m = \frac{1}{2} P_m y_m = 2.2340 \times 10^3 y_m^2$$

Work of dropped weight: $mg(h + y_m) = (2)(9.81)(0.040 + y_m)$

$$= 0.7848 + 19.62 y_m$$

Equating work and energy,

$$0.7848 + 19.62 y_m = 2.2340 \times 10^3 y_m^2$$

$$y_m^2 - 8.7825 \times 10^{-3} y_m - 351.298 \times 10^{-6} = 0$$

$$(a) \quad y_m = \frac{1}{2} \left\{ 8.7825 \times 10^{-3} + \sqrt{(8.7825 \times 10^{-3})^2 + (4)(351.298 \times 10^{-6})} \right\}$$

$$= 23.636 \times 10^{-3} \text{ m} = 23.6 \text{ mm} \quad y_m = 23.6 \text{ mm} \leftarrow$$

$$P_m = (4.4681 \times 10^3)(23.636 \times 10^{-3}) = 105.61 \text{ N}$$

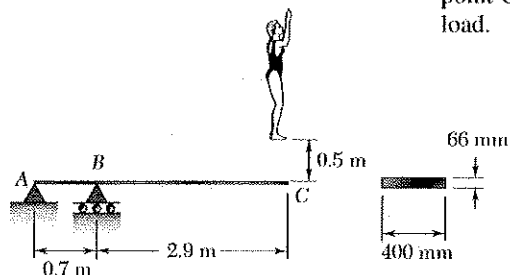
$$(b) \quad M_m = -(105.61)(0.6) = -64.4 \text{ N}\cdot\text{m} \quad |M_m| = 64.4 \text{ N}\cdot\text{m} \leftarrow$$

$$(c) \quad \sigma_m = \frac{|M_m|c}{I} = \frac{(64.4)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 157.6 \times 10^6 \text{ Pa}$$

$$\sigma_m = 157.6 \text{ MPa} \leftarrow$$

Problem 11.55

11.55 A 72-kg diver jumps from a height of 0.5 m onto end C of a diving board having the uniform cross section shown. Assuming that the diver's legs remain rigid and using $E = 12$ GPa, determine (a) the maximum deflection at point C, (b) the maximum normal stress in the board, (c) the equivalent static load.

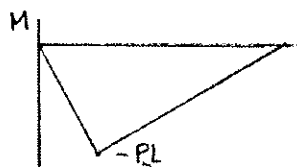
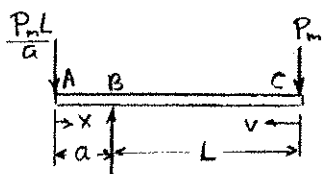


$$W = 72 \times 9.81 = 706.32 \text{ N}$$

$$I = \frac{1}{12} (0.4)(0.066)^3 = 9.583 \times 10^{-6} \text{ m}^4$$

$$L = 2.9 \text{ m}, \quad a = 0.7 \text{ m}$$

$$c = \frac{1}{2} (0.066) = 0.033 \text{ m}$$



Over portion AB: $M = -\frac{P_m L}{a} x$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \frac{P_m^2 L^2}{2EI a^2} \int_0^a x^2 dx = \frac{P_m^2 L^2 a}{6EI}$$

Over portion BC: $M = -P_m v$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dv = \frac{P_m^2}{2EI} \int_0^L v^2 dv = \frac{P_m^2 L^3}{6EI}$$

Total $U = U_{AB} + U_{BC} = \frac{P_m^2 L^2 (a+L)}{6EI}$

$$\frac{1}{2} P_m y_m = U_m \quad y_m = \frac{2U_m}{P_m} = \frac{P_m L^2 (a+L)}{3EI}$$

$$P_m = \frac{3EI}{L^2 (a+L)} y_m = \frac{(3)(12 \times 10^9)(9.583 \times 10^{-6})}{(2.9)^2 (2.9 + 0.7)} y_m = 11394.8 y_m$$

$$U_m = \frac{1}{2} P_m y_m = 5697.4 y_m^2$$

$$\text{Work of weight} = W(h + y_m) = (706.32)(0.5 + y_m) = 353.16 + 706.32 y_m$$

$$\text{Equating: } 353.16 + 706.32 y_m = 5697.4 y_m^2$$

$$y_m^2 - 0.124 y_m - 0.062 = 0$$

$$(a) \quad y_m = \frac{1}{2} \left\{ 0.124 + \sqrt{0.124^2 + (4)(0.062)} \right\} = y_m = 318.6 \text{ mm} \blacktriangleleft$$

$$(c) \quad P_m = (11394.8)(0.3186) = 3630 \text{ N} \quad P_m = 3.63 \text{ kN} \blacktriangleleft$$

$$M_m = -(3630)(2.9) = 10527 \text{ N m}$$

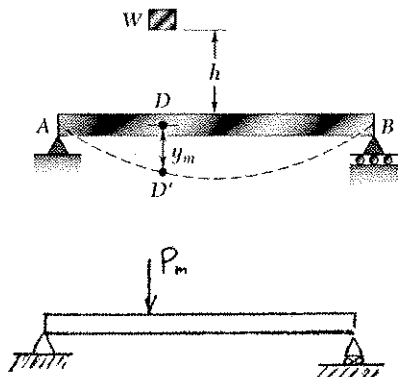
$$(b) \quad \sigma_m = \frac{M_m c}{I} = \frac{(10527)(0.033)}{9.583 \times 10^{-6}} = 36.25 \text{ MPa} \quad \sigma_m = 36.25 \text{ MPa} \blacktriangleleft$$

Problem 11.56

11.56 A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D . (a) Show that the maximum deflection y_m at point D can be expressed as

$$y_m = y_{st} \left(1 + \sqrt{1 + \frac{2h}{y_{st}}} \right)$$

where y_{st} represents the deflection at D caused by a static load W applied at that point and where the quantity in parenthesis is referred to as the *impact factor*. (b) Compute the impact factor for the beam and the impact of Prob. 11.53.



Work of falling weight: $Work = W(h + y_m)$

Strain energy: $U = \frac{1}{2} P y_m = \frac{1}{2} k y_m^2$

where k is the spring constant for a load applied at point D .

Equating work and energy,

$$W(h + y_m) = \frac{1}{2} k y_m^2$$

$$y_m^2 - \frac{2W}{k} y_m - \frac{2W}{k} h = 0$$

$$y_m^2 - 2y_{st} y_m - 2y_{st} h = 0 \quad \text{where } y_{st} = \frac{W}{k}$$

$$(a) \quad y_m = \frac{2y_{st} + \sqrt{4y_{st}^2 + 8y_{st}h}}{2} = y_{st} \left(1 + \sqrt{1 + \frac{2h}{y_{st}}} \right)$$

For Prob. 11.53

$$W = mg = (2)(9.81) = 19.62 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \frac{\pi}{4} \left(\frac{16}{2} \right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$$

$$L = 0.6 \text{ m}$$

$$h = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

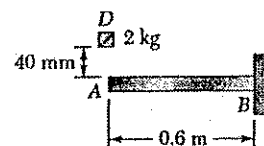
Using Appendix D Case 1

$$y_{st} = \frac{WL^3}{3EI}$$

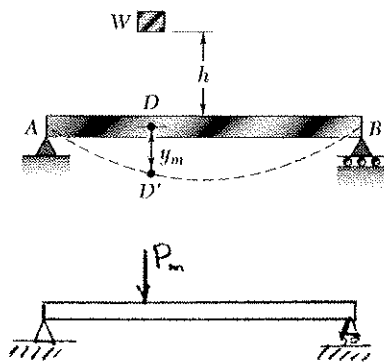
$$y_{st} = \frac{(19.62)(0.6)^3}{(3)(200 \times 10^9)(3.217 \times 10^{-9})} = 2.196 \times 10^{-3} \text{ m}$$

$$\frac{2h}{y_{st}} = \frac{(2)(40 \times 10^{-3})}{2.196 \times 10^{-3}} = 36.44$$

$$(b) \quad \text{impact factor} = 1 + \sqrt{1 + 36.44} = 7.12$$

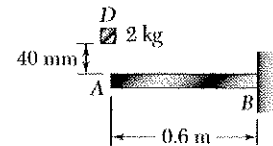


Problem 11.57



11.57 A block of weight W is dropped from a height h onto the horizontal beam AB and hits point D . (a) Denoting by y_m the exact value of the maximum deflection at D and by y'_m the value obtained by neglecting the effect of this deflection on the change in potential energy of the block, show that the absolute value of the relative error is $(y'_m - y_m)/y_m$, never exceeding $y'_m/2h$. (b) Check the result obtained in part a by solving part a of Prob. 11.53 without taking y_m into account when determining the change in potential energy of the load, and comparing the answer obtained in this way with the exact answer to that problem.

11.53 and 11.54 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end A , (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



$$U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2 \quad \text{where } k \text{ is the spring constant for a load at point } D.$$

$$\text{Work of falling weight:} \quad \begin{array}{l} \text{exact: } \text{Work} = W(h + y_m) \\ \text{approximate: } \text{Work} \approx Wh \end{array}$$

$$\text{Equating work and energy,} \quad \begin{array}{l} \frac{1}{2} k y_m^2 = W(h + y_m) \quad (1) \text{ exact} \\ \frac{1}{2} k y_m'^2 = Wh \quad (2) \text{ approximate} \end{array}$$

where y'_m is the approximate value for y_m .

$$\text{Subtracting,} \quad \frac{1}{2} k (y_m^2 - y_m'^2) = W y_m$$

$$y_m^2 - y_m'^2 = (y_m - y_m')(y_m + y_m') = \frac{2W}{k} y_m$$

$$\text{Relative error: } \frac{y_m - y_m'}{y_m} = \frac{2W}{k(y_m + y_m')}$$

$$\text{But, } \frac{2W}{k} = \frac{y_m'^2}{h} \quad \text{from equation (2).}$$

$$(a) \text{ Relative error} = \frac{y_m - y_m'}{y_m} = \frac{y_m'^2}{h(y_m + y_m')} < \frac{y_m'}{2h}$$

$$(b) \text{ From the solution to Prob. 11.53, } y_m = 15.63 \text{ mm}$$

$$\text{Approximate solution: } W = mg = (2)(9.81) = 19.62 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa} \quad I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$$

$$L = 0.6 \text{ m}, \quad h = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$k = \frac{3EI}{L^3} = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3} = 8.936 \times 10^3 \text{ N/m}$$

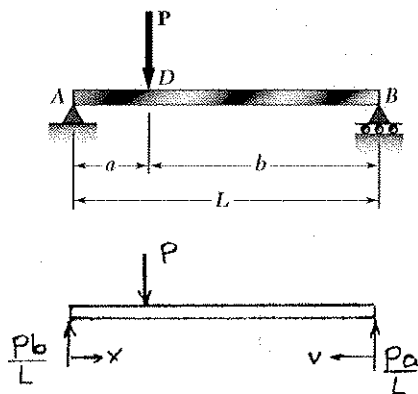
$$y_m'^2 = \frac{2Wh}{k} = \frac{(2)(19.62)(40 \times 10^{-3})}{8.936 \times 10^3} = 175.65 \times 10^{-6} \text{ m}^2$$

$$y_m' = 13.25 \times 10^{-3} \text{ m} = 13.25 \text{ mm}$$

$$\text{relative error} = \frac{15.63 - 13.25}{15.63} = 0.152 \quad \frac{y_m'}{2h} = 0.166$$

Problem 11.58

11.58 and 11.59 Using the method of work and energy, determine the deflection at point D caused by the load P.



Reactions: $R_A = \frac{Pb}{L}$, $R_B = \frac{Pa}{L}$

Over AD: $M = R_A x = \frac{Pbx}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2 b^2}{2EI L^2} \int_0^a x^2 dx = \frac{P^2 b^2 a^3}{6EI L^2}$$

Over DB: $M = R_B v = \frac{Pav}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^b v^2 dv = \frac{P^2 a^2 b^3}{6EI L^2}$$

Total: $U = U_{AD} + U_{DB} = \frac{P^2 a^2 b^2 (a+b)}{6EI L^2} = \frac{P^2 a^2 b^2}{6EI L}$

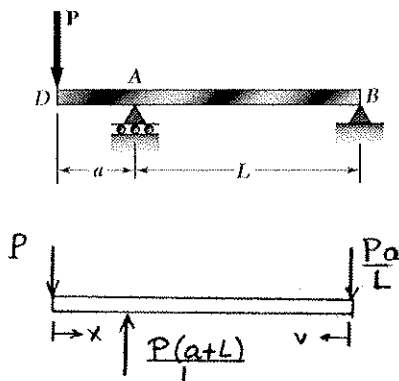
$\frac{1}{2} P S_D = U$

$S_D = \frac{2U}{P}$

$S_D = \frac{Pa^2 b^2}{3EI} \downarrow$

Problem 11.59

11.58 and 11.59 Using the method of work and energy, determine the deflection at point D caused by the load P.



$\sum M_A = 0: Pa + R_B L = 0 \quad R_B = -\frac{Pa}{L}$

Over portion DA: $M = -Px$

$$U_{DA} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over portion AB: $M = -\frac{Pav}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^L v^2 dv = \frac{Pa^2 L}{6EI}$$

Total: $U = U_{DA} + U_{AB} = \frac{P^2 a^2 (a+L)}{6EI}$

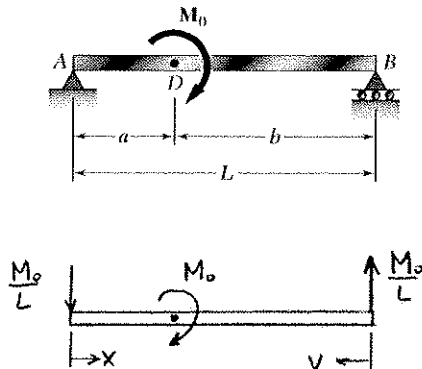
$\frac{1}{2} P S_D = U$

$S_D = \frac{2U}{P}$

$S_D = \frac{Pa^2(a+L)}{3EI} \downarrow$

Problem 11.60

11.60 and 11.61 Using the method of work and energy, determine the slope at point D caused by the couple M_0 .



Reactions: $R_A = \frac{M_0}{L} \downarrow$ $R_B = \frac{M_0}{L} \uparrow$

Over portion AD: $M = -\frac{M_0 x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^a x^2 dx$$

$$= \frac{M_0^2 a^3}{6EI L^2}$$

Over portion DB: $M = \frac{M_0 v}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{M_0^2}{2EI L^2} \int_0^b v^2 dv$$

$$= \frac{M_0^2 b^3}{6EI L^2}$$

Total: $U = U_{AD} + U_{DB} = \frac{M_0^2 (a^3 + b^3)}{6EI L^2}$

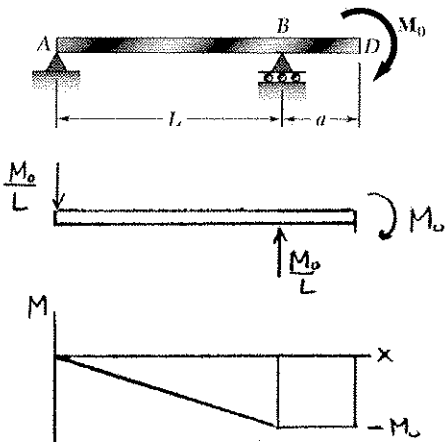
$$\frac{1}{2} M_0 \theta_D = U$$

$$\theta_D = \frac{2U}{M_0}$$

$$\theta_D = \frac{M_0 (a^3 + b^3)}{3EI L^2} \quad \swarrow$$

Problem 11.61

11.60 and 11.61 Using the method of work and energy, determine the slope at point D caused by the couple M_0 .



Reactions: $R_A = \frac{M_0}{L} \downarrow$ $R_B = \frac{M_0}{L} \uparrow$

Over portion AB: $M = -\frac{M_0 x}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^L x^2 dx$$

$$= \frac{M_0^2 L}{6EI}$$

Over portion BD: $M = -M_0$

$$U_{BD} = \frac{M_0^2 a}{2EI}$$

Total: $U = U_{AB} + U_{BD} = \frac{M_0^2 (L + 3a)}{6EI}$

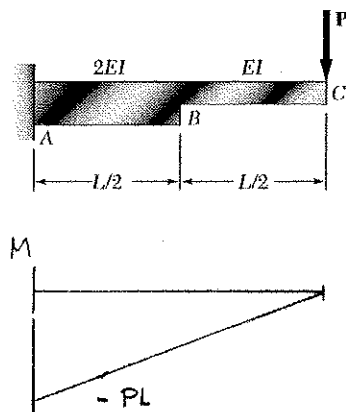
$$\frac{1}{2} M_0 \theta_D = U$$

$$\theta_D = \frac{2U}{M_0}$$

$$\theta_D = \frac{M_0 (L + 3a)}{3EI} \quad \swarrow$$

Problem 11.62

11.62 and 11.63 Using the method of work and energy, determine the deflection at point C caused by the load P.



Bending moment. $M = -Pv$

Over AB:

$$U_{AB} = \int_{\frac{L}{2}}^L \frac{M^2}{4EI} dv = \frac{P^2}{4EI} \int_{\frac{L}{2}}^L v^2 dv$$

$$= \frac{P^2}{12EI} \left[L^3 - \left(\frac{L}{2}\right)^3 \right] = \frac{7}{96} \frac{P^2 L^3}{EI}$$

Over BC: $U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_0^{\frac{L}{2}} v^2 dv$

$$= \frac{1}{48} \frac{P^2 L^3}{EI}$$

Total: $U = U_{AB} + U_{BC} = \frac{3}{32} \frac{P^2 L^3}{EI}$

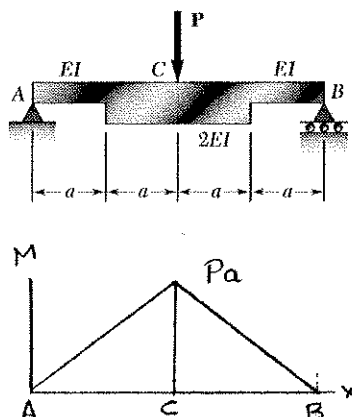
$$\frac{1}{2} P S_c = U$$

$$S_c = \frac{2U}{P}$$

$$S_c = \frac{3 PL^3}{16 EI} \downarrow \blacktriangleleft$$

Problem 11.63

11.62 and 11.63 Using the method of work and energy, determine the deflection at point C caused by the load P.



Symmetric beam and loading: $R_A = R_B = \frac{1}{2} P$

From A to C: $M = R_A x = \frac{1}{2} Px$

$$U_{AC} = \int_0^a \frac{M^2}{2EI} dx + \int_a^{2a} \frac{M^2}{4EI} dx$$

$$= \frac{P^2}{8EI} \int_0^a x^2 dx + \frac{P^2}{16EI} \int_a^{2a} x^2 dx$$

$$= \frac{P^2 a^3}{24EI} + \frac{P^2}{48EI} [(2a)^3 - a^3] = \frac{3}{16} \frac{P^2 a^3}{EI}$$

By symmetry, $U_{CB} = U_{AB} = \frac{3}{16} \frac{P^2 a^3}{EI}$

Total: $U = U_{AB} + U_{BC} = \frac{3}{8} \frac{P^2 a^3}{EI}$

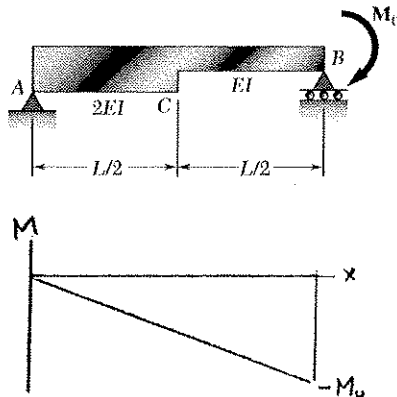
$$\frac{1}{2} P S_c = U$$

$$S_c = \frac{2U}{P}$$

$$S_c = \frac{3 Pa^3}{4 EI} \downarrow \blacktriangleleft$$

Problem 11.64

11.64 Using the method of work and energy, determine the slope at point B caused by the couple M_0 .



$$\sum M_A = 0: -R_B L - M_0 = 0 \quad R_B = -\frac{M_0}{L}$$

$$M = R_B x = -\frac{M_0}{L} x$$

Over portion AC: $U_{AC} = \int_0^{L/2} \frac{M^2}{2(EI)} dx$

$$U_{AC} = \frac{M_0^2}{4EI L^2} \int_0^{L/2} x^2 dx = \frac{1}{96} \frac{M_0^2 L}{EI}$$

Over portion CB: $U_{CB} = \int_{L/2}^L \frac{M^2}{2EI} dx$

$$U_{CB} = \frac{M_0^2}{2EI L^2} \int_{L/2}^L x^2 dx = \frac{M_0^2}{6EI L^2} \left[L^3 - \left(\frac{L}{2}\right)^3 \right]$$

$$= \frac{7}{48} \frac{M_0^2 L}{EI}$$

Total: $U = U_{AC} + U_{CB} = \frac{5}{32} \frac{M_0^2 L}{EI}$

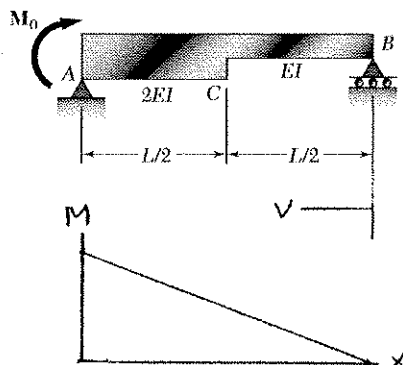
$$\frac{1}{2} M_0 \theta_B = U$$

$$\theta_B = \frac{2U}{M_0}$$

$$\theta_B = \frac{5M_0 L}{16EI} \quad \swarrow$$

Problem 11.65

11.65 Using the method of work and energy, determine the slope at point A caused by the couple M_0 .



$$R_B = \frac{M_0}{L}$$

$$M = R_B v = \frac{M_0}{L} v$$

Over AC: $U_{AC} = \int_{L/2}^L \frac{M^2}{2(2EI)} dv$

$$U_{AC} = \frac{M_0^2}{4EI L^2} \int_{L/2}^L v^2 dv = \frac{M_0^2}{12EI L^2} \left[L^3 - \left(\frac{L}{2}\right)^3 \right]$$

$$= \frac{7}{96} \frac{M_0^2 L}{EI}$$

Over CB: $U_{CB} = \int_0^{L/2} \frac{M^2}{2EI} dv$

$$U_{CB} = \frac{M_0^2}{2EI L^2} \int_0^{L/2} v^2 dv = \frac{1}{48} \frac{M_0^2 L}{EI}$$

Total: $U = U_{AC} + U_{CB} = \frac{3}{32} \frac{M_0^2 L}{EI}$

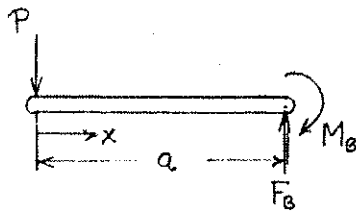
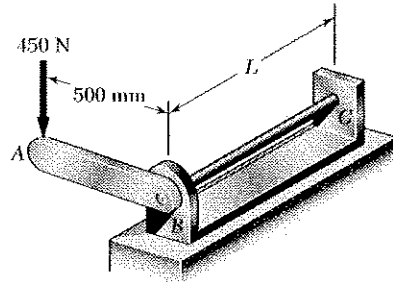
$$\frac{1}{2} M_0 \theta_A = U$$

$$\theta_A = \frac{2U}{M_0}$$

$$\theta_A = \frac{3M_0 L}{16EI} \quad \swarrow$$

Problem 11.66

11.66 The 20-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C . The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the deflection of point A when $L = 600$ mm. Use $E = 200$ GPa and $G = 77.2$ GPa.



Member AB. (Bending)

$$I = \frac{1}{12} (10)(30)^3 = 22.5 \times 10^3 \text{ mm}^4 = 22.5 \times 10^{-9} \text{ m}^4$$

$$a = 500 \text{ mm} = 0.500 \text{ m}$$

$$M_B = Pa = (450)(0.500) = 225 \text{ N}\cdot\text{m}$$

$$M = Px$$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \int_0^a \frac{P^2 x^2}{2EI} dx = \frac{P^2 a^3}{6EI}$$

$$= \frac{(450)^2 (0.500)^3}{(6)(200 \times 10^9)(22.5 \times 10^{-9})} = 0.9375 \text{ J}$$

Member BC. (Torsion)

$$T = M_B = 225 \text{ N}\cdot\text{m} \quad c = \frac{1}{2}d = 10 \text{ mm} \quad J = \frac{\pi}{2} c^4 = 15.708 \times 10^3 \text{ mm}^4 = 15.708 \times 10^{-9} \text{ m}^4$$

$$L = 600 \text{ mm} = 0.600 \text{ m}$$

$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(225)^2 (0.600)}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 12.5242 \text{ J}$$

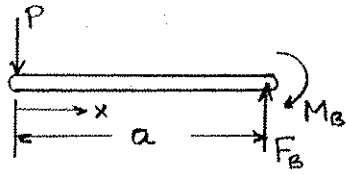
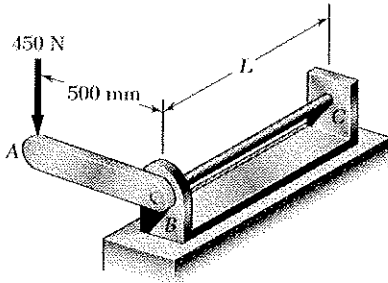
$$\text{Total: } U = U_{AB} + U_{BC} = 0.9375 + 12.5242 = 13.4617 \text{ J}$$

$$\frac{1}{2} P \delta_A = U \quad \delta_A = \frac{2U}{P} = \frac{(2)(13.4617)}{450} = 59.8 \times 10^{-3} \text{ m}$$

$$\delta_A = 59.8 \text{ mm} \quad \blacktriangleleft$$

Problem 11.67

11.67 The 20-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C . The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the length L of the rod BC for which the deflection at point A is 40 mm. Use $E = 200$ GPa and $G = 77.2$ GPa.



Member AB. (Bending)

$$I = \frac{1}{12}(10)(30)^3 = 22.5 \times 10^3 \text{ mm}^4 = 22.5 \times 10^{-9} \text{ m}^4$$

$$a = 500 \text{ mm} = 0.500 \text{ m}$$

$$M_B = Pa = (450)(0.500) = 225 \text{ N}\cdot\text{m}$$

$$M = Px$$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \int_0^a \frac{P^2 x^2}{2EI} dx = \frac{P^2 a^3}{6EI}$$

$$= \frac{(450)^2 (0.500)^3}{(6)(200 \times 10^9)(22.5 \times 10^{-9})} = 0.9375 \text{ J}$$

Member BC (Torsion)

$$T = M_B = 225 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 10 \text{ mm} \quad J = \frac{\pi}{2}c^4 = 15.708 \times 10^3 \text{ mm}^4 = 15.708 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(225)^2 L}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 20.874 L$$

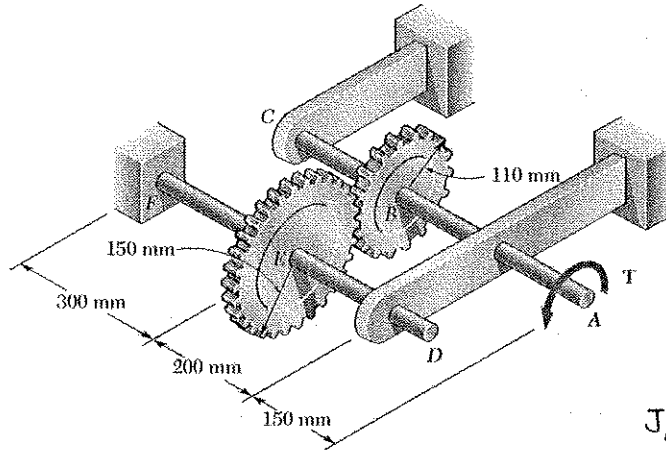
$$\frac{1}{2} P \delta_A = U_{AB} + U_{BC}$$

$$\frac{1}{2}(450)(40 \times 10^{-3}) = 0.9375 + 20.874 L$$

$$L = 0.386 \text{ m} = 386 \text{ mm}$$

Problem 11.68

11.68 Two steel shafts, each of 22-mm diameter, are connected by the gears shown. Knowing that $G = 77$ GPa and that shaft DF is fixed at F , determine the angle through which end A rotates when a $135\text{-N}\cdot\text{m}$ torque is applied at A . Ignore the strain energy due to the bending of the shafts.



Work-energy equation:

$$\frac{1}{2} T_A \phi_A = U$$

$$\phi_A = \frac{2U}{T_A}$$

Portion AB of shaft ABC:

$$T_{AB} = T_A = 135 \text{ N}\cdot\text{m}$$

$$L_{AB} = 0.2 + 0.15 = 0.35 \text{ m}$$

$$J_{AB} = \frac{\pi}{2} \left(\frac{d}{2} \right)^4 = \frac{\pi}{2} (11)^4 = 22998 \text{ mm}^4$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(135)^2 (0.35)}{(2)(77 \times 10^9)(22998 \times 10^{-12})} = 1.8 \text{ J}$$

Portion BC of shaft ABC: $U_{BC} = 0$

$$\text{Gear B: } F_{BE} = \frac{T_B}{r_B} = \frac{T_{AB}}{r_B} = \frac{135}{0.11} = 1227.3 \text{ N}$$

$$\text{Gear E: } T_E = r_E F_{BE} = (0.15)(1227.3) = 184 \text{ N}\cdot\text{m}$$

Portion DE of shaft DEF: $U_{DE} = 0$

Portion EF of shaft DEF: $T_{EF} = T_E = 184 \text{ N}\cdot\text{m}$

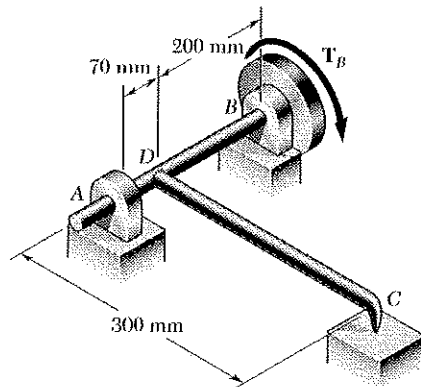
$$L_{EF} = 12 \text{ in. } J_{EF} = J_{AB} = 22998 \text{ mm}^4$$

$$U_{EF} = \frac{T_{EF}^2 L_{EF}}{2GJ_{EF}} = \frac{(184)^2 (0.3)}{(2)(77 \times 10^9)(22998 \times 10^{-12})} = 2.87 \text{ J}$$

$$\text{Total: } U = U_{AB} + U_{BC} + U_{DE} + U_{EF} = 4.67 \text{ J}$$

$$\phi_A = \frac{2U}{T_A} = \frac{(2)(4.67)}{135} = 69.19 \times 10^{-3} \text{ rad} = 3.96^\circ$$

Problem 11.69



11.69 The 20-mm-diameter steel rod CD is welded to the 20-mm-diameter steel shaft AB as shown. End C of rod CD is touching the rigid surface shown when a couple T_B is applied to a disk attached to shaft AB . Knowing that the bearings are self aligning and exert no couples on the shaft, determine the angle of rotation of the disk when $T_B = 400 \text{ N}\cdot\text{m}$. Use $E = 200 \text{ GPa}$ and $G = 77.2 \text{ GPa}$. (Consider the strain energy due to both bending and twisting in shaft AB and to bending in arm CD .)

$$\sum M_{AB} = 0: \quad r_{CD} F_C = T_B \quad F_C = \frac{T_B}{r_{CD}}$$

$$F_C = \frac{400}{300 \times 10^{-3}} = 1333.3 \text{ N}, \quad F_D = 1333.3 \text{ N}$$

Bending of rod CD :

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{20}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$M = F_C x$$

$$U = \int_0^{L_{CD}} \frac{(F_C x)^2}{2EI} dx = \frac{F_C^2 L_{CD}^3}{6EI}$$

$$= \frac{(1333.3)^2 (300 \times 10^{-3})^3}{(6)(200 \times 10^9)(7.854 \times 10^{-9})} = 5.093 \text{ J}$$

Bending of shaft ADB :

$$\sum M_B = 0: \quad -F_A L_{AB} + F_D b = 0 \quad F_A = \frac{F_D b}{L_{AB}}$$

$$\sum M_A = 0: \quad +F_A L_{AB} - F_D a = 0 \quad F_A = \frac{F_D a}{L_{AB}}$$

$$U = \frac{1}{2EI} \left\{ \int_0^a \left(\frac{F_D b}{L_{AB}}\right)^2 dx + \int_0^b \left(\frac{F_D a}{L_{AB}}\right)^2 dx \right\} = \frac{F_D^2 a^2 b^2}{6EI L_{AB}}$$

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = (270 \times 10^{-3}) \text{ m}$$

$$U = \frac{(1333.3)^2 (70 \times 10^{-3})^2 (200 \times 10^{-3})^2}{(6)(200 \times 10^9)(7.854 \times 10^{-9})(270 \times 10^{-3})} = 0.137 \text{ J}$$

Torsion: Only portion DB carries torque. $J = 2J = 15.708 \times 10^{-9} \text{ m}^4$

$$U = \frac{T_B^2 L_{DB}}{2GJ} = \frac{(400)^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 13.194 \text{ J}$$

$$\text{Total: } U = 5.093 + 0.137 + 13.194 = 18.424 \text{ J}$$

$$\frac{1}{2} T_B \phi_B = U$$

$$\phi_B = \frac{2U}{T_B} = \frac{(2)(18.424)}{400} = 92.1 \times 10^{-3} \text{ rad}$$

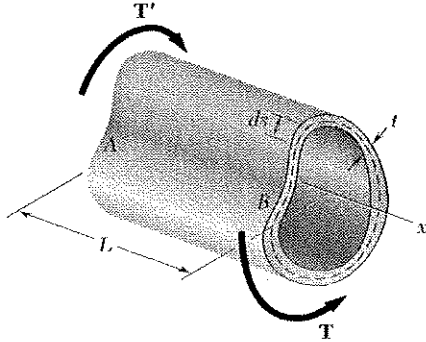
$$\phi_B = 5.28^\circ$$

Problem 11.70

11.70 The thin-walled hollow cylindrical member AB has a noncircular cross section of nonuniform thickness. Using the expression given in Eq. (3.53) of Sec. 3.13, and the expression for the strain-energy density given in Eq. (11.19), show that the angle of twist of member AB is

$$\phi = \frac{TL}{4GJ} \oint \frac{ds}{t}$$

where ds is an element of the center line of the wall cross section and O is the area enclosed by that center line.



From equation (3.53), $\gamma = \frac{T}{2ta}$

Strain energy density:

$$U = \frac{\gamma^2}{2G} = \frac{T^2}{8Gt^2a^2}$$

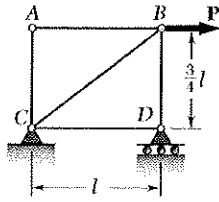
$$U = \int_0^L \oint U t ds dx$$

$$= \int_0^L \frac{T^2}{8Ga^2} \oint \frac{ds}{t} dx = \frac{T^2 L}{8Ga^2} \oint \frac{ds}{t}$$

$$\text{Work of torque} = \frac{1}{2} T \phi = \frac{T^2 L}{8Ga^2} \oint \frac{ds}{t} \quad \phi = \frac{TL}{4Ga^2} \oint \frac{ds}{t}$$

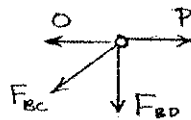
Problem 11.71

11.71 and 11.72 Each member of the truss shown has a uniform cross-sectional area A . Using the method of work and energy, determine the horizontal deflection of the point of application of the load P .



Members AB , AC and CD are zero force members

Joint B



$$+\rightarrow \sum F_x = 0:$$

$$P - \frac{4}{5} F_{BC} = 0$$

$$F_{BC} = \frac{5}{4} P$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{BD} - \frac{3}{5} F_{BC} = 0$$

$$F_{BD} = -\frac{3}{4} P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$= \frac{19}{16} \frac{P^2 l}{EA}$$

$$\text{Work of } P = \frac{1}{2} P \Delta = U$$

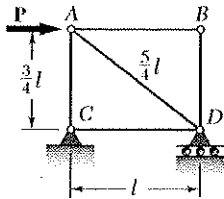
$$\Delta = \frac{2U}{P} = \frac{19}{8} \frac{Pl}{EA}$$

Member	F	L	$F^2 L$
AB	0	l	0
AC	0	$\frac{3}{4} l$	0
CD	0	l	0
BC	$\frac{5}{4} P$	$\frac{5}{4} l$	$\frac{125}{64} P^2 l$
BD	$-\frac{3}{4} P$	$\frac{3}{4} l$	$\frac{27}{64} P^2 l$
Σ			$\frac{19}{8} P^2 l$

$$\Delta = 2.375 \frac{Pl}{EA} \rightarrow \blacktriangleleft$$

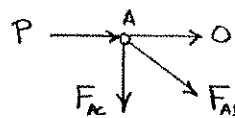
Problem 11.72

11.71 and 11.72 Each member of the truss shown has a uniform cross-sectional area A . Using the method of work and energy, determine the horizontal deflection of the point of application of the load P .



Members AB and BD are zero force members.

Joint A



$$+\rightarrow \sum F_x = 0:$$

$$\frac{4}{5} F_{AD} + P = 0$$

$$F_{AD} = -\frac{5}{4} P$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{AC} - \frac{3}{5} F_{AD} = 0$$

$$F_{AC} = \frac{3}{4} P$$

$$\text{Joint D } +\rightarrow \sum F_x = 0:$$

$$\frac{4}{5} \cdot \frac{5}{4} P - F_{CD} = 0$$

$$F_{CD} = P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$= \frac{27}{16} \frac{P^2 l}{EA}$$

Member	F	L	$F^2 L$
AB	0	l	0
BD	0	$\frac{3}{4} l$	0
AD	$-\frac{5}{4} P$	$\frac{5}{4} l$	$\frac{125}{64} P^2 l$
CD	P	l	$P^2 l$
AC	$\frac{3}{4} P$	$\frac{3}{4} l$	$\frac{27}{64} P^2 l$
Σ			$\frac{27}{8} P^2 l$

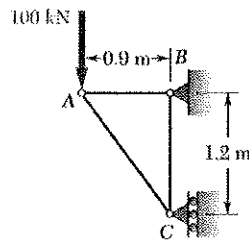
$$\text{Work of } P = \frac{1}{2} P \Delta = U$$

$$\Delta = \frac{2U}{P} = \frac{27}{8} \frac{Pl}{EA}$$

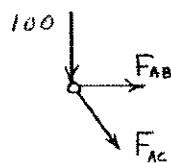
$$\Delta = 3.375 \frac{Pl}{EA} \rightarrow \blacktriangleleft$$

Problem 11.73

11.73 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 1945 mm^2 . Using $E = 200 \text{ GPa}$, determine the vertical deflection of joint A caused by the application of the 100-kN load.



Joint A



$$+\uparrow \sum F_y = 0$$

$$-100 - \frac{0.9}{1.2} F_{AC} = 0$$

$$F_{AC} = -133.3 \text{ kN}$$

$$+\rightarrow \sum F_x = 0$$

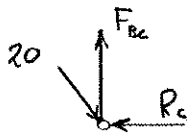
$$\frac{0.9}{1.5} F_{AC} + F_{AB} = 0$$

$$F_{AB} = 80 \text{ kN}$$

Joint C

$$+\uparrow \sum F_y = 0 \quad F_B - \frac{0.9}{1.2} (133.3) = 0$$

$$F_{BC} = 100 \text{ kN}$$



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$A = 1945 \text{ mm}^2$$

Member	F (kN)	L (m)	F ² L (kN ² m)
AB	80	0.9	5760
AC	-133.3	1.5	26653
BC	100	1.2	12000
Σ			44413

$$U = \frac{44413 \times 10^6}{(2)(200 \times 10^9)(1945 \times 10^{-6})} = 57.09 \text{ J}$$

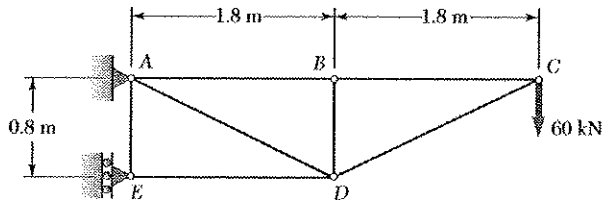
$$\frac{1}{2} P \Delta = U$$

$$\Delta = \frac{2U}{P} = \frac{(2)(57.09)}{100000} = 1.14 \times 10^{-3} \text{ m}$$

$$= 1.14 \text{ mm} \downarrow$$

Problem 11.74

11.74 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 3220 mm². Using $E = 200$ GPa, determine the vertical deflection of joint C caused by the application of the 60-kN load.



Members BD and AE are zero force members.

For entire truss $\sum M_A = 0$:

$$0.8 R_D - (60)(3.6) = 0$$

$$R_D = 270 \text{ kN}$$

For equilibrium of joint E

$$F_{ED} = -R_D = -270 \text{ kN}$$

Joint C

$$+\uparrow \sum F_y = 0$$

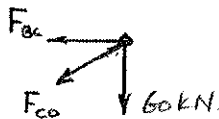
$$-\frac{0.8}{1.97} F_{CD} - 60 = 0$$

$$F_{CD} = -147.75$$

$$+\rightarrow \sum F_x = 0$$

$$-\frac{1.8}{1.97} F_{CD} - F_{BC} = 0$$

$$F_{BC} = 135 \text{ kN}$$



Joint D

$$+\rightarrow \sum F_x = 0$$

$$-270 - \frac{1.8}{1.97} (F_{AD} + 147.75) = 0$$

$$F_{AD} = 147.75 \text{ kN}$$

Joint B $\sum F_x = 0$:

$$-F_{AB} + F_{BC} = 0$$

$$F_{AB} = 135 \text{ kN}$$



$$\text{Strain energy: } U_m = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

Member	F (kN)	L (m)	F ² L (kN ² m)
AB	135	1.8	32805
BC	135	1.8	32805
CD	-147.75	1.97	43005
DE	-120	1.8	25920
BD	0	0.8	0
AE	0	0.8	0
AD	147.75	1.97	43005
Σ			177540

$$\text{Data: } E = 200 \times 10^6 \text{ kPa}$$

$$A = 3220 \text{ mm}^2$$

$$U_m = \frac{177540}{(2)(200 \times 10^6)(3220 \times 10^{-6})}$$

$$= 0.2757 \text{ kJ}$$

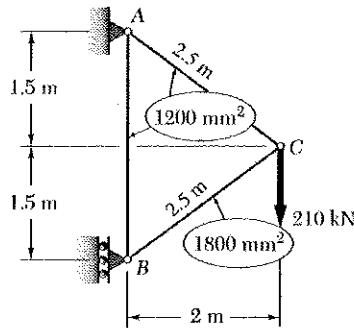
$$\frac{1}{2} P_m \Delta_m = U$$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(0.2757)}{60} = 0.00919 \text{ m}$$

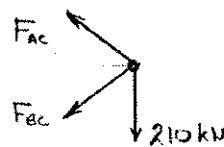
$$= 9.2 \text{ mm} \downarrow$$

Problem 11.75

11.75 Members of the truss shown are made of steel and have the cross-sectional areas shown. Using $E = 200 \text{ GPa}$, determine the vertical deflection of joint C caused by the application of the 210-kN load.



Joint C



$$+\rightarrow \Sigma F_x = 0:$$

$$-\frac{4}{5} F_{AC} - \frac{4}{5} F_{BC} = 0$$

$$+\uparrow \Sigma F_y = 0:$$

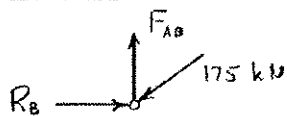
$$\frac{3}{5} F_{AC} - \frac{3}{5} F_{BC} - 210 = 0$$

Solving simultaneously,

$$F_{AC} = 175 \text{ kN}$$

$$F_{BC} = -175 \text{ kN}$$

Joint B



$$+\uparrow \Sigma F_y = 0:$$

$$F_{AB} - \left(\frac{3}{5}\right)(175) = 0$$

$$F_{AB} = 105 \text{ kN}$$

$$U_m = \Sigma \frac{F^2 L}{2EA}$$

Member	F (kN)	L (m)	A (10^{-6} m^2)	$F^2 L / A$ (N^2/m)
AB	105	3.0	1200	27.5625×10^{12}
AC	175	2.5	1200	63.8021×10^{12}
BC	-175	2.5	1800	42.5347×10^{12}
Σ				133.8993×10^{12}

$$U_m = \frac{1}{2E} \Sigma \frac{F^2 L}{A} = \frac{133.8993 \times 10^{12}}{(2)(200 \times 10^9)} = 334.75 \text{ J}$$

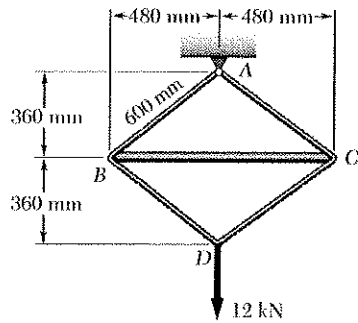
$$\frac{1}{2} P_m \Delta_m = U_m$$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(334.75)}{210 \times 10^3} = 3.19 \times 10^{-3} \text{ m}$$

$$\Delta_m = 3.19 \text{ mm} \downarrow$$

Problem 11.76

11.76 The steel rod BC has a 24-mm diameter and the steel cable $ABDCA$ as a 12-mm diameter. Using $E = 200$ GPa, determine the deflection of joint D caused by the 12-kN load.



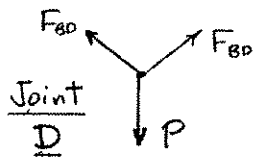
Owing to symmetry, $F_{AB} = F_{BD} = F_{DC} = F_{CA}$

$$U_{AB} = U_{BD} = U_{DC} = U_{CA}$$

$$U = 4 U_{BD} + U_{BC} = 4 \frac{F_{BD}^2 L_{BD}}{2EA_{BD}} + \frac{F_{BC}^2 L_{BC}}{2EA_{BC}}$$

Let P be the load at D .

$$\delta_D = \frac{\partial U}{\partial P} = 4 \frac{F_{BD} L_{BD}}{EA_{BD}} \frac{\partial F_{BD}}{\partial P} + \frac{F_{BC} L_{BC}}{EA_{BC}} \frac{\partial F_{BC}}{\partial P}$$

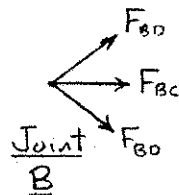


$$+\uparrow \sum F_y = 0:$$

$$2 \left(\frac{3}{5} F_{BD} \right) - P = 0$$

$$F_{BD} = \frac{5}{6} P$$

$$\frac{\partial F_{BD}}{\partial P} = \frac{5}{6}$$



$$+\rightarrow \sum F_x = 0:$$

$$F_{BC} + (2) \left(\frac{4}{5} F_{BD} \right) = 0$$

$$F_{BC} = -\frac{8}{5} F_{BD} = -\frac{4}{3} P$$

$$\frac{\partial F_{BC}}{\partial P} = -\frac{4}{3}$$

$$\delta_D = 4 \left(\frac{5}{6} \right)^2 \frac{P L_{BD}}{EA_{BD}} + \left(\frac{4}{3} \right)^2 \frac{P L_{BC}}{EA_{BC}} = \frac{P}{E} \left\{ \frac{25}{9} \frac{L_{BD}}{A_{BD}} + \frac{16}{9} \frac{L_{BC}}{A_{BC}} \right\}$$

Data: $P = 12 \times 10^3$ N

$E = 200 \times 10^9$ Pa

$L_{BD} = 600 \times 10^{-3}$ m

$A_{BD} = \frac{\pi}{4} (12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$

$L_{BC} = 960 \times 10^{-3}$ m

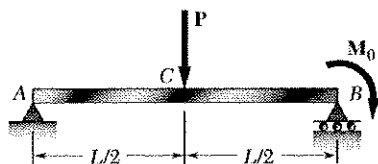
$A_{BC} = \frac{\pi}{4} (24)^2 = 452.39 \text{ mm}^2 = 452.39 \times 10^{-6} \text{ m}^2$

$$\delta_D = \frac{12 \times 10^3}{200 \times 10^9} \left\{ \frac{25}{9} \cdot \frac{600 \times 10^{-3}}{113.097 \times 10^{-6}} + \frac{16}{9} \cdot \frac{960 \times 10^{-3}}{452.39 \times 10^{-6}} \right\} = 1.111 \times 10^{-3} \text{ m}$$

$$\delta_D = 1.111 \text{ mm} \downarrow$$

Problem 11.77

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M is applied first.



From Appendix D, Case 4.

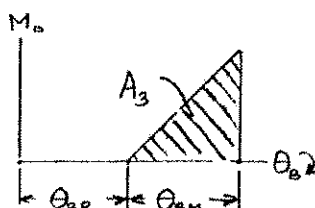
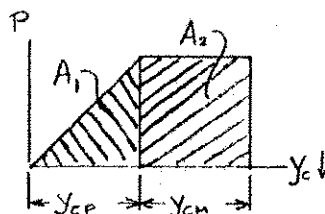
$$\downarrow y_c = \frac{PL^3}{48EI} \quad \curvearrowleft \theta_B = -\frac{PL^2}{16EI}$$

From Appendix D, Case 7.

$$\downarrow y_c = \frac{M_0}{6EI} \left((L/2)^3 - L^2(L/2) \right) = -\frac{M_0 L^2}{16EI}$$

$$\curvearrowright \theta_B = \frac{M_0 L}{3EI}$$

(a) First P , then M_0 .

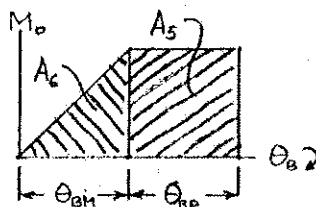
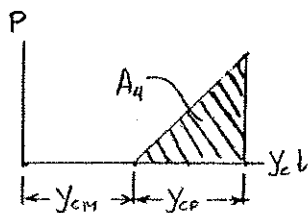


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2} P y_{CP} + P y_{CM} + \frac{1}{2} M_0 \theta_{BM}$$

$$= \frac{P^2 L^3}{96EI} - \frac{PM_0 L^2}{16EI} + \frac{M_0^2 L}{6EI}$$

(b) First M_0 , then P .



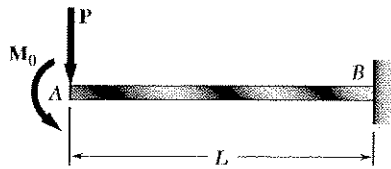
$$U = A_4 + A_5 + A_6$$

$$= \frac{1}{2} P y_{CP} + M_0 \theta_{BP} + \frac{1}{2} M_0 \theta_{BM}$$

$$= \frac{P^2 L^3}{96EI} - \frac{M_0 P L^2}{16EI} + \frac{M_0^2 L}{6EI}$$

Problem 11.78

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M is applied first.



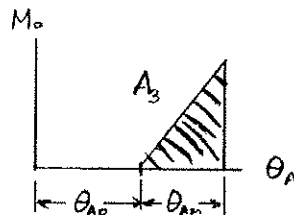
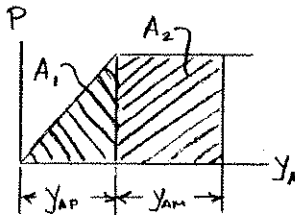
From Appendix D, Case 1.

$$y_{AP} = \frac{PL^3}{3EI} \quad \theta_{AP} = \frac{PL^2}{2EI}$$

From Appendix D, Case 3.

$$y_{AM} = \frac{M_0 L^2}{2EI} \quad \theta_{AM} = \frac{M_0 L}{EI}$$

(a) First P , then M_0 .

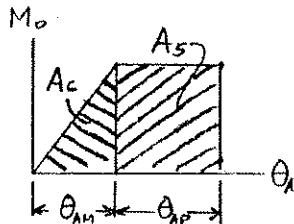
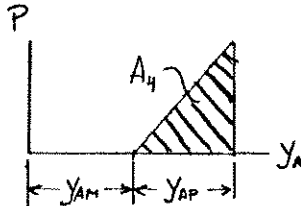


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2} P y_{AP} + P y_{AM} + \frac{1}{2} M_0 \theta_{AM}$$

$$= \frac{P^2 L^3}{6EI} + \frac{P M_0 L^2}{2EI} + \frac{M_0^2 L}{2EI}$$

(b) First M_0 , then P .



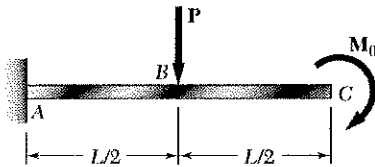
$$U = A_4 + A_5 + A_6$$

$$= \frac{1}{2} P y_{AP} + M_0 \theta_{AP} + \frac{1}{2} M_0 \theta_{AM}$$

$$= \frac{P^2 L^3}{6EI} + \frac{M_0 P L^2}{2EI} + \frac{M_0^2 L}{2EI}$$

Problem 11.79

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M is applied first.



Appendix D Cases 1 and 3.

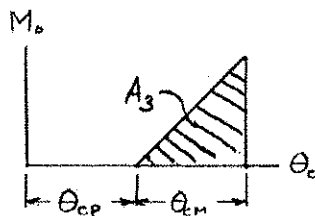
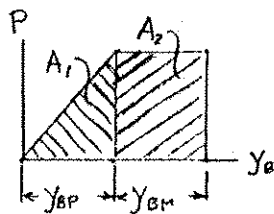
$$y_{BP} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\theta_{CP} = \frac{P(L/2)^2}{2EI} = \frac{PL^2}{8EI}$$

$$y_{BM} = \frac{M_0(L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$\theta_{BM} = \frac{M_0 L}{EI}$$

(a) First P , then M_0 .

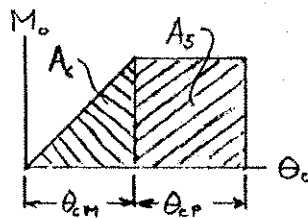
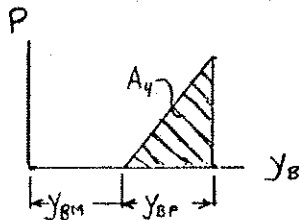


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2} P y_{BP} + P y_{BM} + \frac{1}{2} M_0 \theta_{CM}$$

$$= \frac{P^2 L^3}{48EI} + \frac{P M_0 L^2}{8EI} + \frac{M_0^2 L}{2EI}$$

(b) First M_0 , then P .



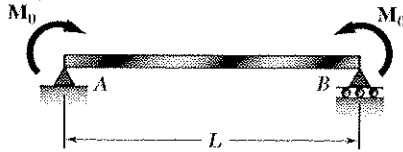
$$U = A_4 + A_5 + A_6$$

$$= \frac{1}{2} P y_{BP} + M_0 \theta_{CP} + \frac{1}{2} M_0 \theta_{CM}$$

$$= \frac{P^2 L^3}{48EI} + \frac{M_0 P L^2}{8EI} + \frac{M_0^2 L}{2EI}$$

Problem 11.80

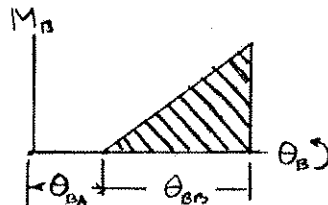
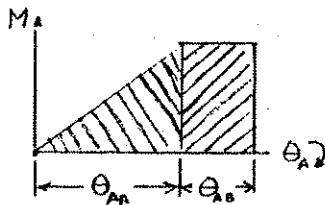
11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.



(a) Label the couples M_A and M_B .

Using Appendix D, Case 7.

$$C\theta_{AA} = \frac{M_A L}{3EI} \quad C\theta_{BA} = \frac{M_A L}{6EI} \quad C\theta_{BB} = \frac{M_B L}{3EI} \quad C\theta_{AB} = \frac{M_B L}{6EI}$$



Apply M_A first, then M_B .

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} M_A \theta_{AA} + M_A \theta_{AB} + \frac{1}{2} M_B \theta_{BB} = \frac{1}{6} \frac{M_A^2 L}{EI} + \frac{1}{6} \frac{M_A M_B L}{EI} + \frac{1}{6} \frac{M_B^2 L}{EI}$$

With $M_A = M_B = M_0$

$$U = \frac{1}{2} \frac{M_0^2 L}{EI}$$

$$U = \frac{M_0^2 L}{2EI}$$

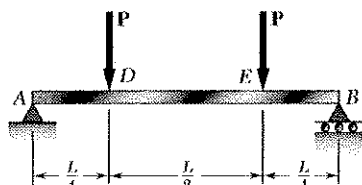
(b) Bending moment

$$M = M_0$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2 L}{2EI}$$

$$U = \frac{M_0^2 L}{2EI}$$

Problem 11.81



11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.

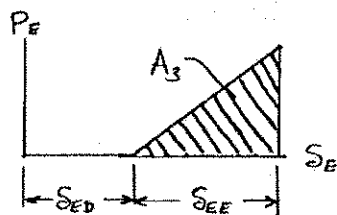
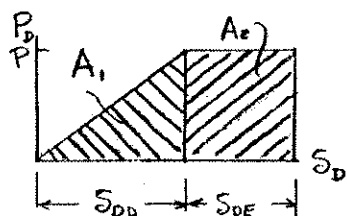
(a) Label the forces P_D and P_E .

Using Appendix D Case 5,

$$S_{EE} = \frac{P_E a^2 b^2}{3EI L} = \frac{P_E \left(\frac{3L}{4}\right)^2 \left(\frac{L}{4}\right)^2}{3EI L} = \frac{3}{256} \frac{P_E L^3}{EI}$$

$$S_{DE} = \frac{P_E b}{6EI L} \left[(L^2 - b^2)x - x^3 \right] = \frac{P_E \left(\frac{L}{4}\right)}{6EI L} \left[\left(L^2 - \left(\frac{L}{4}\right)^2 \right) \left(\frac{L}{4}\right) - \left(\frac{L}{4}\right)^3 \right] = \frac{7}{768} \frac{P_E L^3}{EI}$$

$$\text{Likewise, } S_{DD} = \frac{3}{256} \frac{P_D L^3}{EI} \quad \text{and} \quad S_{ED} = \frac{7}{768} \frac{P_D L^3}{EI}$$



Let P_D be applied first.

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} P_D S_{DD} + P_D S_{DE} + \frac{1}{2} P_E S_{EE} = \frac{3}{512} \frac{P_D^2 L^3}{EI} + \frac{7}{768} \frac{P_D P_E L^3}{EI} + \frac{3}{512} \frac{P_E^2 L^3}{EI}$$

$$\text{With } P_D = P_E = P \quad U = \frac{1}{48} \frac{P^2 L^3}{EI}$$

$$U = \frac{P^2 L^3}{48 EI}$$

(b) Reactions: $R_A = R_B = P$

Over portion AD: $(0 < x < \frac{L}{4})$; $M = Px$

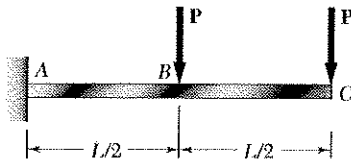
$$U_{AD} = \int_0^{\frac{L}{4}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{\frac{L}{4}} x^2 dx = \frac{P^2}{2EI} \cdot \frac{1}{3} \left(\frac{L}{4}\right)^3 = \frac{1}{384} \frac{P^2 L^3}{EI}$$

$$\text{Over portion DE: } M = \frac{PL}{4} \quad U_{DE} = \frac{M^2 \left(\frac{L}{2}\right)}{2EI} + \frac{P^2 L^2}{2EI} \cdot \frac{1}{16} \cdot \frac{L}{2} = \frac{P^2 L^3}{64 EI}$$

$$\text{Over portion EB: By symmetry, } U_{EB} = U_{AD} = \frac{1}{384} \frac{P^2 L^3}{EI}$$

$$\text{Total: } U = U_{AD} + U_{DE} + U_{EB} = \left(\frac{1}{384} + \frac{1}{64} + \frac{1}{384} \right) \frac{P^2 L^3}{EI} = \frac{1}{48} \frac{P^2 L^3}{EI}$$

Problem 11.82



11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.

(a) Label the forces P_B and P_C .

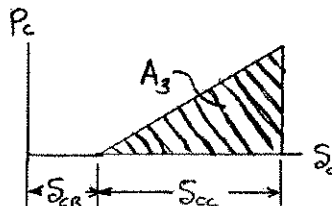
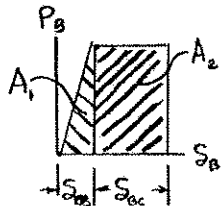
Using Appendix D Case 1

$$\delta_{BB} = \frac{P_B (L/2)^3}{3EI} = \frac{1}{24} \frac{P_B L^3}{EI}$$

$$\delta_{CB} = \delta_{BB} + \frac{L}{2} \theta_B = \frac{1}{24} \frac{P_B L^3}{EI} + \frac{L}{2} \frac{P_B (L/2)^2}{2EI} = \left(\frac{1}{24} + \frac{1}{16} \right) \frac{P_B L^3}{EI} = \frac{5}{48} \frac{P_B L^3}{EI}$$

$$\delta_{CC} = \frac{1}{3} \frac{P_C L^3}{EI}$$

$$\delta_{BC} = \frac{P_C}{6EI} (3Lx^2 - x^3) = \frac{P_C}{6EI} \left(3L \left(\frac{L}{2} \right)^2 - \left(\frac{L}{2} \right)^3 \right) = \frac{5}{48} \frac{P_C L^3}{EI}$$



Apply P_B first, then P_C

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} P_B \delta_{BB} + P_B \delta_{BC} + \frac{1}{2} P_C \delta_{CC} = \frac{1}{48} \frac{P_B L^3}{EI} + \frac{5}{48} \frac{P_B P_C L^3}{EI} + \frac{1}{6} \frac{P_C^2 L^3}{EI}$$

$$\text{With } P_B = P_C = P \quad U = \left(\frac{1}{48} + \frac{5}{48} + \frac{1}{6} \right) \frac{P^2 L^3}{EI} \quad U = \frac{7P^2 L^3}{24EI}$$

(b) Over AB: $M = Pv + P(v - \frac{L}{2}) = P(2v - \frac{L}{2})$

$$\begin{aligned} U_{AB} &= \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_{\frac{L}{2}}^L \left(4v^2 - 2Lv + \frac{1}{4} L^2 \right) dv \\ &= \frac{P^2}{2EI} \left\{ \frac{4}{3} \left[L^3 - \left(\frac{L}{2} \right)^3 \right] - 2L \cdot \frac{1}{2} \left[L^2 - \left(\frac{L}{2} \right)^2 \right] + \frac{1}{4} L^2 \left[L - \frac{L}{2} \right] \right\} \\ &= \frac{P^2}{2EI} \left\{ \frac{7}{6} L^3 - \frac{3}{4} L^3 + \frac{1}{8} L^3 \right\} = \frac{13}{48} \frac{P^2 L^3}{EI} \end{aligned}$$

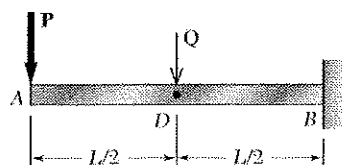
Over BC: $M = Pv \quad U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_0^{\frac{L}{2}} v^2 dv = \frac{P^2}{2EI} \cdot \frac{1}{3} \left(\frac{L}{2} \right)^3$

$$= \frac{P^2 L^3}{48EI}$$

Total: $U = U_{AB} + U_{BC} = \left(\frac{13}{48} + \frac{1}{48} \right) \frac{P^2 L^3}{EI} \quad U = \frac{7P^2 L^3}{24EI}$

Problem 11.83

11.83 and 11.84 For the prismatic beam shown, determine the deflection of point D.



Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_D = \frac{\partial U}{\partial Q} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx$$

Over portion AD: ($0 < x < \frac{L}{2}$) $M = -Px$, $\frac{\partial M}{\partial Q} = 0$

Over portion DB: ($\frac{L}{2} < x < L$) $M = -Px - Q(x - \frac{L}{2})$, $\frac{\partial M}{\partial Q} = -(x - \frac{L}{2})$

Set $Q = 0$.

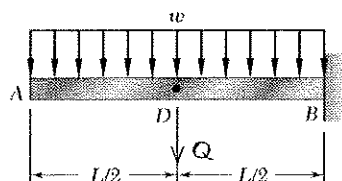
$$S_D = \frac{1}{EI} \int_0^{\frac{L}{2}} (-Px)(0) dx + \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px) \left[-(x - \frac{L}{2}) \right] dx$$

$$= \frac{P}{EI} \int_{\frac{L}{2}}^L (x^2 - \frac{L}{2}x) dx = \frac{P}{EI} \left\{ \frac{1}{3}L^3 - \frac{1}{3}(\frac{L}{2})^3 - (\frac{L}{2})\frac{1}{2}L^2 + \frac{L}{2}\frac{1}{2}(\frac{L}{2})^2 \right\}$$

$$= \left(\frac{1}{3} - \frac{1}{24} - \frac{1}{4} + \frac{1}{16} \right) \frac{PL^3}{EI} \quad S_D = \frac{5PL^3}{48EI} \downarrow$$

Problem 11.84

11.83 and 11.84 For the prismatic beam shown, determine the deflection of point D.



Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_D = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx$$

Over portion AD: ($0 < x < \frac{L}{2}$) $M = -\frac{1}{2}wx^2$, $\frac{\partial M}{\partial Q} = 0$

Over portion DB: ($\frac{L}{2} < x < L$) $M = -\frac{1}{2}wx^2 - Q(x - \frac{L}{2})$, $\frac{\partial M}{\partial Q} = -(x - \frac{L}{2})$

Set $Q = 0$.

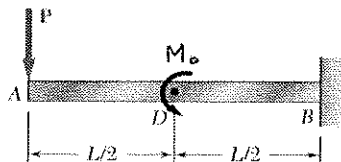
$$S_D = \frac{1}{EI} \int_0^{\frac{L}{2}} (-\frac{1}{2}wx^2)(0) dx + \frac{1}{EI} \int_{\frac{L}{2}}^L (-\frac{1}{2}wx^2) \left[-(x - \frac{L}{2}) \right] dx$$

$$= \frac{w}{2EI} \int_{\frac{L}{2}}^L (x^3 - \frac{L}{2}x^2) dx = \frac{w}{2EI} \left\{ \frac{1}{4}L^4 - \frac{1}{4}(\frac{L}{2})^4 - (\frac{L}{2})\frac{1}{3}L^3 + (\frac{L}{2})\frac{1}{3}(\frac{L}{2})^3 \right\}$$

$$= \frac{1}{2} \left(\frac{1}{4} - \frac{1}{64} - \frac{1}{6} + \frac{1}{48} \right) \frac{wL^4}{EI} = \frac{17}{384} \frac{wL^4}{EI} \quad S_D = 0.0443 \frac{wL^4}{EI} \downarrow$$

Problem 11.85

11.85 and 11.86 For the prismatic beam shown, determine the slope at point D.



Add couple M_0 at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_0} dx$$

Over portion AD: ($0 < x < \frac{L}{2}$)

$$M = -Px$$

$$\frac{\partial M}{\partial M_0} = 0$$

Over portion DB: ($\frac{L}{2} < x < L$)

$$M = -Px - M_0$$

$$\frac{\partial M}{\partial M_0} = -1$$

$$\text{Set } M_0 = 0. \quad \theta_D = \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px)(-1) dx + \frac{1}{EI} \int_0^{\frac{L}{2}} (-Px)(0) dx$$

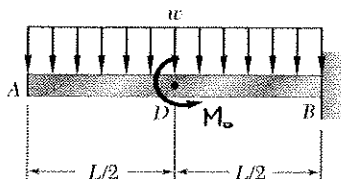
$$= \frac{P}{EI} \int_{\frac{L}{2}}^L x dx = \frac{P}{EI} \left[\frac{1}{2} L^2 - \frac{1}{2} \left(\frac{L}{2} \right)^2 \right]$$

$$= \left(\frac{1}{2} - \frac{1}{8} \right) \frac{PL^2}{EI}$$

$$\theta_D = \frac{3PL^2}{8EI} \quad \swarrow \quad \blacktriangleleft$$

Problem 11.86

11.85 and 11.86 For the prismatic beam shown, determine the slope at point D.



Add couple M_0 at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_0} dx$$

Over portion AD: ($0 < x < \frac{L}{2}$)

$$M = -\frac{1}{2}wx^2$$

$$\frac{\partial M}{\partial M_0} = 0$$

Over portion DB: ($\frac{L}{2} < x < L$)

$$M = -\frac{1}{2}wx^2 - M_0$$

$$\frac{\partial M}{\partial M_0} = -1$$

$$\text{Set } M_0 = 0. \quad \theta_D = \frac{1}{EI} \int_{\frac{L}{2}}^L \left(-\frac{1}{2}wx^2 \right) (-1) dx + \frac{1}{EI} \int_0^{\frac{L}{2}} \left(-\frac{1}{2}wx^2 \right) (0) dx$$

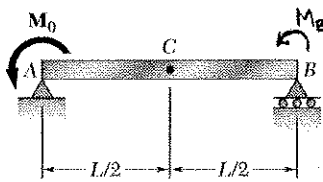
$$= \frac{w}{2EI} \int_{\frac{L}{2}}^L x^2 dx = \frac{w}{2EI} \left[\frac{1}{3} L^3 - \frac{1}{3} \left(\frac{L}{2} \right)^3 \right]$$

$$= \frac{1}{6} \left(1 - \frac{1}{8} \right) \frac{wL^3}{EI}$$

$$\theta_D = \frac{7wL^3}{48EI} \quad \swarrow \quad \blacktriangleleft$$

Problem 11.87

11.87 For the prismatic beam shown, determine the slope at point B.



Add couple M_B at point B as shown.

Reactions. $R_A = \frac{1}{L}(M_0 + M_B) \uparrow$

Strain energy. $U = \int_0^L \frac{M^2}{2EI} dx$

Slope at point B. $\theta_B = \frac{\partial U}{\partial M_B}$

$$M = R_A x - M_0 = (M_0 + M_B) \frac{x}{L} - M_0$$

$$\frac{\partial M}{\partial M_B} = \frac{x}{L} \quad \text{With } M_B = 0 \quad M = M_0 \left(\frac{x}{L} - 1 \right)$$

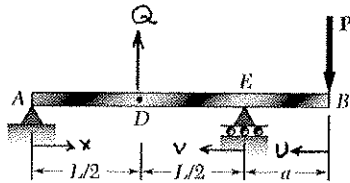
$$\begin{aligned} \frac{\partial U}{\partial M_B} &= \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_B} dx = \frac{M_0}{EI} \int_0^L \left(\frac{x}{L} - 1 \right) \frac{x}{L} dx = \frac{M_0}{EIL^2} \int_0^L (x - L)x dx \\ &= \frac{M_0}{EIL^2} \int_0^L (x^2 - Lx) dx = \frac{M_0}{EI} \left(\frac{x^3}{3} - \frac{Lx^2}{2} \right) \Big|_0^L = -\frac{M_0 L}{6EI} \end{aligned}$$

$$\theta_B = -\frac{M_0 L}{6EI}$$

$$\theta_B = \frac{M_0 L}{6EI} \quad \swarrow \blacktriangleleft$$

Problem 11.88

11.88 and 11.89 For the prismatic beam shown, determine the deflection at point D.



Add force Q at point D.

$$\text{Reactions: } R_A = -\frac{Pa}{L} - \frac{1}{2}Q, \quad R_E = \frac{P(a+L)}{L} - \frac{1}{2}Q$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad S_D = \frac{\partial U}{\partial Q}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = -\frac{Pa}{L}x - \frac{1}{2}Qx, \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}x$$

$$\begin{aligned} \text{Set } Q = 0. \quad \frac{\partial U_{AD}}{\partial Q} &= \frac{1}{EI} \int_0^{L/2} \left(-\frac{Pa}{L}x\right) \left(-\frac{1}{2}x\right) dx = \frac{Pa}{2EIL} \int_0^{L/2} x^2 dx \\ &= \frac{Pa}{2EIL} \frac{1}{3} \left(\frac{L}{2}\right)^3 = \frac{PaL^2}{48EI} \end{aligned}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv, \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$$

$$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{1}{2}Qv - P(a+v) = \frac{Pa}{L}v - Pa - \frac{1}{2}Qv$$

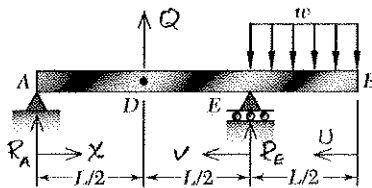
$$\begin{aligned} \text{Set } Q = 0. \quad \frac{\partial U_{DE}}{\partial Q} &= \frac{1}{EI} \int_0^{L/2} \left(\frac{Pa}{L}v - Pa\right) \left(-\frac{1}{2}v\right) dv = \frac{Pa}{2EIL} \int_0^{L/2} (-v^2 + Lv) dv \\ &= \frac{Pa}{2EIL} \left[-\frac{1}{3}\left(\frac{L}{2}\right)^3 + (L)\frac{1}{2}\left(\frac{L}{2}\right)^2\right] = \frac{Pa}{2EIL} \left[-\frac{L^3}{24} + \frac{L^3}{8}\right] \\ &= \frac{1}{24} \frac{PaL^2}{EI} \end{aligned}$$

$$\text{Over portion EB: } M = -Pv, \quad \frac{\partial M}{\partial Q} = 0, \quad \frac{\partial U_{EB}}{\partial Q} = 0$$

$$S_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{PaL^2}{48EI} + \frac{PaL^2}{24EI} + 0 \quad S_D = \frac{PaL^2}{16EI} \uparrow \blacktriangleleft$$

Problem 11.89

11.88 and 11.89 For the prismatic beam shown, determine the deflection at point D.



Add force Q at point D.

$$\text{Reactions: } R_A = -\frac{1}{8}wL - \frac{1}{2}Q, \quad R_E = \frac{5}{8}wL - \frac{1}{2}Q$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad S_D = \frac{\partial U}{\partial Q}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = -\frac{1}{8}wLx - \frac{1}{2}Qx, \quad \frac{\partial M}{\partial x} = -\frac{1}{2}x$$

$$\text{Set } Q = 0. \quad M = -\frac{1}{8}wLx$$

$$\frac{\partial U_{AD}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dx = \int_0^{L/2} \left(-\frac{wLx}{8EI} \right) \left(-\frac{1}{2}x \right) dx = \frac{wL}{16EI} \int_0^{L/2} x^2 dx = \frac{wL^4}{384EI}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$M = R_E v - \frac{wL}{2} \left(v + \frac{L}{4} \right) = \frac{1}{8}wL(v - L) - \frac{1}{2}Qv \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$$

$$\text{Set } Q = 0. \quad M = -\frac{wL}{8}(L - v)$$

$$\begin{aligned} \frac{\partial U}{\partial Q} &= \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dv = \frac{wL}{16EI} \int_0^{L/2} (L - v)v dv = \frac{wL}{16EI} \left(\frac{Lv^2}{2} - \frac{v^3}{3} \right) \bigg|_0^{L/2} \\ &= \frac{wL^4}{16EI} \left(\frac{1}{8} - \frac{1}{24} \right) = \frac{wL^4}{192EI} \end{aligned}$$

$$\text{Over portion EB: } M = -\frac{1}{2}wU^2 \quad \frac{\partial M}{\partial Q} = 0 \quad U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} du$$

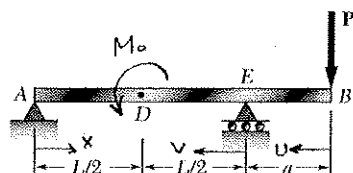
$$\frac{\partial U_{EB}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} du = 0$$

$$S_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{wL^4}{384EI} + \frac{wL^4}{192EI} + 0$$

$$S_D = \frac{wL^4}{128EI} \uparrow \blacktriangleleft$$

Problem 11.90

11.90 and 11.91 For the prismatic beam shown, determine the slope at point D.



Add couple M_o at point D.

$$\text{Reactions: } R_A = -\frac{Pa}{L} + \frac{M_o}{L}, \quad R_B = \frac{P(a+L)}{L} - \frac{M_o}{L}$$

$$U = U_{AD} + U_{DE} + U_{EB} \quad \theta_D = \frac{\partial U}{\partial M_o}$$

Over portion AD: $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = -\frac{Pa}{L}x + \frac{M_o}{L}x, \quad \frac{\partial M}{\partial M_o} = \frac{1}{L}x$

$$\text{Set } M_o = 0. \quad \frac{\partial U_{AD}}{\partial M_o} = \frac{1}{EI} \int_0^{L/2} \left(-\frac{Pa}{L}x\right)\left(\frac{1}{L}x\right) dx = -\frac{Pa}{EIL^2} \int_0^{L/2} x^2 dx$$

$$= -\frac{Pa}{EIL^2} \cdot \frac{1}{3}\left(\frac{L}{2}\right)^3 = -\frac{PaL}{24EI}$$

Over portion DE: $U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv, \quad \frac{\partial M}{\partial M_o} = -\frac{1}{L}v$

$$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{M_o}{L}v - P(a+v) = \frac{Pa}{L}v - Pa - \frac{M_o}{L}v$$

$$\text{Set } M_o = 0. \quad \frac{\partial U_{DE}}{\partial M_o} = \frac{1}{EI} \int_0^{L/2} \left(\frac{Pa}{L}v - Pa\right)\left(-\frac{1}{L}v\right) dv = -\frac{Pa}{EIL^2} \int_0^{L/2} (v^2 - Lv) dv$$

$$= -\frac{Pa}{EIL^2} \left[\frac{1}{3}\left(\frac{L}{2}\right)^3 - L \cdot \frac{1}{2}\left(\frac{L}{2}\right)^2 \right] = -\frac{Pa}{EIL^2} \left[\frac{1}{24}L^3 - \frac{1}{8}L^3 \right]$$

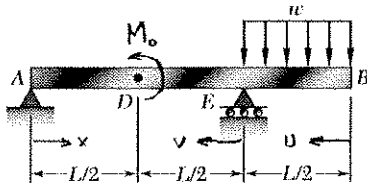
$$= \frac{1}{12} \frac{PaL}{EI}$$

Over portion EB: $M = -Pu, \quad \frac{\partial M}{\partial M_o} = 0, \quad \frac{\partial U_{EB}}{\partial M_o} = 0$

Total: $\theta_D = \frac{\partial U_{AD}}{\partial M_o} + \frac{\partial U_{DE}}{\partial M_o} + \frac{\partial U_{EB}}{\partial M_o} = -\frac{1}{24} \frac{PaL}{EI} + \frac{1}{12} \frac{PaL}{EI} + 0 \quad \theta_D = \frac{PaL}{24EI}$

Problem 11.91

11.90 and 11.91 For the prismatic beam shown, determine the slope at point D.



Add couple M_0 at point D.

$$\text{Reactions: } R_A = \frac{M_0}{L} - \frac{wL}{8}, \quad R_E = -\frac{M_0}{L} + \frac{5wL}{8}$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad \theta_D = \frac{\partial U}{\partial M_0}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = \frac{M_0 x}{L} - \frac{wLx}{8} \quad \frac{\partial M}{\partial M_0} = \frac{x}{L}$$

$$\text{Set } M_0 = 0, \quad M = -\frac{wLx}{8}$$

$$\frac{\partial U_{AD}}{\partial M_0} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \int_0^{L/2} \left(-\frac{wLx}{8EI}\right) \left(\frac{x}{L}\right) dx = -\frac{w}{8EI} \int_0^{L/2} x^2 dx = -\frac{wL^3}{192EI}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$M = R_E v - \frac{wL}{2} \left(v + \frac{L}{4}\right) = -\frac{M_0 v}{L} + \frac{wL}{8} (v - L) \quad \frac{\partial M}{\partial M_0} = -\frac{v}{L}$$

$$\text{Set } M_0 = 0, \quad M = -\frac{1}{8} wL (L - v)$$

$$\frac{\partial U_{DE}}{\partial M_0} = \frac{w}{8EI} \int_0^{L/2} (L - v) v dv = \frac{w}{8EI} \left(\frac{Lv^2}{2} - \frac{v^3}{3} \right) \Big|_0^{L/2} = \frac{wL^3}{96EI}$$

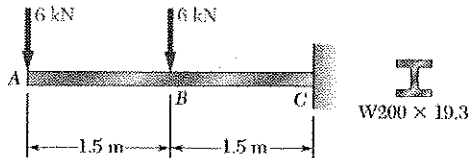
$$\text{Over portion EB: } U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} du, \quad M = -\frac{1}{2} wu^2 \quad \frac{\partial M}{\partial M_0} = 0$$

$$\frac{\partial U_{EB}}{\partial M_0} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial M_0} du = 0$$

$$\theta_D = \frac{\partial U_{AD}}{\partial M_0} + \frac{\partial U_{DE}}{\partial M_0} + \frac{\partial U_{EB}}{\partial M_0} = -\frac{wL^3}{192EI} + \frac{wL^3}{96EI} + 0 \quad \theta_D = \frac{wL^3}{192EI} \quad \swarrow$$

Problem 11.92

11.92 For the beam and loading shown, determine the deflection of point A. Use $E = 200 \text{ GPa}$.



Add force Q at point A.

Units: forces in kN, lengths in m

$$E = 200 \text{ GPa} \quad I = 16.4 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(16.4 \times 10^{-6}) = 3.28 \times 10^6 \text{ Nm}^2$$

$$U = \int_0^{1.5} \frac{M^2}{2EI} dx + \int_0^{1.5} \frac{M^2}{2EI} dv \quad \delta_A = \frac{\partial U}{\partial Q} = \frac{1}{EI} \left\{ \int_0^{1.5} M \frac{\partial M}{\partial Q} dx + \int_0^{1.5} M \frac{\partial M}{\partial Q} dv \right\}$$

$$\text{Over AB: } M = -6x, \quad \frac{\partial M}{\partial Q} = 0 \quad \int_0^{1.5} M \frac{\partial M}{\partial Q} dx = 0$$

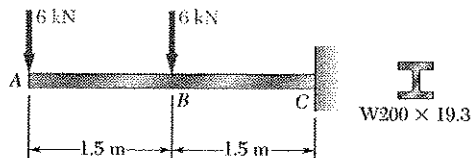
$$\text{Over BC: } M = -6(v+1.5) - 6v - Qv = -12v - 9 - Qv, \quad \frac{\partial M}{\partial Q} = -v$$

$$\int_0^{1.5} M \frac{\partial M}{\partial Q} dv = \int_0^{1.5} (12v^2 + 9v) dv = (4)(1.5)^3 + (4.5)(1.5)^3 = 28.6875$$

$$\delta_A = \frac{1}{EI} \{ 0 + 28.6875 \} = \frac{28.6875}{3.28 \times 10^3} = 8.75 \times 10^{-3} \text{ m} = 8.75 \text{ mm} \quad \downarrow$$

Problem 11.93

11.93 For the beam and loading shown, determine the deflection of point B. Use $E = 200 \text{ GPa}$.



Add force Q at point B.

Units: forces in kN, lengths in m

$$E = 200 \text{ GPa}, \quad I = 16.4 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(16.4 \times 10^{-6}) = 3.28 \times 10^6 \text{ Nm}^2$$

$$U = \int_0^3 \frac{M^2}{2EI} dx \quad \delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^3 M \frac{\partial M}{\partial Q} dx$$

$$\text{Over portion AB } 0 < x < 1.5, \quad M = -6x - Qx \quad \frac{\partial M}{\partial Q} = -x$$

$$\int_0^{1.5} M \frac{\partial M}{\partial Q} dx = \int_0^{1.5} (6x)(x) dx = 6 \int_0^{1.5} x^2 dx = (6 \times \frac{1}{3})(1.5)^3 = 6.75$$

$$\text{Over portion BC } 1.5 < x < 3 \quad M = -6x - 6(x-1.5) - Qx$$

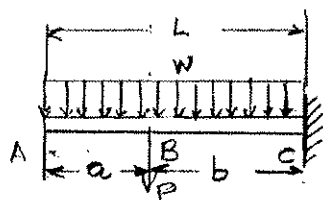
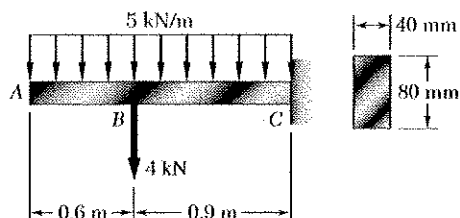
$$M = -12x + 9 - Qx \quad \frac{\partial M}{\partial Q} = -x$$

$$\int_{1.5}^3 M \frac{\partial M}{\partial Q} dx = \int_{1.5}^3 (12x^2 - 9x) dx = (12 \times \frac{1}{3})(3^3 - 1.5^3) - (9 \times \frac{1}{2})(3^2 - 1.5^2) = 64.125$$

$$\delta_B = \frac{1}{EI} \{ 6.75 + 64.125 \} = \frac{70.875}{3.28 \times 10^3} = 21.6 \times 10^{-3} \text{ m} = 21.6 \text{ mm} \quad \downarrow$$

Problem 11.94

11.94 and 11.95 For the beam and loading shown, determine the deflection at point B. Use $E = 200 \text{ GPa}$.



$$U = \int_0^a \frac{M^2}{2EI} dx + \int_a^L \frac{M^2}{2EI} dx$$

$$\delta_B = \frac{\partial U}{\partial P} = \int_0^a \frac{M}{EI} \frac{\partial M}{\partial P} dx + \int_a^L \frac{M}{EI} \frac{\partial M}{\partial P} dx$$

Portion AB: ($0 \leq x \leq a$)

$$M = -\frac{1}{2}wx^2 \quad \frac{\partial M}{\partial P} = 0$$

$$\int_0^a \frac{M}{EI} \frac{\partial M}{\partial P} dx = 0$$

Portion BC: ($a < x \leq L$)

$$M = -\frac{1}{2}wx^2 - P(x-a)$$

$$\frac{\partial M}{\partial P} = -(x-a)$$

$$\begin{aligned} \int_a^L \frac{M}{EI} \frac{\partial M}{\partial P} dx &= \frac{w}{2EI} \int_a^L x^2(x-a) dx + \frac{P}{EI} \int_a^L (x-a)^2 dx \\ &= \frac{w}{2EI} \int_a^L (x^3 - ax^2) dx + \frac{P}{EI} \int_0^b v^2 dv \\ &= \frac{w}{2EI} \left(\frac{L^4}{4} - \frac{aL^3}{3} - \frac{a^4}{4} + \frac{a^4}{3} \right) + \frac{Pb^3}{3EI} \end{aligned}$$

$$\delta_B = 0 + \frac{w}{2EI} \left(\frac{L^4}{4} - \frac{aL^3}{3} + \frac{a^4}{12} \right) + \frac{Pb^3}{3EI}$$

Data: $a = 0.6 \text{ m}$, $b = 0.9 \text{ m}$, $L = a + b = 1.5 \text{ m}$

$$w = 5 \times 10^3 \text{ N/m} \quad P = 4 \times 10^3 \text{ N}$$

$$I = \frac{1}{12}(40)(80)^3 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(1.70667 \times 10^{-6}) = 341333 \text{ N}\cdot\text{m}^2$$

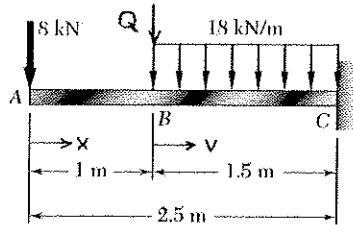
$$\delta_B = 0 + \frac{5 \times 10^3}{(2)(341333)} \left[\frac{(1.5)^4}{4} - \frac{(0.6)(1.5)^3}{3} + \frac{(0.6)^4}{12} \right] + \frac{(4 \times 10^3)(0.9)^3}{(3)(341333)}$$

$$= 7.25 \times 10^{-3} \text{ m}$$

$$\delta_B = 7.25 \text{ mm} \downarrow$$

Problem 11.95

11.94 and 11.95 For the beam and loading shown, determine the deflection at point B. Use $E = 200 \text{ GPa}$.



W250 × 22.3

Add force Q at point B.

Units: Forces in kN, lengths in m.

Over AB: $M = -8x$ $\frac{\partial M}{\partial Q} = 0$

Over BC: $M = -8(v+1) - \frac{1}{2}(18)v^2 - Qv$ $\frac{\partial M}{\partial Q} = -v$

$E = 200 \times 10^9 \text{ Pa}$, $I = 28.9 \times 10^6 \text{ mm}^4 = 28.9 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6 \text{ N}\cdot\text{m}^2 = 5780 \text{ kN}\cdot\text{m}^2$

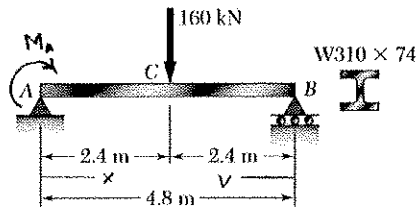
$U = \int_0^1 \frac{M^2}{2EI} dx + \int_0^{1.5} \frac{M^2}{2EI} dv$

$$\begin{aligned} \delta_B &= \frac{\partial U}{\partial Q} = \frac{1}{EI} \left\{ \int_0^1 M \frac{\partial M}{\partial Q} dx + \int_0^{1.5} M \frac{\partial M}{\partial Q} dv \right\} \\ &= \frac{1}{EI} \left\{ 0 + \int_0^{1.5} [-8(v+1) - \frac{1}{2}(18)v^2](-v) dv \right\} = \frac{1}{EI} \int_0^{1.5} (9v^3 + 8v^2 + 8v) dv \\ &= \frac{1}{EI} \left\{ \frac{9}{4}(1.5)^4 + \frac{8}{3}(1.5)^3 + \frac{8}{2}(1.5)^2 \right\} = \frac{29.391}{EI} = \frac{29.391}{5780} \end{aligned}$$

$\delta_B = 5.08 \times 10^{-3} \text{ m} = 5.08 \text{ mm} \downarrow$

Problem 11.96

11.96 For the beam and loading shown, determine the slope at end A.
Use $E = 200 \text{ GPa}$.



Add couple M_A at point A.

Units: forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}, I = 165 \times 10^6 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(165 \times 10^{-6}) = 33 \times 10^6 \text{ N}\cdot\text{m}^2 = 33000 \text{ kN}\cdot\text{m}^2$$

$$\text{Reactions: } R_A = 80 - \frac{M_A}{4.8} \quad R_B = 80 + \frac{M_A}{4.8}$$

$$U = U_{AB} + U_{BC} = \int_0^{2.4} \frac{M^2}{2EI} dx + \int_0^{2.4} \frac{M^2}{2EI} dv \quad \partial \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_A}$$

$$\text{Over AB: } M = M_A + R_A x = M_A + 80x - \frac{M_A}{4.8} x \quad \frac{\partial M}{\partial M_A} = (1 - \frac{x}{4.8})$$

$$\text{Set } M_A = 0. \quad \frac{\partial U_{AB}}{\partial M_A} = \frac{1}{EI} \int_0^{2.4} (80x)(1 - \frac{x}{4.8}) dx = \frac{1}{EI} \int_0^{2.4} (80x - 16.6667x^2) dx$$

$$= \frac{1}{EI} \left\{ (80)(\frac{1}{2})(2.4)^2 - (16.6667)(\frac{1}{3})(2.4)^3 \right\} = \frac{153.6}{EI}$$

$$\text{Over BC: } M = R_B v = 80v + \frac{M_A}{4.8} v, \quad \frac{\partial M}{\partial M_A} = \frac{1}{4.8} v$$

$$\text{Set } M_A = 0. \quad \frac{\partial U_{BC}}{\partial M_A} = \frac{1}{EI} \int_0^{2.4} (80v)(\frac{1}{4.8} v) dv = \frac{16.6667}{EI} \int_0^{2.4} v^2 dv$$

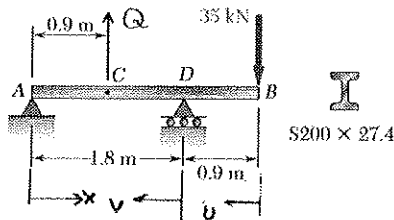
$$= \frac{(16.6667)(2.4)^3}{3EI} = \frac{76.8}{EI}$$

$$\partial \theta_A = \frac{1}{EI} \{ 153.6 + 76.8 \} = \frac{230.4}{33000}$$

$$\theta_A = 6.98 \times 10^{-3} \text{ rad} \quad \swarrow$$

Problem 11.97

11.97 For the beam and loading shown, determine the deflection at point C. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa} \quad I = 23.9 \times 10^{-6} \text{ m}^4$$

$$EI = 200 \times 23.9 = 4780 \text{ kN} \cdot \text{m}^2$$

Add dummy force Q at point C. Reactions $R_A = 17.5 + \frac{1}{2}Q \downarrow$, $R_D = 52.5 - \frac{1}{2}Q \uparrow$

$$U = U_{AC} + U_{CD} + U_{DB} \quad \delta_c = \frac{\partial U}{\partial Q} = \frac{\partial U_{AC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} + \frac{\partial U_{DB}}{\partial Q}$$

Over AC $0 < x < 0.9$ $M = -(17.5 + \frac{1}{2}Q)x$ $\frac{\partial M}{\partial Q} = -\frac{1}{2}x$ Set $Q = 0$.

$$\frac{\partial U_{AC}}{\partial Q} = \frac{1}{EI} \int_0^{0.9} (17.5x)(\frac{1}{2}x) dx = \frac{8.75}{EI} \int_0^{0.9} x^2 dx = \frac{(8.75)(0.9)^3}{3EI} = \frac{2.12625}{EI}$$

Over CD $0 < v < 0.9$ $M = R_D v - 35(v+1.5) = 52.5v - \frac{1}{2}Q - 35v - 52.5$

$$= 17.5v - 52.5 - \frac{1}{2}Qv$$

$$\frac{\partial M}{\partial Q} = -\frac{1}{2}v \quad \text{Set } Q = 0$$

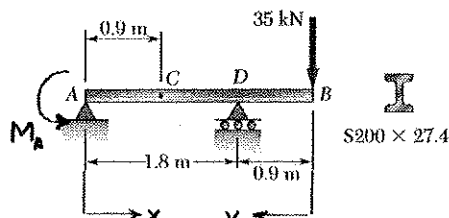
$$\begin{aligned} \frac{\partial U_{CD}}{\partial Q} &= \frac{1}{EI} \int_0^{0.9} (52.5 - 17.5v)(\frac{1}{2}v) dv = \frac{1}{EI} \int_0^{0.9} (26.25v - 8.75v^2) dv \\ &= \frac{1}{EI} \left\{ (26.25)(\frac{0.9^2}{2}) - (8.75)(\frac{0.9^3}{3}) \right\} \\ &= \frac{8.00625}{EI} \end{aligned}$$

Over DB $0 < u < 0.9$ $M = -35u$ $\frac{\partial M}{\partial Q} = 0$ $\frac{\partial U_{DB}}{\partial Q} = 0$

$$\delta_c = \frac{2.12625}{EI} + \frac{8.00625}{EI} + 0 = \frac{10.1325}{4780} = 2.12 \times 10^{-3} \text{ m} = 2.1 \text{ mm} \uparrow$$

Problem 11.98

11.98 For the beam and loading shown, determine the slope at end A. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa} \quad I = 23.9 \times 10^{-6} \text{ m}^4$$

$$EI = 200 \times 23.9 = 4780 \text{ kNm}^2$$

Add dummy couple M_A at end A. Reactions: $R_A = -17.5 + \frac{M_A}{1.8}$, $R_D = 52.5 - \frac{M_A}{1.8}$

$$U = U_{AD} + U_{DB} \quad \delta \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AD}}{\partial M_A} + \frac{\partial U_{DB}}{\partial M_A}$$

Over AD $0 < x < 1.8$ $M = -M_A + R_A x = -M_A - 17.5x + \frac{M_A}{1.8}x$

$$\frac{\partial M}{\partial M_A} = -\left(1 - \frac{x}{1.8}\right) \quad \text{Set } M_A = 0.$$

$$\begin{aligned} \frac{\partial U_{AD}}{\partial M_A} &= \frac{1}{EI} \int_0^{1.8} (17.5x)\left(1 - \frac{x}{1.8}\right) dx = \frac{1}{EI} \int_0^{1.8} \left(17.5x - \frac{17.5}{1.8}x^2\right) dx = \frac{1}{EI} \left\{ (17.5)\frac{(1.8)^2}{2} - \frac{17.5}{1.8} \frac{(1.8)^3}{3} \right\} \\ &= \frac{17.85}{EI} \end{aligned}$$

Over DB $0 < v < 0.9$ $M = -35v$ $\frac{\partial M}{\partial M_A} = 0$ $\frac{\partial U_{DB}}{\partial M_A} = 0$

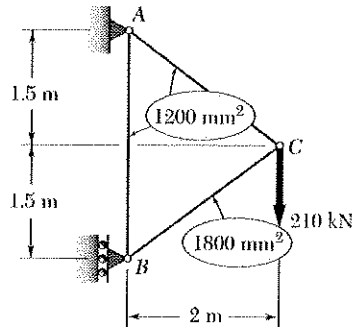
$$\delta \theta_A = \frac{17.85}{EI} + 0 = \frac{17.85}{4780} = 3.73 \times 10^{-3} \text{ rad}$$



Problem 11.99

11.99 and 11.100 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

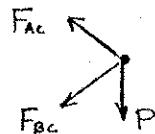
11.99 Vertical deflection of joint C.



Call the vertical load P . The vertical deflection of joint C is δ_P .

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

Joint C: $\rightarrow +\sum F_x = 0: -\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} = 0$



$\uparrow +\sum F_y = 0: \frac{3}{5}F_{AC} - \frac{3}{5}F_{BC} - P = 0$

Solving simultaneously,

Joint B: $\uparrow +\sum F_y = 0:$

$$F_{AC} = \frac{5}{6}P$$

$$F_{BC} = -\frac{5}{6}P$$

$F_{AB} - \frac{3}{5} \cdot \frac{5}{6}P = 0$
 $F_{AB} = \frac{1}{2}P$

Member	F	$\partial F / \partial P$	L (m)	A (10^{-6} m^2)	$F(\partial F / \partial P)L/A$
AB	$\frac{1}{2}P$	$\frac{1}{2}$	3	1200	$625 P$
AC	$\frac{5}{6}P$	$\frac{5}{6}$	2.5	1200	$1446.76 P$
BC	$-\frac{5}{6}P$	$-\frac{5}{6}$	2.5	1800	$964.51 P$
Σ					$3036.27 P$

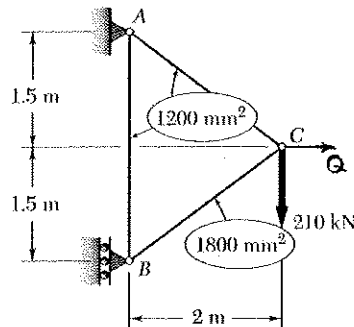
$$\delta_P = \frac{1}{E} (3036.27 P) = \frac{(3036.27)(210 \times 10^3)}{200 \times 10^9} = 3.19 \times 10^{-3} \text{ m}$$

$$\delta_P = 3.19 \text{ mm} \downarrow$$

Problem 11.100

11.99 and 11.100 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

11.100 Horizontal deflection of joint C.



Call the vertical force P . Add a horizontal force Q (positive $\rightarrow$) at joint C. The horizontal deflection of joint C is δ_Q .

$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EI} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

Joint C $\rightarrow \sum F_x = 0:$

$$-\frac{4}{5} F_{AC} - \frac{4}{5} F_{BC} + Q = 0$$

$\uparrow \sum F_y = 0:$

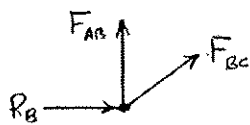
$$\frac{3}{5} F_{AC} - \frac{3}{5} F_{BC} - P = 0$$

Solving simultaneously, $F_{AC} = \frac{5}{6} P + \frac{5}{8} Q$ $F_{BC} = -\frac{5}{6} P + \frac{5}{8} Q$

Joint B $\uparrow \sum F_y = 0:$

$$F_{AB} + \frac{3}{5} F_{BC} = 0$$

$$F_{AB} = -\frac{3}{5} F_{BC} = \frac{1}{2} P - \frac{3}{8} Q$$



Member	F	$\partial F / \partial Q$	L (m)	A (10^6 m^2)	with Q = 0 $F (\partial F / \partial Q) L / A$
AB	$\frac{1}{2} P - \frac{3}{8} Q$	$-\frac{3}{8}$	3	1200	$-468.75 P$
AC	$\frac{5}{6} P + \frac{5}{8} Q$	$+\frac{5}{8}$	2.5	1200	$1085.07 P$
BC	$-\frac{5}{6} P + \frac{5}{8} Q$	$+\frac{5}{8}$	2.5	1800	$-723.38 P$
					$-107.06 P$

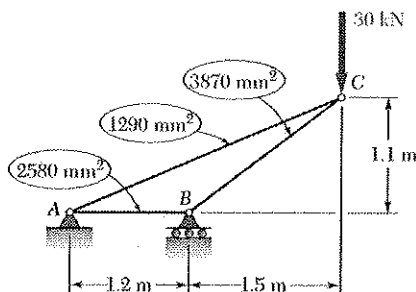
$$\delta_Q = \frac{1}{E} (-107.06 P) = -\frac{(107.06)(210 \times 10^3)}{200 \times 10^9} = -0.1124 \times 10^{-3} \text{ m}$$

$$\delta_Q = 0.1124 \text{ mm} \leftarrow$$

Problem 11.101

11.101 and 11.102 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

11.101 Vertical deflection of joint C.

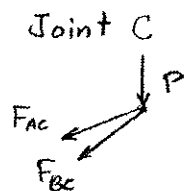


Call the vertical load P . The vertical deflection of joint C is δ_P .

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

$$\text{Geometry} \quad \overline{AC} = \sqrt{2.7^2 + 1.1^2} = 2.91 \text{ m}$$

$$\overline{BC} = \sqrt{1.5^2 + 1.1^2} = 1.86 \text{ m}$$

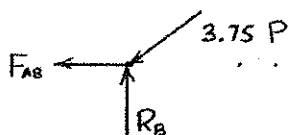


$$+\rightarrow \sum F_x = 0: -\frac{2.7}{2.91} F_{AC} - \frac{1.5}{1.86} F_{BC} = 0$$

$$+\uparrow \sum F_y = 0: -\frac{1.1}{2.91} F_{AC} - \frac{1.1}{1.86} F_{BC} - P = 0$$

Solving simultaneously, $F_{AC} = 3.3 P$, $F_{BC} = -3.8 P$

Joint B



$$+\rightarrow \sum F_x = 0: -F_{AB} - \frac{1.5}{1.86} F_{BC} = 0$$

$$F_{AB} = -3.06 P$$

Member	F	$\partial F / \partial P$	L (m)	$A (\text{mm}^2) \times 10^{-6}$	$F(\partial F / \partial P) L / A$
AB	$-3.06 P$	3.06	1.2	2580	$4355 P$
AC	$3.3 P$	3.3	2.91	1290	$24566 P$
BC	$-3.8 P$	3.8	1.86	3870	$6940 P$

Σ

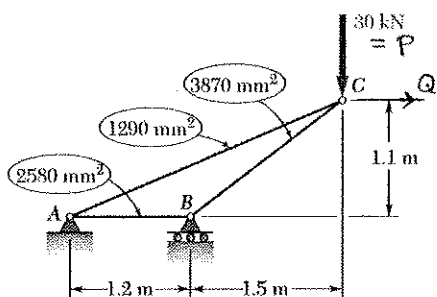
$$35861 P$$

$$\delta_P = \frac{35861 P}{E} = \frac{(35861)(30)}{200 \times 10^6} = 5.379 \times 10^{-3} \text{ m} = 5.38 \text{ mm} \downarrow$$

Problem 11.102

11.101 and 11.102 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

11.102 Horizontal deflection of point C.



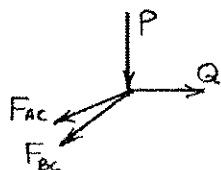
Call the vertical load P . Add horizontal dummy load Q at joint C. The horizontal deflection of joint C is δ_Q .

$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

$$\text{Geometry: } \bar{AC} = \sqrt{2.7^2 + 1.1^2} = 2.91 \text{ m}$$

$$\bar{BC} = \sqrt{1.5^2 + 1.1^2} = 1.86 \text{ m}$$

Joint C



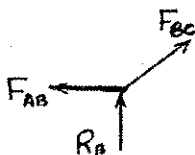
$$+\rightarrow \sum F_x = 0: -\frac{2.7}{2.91} F_{AC} - \frac{1.5}{1.86} F_{BC} + Q = 0$$

$$+\uparrow \sum F_y = 0: -\frac{1.1}{2.91} F_{AC} - \frac{1.1}{1.86} F_{BC} - P = 0$$

$$\text{Solving simultaneously, } F_{AC} = 3.3 P + 2.475 Q$$

$$F_{BC} = -3.8 P - 1.583 Q$$

Joint B



$$+\rightarrow \sum F_x = 0: \frac{1.5}{1.86} F_{AC} - F_{AB} = 0$$

$$F_{AB} = \frac{1.5}{1.86} F_{BC} = -3.06 P - 1.275 Q$$

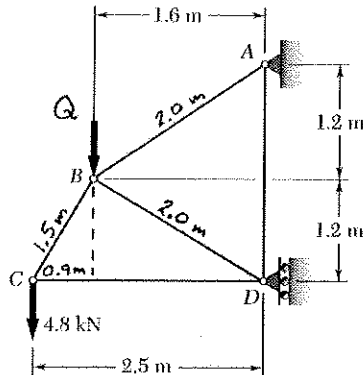
Member	F	$\partial F / \partial Q$	L (m)	A ($\times 10^{-6} \text{ m}^2$)	$F (\partial F / \partial Q) L / A$
AB	$-3.06 P - 1.275 Q$	-1.275	1.2	2580	$1814.7 P$
AC	$3.3 P + 2.475 Q$	2.475	2.91	1290	$18424.4 P$
BC	$-3.8 P - 1.583 Q$	-1.583	1.86	3870	$2891.1 P$
Σ					$581.67 P$

$$\delta_Q = \frac{23130.2 P}{E} = \frac{(23130.2)(30)}{200 \times 10^6} = 0.00347 \text{ m} = 3.5 \text{ mm} \rightarrow$$

Problem 11.103

11.103 and 11.104 Each member of the truss shown is made of steel and has a cross-sectional area of 500mm^2 . Using $E = 200\text{ GPa}$, determine the deflection indicated.

11.103 Vertical deflection of joint B.



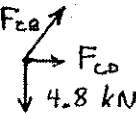
Find the length of each member as shown.

Add vertical force Q at joint B.

$$S_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial Q} L$$

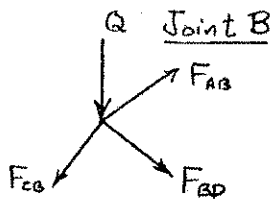
Joint C $+ \uparrow \sum F_y = 0: \frac{4}{5} F_{CB} - 4.8 = 0$

$$F_{CB} = 6.0 \text{ kN}$$



$+ \rightarrow \sum F_x = 0: \frac{3}{5} F_{CB} + F_{CD} = 0$

$$F_{CD} = -3.6 \text{ kN}$$



$+ \rightarrow \sum F_x = 0: \frac{4}{5} F_{AB} + \frac{4}{5} F_{BD} - 3.6 = 0$

$+ \uparrow \sum F_y = 0: \frac{3}{5} F_{AB} - \frac{3}{5} F_{BD} - 4.8 - Q = 0$

Solving simultaneously, $F_{AB} = 6.25 + 0.8333 Q \text{ kN}$

$$F_{BD} = -1.75 - 0.8333 Q \text{ kN}$$

Joint D



$+ \uparrow \sum F_y = 0: \frac{3}{5} F_{BD} + F_{AD} = 0$

$$F_{AD} = -\frac{3}{5} F_{BD} = 1.05 + 0.5 Q$$

Member	F (10^3 N)	$\partial F / \partial Q$	L (m)	with $Q = 0$ $F(\partial F / \partial Q) L$ ($10^3 \text{ N}\cdot\text{m}$)
AB	$6.25 + 0.8333 Q$	0.8333	2.0	10.4167
AD	$1.05 + 0.5 Q$	0.5	2.4	1.26
BD	$-1.75 - 0.8333 Q$	-0.8333	2.0	2.9167
BC	6.0	0	1.5	0
CD	-3.6	0	2.5	0
Σ				14.593

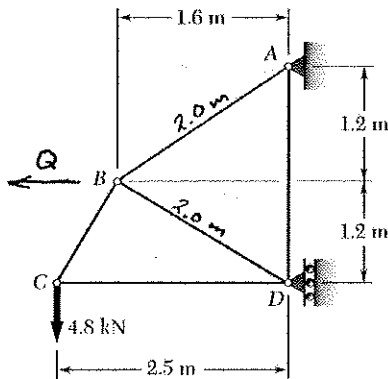
$$S_B = \frac{1}{EA} \sum F(\partial F / \partial Q) L = \frac{14.593 \times 10^3}{(200 \times 10^9)(500 \times 10^{-6})} = 145.9 \times 10^{-6} \text{ m}$$

$$S_B = 0.1459 \text{ mm} \downarrow$$

Problem 11.104

11.103 and 11.104 Each member of the truss shown is made of steel and has a cross-sectional area of 500mm^2 . Using $E = 200\text{ GPa}$, determine the deflection indicated.

11.104 Horizontal deflection of joint B.



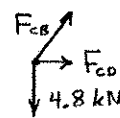
Find the length of each member as shown.

Add horizontal force Q at joint B.

$$\delta_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial Q} L$$

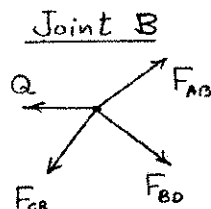
Joint C $\uparrow \sum F_y = 0: \frac{4}{5} F_{CB} - 4.8 = 0$

$$F_{CB} = 6.0 \text{ kN}$$



$\rightarrow \sum F_x = 0: \frac{3}{5} F_{CB} + F_{CD} = 0$

$$F_{CD} = -3.6 \text{ kN}$$



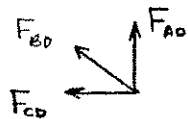
$\rightarrow \sum F_x = 0: \frac{4}{5} F_{AB} + \frac{4}{5} F_{BD} - 3.6 - Q = 0$

$\uparrow \sum F_y = 0: \frac{3}{5} F_{AB} - \frac{3}{5} F_{BD} - 4.8 = 0$

Solving simultaneously $F_{AB} = 6.25 + 0.625 Q \text{ kN}$

$$F_{BD} = -1.75 + 0.625 Q \text{ kN}$$

Joint D



$\uparrow \sum F_y = 0: \frac{3}{5} F_{BD} + F_{AD} = 0$

$$F_{AD} = -\frac{3}{5} F_{BD} = 1.05 - 0.375 Q$$

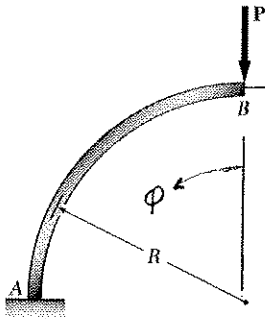
Member	F 10^3 N	$\partial F / \partial Q$	L (m)	$F(\partial F / \partial Q)L$ ($10^3 \text{ N}\cdot\text{m}$)
AB	$6.25 + 0.625 Q$	0.625	2.0	7.8125
AD	$1.05 + 0.375 Q$	-0.375	2.4	-0.9450
BD	$-1.75 + 0.625 Q$	0.625	2.0	-2.1875
BC	6.0	0	1.5	0
CD	-3.6	0	2.5	0
Σ				4.680

$$\delta_B = \frac{1}{EA} \sum F(\partial F / \partial Q)L = \frac{4.680 \times 10^3}{(200 \times 10^9)(500 \times 10^{-6})} = 46.8 \times 10^{-6} \text{ m}$$

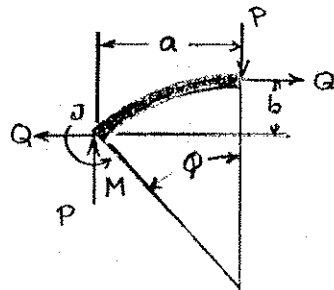
$$\delta_B = 0.0468 \text{ mm} \leftarrow$$

Problem 11.105

11.105 For the beam and loading shown and using Castigliano's theorem, determine (a) the horizontal deflection of point B, (b) the vertical deflection of point B.



Add horizontal force Q at point B.
Use polar coordinate φ .



$$U = \int_0^{\frac{\pi}{2}} \frac{M^2}{2EI} R d\varphi$$

Bending moment.

$$+\circlearrowleft \sum M_J = 0: M - Pa - Qb = 0$$

$$M = Pa + Qb \\ = PR \sin \varphi + QR(1 - \cos \varphi)$$

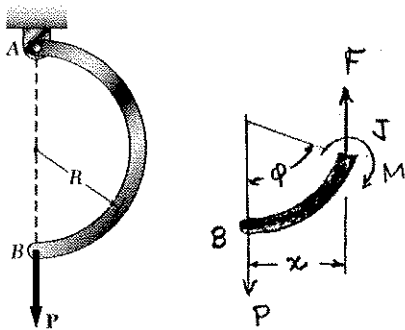
$$\frac{\partial M}{\partial P} = R \sin \varphi \quad \frac{\partial M}{\partial Q} = R(1 - \cos \varphi) \quad \text{Set } Q = 0$$

$$\begin{aligned} (a) \quad \delta_a &= \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial Q} R d\varphi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} PR \sin \varphi R(1 - \cos \varphi) R d\varphi \\ &= \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} (\sin \varphi - \sin \varphi \cos \varphi) d\varphi = \frac{PR^3}{EI} \left(-\cos \varphi - \frac{1}{2} \sin^2 \varphi \right) \Big|_0^{\frac{\pi}{2}} \\ &= \frac{PR^3}{EI} \left(-\cos \frac{\pi}{2} + \cos 0 - \frac{1}{2} \sin^2 \frac{\pi}{2} + \frac{1}{2} \sin^2 0 \right) \\ &= \frac{PR^3}{EI} \left(0 + 1 - \frac{1}{2} + 0 \right) \end{aligned} \quad \delta_a = \frac{PR^3}{2EI} \rightarrow \blacktriangleleft$$

$$\begin{aligned} (b) \quad \delta_p &= \frac{\partial U}{\partial P} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial P} R d\varphi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} PR \sin \varphi R \sin \varphi R d\varphi \\ &= \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} \sin^2 \varphi d\varphi = \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos 2\varphi) d\varphi \\ &= \frac{PR^3}{EI} \left(\frac{1}{2} \varphi - \frac{1}{2} \sin 2\varphi \right) \Big|_0^{\frac{\pi}{2}} = \frac{PR^3}{EI} \left(\frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \cdot 0 - \frac{1}{2} \sin \pi + \frac{1}{2} \cdot \sin 0 \right) \\ &= \frac{PR^3}{EI} \left(\frac{\pi}{4} - 0 - 0 + 0 \right) \end{aligned} \quad \delta_p = \frac{\pi PR^3}{4EI} \downarrow \blacktriangleleft$$

Problem 11.106

11.106 For the uniform rod and loading shown and using Castigliano's theorem, determine the deflection of point B.



Use polar coordinate ϕ .

Calculate the bending moment $M(\phi)$ using free body B.J.

$$+\circlearrowleft \sum M_J = 0 : Px - M = 0$$

$$M = Px = PR \sin \phi$$

Strain energy: $U = \int_0^L \frac{M^2}{2EI} ds$

$$U = \int_0^\pi \frac{(PR \sin \phi)^2}{2EI} (R d\phi) = \frac{P^2 R^3}{2EI} \int_0^\pi \sin^2 \phi d\phi$$

$$= \frac{P^2 R^3}{2EI} \int_0^\pi \frac{1 - \cos 2\phi}{2} d\phi$$

$$= \frac{P^2 R^3}{2EI} \left(\frac{1}{2} \phi \Big|_0^\pi - \frac{1}{4} \sin 2\phi \Big|_0^\pi \right) = \frac{\pi P^2 R^3}{4EI}$$

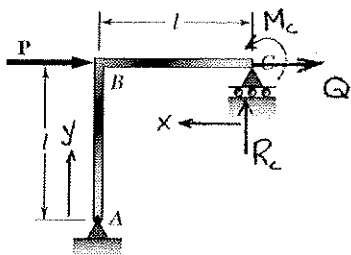
By Castigliano's theorem,

$$\delta = \frac{\partial U}{\partial P}$$

$$\delta = \frac{\pi P R^3}{2EI} \downarrow$$

Problem 11.107

11.107 Two rods AB and BC of the same flexural rigidity EI are welded together at B . For the loading shown, determine (a) the deflection of point C , (b) the slope of member BC at point C .



Add horizontal force Q and couple M_c at C .

$$\circlearrowleft \sum M_A = 0: R_c l + M_c - (P+Q)l = 0$$

$$R_c = P + Q + \frac{M_c}{l}$$

$$\rightarrow \sum F_x = 0: P + Q + R_{Ax} = 0 \quad R_{Ax} = P + Q \leftarrow$$

Member AB : $M = R_{Ax}y = (P+Q)y, \quad \frac{\partial M}{\partial Q} = y, \quad \frac{\partial M}{\partial M_c} = 0$

$$U_{AB} = \int_0^l \frac{M^2}{2EI} dy$$

Set $Q = 0$ and $M_c = 0$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dy = \frac{1}{EI} \int_0^l (Py)(y) dy = \frac{1}{3} \frac{Pl^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = 0$$

Member BC : $M = M_c + R_c x = M_c + (P + Q + \frac{M_c}{l})x$

$$\frac{\partial M}{\partial Q} = x, \quad \frac{\partial M}{\partial M_c} = 1 + \frac{x}{l}$$

$$U_{BC} = \int_0^l \frac{M^2}{2EI} dx$$

Set $Q = 0$ and $M_c = 0$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^l (Px)x dx = \frac{1}{3} \frac{Pl^3}{EI}$$

$$\begin{aligned} \frac{\partial U}{\partial M_c} &= \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = \frac{1}{EI} \int_0^l (Px) \left(1 + \frac{x}{l}\right) dx = \frac{P}{EI} \int_0^l \left(x + \frac{x^2}{l}\right) dx \\ &= \frac{P}{EI} \left(\frac{1}{2}l^2 + \frac{1}{3}l^2\right) = \frac{1}{6} \frac{Pl^3}{EI} \end{aligned}$$

(a) Deflection at C : $\delta_c = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q}$

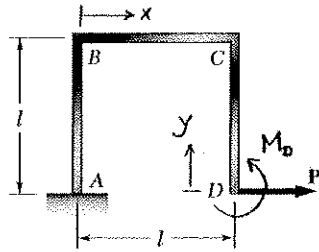
$$\frac{2Pl^3}{3EI} \rightarrow \blacktriangleleft$$

(b) Slope at C : $\theta_c = \frac{\partial U_{AB}}{\partial M_c} + \frac{\partial U_{BC}}{\partial M_c}$

$$\frac{Pl^2}{6EI} \rightarrow \blacktriangleleft$$

Problem 11.108

11.108 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the horizontal deflection of point D, (b) the slope at point D.



Add couple M_D at point D.

Reactions at A: $R_{Ay} = 0$, $R_{Ax} = P \leftarrow$, $M_A = M_D \curvearrowright$

Member AB: $M = M_A + R_{Ay} = M_D + Py$ $\frac{\partial M}{\partial P} = y$, $\frac{\partial M}{\partial M_D} = 1$
 $U_{AB} = \int_0^l \frac{M^2}{2EI} dy$ Set $M_D = 0$.

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI} \int_0^l (Py)(y) dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{AB}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dy = \frac{1}{EI} \int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

Member BC: $M = M_A + R_A l = M_D + Pl$ $\frac{\partial M}{\partial P} = l$, $\frac{\partial M}{\partial M_D} = 1$
 $U_{BC} = \int_0^l \frac{M^2}{2EI} dx$ Set $M_D = 0$.

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dx = \frac{1}{EI} \int_0^l (Pl)(l) dx = \frac{Pl^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dx = \frac{1}{EI} \int_0^l (Pl)(1) dx = \frac{Pl^2}{EI}$$

Member CD: $M = M_D + Py$ $\frac{\partial M}{\partial P} = y$, $\frac{\partial M}{\partial M_D} = 1$
 $U_{CD} = \int_0^l \frac{M^2}{2EI} dy$ Set $M_D = 0$.

$$\frac{\partial U_{CD}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI} \int_0^l (Py)(y) dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{CD}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dy = \frac{1}{EI} \int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

(a) Horizontal deflection of point D:

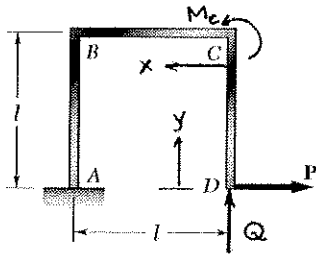
$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P} = \left(\frac{1}{3} + 1 + \frac{1}{3}\right) \frac{Pl^3}{EI} \quad \delta_P = \frac{5Pl^3}{3EI} \rightarrow \leftarrow$$

(b) Slope at point D:

$$\theta_D = \frac{\partial U_{AB}}{\partial M_D} + \frac{\partial U_{BC}}{\partial M_D} + \frac{\partial U_{CD}}{\partial M_D} = \left(\frac{1}{2} + 1 + \frac{1}{2}\right) \frac{Pl^2}{EI} \quad \theta_D = \frac{3Pl^2}{EI} \curvearrowright \leftarrow$$

Problem 11.109

11.109 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the vertical deflection of point D, (b) the slope of BC at point C.



Add vertical force Q at point D and couple M_c at point C.

$$\text{Reactions at A: } R_{Ax} = P \leftarrow, R_{Ay} = Q \downarrow \\ M_A = Ql + M_c \curvearrowright$$

Member AB: $M = M_A + R_{Ay}y = Ql + M_c + Py$, $\frac{\partial M}{\partial Q} = l$, $\frac{\partial M}{\partial M_c} = 1$

$$U_{AB} = \int_0^l \frac{M^2}{2EI} dy$$

Set $Q = 0$, and $M_c = 0$.

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dy = \frac{1}{EI} \int_0^l (Py)(l) dy = \frac{Pl^3}{2EI}$$

$$\frac{\partial U_{AB}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dy = \frac{1}{EI} \int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

Member BC: $M = M_c + Pl + Qx$, $\frac{\partial M}{\partial Q} = x$, $\frac{\partial M}{\partial M_c} = 1$

$$U_{BC} = \frac{1}{EI} \int_0^l \frac{M^2}{2EI} dx$$

Set $Q = 0$, and $M_c = 0$.

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^l (Pl)(x) dx = \frac{Pl^3}{2EI}$$

$$\frac{\partial U_{BC}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = \frac{1}{EI} \int_0^l (Pl)(1) dx = \frac{Pl^2}{EI}$$

Member CD: $M = Py$, $\frac{\partial M}{\partial Q} = 0$, $\frac{\partial M}{\partial M_c} = 0$

$$\frac{\partial U_{CD}}{\partial Q} = 0, \quad \frac{\partial U_{CD}}{\partial M_c} = 0$$

(a) Vertical deflection of point D:

$$\Delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} = \left(\frac{1}{2} + \frac{1}{2} + 0\right) \frac{Pl^3}{EI}$$

$$\Delta_Q = \frac{Pl^3}{EI} \uparrow$$

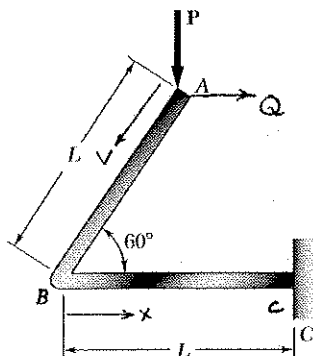
(b) Slope of BC at C:

$$\theta_c = \frac{\partial U_{AB}}{\partial M_c} + \frac{\partial U_{BC}}{\partial M_c} + \frac{\partial U_{CD}}{\partial M_c} = \left(\frac{1}{2} + 1 + 0\right) \frac{Pl^2}{EI}$$

$$\theta_c = \frac{3Pl^2}{2EI} \curvearrowright$$

Problem 11.110

11.110 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the vertical deflection of point A, (b) the horizontal deflection of point A.



Add horizontal force Q at point A.

Over AB: $M = \frac{1}{2} P v + \frac{\sqrt{3}}{2} Q v$

$$\frac{\partial M}{\partial P} = \frac{1}{2} v \quad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2} v$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv \quad \text{Set } Q = 0$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dv = \frac{1}{EI} \int_0^L \left(\frac{1}{2} P v \right) \left(\frac{1}{2} v \right) dv$$

$$= \frac{1}{12} \frac{P L^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dv = \frac{1}{EI} \int_0^L \left(\frac{1}{2} P v \right) \frac{\sqrt{3}}{2} dv$$

$$= \frac{\sqrt{3}}{12} \frac{P L^3}{EI}$$

Over BC: $M = -P \left(x - \frac{L}{2} \right) + \frac{\sqrt{3}}{2} Q L$, $\frac{\partial M}{\partial P} = -\left(x - \frac{L}{2} \right)$, $\frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2} L$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx \quad \text{Set } Q = 0.$$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dx = \frac{1}{EI} \int_0^L P \left(x - \frac{L}{2} \right)^2 dx = \frac{P}{3EI} \left(x - \frac{L}{2} \right)^3 \Big|_0^L = \frac{1}{12} \frac{P L^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L P \left(x - \frac{L}{2} \right) \left(\frac{\sqrt{3}}{2} \right) L dx = -\frac{\sqrt{3} P}{4EI} \left(x - \frac{L}{2} \right)^2 \Big|_0^L = 0$$

(a) Vertical deflection of point A.

$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P}$$

$$\delta_P = \frac{P L^3}{6EI} \downarrow$$

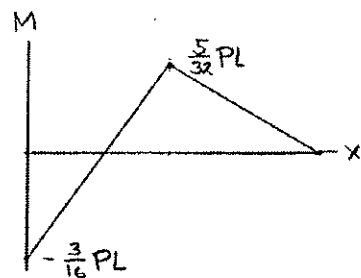
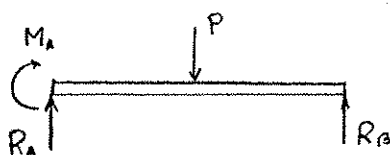
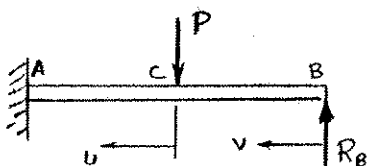
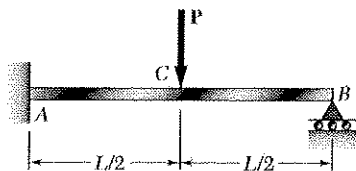
(b) Horizontal deflection of point A.

$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{\sqrt{3}}{12} \frac{P L^3}{EI}$$

$$\delta_Q = 0.1443 \frac{P L^3}{EI} \rightarrow$$

Problem 11.111

11.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Remove support B and add reaction R_B as a load:

$$U = U_{AC} + U_{CB} = \int_0^{L/2} \frac{M^2}{2EI} du + \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{\partial U_{AB}}{\partial R_B} + \frac{\partial U_{CB}}{\partial R_B} = 0$$

Over AC: $M = R_B(u + \frac{L}{2}) - Pu$, $\frac{\partial M}{\partial R_B} = (u + \frac{L}{2})$

$$\frac{\partial U_{AB}}{\partial R_B} = \frac{1}{EI} \int_0^{L/2} [R_B(u + \frac{L}{2}) - Pu](u + \frac{L}{2}) du$$

$$= \frac{R_B}{EI} \int_0^{L/2} (u + \frac{L}{2})^2 du - \frac{P}{EI} \int_0^{L/2} u(u + \frac{L}{2}) du$$

$$= \frac{R_B}{3EI} [L^3 - (\frac{L}{2})^3] - \frac{P}{EI} [\frac{1}{3}(\frac{L}{2})^3 + \frac{L}{2} \cdot \frac{L}{2}(\frac{L}{2})]$$

$$= \frac{7}{24} \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI}$$

Over CB: $M = R_B v$, $\frac{\partial M}{\partial R_B} = v$

$$\frac{\partial U_{CB}}{\partial R_B} = \frac{1}{EI} \int_0^{L/2} (R_B v) v dv = \frac{R_B}{3EI} (\frac{L}{2})^3 = \frac{1}{24} \frac{R_B L^3}{EI}$$

$$y_B = (\frac{7}{24} + \frac{1}{24}) \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI} = 0$$

$$R_B = \frac{5}{16} P \quad \uparrow$$

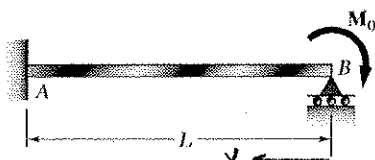
$$M_C = R_B \frac{L}{2} = \frac{5}{32} PL$$

$$M_A = R_B L - P \frac{L}{2} = (\frac{5}{16} - \frac{1}{2}) PL = -\frac{3}{16} PL$$

$$M_B = 0$$

Problem 11.112

1.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.

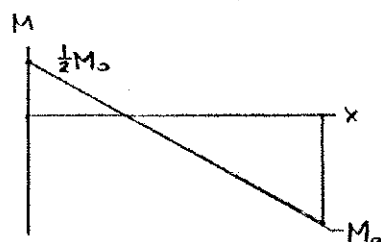
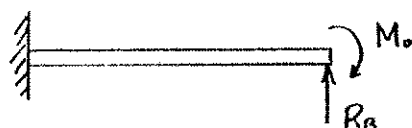


Remove support B and add reaction R_B as a load.

$$U = \int_0^L \frac{M^2}{2EI} dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_B} dv = 0$$

$$M = R_B v - M_0 \quad \frac{\partial M}{\partial R_B} = v$$



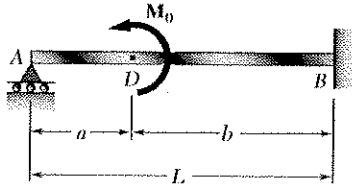
$$\begin{aligned} y_B &= \frac{1}{EI} \int_0^L (R_B v - M_0) v dv \\ &= \frac{R_B}{EI} \int_0^L v^2 dv - \frac{M_0}{EI} \int_0^L v dv \\ &= \frac{R_B L^3}{3EI} - \frac{M_0 L^2}{2EI} = 0 \end{aligned}$$

$$R_B = \frac{3}{2} \frac{M_0}{L} \uparrow$$

$$M_A = R_B - M_0 = \frac{3}{2} M_0 - M_0 = \frac{1}{2} M_0$$

Problem 11.113

11.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_A} dx = 0$$

Portion AD: ($0 < x < a$) $M = R_A x$ $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI} \int_0^a (R_A x)(x) dx = \frac{R_A a^3}{3EI}$$

Portion DB: ($a < x < L$) $M = R_A x - M_0$ $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI} \int_a^L (R_A x - M_0)(x) dx = \frac{1}{EI} \left\{ \frac{1}{3} R_A (L^3 - a^3) - \frac{1}{2} M_0 (L^2 - a^2) \right\}$$

$$S_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI} \left\{ R_A \left(\frac{1}{3} a^3 + \frac{1}{3} L^3 - \frac{1}{3} a^3 \right) - \frac{1}{2} M_0 (L^2 - a^2) \right\} = 0$$

$$R_A = \frac{3}{2} \frac{M_0 (L^2 - a^2)}{L^3}$$

$$R_A' = \frac{3 M_0 b (L + a)}{2 L^3} \quad \blacktriangleleft$$

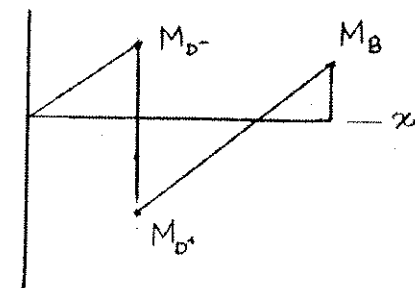
$$M_A = 0$$

$$M_{D-} = R_A a = \frac{3}{2} \frac{M_0 a b (L + a)}{L^3}$$

$$M_{D+} = M_{D-} - M_0 = \frac{3}{2} \frac{M_0 a b (L + a)}{L^3} - M_0$$

$$M_B = R_A L - M_0 = \frac{3}{2} \frac{M_0 b (L + a)}{L^2} - M_0$$

Bending moment diagram drawn to scale for $a = \frac{1}{3}L$.

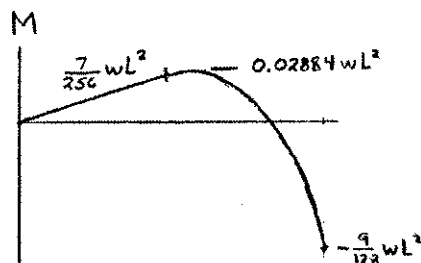
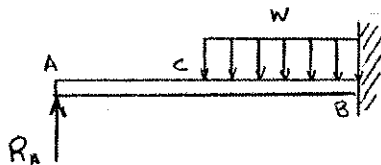
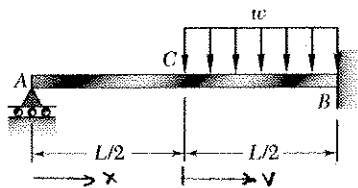


By singularity functions,

$$M = 3 M_0 b (L + a) x / 2 L^3 - M_0 \langle L - a \rangle^0 \quad \blacktriangleleft$$

Problem 11.114

11.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Remove support A and add reaction R_A as a load.

$$U = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx + \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dx + \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dv = 0$$

Portion AC: $(0 < x < \frac{L}{2})$ $M = R_A x$ $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AC}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} (R_A x)(x) dx = \frac{R_A L^3}{24 EI}$$

Portion CB: $(0 < v < \frac{L}{2})$

$$M = R_A (v + \frac{L}{2}) - \frac{1}{2} w v^2 \quad \frac{\partial M}{\partial R_A} = (v + \frac{L}{2})$$

$$\begin{aligned} \frac{\partial U_{CB}}{\partial R_A} &= \frac{1}{EI} \int_0^{\frac{L}{2}} [R_A (v + \frac{L}{2}) - \frac{1}{2} w v^2] (v + \frac{L}{2}) dv \\ &= \frac{1}{EI} \left\{ R_A \int_0^{\frac{L}{2}} (v + \frac{L}{2})^2 dv - \frac{1}{2} w \int_0^{\frac{L}{2}} (v^3 + \frac{L}{2} v^2) dv \right\} \\ &= \frac{R_A}{EI} \left[\frac{1}{3} L^3 - \frac{1}{3} (\frac{L}{2})^3 \right] - \frac{w}{2EI} \left[\frac{1}{4} (\frac{L}{2})^4 + \frac{L}{2} \frac{1}{3} (\frac{L}{2})^3 \right] \\ &= (\frac{1}{3} - \frac{1}{24}) \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} \end{aligned}$$

$$\delta_A = \frac{\partial U_{AC}}{\partial R_A} + \frac{\partial U_{CB}}{\partial R_A} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} = 0$$

$$R_A = \frac{7}{128} w L \quad \uparrow$$

Bending moments:

Over AC: $M = \frac{7}{128} w L x$

$$M_c = \frac{7}{256} w L^2 = 0.02734 w L^2$$

Over CB: $M = \frac{7}{128} w L (v + \frac{L}{2}) - \frac{1}{2} w v^2$

$$\begin{aligned} M_B &= \frac{7}{128} w L^2 - \frac{1}{2} w (\frac{L}{2})^2 = -\frac{9}{128} w L^2 \\ &= -0.07031 w L^2 \end{aligned}$$

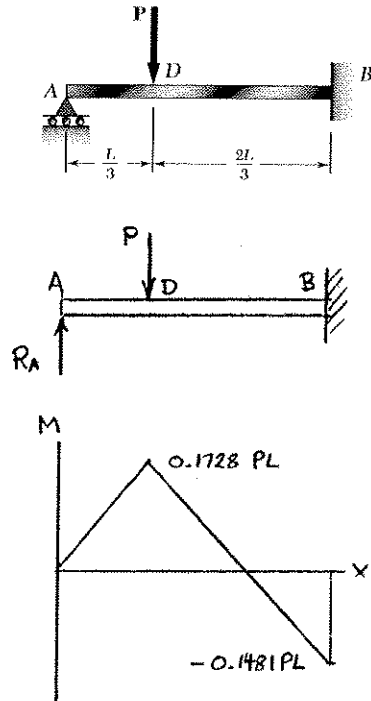
$$\frac{dM}{dv} = \frac{7}{128} w L - w v_m = 0 \quad v_m = \frac{7}{128} L$$

$$\begin{aligned} M_m &= \frac{7}{128} w L \left(\frac{7}{128} L + \frac{L}{2} \right) - \frac{1}{2} w \left(\frac{7}{128} L \right)^2 \\ &= \frac{945}{32768} w L^2 = 0.02884 w L^2 \end{aligned}$$

on $M = \frac{7}{128} w L x - w \langle x - L/2 \rangle^2 / 2$

Problem 11.115

11.115 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and load shown.



Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_A} dx$$

Portion AD: $(0 < x < \frac{L}{3}) \quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$

$$\begin{aligned} \frac{\partial U_{AD}}{\partial R_A} &= \frac{1}{EI} \int_0^{\frac{L}{3}} M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_0^{\frac{L}{3}} (R_A x)(x) dx \\ &= \frac{R_A}{3EI} \left(\frac{L}{3}\right)^3 = \frac{1}{81} \frac{R_A L^3}{EI} \end{aligned}$$

Portion DB: $(\frac{L}{3} < x < L) \quad M = R_A x - P(x - \frac{L}{3})$
 $\frac{\partial M}{\partial R_A} = x$

$$\begin{aligned} \frac{\partial U_{DB}}{\partial R_A} &= \frac{1}{EI} \int_{\frac{L}{3}}^L M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_{\frac{L}{3}}^L [R_A x - P(x - \frac{L}{3})] x dx \\ &= \frac{R_A}{EI} \int_{\frac{L}{3}}^L x^2 dx - \frac{P}{EI} \int_{\frac{L}{3}}^L (x^2 - \frac{L}{3} x) dx \\ &= \frac{R_A}{3EI} \left[L^3 - \left(\frac{L}{3}\right)^3 \right] - \frac{P}{EI} \left[\frac{1}{3} \left(L^3 - \left(\frac{L}{3}\right)^3 \right) - \frac{L}{6} \left(L^2 - \left(\frac{L}{3}\right)^2 \right) \right] \\ &= \left(\frac{1}{3} - \frac{1}{81} \right) \frac{R_A L^3}{EI} - \left(\frac{1}{3} - \frac{1}{81} - \frac{1}{6} + \frac{1}{54} \right) \frac{PL^3}{EI} \end{aligned}$$

$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \left(\frac{1}{81} + \frac{1}{3} - \frac{1}{81} \right) \frac{R_A L^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = 0$$

$$R_A = \frac{14}{27} P \uparrow$$

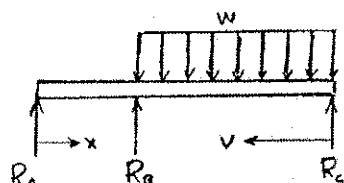
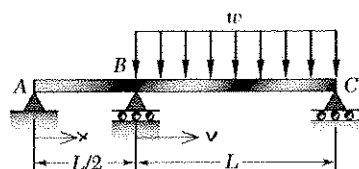
Bending moments: $M_D = R_A \left(\frac{L}{3}\right) = \frac{14}{81} PL = 0.1728 PL$

$$M_B = R_A L - P\left(\frac{2L}{3}\right) = -\frac{4}{27} PL = -0.1481 PL$$

By singularity functions, $M = \frac{14}{27} Px - P\langle x - L/3 \rangle'$

Problem 11.116

11.116 For the uniform beam and loading shown, determine the reaction at each support.



Remove support A and add reaction R_A as a load.

$$\sum M_B = 0 \quad -R_A \frac{L}{2} - \frac{1}{2} w L^2 + R_C L = 0$$

$$R_C = \frac{1}{2} R_A + \frac{1}{2} w L$$

$$U = U_{AB} + U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx + \int_0^L \frac{M^2}{2EI} dv$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = 0$$

Portion AB: $M = R_A x, \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AB}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_0^{\frac{L}{2}} (R_A x)(x) dx = \frac{R_A}{3EI} \left(\frac{L}{2} \right)^3 = \frac{1}{24} \frac{R_A L^3}{EI}$$

Portion BC: $M = R_C v - \frac{1}{2} w v^2 = \frac{1}{2} R_A v + \frac{1}{2} w L v - \frac{1}{2} w L v^2$

$$\frac{\partial M}{\partial R_A} = \frac{1}{2} v$$

$$\frac{\partial U_{BC}}{\partial R_A} = \frac{1}{EI} \int_0^L \left[\frac{1}{2} R_A v + \frac{1}{2} w (L v - v^2) \right] \left(\frac{1}{2} v \right) dv = \frac{1}{4EI} \int_0^L [R_A v^2 + w (L v^2 - v^3)] dv$$

$$= \frac{1}{4EI} \left[R_A \frac{L^3}{3} + w \left(\frac{L^4}{3} - \frac{L^4}{4} \right) \right] = \frac{R_A L^3}{12EI} + \frac{w L^4}{48EI}$$

$$S_A = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = \left(\frac{1}{24} + \frac{1}{12} \right) \frac{R_A L^3}{EI} + \frac{w L^4}{48EI} = 0$$

$$R_A = -\frac{1}{6} w L =$$

$$R_A = \frac{1}{6} w L \downarrow \quad \blacktriangleleft$$

$$R_C = \frac{1}{2} \left(-\frac{1}{6} w L \right) + \frac{1}{2} w L$$

$$R_C = \frac{5}{12} w L \uparrow \quad \blacktriangleleft$$

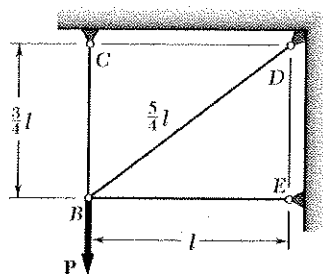
$$+\uparrow \sum F_y = 0: \quad R_A + R_B + R_C - w L = 0$$

$$-\frac{1}{6} w L + R_B + \frac{5}{12} w L - w L = 0$$

$$R_B = \frac{3}{4} w L \uparrow \quad \blacktriangleleft$$

Problem 11.117

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .

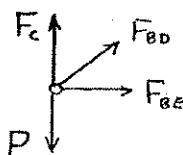


Detach member BC from support C . Add reaction F_c as a load.

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$S_c = \frac{\partial U}{\partial F_c} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_c} L$$

Joint B



$$+\uparrow \sum F_y = 0: F_c - P + \frac{3}{5} F_{BD} = 0 \quad F_{BD} = \frac{5}{3} P - \frac{5}{3} F_c$$

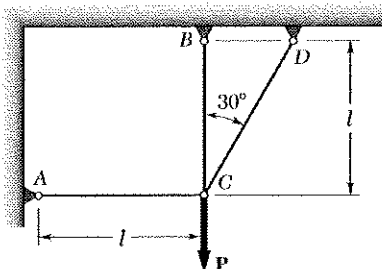
$$\pm \sum F_x = 0: F_{BE} + \frac{4}{5} F_{BD} = 0 \quad F_{BE} = -\frac{4}{3} P + \frac{4}{3} F_c$$

Member	F	$\partial F / \partial F_c$	L	$F(\partial F / \partial F_c) L$
BC	F_c	1	$\frac{3}{4} l$	$\frac{3}{4} F_c l$
BD	$\frac{5}{3} P - \frac{5}{3} F_c$	$-\frac{5}{3}$	$\frac{5}{4} l$	$-\frac{125}{36} P l + \frac{125}{36} F_c l$
BE	$-\frac{4}{3} P + \frac{4}{3} F_c$	$\frac{4}{3}$	l	$-\frac{16}{9} P l + \frac{16}{9} F_c l$
Σ				$-\frac{21}{4} P l + 6 F_c l$

$$S_c = \frac{1}{EA} \left(-\frac{21}{4} P l + 6 F_c l \right) = 0 \quad F_c = \frac{7}{8} P \quad F_{BC} = F_c \quad F_{BC} = \frac{7}{8} P$$

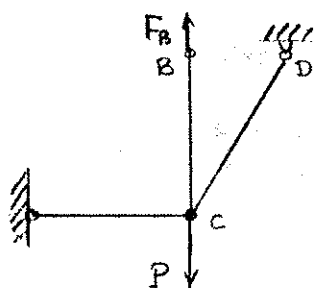
Problem 11.118

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .

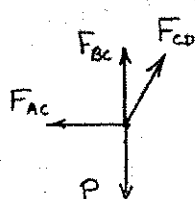


Cut member BC at end B and replace member force F_{BC} by load F_B acting on member BC at B .

$$S_B = \frac{\partial U}{\partial F_B} = \frac{\partial}{\partial F_B} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_B} L$$



Joint C



$$+\uparrow \Sigma F_y = 0: \frac{\sqrt{3}}{2} F_{CD} + F_{BC} - P = 0$$

$$F_{CD} = \frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$$

$$\pm \Sigma F_x = 0: F_{AC} - \frac{1}{2} F_{CD} = 0$$

$$F_{AC} = \frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$$

Member	F	$\partial F / \partial F_B$	L	$F(\partial F / \partial F_B) L$
AC	F_B	1	l	$F_B l$
BC	$\frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$	$-\frac{1}{\sqrt{3}}$	l	$-\frac{1}{3} P l + \frac{1}{3} F_B l$
CD	$\frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}} l$	$-\frac{8}{3} P l + \frac{8}{3} F_B l$
Σ				$-(\frac{1}{3} + \frac{8}{3}) P l + (\frac{4}{3} + \frac{8}{3}) F_B l$

$$S_B = -(\frac{1}{3} + \frac{8}{3}) \frac{P l}{EA} + (\frac{4}{3} + \frac{8}{3}) \frac{F_B l}{EA}$$

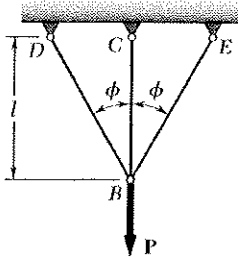
$$F_B = \frac{\frac{1}{3} + \frac{8}{3}}{\frac{4}{3} + \frac{8}{3}} P = \frac{8 + \sqrt{3}}{8 + 4\sqrt{3}} P = 0.652 P$$

$$F_{BC} = F_B$$

$$F_{BC} = 0.652 P \quad \blacktriangleleft$$

Problem 11.119

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .



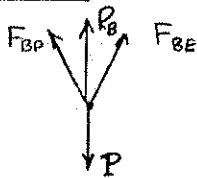
Detach member BC at support C .

Add reaction R_c as a load.

$$U = \sum \frac{F^2 L}{2EA} \quad y_c = \frac{\partial U}{\partial R_c} = \sum \frac{FL}{EA} \frac{\partial F}{\partial R_c} = 0$$

Joint C $F_{BC} = R_c$

Joint B



$$+\rightarrow \sum F_x = 0: F_{BE} \sin \phi - F_{BD} \sin \phi = 0 \quad F_{BE} = F_{BD}$$

$$+\uparrow \sum F_y = 0: F_{BD} \cos \phi + F_{BE} \cos \phi + R_B - P$$

$$F_{BD} = F_{BE} = \frac{P - R_B}{2 \cos \phi}$$

Member	F	$\partial F / \partial R_B$	L	$(FL/EA) \partial F / \partial R_B$
BD	$(P - R_B) / 2 \cos \phi$	$-1 / 2 \cos \phi$	$l / \cos \phi$	$(R_B - P) l / 4EA \cos^3 \phi$
BE	$(P - R_B) / 2 \cos \phi$	$-1 / 2 \cos \phi$	$l / \cos \phi$	$(R_B - P) l / 4EA \cos^3 \phi$
BC	R_B	1	l	$R_B l / EA$

$$y_B = -Pl / 2EA \cos^3 \phi + R_B l / 2EA \cos^3 \phi + R_B l / EA = 0$$

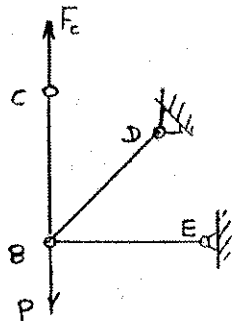
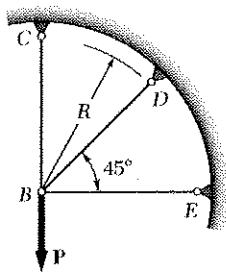
$$R_B = \frac{P}{1 + 2 \cos^3 \phi}$$

$$F_{BC} = R_B$$

$$F_{BC} = \frac{P}{1 + 2 \cos^3 \phi}$$

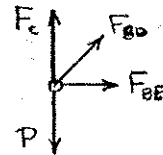
Problem 11.120

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .



Detach member BC from its support at point C . Add reaction F_c as a load.

Joint B



$$\uparrow \sum F_y = 0 :$$

$$\frac{\sqrt{2}}{2} F_{BD} + F_c - P = 0$$

$$F_{BD} = \sqrt{2} P - \sqrt{2} F_c$$

$$\rightarrow \sum F_x = 0 :$$

$$\frac{\sqrt{2}}{2} F_{BD} + F_{BE} = 0$$

$$F_{BE} = -P + F_c$$

$$S_c = \frac{R}{EA} (-3P + 4F_c) = 0$$

$$F_c = \frac{3}{4} P$$

$$F_{BC} = F_c \quad F_{BC} = \frac{3}{4} P \quad \blacktriangleleft$$

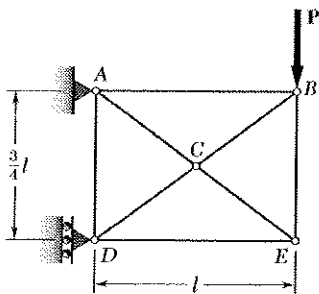
$$U = \sum \frac{F^2 R}{2EA} = \frac{R}{2EA} \sum F^2$$

$$S_c = \frac{\partial U}{\partial F_c} = \frac{R}{EA} \sum F \frac{\partial F}{\partial F_c} = 0$$

Member	F	$\partial F / \partial F_c$	$F(\partial F / \partial F_c)$
BC	F_c	1	F_c
BD	$\sqrt{2} P - \sqrt{2} F_c$	$-\sqrt{2}$	$-2P + 2F_c$
BE	$-P + F_c$	1	$-P + F_c$
Σ			$-3P + 4F_c$

Problem 11.121

11.121 and 11.122 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.



Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$

Joint B $\rightarrow \sum F_x = 0: -F_A - \frac{4}{5} F_{BD} = 0 \quad F_{BD} = -\frac{5}{4} F_A$

$\uparrow \sum F_y = 0: -P - F_{BE} - \frac{3}{5} F_{BD} = 0 \quad F_{BE} = -P + \frac{3}{4} F_A$

Joint E $\uparrow \sum F_y = 0: F_{BE} + \frac{3}{5} F_{AE} = 0 \quad F_{AE} = \frac{5}{3} P - \frac{5}{4} F_A$

$\rightarrow \sum F_x = 0: -\frac{4}{5} F_{AE} - F_{DE} = 0 \quad F_{DE} = -\frac{4}{3} P + F_A$

Joint D $\uparrow \sum F_y = 0: F_{AD} + \frac{3}{5} F_{BD} = 0$

$F_{AD} = \frac{3}{4} F_A$

Member	F	$\partial F / \partial F_A$	L	$F(\partial F / \partial F_A) L$
AB	F_A	1	l	$F_A l$
AD	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{48} P l + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$\frac{125}{64} F_A l$
BE	$-P + \frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$-\frac{9}{16} P l + \frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	l	$-\frac{4}{3} P l + F_A l$
Σ				$-\frac{9}{2} P l + \frac{27}{4} F_A l$

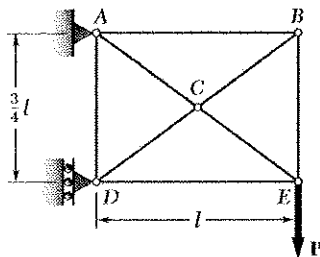
$$S_A = \frac{1}{EA} \left(-\frac{9}{2} P l + \frac{27}{4} F_A l \right) = 0 \quad F_A = \frac{2}{3} P$$

$$F_{AB} = F_A$$

$$F_{AB} = \frac{2}{3} P \quad \blacktriangleleft$$

Problem 11.122

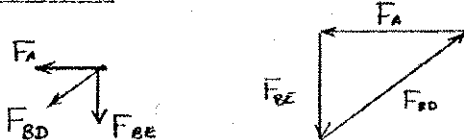
11.121 and 11.122 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.



Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

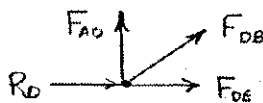
$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$

Joint B



$$F_{BD} = -\frac{5}{4} F_A \quad F_{BE} = \frac{3}{4} F_A$$

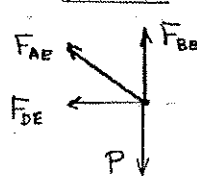
Joint D



$$+\uparrow \sum F_y = 0: F_{AD} + \frac{3}{5} F_{DB} = 0$$

$$F_{AD} = -\frac{3}{5} F_{DB} = -\frac{3}{4} F_A$$

Joint E



$$+\uparrow \sum F_y = 0:$$

$$F_{BE} - P + \frac{3}{5} F_{AE} = 0$$

$$F_{AE} = \frac{5}{3} P - \frac{5}{3} F_{BE} = \frac{5}{3} P - \frac{5}{4} F_A$$

$$+\rightarrow \sum F_x = 0: -\frac{4}{5} F_{AE} - F_{DE} = 0$$

$$F_{DE} = -\frac{4}{5} F_{AE} = -\frac{4}{3} P + F_A$$

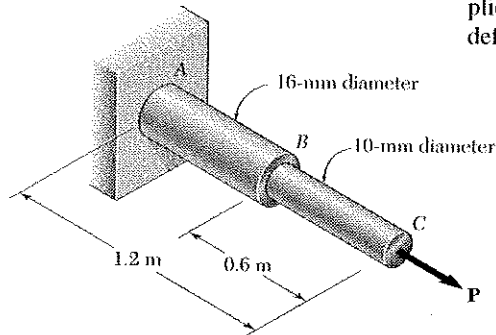
Member	F	$\partial F / \partial F_A$	L	$F(\partial F / \partial F_A) L$
AB	F_A	1	l	$F_A l$
AD	$-\frac{3}{4} F_A$	$-\frac{3}{4}$	$\frac{3}{4} l$	$-\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{48} P l + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{64} F_A l$
BE	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	l	$-\frac{4}{3} P l + F_A l$
Σ				$-\frac{63}{16} P l + \frac{27}{4} F_A l$

$$S_A = \frac{1}{EA} \left(-\frac{63}{16} P l + \frac{27}{4} F_A l \right) = 0 \quad F_A = \frac{7}{12} P$$

$$F_{AB} = F_A =$$

$$F_{AB} = \frac{7}{12} P \quad \leftarrow$$

Problem 11.123



11.123 Rod AB is made of a steel for which the yield strength is $\sigma_Y = 450$ MPa and the modulus of elasticity is $E = 200$ GPa. Knowing that a strain energy of $7 \text{ N} \cdot \text{m}$ must be acquired by the rod as the axial load P is applied, determine the factor of safety of the rod with respect to permanent deformation.

$$A_{AB} = \frac{\pi}{4} (16)^2 = 201.06 \text{ mm}^2$$

$$A_{BC} = \frac{\pi}{4} (10)^2 = 78.54 \text{ mm}^2$$

$$P_Y = \sigma_Y A_{\min} = (450)(78.54) = 35,343 \text{ N}$$

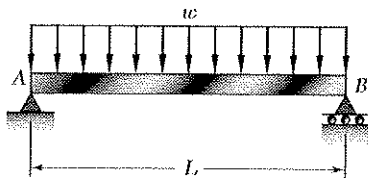
$$U_r = \sum \frac{P_r^2 \ell}{EA} = \frac{P_Y^2}{2E} \sum \frac{\ell}{A}$$

$$U_r = \frac{(35,343)^2}{2(200 \times 10^9)} \left[\frac{1.2 - 0.6}{201.07 \times 10^{-6}} + \frac{0.6}{78.54 \times 10^{-6}} \right] = 33.18 \text{ Nm}$$

$$F.S. = \frac{U_r}{U_{\text{design}}} = \frac{33.18}{7} = 4.74$$

Problem 11.124

11.124 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\sum M_B = 0: -R_A L + (wL)\left(\frac{L}{2}\right) = 0 \quad R_A = \frac{wL}{2}$$

$$\text{Bending moment: } M = R_A x - \frac{1}{2} wx^2 \\ = \frac{w}{2} (Lx - x^2)$$

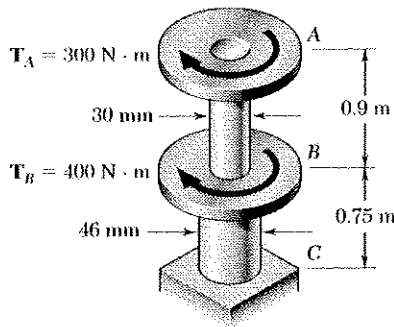
$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{w^2}{8EI} \int_0^L (Lx - x^2)^2 dx$$

$$= \frac{w^2}{8EI} \int_0^L (L^2 x^2 - 2Lx^3 + x^4) dx = \frac{w^2}{8EI} \left[\frac{L^2 x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right] \Big|_0^L$$

$$= \frac{w^2 L^5}{8EI} \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right]$$

$$U = \frac{w^2 L^5}{240EI}$$

Problem 11.125



11.125 In the assembly shown torques T_A and T_B are exerted on disks A and B , respectively. Knowing that both shafts are solid and made of aluminum ($G = 73$ GPa), determine the total strain energy acquired by the assembly.

Over portion AB:

$$T_{AB} = T_A = 300 \text{ N}\cdot\text{m}$$

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{30}{2}\right)^4 = 79.52 \times 10^3 \text{ mm}^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = 0.9 \text{ m}$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2 G J_{AB}} = \frac{(300)^2 (0.9)}{(2)(73 \times 10^9)(79.52 \times 10^{-9})} = 6.977 \text{ J}$$

Over portion BC: $T_{BC} = T_A + T_B = 300 + 400 = 700 \text{ N}\cdot\text{m}$, $L_{BC} = 0.75 \text{ m}$

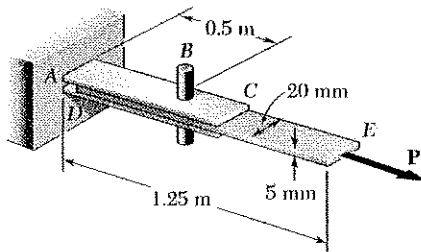
$$J_{BC} = \frac{\pi}{2} \left(\frac{46}{2}\right)^4 = 439.57 \times 10^3 \text{ mm}^4 = 439.57 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T_{BC}^2 L_{BC}}{2 G J_{BC}} = \frac{(700)^2 (0.75)}{(2)(73 \times 10^9)(439.57 \times 10^{-9})} = 5.726 \text{ J}$$

Total: $U = U_{AB} + U_{BC} = 6.977 + 5.726$

$$U = 12.70 \text{ J} \quad \blacktriangleleft$$

Problem 11.126



11.126 A single 6-mm-diameter steel pin B is used to connect the steel strip DE to two aluminum strips, each of 20-mm width and 5-mm thickness. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that for the pin at B the allowable shearing stress is $\tau_{all} = 85$ MPa, determine, for the loading shown, the maximum strain energy that can be acquired by the assembled strips.

$$A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (6)^2 = 28.274 \text{ mm}^2 = 28.274 \times 10^{-6} \text{ m}^2$$

$$\tau_{all} = 85 \times 10^6 \text{ Pa}$$

$$\text{Double shear: } P = 2A\tau = (2)(28.274 \times 10^{-6})(85 \times 10^6) = 4.8066 \times 10^3 \text{ N}$$

For strips AB, DB, BE , $A = (20)(5) = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$

$$F_{AB} = F_{DB} = \frac{1}{2} P = 2.4033 \times 10^3 \text{ N}$$

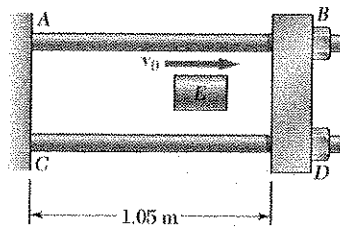
$$U_{AB} = U_{DB} = \frac{F_{AB}^2 L_{AB}}{2 E_A A_{AB}} = \frac{(2.4033 \times 10^3)^2 (0.5)}{(2)(70 \times 10^9)(100 \times 10^{-6})} = 0.2063 \text{ J}$$

$$U_{BE} = \frac{F_{BE}^2 L_{BE}}{2 E_S A_{BE}} = \frac{(4.8066 \times 10^3)^2 (1.25 - 0.5)}{(2)(200 \times 10^9)(100 \times 10^{-6})} = 0.4332 \text{ J}$$

Total: $U = U_{AB} + U_{DB} + U_{BE} = 0.846 \text{ J}$

$$U = 0.846 \text{ J} \quad \blacktriangleleft$$

Problem 11.127



11.127 The cylindrical block E has a speed $v_0 = 4.8$ m/s when it strikes squarely the yoke BD that is attached to the 22-mm-diameter rods AB and CD . Knowing that the rods are made of a steel for which $\sigma_Y = 350$ MPa and $E = 200$ GPa, determine the weight of block E for which the factor of safety is 5 with respect to permanent deformation of the rods.

At the onset of yielding the force in each rod is

$$F = \sigma_Y A$$

Corresponding strain energy.

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_Y^2 A^2 L}{2EA} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_m = U_{AB} + U_{CD} = \frac{\sigma_Y^2 AL}{E}$$

$$U_m = \left(\frac{1}{2} m v_0^2\right) (\text{F.S.}) = \left(\frac{1}{2} \frac{W}{g} v_0^2\right) (\text{F.S.})$$

$$\text{Solving for } W; \quad W = \frac{2g U_m}{v_0^2 (\text{F.S.})} = \frac{2g \sigma_Y^2 AL}{v_0^2 (\text{F.S.}) E}$$

$$\text{Data: } g = 9.81 \text{ m/s}^2, \quad \sigma_Y = 350 \text{ MPa},$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (22)^2 = 380 \text{ mm}^2$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$L = 1.05 \text{ m}$$

$$\text{F.S.} = 5$$

$$v_0 = 4.8 \text{ m/s}$$

$$W = \frac{(2)(9.81)(350 \times 10^6)^2 (380 \times 10^{-6})(1.05)}{(4.8)^2 (5)(200 \times 10^9)} = 41.6 \text{ N}$$

Problem 11.128

11.128 A block of weight W is placed in contact with a beam at some given point D and released. Show that the resulting maximum deflection at point D is twice as large as the deflection due to a static load W applied at D .

Consider dropping the weight from a height h above the beam. The work done by the weight is

$$\text{Work} = W(h + y_m)$$

$$\text{Strain energy: } U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2$$

where k is the spring constant of the beam for loading at point D .

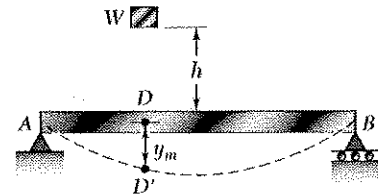
$$\text{Equating work and energy, } W(h + y_m) = \frac{1}{2} k y_m^2$$

$$\text{Setting } h = 0, \quad W y_m = \frac{1}{2} k y_m^2, \quad y_m = \frac{2W}{k}$$

The static deflection at point D due to weight applied at D is

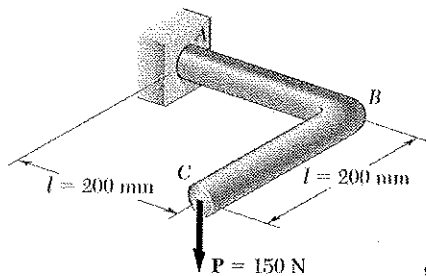
$$S_{st} = \frac{W}{k}$$

$$y_m = 2 S_{st} \quad \blacktriangleleft$$



Problem 11.129

11.129 The 12-mm-diameter steel rod ABC has been bent into the shape shown. Knowing that $E = 200$ GPa and $G = 77.2$ GPa, determine the deflection of end C caused by the 150-N force.

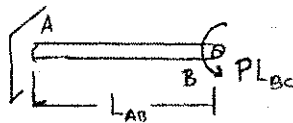
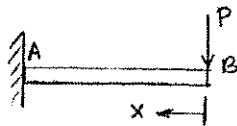


$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} \left(\frac{12}{2} \right)^4 = 2.0358 \times 10^3 \text{ mm}^4 = 2.0358 \times 10^{-9} \text{ m}^4$$

$$I = \frac{1}{2} J = 1.0179 \times 10^{-9} \text{ m}^4$$

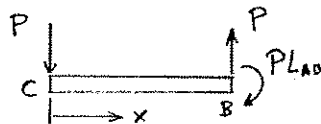
Portion AB: bending $M = -Px$

$$U_{AB,b} = \int_0^{L_{AB}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L_{AB}} x^2 dx = \frac{P^2 L_{AB}^3}{6EI} = \frac{(150)^2 (200 \times 10^{-3})^3}{(6)(200 \times 10^9)(1.0179 \times 10^{-9})} = 0.14736 \text{ J}$$



torsion $T = PL_{BC}$

$$U_{AB,t} = \frac{T^2 L_{AB}}{2GJ} = \frac{P^2 L_{BC}^2 L_{AB}}{2GJ} = \frac{(150)^2 (200 \times 10^{-3})^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(2.0358 \times 10^{-9})} = 0.57265 \text{ J}$$



Portion BC: $M = -Px$

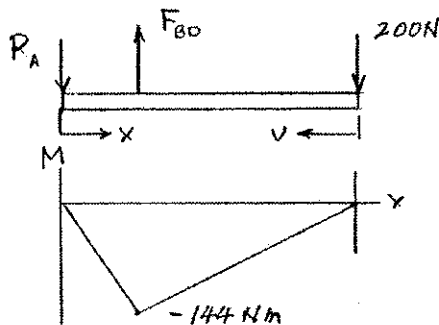
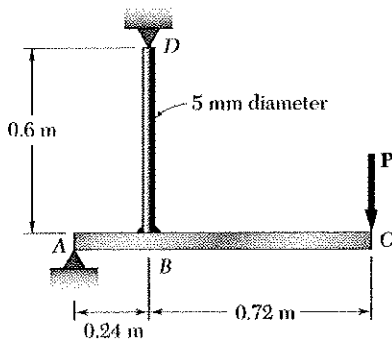
$$U_{BC} = \int_0^{L_{BC}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L_{BC}} x^2 dx = \frac{P^2 L_{BC}^3}{6EI} = \frac{(150)^2 (200 \times 10^{-3})^3}{(6)(200 \times 10^9)(1.0179 \times 10^{-9})} = 0.14736 \text{ J}$$

Total: $U = U_{AB,b} + U_{AB,t} + U_{BC} = 0.86737 \text{ J}$

Work-energy: $\frac{1}{2} PS = U$ $S = \frac{2U}{P} = \frac{(2)(0.86737)}{150} = 11.57 \times 10^{-3} \text{ m} = 11.57 \text{ mm} \downarrow$

Problem 11.130

11.130 The steel bar ABC has a square cross section of side 18 mm and is subjected to a 200-N load P . Using $E = 200$ GPa, determine the deflection of point C .



Assume member BD is a two-force member.

$$\sum M_A = 0: 0.24 F_{BD} - (0.96)(200) = 0 \quad F_{BD} = 800 \text{ N}$$

$$A_{BD} = \frac{\pi}{4} (0.005)^2 = 19.635 \times 10^{-6} \text{ m}^2$$

$$U_{BD} = \frac{F_{BD}^2 L_{BD}}{2EA} = \frac{(800)^2 (0.6)}{(2)(200 \times 10^9)(19.635 \times 10^{-6})} = 0.04889 \text{ J}$$

Member ABC :

$$I = \frac{1}{12} (0.018)(0.018)^3 = 8.748 \times 10^{-9} \text{ m}^4$$

Portion AB : $M = -144 \frac{x}{0.24} = -600x$

$$U_{AB} = \int_0^{0.24} \frac{M^2}{2EI} dx = \frac{600^2}{2EI} \int_0^{0.24} x^2 dx = \frac{(600)^2 (0.24^3)}{(2)(200 \times 10^9)(8.748 \times 10^{-9})(3)} = 0.47407 \text{ J}$$

Portion BC : $M = -200v$

$$U_{BC} = \int_0^{0.72} \frac{M^2}{2EI} dv = \frac{200^2}{2EI} \int_0^{0.72} v^2 dv = \frac{(200)^2 (0.72^3)}{(2)(200 \times 10^9)(8.748 \times 10^{-9})(3)} = 1.42222 \text{ J}$$

Total: $U = U_{BD} + U_{AB} + U_{BC} = 1.94516 \text{ J}$

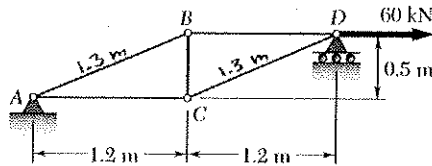
$$\frac{1}{2} P \delta_c = U$$

$$\delta_c = \frac{2U}{P} = \frac{(2)(1.94516)}{200} = 0.0194516$$

$$\delta_c = 19.5 \text{ mm} \quad \downarrow \quad \blacktriangleleft$$

Problem 11.131

11.131 Each member of the truss shown is made of steel; the cross-sectional area of member BC is 800 mm² and for all other members the cross-sectional area is 400 mm². Using $E = 200$ GPa, determine the deflection of point D caused by the 60-kN load.



Entire truss $\sum M_A = 0 :$

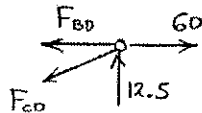
$$2.4 R_D - (0.5)(60) = 0 \quad R_D = 12.5 \text{ kN}$$

Joint D

$$+\uparrow \sum F_y = 0 :$$

$$12.5 - \frac{0.5}{1.3} F_{CD} = 0$$

$$F_{CD} = 32.5 \text{ kN}$$



$$+\rightarrow \sum F_x = 0 :$$

$$60 - F_{BD} - \frac{1.2}{1.3} F_{CD} = 0$$

$$F_{BD} = 30 \text{ kN}$$

Joint B

$$+\rightarrow \sum F_x = 0 :$$

$$30 - \frac{1.2}{1.3} F_{AB} = 0$$

$$F_{AB} = 32.5 \text{ kN}$$



$$+\uparrow \sum F_y = 0 :$$

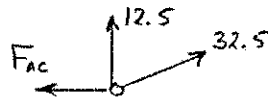
$$-\frac{0.5}{1.3} F_{AB} + F_{BC} = 0$$

$$F_{BC} = 12.5 \text{ kN}$$

Joint C

$$+\rightarrow \sum F_x = 0 : -F_{AC} + \frac{1.2}{1.3}(32.5) = 0$$

$$F_{AC} = 30 \text{ kN}$$



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10 ⁶ m ²)	F ² L/A (N ² /m)
CD	32.5	1.3	400	3.4328 × 10 ¹²
BD	30	1.2	400	2.7 × 10 ¹²
AB	32.5	1.3	400	3.4328 × 10 ¹²
BC	12.5	0.5	800	0.0977 × 10 ¹²
AC	30	1.2	400	2.7 × 10 ¹²
Σ				12.3633 × 10 ¹²

$$E = 200 \times 10^9 \text{ Pa}$$

$$U = \frac{12.3633 \times 10^{12}}{(2)(200 \times 10^9)} = 30.908 \text{ J}$$

Work-energy:

$$\frac{1}{2} P \Delta = U$$

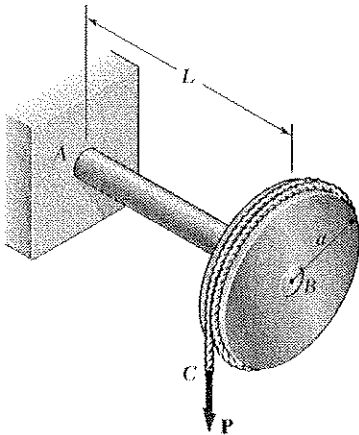
$$\Delta = \frac{2U}{P} = \frac{(2)(30.908)}{60 \times 10^3} = 1.030 \times 10^{-3} \text{ m}$$

$$\Delta = 1.030 \text{ mm} \rightarrow$$

Problem 11.132

11.132 A disk of radius a has been welded to end B of the solid steel shaft AB . A cable is then wrapped around the disk and a vertical force P is applied to end C of the cable. Knowing that the radius of the shaft is r and neglecting the deformations of the disk and of the table, show that the deflection of point C caused by the application of P is

$$\delta_c = \frac{PL^3}{3EI} \left(1 + 1.5 \frac{Er^2}{GL^2} \right)$$



Torsion: $T = Pa$

$$U_t = \frac{T^2 L}{2GJ} = \frac{P^2 a^2 L}{2GJ}$$

Bending: $M = Pv$

$$U_b = \int_0^L \frac{M^2 dv}{2EI} = \int_0^L \frac{P^2 v^2 dv}{2EI}$$

$$= \frac{P^2 L^3}{6EI}$$



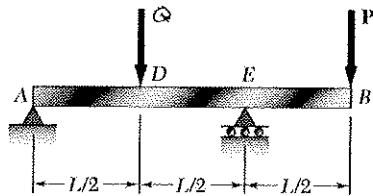
Total: $U = \frac{P^2 a^2 L}{2GJ} + \frac{P^2 L^3}{6EI} = \frac{1}{2} P \delta_c$

$$\delta_c = \frac{Pa^2 L}{GJ} + \frac{PL^3}{3EI} = \frac{PL^3}{3EI} \left(1 + \frac{3EIa^2}{GJL^2} \right)$$

Since $J = 2I$, $\delta_c = \frac{PL^3}{3EI} \left(1 + 1.5 \frac{Er^2}{GL^2} \right)$

Problem 11.133

11.133 For the prismatic beam shown, determine the deflection of point D.



Change force at D from P to Q.

Reactions: $R_A = \frac{1}{2}(Q-P) \uparrow$, $R_E = \frac{1}{2}(3P+Q) \uparrow$

Strain energy: $U = U_{AD} + U_{DE} + U_{EB}$

Deflection of point D (Formula).

$$\delta_D = \frac{\partial U}{\partial Q} = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q}$$

Portion AD: $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx$

$$M = R_A x = \frac{1}{2}(Q-P)x$$

$$\frac{\partial M}{\partial Q} = \frac{1}{2}x \quad \text{With } Q=P, M=0$$

$$\frac{\partial U_{AD}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dx = 0$$

Portion DE: $U_{DE} = \int_{L/2}^L \frac{M^2}{2EI} dx$

$$M = R_A x - Q(x - \frac{L}{2}) = \frac{1}{2}(Q-P)x - Q(x - \frac{L}{2}) = \frac{1}{2}Q(L-x) - \frac{1}{2}Px$$

$$\frac{\partial M}{\partial Q} = \frac{1}{2}(L-x)$$

$$\text{With } Q=P, M = -\frac{1}{2}P(2x-L)$$

$$\frac{\partial U_{DE}}{\partial Q} = \int_{L/2}^L \frac{M}{EI} \frac{\partial M}{\partial Q} dx = -\frac{P}{4EI} \int_{L/2}^L (2x-L)(L-x) dx$$

$$= -\frac{P}{4EI} \int_{L/2}^L (3Lx - 2x^2 - L^2) dx$$

$$= -\frac{P}{4EI} \left(\frac{3}{2}Lx^2 - \frac{2}{3}x^3 - L^2x \right) \Big|_{L/2}^L$$

$$= -\frac{PL^3}{4EI} \left[\frac{3}{2} - \frac{2}{3} - 1 - \left(\frac{3}{2} \left(\frac{1}{4} \right) - \left(\frac{2}{3} \right) \left(\frac{1}{8} \right) + \frac{1}{2} \right) \right] = -\frac{PL^3}{96EI}$$

Portion EB: $U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} dv$

$$M = -Pv$$

$$\frac{\partial M}{\partial Q} = 0$$

$$\frac{\partial U_{EB}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dv = 0$$

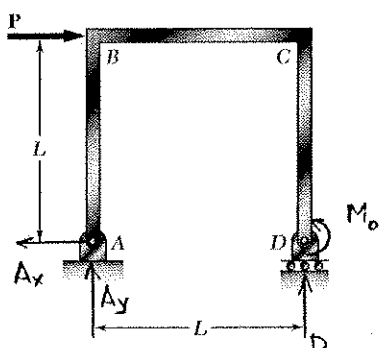
Deflection of Point D.

$$\delta_D = 0 - \frac{PL^3}{96EI} + 0$$

$$\delta_D = \frac{PL^3}{96EI} \uparrow$$

Problem 11.134

11.134 Three rods, each of the same flexural rigidity EI , are welded to form the frame $ABCD$. For the loading shown, determine the angle formed by the frame at point D .



Add couple M_0 at point D .

$$\text{Statics} \quad +\circlearrowleft M_A = 0: M_0 + DL - PL = 0$$

$$D = P - \frac{M_0}{L} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: -A_x + P = 0 \quad A_x = P \leftarrow$$

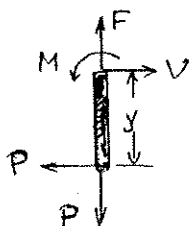
$$+\uparrow \Sigma F_y = 0: A_y + D = 0 \quad A_y = P - \frac{M_0}{L} \downarrow$$

$$U = U_{AB} + U_{BC} + U_{CD}$$

$$\text{By Castigliano's theorem, } \theta_D = \frac{\partial U}{\partial M_0}$$

$$\theta_D = \frac{\partial U_{AB}}{\partial M_0} + \frac{\partial U_{BC}}{\partial M_0} + \frac{\partial U_{CD}}{\partial M_0}$$

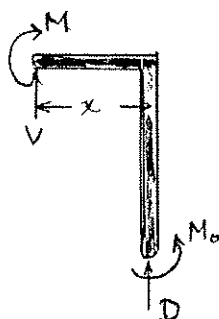
Member AB:



$$M = Py \quad \frac{\partial M}{\partial M_0} = 0 \quad U_{AB} = \int_0^L \frac{M^2}{2EI} dy$$

$$\frac{\partial U_{AB}}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dy = 0$$

Member BC:



$$M = M_0 + Dx = M_0 + Px - \frac{Mx}{L}$$

$$\text{Set } M_0 = 0 \quad M = Px$$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx$$

$$\frac{\partial U_{BC}}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{P}{EI} \int_0^L x \left(1 - \frac{x}{L}\right) dx = \frac{PL^2}{6EI}$$

$$\text{Member CD:} \quad M = M_0 \quad \frac{\partial M}{\partial M_0} = 1$$

$$\text{Set } M_0 = 0 \quad M = 0$$

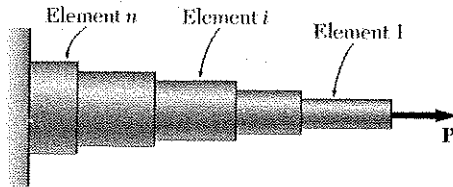
$$U = \int_0^L \frac{M^2}{2EI} dy$$

$$\frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = 0$$

$$\theta_D = 0 + \frac{PL^2}{6EI} + 0$$

$$\theta_D = \frac{PL^2}{6EI} \curvearrowright \blacktriangle$$

PROBLEM 11.C1



11.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to a load P applied at its free end. The length of element i is denoted by L_i and its diameter by d_i . (a) Denoting by E the modulus of elasticity of the material used in the rod, write a computer program that can be used to determine the strain energy acquired by the rod and the deformation measured at the free end. (b) Use this program to determine the strain energy and deformation of the rods of Probs. 11.9 and 11.10.

SOLUTION

ENTER: P AND E

FOR EACH ELEMENT

ENTER A_i AND D_i

COMPUTE: NORMAL STRESS: $\sigma_i = \frac{P}{A_i}$

STRAIN ENERGY: $U_i = \frac{P^2 L_i}{2 A_i E}$

STRAIN ENERGY DENSITY: $u = \frac{\sigma_i^2}{2E}$

TOTAL STRAIN ENERGY

UPDATE THROUGH n ELEMENTS

$$U = U + U_i$$

TOTAL DEFORMATION

$$\frac{1}{2} P \Delta = U \quad : \quad \Delta = \frac{2U}{P}$$

PROGRAM OUTPUT

Problem 11.9

Axial load = 35 kN

Modulus of elasticity = 200 GPa

Element	Length m	delta L mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m ³
1	0.6	0.52	174.1	9.093	75.777
2	1.5	0.836	111.4	14.622	31.025

Total Strain Energy = 23.715 J

Total Deformation = 1.356 mm

Problem 11.10

Axial load = 25.000 kN

Modulus of elasticity = 200 GPa

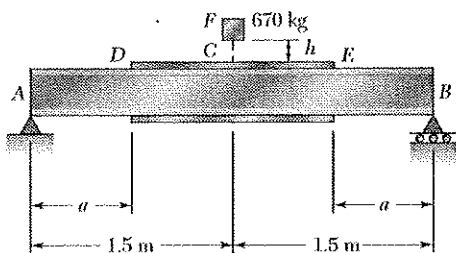
Element	Length m	delta L mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m ³
1	0.80	0.497	124.34	6.22	38.65
2	1.20	0.477	79.58	5.97	15.83

Total Strain Energy = 12.1853 J

Total Deformation = 0.9748 mm

PROBLEM 11.C2

11.C2 Two 18×150 -mm cover plates are welded to a $W200 \times 26.6$ rolled-steel beam as shown. The 670-kg block is to be dropped from a height $h = 50$ mm onto the beam. (a) Write a computer program to calculate the maximum normal stress on transverse sections just to the left of D and at the center of the beam for values of a from 0 to 1.5 m using 125-mm increments. (b) From the values considered in part a, select the distance a for which the maximum normal stress is as small as possible. Use $E = 200$ GPa.



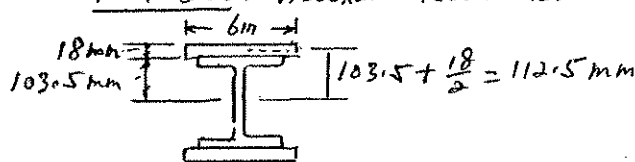
SOLUTION

COMPUTE AND ENTER MOMENTS OF INERTIA AND SECTION MODULI

FOR AD AND EB: $W200 \times 26.6$.

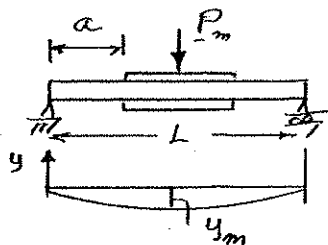
$$I_1 = 25.8 \times 10^6 \text{ mm}^4 \quad S_1 = 249 \times 10^3 \text{ mm}^3 \quad \triangleleft$$

FOR DCE: $W200 \times 26.6$ PLUS COVER PLATES



$$I_2 = 25.8 \times 10^6 + 2(18 \times 150)(112.5)^2 = 94.144 \times 10^6 \text{ mm}^4 \quad \triangleleft$$

$$S_2 = \frac{I_2}{(103.5 + 18)} = \frac{94.144 \times 10^6}{121.5} = 774.85 \times 10^3 \text{ mm}^3 \quad \triangleleft$$



$$y_m = P_m \alpha$$

WHERE $\alpha = \text{INFLUENCE COEFFICIENT}$
SEE NEXT PAGE FOR DETERMINATION OF α

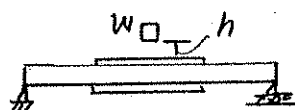
$P_m = \text{EQUIVALENT STATIC LOAD}$

$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

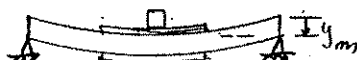
WORK DONE BY W IS $W(h + y_m)$

$$\frac{1}{2} \frac{y_m^2}{\alpha} = Wh + Wy_m$$

$$\text{OR: } y_m^2 - 2W\alpha y_m - 2Wh\alpha \quad \textcircled{A}$$



POSITION 1



POSITION 2

PROGRAM SOLUTION OF $\textcircled{A}$ FOR y_m

ENTER $L = 3$ m, $h = 50$ mm, $W = 670 \times 9.81 = 6573$ N, $E = 200$ GPa.

FOR $a = 0$ TO 1.5 m, STEP 125 mm:

SOLVE $\textcircled{A}$ FOR y_m , $P_m = y_m / \alpha$, $y_{ST} = W\alpha$

$$\sigma_D = \sigma_1 = \frac{1}{2} P_m a / S_1 \quad ; \quad \sigma_C = \sigma_2 = \frac{1}{4} P_m L / S_2$$

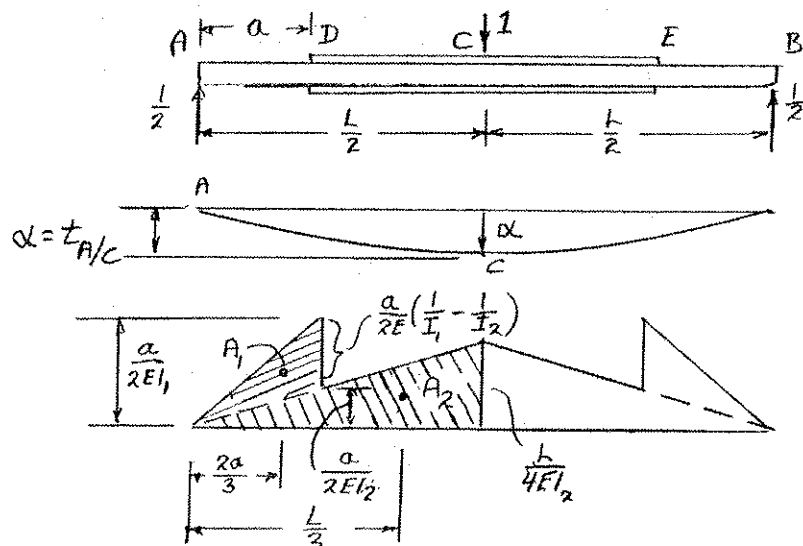
PRINT: a , y_{ST} , y_m , P_m , σ_1 , σ_2 , AND $(\sigma_1 - \sigma_2)$

REPEAT WITH SMALLER INTERVALS TO FIND a FOR $(\sigma_1 - \sigma_2) = 0$
THIS IS THE DISTANCE a FOR τ_{max} AS SMALL AS POSSIBLE

CONTINUED

PROBLEM 11.C2 - CONTINUED

DETERMINATION OF Δ : Δ IS DEFLECTION AT C FOR A UNIT LOAD AT C.



$$\Delta = z_{A/C} = A_1 \left(\frac{2a}{3} \right) + A_2 \left(\frac{L}{3} \right) = \left[\frac{1}{2} \frac{a}{2EI_1} \left(\frac{1}{I_1} - \frac{1}{I_2} \right) \frac{a}{2} \right] \frac{2a}{3} + \left[\frac{1}{2} \cdot \frac{L}{4EI_2} \cdot \frac{L}{2} \right] \frac{L}{3}$$

$$\Delta = \left[\left(\frac{1}{I_1} - \frac{1}{I_2} \right) a^3 + \frac{1}{8I_2} L^3 \right] / 6E$$

PROGRAM OUTPUT

Beam = W 200x26.6 with two 18 by 158 mm cover plates
 h = 50 mm W = 6573 N L = 3 m.

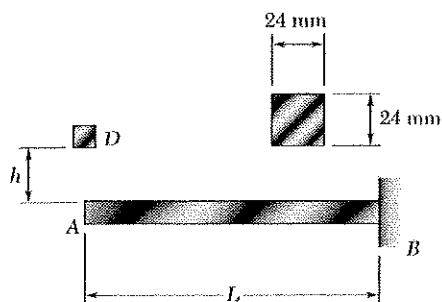
a m	y _{stat} mm	y _{max} mm	P _{max} kN	σ ₁ MPa	σ ₂ MPa	σ ₁ - σ ₂ MPa
0	0.1943	4.605	1510	0	150.22	-150.22
0.5	0.2148	4.855	1525	156.1	143.15	12.95
0.75	0.264	5.4075	1383	212.31	129.78	82.53
1.0	0.3595	6.365	1195	244.65	112.14	132.51
1.5	0.752	9.4575	848.8	260.61	79.66	180.95

Use smaller increments to seek the smallest maximum normal stress

0.4582	0.21	4.7975	145.43	144.60	144.66	-0.06
0.4585	0.21	4.8	145.42	144.67	144.65	0.02
0.4588	0.2103	4.8	145.41	144.74	144.64	0.1

Max stress small as possible for a = 0.4585 m
 Smallest max stress = 144.6 MPa

PROBLEM 11.C3



11.C3 The 16-kg block *D* is dropped from a height *h* onto the free end of the steel bar *AB*. For the steel used $\sigma_{all} = 120 \text{ MPa}$ and $E = 200 \text{ GPa}$. (a) Write a computer program to calculate the maximum allowable height *h* for values of the length *L* from 100 mm to 1.2 m, using 100-mm increments. (b) From the values considered in part a, select the length corresponding to the largest allowable height.

SOLUTION

ENTER $\sigma_{all} = 120 \text{ MPa}$, $E = 200 \text{ GPa}$, $d = 0.024 \text{ m}$
 $m = 16.8 \text{ kg}$, $g = 9.81 \text{ m/s}^2$

$$I = d^4/12 \quad S = \frac{I}{c} = \frac{I}{d/2} = \frac{d^3}{6}$$

FOR $L = 100 \text{ mm}$ TO 1200 mm STEP 100 mm

$$L = L/1000$$

$$y_{st} = mgL^3/3EI$$

$$M_{max} = J_{all} S$$

$$P_{max} = M_{max}/L$$

$$y_{max} = P_{max} L^3/3EI$$

FROM PROB. 11.69, page 705

$$y_m = y_{st} \left[1 + \sqrt{1 + \frac{2h}{y_{st}}} \right] \xrightarrow{\text{SOLVE FOR}} h = \left[\left(\frac{y_{max}}{y_{st}} - 1 \right)^2 - 1 \right] \frac{y_{st}}{2}$$

PRINT: L , y_{st} , y_{max} , P_{max} , M_{max} , h

RETURN

PROGRAM OUTPUT

Problem 11.C3

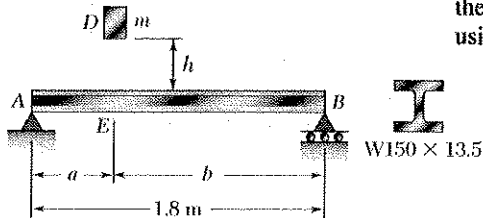
$m = 16.0 \text{ kg}$ $d = 24 \text{ mm}$ $\sigma = 120 \text{ MPa}$ $E = 200 \text{ GPa}$

L mm	y _{stat} mm	y _{max} mm	P _{max} N	M _{max} N·m	h mm
100	0.00946	0.167	2764.8	276.48	1.301
200	0.07569	0.667	1382.4	276.48	2.269
300	0.25547	1.500	921.6	276.48	2.904
400	0.60556	2.667	691.2	276.48	3.205
500	1.18273	4.167	553.0	276.48	3.173
600	2.04375	6.000	460.8	276.48	2.807
700	3.24540	8.167	395.0	276.48	2.109
800	4.84445	10.667	345.6	276.48	1.076
900	6.89766	13.500	307.2	276.48	-0.289
1000	9.46181	16.667	276.5	276.48	-1.988
1100	12.59367	20.167	251.3	276.48	-4.020
1200	16.35000	24.000	230.4	276.48	-6.385

Use smaller increments to seek the largest height *h*

435	0.77883	3.154	635.6	276.48	3.2316
440	0.80599	3.227	628.4	276.48	3.2320
445	0.83378	3.300	621.3	276.48	3.2317

PROBLEM 11.C4



11.C4 The block D of mass $m = 8 \text{ kg}$ is dropped from a height $h = 750 \text{ mm}$ onto the rolled-steel beam AB . Knowing that $E = 200 \text{ GPa}$, write a computer program to calculate the maximum deflection of point E and the maximum normal stress in the beam for values of a from 100 to 900 mm, using 100-mm increments.

SOLUTION

ENTER: $L = 1.8 \text{ m}$, $E = 200 \text{ GPa}$, $h = 0.75 \text{ m}$
 $m = 8 \text{ kg}$, $g = 9.81 \text{ m/s}^2$
 $I = 6.87 \times 10^{-6} \text{ m}^4$
 $S = 91.6 \times 10^{-6} \text{ m}^3$

FOR $a = 100 \text{ mm}$ TO 900 mm STEP 100 mm

$$a = a/1000$$

$$b = L - a$$

$$y_{st} = m g a^2 b^2 / 3 E I L$$

$$\alpha = a^2 b^2 / 3 E I L$$

$$y_m = y_{st} \left[1 + \sqrt{1 + \frac{2h}{y_{st}}} \right]$$

$$P_{max} = y_m / \alpha$$

$$M_{max} = P_{max} a b / L$$

$$\tau_{max} = M_{max} / S$$

PRINT: α , y_{st} , y_m , P_{max} , τ_{max}

RETURN

SEE PROB. 11.71, page 705 →

INFLUENCE COEFFICIENT FOR Δ_E } →
 FOR UNIT LOAD AT E

SEE PROB. 11.69, page 705 →

Problem 11.C4

Beam: W 150 x 13.5

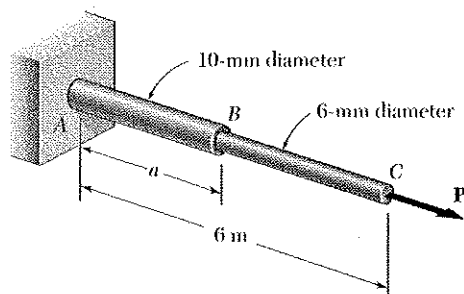
$$I = 6.87 \times 10^{-6} \text{ m}^4 \quad S = 91.6 \times 10^{-6} \text{ m}^3$$

$$L = 1.8 \text{ m} \quad h = 750 \text{ mm} \quad m = 8 \text{ kg} \quad g = 9.81 \text{ m/s}^2$$

a mm	y _{stat} mm	y _{max} mm	P _{max} N	σ _{max} MPa
100	0.0003	0.6775	173.93	179.33
200	0.0011	1.2757	92.43	179.40
300	0.0021	1.7946	65.75	179.46
400	0.0033	2.2339	52.85	179.51
500	0.0045	2.5936	45.55	179.55
600	0.0055	2.8734	41.13	179.59
700	0.0063	3.0734	38.46	179.61
800	0.0068	3.1934	37.02	179.63
900	0.0069	3.2334	36.56	179.63

NOTE: THE SMALL VARIATION IN τ_{max} . THIS IS DUE TO THE ENERGY ACQUIRED BY THE MASS AS IT FALLS THROUGH y_{max} . SEE PROB. 11.147, page 731, FOR A CASE WHERE ENERGY DELIVERED IS CONSTANT AND τ_{max} IS ALSO CONSTANT.

PROBLEM 11.C5



11.C5 The steel rods *AB* and *BC* are made of a steel for which $\sigma_Y = 300 \text{ MPa}$ and $E = 200 \text{ GPa}$. (a) Write a computer program to calculate, for values of *a* from 0 to 6 m, using 1-m increments, the maximum strain energy that can be acquired by the assembly without causing any permanent deformation. (b) For each value of *a* considered, calculate the diameter of a uniform rod of length 6 m and of the same mass as the original assembly, and the maximum strain energy that could be acquired by this uniform rod without causing permanent deformation.

SOLUTION

ENTER: $\sigma_Y = 300 \text{ MPa}$, $E = 200 \text{ GPa}$, $L = 6 \text{ m}$

$$\text{AREA}_{AB} = \frac{\pi}{4} (0.010 \text{ m})^2; \text{AREA}_{BC} = \frac{\pi}{4} (0.006 \text{ m})^2$$

$$P_m = \sigma_Y \text{ AREA}_{BC}$$

FOR $a = 0$ TO 6 m STEP 1 m

$$U = \frac{P_m^2}{2E} \left(\frac{a}{\text{AREA}_{AB}} + \frac{L-a}{\text{AREA}_{BC}} \right)$$

FOR UNIFORM ROD OF SAME VOLUME

$$\text{VOL} = a (\text{AREA}_{AB}) + (L-a) (\text{AREA}_{BC})$$

$$d = \sqrt{\frac{4 \text{ VOL}}{\pi L}}$$

$$\text{AREA}_{\text{NEW}} = \frac{\pi}{4} d^2$$

$$P_{\text{NEW}} = \sigma_Y (\text{AREA}_{\text{NEW}})$$

$$U_{\text{NEW}} = \frac{P_{\text{NEW}}^2 L}{2E (\text{AREA}_{\text{NEW}})}$$

PRINT a , U , VOL , d , P_{NEW} , U_{NEW}

RETURN

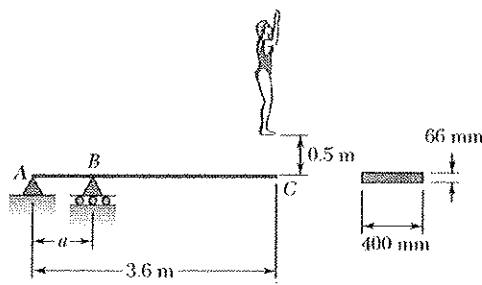
PROGRAM OUTPUT

Problem 11C5

$\sigma_Y = 300 \text{ MPa}$, $P_m = 8482 \text{ N}$, $L = 6 \text{ m}$, $E = 200 \text{ GPa}$

a m	U J	Vol m^3	d mm	New P N	new U J
0.00	38.17	169.65	6.00	8482.30	38.17
1.00	34.10	219.91	6.83	10995.58	49.48
2.00	30.03	270.18	7.57	13508.85	60.79
3.00	25.96	320.44	8.25	16022.12	72.10
4.00	21.88	370.71	8.87	18535.40	83.41
5.00	17.81	420.97	9.45	21048.67	94.72
6.00	13.74	471.24	10.00	23561.95	106.03

PROBLEM 11.C6



11.C6 A 72-kg diver jumps from a height of 0.5 m onto end C of a diving board having the uniform cross section shown. Write a computer program to calculate for values of a from 250 to 1250 mm, using 250-mm increments, (a) the maximum deflection of point C, (b) the maximum bending moment in the board, (c) the equivalent static load. Assume that the diver's legs remain rigid and use $E = 12 \text{ GPa}$.

SOLUTION

ENTER: $L = 3.6 \text{ m}$, $h = 500 \text{ mm}$, $W = (72 \times 9.81) = 706 \text{ N}$
 $E = 12 \text{ GPa}$.

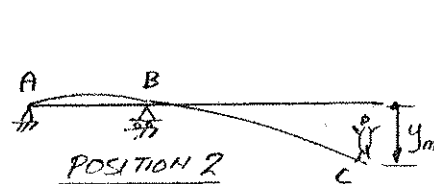
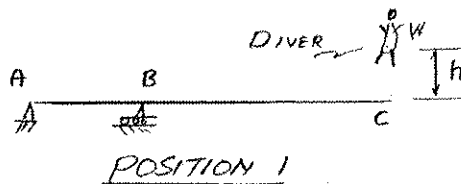
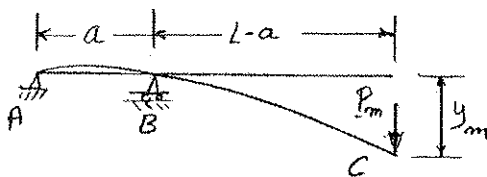
$$I = (0.4)(0.066)^3/12$$

$$S = (0.4)(0.066)^2/6$$

$$y_m = P_m \alpha \quad \text{WHERE } \alpha = \text{INFLUENCE COEFFICIENT}$$

SEE BELOW FOR DETERMINATION OF α

WHERE $P_m = \text{EQUIVALENT STATIC LOAD}$



$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

$$\text{WORK} = W(h + y_m)$$

$$\text{WORK} = U_2$$

$$W(h + y_m) = \frac{1}{2} \frac{y_m^2}{\alpha} \quad A$$

PROGRAM SOLUTION OF a FOR y_m . ENTER α
 FOR $a = 10 \text{ in. TO } 50 \text{ in. STEP } 10 \text{ in.}$

SOLVE A FOR y_m , $P_m = y_m / \alpha$

$$M_{\max} = M_B = P_m (L - a)$$

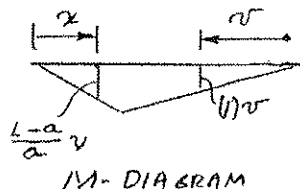
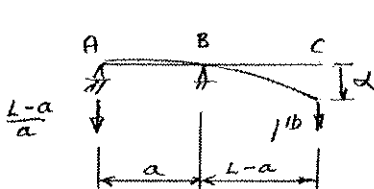
$$T = M_{\max} / S$$

PRINT α , y_m , P_m , M_m , T

PROGRAM OUTPUT

$a(\text{in})$	$y_m(\text{mm})$	$P_m(\text{kN})$	Max M (kNm)	Sigma (MPa)
0.25	365.55	3.370	11.473	37.954
0.5	331.55	3.570	11.246	37.198
0.75	298.75	3.806	11.022	36.456
1.0	267.08	4.088	10.801	35.728
1.25	236.55	4.432	10.584	35.007

DETERMINATION OF INFLUENCE COEFFICIENT α



$$U = \frac{1}{2} (1/b) \alpha = \sum \int \frac{M^2}{2EI} dv$$

$$\frac{\alpha}{2} = \frac{1}{2EI} \left[\int_0^{L-a} \left(\frac{L-a}{a} x^2 \right)^2 dv + \int_{L-a}^L x^2 dv \right]$$

$$\alpha = \frac{1}{EI} \left[\frac{(L-a)^2 a^3}{3} + \frac{(L-a)^3}{3} \right]$$

$$\alpha = \frac{1}{3EI} \left[(L-a)^2 a + (L-a)^3 \right] \quad \Delta$$

